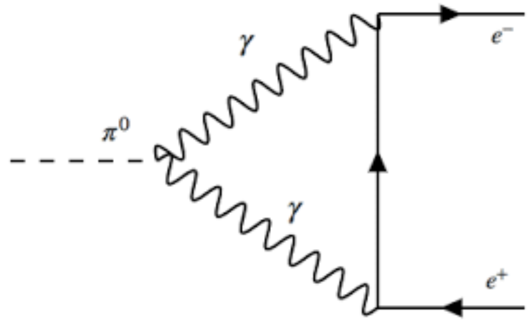
Why zero-modes matter: the role of the chiral anomaly (and chiral symmetry breaking) in polarized DIS



$$\Gamma = (1.19 \pm 0.08) \times 10^{16}\ \mathrm{s}^{-1}$$

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Workshop on Precision physics at the EIC: CFNS, Stony Brook, August 1-5, 2022

Work* in collaboration with Andrey Tarasov (The OSU and CFNS)

* Published in PRD: https://arxiv.org/abs/2008.08104
and arXiv: https://arxiv.org/abs/2109.10370

Talk outline

The chiral anomaly - and an inconvenient pole

The WZW term for the prodigal ninth Goldstone: an axionlike effective action

Spin and the $U_A(1)$ problem: The Goldberger- Treiman relation and topological mass generation of the η'

Worldline computation of box diagram in Bjorken and Regge asymptotics uncovers the anomaly pole

Spin damping at small x: sphaleron transitions induced by gluon saturation

Takeaway: The proton's spin is deeply influenced by the topology of the QCD vacuum
-in particular, its features that are responsible for the large mass of the η' meson

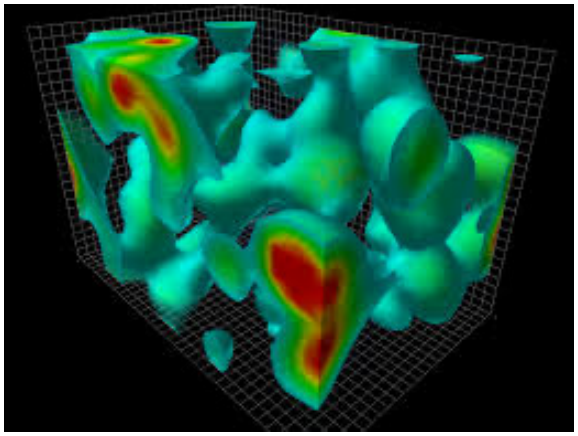
Polarized DIS at the Electron-Ion Collider can uncover first evidence for sphaleron (topological) transitions

Isosinglet axial vector current and the chiral anomaly

$$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right)$$

$$S^\mu\,\Delta\Sigma \;=\; \langle P,S\,|\,\bar{\psi}\,\gamma^\mu\gamma_5\,\psi\,|\,P,S\rangle \;\equiv\; \langle P,S\,|\,j_5^\mu\,|\,P,S\rangle$$

J_5^μ is the isosinglet axial vector current



Divergence of C-S current $\propto F\tilde{F}$

topological charge density

$U_A(1)$ violation from the chiral anomaly:

$$\partial_\mu J_5^\mu = 2n_f \partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i \bar{q}_i \gamma_5 q_i$$

Chern-Simons current

$$K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma\right)\right]$$

Spin "crisis": Why is ΔΣ small?

For massless quarks, conserve $J_5^\mu - 2\,n_f K^\mu$ ⟷ $\Delta\Gamma(Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} N_f \Delta g(Q^2)$

So perhaps then the "real" ΔΣ is $\Sigma(Q^2) = \tilde{\Sigma}(Q^2) - \frac{\alpha_s(Q^2)}{2\pi} N_f \Delta g(Q^2)$

Offers a possible explanation of empirical small ΔΣ (in addition to flavor SU(3) violation)

ca., 1988-90, Efremov, Teryaev; Altarelli, Ross ; Carlitz, Collins, Mueller

Problem: Identification of CS charge with ΔG intrinsically ambiguous: latter is gauge invariant, former is not

Jaffe-Manohar (1990)

$$K_\mu \to K_\mu \; + \; i\frac{g}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\left(U^\dagger\partial^\alpha U A^\beta\right) \; + \; \frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta}\left[(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)\right]$$

Large gauge transformation: integer topological windings of the homotopy group: $SU(N_C) \to S^3$
Distinguishes energy degenerate but topologically inequivalent gauge field configurations

The spin crisis is deeply related to the $U_A(1)$ problem

UA(1) problem: why is there no isosinglet Goldstone boson
or why is the η' so massive (957 MeV!)?

R. L. Jaffe

A. Manohar

The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the η' a mass★.

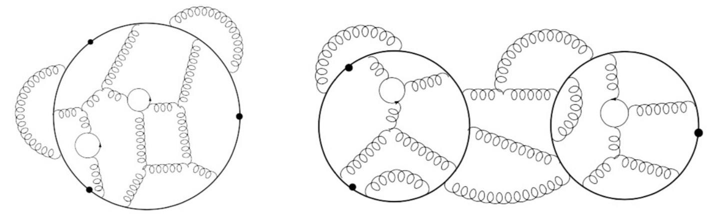
Jaffe,Manohar (1990)

Modern perspective on the η' as the 9th "prodigal" Goldstone boson: nonet Chiral Perturbation theory

Leutwyler, hep-ph/9601234
Herrera-Siklody et al, hep-ph/9610549
Kaiser, Leutwyler, hep-ph/0007101

Alternative picture: topological charge screening of spin

Veneziano (1989); Shore, Veneziano, PLB (1990); NPB (1992)
Narison, Shore, Veneziano, hep-ph/9812333



Employ anomalous chiral Ward identities +extended PCAC in systematic 1/N_C expansion

Famous example: Witten-Veneziano formula $m^2_{\eta'} = \frac{2\,n_f}{f^2_\pi}\chi_{\rm YM}(0) + O((\frac{n_f}{N_c})^2)$ → 0 when $N_c \to \infty$

where the YM topological susceptibility $\chi_{\rm YM}(l^2) = i\int dx\, e^{il\cdot x}\langle 0|T(\Omega(x)\,\Omega(0))|0\rangle$

with $\Omega(x) = \frac{\alpha_S}{8\pi}\,{\rm Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$ topological charge density

$$\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$$

In chiral limit $\chi(0) \to 0$, $\Delta\Sigma$ "controlled" by the slope χ' at l^2=0 – explained small value of $\Delta\Sigma$

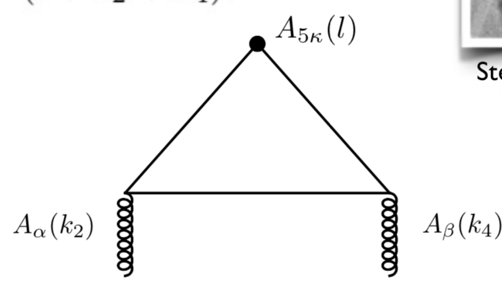
G. Veneziano

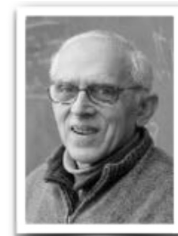
Similar ideas in instanton frameworks:
Forte, Shuryak (1991); Zahed et al. (2016-)

The Adler-Bell-Jackiw chiral (triangle) anomaly

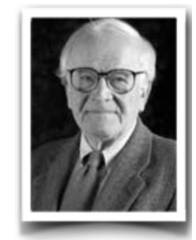
$$\langle P', S| J_5^\kappa |P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

$$= \frac{1}{4\pi^2}\frac{l^\kappa}{l^2}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\, \mathrm{Tr_c} F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\ (2\pi)^4\delta^4(l+k_2+k_4)$$




Steven Adler


John S. Bell


Roman Jackiw

Famous infrared pole of anomaly.
One loop exact: Adler-Bardeen theorem


William A. Bardeen

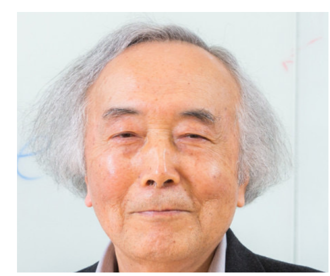
Key insight from Fujikawa:
Anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations

$$e^{iW} = \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\ \exp\left[i\int dx(\mathcal{L}_{\mathrm{QCD}} + V_5^{\mu a}J_{\mu 5}^a + V^{\mu a}J_\mu^a + \theta Q + S_5^a\phi_5^a + S^a\phi^a)\right]$$

$$\rightarrow \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\ \left[\partial^\mu J_{\mu 5}^a - \sqrt{2n_f}\delta^{a0}Q - d_{abc}m^b\phi_5^c - \delta\left(\int d^4x\mathcal{L}_{\mathrm{QCD}}\right)\right]\ \exp\left[\ldots\right] = 0$$

Anomalous functional Ward identities from Wess-Zumino action

Wess, Zumino (1971)


Kazuo Fujikawa

Thinking properly about anomalies with worldlines

Review: Schubert, Phys. Repts. (2001)
N. Mueller, RV: 1701.03331.1702.01233,1901.10492
Tarasov, RV: 1903.11624, 2008.08104 and in preparation

The worldline formulation of QFT is equivalent to the string amplitude formalism of Bern and Kosower, as shown by Strassler - provides a powerful "first quantized"intuition especially for internal symmetries

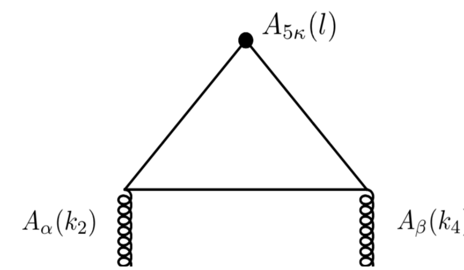
Bern,Kosower, NPB 379 (1992) 145;
Bern, TASI lectures, hep-ph/9304249
Strassler, NPB 385 (1992) 145

Recent example: reformulation of QED to all orders as a first quantized theory of worldline-superpairs
- allowing a proof of infrared safety of the Faddeev-Kulish S-matrix to all orders

X.Feal, A. Tarasov, RV, arXiv:2206.04188

The triangle anomaly in the worldline formalism

The axial vector couplings project out the imaginary part of the effective action



Point particle Bose and Grassmann path integrals

$$\Gamma[A, A_5] = -\frac{1}{2}\mathrm{Tr_c}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \int_{AP} \mathcal{D}\psi$$

$$\times \exp\Big\{ -\int_0^T d\tau \Big(\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu\dot{\psi}^\mu + ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu} - 2i\psi_5\dot{x}^\mu\psi_\mu\psi_\nu A_5^\nu + i\psi_5\partial_\mu A_5^\mu + (D-2)A_5^2\Big)\Big\}$$

Wilson line

Spin precession

“re-exponentiated” axial vector couplings from imaginary part of QCD effective action

$$\langle P', S|J_5^\kappa|P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)}\Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

$$= \frac{1}{4\pi^2}\frac{l^\kappa}{l^2}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\,\mathrm{Tr_c} F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\;(2\pi)^4\delta^4(l+k_2+k_4)$$

Anomalies in the worldline formulation of QFT

Fermion action in background of scalar, pseudoscalar, vector and axial vector fields:
(with focus on $U_A(1)$ sector)

$$S_{\rm fermion}[\bar\Psi,\Phi,\Pi,A,B,\Psi] = \int d^4x\,\bar\Psi^I\left[i\partial\!\!\!/ - \Phi + i\gamma^5\Pi + A\!\!\!/ + \gamma^5 B\!\!\!/\right]^{IJ}\Psi^J$$

Effective action: $-\mathcal{W}[A,B,\Phi,\Pi] = {\rm Ln\,Det}\,[\mathcal{D}]$ with $\mathcal{D} = p\!\!\!/ - i\Phi(x) - \gamma_5\,\Pi - A\!\!\!/ - \gamma_5 B\!\!\!/$

Split into real and imaginary parts: $\mathcal{W}_R = -\frac{1}{2}{\rm Ln}\left(\mathcal{D}^\dagger\mathcal{D}\right)$; $\mathcal{W}_I = \frac{1}{2}{\rm Arg\,Det}\left(\mathcal{D}^2\right)$

Entire dynamics of the anomaly comes from W_I - the phase of the Dirac determinant

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Entire dynamics of the anomaly comes from W_I - the phase of the Dirac determinant

Remarkable observation:
W_I can also be expressed as a worldline Lagrangian of 0+1- bosonic (coordinate) and Grassmann fields

D'Hoker, Gagne, hep-th/9508131

Jacobian for zero modes multiplied by G-parity factor

$$W_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int_{\text{PBC}}\mathcal{D}x\,\mathcal{D}\psi\;\mathrm{tr}\;\chi\,\bar{\omega}(0)\exp\left[-\int_0^T d\tau\,\mathcal{L}_{(\alpha)}(\tau)\right]$$

Worldline Lagrangian
with chiral symmetry breaking interpolating parameter α

Tarasov,RV, arXiv:2109.10370

A big role for a phase: The WZW isosinglet contribution

Explicit computation of the imaginary part of the one loop effective action reveals

$$S^{\bar\eta}_{\rm WZW} = -i\frac{\sqrt{2n_f}}{F_{\bar\eta}}\int d^4x\,\bar\eta\,\Omega$$

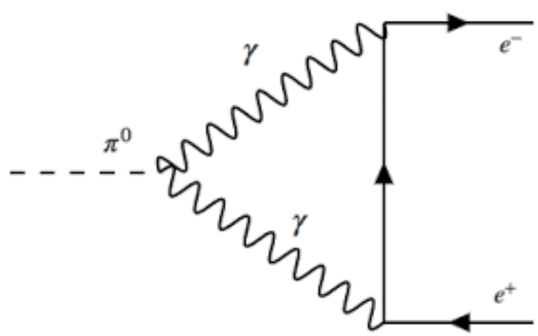
Ω is the topological charge density and F_η is the η' decay constant

Agrees exactly with the expression found in the Ch. PT literature

Kaiser, Leutwler (2000)

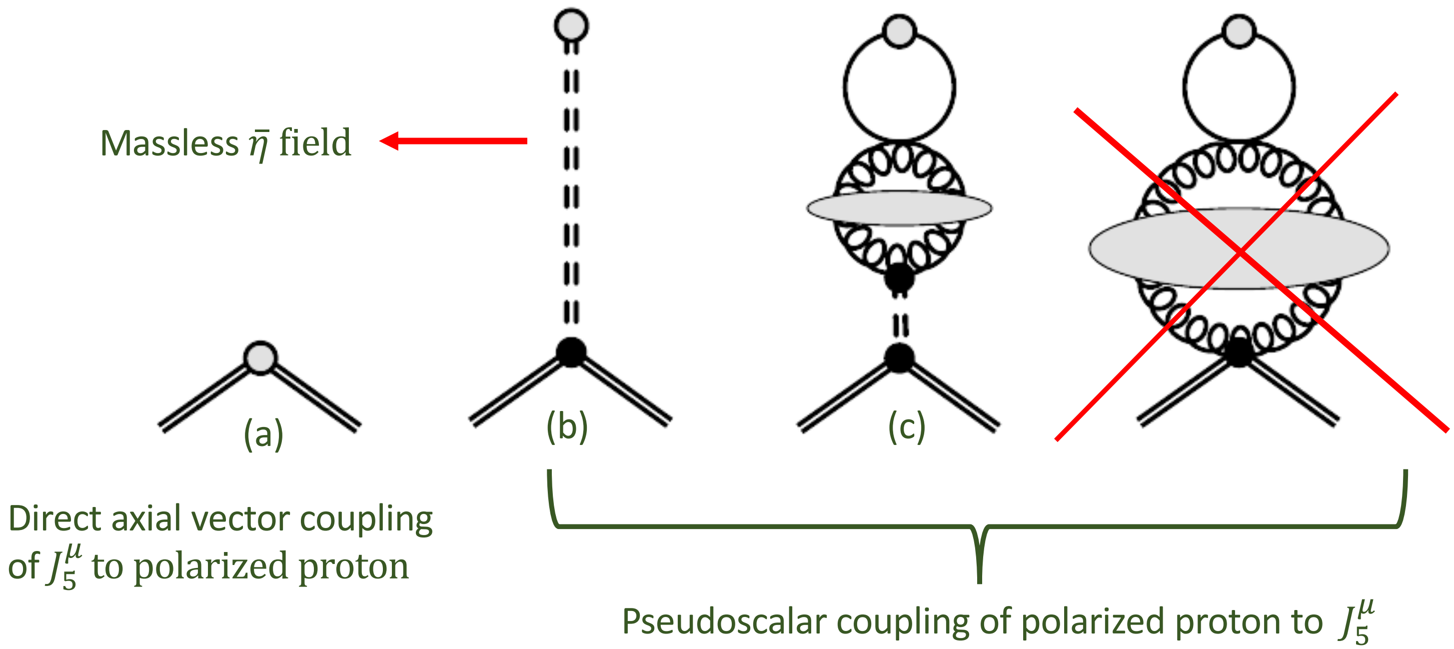
Another famous WZW term derived similarly from the imaginary part of the worldine effective action is that responsible for $\pi^0 \to 2\,\gamma$. This corresponds to the anomaly in the isotriplet axial vector current

D'Hoker,Gagne,hep-ph/9508131



Non-perturbative couplings of isosinglet axial current to the proton

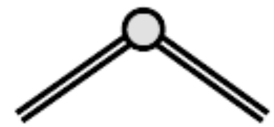
$$\langle P', S|J_5^\mu|P, S\rangle = \bar{u}(P', S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\Big]u(P, S)$$



Tarasov,Venugopalan, arXiv:2109.10370

Isosinglet axial charge and proton helicity

Consider first (a) the direct axial vector coupling



Since there $G_P(0)$ cannot have a pole $\lim_{l\to 0}\left[\langle P', S|J_5^\mu|P, S\rangle|_{\mathrm{Fig.2b}} + \langle P', S|J_5^\mu|P, S\rangle|_{\mathrm{Figs.2c+2d}}\right] = 0$

Hence, "trivially", $\langle P, S|J_5^\mu|P, S\rangle = \langle P, S|J_5^\mu|P, S\rangle|_{\mathrm{Fig.2a}} = 2M_N\, G_A(0)\, S^\mu$

=> $\Sigma(Q^2) = 2\, G_A(0)$ The helicity of the proton is twice its axial vector charge

We will now establish the transitive property a ↔ b ↔ c

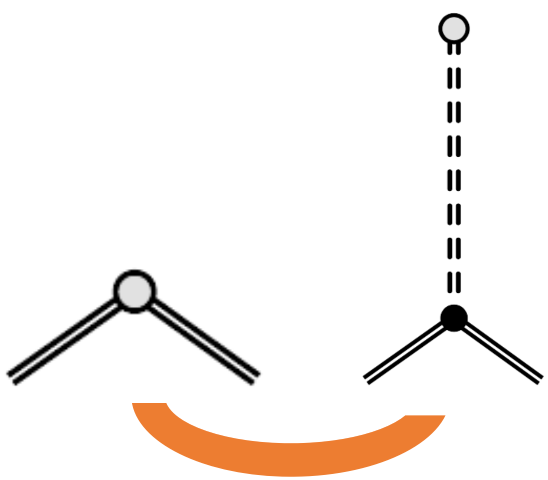
Goldberger-Treiman relation

$$\langle P', S|J_5^\mu|P, S\rangle|_{\mathrm{Fig.2b}} = g_{\eta_0 NN}\,\bar{u}(P', S)\,\gamma_5\,u(P, S)\cdot\frac{i}{l^2}\cdot i\sqrt{2\tilde{n}_f}\,l^\mu F_{\bar{\eta}}(l^2)$$

$$\langle P', S|J_5^\mu|P, S\rangle|_{\mathrm{Figs.2c+2d}} = -i\frac{l^\mu}{l^2}\langle P', S|2\,n_f\,\Omega|P, S\rangle$$

The G-T relation then follows from the anomaly equation+Dirac equation:

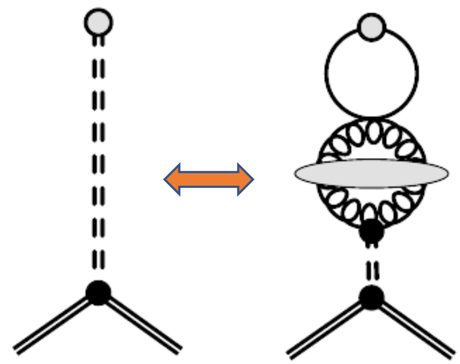
$$iG_A(l^2)\bar{u}(P', S)\not{l}\gamma_5 u(P, S) - ig_{\eta_0 NN}\bar{u}(P', S)\gamma_5 u(P, S)\sqrt{2\tilde{n}_f}F_{\bar{\eta}}(l^2) + \langle P', S|2n_f\Omega|P, S\rangle = \langle P', S|2n_f\Omega|P, S\rangle$$

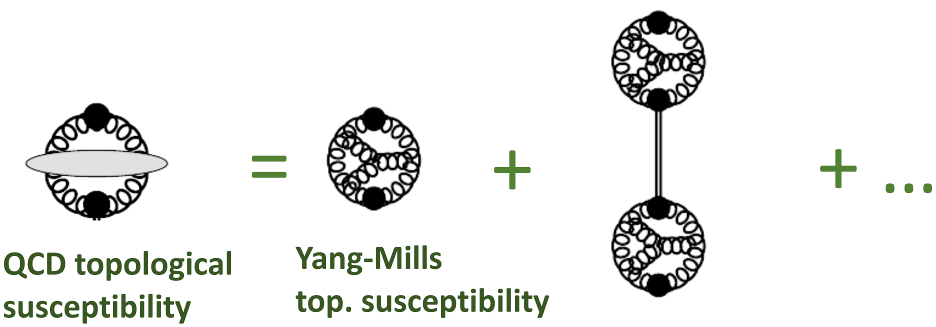


$$G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}F_{\bar{\eta}}\,g_{\eta_0 NN}$$

Topological susceptibility: from Yang-Mills to QCD

Absence of a pseudoscalar pole also implies…





$1/N_C$ corrections to the YM susceptibility
induced by the WZW action
generate the QCD topological susceptibility

$$\chi(l^2) = i \int d^4x \, e^{ilx} \langle 0 | T\, \Omega(x) \Omega(0) | 0 \rangle$$

$$\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\rm YM}(l^2)$$ with $$m_{\eta'}^2 \equiv - \frac{2 n_f}{F_{\bar\eta}^2} \chi_{\rm YM}(0)$$ Witten-Veneziano formula

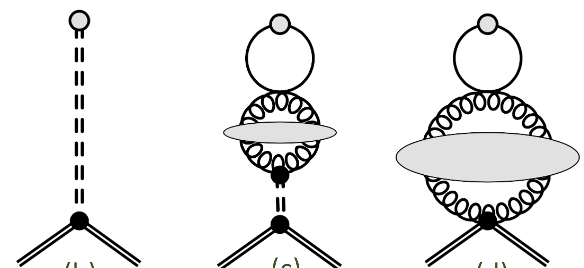
$\chi(l^2) \to 0 \; when \; l^2 \to 0$

Topological generation of a mass greater than the proton's mass ….

Anomaly cancellation and topological screening-I

$$\langle P',S|J_5^\mu|P,S\rangle|_{\mathrm{Fig.2c}} = -i\frac{l^\mu}{l^2}\langle P',S|2n_f\Omega|P,S\rangle|_{\mathrm{Fig.2c}} = i\frac{l^\mu}{l^2}2n_f\cdot\langle 0|T\,\Omega\eta_0|0\rangle\cdot g_{\eta_0 NN}\,\bar{u}(P',S)\,\gamma_5\,u(P,S)$$

$$\langle P',S|J_5^\mu|P,S\rangle|_{\mathrm{Fig.2d}} = -i\frac{l^\mu}{l^2}\langle P',S|2n_f\Omega|P,S\rangle|_{\mathrm{Fig.2d}} = i\frac{l^\mu}{l^2}2n_f\cdot\langle 0|T\,\Omega\Omega|0\rangle\cdot g_{\Omega NN}\,\bar{u}(P',S)\,\gamma_5\,u(P,S)$$



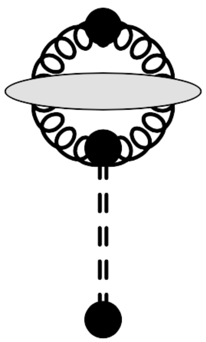
The absence of a pseudoscalar pole results requires that (b), (c) and (d) satisfy

$$\sqrt{2\tilde{n}_f}\,F_{\bar{\eta}}\,g_{\eta_0 NN} = 2n_f\,\lim_{l\to 0}\left[i\,\langle 0|T\,\Omega\eta_0|0\rangle\,g_{\eta_0 NN} + i\,\underbrace{\langle 0|T\,\Omega\Omega|0\rangle}\,g_{\Omega NN}\right]$$

vanishes in the chiral limit

$$\longrightarrow \quad \boxed{\sqrt{2\tilde{n}_f}\,F_{\bar{\eta}} = 2n_f\,\lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle\,.}$$

Anomaly cancellation and topological screening-II



$$\langle 0|T\Omega\eta_0|0\rangle|_{\mathrm{Fig.4b}} = -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\bar{\eta}}}\chi(l^2)$$ Can expand χ in a Taylor series – since first term vanishes,

$$F_{\bar{\eta}}^2 = 2n_f\chi'(0)$$

Hence from the transitive property a ↔ b ↔ c

$$\langle P', S|J_5^\mu|P,S\rangle = \sqrt{\frac{2}{3}}2n_f\, g_{\eta_0 NN}\sqrt{\chi'(0)}\, S^\mu \rightarrow \boxed{\Sigma(Q^2) = \sqrt{\frac{2}{3}}\frac{2n_f}{M_N}g_{\eta_0 NN}\sqrt{\chi'(0)}}$$

Shore, Veneziano (1992)

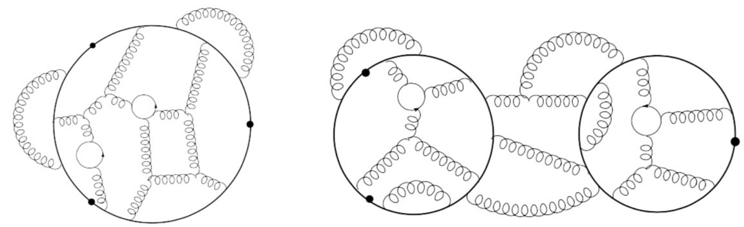
Goldberger-Treiman links the axial vector and pseudo-scalar sectors of QCD in the infrared

The net result is that the quark helicity is directly related to the topological susceptibility of the QCD vacuum

Anomaly cancellation and topological screening-IV

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}\,.$$

Magnitude of of OZI violation $\dfrac{a^0(Q^2)}{a^8} \simeq \dfrac{\sqrt{6}}{f_\pi}\sqrt{\chi'(0)}$



Computations on the lattice…

In my view, it is crucial that lattice configurations used to compute spin also get topology right – a good (but difficult) test is to get the η and the η' masses right simultaneously

Giusti,Rossi,Testa,Veneziano, hep-lat/0108009

Bali et al., arXiv:2106.05398

$G_A|_{model}$ = 0.33±0.05

Sum rule analysis in good agreement with HERMES and COMPASS data

HERMES (Q^2= 5 GeV^2) $0.330 \pm 0.011(th) \pm 0.025(exp) \pm 0.028(evol)$

COMPASS (Q^2=3 GeV^2) $0.35 \pm 0.03(stat) \pm 0.05(syst)$

Narison,Shore,Veneziano (1998)

Axion-like effective action

As suggested by Shore and Veneziano, and following from our discussion as well,

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2} (\partial_\mu \bar\eta)(\partial^\mu \bar\eta) + \left(\theta - \frac{\sqrt{2n_f}}{F_{\bar\eta}} \bar\eta \right) \Omega + \frac{\chi_{\rm YM}}{2} \theta^2 \right]$$

Since θ is not dynamical, can get rid of it from the equations of motion,

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2} (\partial_\mu \bar\eta)(\partial^\mu \bar\eta) - \frac{\sqrt{2n_f}}{F_{\bar\eta}} \bar\eta\, \Omega - \frac{\Omega^2}{2\,\chi_{\rm YM}} \right]$$

Axion-like effective action for $\bar\eta$

Defining $\eta' = \frac{F_{\eta'}}{F_{\bar\eta}} \bar\eta$ *and* $G = \Omega + \frac{\sqrt{2n_f}}{F_{\eta'}} \chi_{\rm YM}\, \eta'$

$$S_{\eta'} = \int d^4x \left[-\frac{1}{2} \eta' \left(\partial^2 + m_{\eta'}^2 \right) \eta' - \frac{G^2}{2\chi_{\rm YM}} \right]$$

Re-express in terms of the η' and a non-propagating glueball that decouples from the physical spectrum

Shore,Veneziano (1990); Hatsuda (1990)
Dvali,Jackiw,Pi (1995)

In the instanton framework, χ_{YM} is saturated by such classical configurations

t'Hooft (1976); Schafer-Shuryak (1996)

Several spin discussions by multiple groups in this framework:

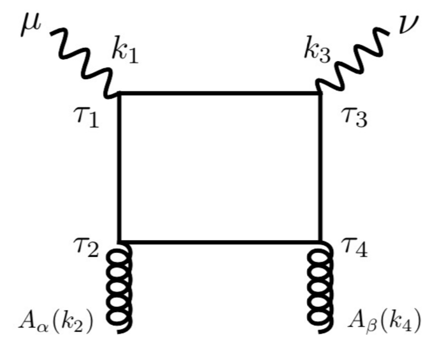
Forte, Shuryak (1990); Qian, Zahed (2016); …

What about g_1?

The box diagram for polarized DIS

Hadron tensor in DIS: $W^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4x\, e^{iqx}\langle P,S|j^\mu(x)j^\nu(0)|P,S\rangle$

Anti-symmetric part: $\tilde{W}_{\mu\nu}(q,P,S) = \frac{2M_N}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\Big\{S^\beta g_1(x_B,Q^2) + \Big[S^\beta - \frac{(S\cdot q)P^\beta}{P\cdot q}\Big]g_2(x_B,Q^2)\Big\}$



$g_1 \propto \;\; \Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\,\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\;\mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$

Polarization tensor (antisymmetric piece)

Box diagram

$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] = -\frac{g^2e^2e_f^2}{2}\int_0^\infty\frac{dT}{T}\int\mathcal{D}x\int\mathcal{D}\psi\;\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\prod_{k=1}^4\int_0^T d\tau_k\,\Big[\sum_{n=1}^9 \mathcal{C}^{\mu\nu\alpha\beta}_{n;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] - (\mu\leftrightarrow\nu)\Big]e^{i\sum_{i=1}^4 k_ix_i}\,.$$

Using the worldline formalism, compute box diagram (no kinematic approximations of internal variables) in both Bjorken limit ($Q^2\to\infty, s\to\infty, x=\text{fixed}$) and Regge limit ($x\to 0, s\to\infty, Q^2=\text{fixed}$)

The latter result is new

The box diagram for polarized DIS ($g_1(x,Q^2)$)

$$\mathcal{C}^{\mu\nu\alpha\beta}_{1;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2;$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{2;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4;$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{3;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4;$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{4;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2$$

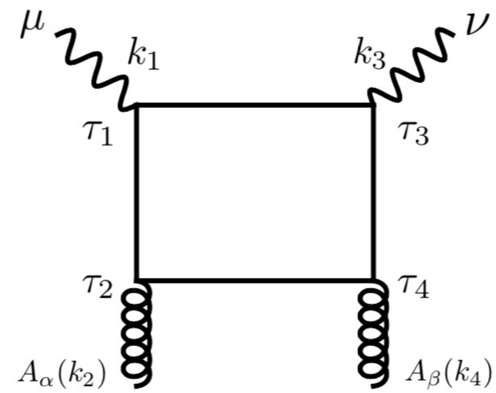
$$\mathcal{C}^{\mu\nu\alpha\beta}_{5;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4;$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{6;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{7;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3;$$

$$\mathcal{C}^{\mu\nu\alpha\beta}_{8;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3$$

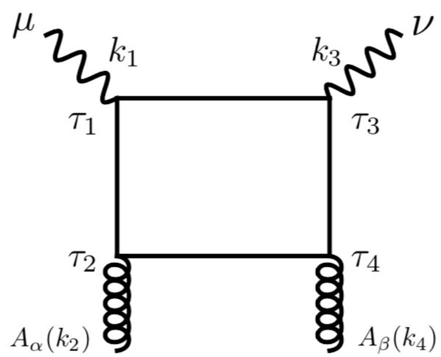
$$\mathcal{C}^{\mu\nu\alpha\beta}_{9;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = 16\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4$$

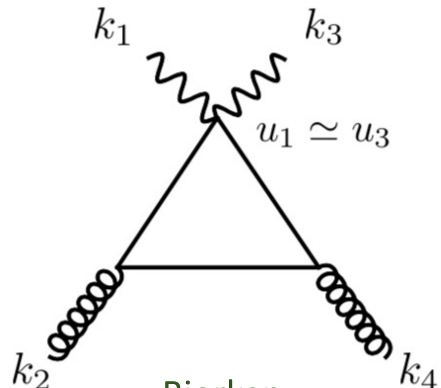


Can compute these explicitly using worldline integration techniques

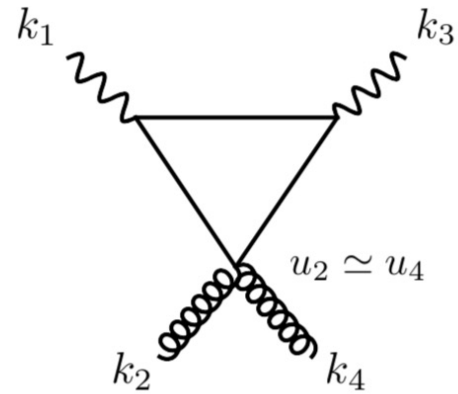
Tarasov, RV, 1903.11624, 2008.08104

Finding triangles in boxes in Bjorken and Regge asymptotics





Bjorken

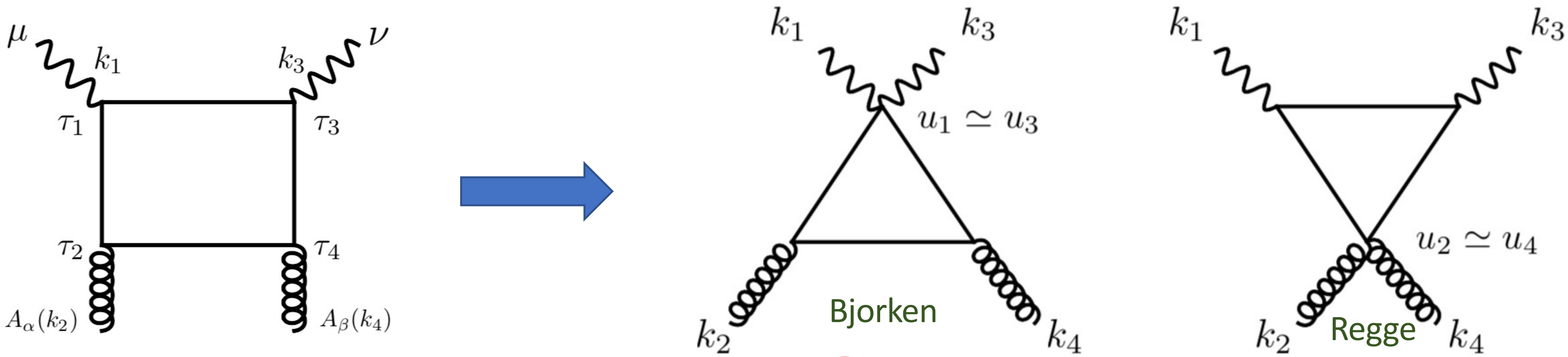


Regge

Tarasov, RV, arXiv:2008.08104

Remarkably, box diagram has same structure in both limits, dominated by the triangle anomaly!
This strongly suggests that the underlying physics is governed by topology
… as is the case for the first moment

Finding triangles in boxes in Bjorken and Regge asymptotics



$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole

Tarasov, RV, arXiv:2008.08104

$$S^\mu g_1(x_B,Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole

Notes:

i) the interpretation of the r.h.s as a contribution to g_1 is ambiguous since it is proportional to l^μ and not S^μ

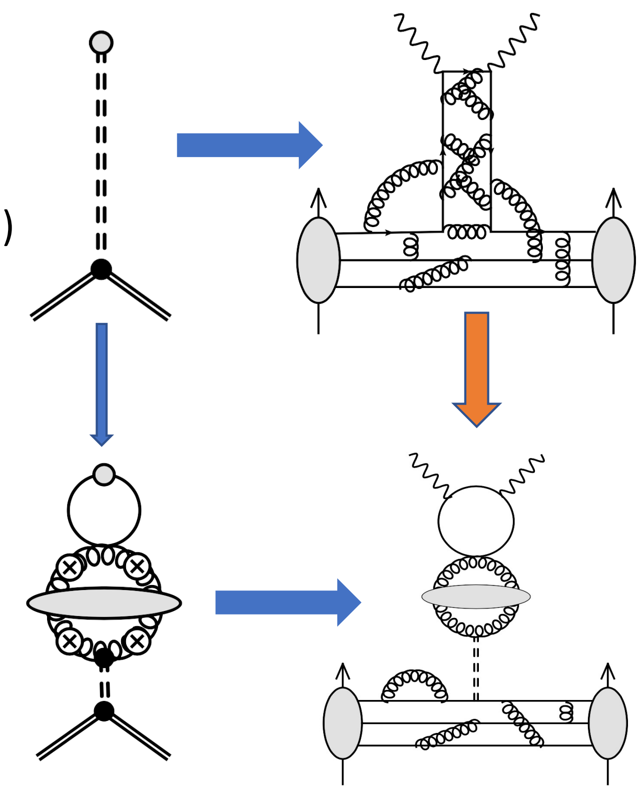
However, integral over x_B gives anomaly equation for Σ - G-T relation shows axial and pseudoscalar sectors are tied

ii) There can be finite/logarithmic pieces that contribute to g1 and not to Σ – these will contribute to renormalization of the former

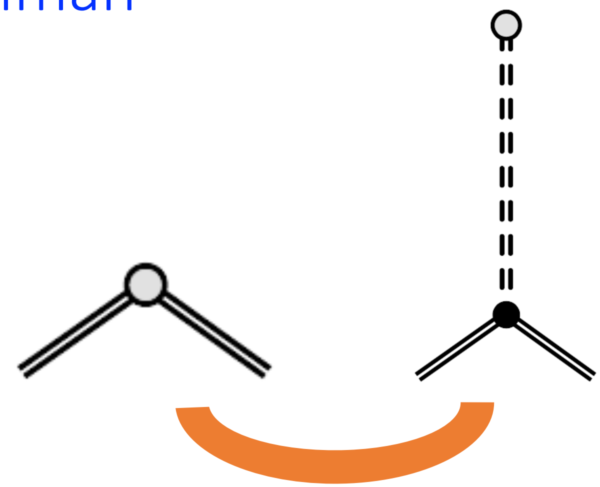
But the pole trumps all and must be resolved – it is the elephant in the room…

Pole cancellation beyond the first moment

Isosinglet exchange only known mechanism to cancel anomaly pole so the absence of a physical pole and the $1/N_C$ expansion of the topological susceptibility must go through as for the first moment (triangle)



Generalized Goldberger-Treiman



Likewise, for the G-T relation to be valid, one requires the anomaly equation to be valid for the “smeared” topological charge density

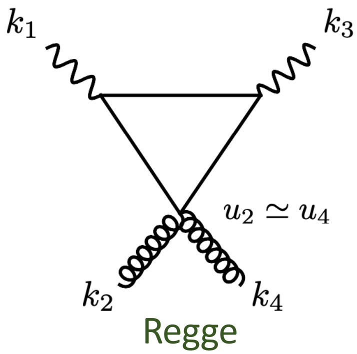
This is not unexpected from the perspective of the point-splitting approach to derivations of the anomaly equation

If this generalized Goldberger-Treiman relation holds, everything follows as for the first moment, and one can relate g_1 (singlet) directly to the coefficient of the $\frac{l^\mu}{l^2}$ divergent term in the box diagram calculation

$$\langle P,S|J_5^\mu|P,S\rangle = 2n_f \lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle\, g_{\eta_0 NN}\, S^\mu$$

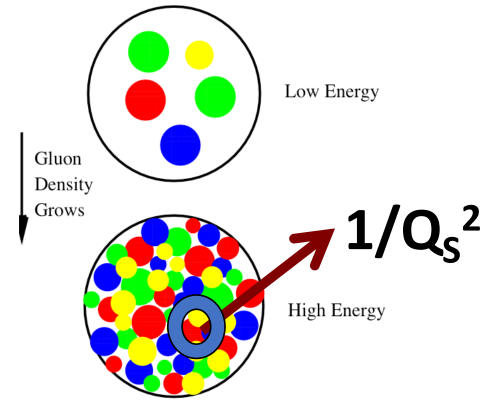
$$\Rightarrow\quad g_1(x_B,Q^2) = \left(\sum_f e_f^2\right)\frac{n_f\alpha_s}{\pi M_N} i\int d^4y\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\int D\bar\eta\,\tilde W_{P,S}[\bar\eta]\int[DA]$$
$$\times\,\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde F^{\alpha\beta}(0)\,\eta_0(y)\,\exp\left(iS_{\rm YM}+i\int d^4x\left[\frac{1}{2}(\partial_\mu\bar\eta)(\partial^\mu\bar\eta)-\frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega\right]\right)$$

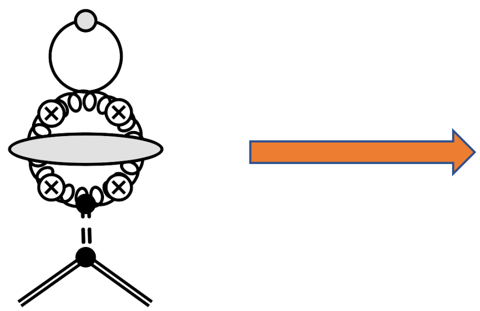
What about g_1 at small x_{Bj}

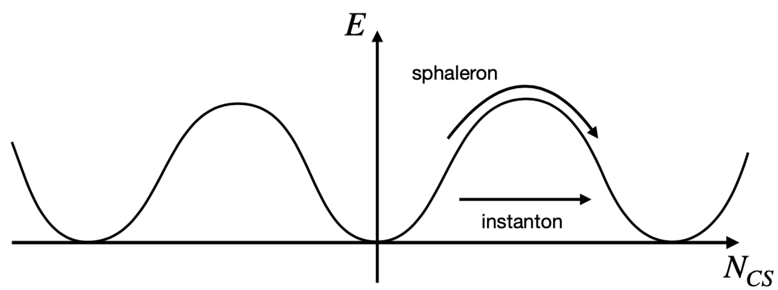


g_1 in the Regge limit is also controlled by the anomaly

How does gluon saturation influence the anomaly ?



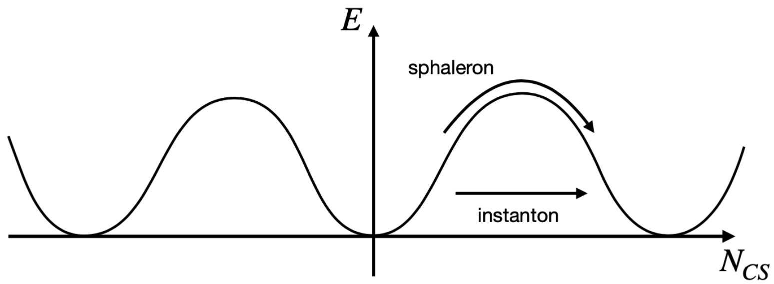


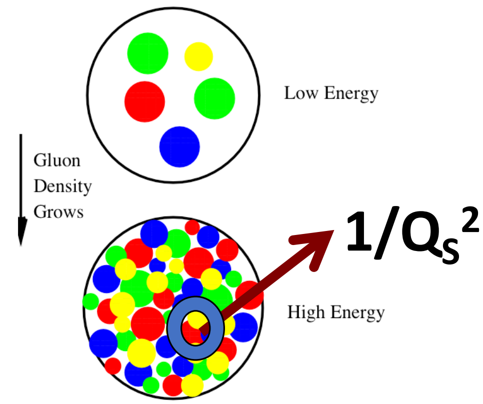


Gluon saturation can induce over the barrier sphaleron-like transitions

Tarasov,Venugopalan, arXiv:2109.10370

What about g_1 at small x_{Bj} ?





Gluon saturation induced over the barrier sphaleron-like transitions estimated by an axion-like effective action and its coupling to the CGC

$$g_1^{\text{Regge}}(x_B, Q^2) = \left(\sum_f e_f^2\right) \frac{n_f \alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x}\left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \int \mathcal{D}\rho\, W_Y[\rho] \int D\bar{\eta}\, \tilde{W}_{P,S}[\bar{\eta}] \int [DA]$$
$$\times \text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0)\, \eta_0(y)\, \exp\left(iS_{\text{CGC}} + i \int d^4x \left[\frac{1}{2}(\partial_\mu \bar{\eta})(\partial^\mu \bar{\eta}) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta}\, \Omega\right]\right)$$

Can be studied it two limits: $Q_S^2 < m_{\eta'}^2$ and $Q_S^2 > m_{\eta'}^2$

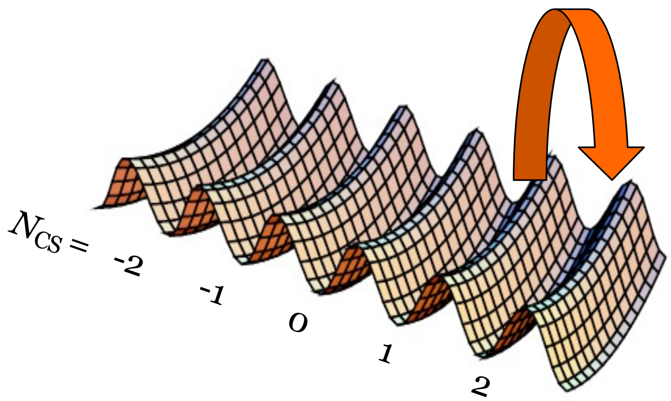
Tarasov,Venugopalan, arXiv:2109.10370

Spin diffusion via sphaleron transitions in topologically disordered media

Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\mathrm{YM}}}{F^2}$

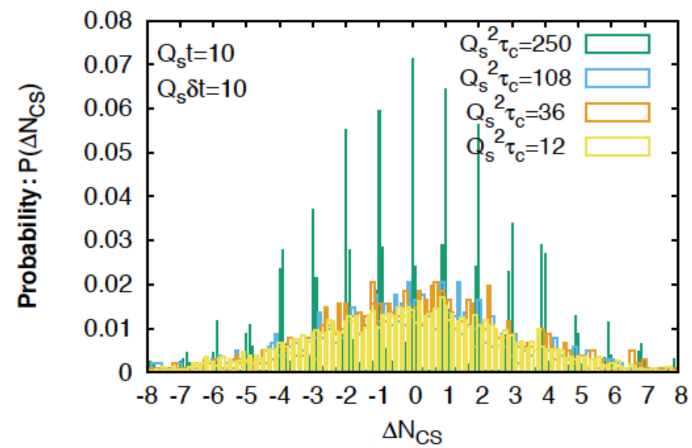
- the gluon saturation scale Q_S

When $Q_S^2 >> m_{\eta'}^2$ over the barrier gauge configurations dominate over instanton configurations



Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum…
characterized by integer valued Chern-Simons #

Topological transitions in overoccupied gauge fields



Mace, Schlichting, RV: PRD (2016) 1601.07342

Axion-like dynamics in a hot QCD plasma - McLerran,Mottola,Shaposhnikov (1990)

Spin diffusion via sphaleron transitions in topologically disordered media

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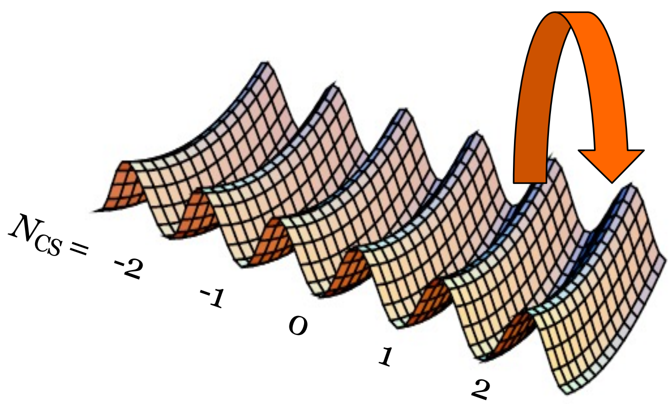
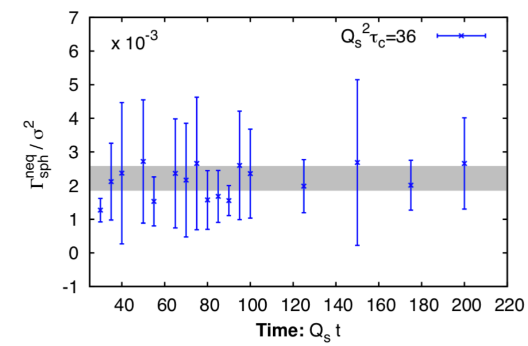
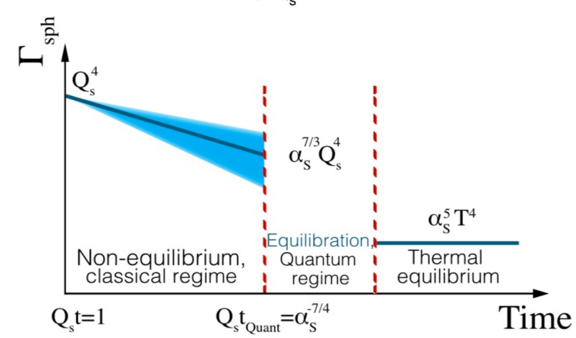


Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum…
characterized by integer valued Chern-Simons #

Axion-like dynamics in a hot QCD plasma - McLerran,Mottola,Shaposhnikov (1990)

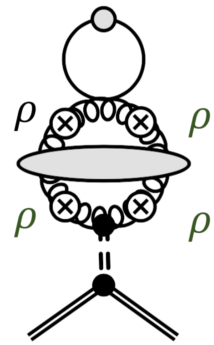
Sphaleron transition rate off-equilibrium





g_1 at small x_{Bj} from sphaleron transitions

For $Q_S^2 < m_{\eta'}^2$
over the barrier transitions



From our small x_B effective action, $\frac{\partial^2 \eta'}{\partial t^2} = -\gamma \frac{\partial \eta'}{\partial t} - m_{\eta'}^2 \eta'$ $\quad \gamma = \frac{2n_f \, \Gamma_{sphaleron}}{F^2{}_{\eta'} \, Qs}$

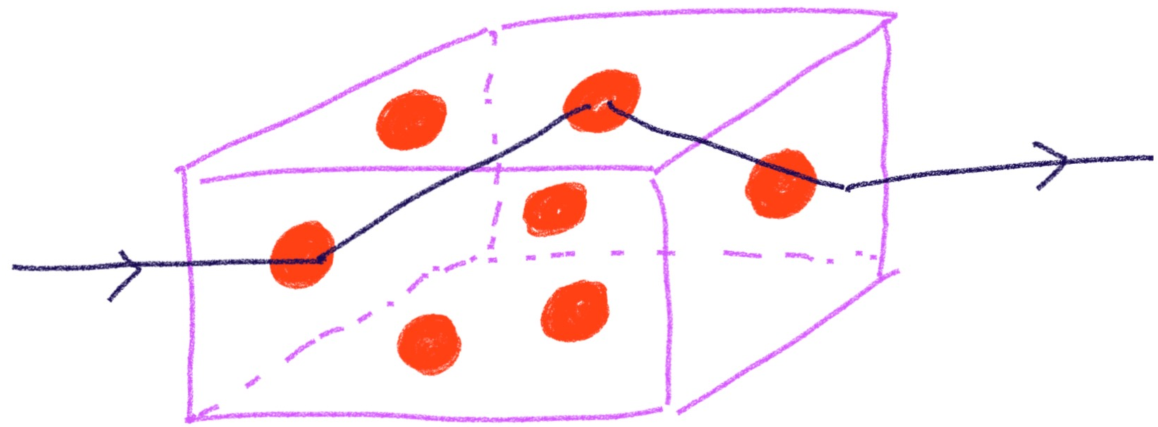
Spin diffusion due to "drag force" on "axion" propagation in the shock wave background
-drag force is proportional to sphaleron transition rate

McLerran,Mottola,Shaposhnikov (1990)

$$g_1^{\text{Regge}}(x_B, Q^2) \propto \text{F}(x_B) \times \frac{Q_S^2 m_{\eta'}^2}{F_{\tilde{\eta}}^3 M_N} \exp\left(-4\, n_f C \, \frac{Q_S^2}{F_{\tilde{\eta}}^2}\right)$$

Very rapid quenching of spin diffusion at small x_{Bj} !

Spin diffusion: sphaleron transitions in topologically disordered media



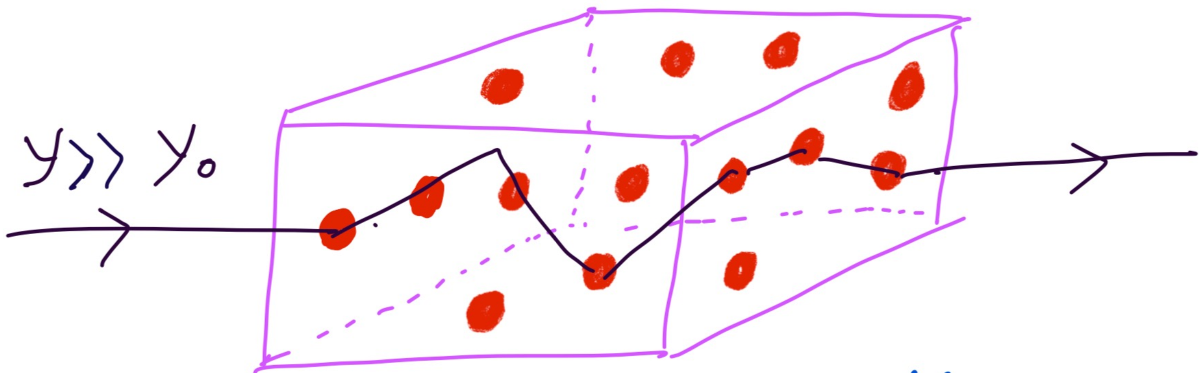
● Dense gluon blob of size $1/\sqrt{\sigma}$ given by $\Gamma^{Y}_{\text{sphaleron}} = \#\, \sigma^2$ & carrying topological charge

Atiyah-Singer index theorem

Helicity flip for massless quarks given by $n_L - n_R = n_f \nu$,

where ν is the topological charge and $\Gamma^{Y}_{\text{sphaleron}} \propto \langle \nu^2 \rangle$

Spin diffusion: sphaleron transitions in topologically disordered media



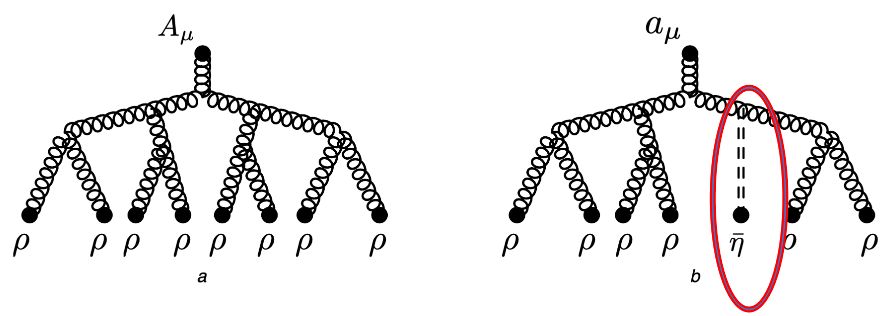
As x decreases ($y >> y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Expect very rapid quenching of g_1 at small x_B:
interplay between QCD evolution of the topological charge and the saturation scale

Thank you for your attention !

What about g_1 at small x_{Bj} ?

For $Q_S^2 > m_{\eta'}^2$,
axion is perturbation
to shock wave background



Calculation analogous to calculation of axion dynamics in the Glasma

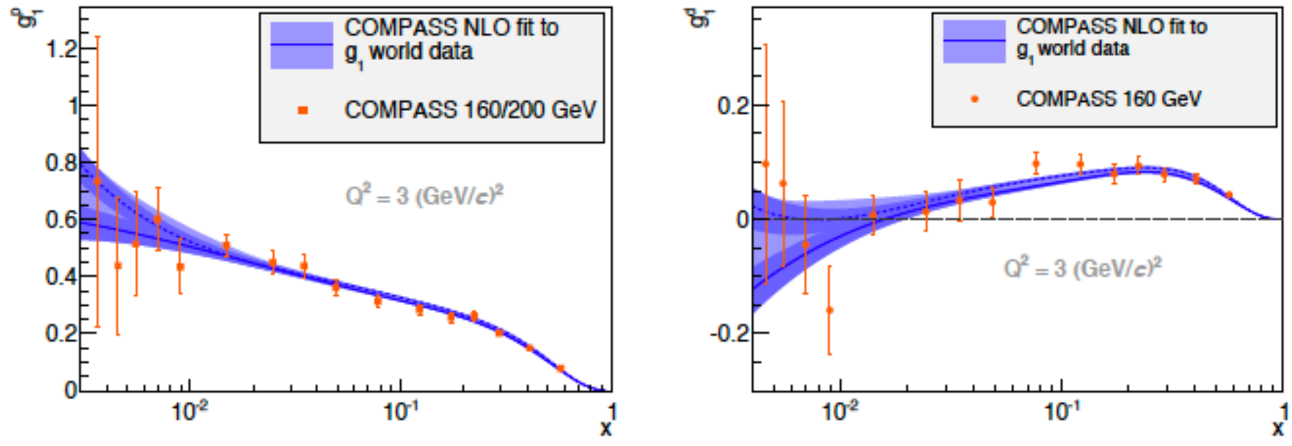
Jokela, Kajantie, and Sarkkinen, arXiv:2012.14160 [hep-ph]

Estimate also gives exponential suppression with increasing Q_S – prefactors remain to be worked out

Tarasov,Venugopalan, arXiv:2109.10370

g_1 at small x_{Bj} from sphaleron transitions

COMPASS: arXiv:1503.08935
arXiv: 1612.00620



The key feature of the topological screening picture is its target independence
However, as we have argued, the result is sensitive to the density of color sources, which is larger for the deuteron – so one anticipates the same behavior for g_1^p as g_1^d at smaller x_B

Other observables: semi-inclusive DIS, g_1^γ …work in progress