GLOBAL ANALYSES OF FRAGMENTATION FUNCTIONS AT NNLO ACCURACY

Ignacio Borsa

In collaboration with D. De Florian, R. Sassot, M. Stratmann and W. Vogelsang

Precision QCD predictions for ep physics at the EIC

CFNS

August 4th 2022

ROADMAP

FFs in a nutshell

Why should we care about hadrons?
Why do we need NNLO sets of FFs?

Parton-to-pion FFs at NNLO accuracy

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Phys.Rev.Lett. 129 (2022) 1, 012002

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FRAGMENTATION FUNCTIONS IN A NUTSHELL

For a review see **Metz, Vossen. Prog.Part.Nucl.Phys. 91 (2016) 136-202**

Based on Collinear Factorization

$$\mathcal{O}^h = \sum_{ab} \int dx \int dz\, f_a(x, \mu_F^2)\, D_b^h(z, \mu_F^2)\, \hat{\sigma}_{ab}(x, z, \alpha_S(\mu_R), \mu_F^2, \mu_R^2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

Parton distribution functions/
Fragmentation Functions
Non-perturbative & Universal

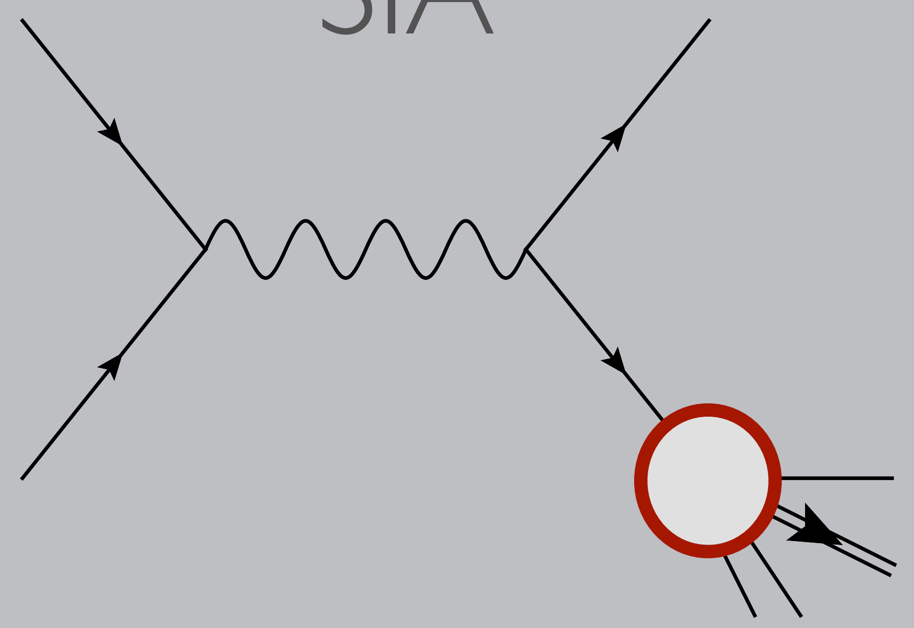
Partonic cross section;
Perturbative & process dependent

Power Corrections

$$\hat{\sigma}_a = \sigma_a^{\rm Born}\left(1 + \frac{\alpha_S}{2\pi}\sigma_a^{(1)} + \left(\frac{\alpha_S}{2\pi}\right)^2\sigma_a^{(2)} + \left(\frac{\alpha_S}{2\pi}\right)^3\sigma_a^{(3)} + \ldots\right)$$

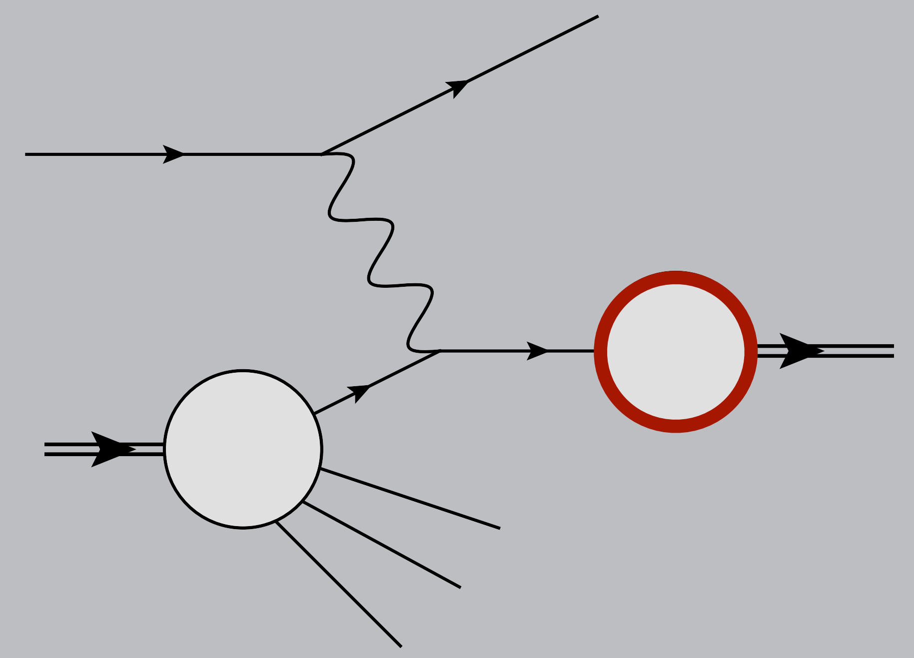
Validated by a large set of unpolarized data

SIA



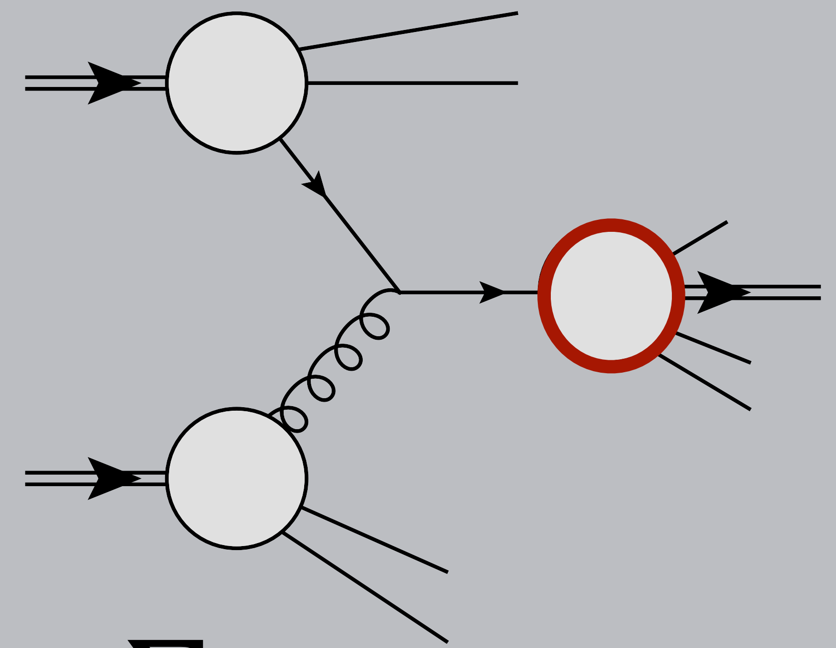
$$d\sigma = \sum_a d\hat{\sigma}_a \otimes D_a^h$$

SIDIS



$$d\sigma = \sum_{ab} f_a \otimes d\hat{\sigma}_{ab} \otimes D_b^h$$

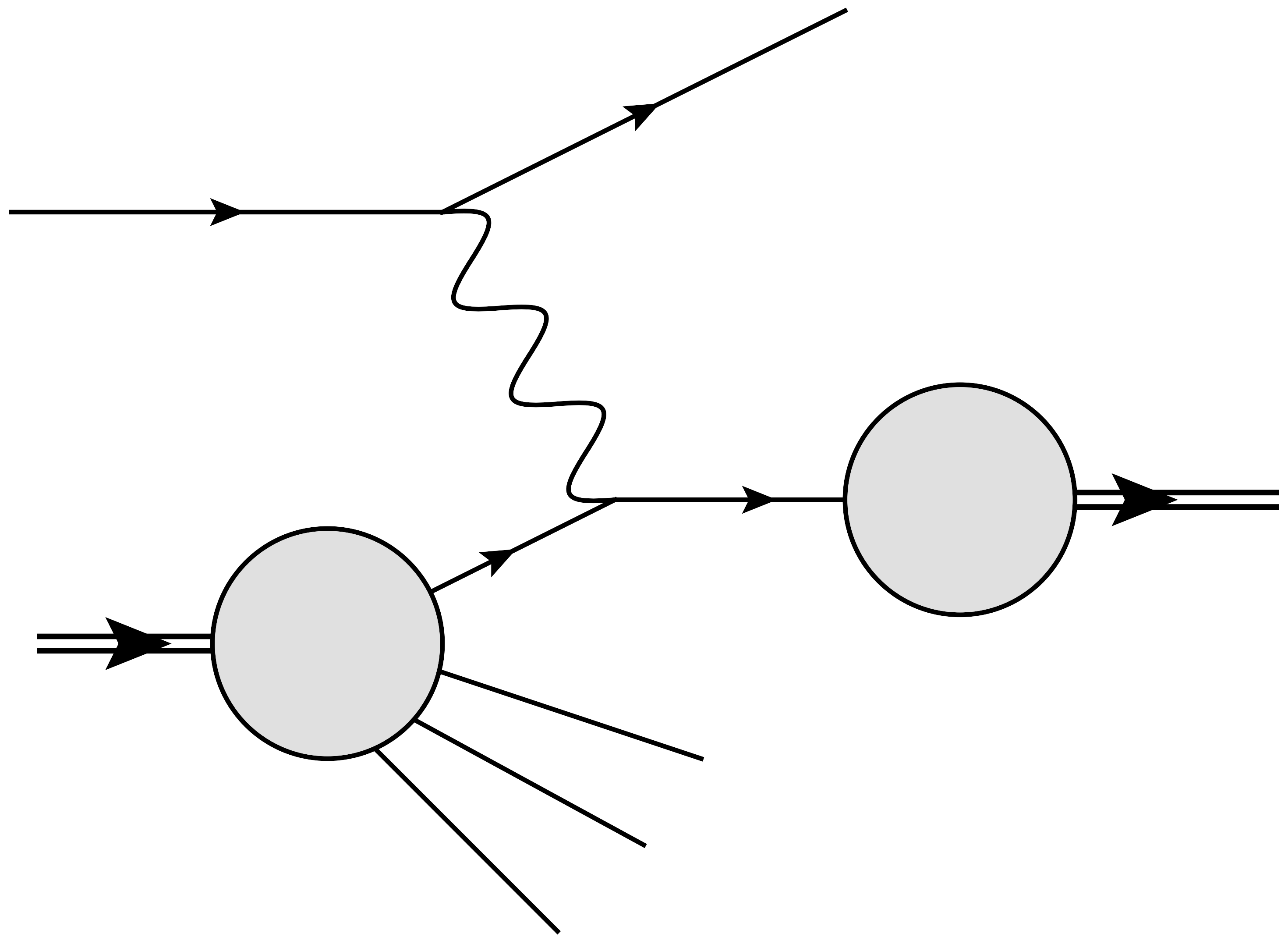
PP



$$d\sigma = \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}_{ab}^c \otimes D_c^h$$

WHY SHOULD WE CARE ABOUT HADRONS?

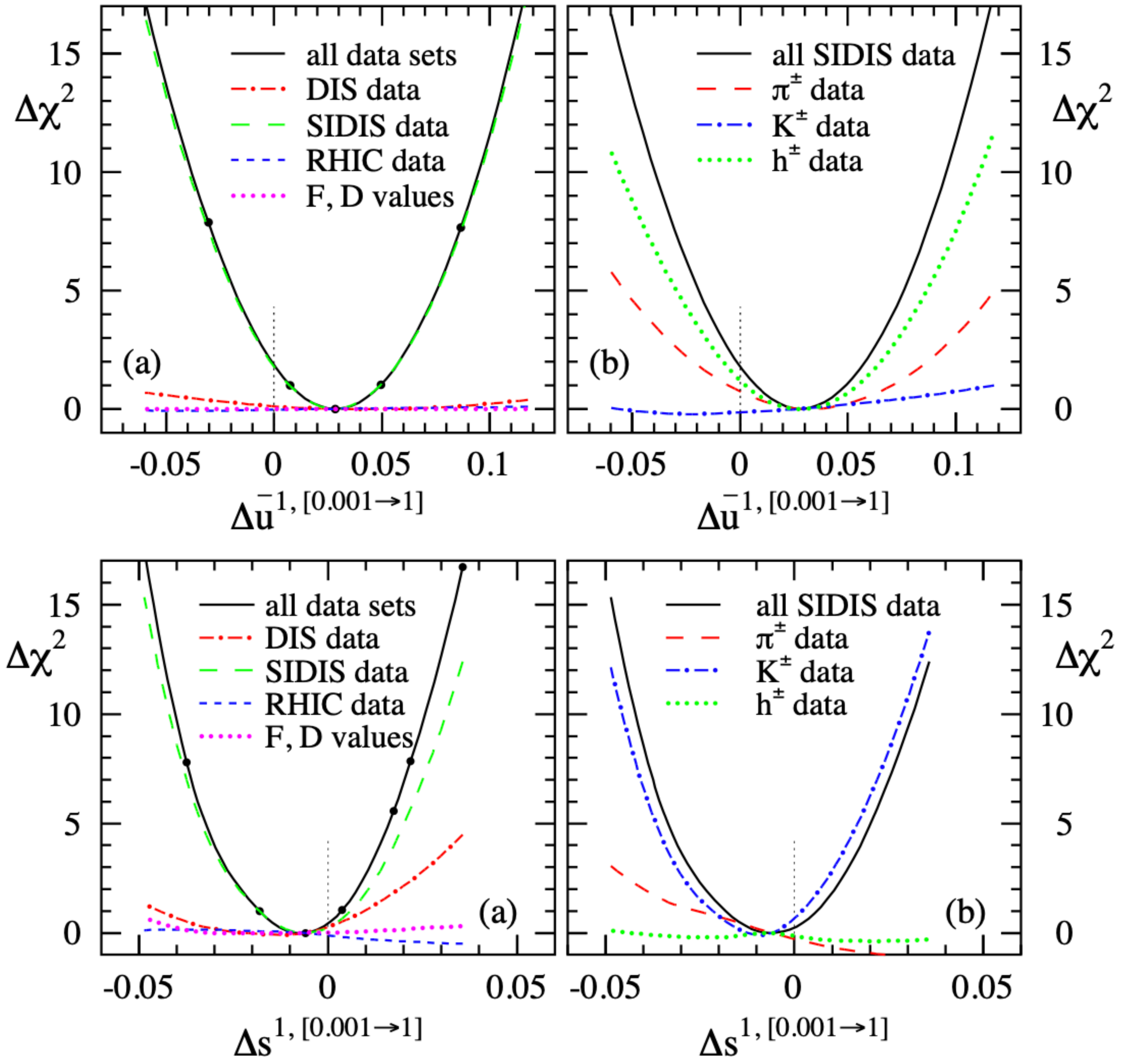
Processes with identified hadrons in the final state are instrumental for the decomposition of the proton spin content in helicity PDFs global analyses



$$\sum_q e_q^2 D_q^H(x, Q^2) \otimes f_q(x, Q^2)$$

Flavor separation of helicity PDF depends crucially on SIDIS results

Important constraints from future EIC measurements



De Florian, Sassot, Stratmann, Vogelsang. *Phys.Rev.D* 80 (2009) 034030

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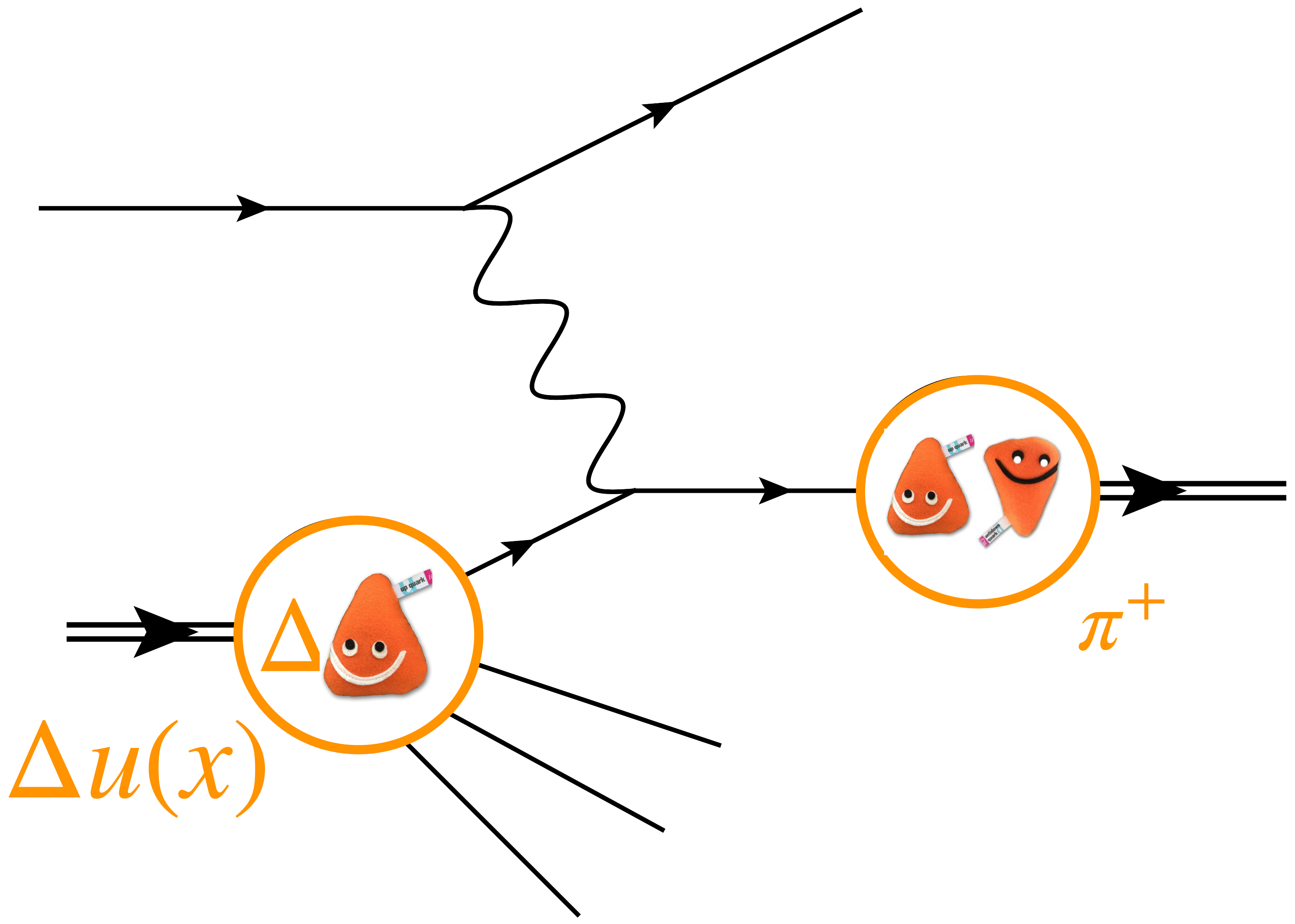

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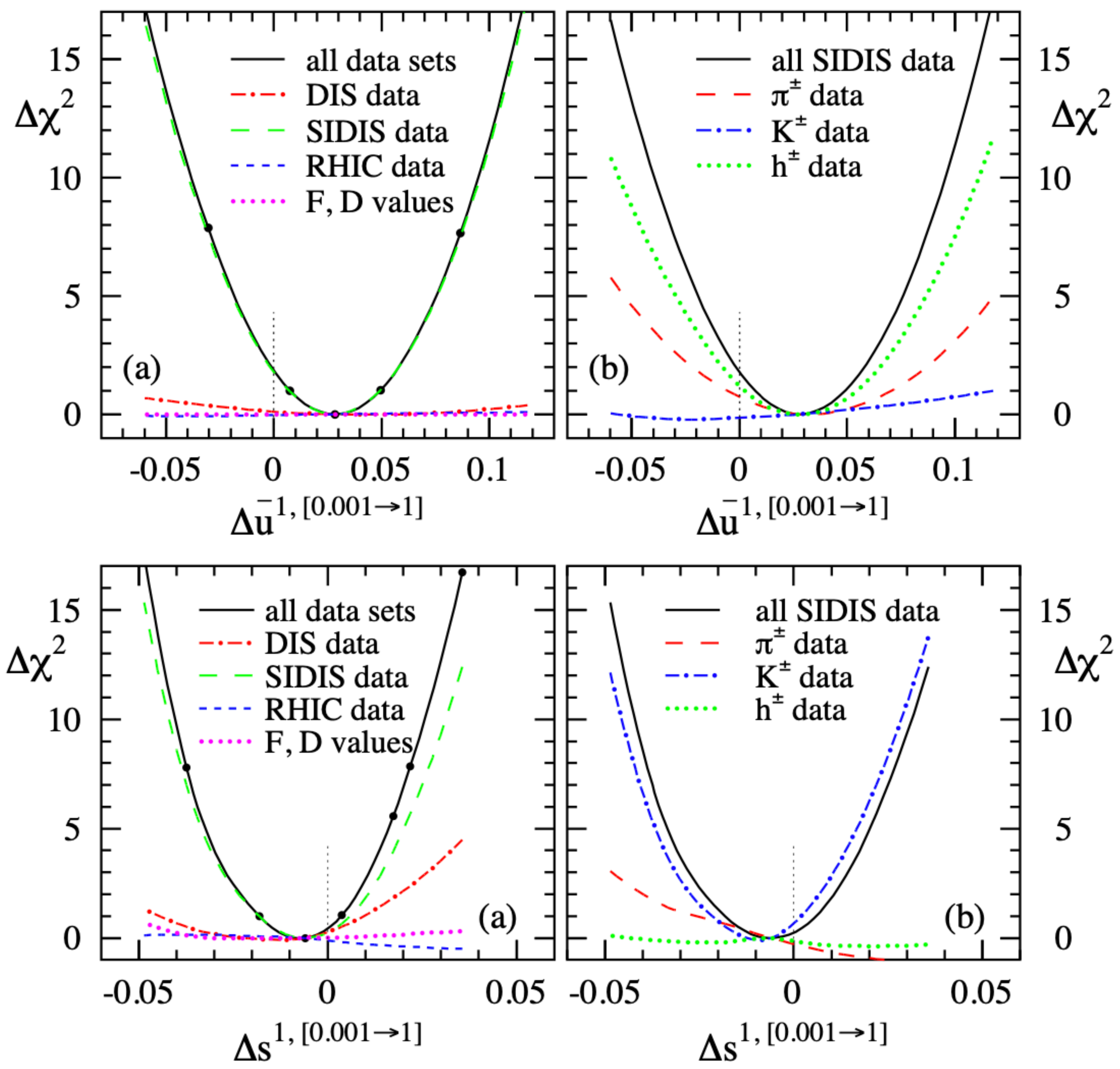

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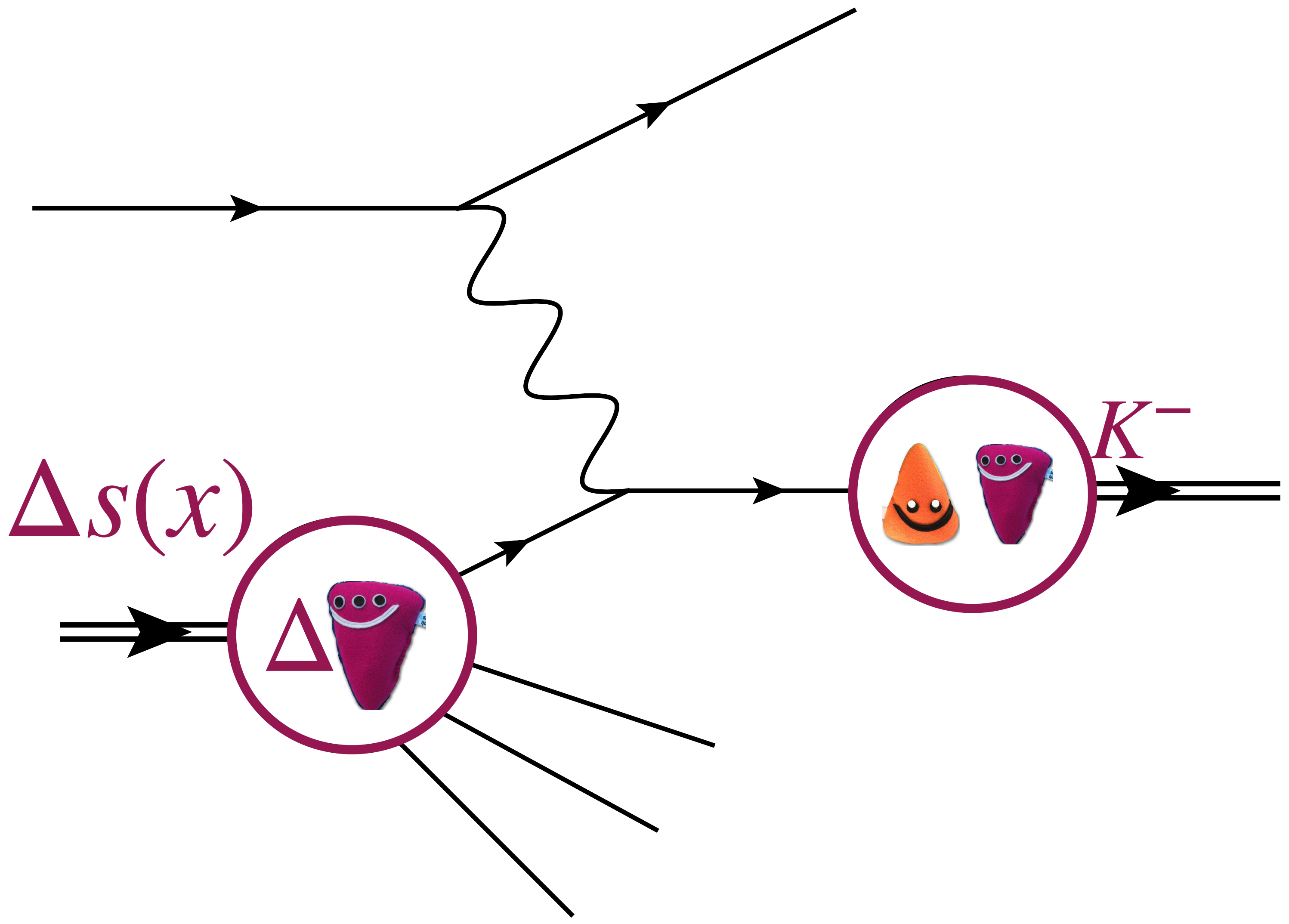

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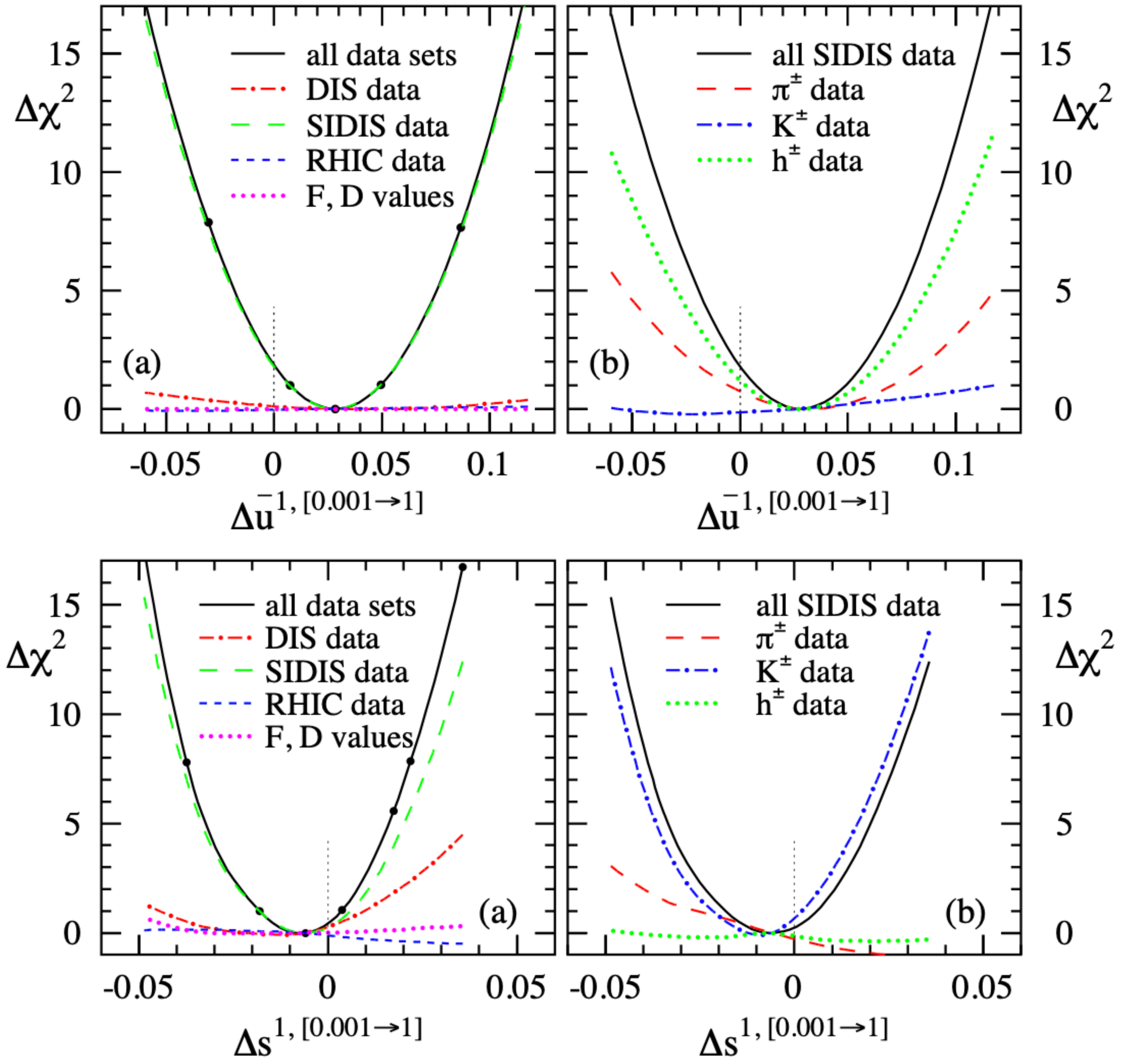

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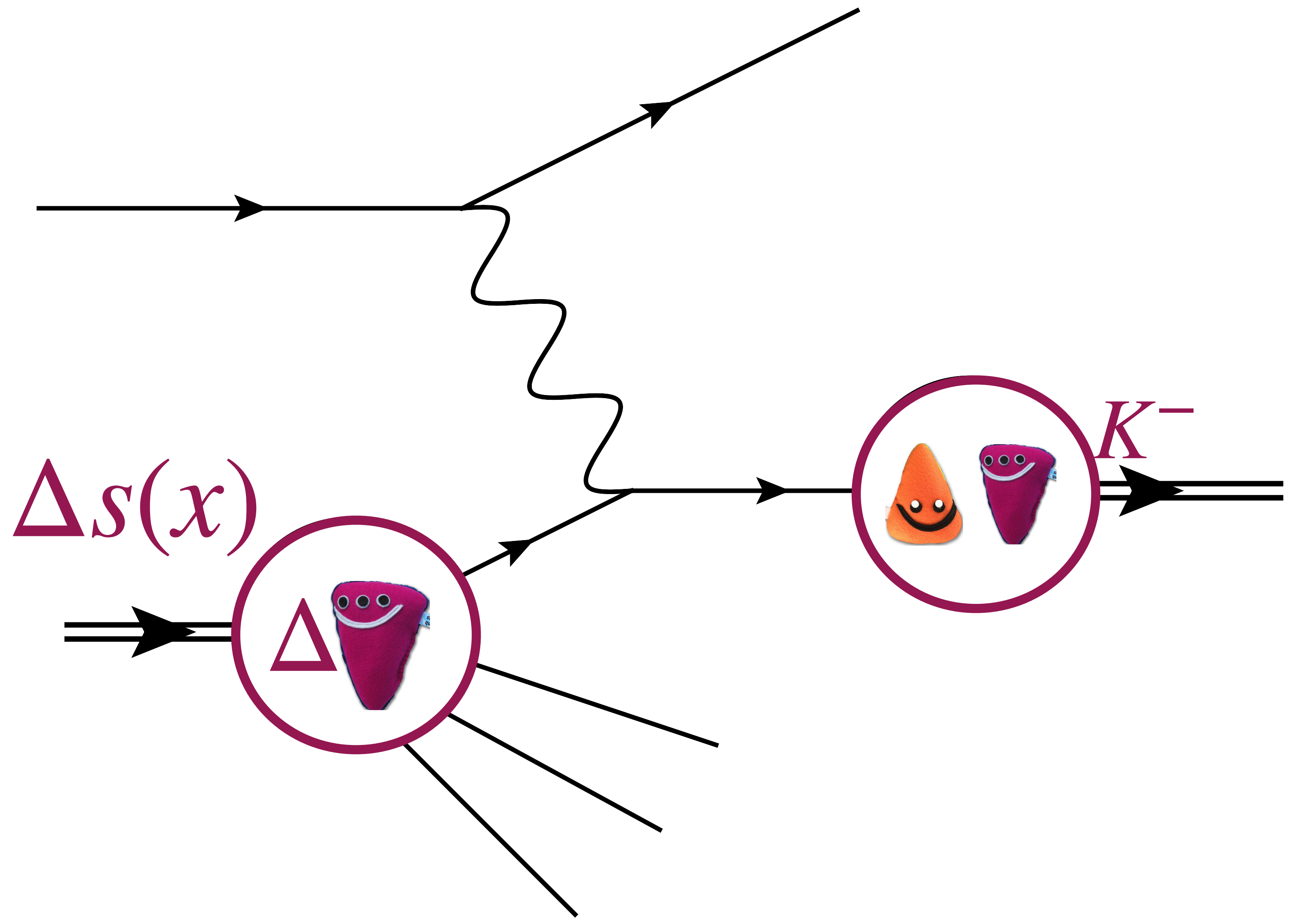

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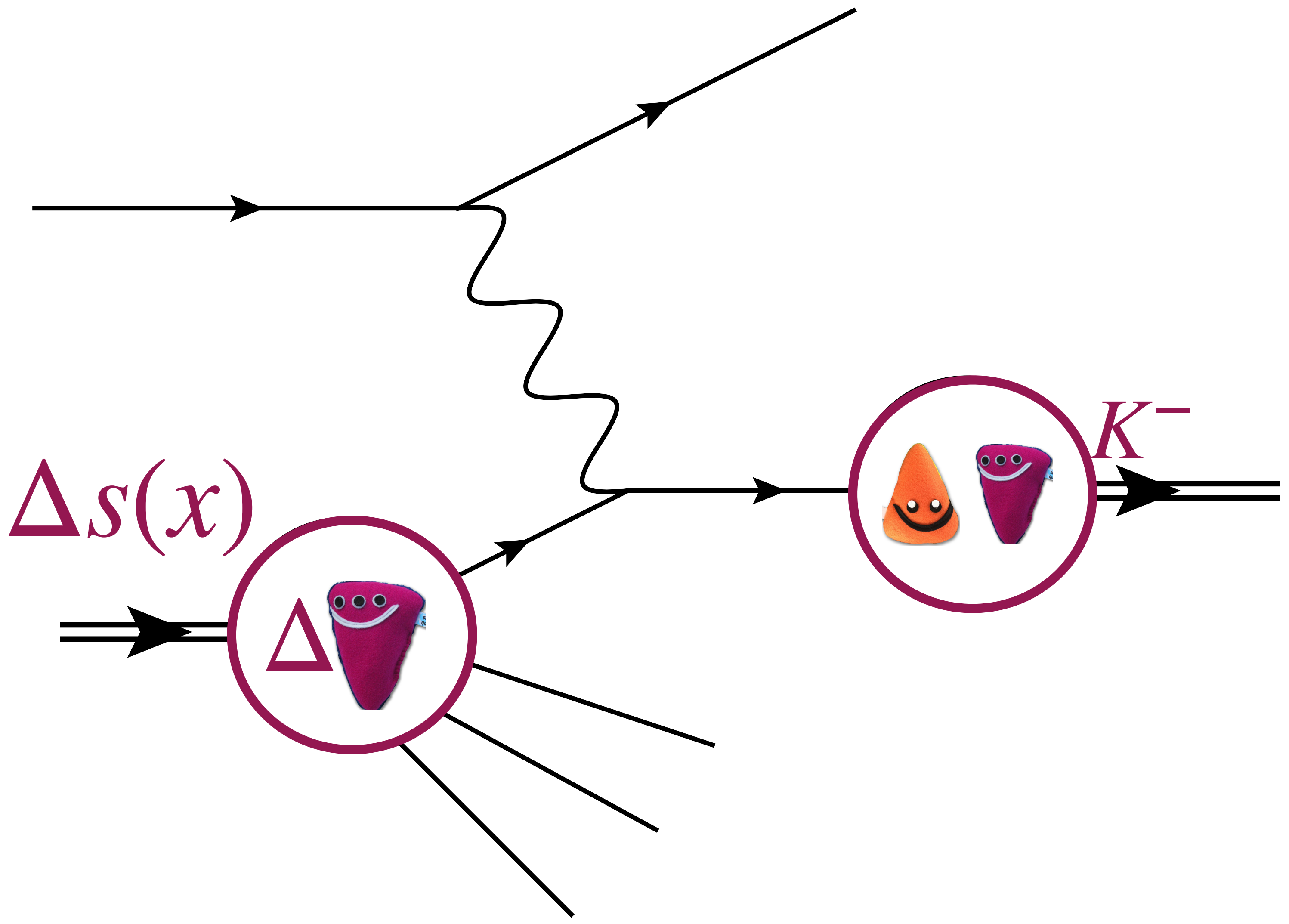

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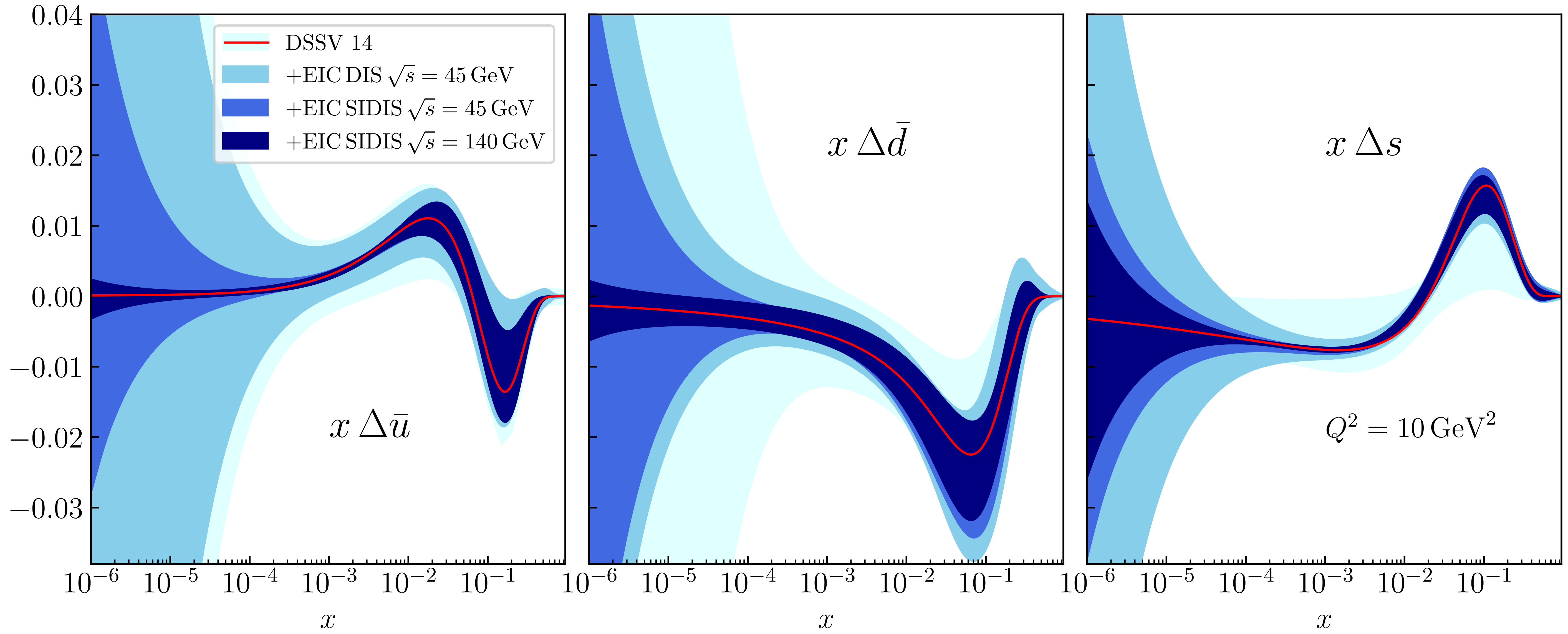
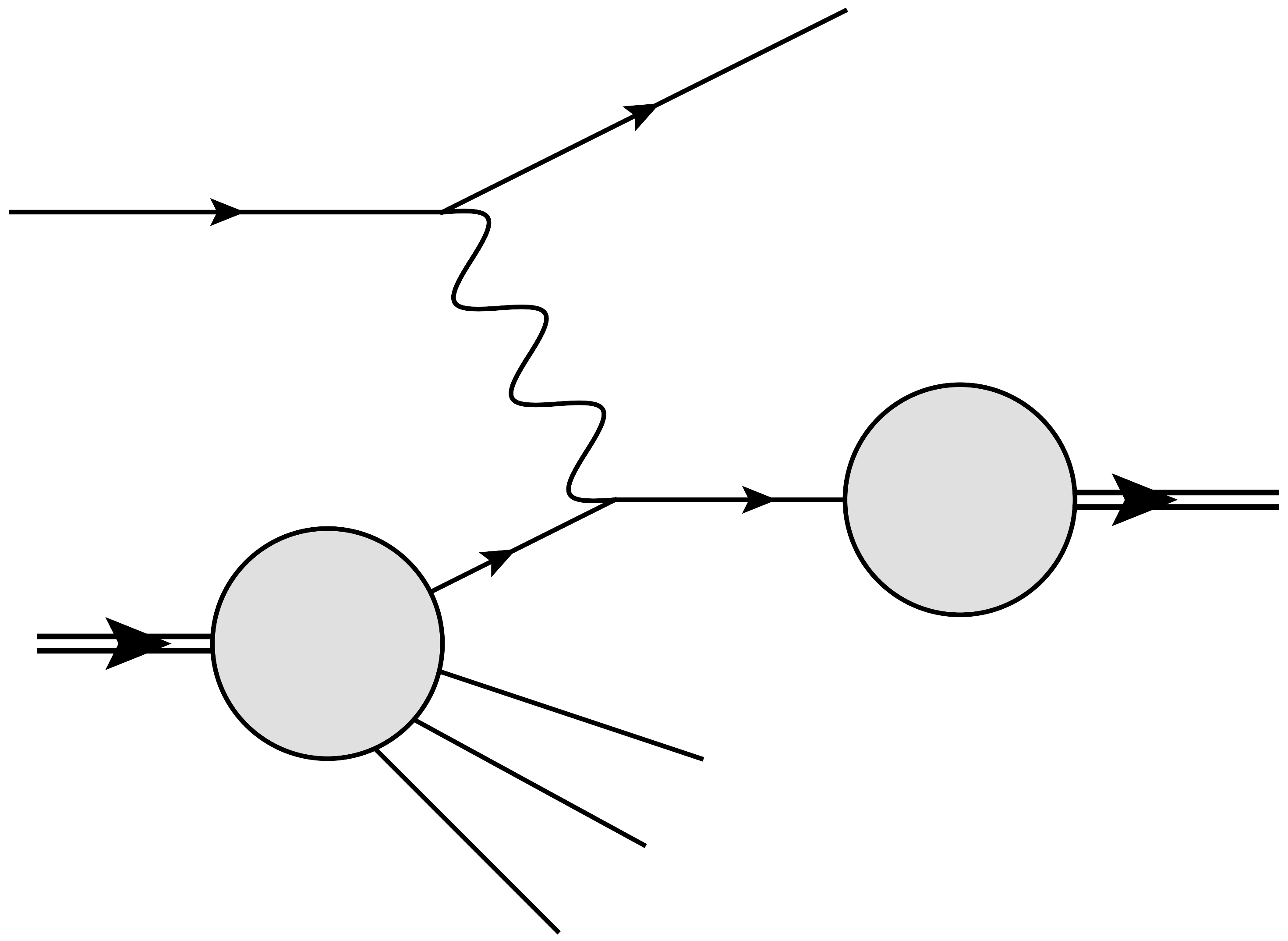


Aschgrauer, IB, Lucero, Nunes, Sassot.
Phys.Rev.D 102 (2020) 9, 094018

WHY SHOULD WE CARE ABOUT HADRONS?

Processes with identified hadrons in the final state can probe PDFs for sea quarks



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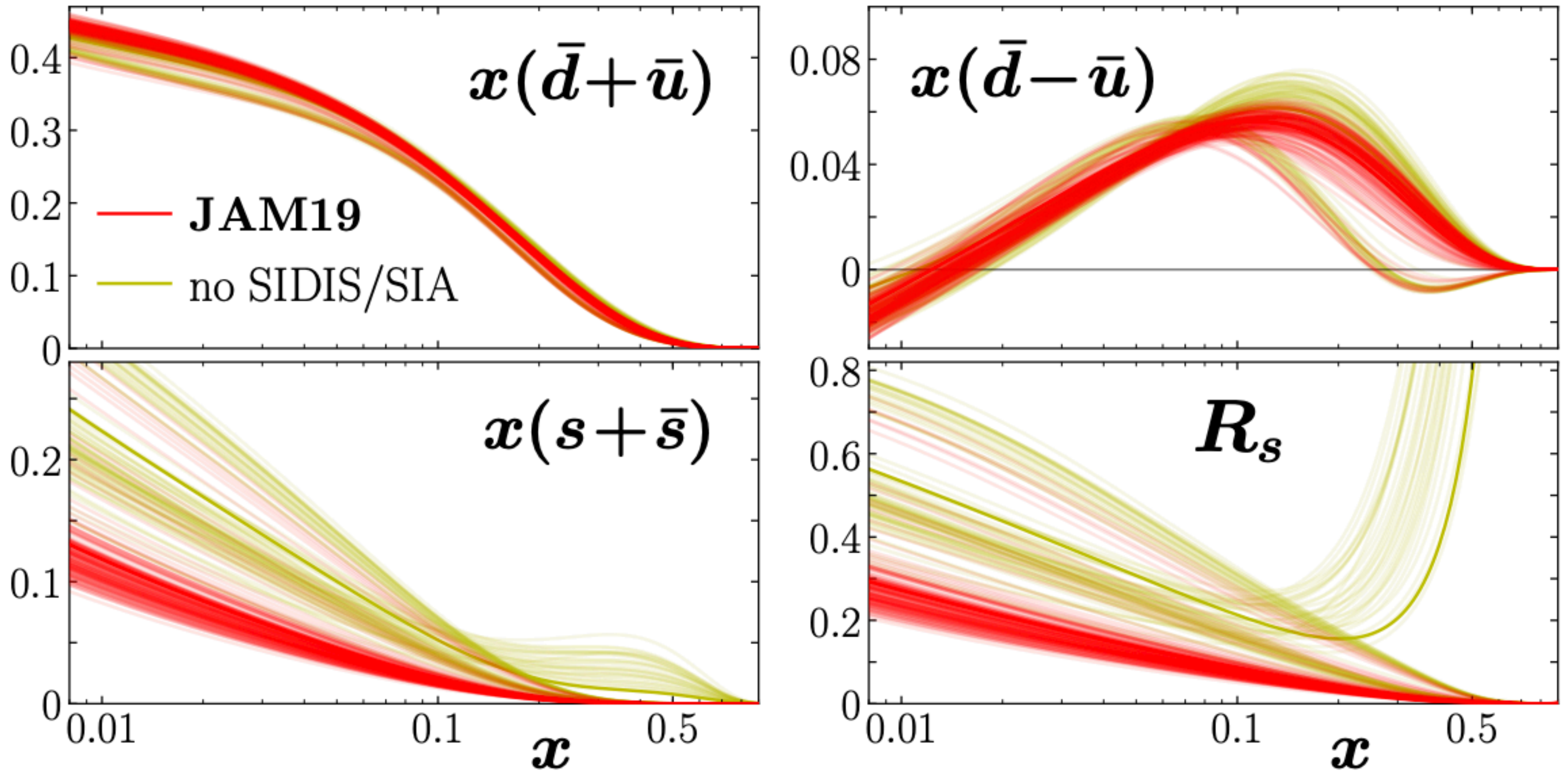
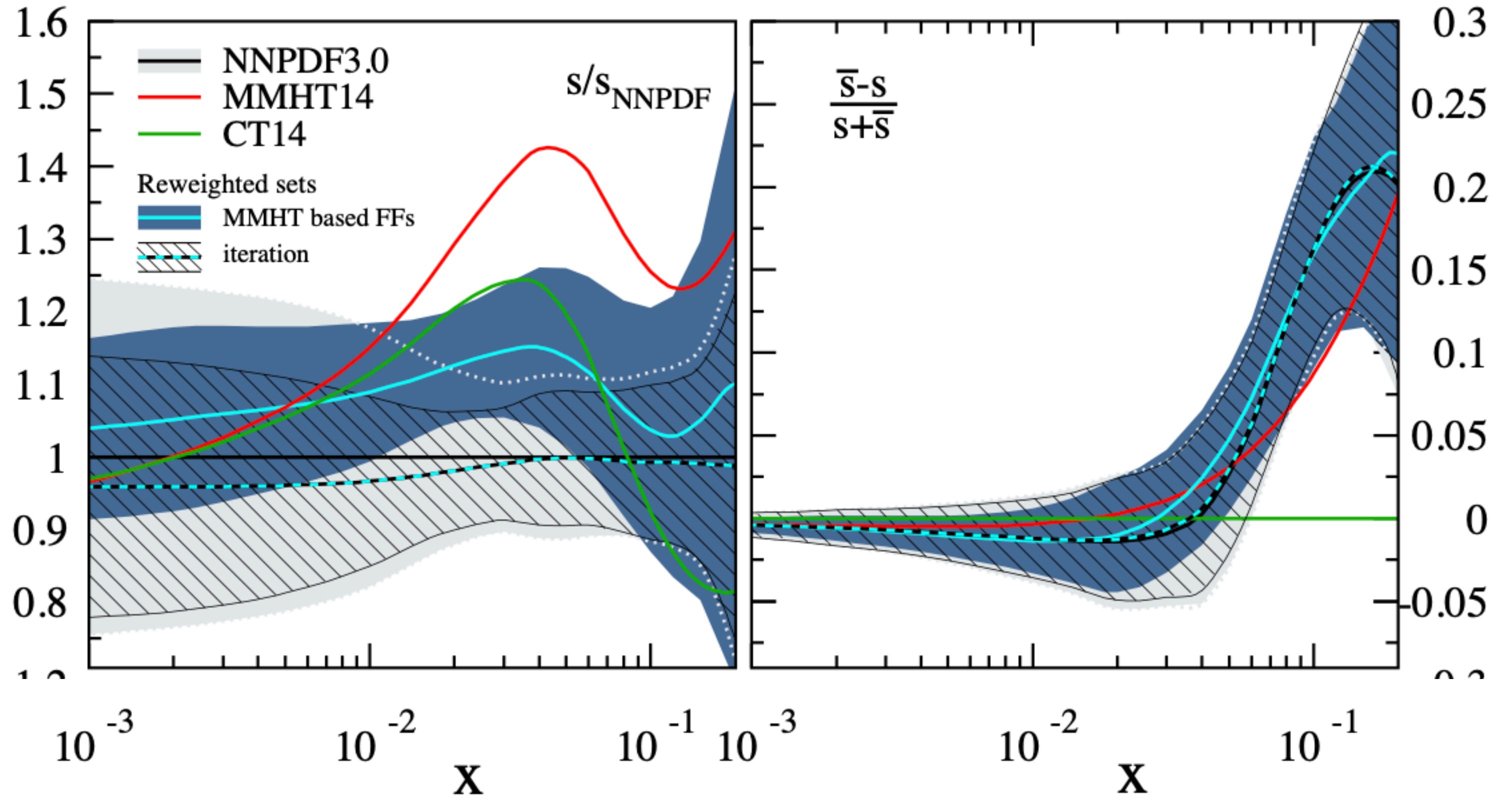
Additional information on sea quarks PDFs

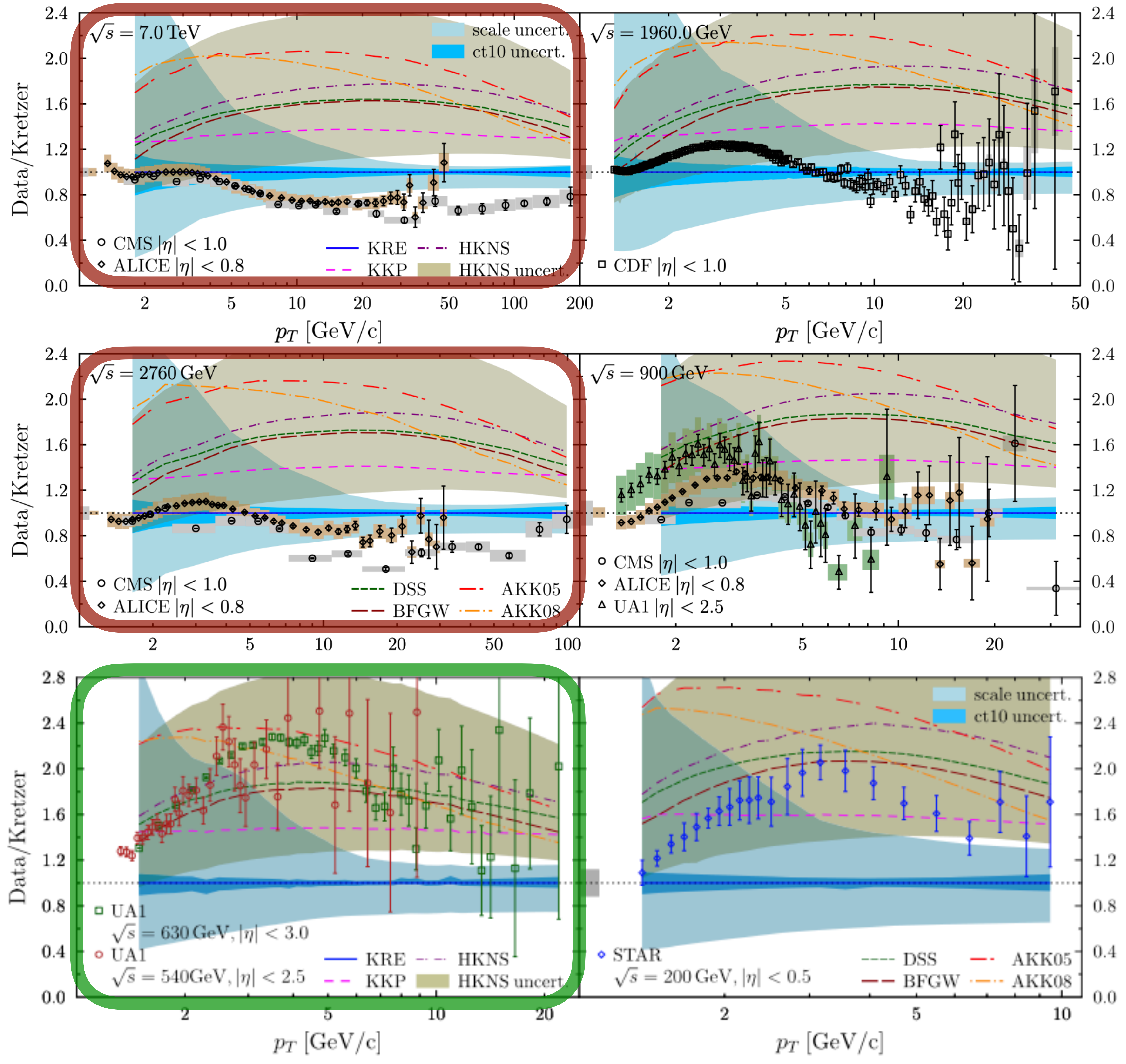


JAM Collaboration. *Phys.Rev.D* 101 (2020) 7, 074020



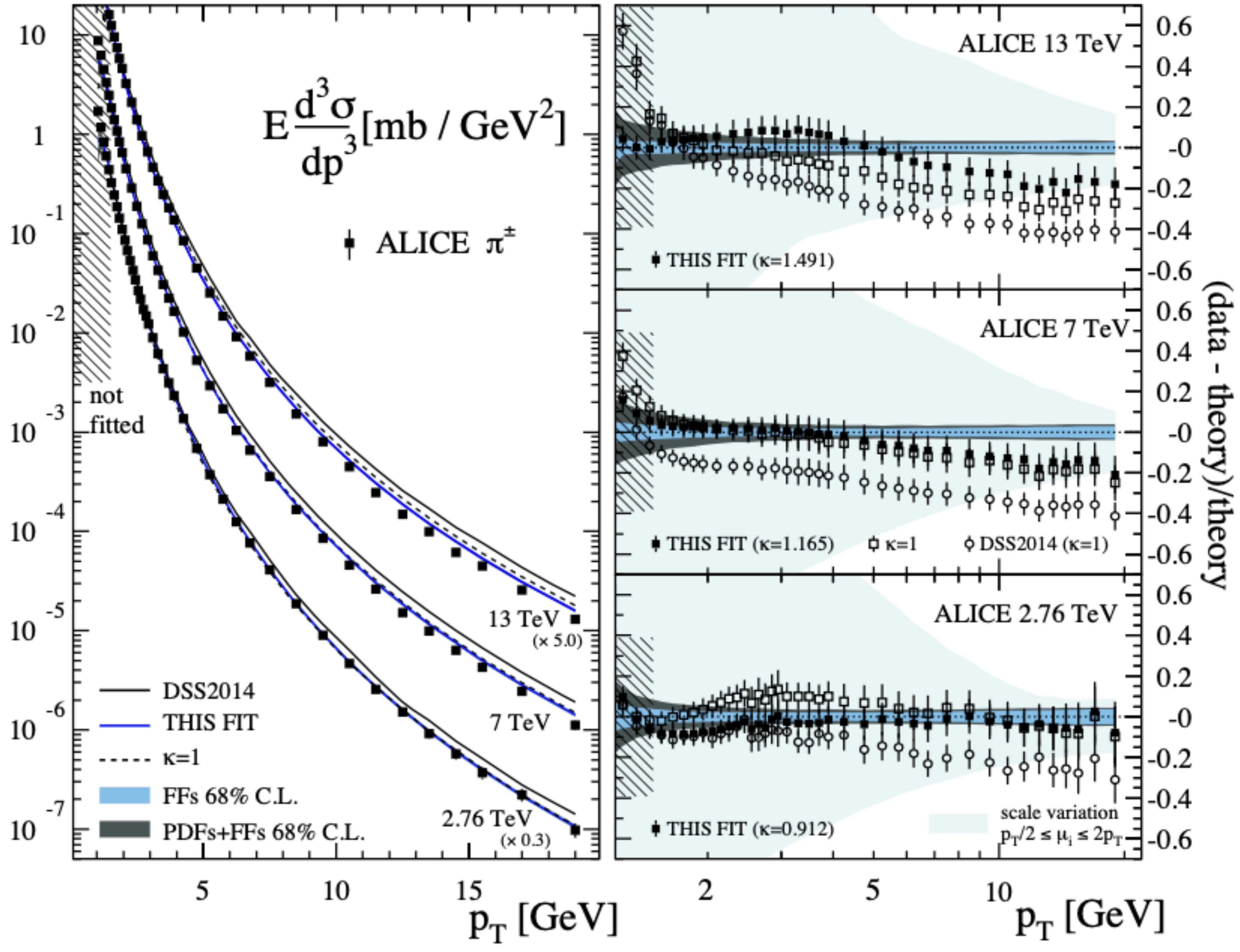
IB, Sassot, Stratmann. *Phys.Rev.D* 96 (2017) 9, 094020

WHY DE WE NEED NNLO SETS OF FFS?



d'Enterria, Eskola, Helenius, Paukkunen.
Nucl.Phys.B 883 (2014) 615-628

Description of the p_T -behaviour deteriorates for higher c.m.s energies
Overestimation of the cross section for high p_T



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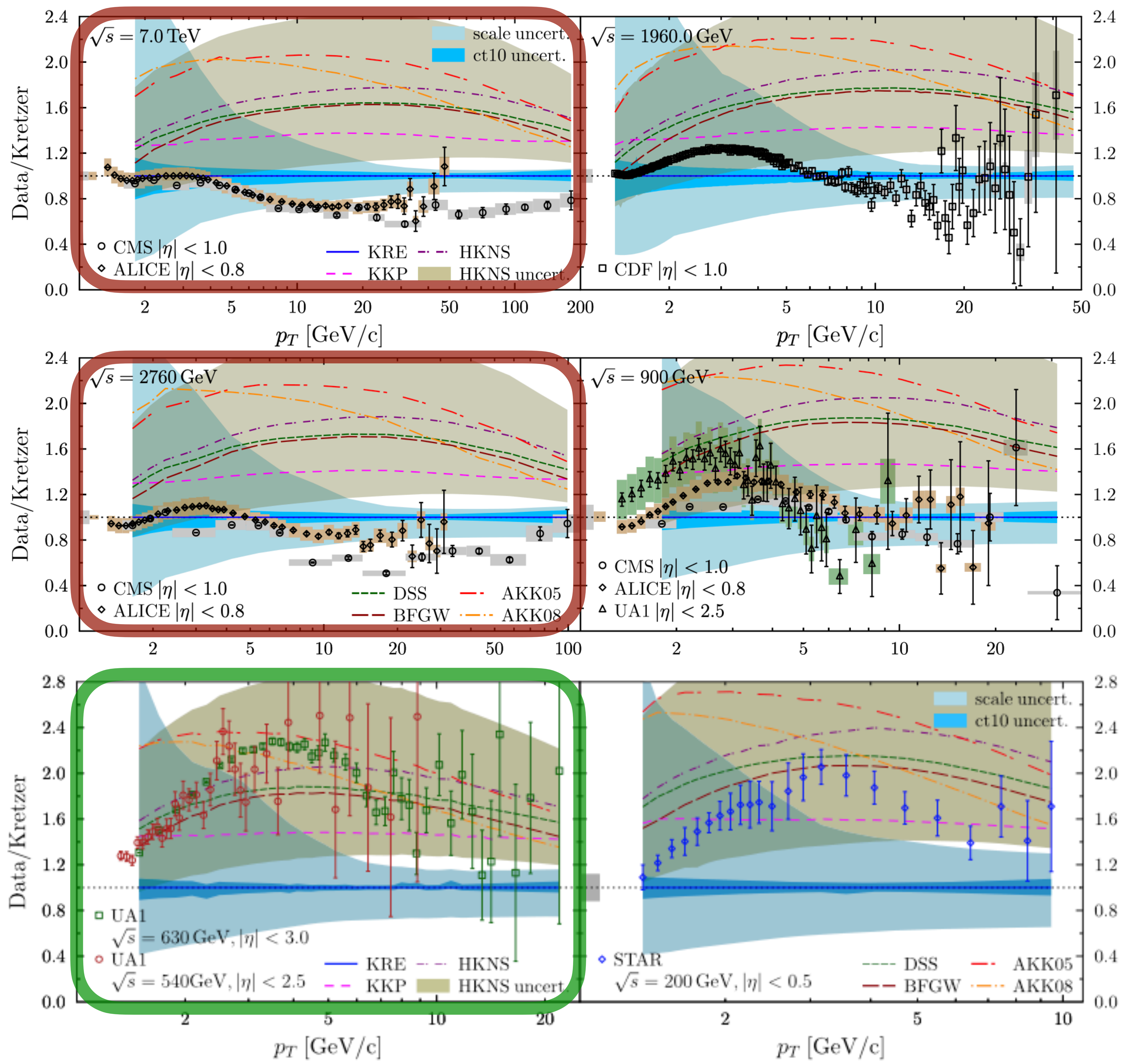

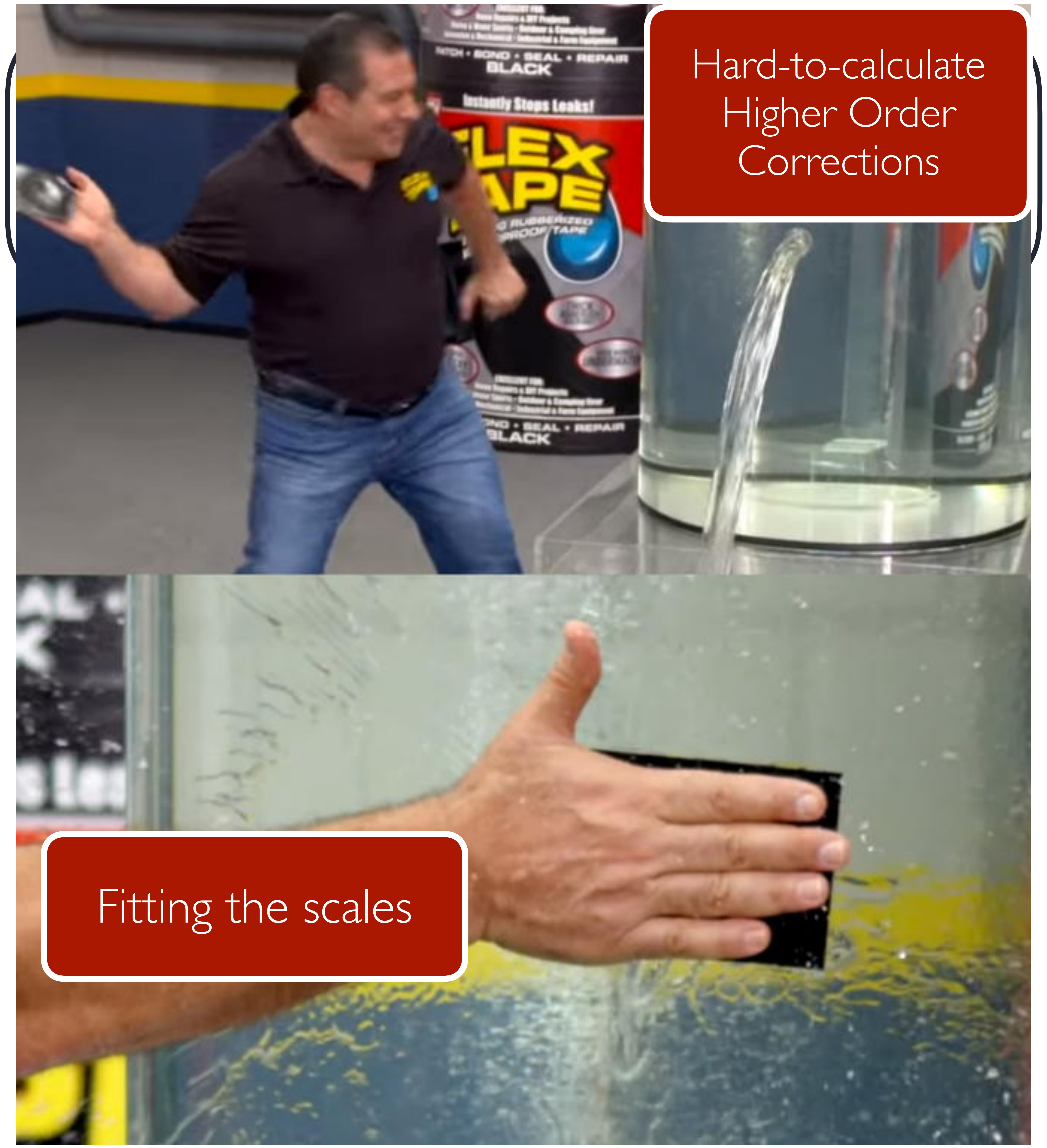

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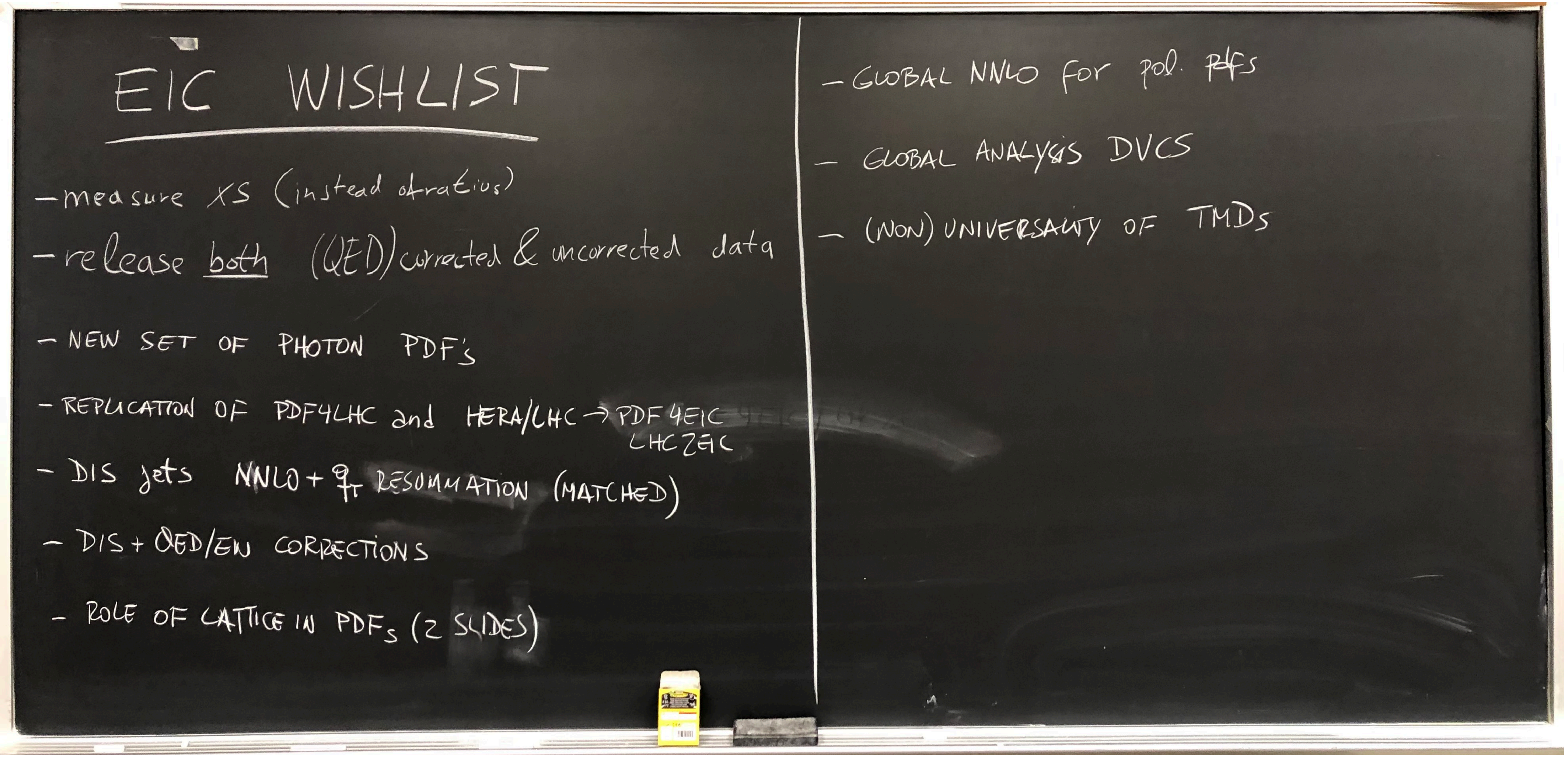

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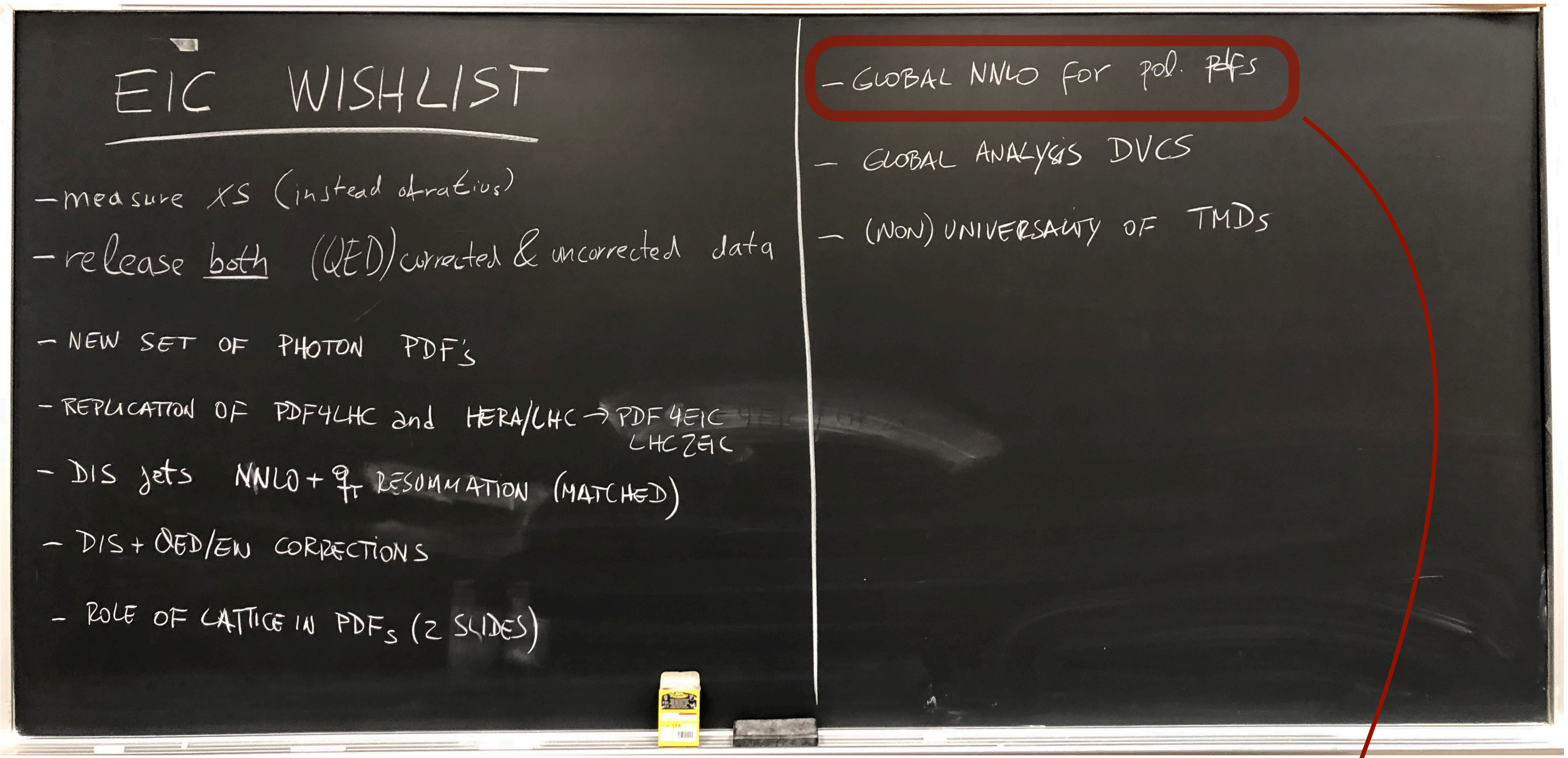

Will require knowledge on the FFs up to NNLO

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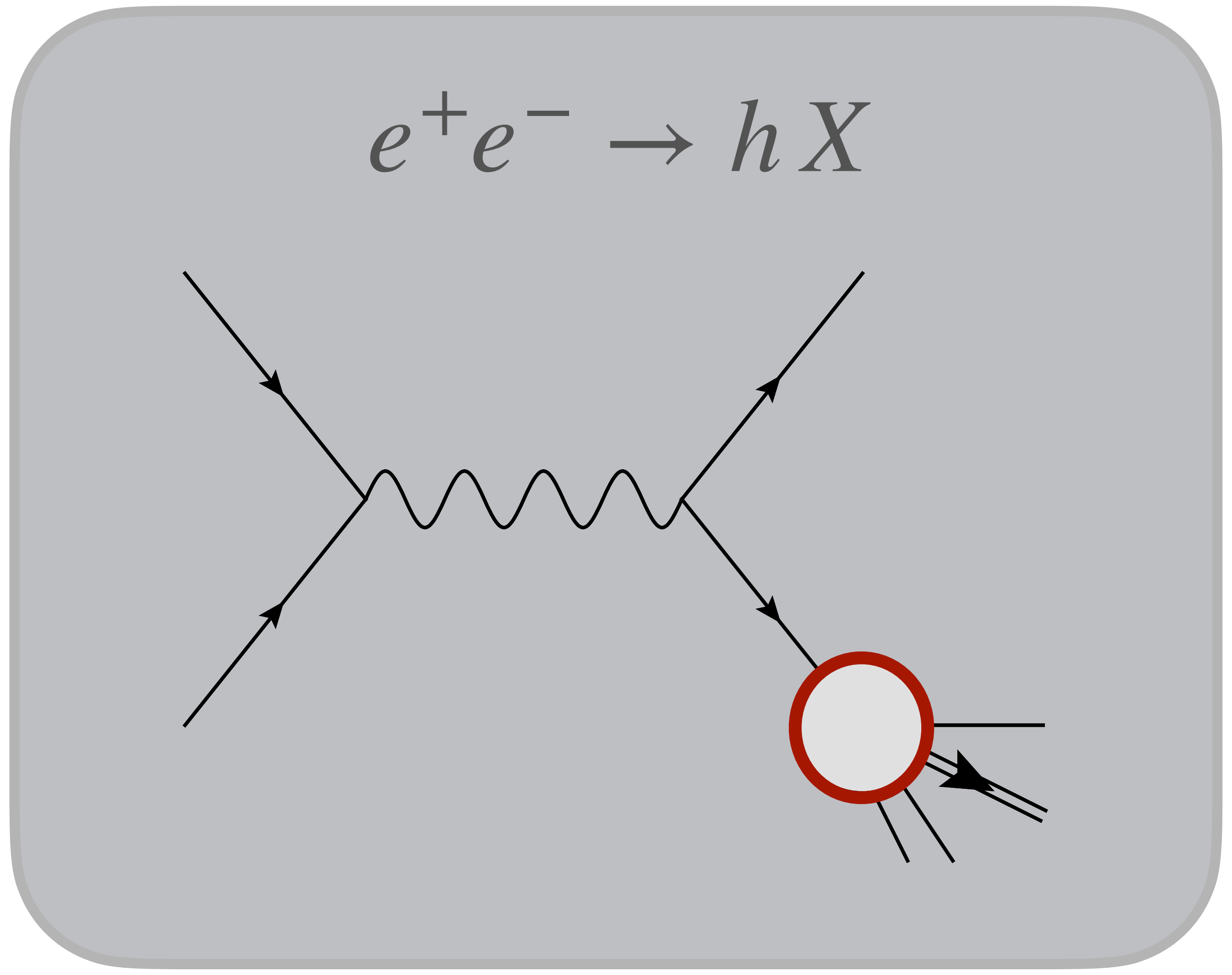
FROM NLO TO NNLO

Parton-to-Pion at NNLO from SIA data



$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma^h}{dz} = \frac{1}{\sigma_{\text{tot}}}\left[\frac{d\sigma_T^h}{dz} + \frac{d\sigma_L^h}{dz}\right]$$

$$\begin{aligned}\frac{d\sigma_k^h}{dz} = \sigma_{\text{tot}}^{(0)}\Big[& D_{\text{S}}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\text{S}}\left(z,\frac{Q^2}{\mu^2}\right) \\ & + D_g^h\left(z,\mu^2\right) \otimes \mathbb{C}_{k,g}^{\text{S}}\left(z,\frac{Q^2}{\mu^2}\right)\Big] \\ & + \sum_q \sigma_q^{(0)}\, D_{\text{NS},q}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\text{NS}}\left(z,\frac{Q^2}{\mu^2}\right)\end{aligned}$$

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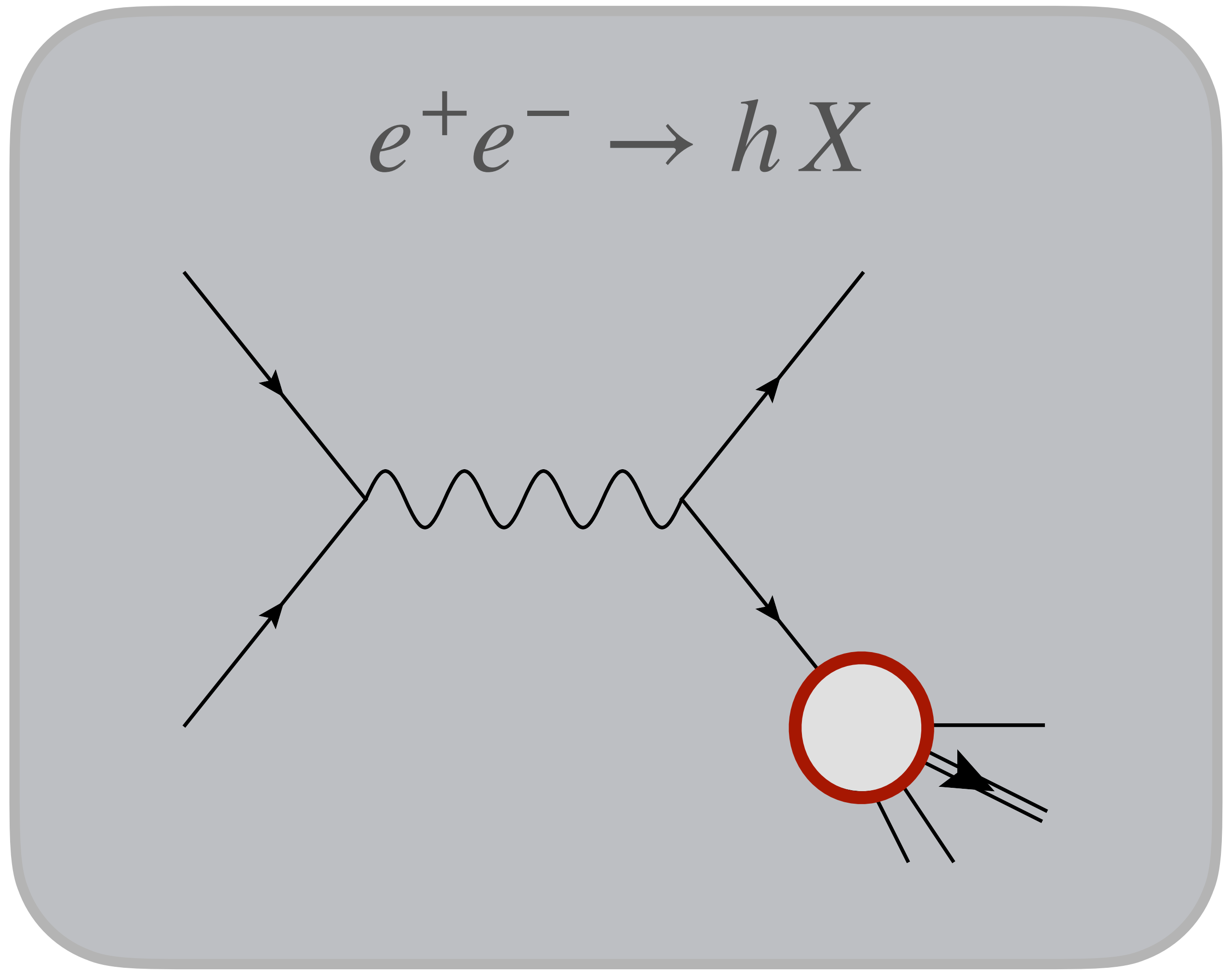

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- Cleanest access to FFs → only non-perturbative quantity
- "Easier" higher order corrections → Coefficient Function known up to NNLO
- Very precise data

FROM NLO TO NNLO

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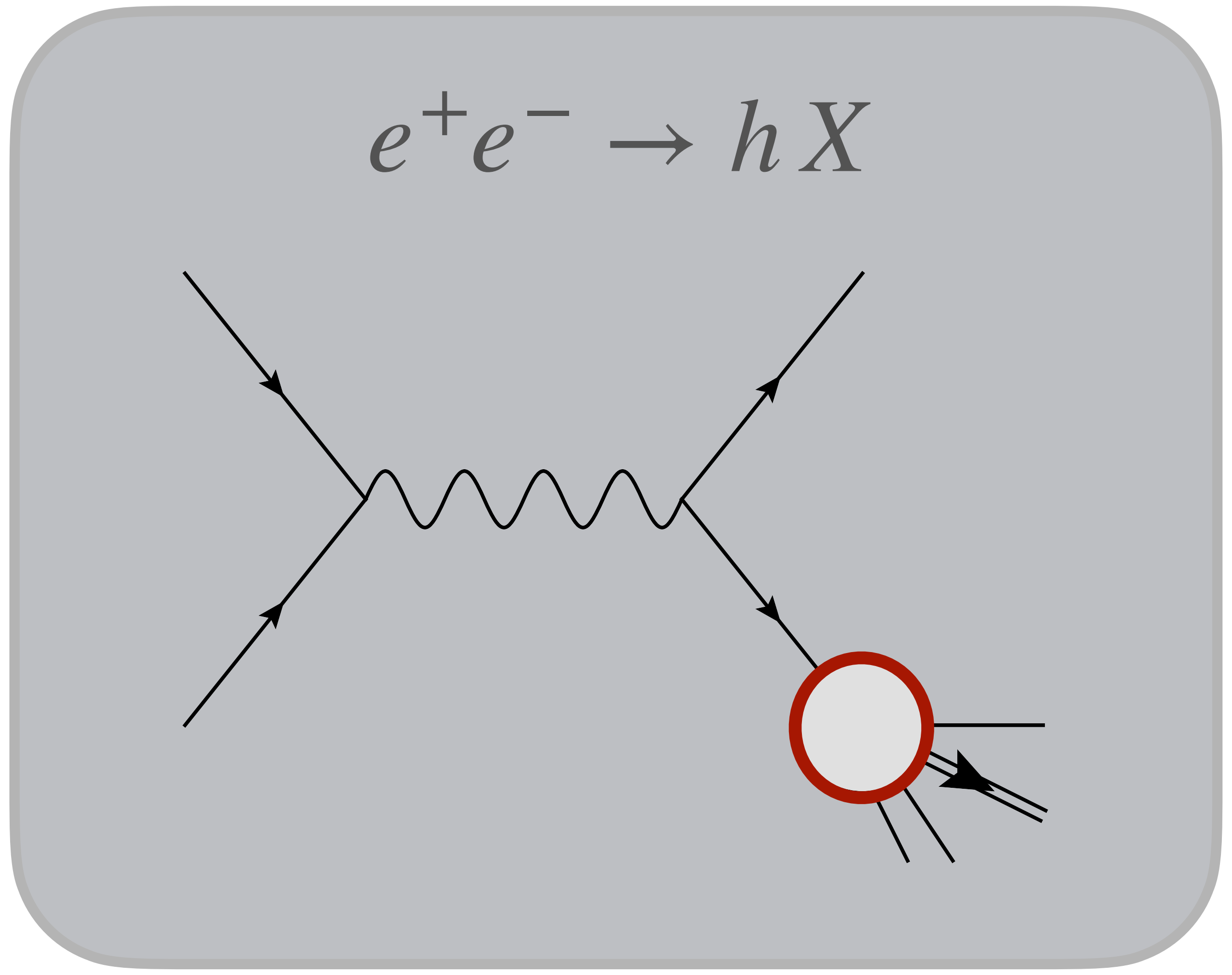

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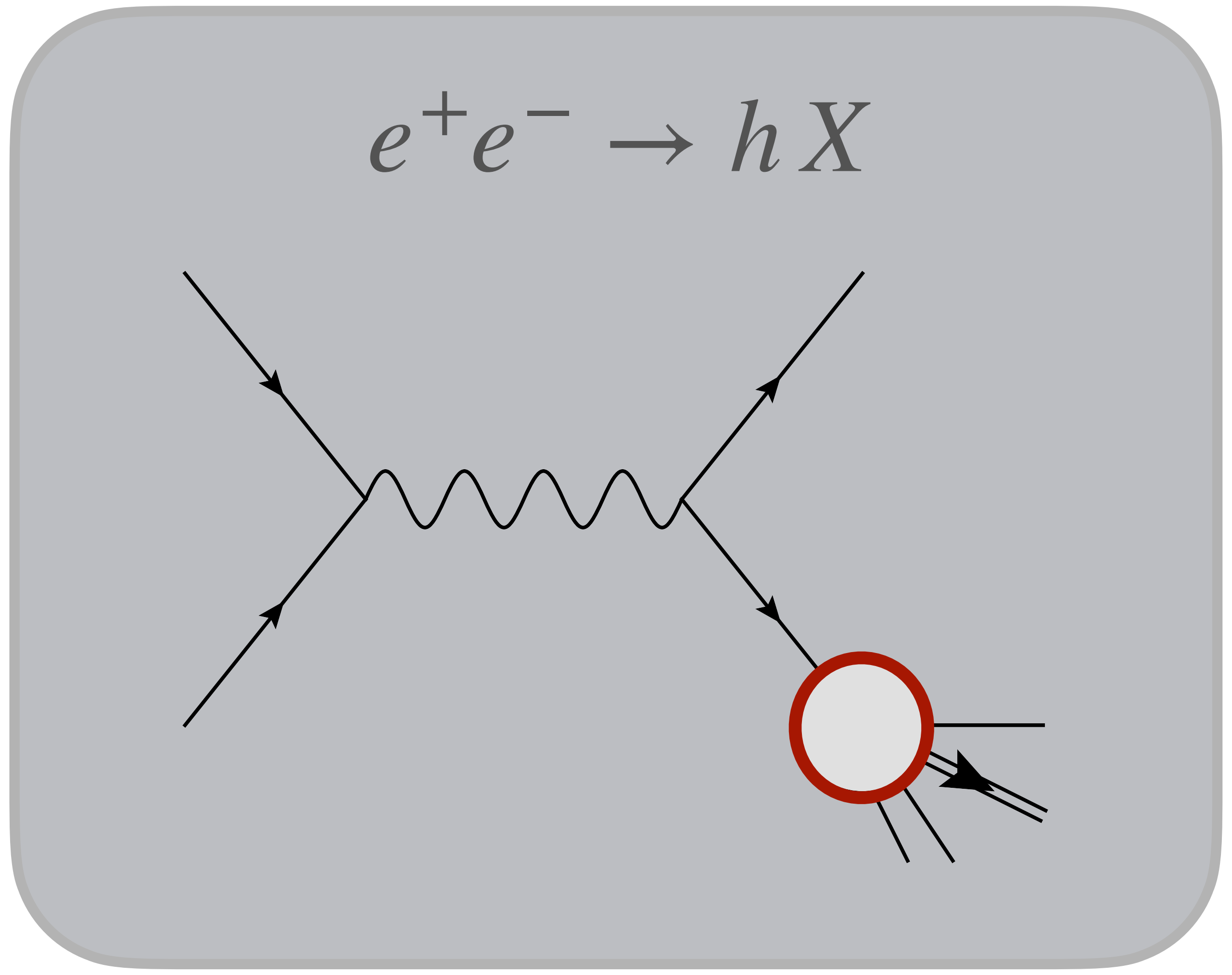
- Cleanest access to FFs → only non-perturbative quantity
- "Easier" higher order corrections → Coefficient Function known up to NNLO
- Very precise data

- Probes the combination $[D_q^h + D_{\bar{q}}^h]$ → limited flavor separation
- Mainly constrain the singlet distribution $D_\Sigma^h = D_u^h + D_{\bar{u}}^h + D_d^h + D_{\bar{d}}^h + D_s^h + D_{\bar{s}}^h + \ldots$
- Gluon fragmentation suppressed in $\alpha_S(Q^2)$ → only indirect constraints through scaling violations
- Weak scale dependence

FROM NLO TO NNLO

Parton-to-Pion at NNLO from SIA data



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Anderle, Ringer, Stratmann. *Phys.Rev.D* 92 (2015) 11, 114017

- Extension of the PEGAUS Evolution Library for FFs evolution: Implementation of time-like evolution kernels, $P_{ij}^{(S)} \to P_{ji}^{(T)}$

  Vogt. Comput.Phys.Commun. 170 (2005) 65-92

- Mellin moments of the NNLO coefficient functions available from

  Mitov, Moch. Nucl.Phys.B 751 (2006) 18-52

FROM NLO TO NNLO

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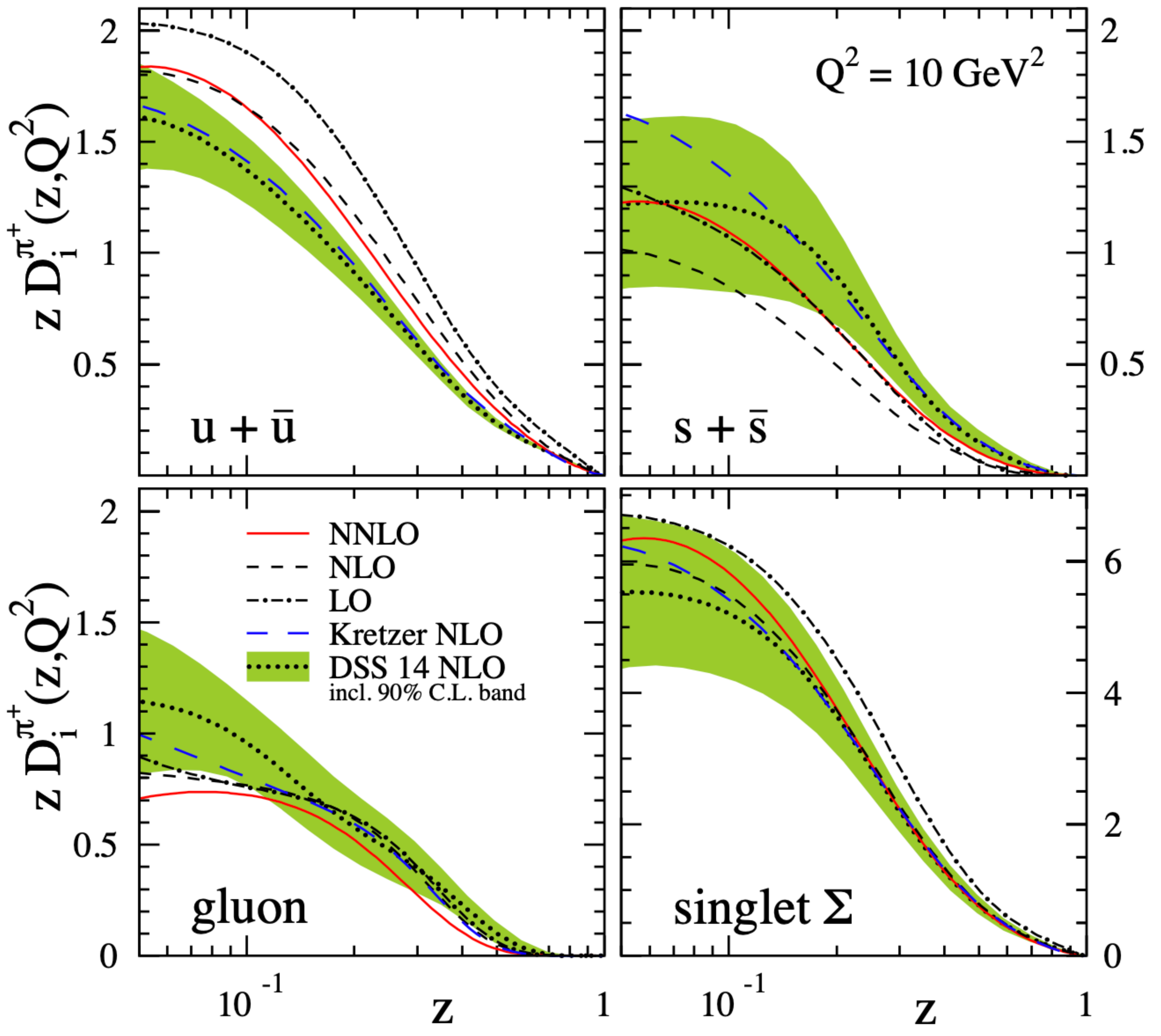
Anderle, Ringer, Stratmann. *Phys.Rev.D* 92 (2015) 11, 114017



experiment	data type	# data in fit	χ^2 LO	NLO	NNLO
SLD [40]	incl.	23	15.0	14.8	15.5
	uds tag	14	9.7	18.7	18.8
	c tag	14	10.4	21.0	20.4
	b tag	14	5.9	7.1	8.4
ALEPH [41]	incl.	17	19.2	12.8	12.6
DELPHI [42]	incl.	15	7.4	9.0	9.9
	uds tag	15	8.3	3.8	4.3
	b tag	15	8.5	4.5	4.0
OPAL [43]	incl.	13	8.9	4.9	4.8
TPC [44]	incl.	13	5.3	6.0	6.9
	uds tag	6	1.9	2.1	1.7
	c tag	6	4.0	4.5	4.1
	b tag	6	8.6	8.8	8.6
BABAR [10]	incl.	41	108.7	54.3	37.1
BELLE [9]	incl.	76	11.8	10.9	11.0
NORM. SHIFTS			7.4	6.8	7.1
TOTAL:		288	241.0	190.0	175.2

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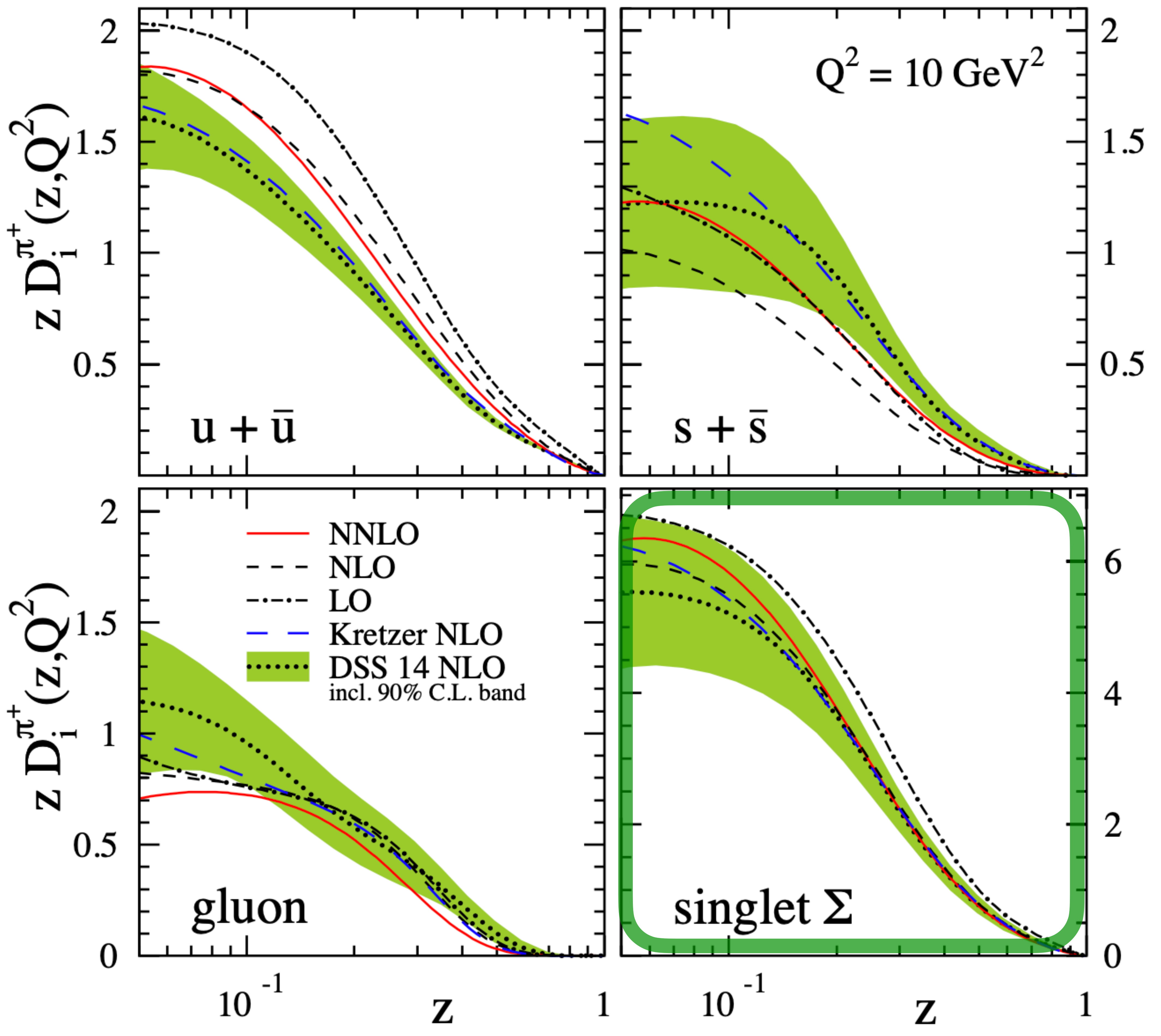

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- Perturbative stability for the FF associated to the singlet combination $D_{\Sigma}^{\pi^+}$
- Weaker constraints $D_{u+\bar{u}}^{\pi^+}$, $D_{s+\bar{s}}^{\pi^+}$ and $D_g^{\pi^+}$

FROM NLO TO NNLO

Parton-to-Pion at NNLO from SIA data

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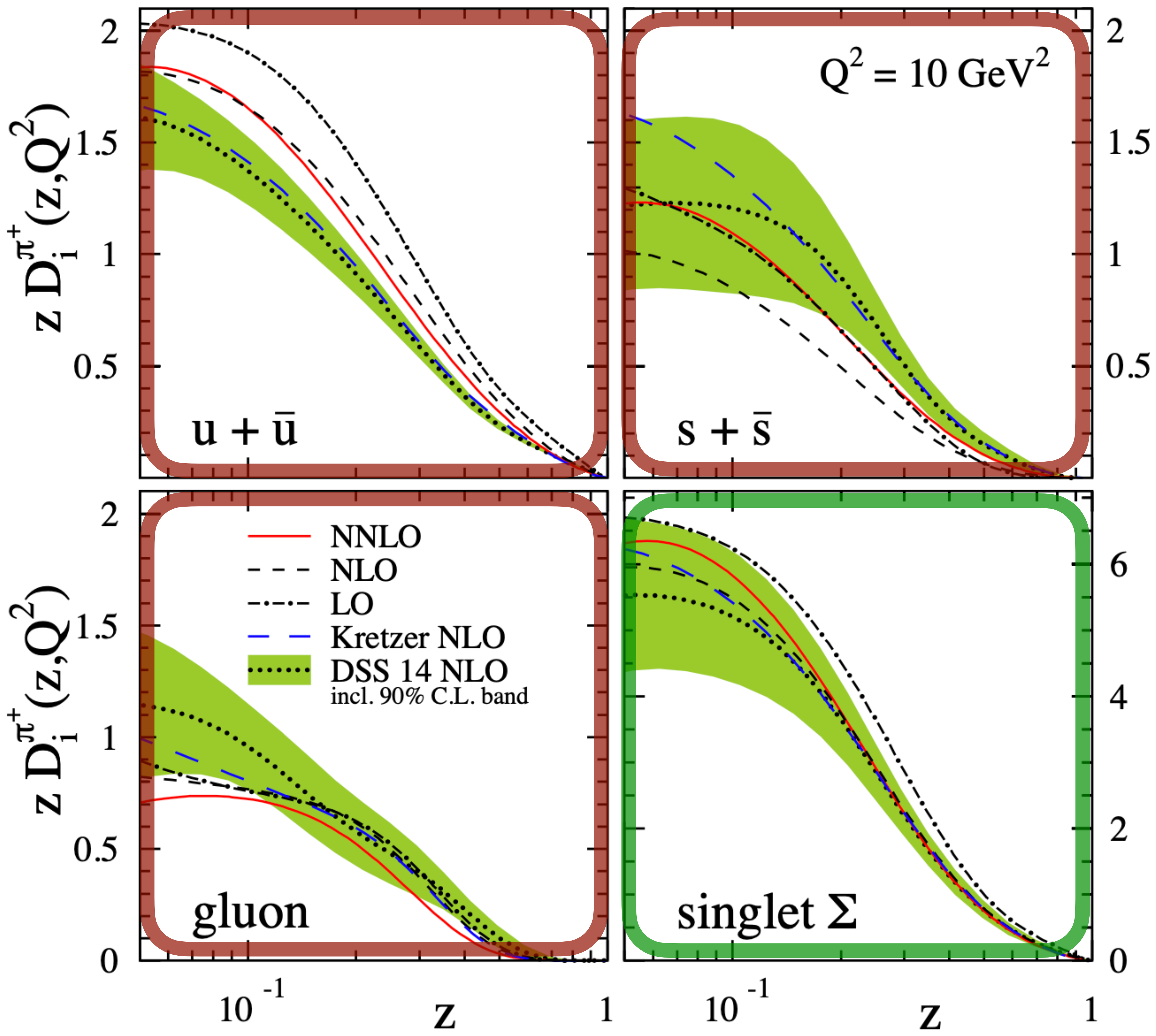

Over-fitting a single data type

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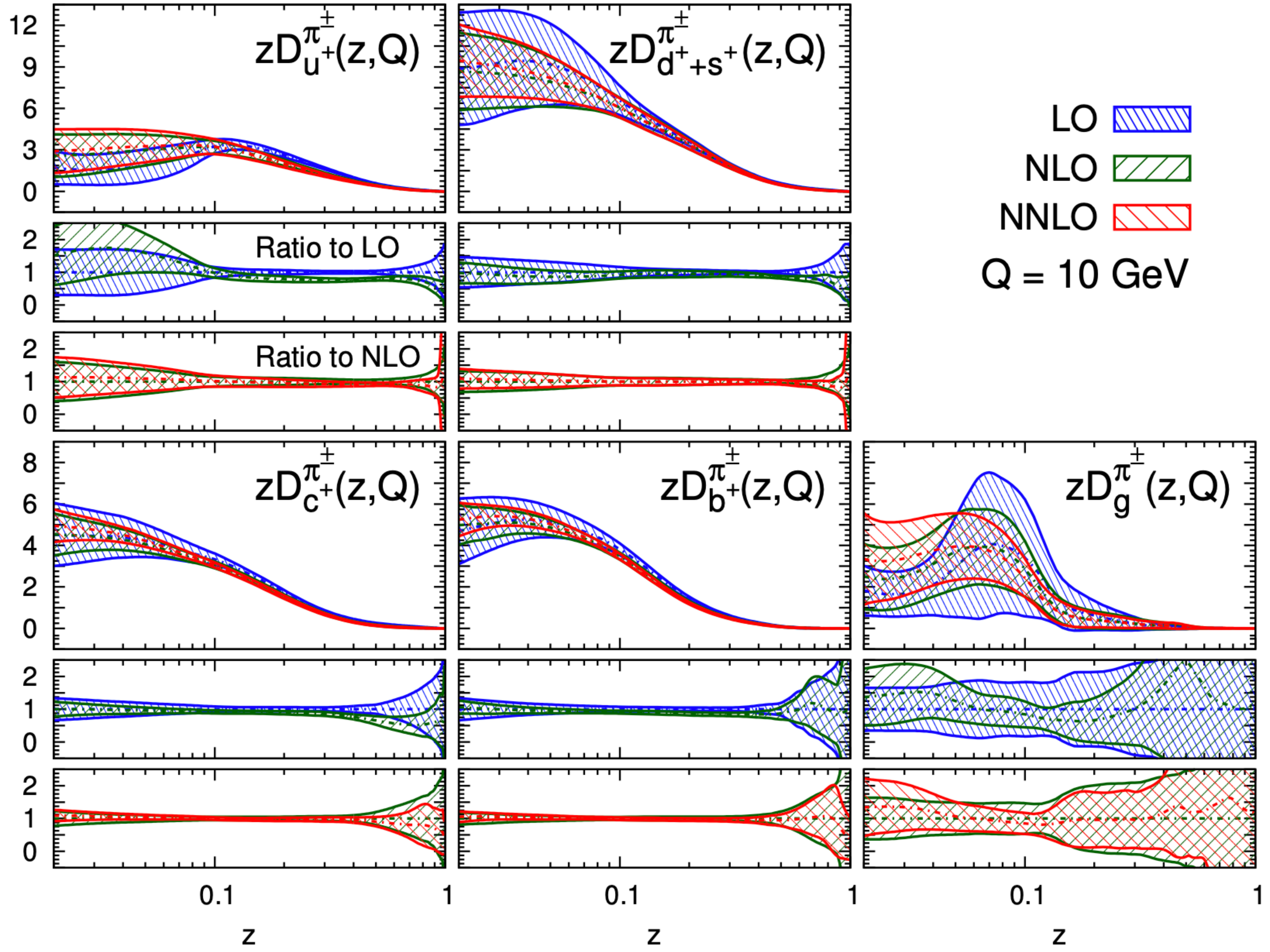
Bertone, Carraza, Hartland, Nocera, Rojo *Eur.Phys.J.C* 77 (2017) 8, 516

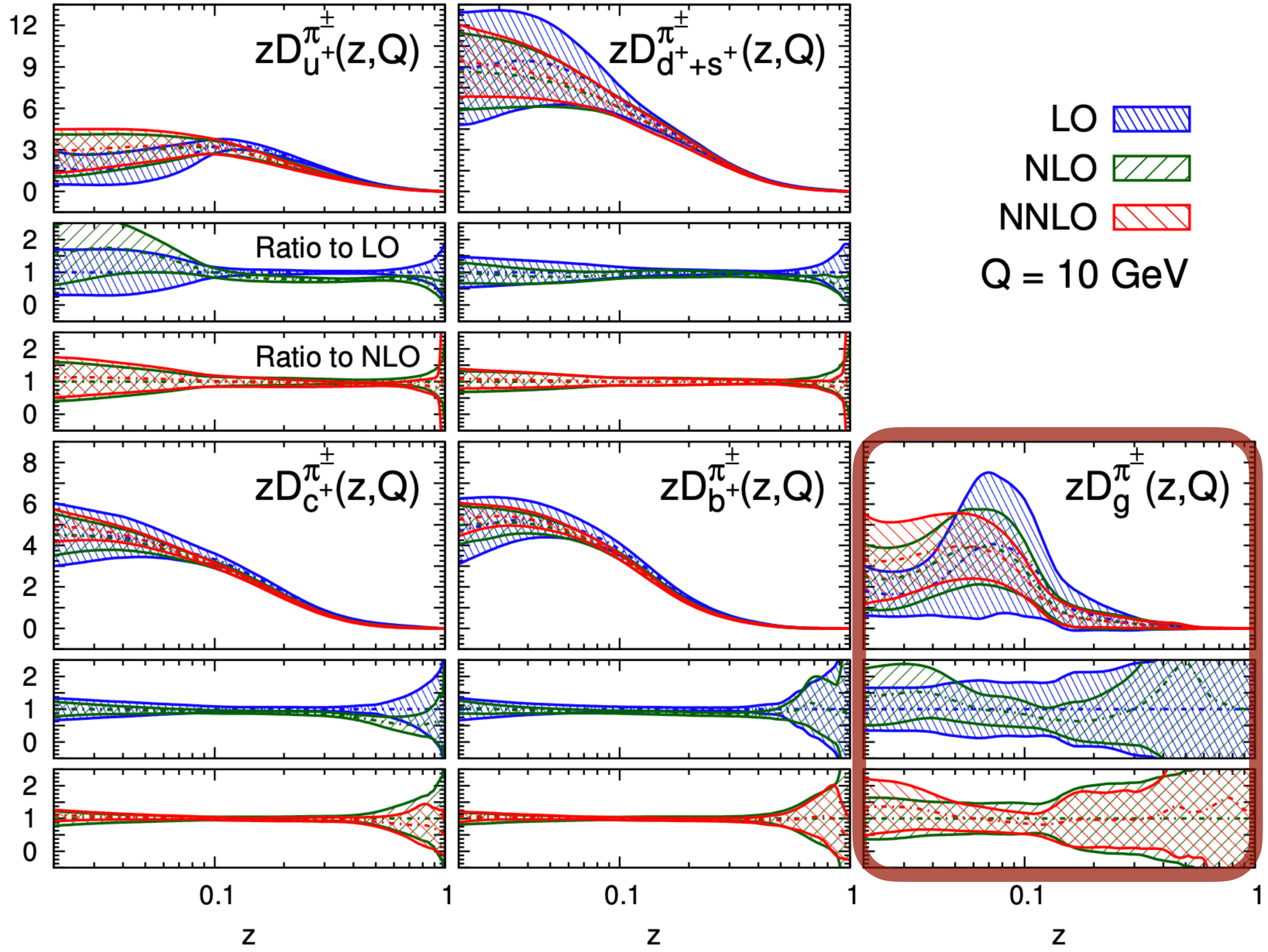


Exp.	$\chi^2/N_{\rm dat}$ $(h=\pi^\pm)$ LO	NLO	NNLO
BELLE	0.60	0.11	0.09
BABAR	1.91	1.77	0.78
TASSO12	0.70	0.85	0.87
TASSO14	1.55	1.67	1.70
TASSO22	1.64	1.91	1.91
TPC (incl.)	0.46	0.65	0.85
TPC (*uds* tag)	0.78	0.55	0.49
TPC (*c* tag)	0.55	0.53	0.52
TPC (*b* tag)	1.44	1.43	1.43
TASSO30	—	—	—
TASSO34	1.16	0.98	1.00
TASSO44	2.01	2.24	2.34
TOPAZ	1.04	0.82	0.80
ALEPH	1.68	0.90	0.78
DELPHI (incl.)	1.44	1.79	1.86
DELPHI (*uds* tag)	1.30	1.48	1.54
DELPHI (*b* tag)	1.21	0.99	0.95
OPAL	2.29	1.88	1.84
SLD (incl.)	2.33	1.14	0.83
SLD (*uds* tag)	0.95	0.65	0.52
SLD (*c* tag)	3.33	1.33	1.06
SLD (*b* tag)	0.45	0.38	0.36
Total dataset	**1.44**	**1.02**	**0.87**

FROM NLO TO NNLO

Parton-to-Pion at NNLO from SIA data

Bertone, Carraza, Hartland, Nocera, Rojo *Eur.Phys.J.C* 77 (2017) 8, 516



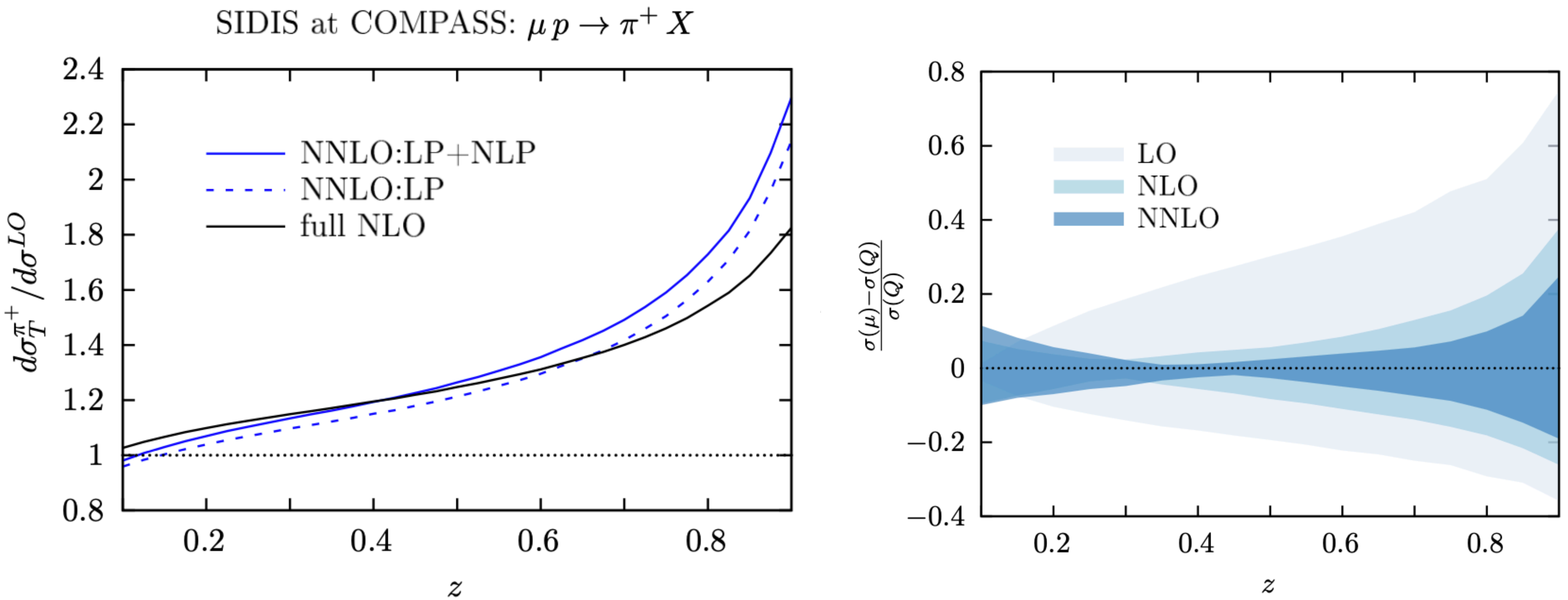
Exp.	$\chi^2/N_{\rm dat}$ $(h=\pi^\pm)$ LO	NLO	NNLO
BELLE	0.60	0.11	0.09
BABAR	1.91	1.77	0.78
TASSO12	0.70	0.85	0.87
TASSO14	1.55	1.67	1.70
TASSO22	1.64	1.91	1.91
TPC (incl.)	0.46	0.65	0.85
TPC (*uds* tag)	0.78	0.55	0.49
TPC (*c* tag)	0.55	0.53	0.52
TPC (*b* tag)	1.44	1.43	1.43
TASSO30	—	—	—
TASSO34	1.16	0.98	1.00
TASSO44	2.01	2.24	2.34
TOPAZ	1.04	0.82	0.80
ALEPH	1.68	0.90	0.78
DELPHI (incl.)	1.44	1.79	1.86
DELPHI (*uds* tag)	1.30	1.48	1.54
DELPHI (*b* tag)	1.21	0.99	0.95
OPAL	2.29	1.88	1.84
SLD (incl.)	2.33	1.14	0.83
SLD (*uds* tag)	0.95	0.65	0.52
SLD (*c* tag)	3.33	1.33	1.06
SLD (*b* tag)	0.45	0.38	0.36
Total dataset	**1.44**	**1.02**	**0.87**

FROM NLO TO NNLO

Approximate NNLO corrections for SIDIS

Abele, de Florian, Vogelsang. *Phys.Rev.D* 104 (2021) 9, 094046

Dominant contribution associated to soft gluon emission as $x, z \sim 1$



- Significant corrections for large z
- Sizable NLP contribution
- Reduction of the scale dependence for $z \gtrsim 0.3$

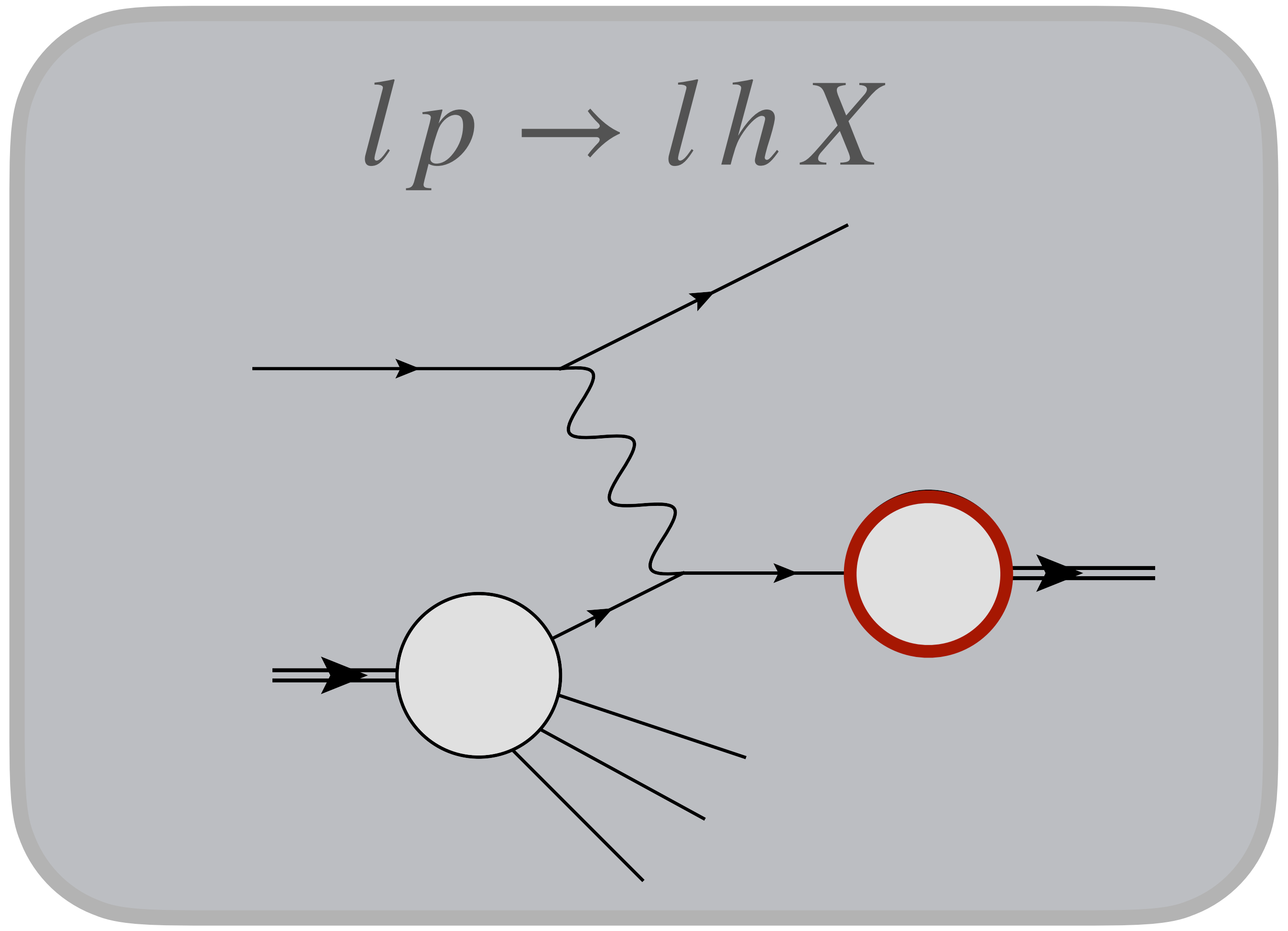
Suitable for numerical implementation in Mellin space

$$\mathcal{F}_i^h(x,z,Q^2) = \int_{\mathcal{C}_M} \frac{dM}{2\pi i}\, z^{-M} \sum_{f,f'} \tilde{D}^h_{f'}(M,\mu_F) \int_{\mathcal{C}_N} \frac{dN}{2\pi i}\, x^{-N}\, \tilde{\omega}^i_{f'f}\left(N, M, \alpha_s(\mu_R), \frac{\mu_R}{Q}, \frac{\mu_F}{Q}\right) \tilde{f}(N,\mu_F)$$

MELLIN INVERSION — FIT — PRECALCULATED GRIDS

Analytical moments of PDFs (NNPDF4.0)

FROM NLO TO NNLO



$$\frac{d\sigma^H}{dx\,dQ^2\,dz_H} = \frac{2\pi\alpha^2}{xsQ^2}\left[\frac{1+(1-y)^2}{y}2F_1^H(x,z_H,Q^2) + \frac{2(1-y)}{y}F_L^H(x,z_H,Q^2)\right]$$

$$F_1^H(x,z,Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2 f_q(x,Q^2) D_q(z,Q^2)$$

$$+\frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2[f_q \otimes C_{qq} \otimes D_q^H + f_q \otimes C_{gq} \otimes D_g^H + f_g \otimes C_{qg} \otimes D_q^H]$$

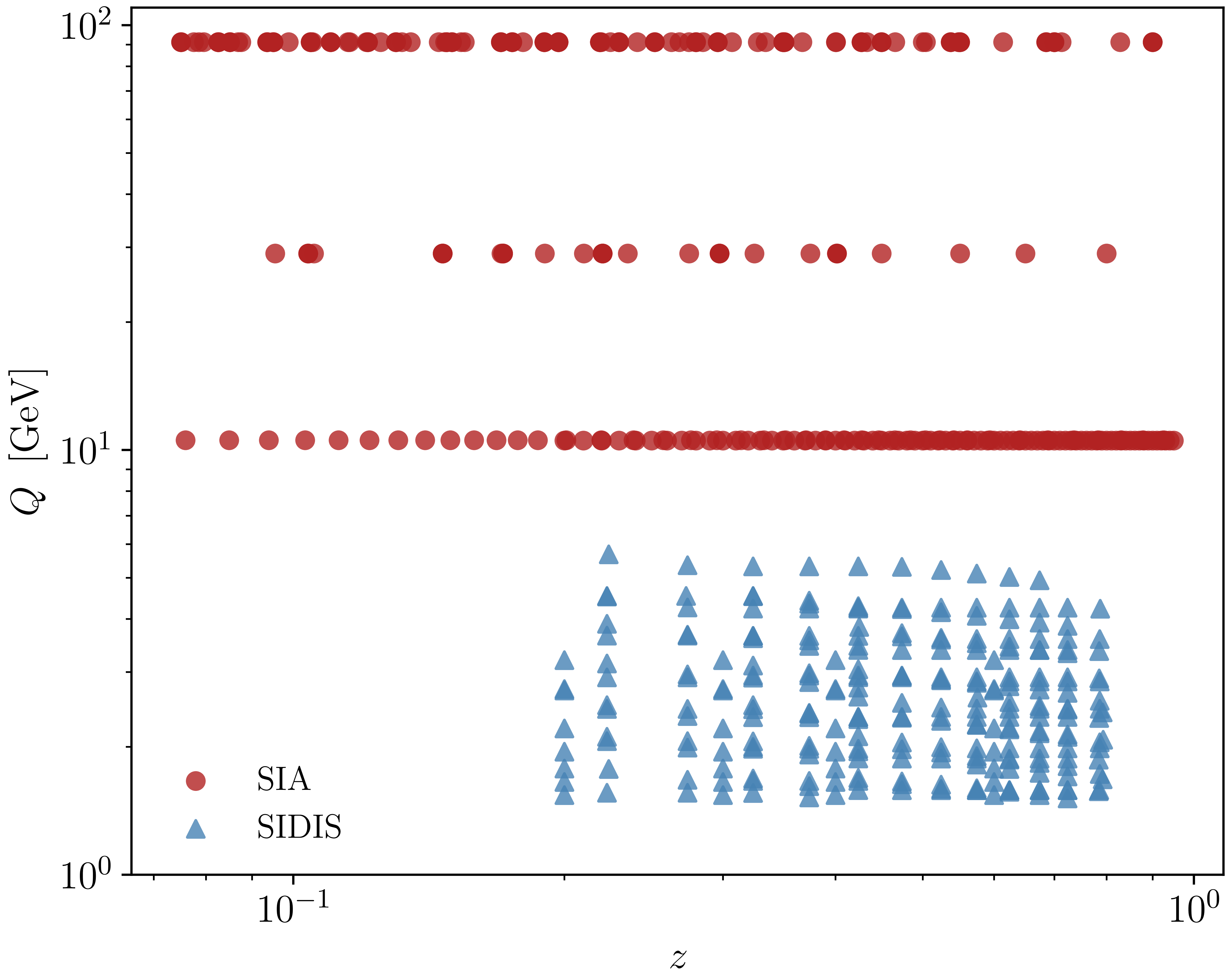
Effective charge

- PDFs act as effective charges → allows for flavor and charge separation
- Different targets and x can be used
- Probe lower values of Q^2 → improve constraints on D_g^h
- Approximate NNLO corrections

- Dependence on the PDFs
- Gluon fragmentation suppressed in $\alpha_S(Q^2)$ → only indirect constraints through scaling violations
- Additional corrections at small Q^2 ?

EXPERIMENTAL DATA

experiment		data type
TPC	29 GeV	incl.
		uds, c, b tag
SLD	91.2 GeV	incl.
		uds, c, b tag
ALEPH	91.2 GeV	incl.
DELPHI	91.2 GeV	incl.
		uds, b tag
OPAL	91.2 GeV	incl.
		u, d, s, c, b tag
BABAR	10.54 GeV	incl.
BELLE	10.52 GeV	incl.
SIA data (sum)		
HERMES	π^+, π^-	(p-Q^2)
	π^+, π^-	(d-Q^2)
	π^+, π^-	(p-x)
	π^+, π^-	(d-x)
COMPASS	π^+, π^-	(d-z)
SIDIS data (sum)		



$$0.075 < z_{\rm SIA} < 0.95$$

$$Q^2_{\rm SIDIS} > 1.5\,{\rm GeV}^2$$

DSS FRAMEWORK

Parametrization for $D_i^h(z, Q_0)$ at input scale Q_0

$$D_i^{\pi^+}(z, \mu_0) = N_i \, z^{\alpha_i} \sum_j^3 \gamma_{ij}(1-z)^{\beta_{ij}}$$

Evolve to the relevant scale using DGLAP Equations

Calculate χ^2

$$\chi^2 = \sum_j^{\text{sets}} \sum_j^{\text{points}} \left(\frac{\sigma_{ji}^{\text{exp}} - \mathcal{N}_j \sigma_{ji}^{\text{exp}}}{\Delta \sigma_{ji}^{\text{exp}}}\right)^2 - \sum_j \left(\frac{1-\mathcal{N}_j}{\Delta \mathcal{N}_j^{\text{exp}}}\right)^2$$

χ^2 minimum?

No → Change parameters

Yes → Optimum set of parameters

+ Prescription for uncertainties

At initial scale

$Q_0^2 = 1$ GeV^2 u,d,s

$Q_0^2 = m_q^2$ c,b

Disscard redundant parameters

$$D_{\bar{u}}^{\pi^+} = D_d^{\pi^+}$$

$$D_{d+\bar{d}}^{\pi^+} = N_{d+\bar{d}} \, D_{u+\bar{u}}^{\pi^+}$$

$$D_{\bar{s}}^{\pi^+} = D_s^{\pi^+} = N_s \, z^{\alpha_s} D_{\bar{u}}^{\pi^+}$$

$$\gamma_{g2} = \gamma_{g3} = 0$$

(Simpler parametrization for $D_s^{\pi^+}$ and $D_g^{\pi^+}$)

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Extension of PEGASUS libraries to evolve FFs (solved in N-space)

α_S from LHAPDF libraries (MSHT20 / NNPDF4.0)

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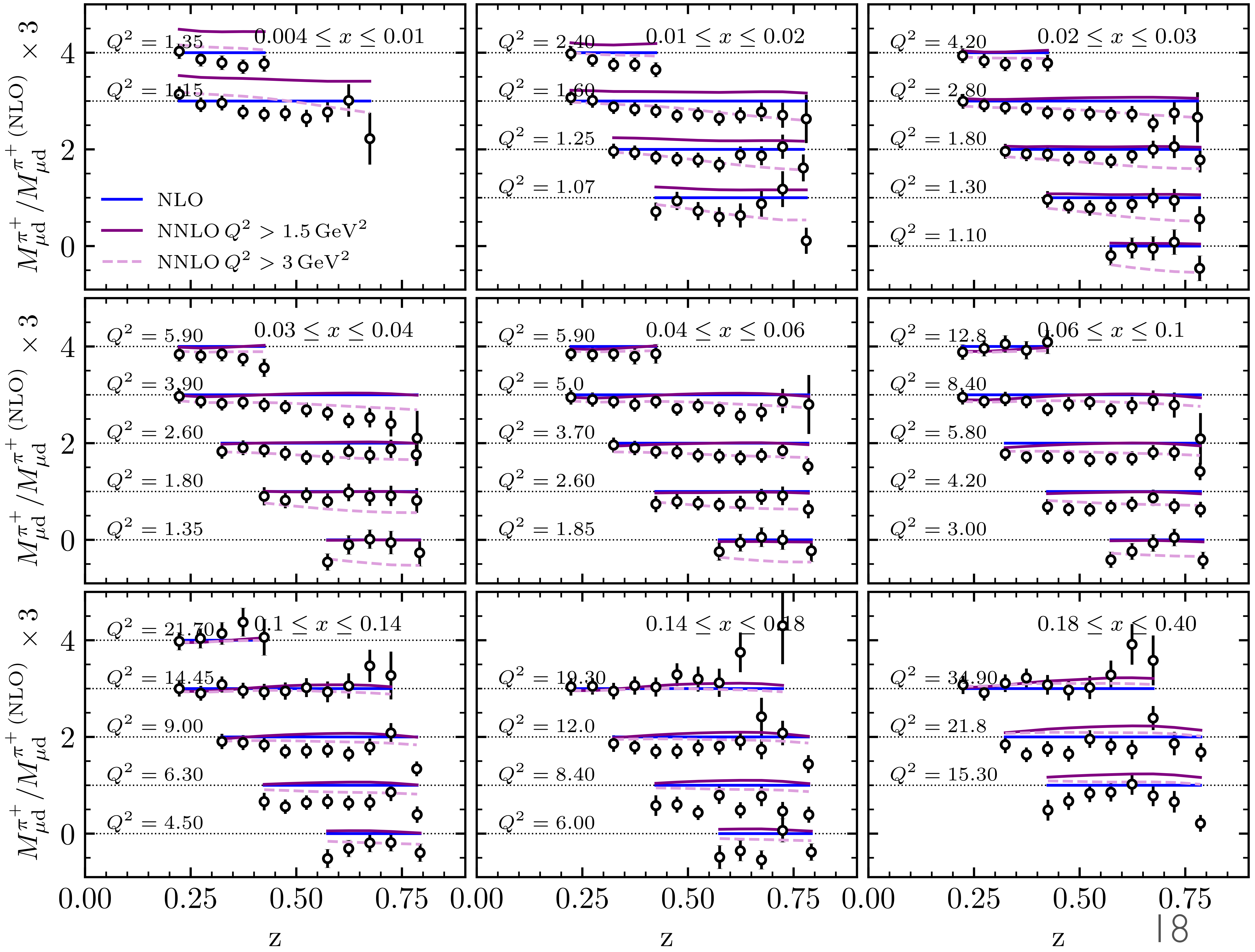
α_S from LHAPDF libraries (MSHT20 / NNPDF4.0)

Fast evaluation in Mellin N-space

Monte-Carlo sampling to estimate FFs uncertainties

RESULTS - SIDIS FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$		
	#data	NLO	NNLO
SIA	288	1.05	0.96
COMPASS	510	0.98	1.14
HERMES	224	2.24	2.27
TOTAL	1022	1.27	1.33



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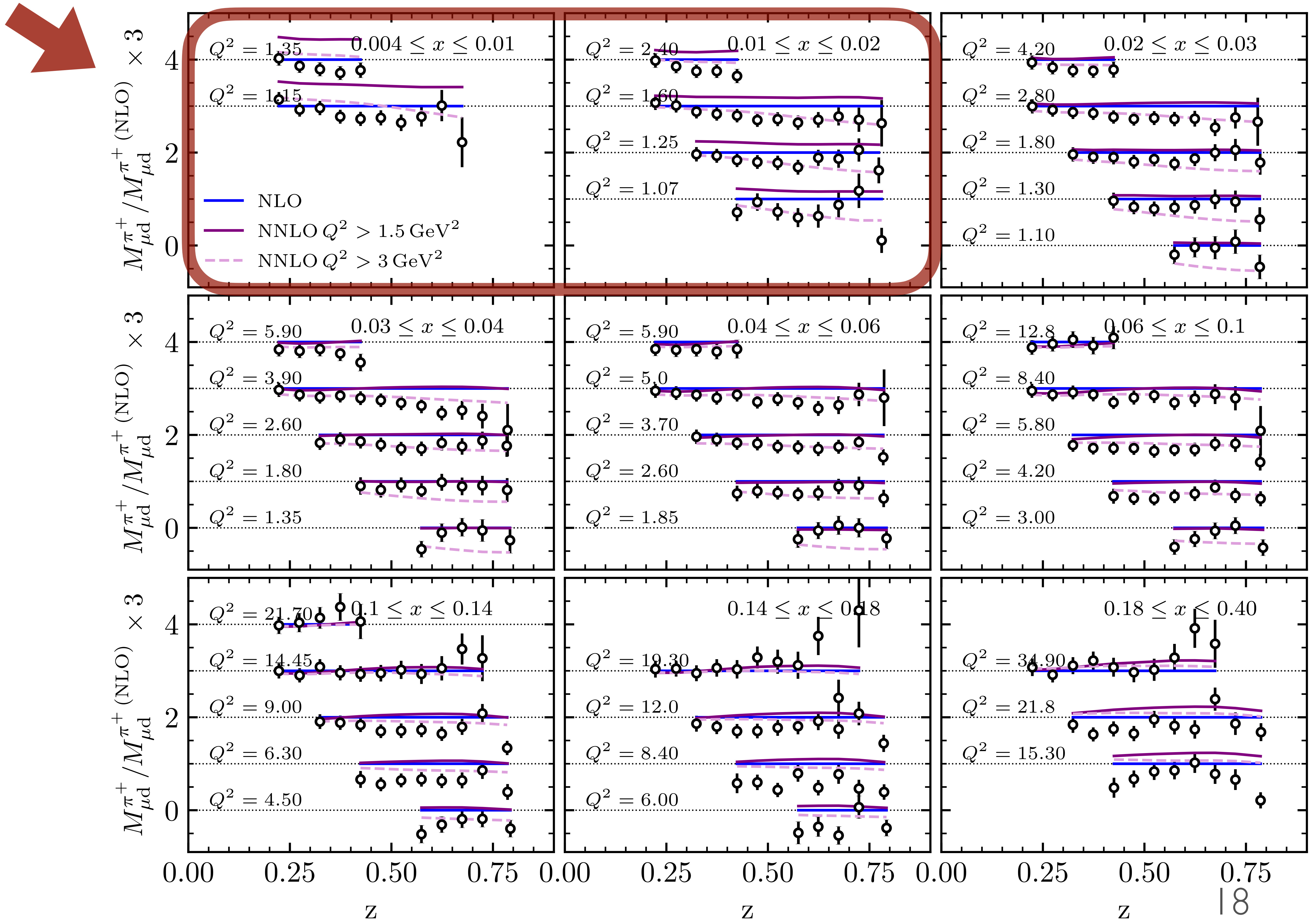
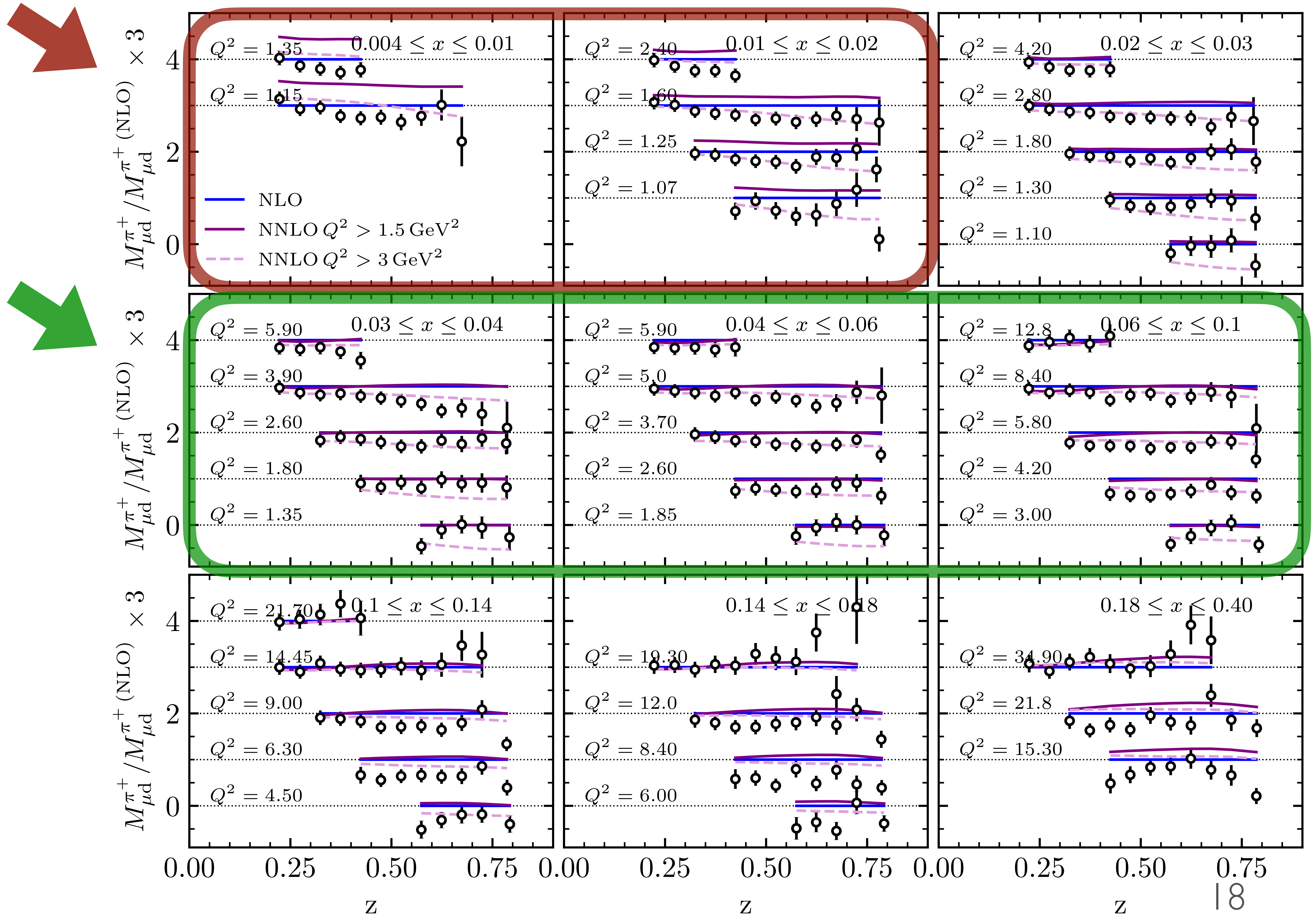


- For $Q^2 \geq 1.5$ GeV^2, better fit quality at NLO →NNLO overestimation of the multiplicities for lower values of x

RESULTS - SIDIS FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07



- For $Q^2 \geq 1.5$ GeV^2, better fit quality at NLO →NNLO overestimation of the multiplicities for lower values of x
- Progressive NNLO improvement as the Q^2-cut is increased
- Better NNLO description of data from $Q^2 \geq 3$ GeV^2

RESULTS - SIDIS FIT QUALITY

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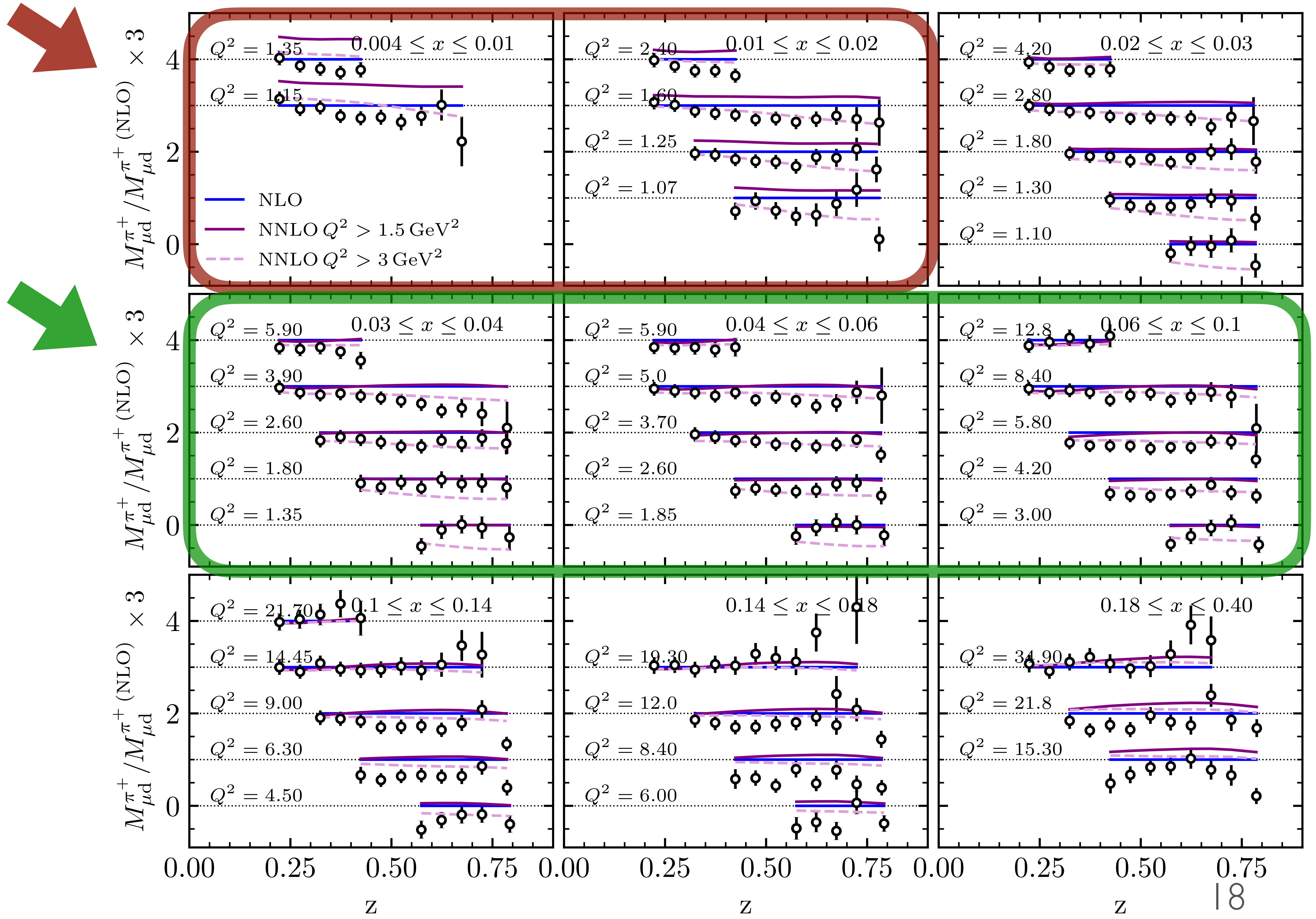

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- Power corrections?
- Extrapolation of PDFs beyond the data-constrained region
- Extrapolation of the approximate NNLO corrections to low x

RESULTS - SIDIS FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
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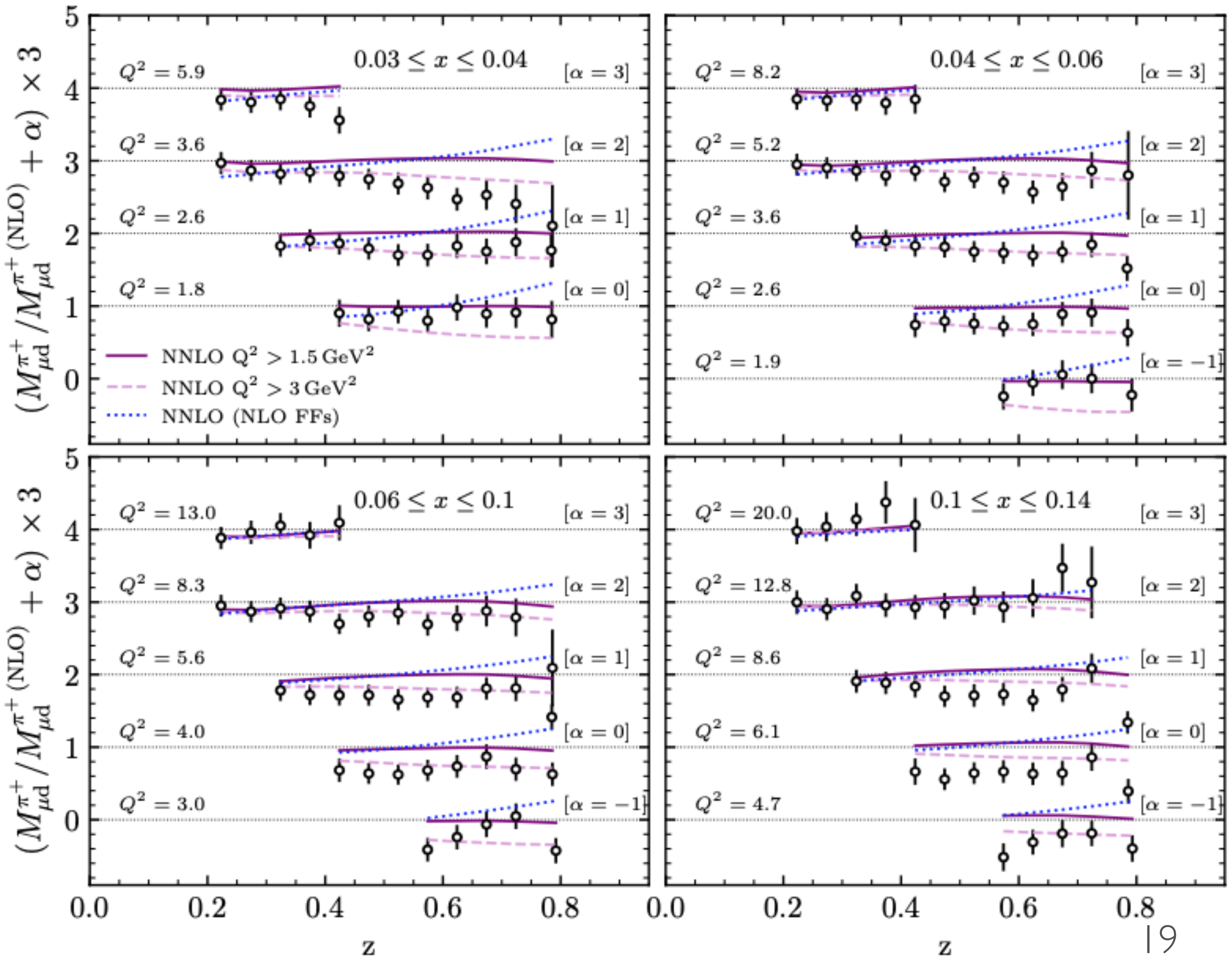

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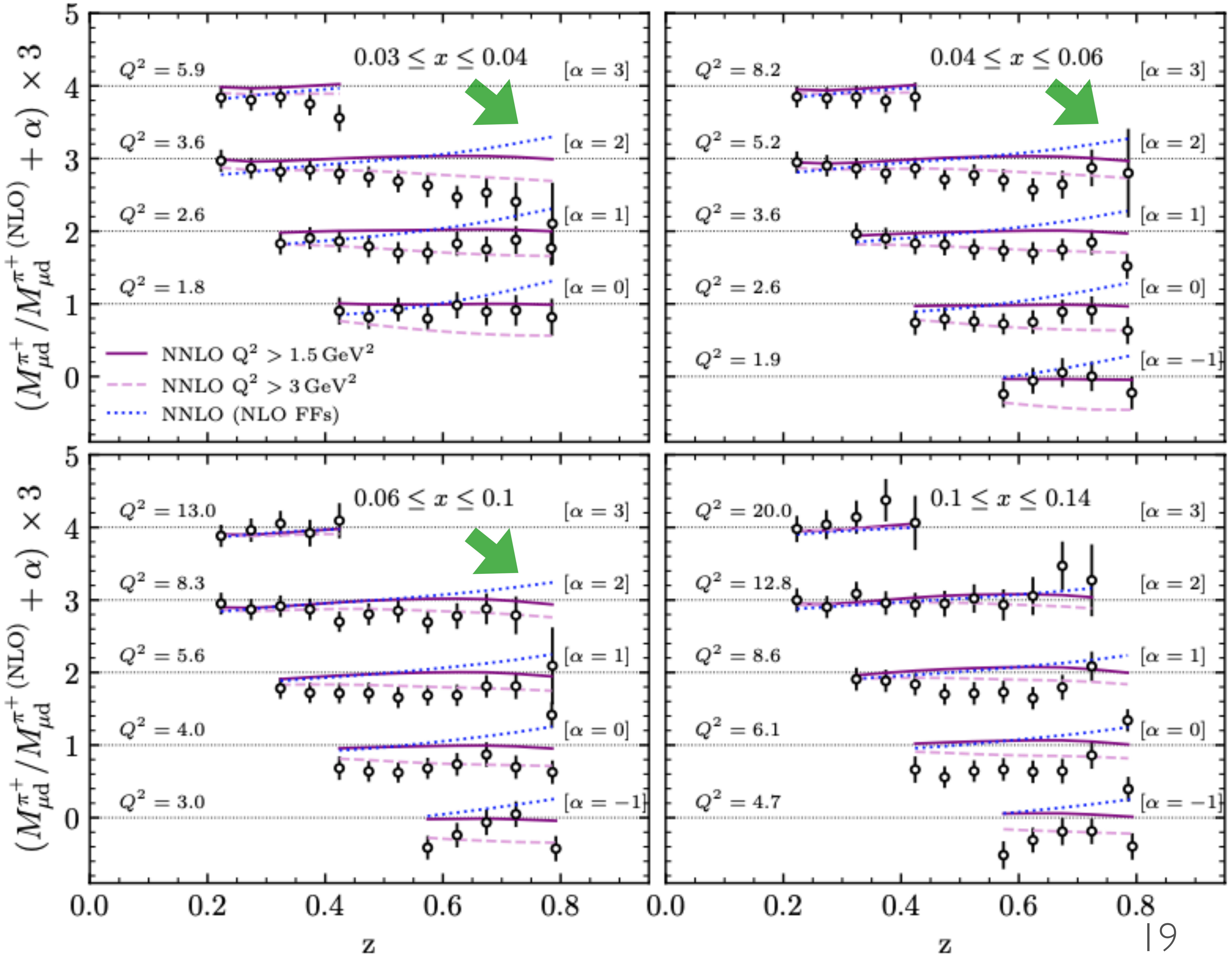

- For $Q^2 \geq 1.5$ GeV^2, better fit quality at NLO →NNLO overestimation of the multiplicities for lower values of x
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- Better NNLO description of data from $Q^2 \geq 3$ GeV^2

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RESULTS - SIDIS FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
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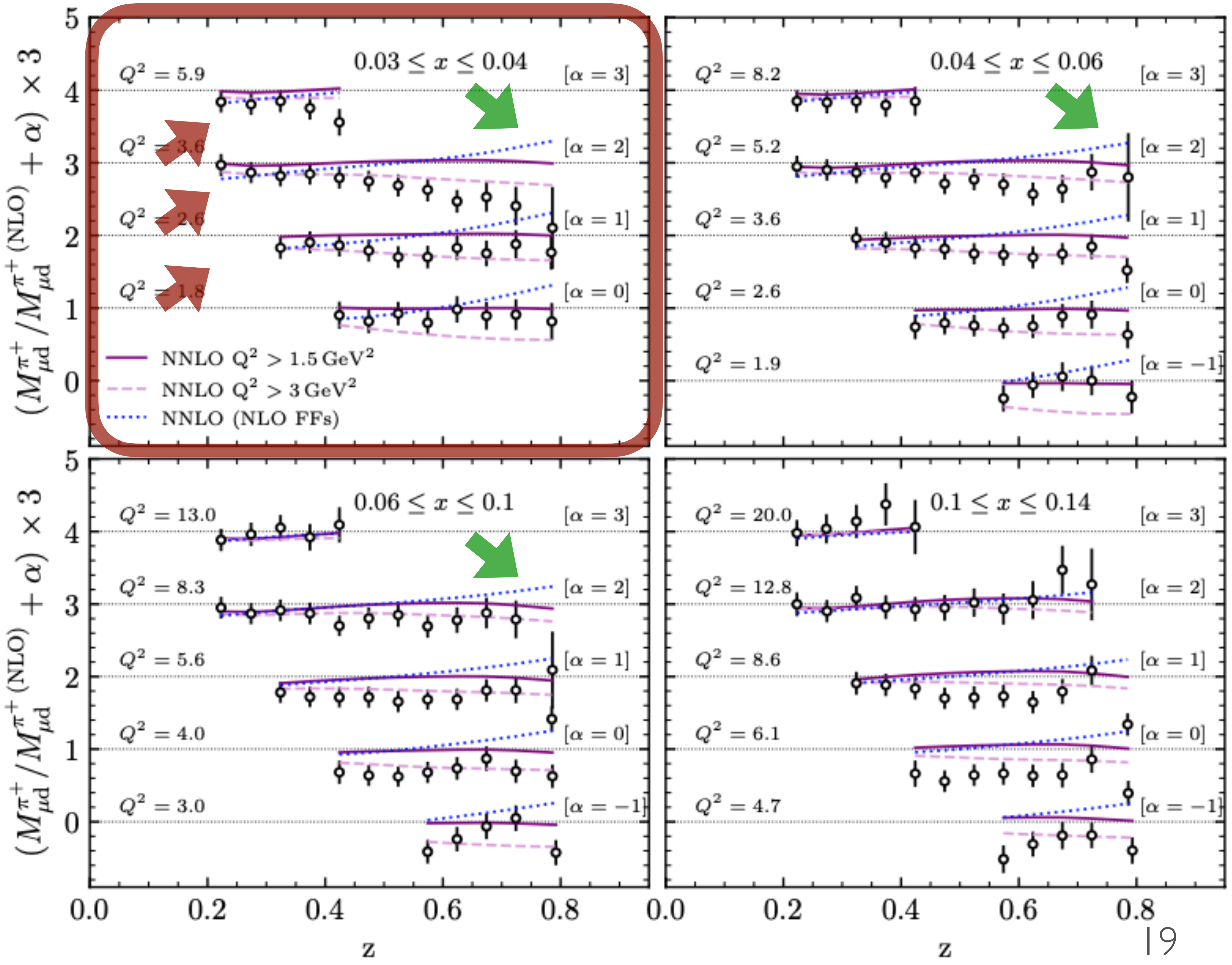

- For $Q^2 \geq 1.5$ GeV^2, better fit quality at NLO →NNLO overestimation of the multiplicities for lower values of x
- Progressive NNLO improvement as the Q^2-cut is increased
- Better NNLO description of data from $Q^2 \geq 3$ GeV^2

- Power corrections?
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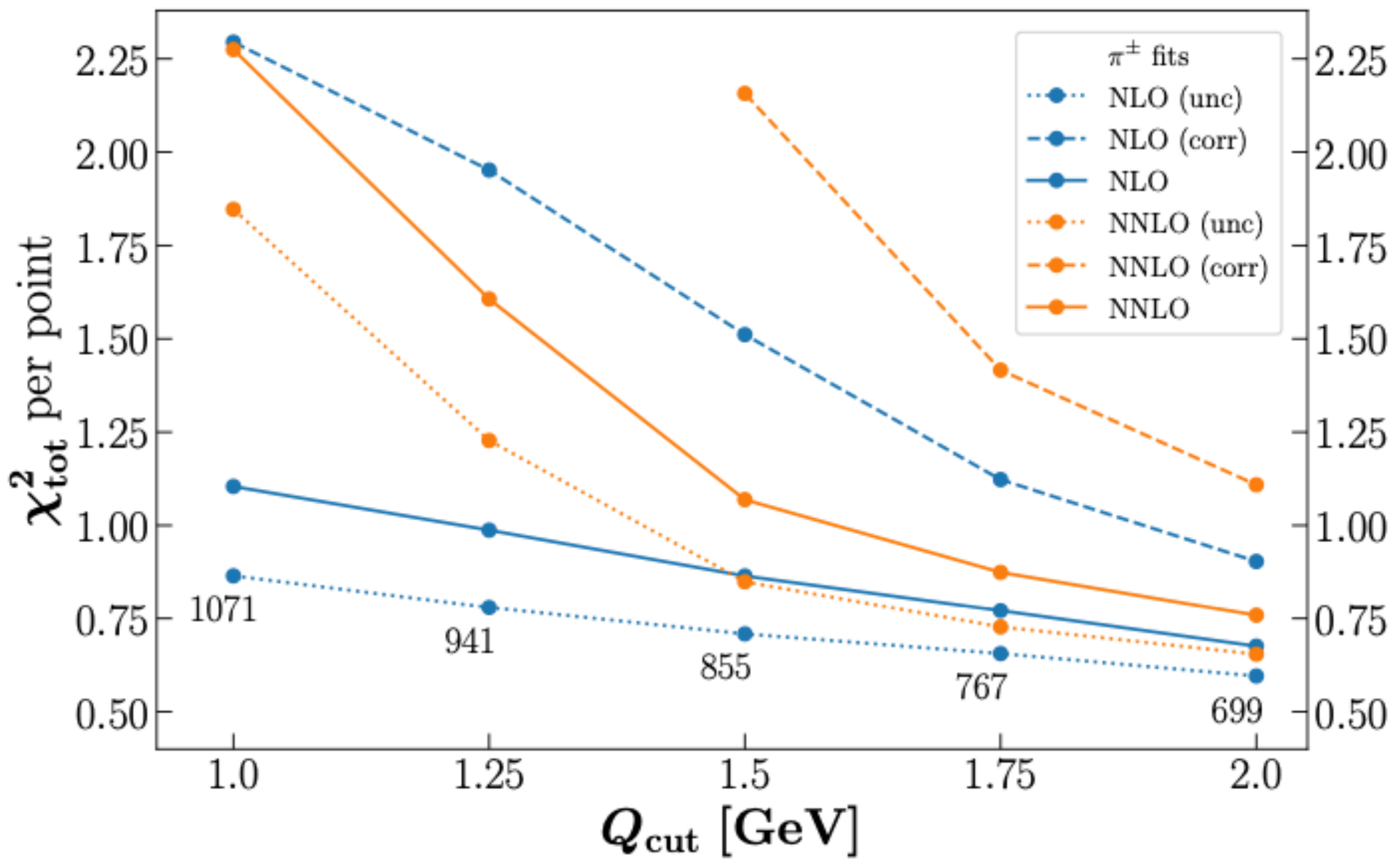
- Extrapolation of the approximate NNLO corrections to low x

RESULTS - SIDIS FIT QUALITY

Recent set of NNLO FFs ($\pi^{\pm}$ & $K^{\pm}$) by the MAP Collaboration

Khalek, Bertone, Khoudli, Nocera. *arXiv* 2204.10331



- For pions FFs, similar deterioration of χ^2 as Q^2-cut is lowered, with a steeper increase in the case of NNLO
- For higher Q^2-cut values, similar χ^2 at NLO and NNLO

- Power corrections?
- Extrapolation of PDFs beyond the data-constrained region
- Extrapolation of the approximate NNLO corrections to low x

RESULTS - SIDIS FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
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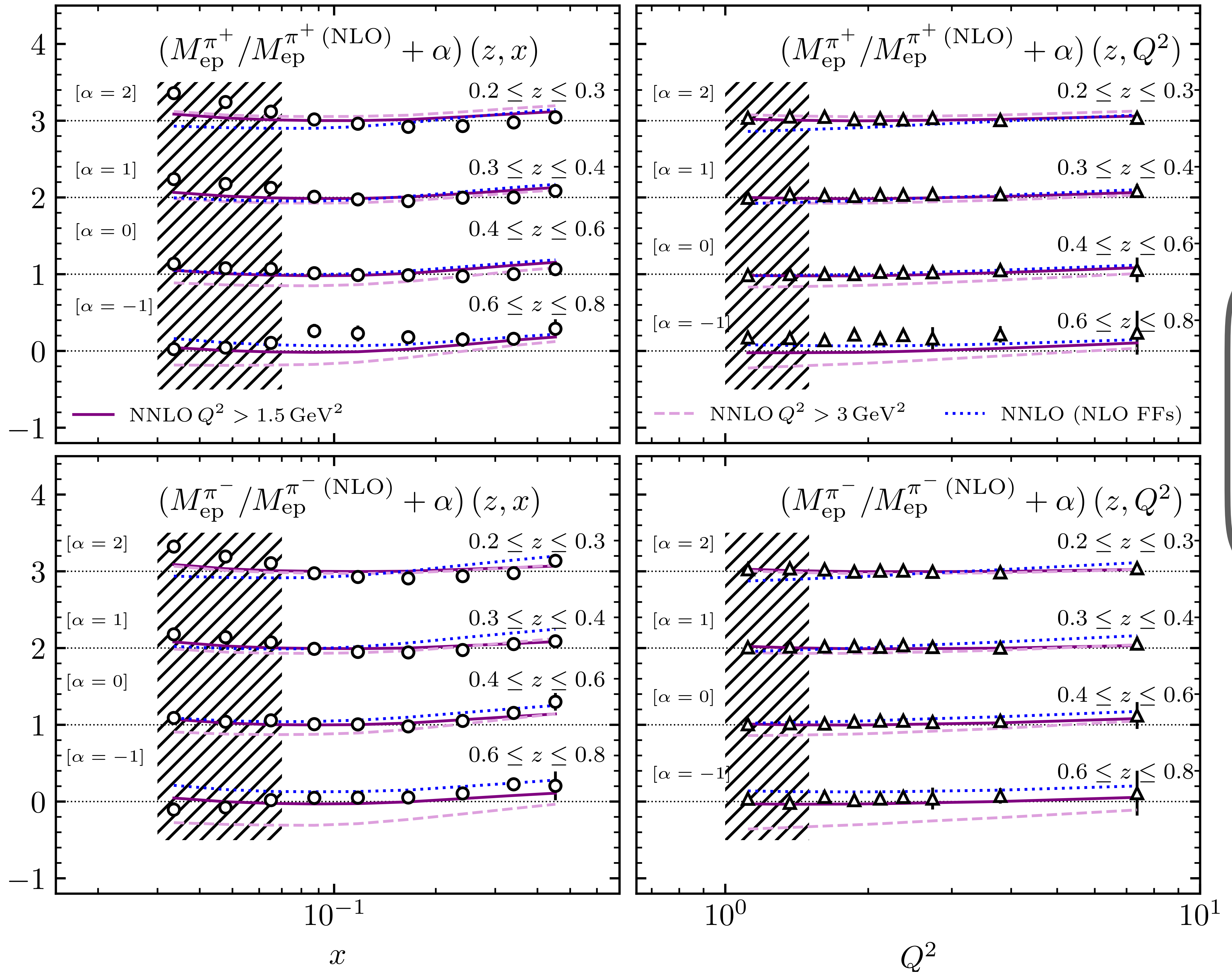

- Same pattern of NNLO correction for HERMES
- No significant improvement in χ^2/d.o.f increasing Q^2-cut, since the fit mainly accommodates COMPASS data

RESULTS - SIA FIT QUALITY

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
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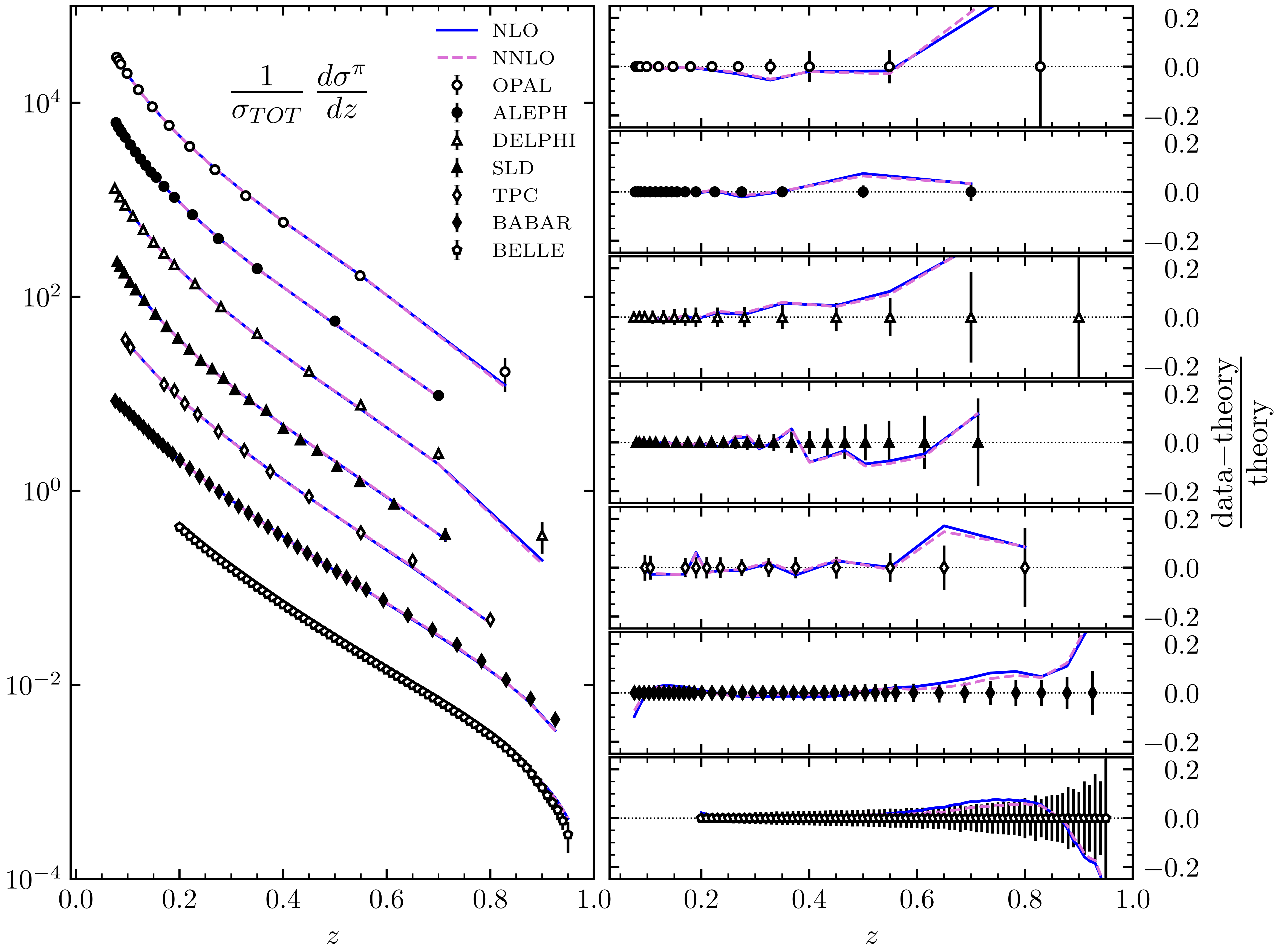
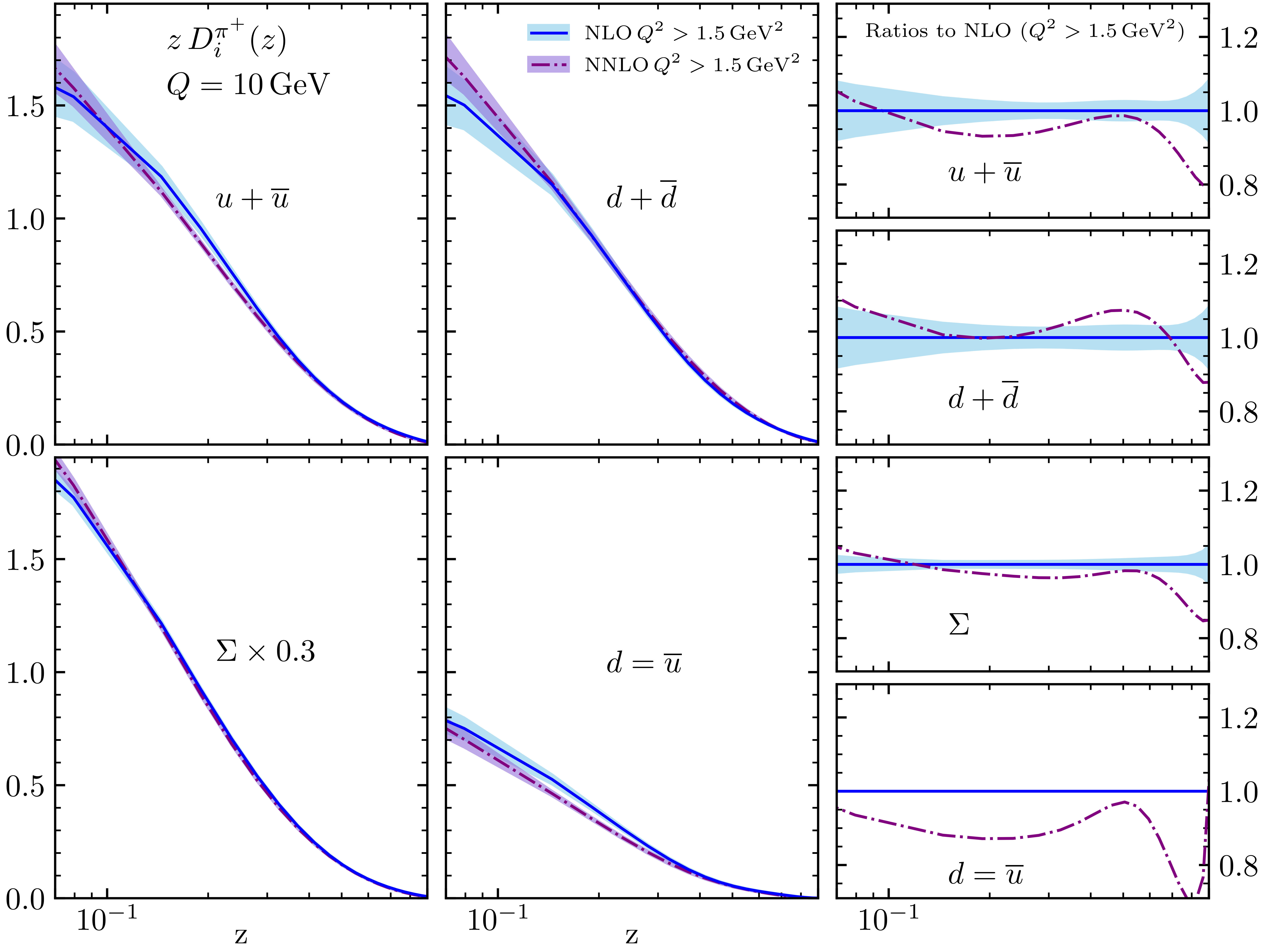
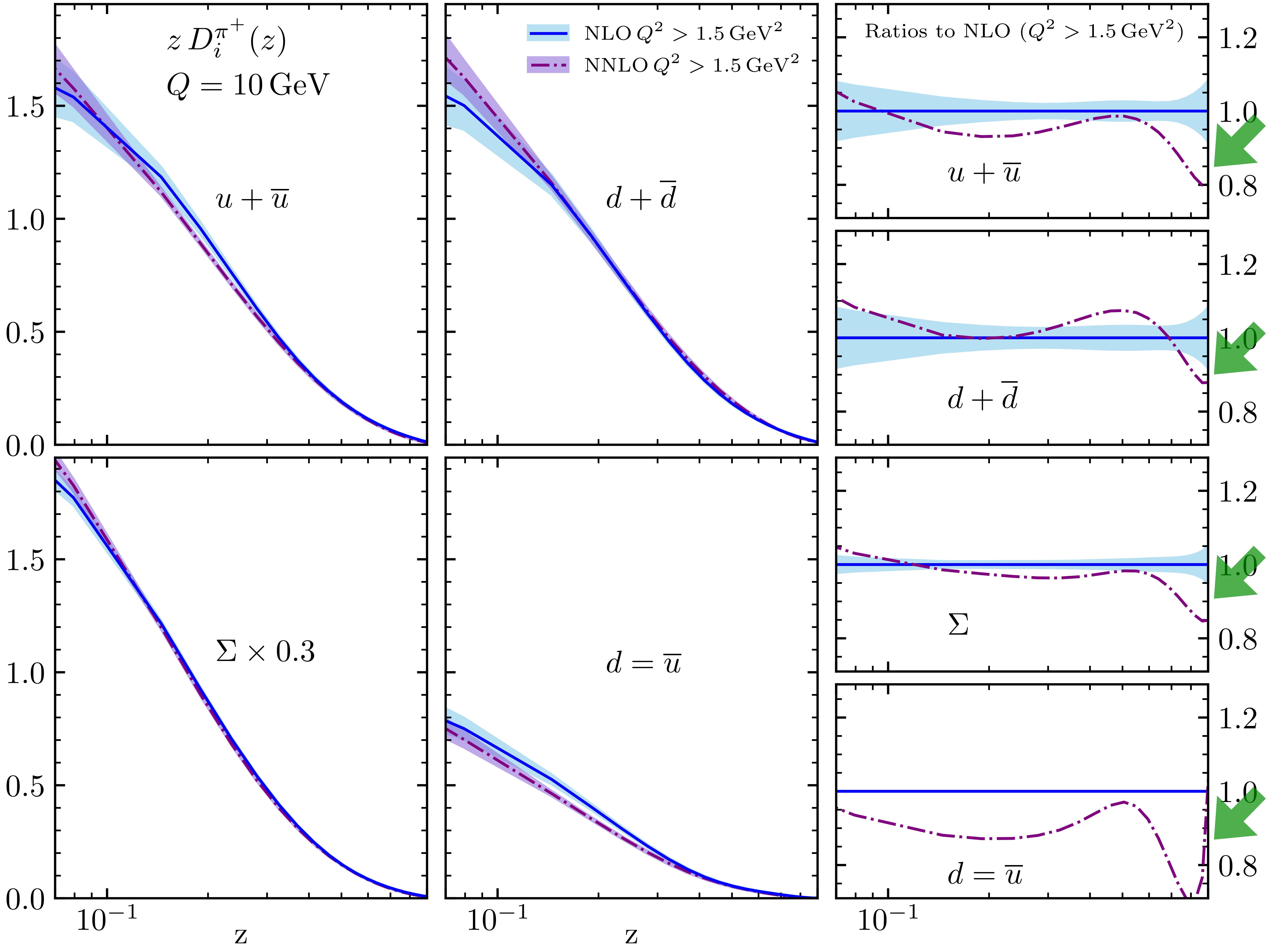


- Slight improvement of the description of SIA data at NNLO
- Reduced tension with SIDIS data as higher Q^2 -cuts are imposed

RESULTS - NNLO FFS

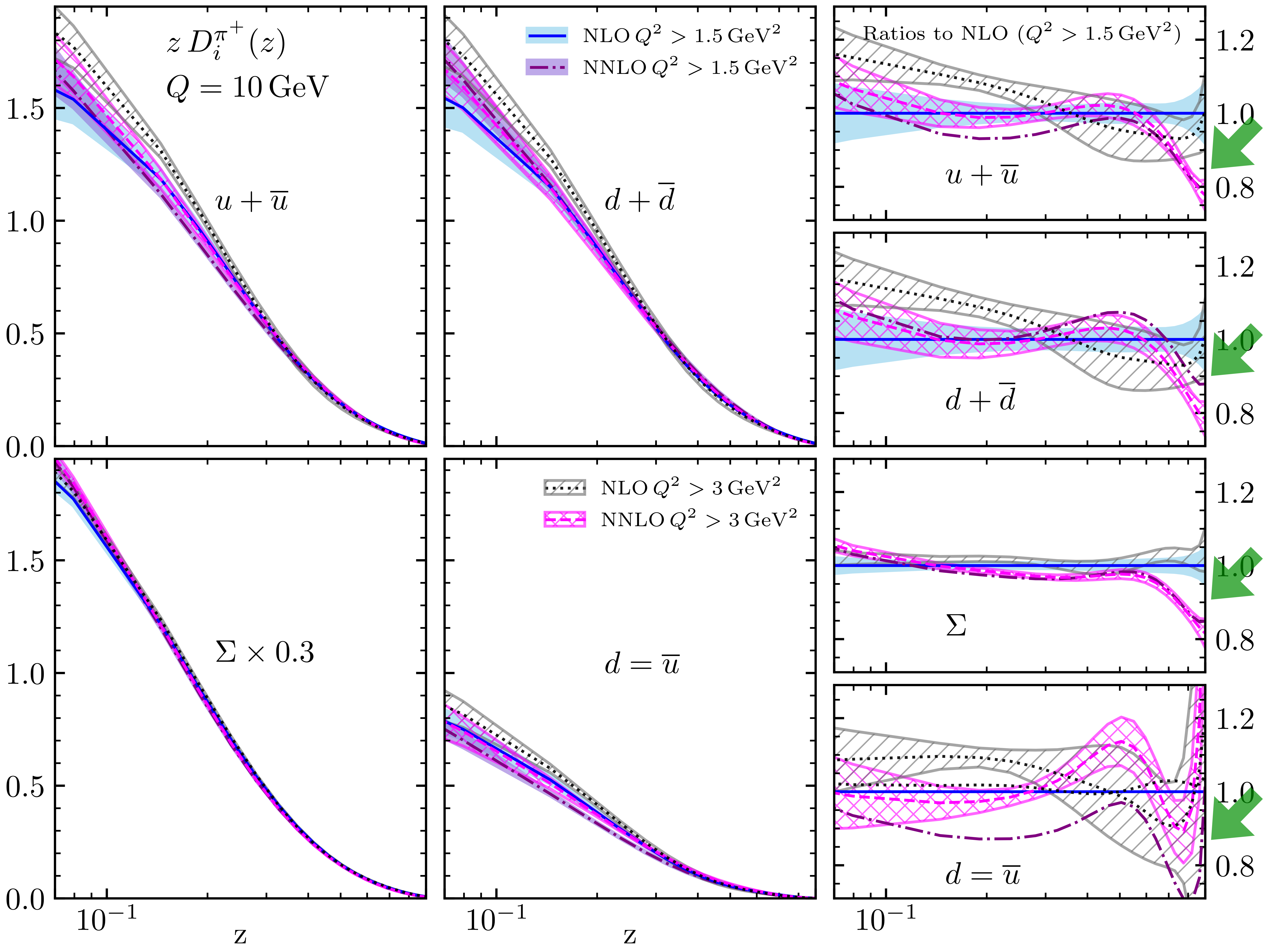


RESULTS - NNLO FFS



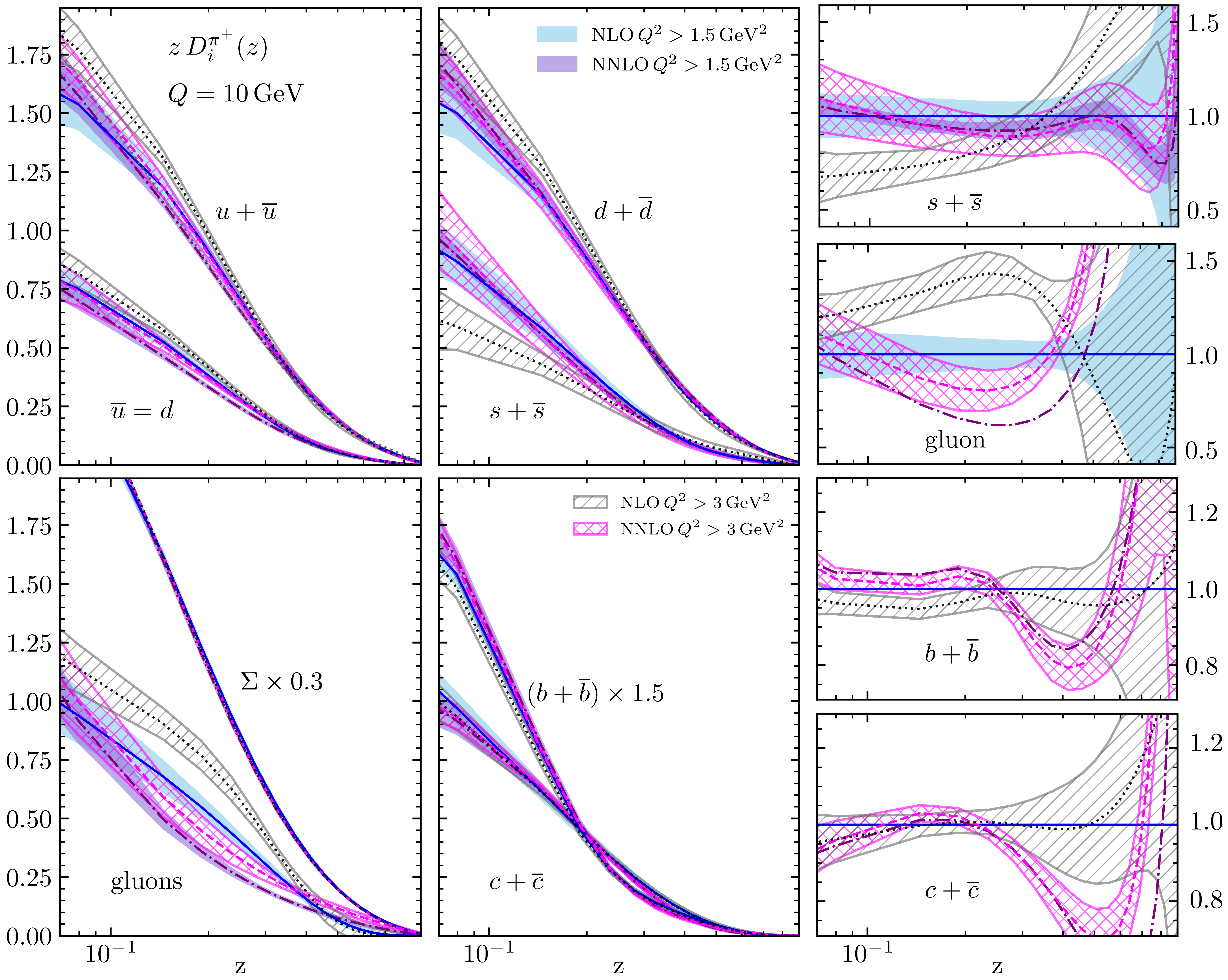
- Suppression of the distributions for $z \gtrsim 0.6$ to compensate the large-z enhancement of the cross sections.

RESULTS - NNLO FFS



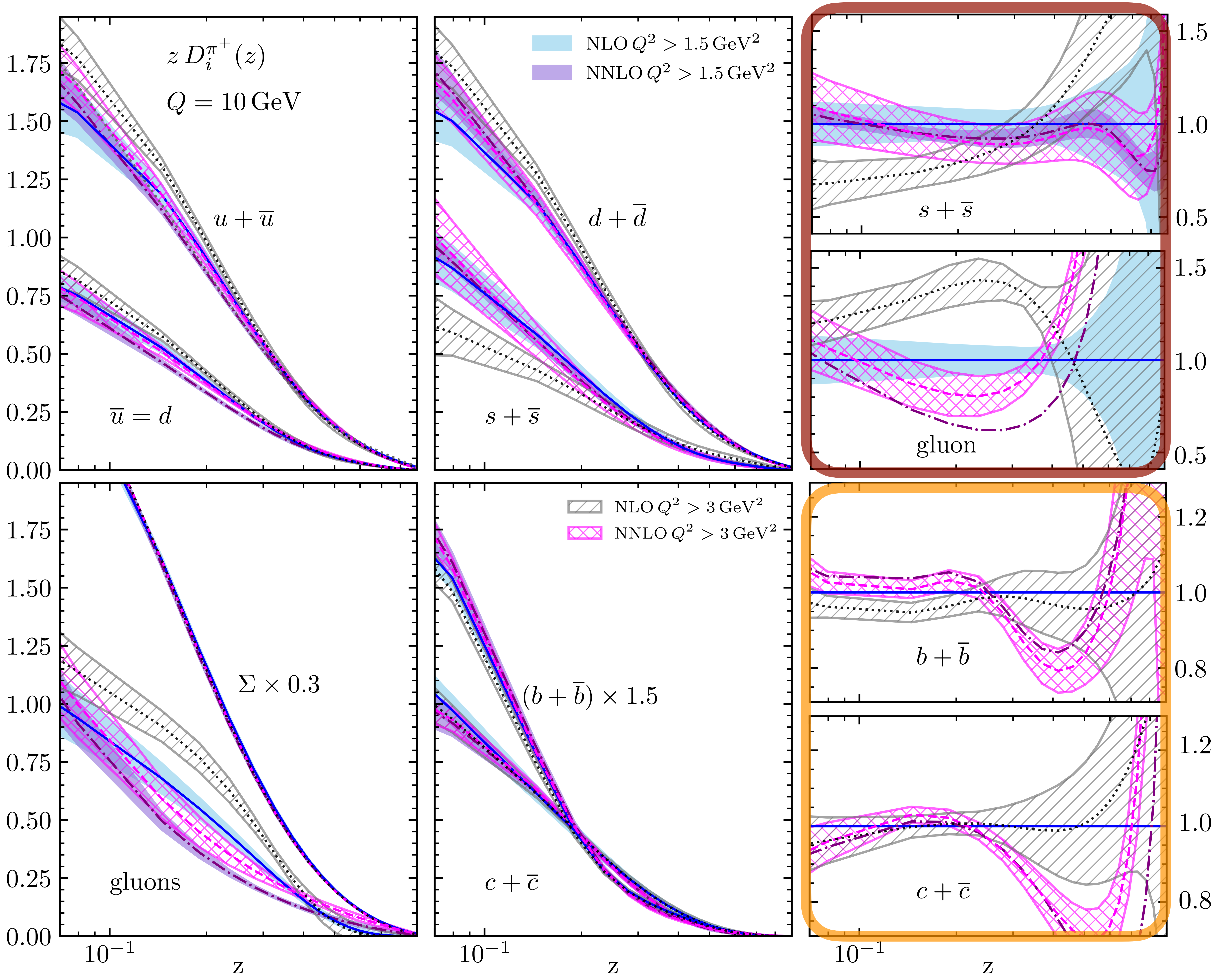
- Suppression of the distributions for $z \gtrsim 0.6$ to compensate the large-z enhancement of the cross sections.
- The cut $Q^2 \geq 3$ GeV^2 leads to an enhancement of $D_{u_{\mathrm{tot}}}^{\pi^+}$ below 0.6, and a z-independent enhancement of $D_{\overline{u}}^{\pi^+}$.
- Similar NNLO & NLO uncertainties for $Q^2 \geq 1.5$ GeV^2. Using the more strict cut of $Q^2 \geq 3$ GeV^2 leads to a reduction in the NNLO uncertainty compared to the NLO one.

RESULTS - NNLO FFS



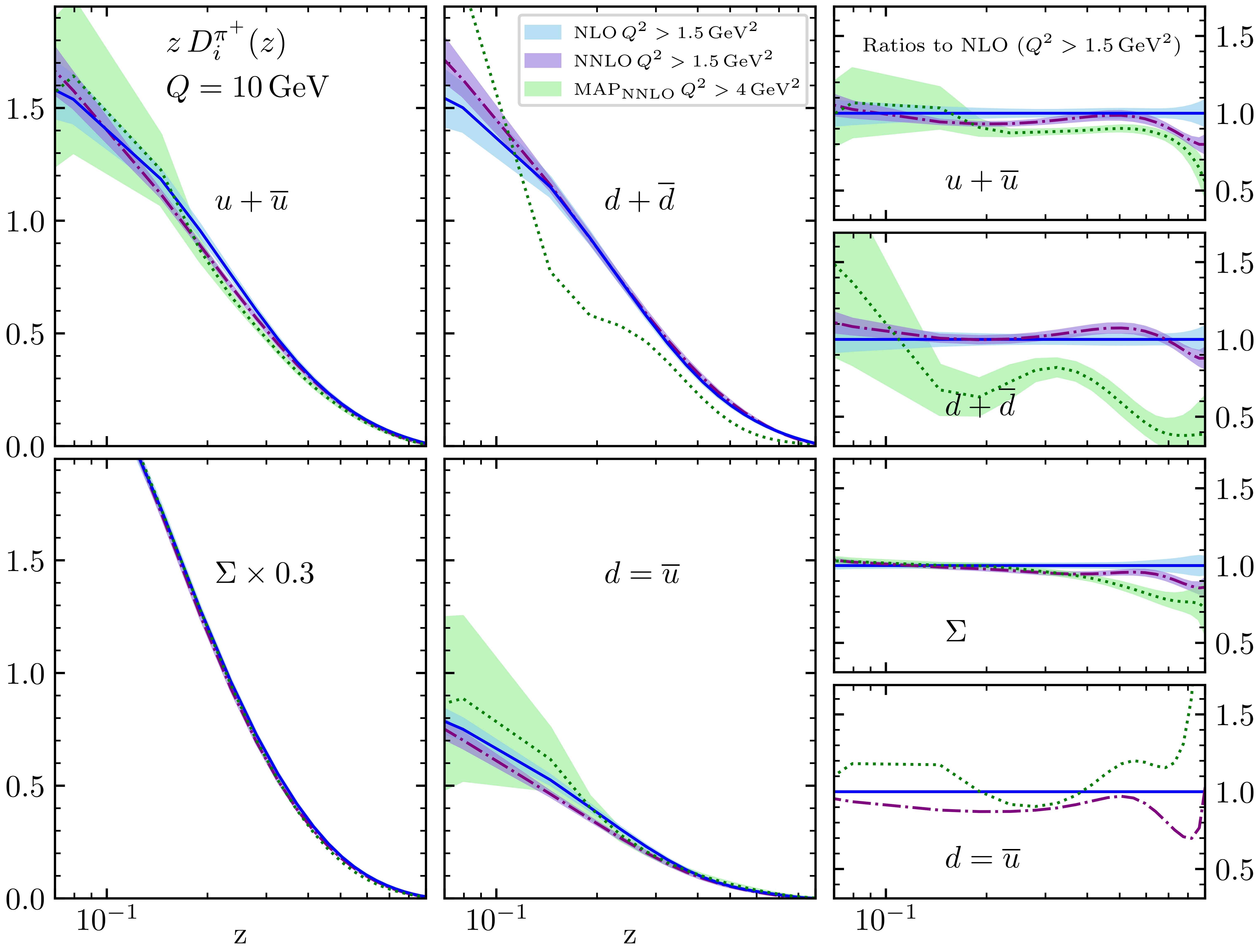
- Largely unconstrained $D_g^{\pi^+}$ (need for pp data!) and $D_{s_{tot}}^{\pi^+}$. Rigid parametrization might lead to underestimation of the uncertainty
- $D_{c_{tot}}^{\pi^+}$, $D_{b_{tot}}^{\pi^+}$ only weakly constrained by tagged SIA data

RESULTS - NNLO FFS



- Largely unconstrained $D_g^{\pi^+}$(need for pp data!) and $D_{s_{\text{tot}}}^{\pi^+}$. Rigid parametrization might lead to underestimation of the uncertainty
- $D_{c_{\text{tot}}}^{\pi^+}$, $D_{b_{\text{tot}}}^{\pi^+}$ only weakly constrained by tagged SIA data

RESULTS - NNLO FFS



- Rather good agreement for $D_\Sigma^{\pi^\pm}$ and $D_{u_{\mathrm{tot}}}^{\pi^\pm}$
- Large difference for $D_{d_{\mathrm{tot}}}^{\pi^\pm}$ (In DSS, $D_{d_{\mathrm{tot}}}^{\pi^\pm}$ constrained through partial SU(2) symmetry)
- Larger uncertainties for MAP FFs (reduced data set & more flexible parametrization)

SUMMARY

Precise knowledge of FFs is needed to probe the proton spin and flavor decomposition, and will play an instrumental role for the EIC spin program

- NNLO fit of polarized PDFs
- N^3LL determination of TMDs

First "proof-of-principle"
NNLO global analysis of FFs based on SIA and SIDIS data:

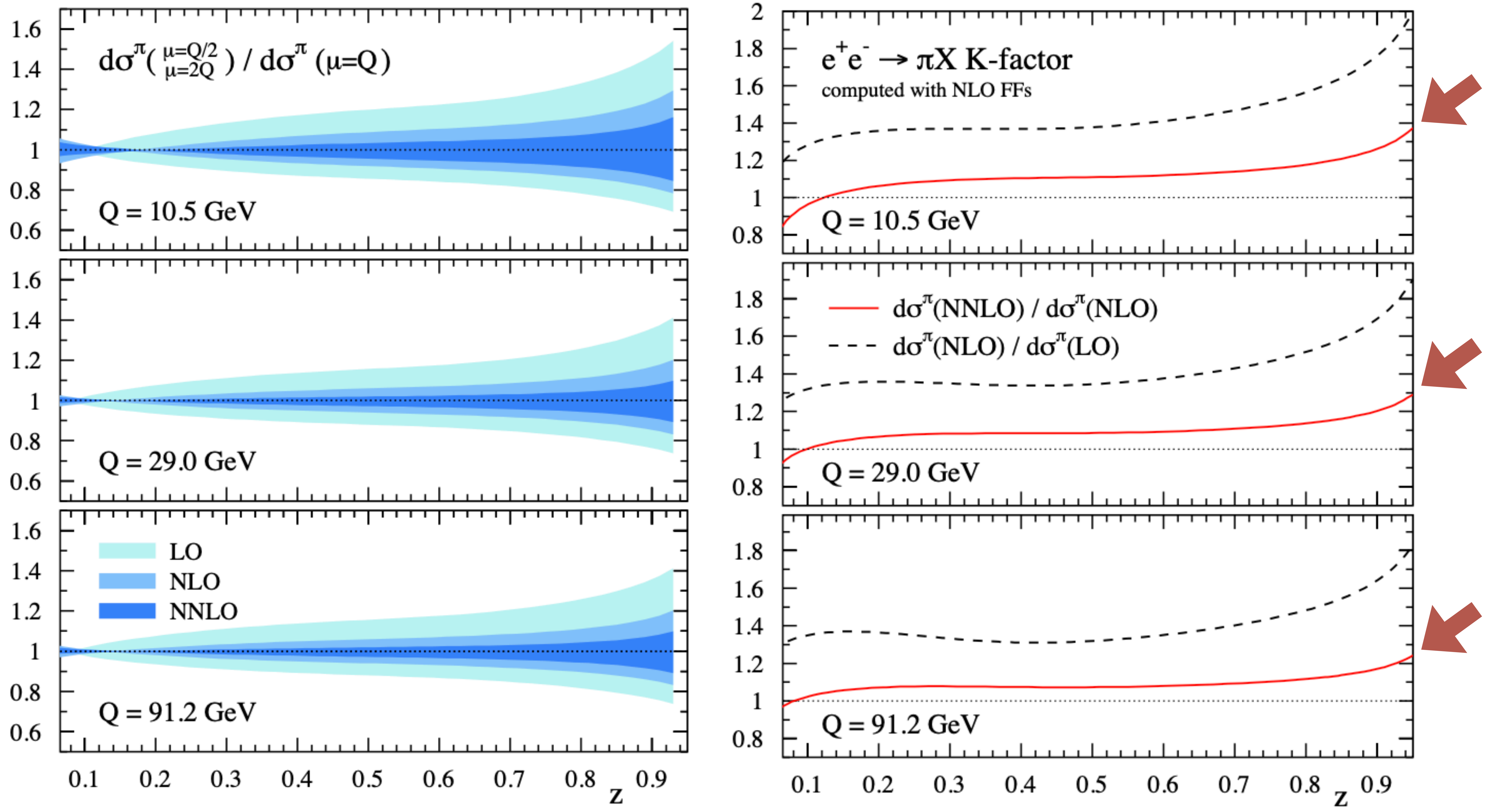
- Inclusion of NNLO corrections improve the quality of the fit, but after imposing a lower cut on Q^2 of at least 2 GeV^2
- Suppression of the FFs in the high-z region
- Unconstrained gluons → Need for pp NNLO corrections

THANK YOU

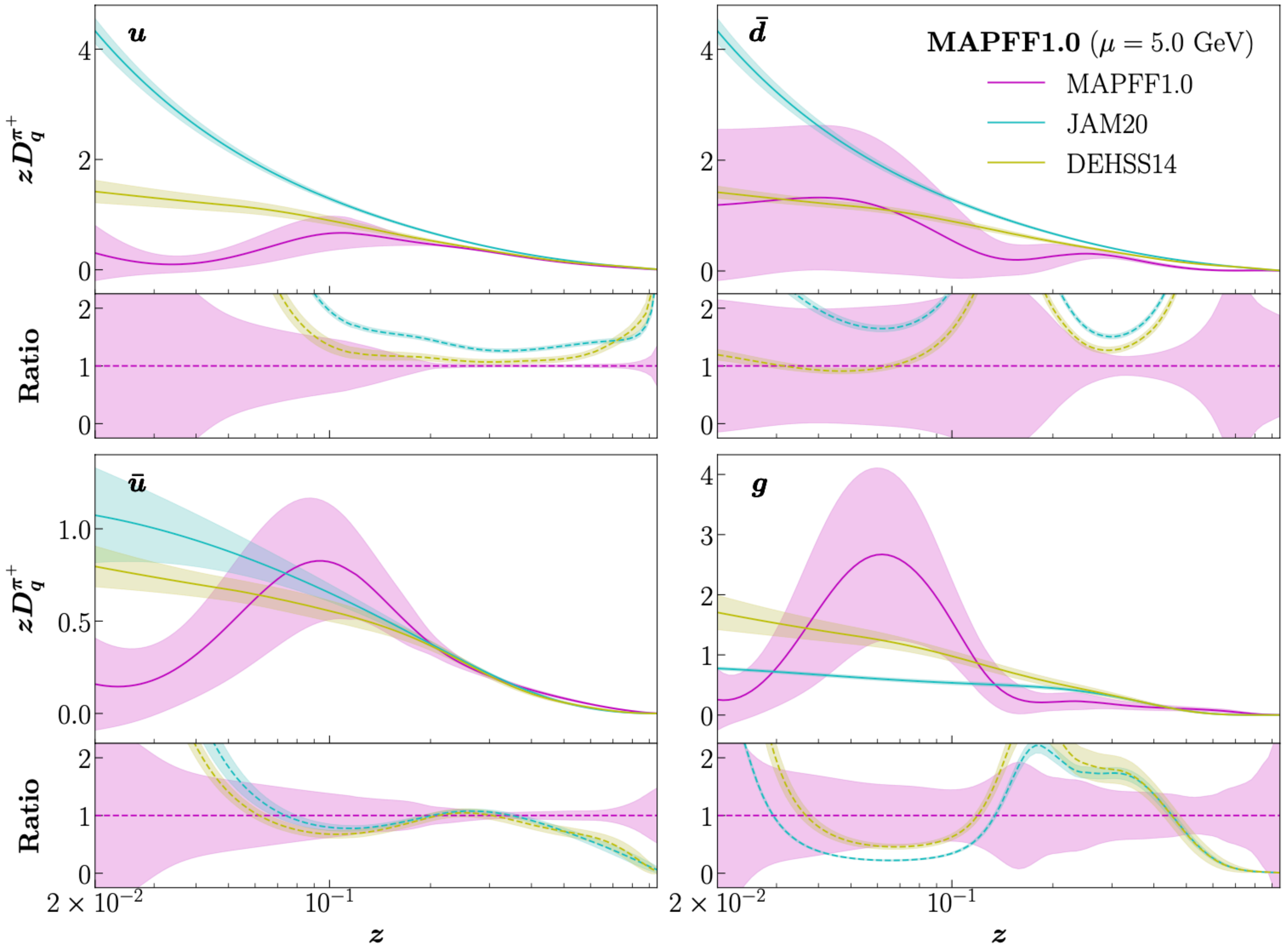
FROM NLO TO NNLO

Parton-to-Pion at NNLO from SIA data

Anderle, Ringer, Stratmann. *Phys.Rev.D* 92 (2015) 11, 114017



FROM NLO TO NNLO



Khalek, Bertone, Nocera. *Phys.Rev.D* 104 (2021) 3, 034007