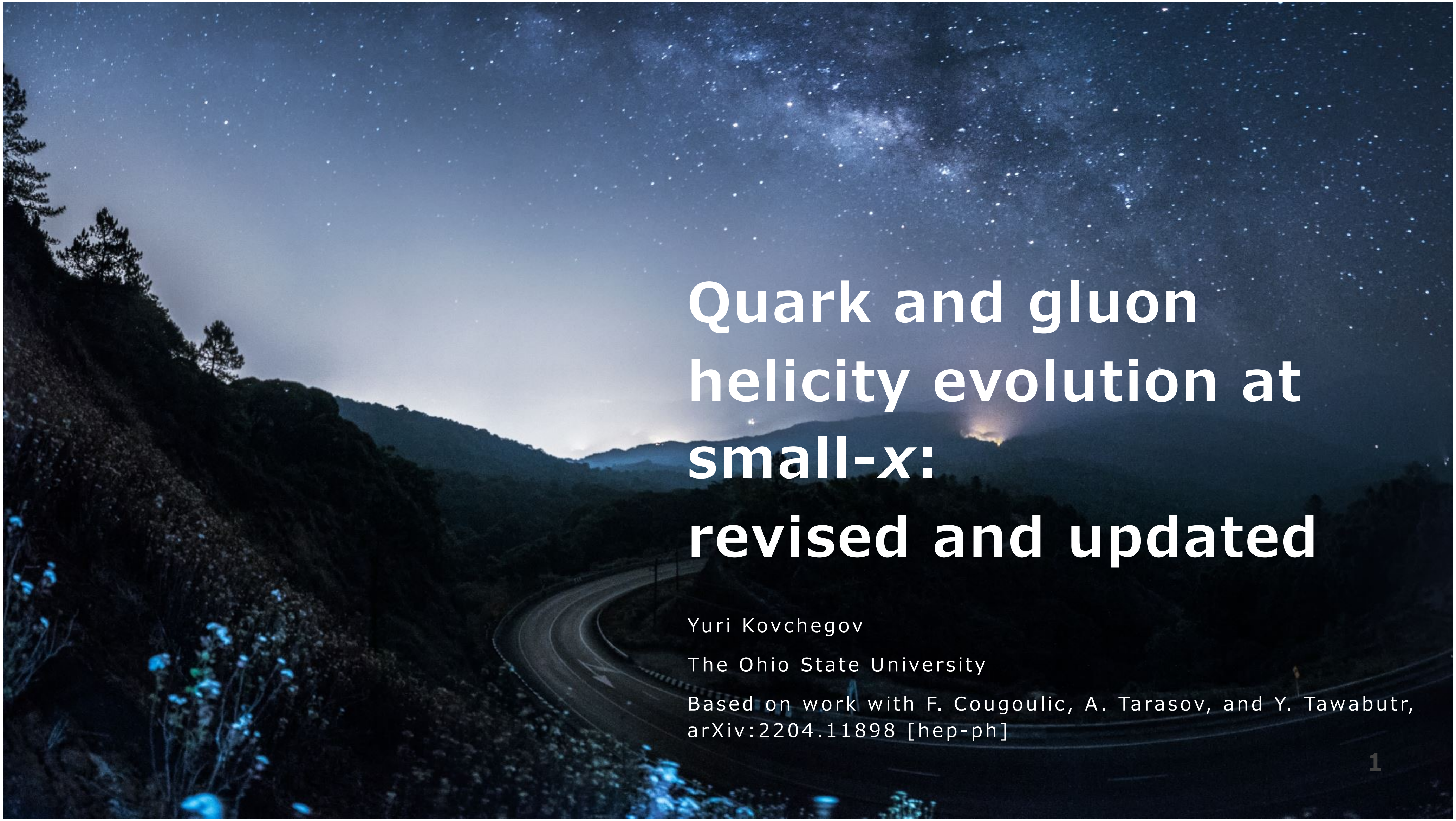

Quark and gluon helicity evolution at small-*x*: revised and updated

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Based on work with F. Cougoulic, A. Tarasov, and Y. Tawabutr, arXiv:2204.11898 [hep-ph]

Preamble

- We revisit helicity evolution at small *x* (compared to D. Pitonyak, M. Sievert & myself, 2015-18).
- Why? We found a new (sub-eikonal) operator that contributes to small-*x* helicity evolution. This operator was not present in helicity evolution in the shock wave picture until now.
- How do we know we are not missing anything else? We have now included <u>all</u> sub-eikonal operators and ran a series of cross checks: calculated evolution in two different ways, agreed with spin-dependent DGLAP to 3 loops, and verified the contribution of ΔG to g_1 structure function (at DLA).

Preamble

- The formalism that D. Pitonyak, M. Sievert and myself (KPS) have developed remains largely the same, with all the machinery developed (sub-eikonal dipole evolution, “neighbor” dipole, light-cone operator treatment (LCOT™), large-N_c&N_f limit, analytic solution methods).
- New operator introduces new terms in the helicity evolution equations, affecting its solution and intercept.
- The flavor non-singlet helicity distribution evolution at large N_c from 2016 is not affected (→ I will not talk about it).

Outline

- Introduction.
- A full list of sub-eikonal operators. What was included and what was missing.
- Spin-dependent quantities at small *x*: g_1 structure function, $\Delta\Sigma$ and ΔG. The expressions for the first two are modified.
- New helicity evolution equations in operator form (not closed).
- New helicity evolution at large-N_c and large-N_c&N_f (closed).
- Cross-checks.
- Solution of the large-N_c equations & the intercept.
- Some small-*x* helicity phenomenology.

Eikonality

- One can classify various TMDs by their small-x asymptotics.
- Eikonal behavior corresponds to (up to ~α_S corrections in the power)

$$f(x, k_T^2) \sim \frac{1}{x}$$

  Examples: unpolarized TMDs, Sivers function for gluons and quarks.

- Sub-eikonal behavior corresponds to

$$g(x, k_T^2) \sim \left(\frac{1}{x}\right)^0 = \mathrm{const}$$

  Example: helicity TMDs.

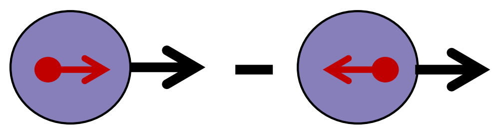
- Sub-sub-eikonal behavior is $h(x, k_T^2) \sim x$
  Example: transversity.

- We've been calling the leading power of x "eikonality".

Introduction and goals

Helicity Distributions

- To quantify the contributions of quarks and gluons to the proton spin on defines helicity distribution functions: number of quarks/gluons with spin parallel to the proton momentum minus the number of quarks/gluons with the spin opposite to the proton momentum:



- The helicity parton distributions are

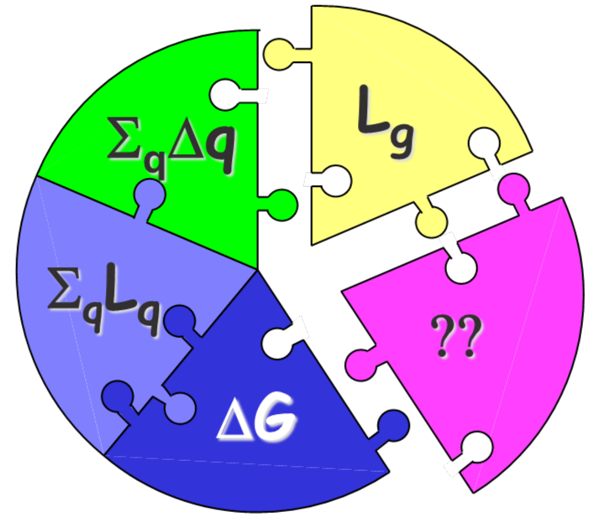
$$\Delta f(x,Q^2) \equiv f^+(x,Q^2) - f^-(x,Q^2)$$

with the net flavor-singlet quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

and $\Delta G(x, Q^2)$ the gluon helicity distribution.

Proton Helicity Sum Rule



- Helicity sum rule (Jaffe & Manohar, 1989):

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- L_q and L_g are the quark and gluon orbital angular momenta

Proton Spin Puzzle

- The spin puzzle began when the EMC collaboration measured the proton g_1 structure function ca 1988. Their data resulted in

$$S_q \approx 0.05$$

- It appeared (constituent) quarks do not carry all of the proton spin (which would have corresponded to $S_q = 1/2$).

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

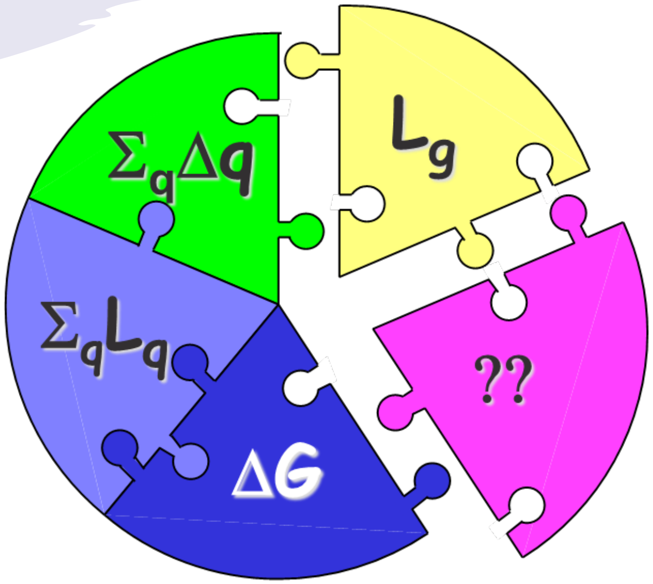
- Missing spin can be
  - Carried by gluons
  - In the orbital angular momenta of quarks and gluons
  - At small x:

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

Can't integrate down to zero, use x_{min} instead!

  - Or all of the above!

Current Knowledge of Proton Spin



- The proton spin carried by the quarks is estimated to be (for $0.001 < x < 1$)

$$S_q(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.15 \div 0.20$$

- The proton spin carried by the gluons is (for $0.05 < x < 1$, STAR+COMPASS+HERMES+…)

$$S_G(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.13 \div 0.26$$

- Unfortunately, the uncertainties are large. Note also that the x-ranges are limited, with more spin (positive or negative) possible at small x.

Our goal

- The goal is to constrain theoretically the amount of proton spin and OAM coming from small x.

- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

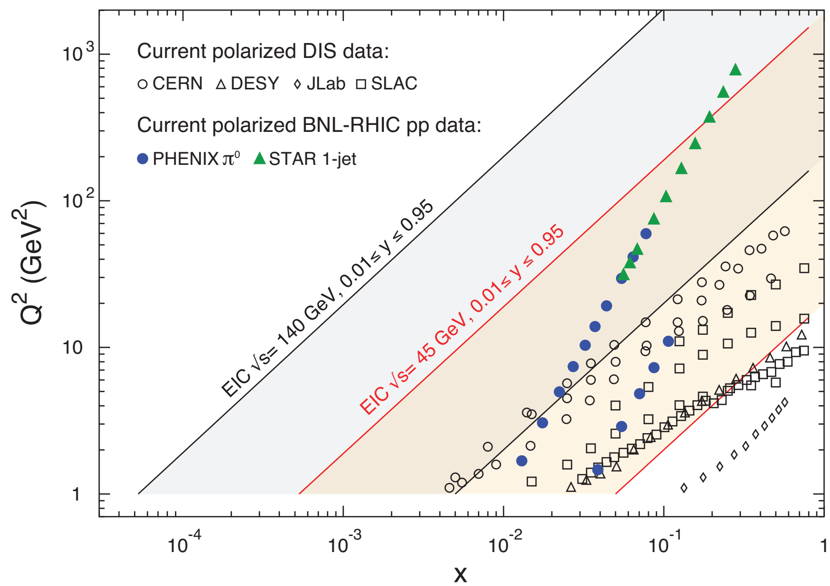
$$L_{q+\bar{q}}(Q^2) = \int_0^1 dx\, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int_0^1 dx\, L_G(x, Q^2)$$

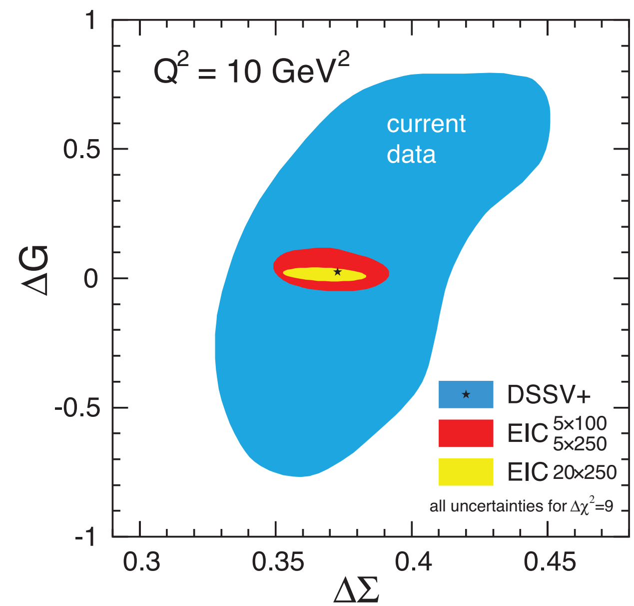
- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

EIC & Spin Puzzle

Still, even with the EIC data we need to extrapolate quark and gluon spin down to smaller x.

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a polarized DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton's spin:





- ΔG and $\Delta\Sigma$ are integrated over x in the $0.001 < x < 1$ interval.

Sub-Eikonal Operators

Dipole picture of DIS

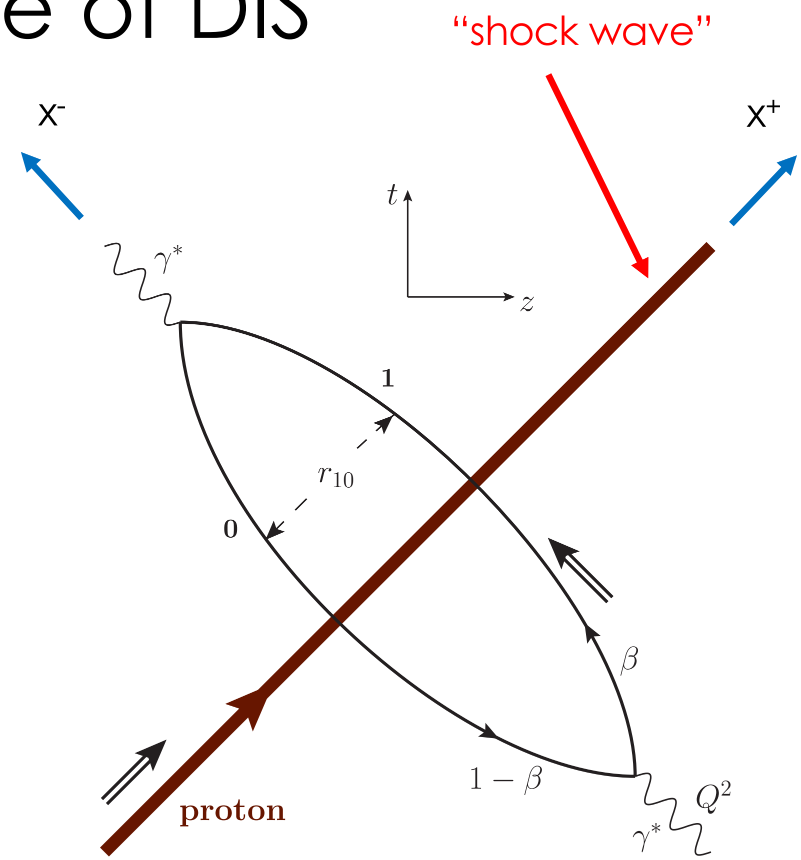
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x \, e^{iq\cdot x} \, \langle P | j^\mu(x) \, j^\nu(0) | P \rangle$$

Large q^- → large x^- separation

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp \right)$$

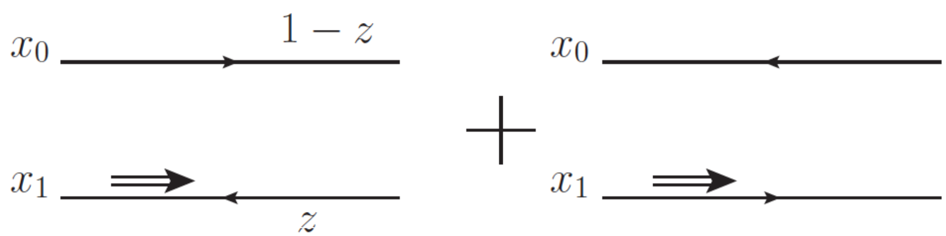
Same is true for PDFs: small x means large x^- spread

$$q(x, Q^2) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dx^- \, e^{ixP^+x^-} \, \langle P | \bar{q}(x^-) \gamma^+ \mathcal{U} \, q(0) | P \rangle$$



Polarized Dipole: non-eikonal small-x physics

- All flavor-singlet small-x helicity observables depend on one object, "polarized dipole amplitude":



$$G_{10}(z) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T\,tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger}\right] + \mathrm{T\,tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger}\right] \right\rangle\!\!\right\rangle(z)$$

unpolarized quark

polarized quark: eikonal propagation, non-eikonal spin-dependent interaction

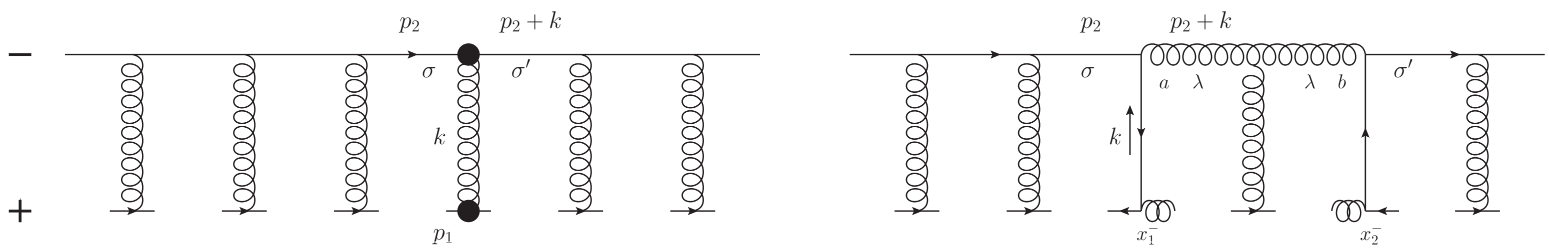
$$V_{\underline{x}} = \mathcal{P} \exp\left[ig \int\limits_{-\infty}^{\infty} dx^- A^+(0^+, x^-, \underline{x})\right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle(z) \equiv zs \left\langle \mathcal{O} \right\rangle(z)$$

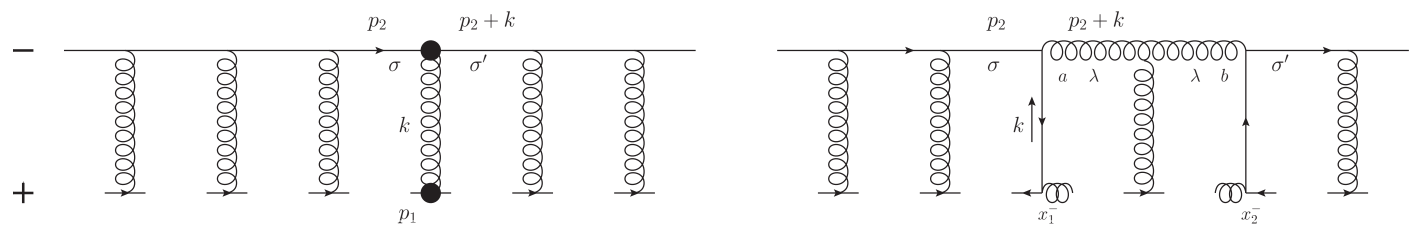
Polarized fundamental "Wilson line"

- To complete the definition of the polarized dipole amplitude, we need to construct the definition of the polarized "Wilson line" V^{pol}, which is the leading helicity-dependent contribution for the quark scattering amplitude on a longitudinally-polarized target proton.



- At the sub-eikonal order we can either exchange one non-eikonal *t*-channel gluon (with quark-gluon vertices denoted by blobs above) to transfer polarization between the projectile and the target, or two *t*-channel quarks, as shown above.

Old: Polarized fundamental "Wilson line"



- In the end one arrives at (KPS '17; YK, Sievert, '18; Chirilli '18; Altinoluk et al, '20)

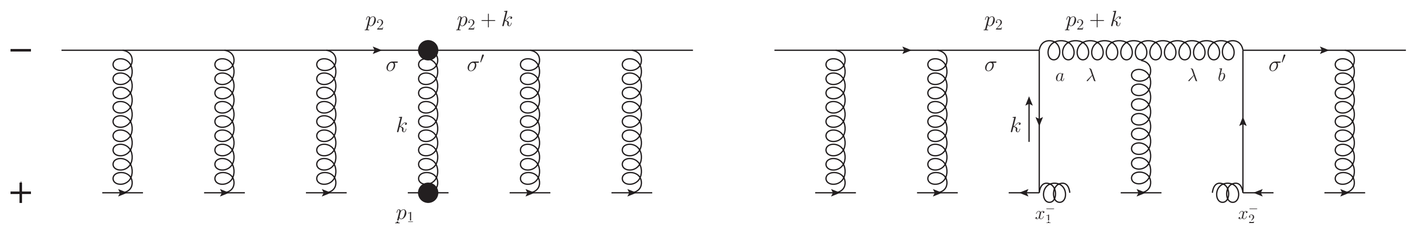
$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2 \, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2} \gamma^+ \gamma^5 \right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

- We have employed an adjoint light-cone Wilson line $U_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp \left[ig \int\limits_{a^-}^{b^-} dx^- \, \mathcal{A}^+(x^+ = 0, x^-, \underline{x}) \right]$
- Note the simple physical meaning of the first term:

$$-\vec{\mu} \cdot \vec{B} = -\mu_z \, B_z = \mu_z \, F^{12}$$

Sub-eikonal quark S-matrix in background gluon and quark fields



- The full sub-eikonal S-matrix for massless quarks is (Balitsky&Tarasov '15; KPS '17; YK, Sievert, '18; Chirilli '18; Altinoluk et al, '20; YK, Santiago '21)

"helicity independent" "helicity dependent"

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\, \delta^2(\underline{x}-\underline{y})\, \delta_{\sigma,\sigma'}$$

$$+ \frac{i\,P^+}{s} \int\limits_{-\infty}^{\infty} dz^- d^2 z\; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z}) \left[-\delta_{\sigma,\sigma'}\, \overleftarrow{D}^i\; D^i + g\,\sigma\,\delta_{\sigma,\sigma'}\, F^{12}\right](z^-,\underline{z})\, V_{\underline{y}}[z^-,-\infty]\, \delta^2(\underline{y}-\underline{z})$$

$$- \frac{g^2 P^+}{2s} \delta^2(\underline{x}-\underline{y}) \int\limits_{-\infty}^{\infty} dz_1^- \int\limits_{z_1^-}^{\infty} dz_2^- V_{\underline{x}}[\infty, z_2^-]\, t^b \psi_\beta(z_2^-,\underline{x})\, U_{\underline{x}}^{ba}[z_2^-, z_1^-] \left[\delta_{\sigma,\sigma'}\gamma^+ - \sigma\delta_{\sigma,\sigma'}\gamma^+\gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(z_1^-,\underline{x})\, t^a V_{\underline{x}}[z_1^-,-\infty]$$

"helicity independent" "helicity dependent"

Sub-eikonal quark S-matrix in background gluon and quark fields

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\,\delta_{\sigma,\sigma'}$$

$$+ \frac{i\,P^+}{s}\int\limits_{-\infty}^{\infty} dz^- d^2z\; V_{\underline{x}}[\infty, z^-]\,\delta^2(\underline{x}-\underline{z})\left[-\delta_{\sigma,\sigma'}\,\overleftarrow{D}^i\,D^i + g\,\sigma\,\delta_{\sigma,\sigma'}\,F^{12}\right](z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z})$$

"helicity independent" "helicity dependent"

$$- \frac{g^2P^+}{2s}\delta^2(\underline{x}-\underline{y})\int\limits_{-\infty}^{\infty} dz_1^- \int\limits_{z_1^-}^{\infty} dz_2^-\, V_{\underline{x}}[\infty, z_2^-]\,t^b\,\psi_\beta(z_2^-,\underline{x})\,U_{\underline{x}}^{ba}[z_2^-,z_1^-]\left[\delta_{\sigma,\sigma'}\gamma^+ - \sigma\delta_{\sigma,\sigma'}\gamma^+\gamma^5\right]_{\alpha\beta}\bar{\psi}_\alpha(z_1^-,\underline{x})\,t^a\,V_{\underline{x}}[z_1^-,-\infty]$$

"helicity independent" "helicity dependent"

- Our KPS papers included only the "helicity-dependent" terms $\sim\sigma\,\delta_{\sigma,\sigma'}$, arguing that the unpolarized terms cannot give a non-zero expectation value in the longitudinally polarized proton state.
- However, DGLAP evolution of the Jaffe-Manohar ΔG is driven by the DD "helicity-independent" term $\sim\delta_{\sigma,\sigma'}$. This realization led us to redo the calculation, keeping the DD term as well. Turns out it contributes to small-x helicity evolution too.

Sub-eikonal quark S-matrix in background gluon and quark fields

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\, \delta^2(\underline{x}-\underline{y})\, \delta_{\sigma,\sigma'}$$

$$+ \frac{i\,P^+}{s} \int\limits_{-\infty}^{\infty} dz^- d^2 z \; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z}) \left[-\delta_{\sigma,\sigma'} \overleftarrow{D}^i D^i + g\,\sigma\,\delta_{\sigma,\sigma'}\, F^{12} \right] (z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z})$$

"helicity independent" "helicity dependent"

$$- \frac{g^2 P^+}{2s} \delta^2(\underline{x}-\underline{y}) \int\limits_{-\infty}^{\infty} dz_1^- \int\limits_{z_1^-}^{\infty} dz_2^- V_{\underline{x}}[\infty, z_2^-] t^b \psi_\beta(z_2^-, \underline{x}) U_{\underline{x}}^{ba}[z_2^-, z_1^-] \left[\delta_{\sigma,\sigma'} \gamma^+ - \sigma \delta_{\sigma,\sigma'} \gamma^+ \gamma^5 \right]_{\alpha\beta} \bar{\psi}_\alpha(z_1^-, \underline{x}) t^a V_{\underline{x}}[z_1^-, -\infty]$$

"helicity independent" "helicity dependent"

- At the same time, the helicity-independent term in the quark sector (γ^+) still does not contribute to helicity evolution.
- Why does DD contribute? We do not quite understand. There are two options:
  - DD is non-local → like OAM?
  - σ is not really helicity, rather projection of spin on the z axis (in LCPT).
- It appears in such calculations all operators of a given eikonality should be considered.

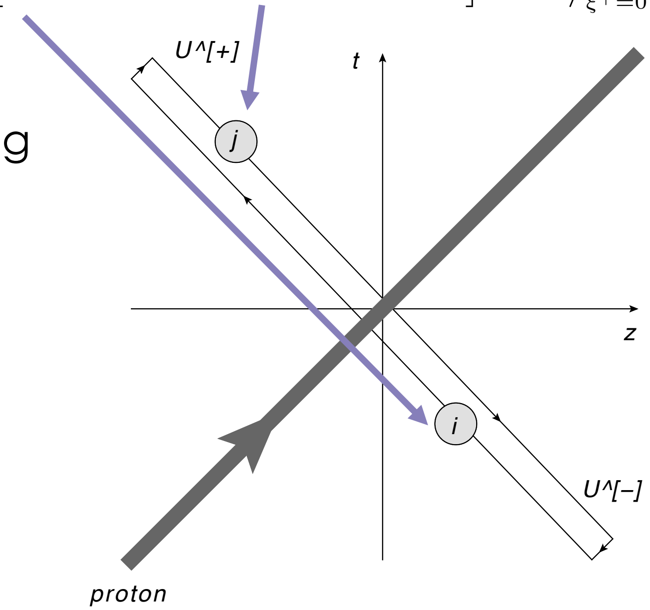
Helicity Observables at Small x

Dipole Gluon Helicity TMD

- We start with the definition of the gluon dipole helicity TMD, corresponding to the Jaffe-Manohar PDF ΔG,

$$g_1^G(x, k_T^2) = \frac{-2i\, S_L}{x\, P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \, e^{ixP^+ \xi^- - i\underline{k}\cdot\underline{\xi}} \left\langle P, S_L | \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0) \, \mathcal{U}^{[+]\dagger}[0, \xi] \, F^{+j}(\xi) \, \mathcal{U}^{[-]}[\xi, 0] \right] | P, S_L \right\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past-pointing Wilson line staples (hence the name \`dipole' TMD, F. Dominguez et al '11 – looks like a dipole scattering on a proton):



Gluon Helicity: (mostly) old results

- A calculation gives

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right) G_2 \left(x_{10}^2, zs = \frac{Q^2}{x} \right) \right]_{x_{10}^2 = \frac{1}{Q^2}}$$

$$g_{1L}^{G\, dip}(x, k_T^2) = \frac{N_c}{\alpha_s 2\pi^4} \int d^2 x_{10}\, e^{-i \underline{k} \cdot \underline{x}_{10}} \left[1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right] G_2 \left(x_{10}^2, zs = \frac{Q^2}{x} \right)$$

- Here we defined a new dipole amplitude G_2 (cf. Hatta et al, 2016; KPS 2017)

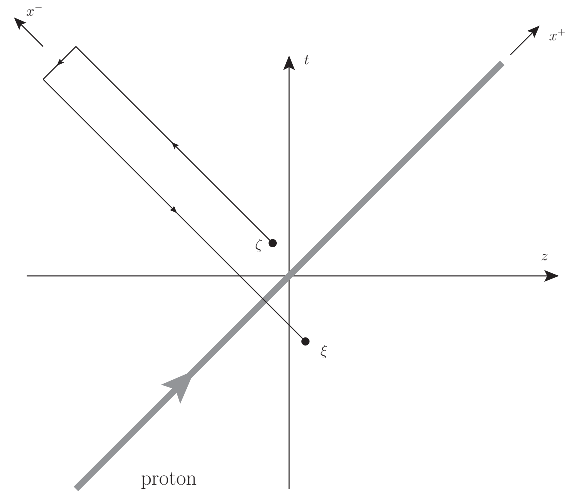
$$\int d^2 \left(\frac{x_1 + x_0}{2} \right) G_{10}^i(zs) = (x_{10})_\perp^i\, G_1(x_{10}^2, zs) + \epsilon^{ij}\, (x_{10})_\perp^j\, G_2(x_{10}^2, zs)$$

$$G_{10}^j(zs) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}^\dagger\, V_{\underline{1}}^{j\,\mathrm{G}[2]} + \left(V_{\underline{1}}^{j\,\mathrm{G}[2]} \right)^\dagger V_{\underline{0}} \right] \right\rangle\!\!\right\rangle$$

$$V_{\underline{z}}^{i\,\mathrm{G}[2]} \equiv \frac{P^+}{2s} \int\limits_{-\infty}^{\infty} dz^-\, V_{\underline{z}}[\infty, z^-] \left[D^i(z^-, \underline{z}) - \overleftarrow{D}^i (z^-, \underline{z}) \right] V_{\underline{z}}[z^-, -\infty]$$

What is this D-D operator? Turns out it is related to the DD operator from before.

Quark Helicity PDF and TMD



- The flavor-singlet quark helicity PDF and SIDIS TMD are

$$\Delta\Sigma(x,Q^2) = -\frac{N_c\,N_f}{2\pi^3}\int\limits_{\Lambda^2/s}^{1}\frac{dz}{z}\int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}}\frac{dx_{10}^2}{x_{10}^2}\,\left[Q(x_{10}^2,zs)+2\,G_2(x_{10}^2,zs)\right]$$

$$g_{1L}^S(x,k_T^2) = \frac{8\,i\,N_c\,N_f}{(2\pi)^5}\int\limits_{\Lambda^2/s}^{1}\frac{dz}{z}\int d^2x_{10}\,e^{i\underline{k}\cdot\underline{x}_{10}}\,\frac{\underline{x}_{10}}{x_{10}^2}\cdot\frac{\underline{k}}{\underline{k}^2}\,\left[Q(x_{10}^2,zs)+2\,G_2(x_{10}^2,zs)\right]$$

- G_2 was defined before. Its contributions to quark helicity TMD and PDF are new (as compared to KPS work). This is the gluon admixture to quark helicity distributions.
- The dipole amplitude Q is the ‘old’ KPS polarized dipole (F^{12} & axial current).
- The contribution of G_2 comes from the DD operator in the quark S-matrix.
- Hence, the DD operator is related to the Jaffe-Manohar distribution.

Amplitude Q

$$Q(x_{10}^2, zs) \equiv \int d^2 \left(\frac{x_0 + x_1}{2} \right) Q_{10}(zs)$$

- The amplitude Q is defined by

$$Q_{10}(zs) \equiv \frac{1}{2N_c} \text{Re} \left\langle\!\!\left\langle \text{T}\,\text{tr} \left[V_{\underline{0}}\, V_{\underline{1}}^{\text{pol}[1]\,\dagger} \right] + \text{T}\,\text{tr} \left[V_{\underline{1}}^{\text{pol}[1]}\, V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle$$

with $V_{\underline{x}}^{\text{pol}[1]} = V_{\underline{x}}^{\text{G}[1]} + V_{\underline{x}}^{\text{q}[1]}$, where

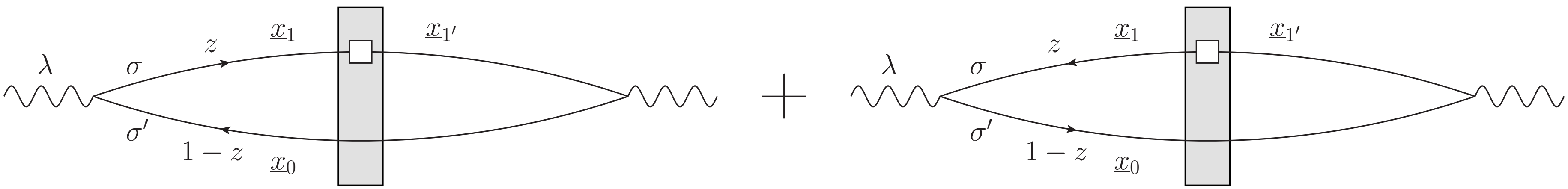
$$V_{\underline{x}}^{\text{G}[1]} = \frac{i\, g\, P^+}{s} \int\limits_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-]\, F^{12}(x^-, \underline{x})\ V_{\underline{x}}[x^-, -\infty]$$

$$V_{\underline{x}}^{\text{q}[1]} = \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\ \left[\gamma^+ \gamma^5\right]_{\alpha\beta}\ \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty]$$

- This is the ‘old’ KPS polarized dipole amplitude.

g_1 structure function

- g_1 structure function is obtained similarly, using DIS in the dipole picture:



- One gets

new

$$g_1(x, Q^2) = -\sum_f \frac{N_c\, Z_f^2}{4\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

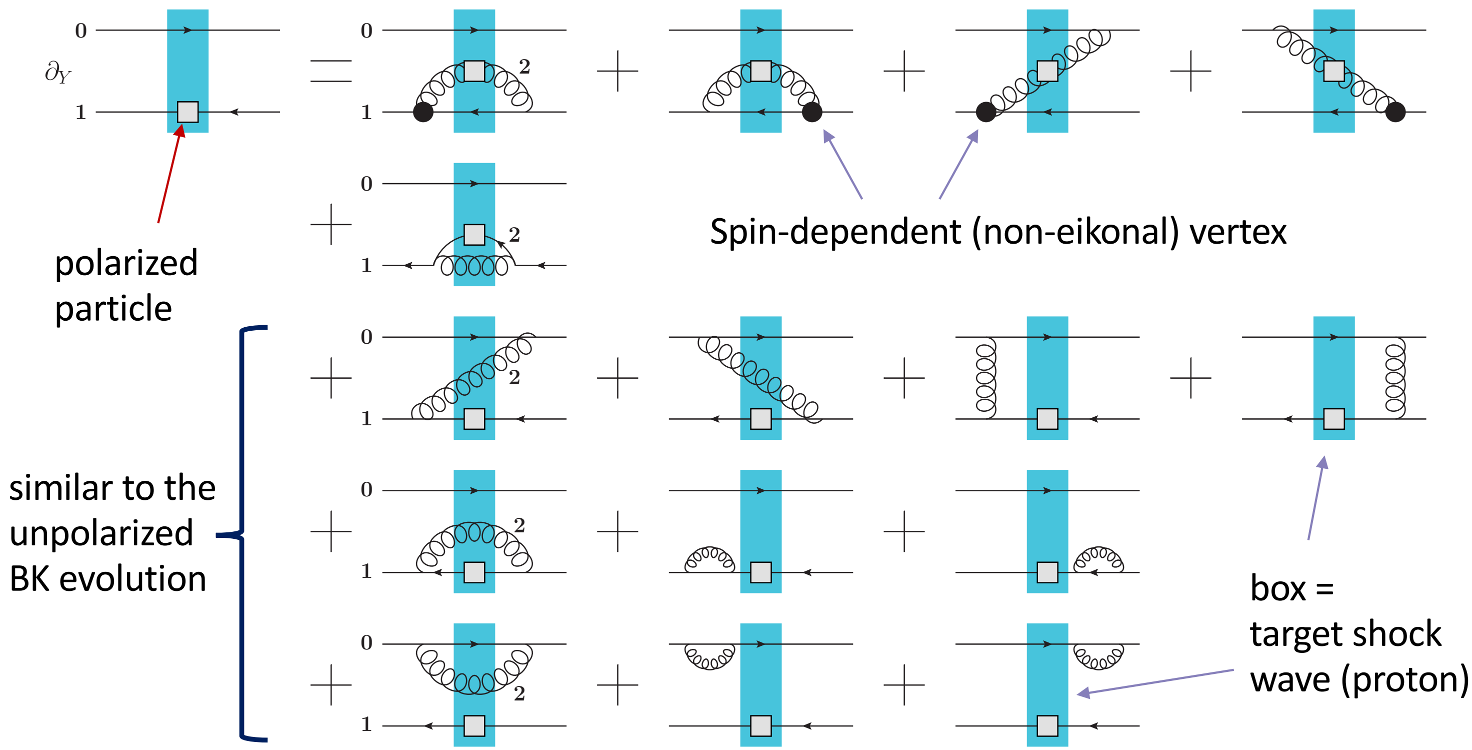
such that one reproduces the standard result $g_1(x, Q^2) = \frac{1}{2} \sum_f Z_f^2\, \Delta q_f^+(x, Q^2)$

$$\Delta q_f^+ = \Delta q_f + \Delta \bar{q}_f$$

Helicity Evolution

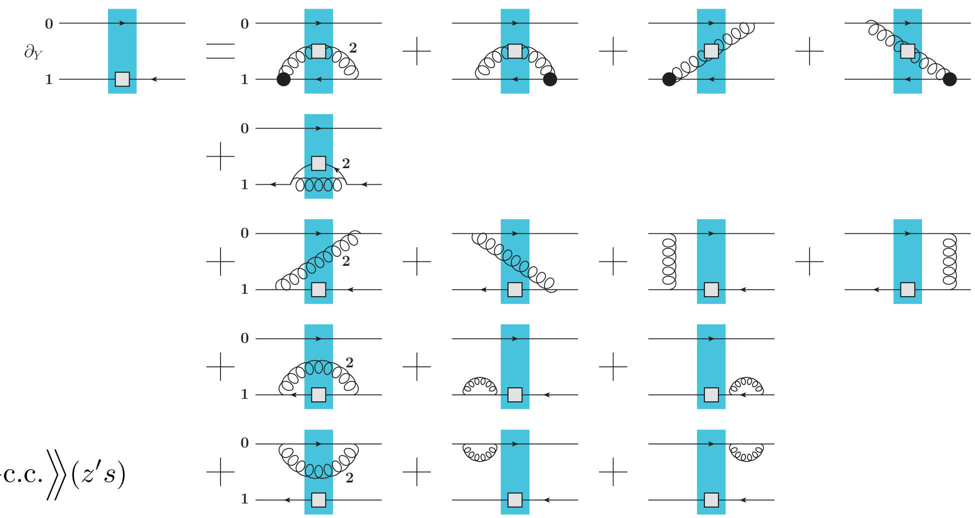
Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Evolution for Polarized Quark Dipole

$$\langle\langle \ldots \rangle\rangle = \frac{1}{z\, s} \langle \ldots \rangle$$



$$\frac{1}{2N_c} \left\langle\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.} \right\rangle\!\right\rangle(zs) = \frac{1}{2N_c} \left\langle\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.} \right\rangle\!\right\rangle_0(zs)$$
$$+ \frac{\alpha_s\, N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \Bigg\{ \left[\frac{1}{x_{21}^2} - \frac{\underline{x}_{21}}{x_{21}^2} \cdot \frac{\underline{x}_{20}}{x_{20}^2}\right] \frac{1}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right] \left(U_{\underline{2}}^{\mathrm{pol}[1]}\right)^{ba} + \mathrm{c.c.} \right\rangle\!\right\rangle(z's)$$
$$+ \left[2 \frac{\epsilon^{ij}\, x_{21}^j}{x_{21}^4} - \frac{\epsilon^{ij}\, (x_{20}^j + x_{21}^j)}{x_{20}^2\, x_{21}^2} - \frac{2\, \underline{x}_{20} \times \underline{x}_{21}}{x_{20}^2\, x_{21}^2} \left(\frac{x_{21}^i}{x_{21}^2} - \frac{x_{20}^i}{x_{20}^2}\right)\right] \frac{1}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right] \left(U_{\underline{2}}^{i\,\mathrm{G}[2]}\right)^{ba} \right\rangle\!\right\rangle(z's) \Bigg\}$$
$$+ \frac{\alpha_s N_c}{4\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int \frac{d^2 x_2}{x_{21}^2} \Bigg\{ \frac{1}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{1}}^{ba} \right\rangle\!\right\rangle(z's) + 2\, \frac{\epsilon^{ij}\, \underline{x}_{21}^j}{x_{21}^2} \frac{1}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{i\,\mathrm{G}[2]\,\dagger}\right] U_{\underline{1}}^{ba} \right\rangle\!\right\rangle(z's) \Bigg\}$$
$$+ \frac{\alpha_s\, N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2\, \frac{x_{10}^2}{x_{21}^2\, x_{20}^2} \Bigg\{ \frac{1}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{2}}^{ba} \right\rangle\!\right\rangle(z's) - \frac{C_F}{N_c^2} \left\langle\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] \right\rangle\!\right\rangle(z's) \Bigg\} +$$
$$\mathrm{c.c.}$$

Equation does not close!

Large N_c

- At large-N_c the equations close (Q → G). Everything with 2 in the subscript (e.g., G_2 and Γ_2) is new compared to KPS papers.

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z' s) + 3\, G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z' s) \Big],$$

KPS

$$\Gamma(x_{10}^2, x_{21}^2, z' s) = G^{(0)}(x_{10}^2, z' s) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z'' s) + 3\, G(x_{32}^2, z'' s) + 2\, G_2(x_{32}^2, z'' s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z'' s) \Big],$$

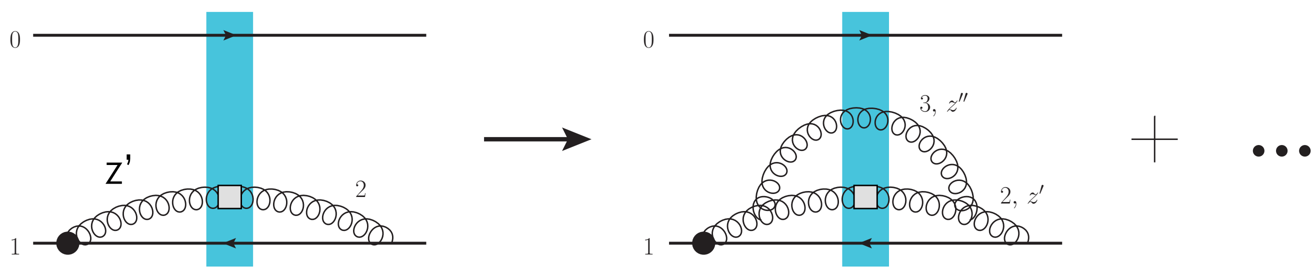
KPS

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z' s}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z' s) = G_2^{(0)}(x_{10}^2, z' s) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z'' s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z'' s) + 2\, G_2(x_{32}^2, z'' s) \right]$$

“Neighbor” dipole

- There is a new object in the evolution equation – **the neighbor dipole amplitude**.
- This is specific for the DLA evolution. Gluon emission may happen in one dipole, but, due to lifetime ordering, may `know’ about another dipole:



$$x_{21}^2\, z' \gg x_{32}^2\, z''$$

- We denote the evolution in the neighbor dipole 02 by $\Gamma_{02,\,21}(z')$

Resummation Parameter

- For helicity evolution the leading resummation parameter is different from BFKL, BK or JIMWLK, which resum powers of leading logarithms (LLA)

$$\alpha_s \, \ln(1/x)$$

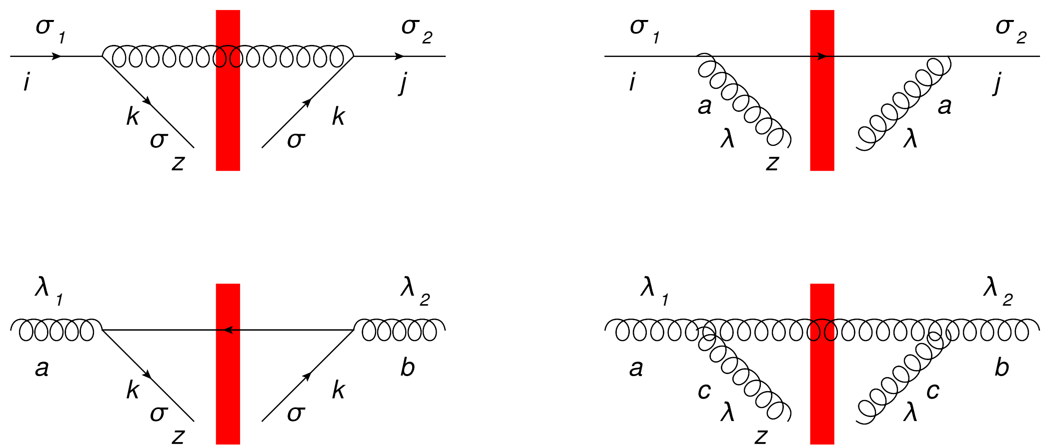
- Helicity evolution resummation parameter is double-logarithmic (DLA):

$$\alpha_s \, \ln^2 \frac{1}{x}$$

- The second logarithm of x arises due to transverse momentum (or transverse coordinate) integration being logarithmic both in the UV and IR.
- This was known before: Kirschner and Lipatov '83; Kirschner '84; Bartels, Ermolaev, Ryskin '95, '96; Griffiths and Ross '99; Itakura et al '03; Bartels and Lublinsky '03.

Helicity Evolution Ingredients

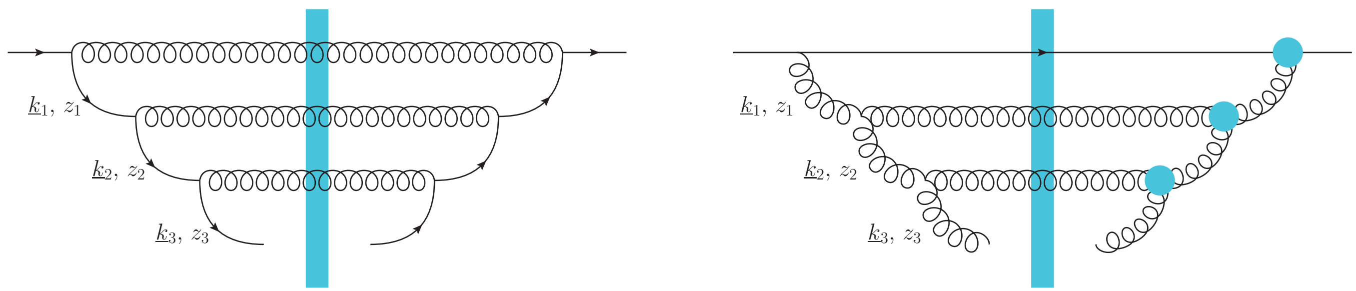
- Unlike the unpolarized evolution, in one step of helicity evolution we may emit a soft gluon or a soft quark (all in $A^{-}=0$ LC gauge of the projectile):



- When emitting gluons, one emission is eikonal, while another one is soft, but non-eikonal, as is needed to transfer polarization down the cascade/ladder.

Helicity Evolution: Ladders

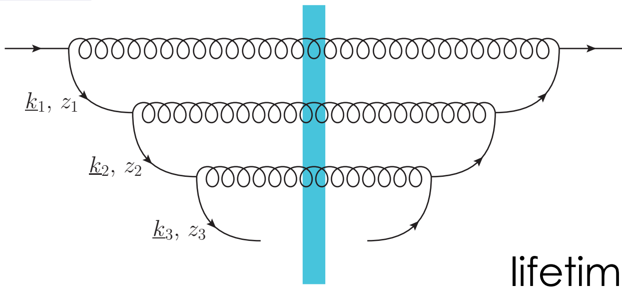
- To get an idea of how the helicity evolution works let us try iterating the splitting kernels by considering ladder diagrams (circles denote non-eikonal gluon vertices):

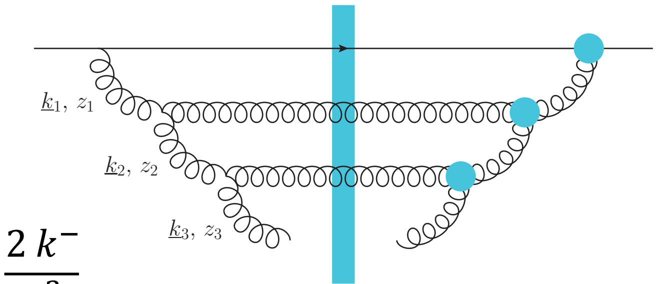


- To get the leading-energy asymptotics we need to order the longitudinal momentum fractions of the quarks and gluons (just like in the unpolarized evolution case) $1 \gg z_1 \gg z_2 \gg z_3 \gg \ldots$

  obtaining a nested integral $$\alpha_s^3 \int\limits_{z_i}^{1} \frac{dz_1}{z_1} \int\limits_{z_i}^{z_1} \frac{dz_2}{z_2} \int\limits_{z_i}^{z_2} \frac{dz_3}{z_3}\, z_3 \otimes \frac{1}{z_3\, s} \sim \frac{1}{s}\, \alpha_s^3 \ln^3 s$$

Helicity Evolution: Ladders





$$\text{lifetime} = \frac{2\,k^-}{k_\perp^2}$$

- However, these are not all the logs of energy one can get here. Transverse momentum (or distance) integrals have UV and IR divergences, which lead to logs of energy as well.
- If we order gluon/quark lifetimes as (Sudakov-β ordering)

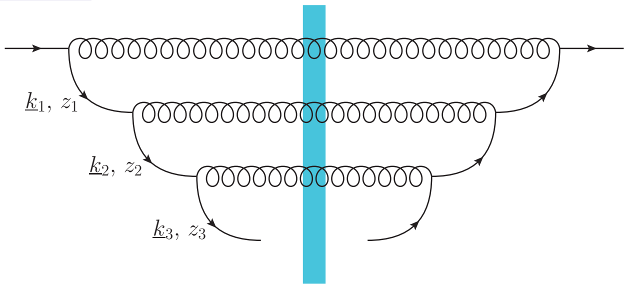
$$\frac{\underline{k}_1^2}{z_1} \ll \frac{\underline{k}_2^2}{z_2} \ll \frac{\underline{k}_3^2}{z_3} \ll \ldots \qquad\qquad z_1\,\underline{x}_1^2 \gg z_2\,\underline{x}_2^2 \gg z_3\,\underline{x}_3^2 \gg \ldots$$

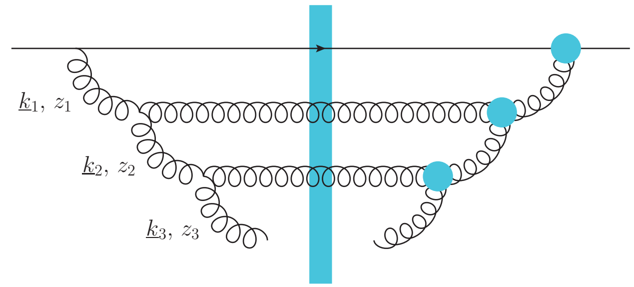
we would get integrals like

$$\int\limits_{1/(z_n\, s)}^{x^2_{n-1,\perp}\, z_{n-1}/z_n} \frac{dx^2_{n,\perp}}{x^2_{n,\perp}}$$

also generating logs of energy.

Helicity Evolution: Ladders





- To summarize, the above ladder diagrams are parametrically of the order

$$\frac{1}{s}\,\alpha_s^3\,\ln^6 s$$

- Note two features:
  - 1/s suppression due to non-eikonal exchange
  - two logs of energy per each power of the coupling!

Large-N_c&N_f

- Equations also close in the large-N_c&N_f (Veneziano) limit.
- This is more realistic as it includes quarks.
- Everything with 2 in the subscript (e.g., G_2 and Γ_2) is new compared to KPS papers.

$$Q(x_{10}^2, zs) = Q^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[2\,\widetilde{G}(x_{21}^2, z's) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) + Q(x_{21}^2, z's) - \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) + 2\,\Gamma_2(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \Big] + \frac{\alpha_s N_c}{4\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/z's}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\overline{\Gamma}(x_{10}^2, x_{21}^2, z's) = Q^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[2\,\widetilde{G}(x_{32}^2, z''s) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + Q(x_{32}^2, z''s) - \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,\Gamma_2(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \Big] + \frac{\alpha_s N_c}{4\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/z''s}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],$$

$$\widetilde{G}(x_{10}^2, zs) = \widetilde{G}^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[3\,\widetilde{G}(x_{21}^2, z's) + \widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{21}^2, z's) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) \Big] - \frac{\alpha_s N_f}{8\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{\max\{x_{10}^2,\, 1/z's\}}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) = \widetilde{G}^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[3\,\widetilde{G}(x_{32}^2, z''s) + \widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{32}^2, z''s) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) \Big] - \frac{\alpha_s N_f}{8\pi} \int\limits_{\Lambda^2/s}^{z' x_{21}^2/x_{10}^2} \frac{dz''}{z''} \int\limits_{\max\{x_{10}^2,\, 1/z''s\}}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\frac{z}{z'} x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\widetilde{G}(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\frac{z'}{z''} x_{21}^2} \frac{dx_{32}^2}{x_{32}^2} \left[\widetilde{G}(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right]$$

Cross Checks

Cross Checks: spin-dependent DGLAP

- Our evolution at large Q^2 can be cross-checked against the small-x limit of the spin-dependent DGLAP equation.
- At DLA, ΔG is simply proportional to G_2:

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right) G_2 \left(x_{10}^2, zs = \frac{Q^2}{x} \right) \right]_{x_{10}^2 = \frac{1}{Q^2}} \approx \frac{2N_c}{\alpha_s \pi^2} G_2 \left(\frac{1}{Q^2}, zs = \frac{Q^2}{x} \right)$$

- We solved our large-N_c equations iteratively, comparing $\ln(Q^2)$ terms with small-x spin-dependent DGLAP evolution.

$$\frac{\partial \Delta G(x, Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} \, \Delta P_{GG}(z) \, \Delta G \left(\frac{x}{z}, Q^2 \right)$$

- We obtained a complete agreement with LO+NLO+NNLO DGLAP at small x (R. Mertig, W.L. van Neerven '95; S. Moch, J.A.M. Vermaseren, A. Vogt, '14).

$$\Delta P_{GG}(z) = \frac{\alpha_s}{2\pi} 4N_c + \left(\frac{\alpha_s}{2\pi} \right)^2 4N_c^2 \ln^2 z + \left(\frac{\alpha_s}{2\pi} \right)^3 \frac{7}{3} N_c^3 \ln^4 z + \ldots$$

We can predict higher orders.

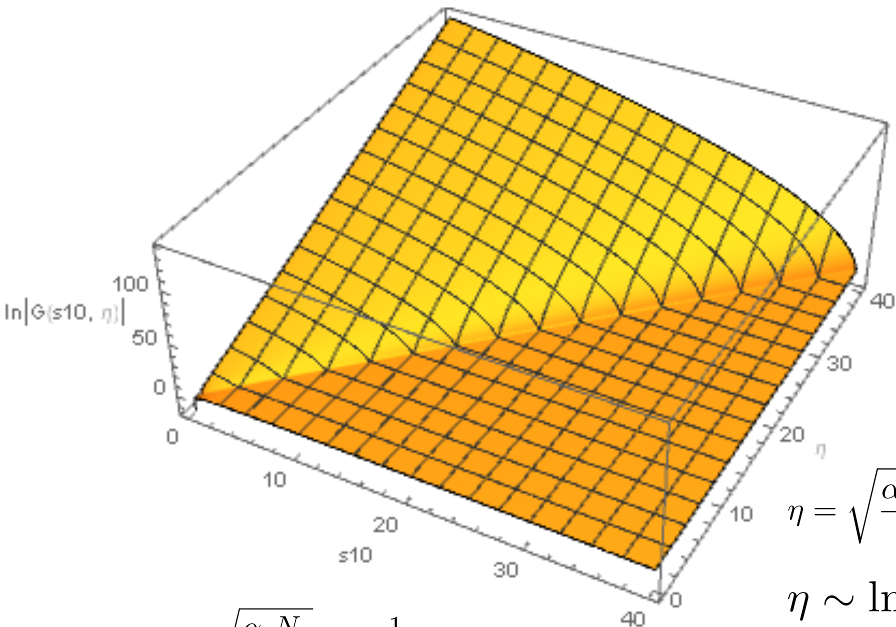
Cross Checks: two methods, ΔG in g_1

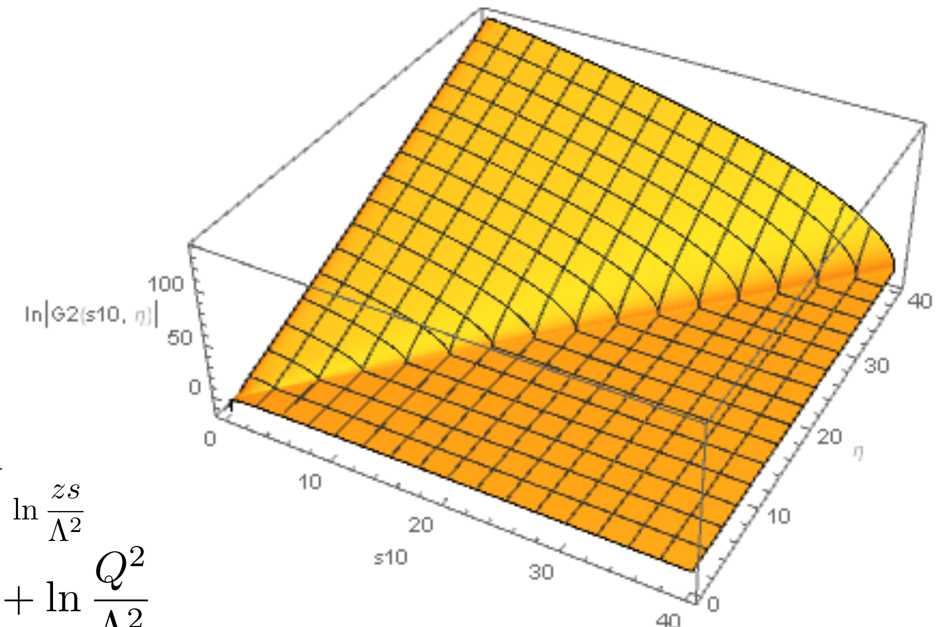
- We rederived our evolution equations using the background field method.
- This is in addition to a blend of Brodsky & Lepage's LCPT and background field method-inspired operator treatment which I presented above. We refer to the latter as the **light-cone operator treatment (LCOT)**.
- Finally, one can even verify that ΔG enters the expression for g_1 with the right coefficient using ΔG ~ G_2 and

$$g_1(x, Q^2) = -\sum_f \frac{N_c\, Z_f^2}{4\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

Solution of the Large-N_C Equations

Solution of the Large-N_c Equations





$$\eta = \sqrt{\frac{\alpha_s N_c}{2\pi}}\,\ln\frac{zs}{\Lambda^2}$$

$$\eta \sim \ln\frac{1}{x} + \ln\frac{Q^2}{\Lambda^2}$$

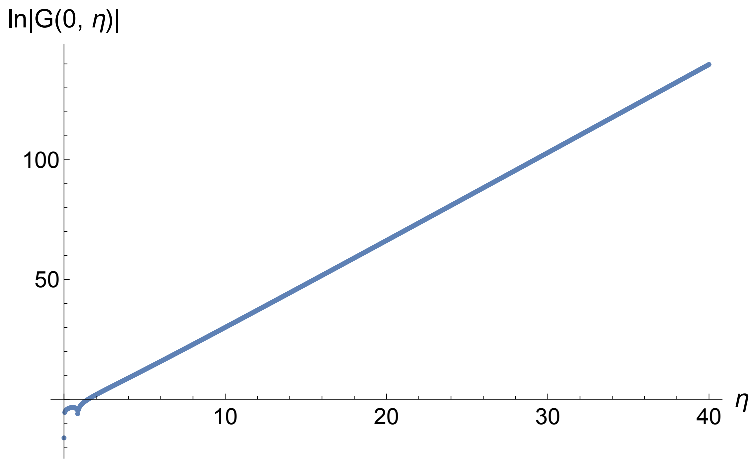
$$s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}}\,\ln\frac{1}{x_{10}^2\,\Lambda^2}$$

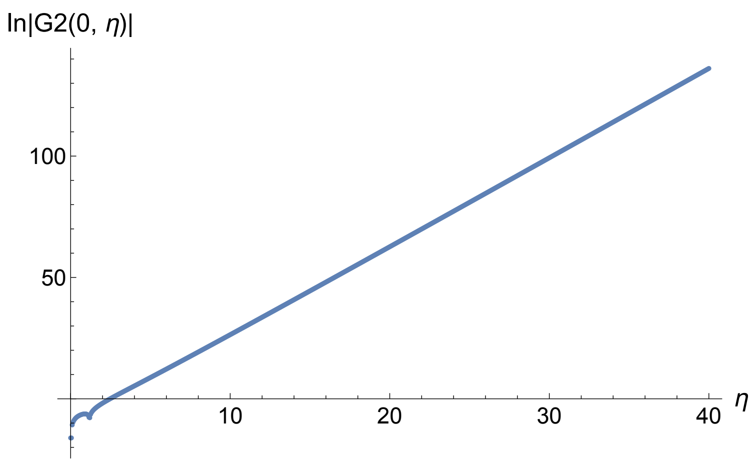
$$s_{10} \sim \ln\frac{Q^2}{\Lambda^2}$$

The large-N_c equations for G and G_2 can be solved numerically (and, possibly, analytically).

Small-x Asymptotics

- Fitting the slope of the log plots of G and G_2 vs e we can read off the small-x intercept:





Small-x Asymptotics for Helicity

- The resulting small-x asymptotics for helicity PDFs and the g_1 structure function is

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

- This intercept is in complete agreement with the work by J. Bartels, B. Ermolaev, and M. Ryskin (1996) using infrared evolution equations (with the analytic intercept constructed by KPS in 2016):

$$\alpha_h = \sqrt{\frac{17+\sqrt{97}}{2}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

- "Peace in the valley."

Helicity at Small x:
Preliminary Phenomenology

D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert, YK, 2102.06159 [hep-ph]

$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

(Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK, 2102.06159 [hep-ph] = JAMsmallx)

Small-x Polarized DIS Data

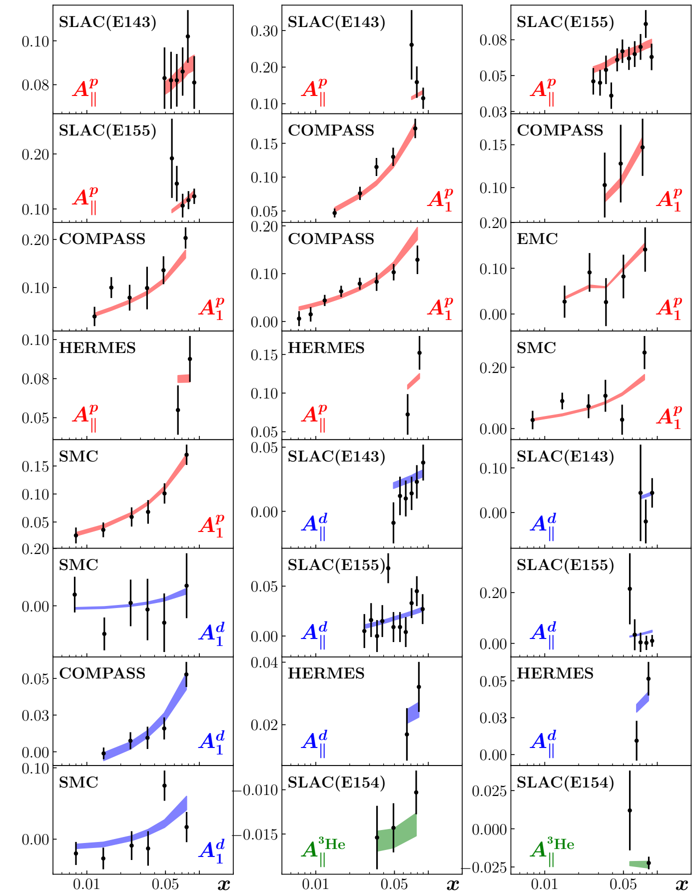
$$A_1 \sim A_{\parallel} = \frac{\sigma_{+-} - \sigma_{++}}{\sigma_{+-} + \sigma_{++}} \sim \frac{g_1}{F_1}$$

- We have analyzed all existing world polarized DIS data with $x<0.1=x_0$, $Q^2 > m_c^2$ (122 data points) using the large-N_C KPS evolution with the Born-inspired initial conditions (8 parameters for 2 flavors, 11 parameters for 3 flavors).

$$G^{(0)}(x_{10}^2, z) \propto a_q \ln \frac{zs}{\Lambda^2} + b_q \ln \frac{1}{x_{10}^2 \Lambda^2} + c_q$$

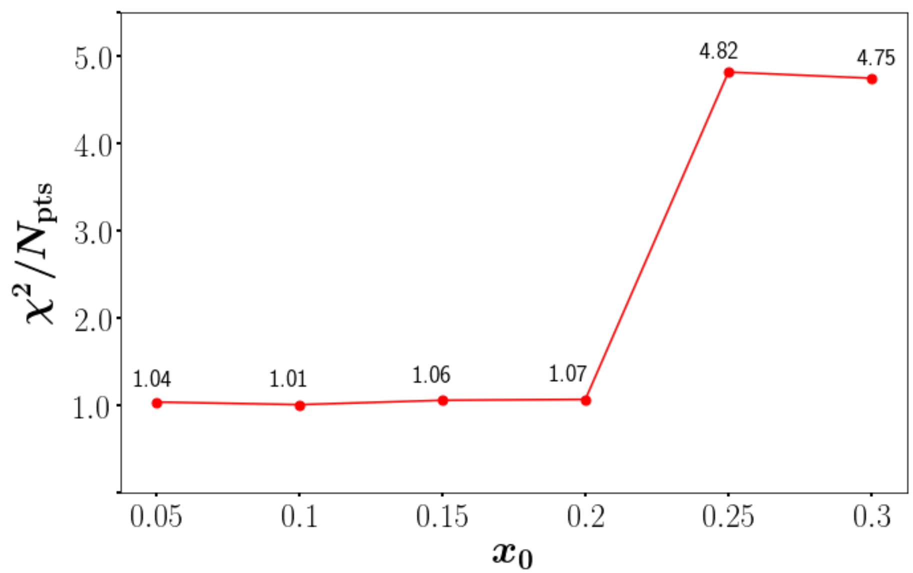
- It worked well, with $\chi^2/N_{pts} = 1.01$ (cf. JAM16: $\chi^2/N_{pts} = 1.07$)

- Small-x evolution starts at $x_0=0.1$! (cf. $x_0=0.01$ for unpolarized BK/JIMWLK evolution) Our approach fails at larger x as expected ($x_0=0.3$ gives $\chi^2/N_{pts} = 4.75$).

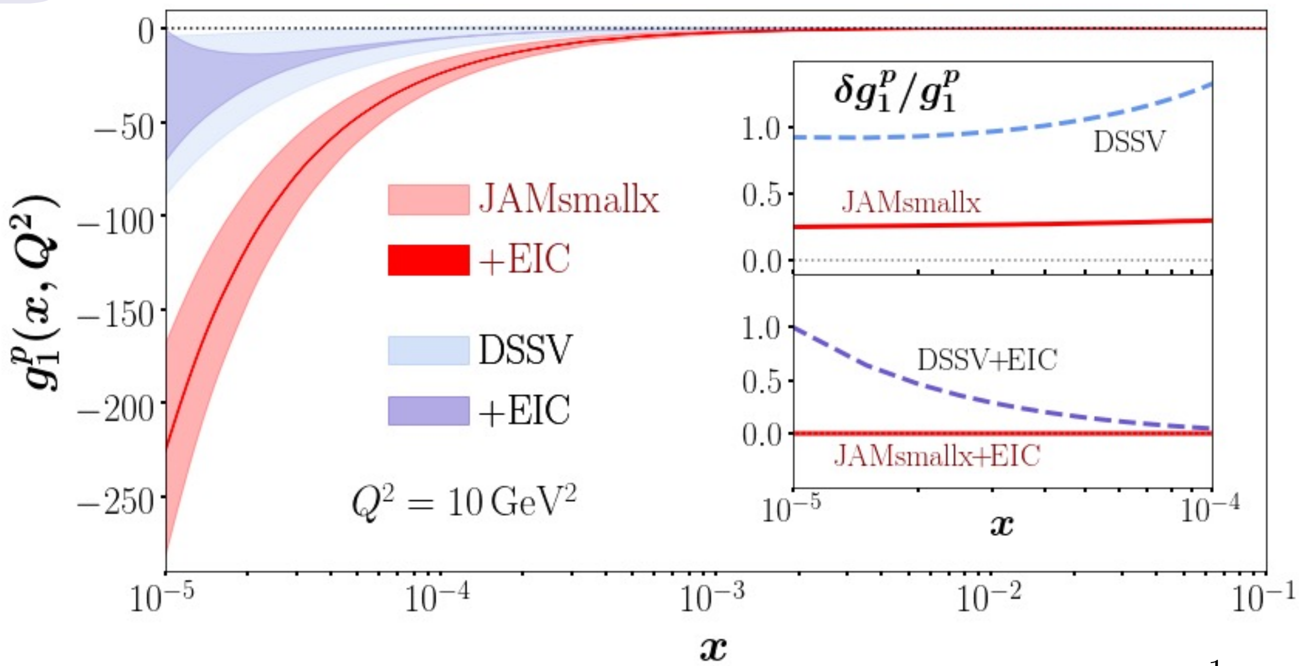


Where to start small-x evolution

- The evolution starts at $x=x_0$, and continues toward smaller x.
- The quality of our fit rapidly deteriorates for $x_0>0.2$, as expected from a small-x approach.
- In unpolarized BK/JIMWLK evolution, typically $x_0=0.01$, so the fact that our fit works up to such a high x_0 is quite remarkable.



Prediction for g_1 structure function

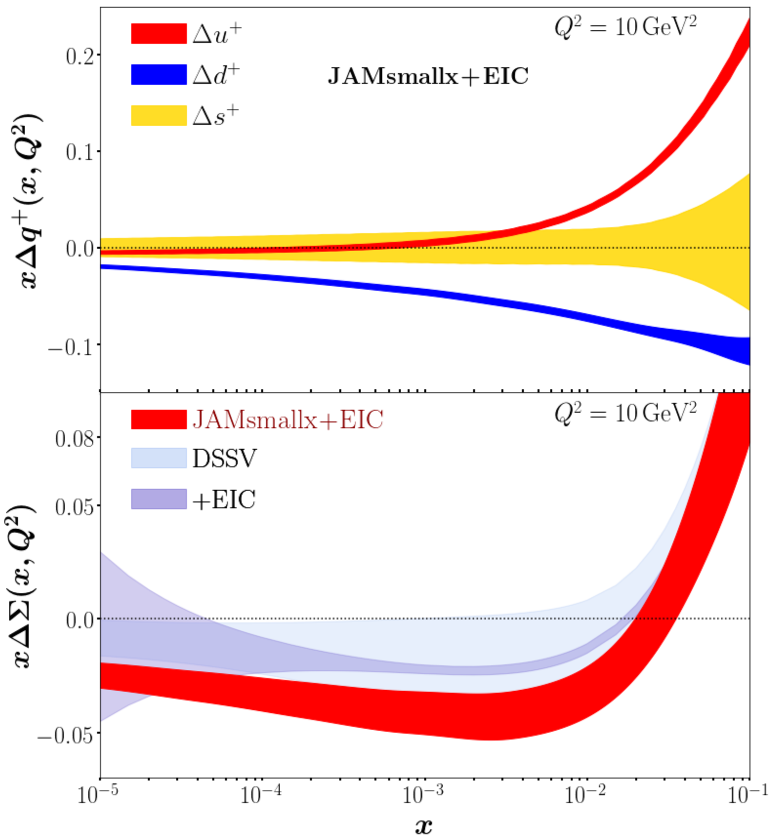


g_1 structure function ≈ spin-dependent part of σ^{e+p}

$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

Thick band: 1σ CL; thin band: impact of EIC data. With the EIC pseudo-data we have 1096 data points.

Predictions for helicity PDFs



D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert & YK, 2102.06159 [hep-ph], in the JAM Collaboration framework.

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma(x, Q^2) = \sum_f \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

If we plug in $\alpha_S = 0.25$ we get $\alpha_h^q = 0.80$

old intercept!

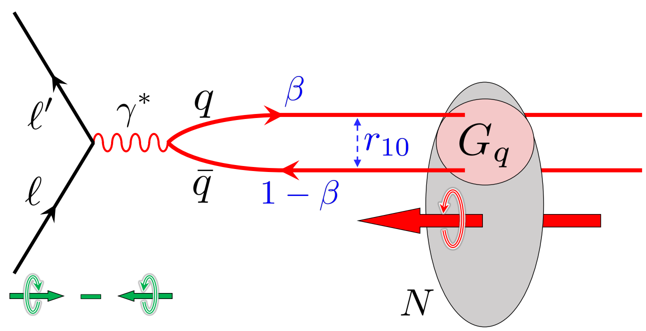
Our (red) error band does not explode in the unmeasured region. We can **predict** spin at small x.

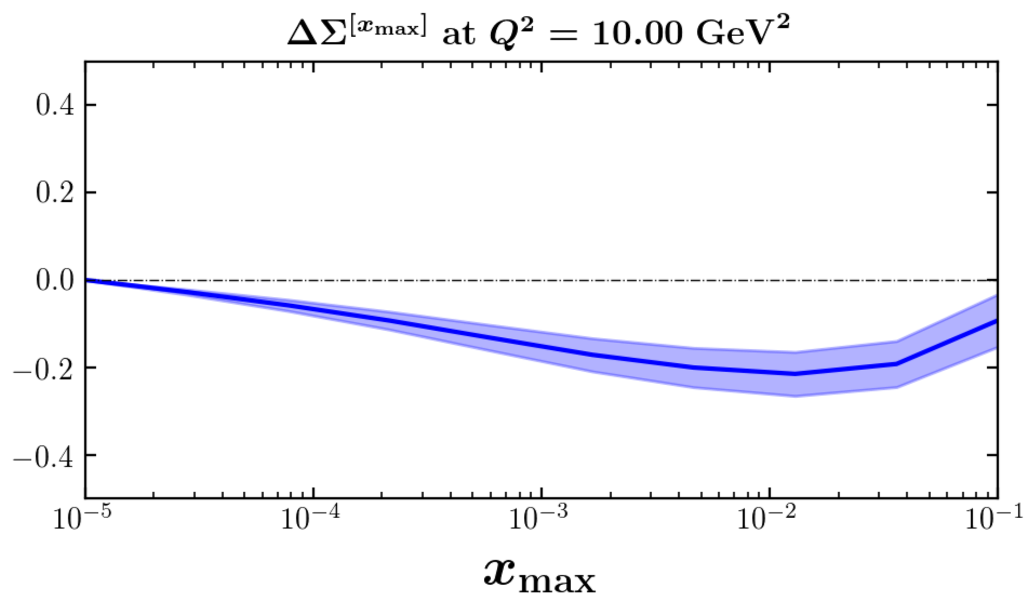
$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Small-x quarks impact on the proton spin

- Potentially negative 10-20% of the proton spin may be carried by small-x quarks (JAMsmallx, preliminary):

$$\Delta\Sigma^{[x_{max}]}(Q^2) = \int_{10^{-5}}^{x_{max}} dx\, \Delta\Sigma(x, Q^2)$$





Speculation on a path to resolving the spin puzzle

- Above we discussed quark helicity at small x. Let's add the orbital angular momentum (OAM) (Hatta & Yang, '18; YK '19):

$$\frac{1}{2}\,\Delta\Sigma(x,Q^2) + L_{q+\bar{q}}(x,Q^2) = -\frac{1}{2}\,\Delta\Sigma(x,Q^2)$$

- So, the net quark (1/2) helicity+OAM = (-1/2) helicity.
- For x<0.001 we thus expect (preliminary!)

$$\left[\frac{1}{2}\Delta\Sigma + L_{q+\bar{q}}\right]_{Q^2=10\,\mathrm{GeV}^2,\ x<0.001} \approx -\frac{1}{2}\,(-0.2) = 0.1$$

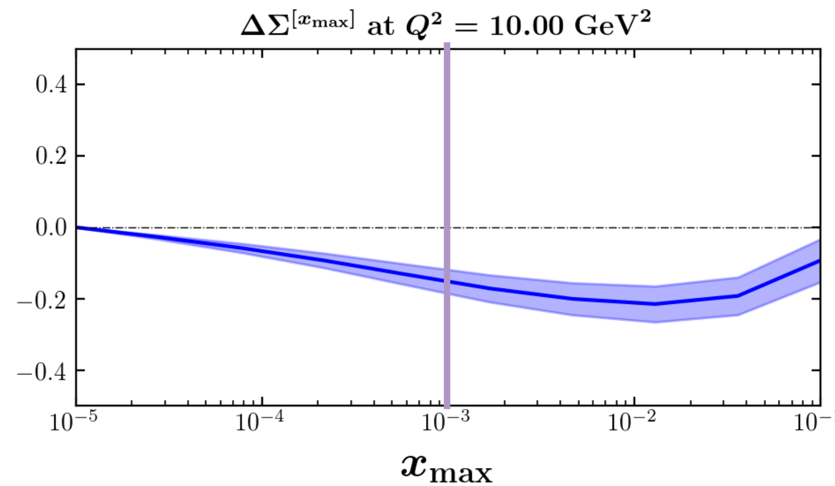
- Add to this the larger-x numbers

$$S_q(Q^2 = 10\,\mathrm{GeV}^2, x > 0.001) \approx 0.18$$
$$S_G(Q^2 = 10\,\mathrm{GeV}^2, x > 0.05) \approx 0.2$$

- We get

$$0.18 + 0.2 + 0.1 = 0.48$$

JAMsmallx, preliminary, Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK



A warning

- Note that the above phenomenology was done using the old version of the evolution equations. Consider it a proof of principle. (Most data was described by the Born-inspired initial conditions for the dipole amplitudes, that is, without evolution.)

- Phenomenology with the new corrected evolution equations is being developed as we speak.

Conclusions

- Small-x helicity evolution equations have been updated due to the mixing of the amplitude G_2 with the previously known (KPS) evolution.
- The physical reason behind the mixing is not clear.
- The new equations pass many independent cross checks.
- The solution of the large-N_c equations agrees with the result by Bartels, Ermolaev & Ryskin (1996).
- Small-x OAM asymptotics, helicity JIMWLK & MV model, single-logarithmic evolution, and phenomenology must be updated too. Stay tuned.