# Parton orbital angular momentum

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# Outline

- Definition of OAM
- Evolution of OAM
- Small-x limit of OAM
- Observables of OAM

# Proton spin decomposition

Jaffe-Manohar sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Enormous work has been done for the parton helicity contributions $\Delta\Sigma, \Delta G$

A big elephant in the room: Orbital angular momentums (OAM) $L^{q,g}_{can}$

What are they exactly?
Are they numerically important?
Are they measurable?

This talk is only about OAM in the JM sum rule, not one in the Ji sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

# Canonical OAM in QCD

Jaffe, Manohar (1990)

Decomposition of the canonical angular momentum tensor operator $M^{\mu\nu\rho}_{can}$

$$\Delta L_q = \frac{1}{2E(2\pi)^3\delta^3(0)} \left\langle p^0_\infty, s^0 \middle| \int \mathrm{d}^3x\, i\psi^\dagger(\boldsymbol{x}\times\boldsymbol{\nabla})^3\psi \middle| p^0_\infty, s^0 \right\rangle,$$

$$\Delta L_g = \frac{1}{2E(2\pi)^3\delta^3(0)} \left\langle p^0_\infty, s^0 \middle| \int \mathrm{d}^3x\, \mathrm{Tr}\{E^k(\boldsymbol{x}\times\boldsymbol{\nabla})^3 A^k\} \middle| p^0_\infty, s^0 \right\rangle.$$

To be understood in the light-cone gauge $A^+ = 0$

# Gauge invariant canonical OAM

YH (2011)

see also, Bashinsky, Jaffe (1999);
Chen, Lu, Sun, Wang, Goldman (2008)

Exact definition of OAM to be used in the Jaffe-Manohar decomposition

$$\lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i\overleftrightarrow{D}^i_{pure}\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{can}$$

$$\lim_{\Delta\to 0}\langle P'S|F^{+\alpha}\overleftrightarrow{D}^i_{pure}A^{phys}_\alpha|PS\rangle = -i\epsilon^{ij}\Delta_{\perp j}S^+L^g_{can}$$

$$A^\mu_{phys}(x) = \frac{1}{D^+}F^{+\mu} = \int_{x^-}^{\infty} dz^- W[x^-,z^-]F^{+\mu}(z^-,x_\perp)$$

$$D^\mu_{pure} = D^\mu - iA^\mu_{phys}$$

The boundary condition for the operator $1/D^+$ does not matter

# OAM from the Wigner distribution

Lorce, Pasquini (2011);
YH (2011);
Lorce, Pasquini, Xiong, Yuan (2011)

Wigner distribution
Phase space distribution of partons in QCD

Belitsky, Ji, Yuan (2004)

$$W(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2}\frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle P-\frac{\Delta}{2}|\bar{q}(b-z/2)\gamma^+ q(b+z/2)|P+\frac{\Delta}{2}\rangle$$

Define

$$L^q = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp\times\vec{k}_\perp)_z W^q(x,\vec{b}_\perp,\vec{k}_\perp)$$

# OAM from the generalized TMD (GTMD)

Fourier transform of Wigner : GTMD $W(x,\vec{k}_\perp,\vec{b}_\perp) \to W(x,\vec{k}_\perp,\vec{\Delta}_\perp)$

$$\int \frac{d^3z}{2(2\pi)^3} e^{ixP^+z^- - i\widetilde{k}_\perp\cdot\widetilde{z}_\perp} \langle p'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|p\rangle$$

$$= \frac{1}{2M}\bar{u}(p')\left[F^q_{1,1} + i\frac{\sigma^{j+}}{P^+}(\widetilde{k}^j_\perp F^q_{1,2} + \widetilde{\Delta}^j_\perp F^q_{1,3}) + i\frac{\sigma^{ij}\widetilde{k}^i_\perp\widetilde{\Delta}^j_\perp}{M^2}F^q_{1,4}\right]u(p)$$

$$L_{q,g} = -\int dx \int d^2k_\perp \frac{k^2_\perp}{M^2} F^{q,g}_{1,4}(x)$$

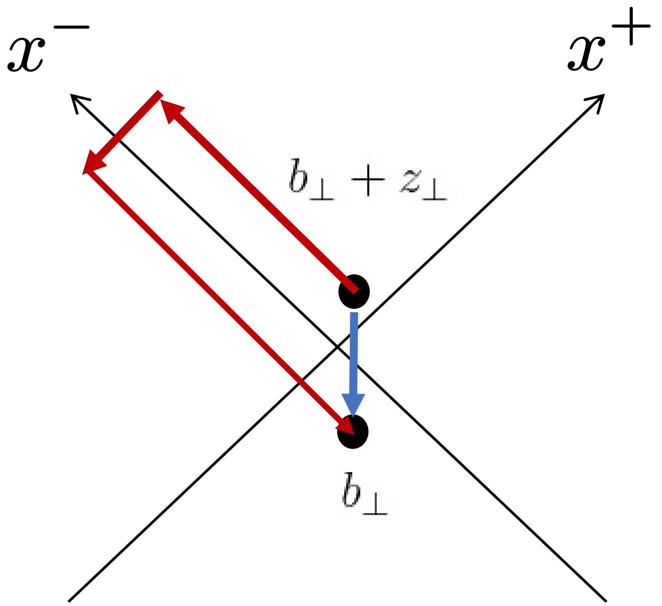
Which OAM is this?

# Wilson lines and OAMs

Canonical (JM) OAM from the light-cone staple Wilson line YH (2011)

$$\int d^2k_\perp (b_\perp \times k_\perp) W_{LC}(b_\perp, k_\perp) = \langle \bar{\psi} b_\perp \times i D_\perp^{pure} \psi \rangle$$

$$D_{pure}^\perp = D^\perp - \frac{i}{D^+} F^{+\perp}$$

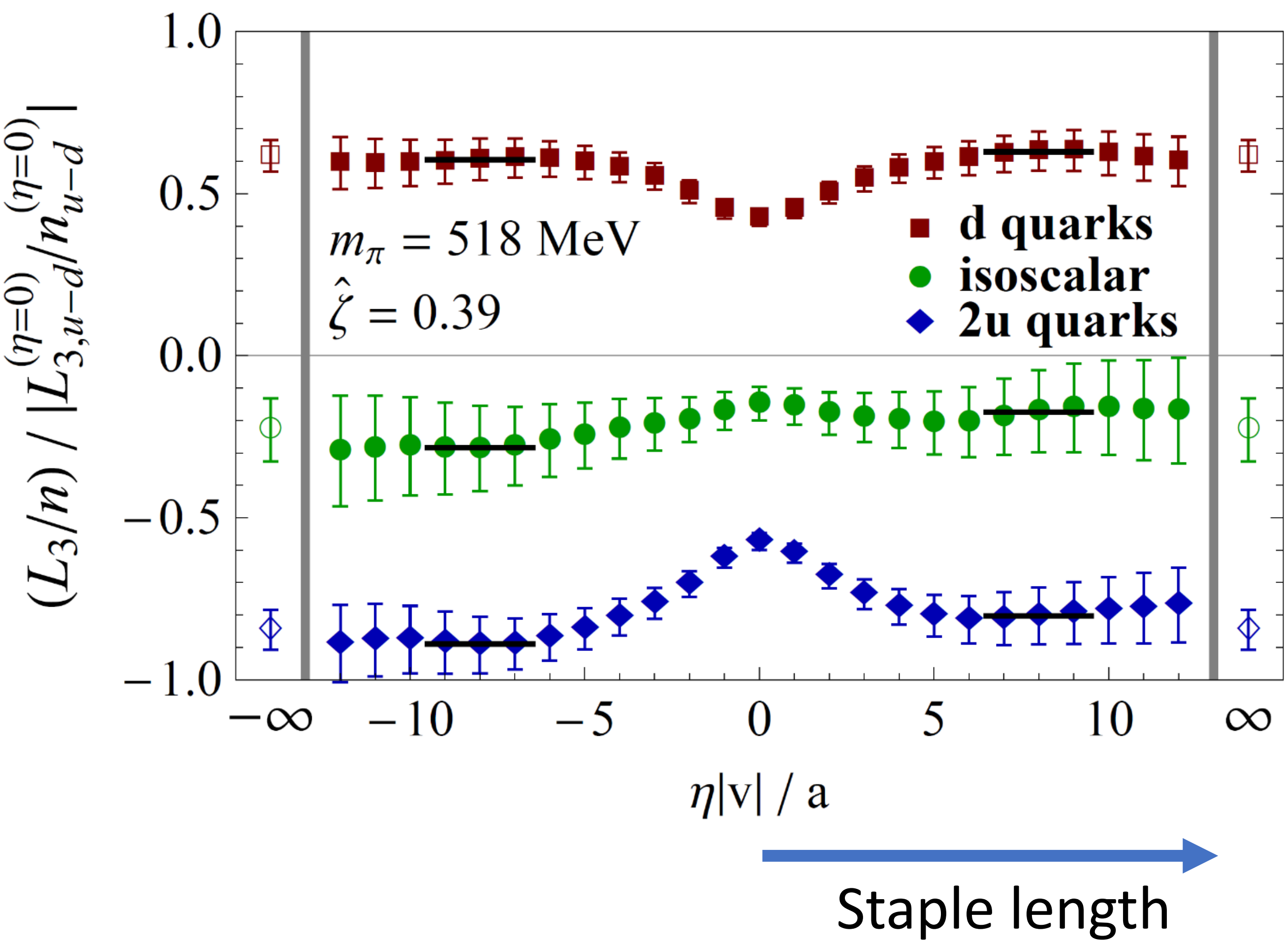



Kinetic (Ji's) OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

# Jaffe-Manohar vs. Ji on a lattice

Engelhardt (2017)
Engelhardt et al. (2020)



# `PDF' of OAM

In order to determine $\Delta\Sigma, \Delta G$ , we first extract the associated PDFs $\Delta q(x), \Delta G(x)$ and integrate over x.

Same with $L^{q,g}_{can}$

OAM from the Wigner distribution

$$L^q_{can} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

Define the x-distribution .

$$\Rightarrow \quad L^q_{can}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

It's a twist-3 PDF, similar to $g_2(x)$.

# Twist structure of OAM PDF

YH, Yoshida (2012)

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x')+E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1-x_2}{x_1^2(x_1-x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)}\,.$$

genuine twist-3

$$\Phi_F \sim \langle P'|\bar{\psi}\gamma^+ F^{+i}\psi|P\rangle$$

$$M_F \sim \langle P'|F^{+\mu}F^{+i}F^+_\mu|P\rangle$$

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$

$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$

$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

# Evolution of $L_{q,g}(x)$ : WW part

$$\frac{d}{d\ln Q^2}\begin{pmatrix}L_q(x)\\ L_g(x)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix}\hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z)\\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z)\end{pmatrix}\begin{pmatrix}L_q(x/z)\\ L_g(x/z)\\ \Delta q(x/z)\\ \Delta G(x/z)\end{pmatrix},$$

Leading order — Hagler, Schafer (1998)
Harindranath, Kundu (1999)
Hoodbhoy, Ji, Lu (1999)

All orders — Boussarie, YH, Yuan (2019)

$$\frac{\partial}{\partial\ln Q^2}\begin{pmatrix}L_q^\omega\\ L_g^\omega\end{pmatrix} = \begin{pmatrix}\gamma_{qq}^{\omega+1} & \gamma_{qg}^{\omega+1}\\ \gamma_{gq}^{\omega+1} & \gamma_{gg}^{\omega+1}\end{pmatrix}\begin{pmatrix}L_\Sigma^\omega\\ L_g^\omega\end{pmatrix}$$

$$+\frac{1}{\omega+1}\begin{pmatrix}\gamma_{qq}^{\omega+1}-\Delta\gamma_{qq}^{\omega} & 2\gamma_{qg}^{\omega+1}-\Delta\gamma_{qg}^{\omega}\\ \gamma_{gq}^{\omega+1}-2\Delta\gamma_{gq}^{\omega} & 2\gamma_{gg}^{\omega+1}-2\Delta\gamma_{gg}^{\omega}\end{pmatrix}\begin{pmatrix}\Delta\Sigma^\omega\\ \Delta G^\omega\end{pmatrix}$$

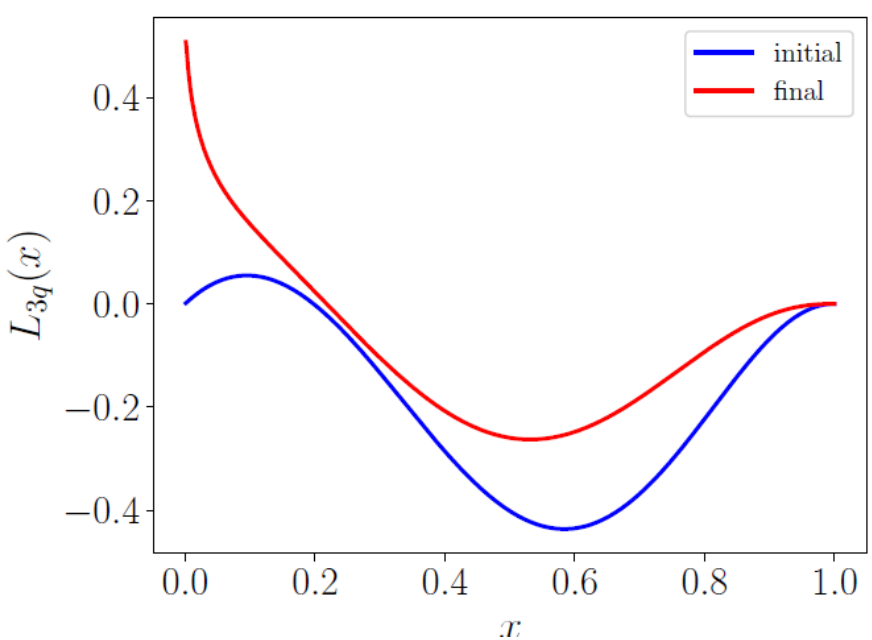
# Evolution of $L_{q,g}(x)$ : genuine twist-3 part

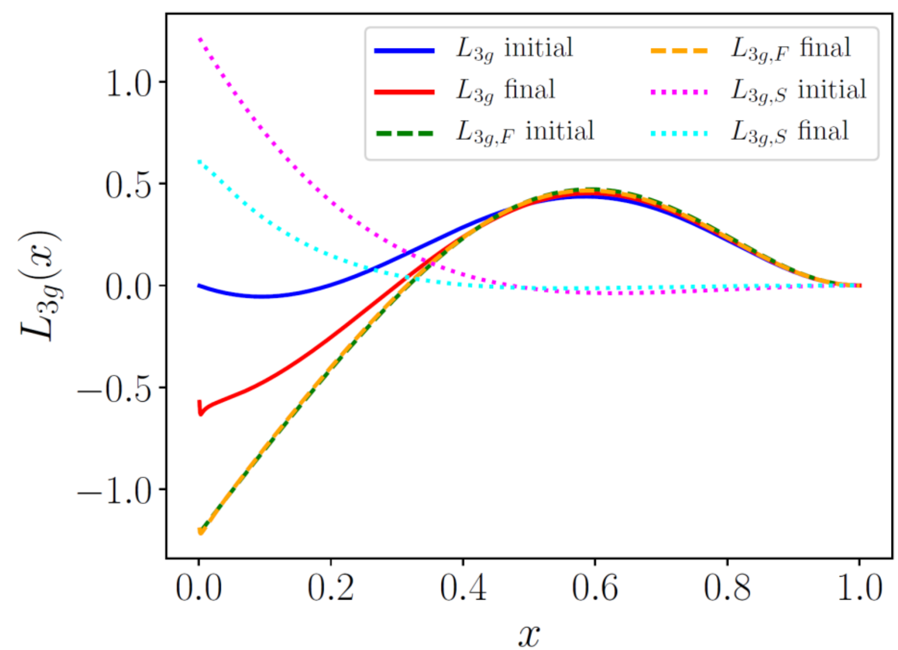
YH, Yao (2019)

Non-forward matrix elements of $\begin{matrix} \bar{\psi}\gamma^+ F^{+i}\psi \\ F^{+\mu}F^{+i}F^+_\mu \end{matrix}$ in the limit $\Delta_\perp \to 0$, zero skewness

→ The same evolution as for the Efremov-Teryaev-Qiu-Sterman function.

Numerical code developed by Pirnay (2013)
Straightforward to adapt to the OAM problem

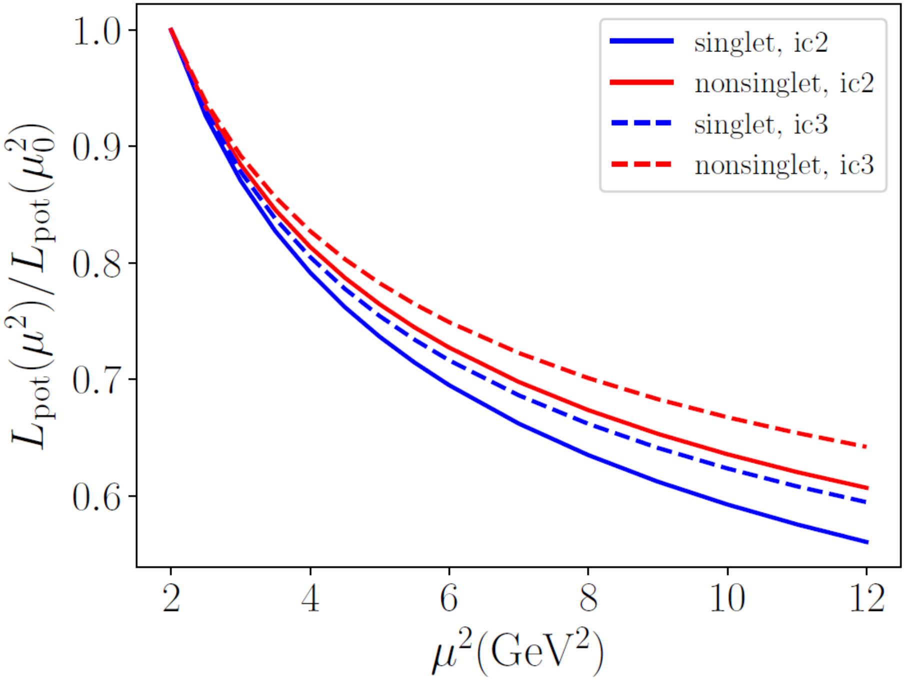


# Scale evolution of the `potential' angular momentum

YH, Yao (2019)

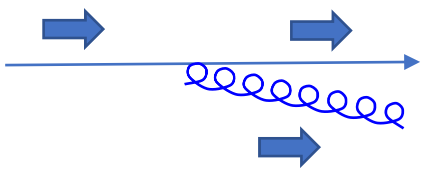
$$L_{pot} = L_{Ji} - L_{can} \sim \langle \bar{\psi}\gamma^+ \epsilon^{ij} x^i \times \frac{1}{D^+} F^{+j} \psi \rangle$$




Cf. $L_{pot} = 0$ to one-loop in QED    Ji, Schafer, Yuan, Zhang, Zhao (2016)

Evolution not characterized by a single anomalous dimension.
Different Mellin moments mix under evolution.

# OAM at small-x



$$x \ll 1$$

Suppose a quark emits a very soft gluon.
Nothing happens to the quark.
From angular momentum conservation, gluon helicity and OAM must cancel.

$$\frac{d}{d\ln Q^2}L_g(x) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}(-2C_F + \cdots)\Delta q(x/z)$$

$$\frac{d}{d\ln Q^2}\Delta G(x) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}(+2C_F + \cdots)\Delta q(x/z)$$

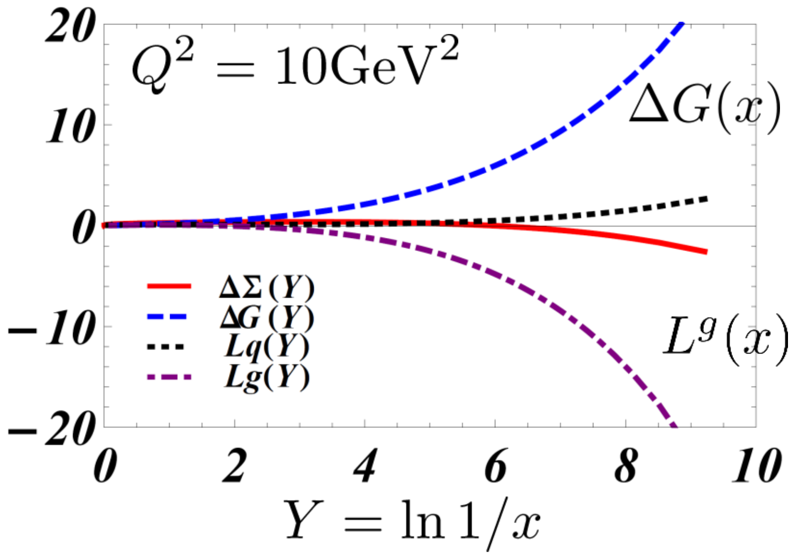
# 1-loop DGLAP evolution

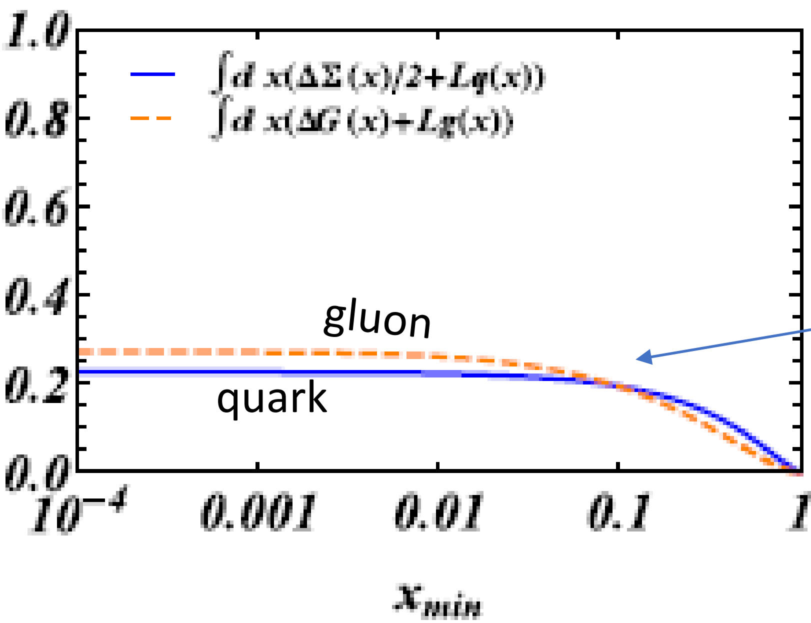
YH, Yang (2018)

`Democratic model'

$$\Delta\Sigma(x,Q_0^2)=\frac{1}{4},\quad \Delta G(x,Q_0^2)=\frac{1}{8},$$

$$L_q(x,Q_0^2)=\frac{1}{8},\quad L_g(x,Q_0^2)=\frac{1}{8},$$




$$\int_{x_{min}}^{1} dx\left(\frac{1}{2}\Delta\Sigma(x)+\Delta G(x)+L_q(x)+L_g(x)\right)$$

saturate due to the cancellation between helicity and OAM

# All-order argument

Boussarie, YH, Yuan (2019)

Start from the exact formula YH, Yoshida (2013)

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

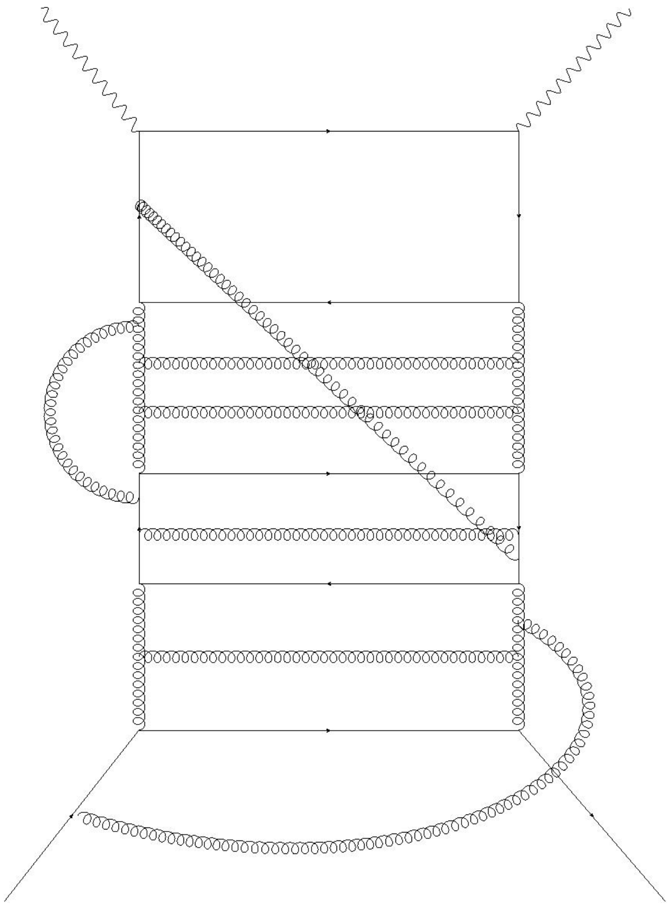
Assume that the helicity term dominates on the rhs
(in the spirit of double-log approximation)

If $\Delta G(x) \sim \frac{1}{x^\alpha}$, then $L_g(x) \approx -\frac{2}{1+\alpha}\Delta G(x)$

# Double logarithmic approximation (DLA)

Higher order diagrams for $\Delta\Sigma(x), \Delta G(x)$ contain double logarithms $(\alpha_s \ln^2 1/x)^n$

The same is expected for OAM at small-x.



Unlike BFKL, we need to resum quark ladders and non-ladder diagrams.

Resummation hard, but can be done.

Kirshner, Lipatov (1983)

Bartels, Ermolaev, Ryskin (1996),
Kovchegov, Pitonyak. Sievert (2015~)
Kovchegov (2019)
Boussarie, YH, Yuan (2019)
Cougoulik, Kovchegov, Tarasov, Tawabutr (2022)

# InfraRed Evolution Equation (IREE) for helicity PDF

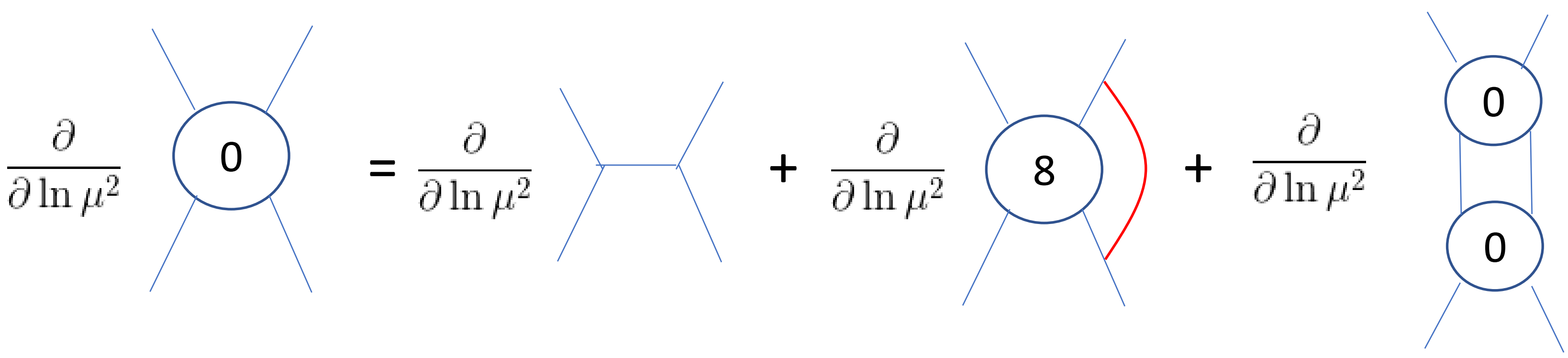
Kirshner, Lipatov (1983)

Bartels, Ermolaev, Ryskin (1996),

$$F_0 = \frac{g^2}{\omega} M_0 - \frac{g^2}{2\pi^2\omega^2} F_8 G_0 + \frac{1}{8\pi^2\omega} F_0^2$$

$$F_8 = \frac{g^2}{\omega} M_8 + \frac{g^2 C_A}{8\pi^2\omega} \frac{d}{d\omega} F_8 + \frac{1}{8\pi^2\omega} F_8^2$$

$$M_0 = \begin{pmatrix} C_F & -2T_f \\ 2C_F & 4C_A \end{pmatrix} \qquad M_8 = \begin{pmatrix} -1/2N_c & -T_f \\ C_A & 2C_A \end{pmatrix} \qquad G_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}$$



# Generalizing IREE to OAM

Boussarie, YH, Yuan (2019)

$$F_0 = \frac{g^2}{\omega} M_0 - \frac{g^2}{2\pi^2\omega^2} F_8 G_0 + \frac{1}{8\pi^2\omega} F_0^2$$

$$F_8 = \frac{g^2}{\omega} M_8 + \frac{g^2 C_A}{8\pi^2\omega} \frac{d}{d\omega} F_8 + \frac{1}{8\pi^2\omega} F_8^2$$

The same coupled equations, but now with 4x4 matrices.

$$M_0 = \begin{pmatrix} C_F & -2T_f & 0 & 0 \\ 2C_F & 4C_A & 0 & 0 \\ -C_F & 2T_f & 0 & 0 \\ -2C_F & -4C_A & 2C_F & 2C_A \end{pmatrix} \quad G_0 = \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_A & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_A \end{pmatrix} \quad M_8 = \begin{pmatrix} -1/2N_c & -T_f & 0 & 0 \\ C_A & 2C_A & 0 & 0 \\ 1/2N_c & T_f & 0 & 0 \\ -C_A & -2C_A & C_A & C_A \end{pmatrix}$$

# Exact solution

$$F_0^{2\times 2} = \frac{g^2}{\omega} M_0^{2\times 2} + \begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \end{pmatrix}$$

← Bartels, Ermolaev, Ryskin solution

$$F_0^{4\times 4} = \frac{g^2}{\omega} M_0^{4\times 4} + \begin{pmatrix} A_1 & A_2 & 0 & 0 \\ B_1 & B_2 & 0 & 0 \\ -A_1 & -A_2 & 0 & 0 \\ -2B_1 & -2B_2 & 0 & 0 \end{pmatrix}$$

$\times(-1)$ $\times(-2)$

Fully consistent with the evolution of WW part in DLA

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} L_q^\omega \\ L_g^\omega \end{pmatrix} = \begin{pmatrix} \gamma_{qq}^{\omega+1} & \gamma_{qg}^{\omega+1} \\ \gamma_{gq}^{\omega+1} & \gamma_{gg}^{\omega+1} \end{pmatrix} \begin{pmatrix} L_\Sigma^\omega \\ L_g^\omega \end{pmatrix}$$

$$+ \frac{1}{\omega+1} \begin{pmatrix} \gamma_{qq}^{\omega+1} - \Delta\gamma_{qq}^\omega & 2\gamma_{qg}^{\omega+1} - \Delta\gamma_{qg}^\omega \\ \gamma_{gq}^{\omega+1} - 2\Delta\gamma_{gq}^\omega & 2\gamma_{gg}^{\omega+1} - 2\Delta\gamma_{gg}^\omega \end{pmatrix} \begin{pmatrix} \Delta\Sigma^\omega \\ \Delta G^\omega \end{pmatrix} + \cdots$$

# Result in DLA

$n_f = 4$

$$\Delta G(x) \approx -2.29\Delta\Sigma(x) \propto x^{-3.45\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

$$L_g(x) \approx -2\Delta G(x), \quad \Delta\Sigma(x) \approx -L_q(x)$$

Compare with the all-order argument

$$L_g(x) \approx -\frac{2}{1+\alpha}\Delta G(x)$$

EIC will allow us to constrain $\Delta G(x)$ at small-x.

However, this will not solve the spin puzzle because whatever helicity we find at small-x will be more than compensated by the OAM.

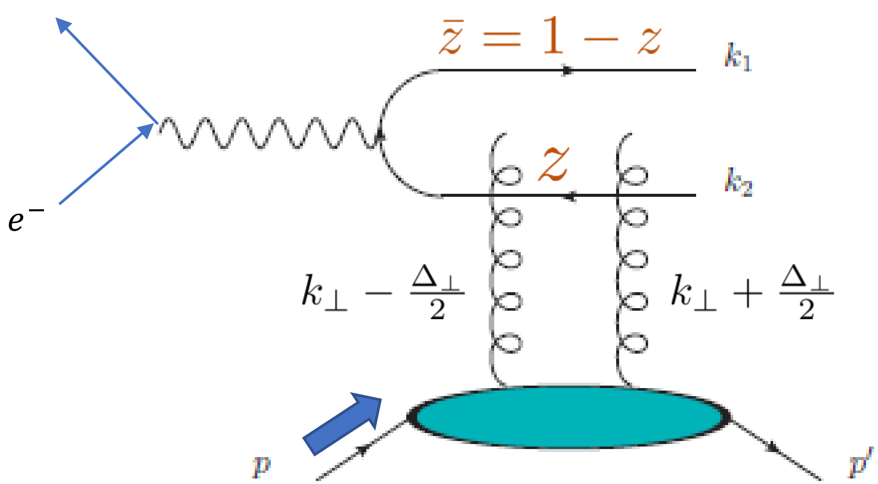
# Observables for OAM

- Nonexistent…until recently
- Still at the level of identifying processes that may be sensitive to OAM at leading order.
- Measuring OAM = Measuring Wigner. Challenging for an all-order factorization

# Longitudinal single spin asymmetry in diffractive dijets

Leading order, unpolarized (twist-2 GPDs) Braun, Ivanov (2005)

Single spin asymmetry Ji, Yuan, Zhao (2016); YH, Nakagawa, Yuan, Xiao, Zhao (2016)




Expand the amplitude to linear order in $k_\perp$ (twist-3 effect)

$$\int d^2k_\perp k_\perp^i f_g(k_\perp, \Delta_\perp) \sim \epsilon^{ij} \Delta_\perp^j L_g$$

→ sensitivity to OAM

Interference between twist-2 and twist-3 amplitudes

$$d\sigma^{h_p} \sim h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp})(z - \bar{z}) \Im\mathrm{m}(A_2 A_3^*)$$

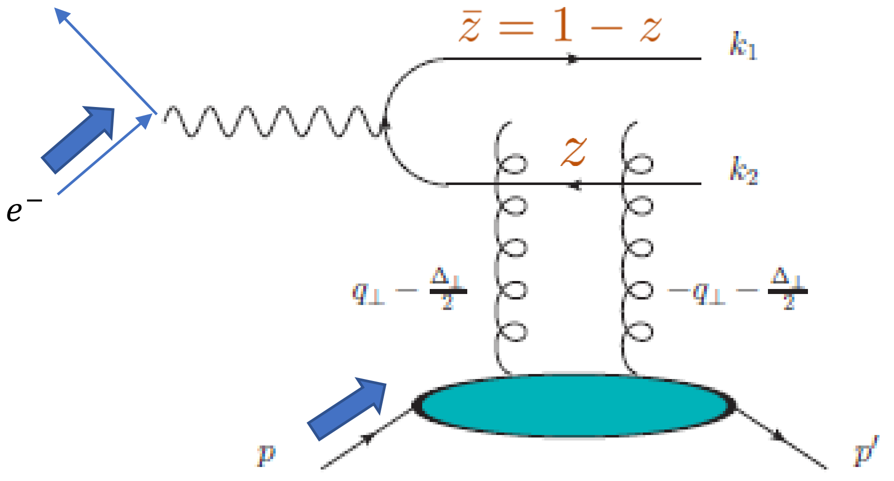
dijet relative momentum

proton recoil momentum

Asymmetry vanishes for symmetric jet configurations z=1/2

# Longitudinal double spin asymmetry in diffractive dijets

Bhattacharya, Boussarie, YH, 2201.08709




Interference between twist-2 (unpol gluon GPD) and twist-3 (gluon OAM) $\mathcal{H}_g \mathcal{L}_g^*$

Interference between longitudinal and transverse photon amplitudes

Another contribution from helicity GPD "kinematical higher twist" effect (also present in SSA) $\mathcal{H}_g \tilde{\mathcal{H}}_g^*$

$$d\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \mathfrak{Re}(A'_2 A'^*_3)$$

lepton momentum

Does not vanish at z=1/2

# Collinear factorization breaking?

In general, the cross section contains Compton form factors with a third pole

$$\int dx \frac{H_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^3}$$

The x-integral is divergent if gluon GPDs contain

$$H_g(x,\xi) \sim \theta(\xi - |x|)(x^2-\xi^2)^2$$

Luckily, all these factorization breaking terms can be eliminated by setting $z = 1/2$

In SSA, one cannot set $z = 1/2$ because the asymmetry vanishes there.

# Complete result

$$\frac{d\sigma}{dydQ^2d\phi_{l_\perp}dzdq_\perp^2d^2\Delta_\perp}=\frac{\alpha_{em}y}{2^{11}\pi^7Q^4}\frac{\int d\phi_{q_\perp}L^{\mu\nu}A_\mu^*A_\nu}{(W^2+Q^2)(W^2-M_J^2)z\bar{z}}$$

$$\int d\phi_{q_\perp}L^{\mu\nu}A_\mu^*A_\nu=-\frac{2^{10}\pi^4}{N_c}h_lh_p\alpha_s^2\alpha_{em}e_q^2\frac{(1+\xi)\xi Q^2}{(q_\perp^2+\mu^2)^2}|l_\perp||\Delta_\perp\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})$$
$$\times\mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*}-\frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*}+\frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*}-\frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g\right.$$
$$\left.+\left(\mathcal{E}_g^{(1)*}+\frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right]$$

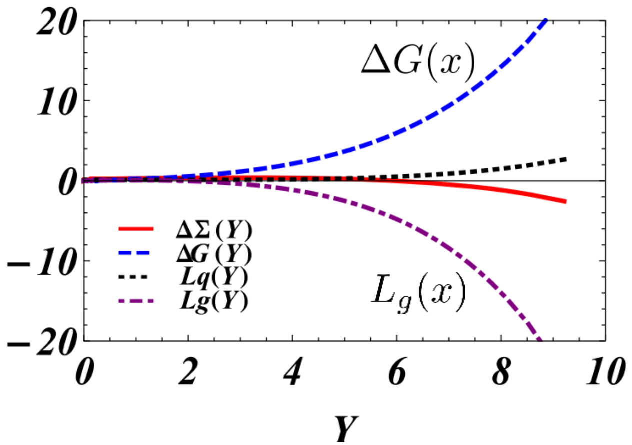
$$+\frac{2^{10}\pi^4}{N_c}h_lh_p\alpha_s^2\alpha_{em}e_q^2\frac{(1-\xi^2)\xi Q^2}{(q_\perp^2+\mu^2)^2}|l_\perp||\Delta_\perp|\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})$$
$$\times\mathfrak{Re}\left[\left(\mathcal{H}_g^{(1)*}-\frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*}\right)\left(\tilde{\mathcal{H}}_g^{(2)}-\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_g^{(2)}\right)\right]$$

# Interplay between OAM and helicity

Cross section proportional to
$$\mathcal{H}_g^{(1)}\left(\tilde{\mathcal{H}}_g^{(2)} + \frac{q_\perp^2 - z\bar{z}Q^2}{q_\perp^2 + z\bar{z}Q^2}\mathcal{L}_g\right)$$

$$\tilde{\mathcal{H}}_g^{(2)}(\xi) = \int dx \frac{x\tilde{H}_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^2} \qquad \mathcal{L}_g(\xi) = \int_{-1}^{1} dx \frac{x^2 L_g(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2}$$

Gluon OAM and helicity opposite signs at small-x
YH, Yang (2018); Boussarie, YH, Yuan (2019)




Depending on the sign of $q_\perp^2 - \frac{Q^2}{4}$, the two contributions add up/cancel.

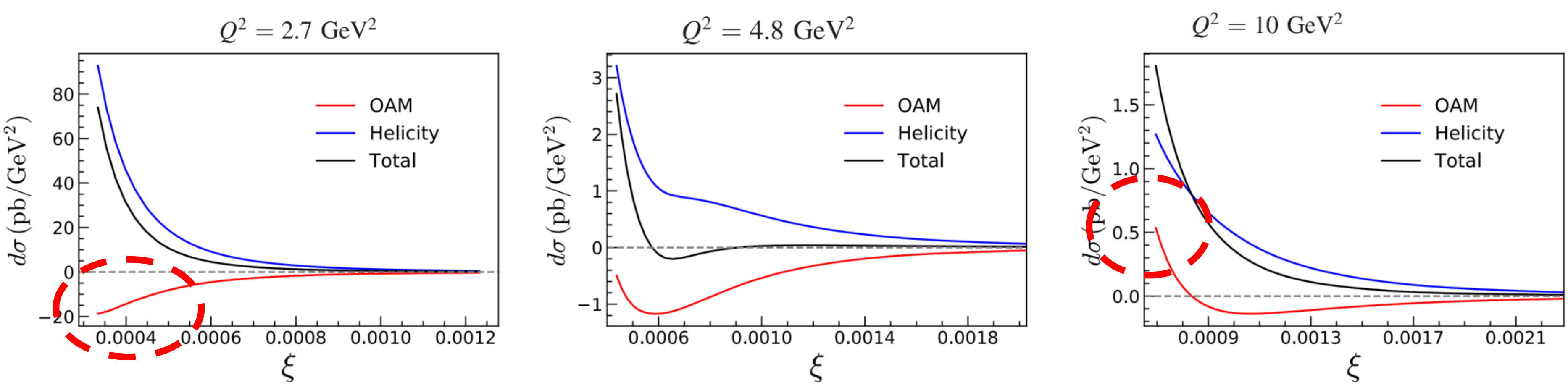
# Prediction for EIC

Use the Wandzura-Wilczek approximation for gluon OAM

$$L_g(x) \approx x \int_x^1 \frac{dx'}{x'^2} x' G(x') - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x')$$

...though our ultimately interest is in the deviation from this formula

Use the double distribution trick (Radyushkin) to model GPDs $H_g(x) \to H_g(x, \xi)$ etc.




First-ever quantitative prediction for an observable sensitive to OAM

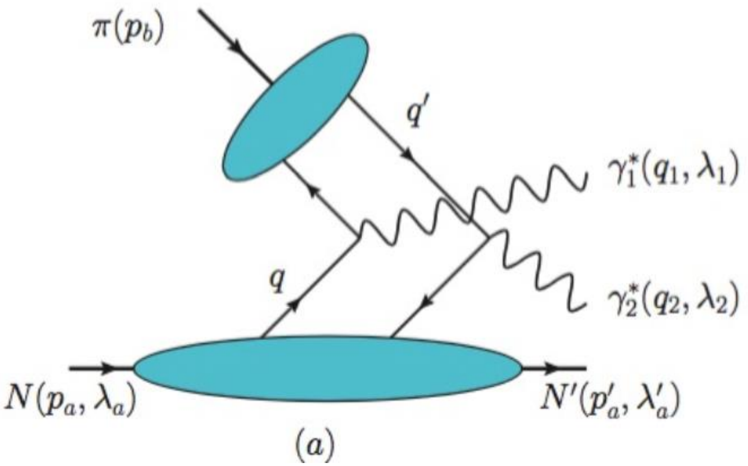
To extract OAM, we need to know $\Delta G(x)$ precisely at small-x

# Quark OAM?

Bhattacharya, Metz, Zhou (2017)

Spin asymmetires in double Drell-Yan

$$\pi N \to (\ell_1^- \ell_1^+)(\ell_2^- \ell_2^+) N'$$




$$\frac{1}{2}(\tau_{LU} + \tau_{UL}) = \frac{1}{2}(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{-,-}|^2) = \frac{4}{M^2}\,\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\,\mathrm{Im}\Big\{C^{(-)}\Big[F_{1,1}\,\Phi_\pi\Big]C^{(+)}\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,F_{1,4}^*\,\Phi_\pi^*\Big] - C^{(+)}\Big[G_{1,4}\,\Phi_\pi\Big]C^{(-)}\Big[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\,G_{1,1}^*\,\Phi_\pi^*\Big]\Big\}.$$

A proof of factorization with GTMDs?

Echevarria. Gutierrez Garcia, Scimemi 2208.00021

# Conclusions

- OAM is an essential component of the spin sum rule.
- Helicity is not RG invariant. OAM is always there.
- Unraveling the proton spin structure is a key mission of EIC. More attention/discussion needed in the community.
- Our proposal: DSA in diffractive dijet. Reduces to a twist-3 GPD observable.