Final-state interactions in e-A collisions from eHIJING

CFNS Workshop – Jet Physics: from RHIC/LHC to EIC
Stony Brook University, Jun 29, 2022

HIJING

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In preparation.

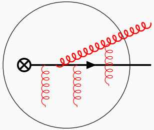
Jet tomography of nuclear matter in *e*-*A* collisions

Distinguish & **determine** different nuclear matter effects using jets & hadrons.

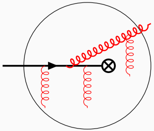
Dynamical effects:

- Medium-modified jet evolution.
- Evolution of the nuclear target.
- Process dependent, for example

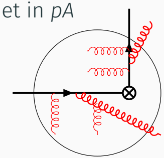
DIS in *eA*



Drell-Yan in *pA*



h/jet in *pA*

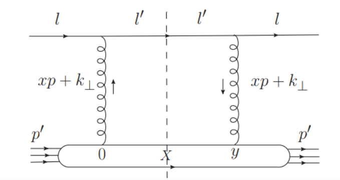


Nuclear non-perturbative input

- Nuclear parton distribution.
- In-medium fragmentation.
- Intrinsic properties of the medium.

In-medium evolution is needed to consistently extract NP inputs.

Final-state interactions in nuclear & *eA* collisions



Final-state interactions are mediated by Glauber gluons $\Leftrightarrow$ k_T-dependent medium gluon density at small x

$$\phi_g(x, \mathbf{k}_\perp) = \int \frac{d\xi^+ d\vec{\xi}_T^2}{2\pi P^-} e^{-ixP^- \xi^+ - i\mathbf{k}_\perp \cdot \vec{xi}_T} \langle F^{i-}(0, \vec{0}) F_i^-(\xi^+, \vec{\xi}_T) \rangle.$$

- Collisional broadening of parton
- In-modified QCD splitting functions modifies both p_T and z-dependence of hadron/jet fragmentation.

$\Rightarrow$ **a modified fragmentation of hadron & jet** $D(z, p_T)$

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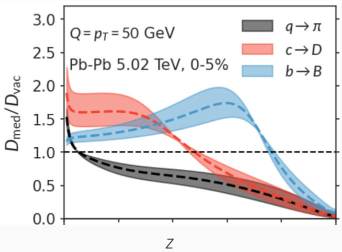

△ Huge effects in nuclear collisions. $\Delta p_T \sim 1$ GeV. But calculations & direct measurement of the 2D modified fragmentation is hard.

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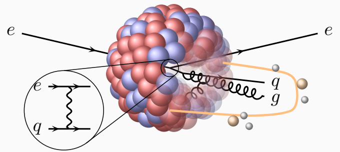
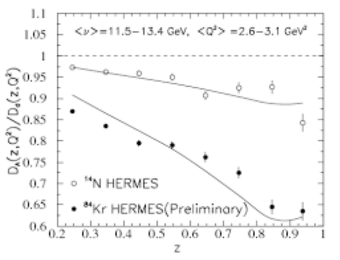
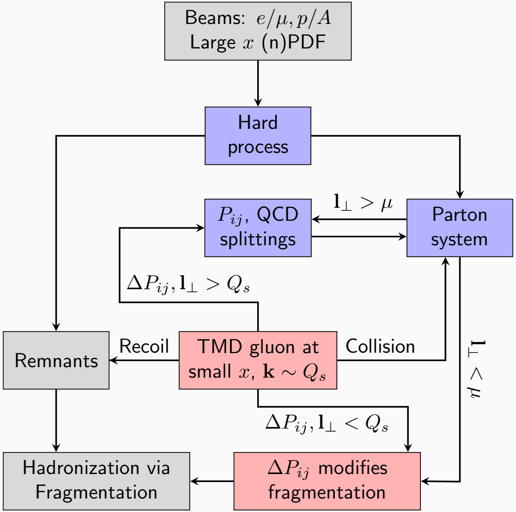




[E Wang, X-N Wang PRL89 162301]

e-*A* provide direct access to $D^h(z,p_T)$ to constrain theory.

eHIJING (electron-Heavy-Ion-Jet-INteraction-Generator)



- Dynamical medium corrections
  - Multiple jet-nucleus collisions.
  - Parton shower development and hadronization with final-state effects.
- *e*-*p* event generation in the vacuum using Pythia8.

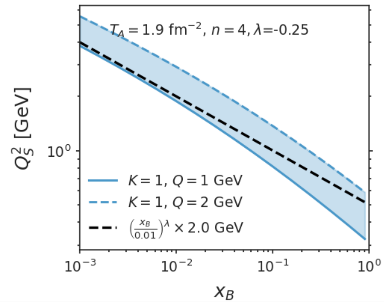
Parametrize the k_T-dependent gluon distribution at small x_g

A saturation-based model of $\phi_g(x, \mathbf{k}_\perp^2)$ [No evolution at the moment!]

$$\phi_g(x_g, \mathbf{k}_\perp) = \frac{N}{\alpha_s} \frac{(1-x)^n x^\lambda}{\mathbf{k}_\perp^2 + Q_s^2(x_B, Q^2)}$$

For a given nuclear thickness $T_A(\mathbf{b})$, Q_s is determined self-consistently [Y-Y Zhang, X-N Wang PRD105(2022)034015; A. Mueller NPB558(1999)285-303].

$$Q_s^2(x_B, Q^2) = T_A \frac{C_A}{d_A} \int_{\Lambda^2}^{Q^2/x_B} d^2\mathbf{k}_\perp \alpha_s \phi_g(x_B \frac{\mathbf{k}_\perp^2}{Q^2}, \mathbf{k}_\perp^2; Q_s^2)$$



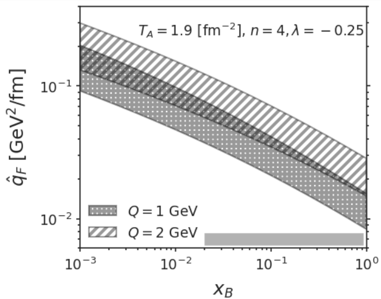
Averaged collisional p_T broadening v.s. the modified p_T spectra

p_T broadening at LO (single hard parton)

$$\langle \Delta p^2_{T,h} \rangle = z_h^2 \frac{C_R}{C_A} Q_s^2(x_B, Q^2)$$

The corresponding jet transport parameter

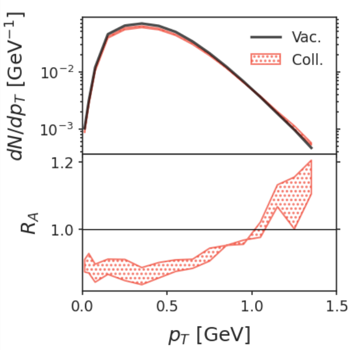
$$\hat{q}_R \equiv \frac{d\langle \Delta p_\perp^2 \rangle}{dL} = \frac{C_R}{C_A} \frac{Q_s^2(x_B, Q^2, T_A)}{L}$$



In a dilute medium, the number of collisions follows a Poisson distribution with large fluctuation.

$$P(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}, \ \langle N \rangle = \int_0^{L_{\max}} \rho(L) dL \int_{\Lambda^2}^{\frac{Q^2}{x_B}} \frac{C_R}{d_A} \frac{\alpha_s \phi_g}{\mathbf{k}_\perp^2} d\mathbf{k}_\perp^2$$

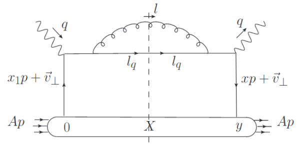
For each shower parton, sample N and individual collisions $(L_1, k_{\perp,1}, x_{g,1}), \cdots, (L_N, k_{\perp,N}, x_{g,N})$.



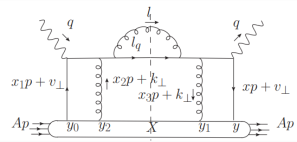
Modifications on top of the p_T distribution of the vacuum shower.

Medium-induced radiation will further modify dN/dp_T

Medium-modified QCD splitting functions at twist-4



$\cdots \times \frac{\alpha_s}{2\pi}\frac{1}{l_\perp^2}P_{qq}(z)$



$\cdots \times \frac{\alpha_s}{2\pi}\frac{1}{l_\perp^2}P_{qq}(z)\int_{\mathbf{k}_\perp}\frac{T_A\phi_g}{\mathbf{k}_\perp^2}\int_0^L N(t)dt$

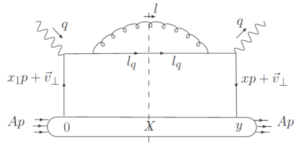
$$\frac{d\sigma^D_{eA}}{dx_B dQ^2 dz d^2l_\perp d^2l_{q\perp}} = \frac{2\pi\alpha_{em}^2}{Q^4}\sum_q e_q^2[1+(1-\frac{Q^2}{x_B s})^2]\frac{\alpha_s}{2\pi}\frac{1+z^2}{1-z}\frac{2\pi\alpha_s}{N_c}\int\frac{d^2k_\perp}{(2\pi)^2}\int d^2b_\perp dy_0^- dy_1^-$$
$$\times\rho_A(y_0^-,b_\perp)\rho_A(y_1^-,b_\perp)q_N(x_B,\vec{v}_\perp,b_\perp)\frac{\phi_N(x_G,\vec{k}_\perp)}{k_\perp^2}\left[\mathcal{N}_g^{\rm qLPM}+\mathcal{N}_g^{\rm gLPM}+\mathcal{N}_g^{\rm nonLPM}\right]. \quad (19)$$

$$\mathcal{N}_g^{\rm qLPM} = \frac{1}{N_c}\left(\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp]\cdot[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}-\frac{1}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2}\right)$$
$$\times\left(1-\cos[(x_L+x_E-x_F)p^+(y_1^- - y_0^-)]\right), \quad (20)$$

$$\mathcal{N}_g^{\rm gLPM} = C_A\left(\frac{2}{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2}-\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]\cdot[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}\right.$$
$$\left.-\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp]\cdot[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2}\right)\times\left(1-\cos[(x_L+\frac{z}{1-z}x_D+x_S-x_F)p^+(y_1^- - y_0^-)]\right), \quad (21)$$

$$\mathcal{N}_g^{\rm nonLPM} = C_F\left(\frac{1}{[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}-\frac{1}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2}\right), \quad (22)$$

[Y-Y Zhang, X-N Wang PRD105(2022)034015]



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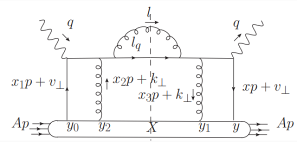

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$$\times(1-\cos[(x_L+x_E-x_F)p^+(y_1^--y_0^-)]), \qquad (20)$$

Suppressed by $1/N_c^2$

$$\mathcal{N}_g^{\rm gLPM} = C_A\left(\frac{2}{[\vec l_\perp-(1-z)\vec v_\perp-\vec k_\perp]^2}-\frac{[\vec l_\perp-(1-z)\vec v_\perp-\vec k_\perp]\cdot[\vec l_\perp-(1-z)(\vec l_\perp+\vec l_{q\perp})]}{[\vec l_\perp-(1-z)\vec v_\perp-\vec k_\perp]^2[\vec l_\perp-(1-z)(\vec l_\perp+\vec l_{q\perp})]^2}\right.$$
$$\left.-\frac{[\vec l_\perp-(1-z)\vec v_\perp]\cdot[\vec l_\perp-(1-z)\vec v_\perp-\vec k_\perp]}{[\vec l_\perp-(1-z)\vec v_\perp]^2[\vec l_\perp-(1-z)\vec v_\perp-\vec k_\perp]^2}\right)\times(1-\cos[(x_L+\frac{z}{1-z}x_D+x_S-x_F)p^+(y_1^--y_0^-)]), \qquad (21)$$

Broadening from LPM-type induced radiation

$$\mathcal{N}_g^{\rm nonLPM} = C_F\left(\frac{1}{[\vec l_\perp-(1-z)(\vec l_\perp+\vec l_{q\perp})]^2}-\frac{1}{[\vec l_\perp-(1-z)\vec v_\perp]^2}\right), \qquad (22)$$

Non-LPM, come from the shift of initial hard quark by $\mathrm{k}_\perp$

[Y-Y Zhang & X-N Wang, 2104.04520]

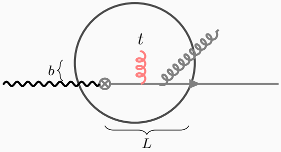
The LPM-type contribution

Depend on the phase factor $\sim 1-\cos(L\tau_f^{-1})$. $\tau_f^{-1} = \frac{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2}{2z(1-z)p^+} = \frac{(p')^-}{z(1-z)}$

$$P_{qq}(x, \mathbf{l}_\perp) = \frac{\alpha_s C_F}{2\pi}\frac{P_{qq}(z)}{l_\perp^2}\left\{1 + \int dL\rho(L)\int_{\mathbf{k}_\perp}\frac{C_A}{d_A}\frac{\alpha_s\phi_g(x_g, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2}\frac{2\mathbf{k}_\perp\cdot\mathbf{l}_\perp}{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2}\left[1-\cos\frac{L}{\tau_f}\right]\right\}$$

- $\tau_f^{-1}L \ll 1$: energetic splitting $z(p')^+ \gg p'_\perp(p'_\perp L)$, rare but modifies $D(z)$.

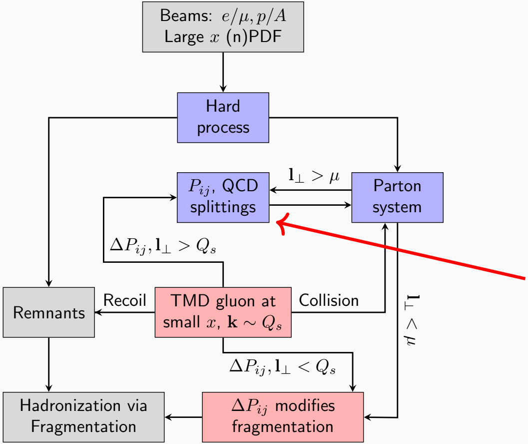
$$\Delta D(x, Q) = \frac{\alpha_s}{2\pi}\int^{Q^2}\frac{d\mathbf{l}_\perp^2}{\mathbf{l}_\perp^2}\int_x^1\frac{dz}{z}D_0\left(\frac{x}{z}\right)\Delta P_{\rm med}(z) + \cdots$$



- $\tau_f^{-1}L \gg 1$: frequent & incoherent emissions, important to the p_T recoil of the hard parton

$$\frac{\alpha_s C_F}{2\pi}\frac{P_{qq}(z)}{l_\perp^2}\int dL\rho(L)\int_{\mathbf{k}_\perp}\frac{C_A}{d_A}\frac{\alpha_s\phi_g(x_g, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2}2\Theta(\mathbf{k}_\perp^2 - \mathbf{l}_\perp^2)$$

Medium modified (DGLAP) parton shower

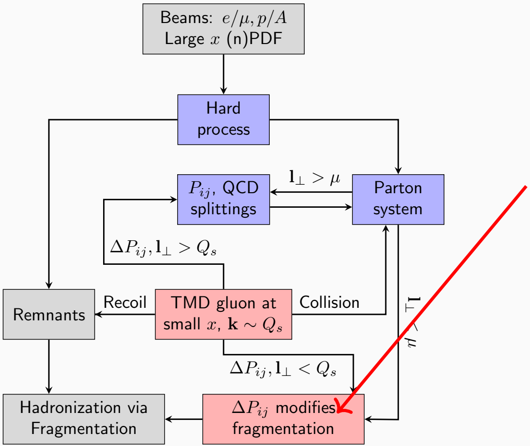


- In the saturation model, typical $\mathbf{k}_\perp \sim Q_s$.
- Induced emissions are qualitatively different in regions $\mathbf{k}_\perp \gg Q_s$ and $\mathbf{k}_\perp < Q_s$.
- For $Q_s \ll \mathbf{l}_\perp$, the modification is considered part of the DGLAP evolution.

  Medium splittings are added to vacuum splitting functions used in Pythia8 parton shower

$$P_{ij}(z, \mathbf{l}_\perp) = P_{ij}^{\mathrm{vac}}(z, \mathbf{l}_\perp) + \Delta P_{ij}^{\mathrm{med}}(z, \mathbf{l}_\perp)\Theta(\mathbf{l}_\perp - Q_s).$$

Medium-modified fragmentation



- For $\mathrm{l}_\perp \lesssim \mathrm{k}_\perp \sim Q_s$. Multiple **medium-induced** gluon emissions are generated from

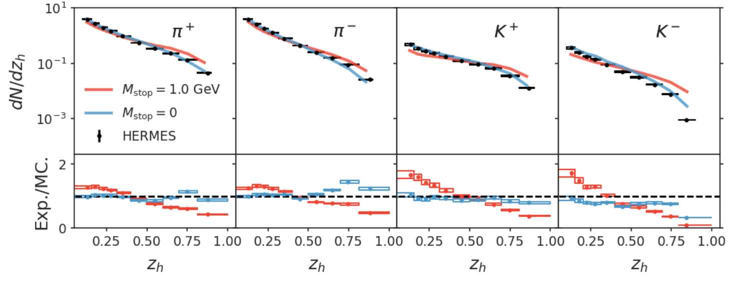
$$\Delta P_{ij}^{\mathrm{med}}(z, \mathrm{l}_\perp)\Theta(Q_s - \mathrm{l}_\perp). \tag{1}$$

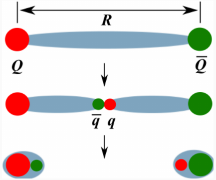
ordered in formation time, $\tau_f = \frac{2z(1-z)p^+}{(\mathrm{l}_\perp - \mathrm{k}_\perp)^2}$

$$1/Q_s \sim \tau_{f,1} < \tau_{f,2} < \cdots < \tau_{f,n}$$

- Hadronization of the parton shower using Lund string fragmentation.
- Gluons generated from the τ_f-ordered shower are attached to medium quark/antiquark to form strings.

The Lund string model in Pythia8: $D(z)$



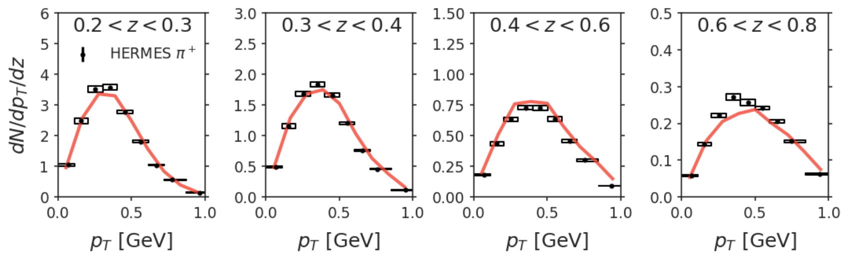


[HERMES, Phys Rev D 87, 074029 (2013)]

- Change a default Pythia8 fragmentation parameter $M_{\rm stop}$ from 1 GeV to 0 to fit π and K spectra in e-d collisions at HERMES energy.
- $M_{\rm stop}$ controls the minimum mass of string to break $W > m_q + m_{\bar{q}'} + M_{\rm stop}$.

The Lund string model in Pythia8: $d^2N_h/dz_h/dp_T$

$d^2N_h/dz_h/dp_T$



[HERMES, Phys Rev D 87, 074029 (2013)]

Reasonable agreement with Pythia8's non-perturbative modeling

- A primordial quark k_T, $k_T \sim e^{-k_T^2/2\sigma_1^2}$ with $\sigma_1 \propto (1 + Q_{1/2}/Q)^{-1}$ [T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053].
- k_T from Lund string fragmentation, $k_T \sim e^{-k_T^2/2\sigma_2^2}$ with $\sigma_2 = 0.335$ GeV as default.

To test the sensitivity of observables to different "theories"

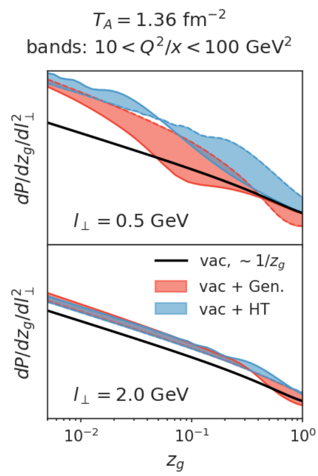
eHIJING implemented two versions of medium-modified splitting functions:

The generalized formula:

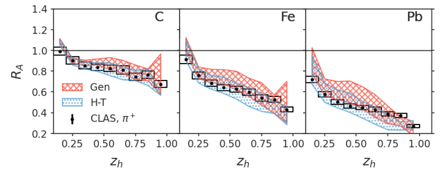
$$\int_0^L dt \int \frac{d^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2}\alpha_s \frac{C_A\rho_0\phi_g(x_g,\mathbf{k}_\perp^2)}{d_A}\frac{2\mathbf{k}_\perp\cdot\mathbf{l}_\perp}{(\mathbf{l}_\perp-\mathbf{k}_\perp)^2}\left(1-\cos\frac{(\mathbf{l}_\perp-\mathbf{k}_\perp)^2 t}{2(1-z)zE}\right)$$

Collinear expanded formula (higher-twist): expand in powers of $\mathbf{k}_\perp^2/\mathbf{l}_\perp^2$ [X-N Wang, X Guo, A. Majumder, etc].

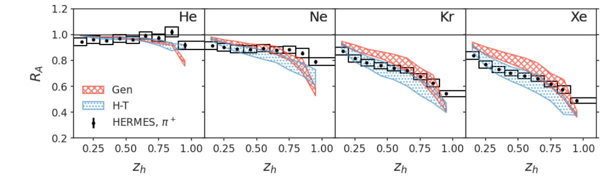
$$\int dL \frac{2\hat{q}'_A}{\mathbf{l}_\perp^2}\left[1-\cos\frac{\mathbf{l}_\perp^2 L}{2(1-z)zE}\right], \quad \hat{q}'_A = \int_0^{\mathbf{l}_\perp^2} d\mathbf{k}_\perp^2 \alpha_s \frac{\pi C_A\rho_0\phi_g(x_g,\mathbf{k}_\perp^2)}{d_A}$$



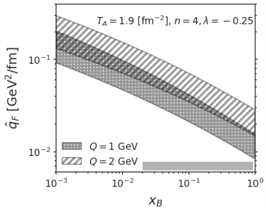
Test of medium-modified hadronization at CLAS and HERMES



[CLAS arXiv:2109.09951]

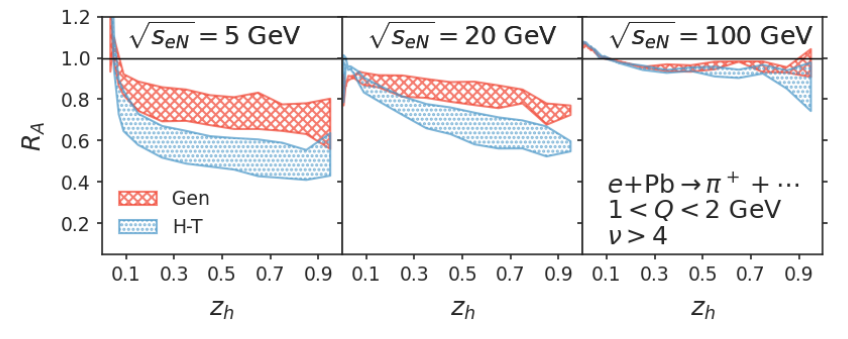


[HERMES, NPB 780, 24 (2007)]



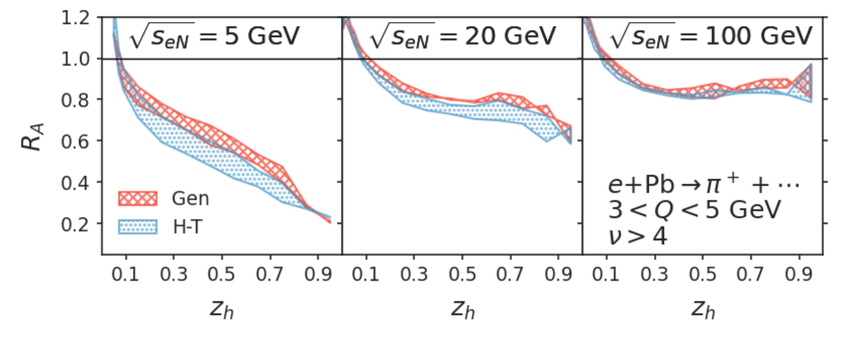
- Nuclear modification
  $R_A = \left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{eA} / \left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{ed}$.
- HT (red) & generalized HT (blue). Bands: $\hat{q}_F$ variation ▷.
- Consistent with the A dependence of data.
- Nuclear PDF EPPS16 [EPJC 77, 163 (2017)] used for hard process.

Effects of modified fragmentation decreases with increasing jet energy. Still, expect sizable correction if one aims for processes with large Q^2 and x_B.

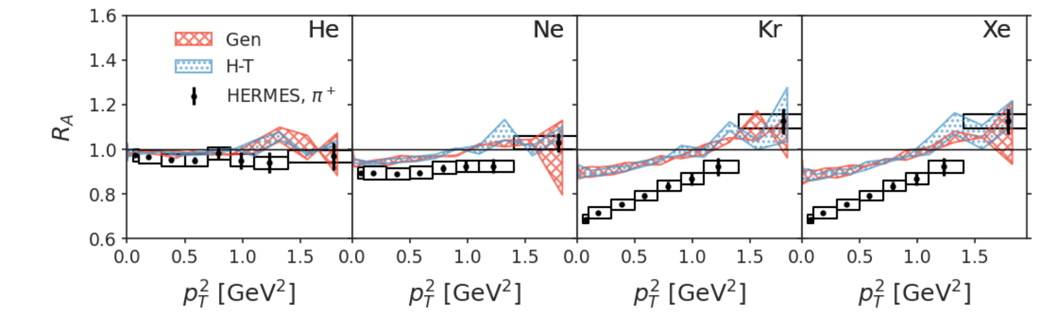


From fixed target to collider energies

Effects of modified fragmentation decreases with increasing jet energy. Still, expect sizable correction if one aims for processes with large Q^2 and x_B.

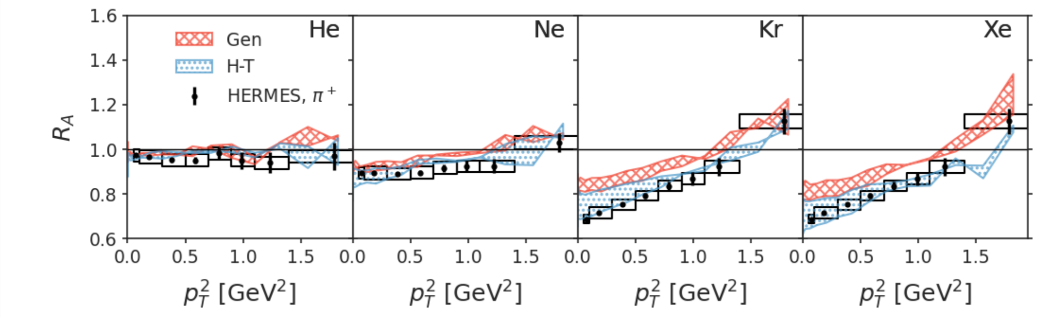


Collisional broadening of the parton shower



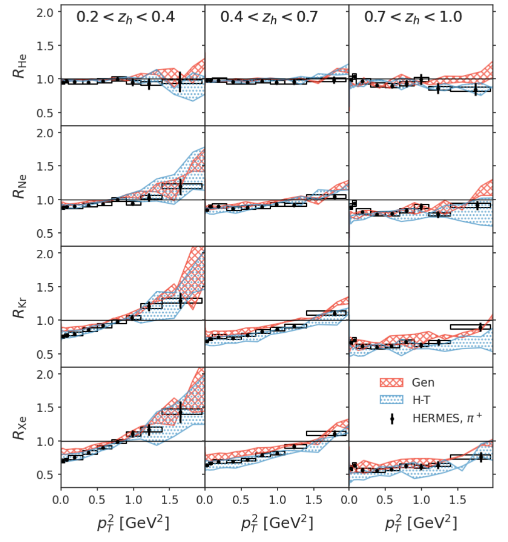
Collisional & radiative contribution to momentum broadening

Broadening from both collisions & induced radiations



- Broadening due to medium-induced radiation is important in large nucleus!
- Sensitive to calculation of details of the in-medium splitting functions.

p_T-dependent modified fragmentation function $D(z_h, p_T)$

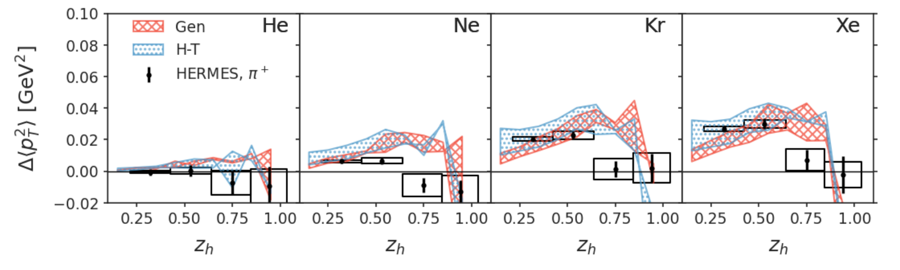


[HERMES, Nuclear Physics B 780, 24 (2007)]

$$R_A = \frac{(N_h(\nu, Q^2; z_h, p_t)/N_\gamma)_{eA}}{(N_h(\nu, Q^2; z_h, p_t)/N_\gamma)_{ed}}$$

- Large z: suppression due to parton energy loss of leading particles.
- Small z: interplay of k_T broadening and the parton shower evolution.
- Partons that stay at large z likely to undergo fewer collisions, $\Rightarrow$ a "survival bias" due to the large flu cation of number of collisions.

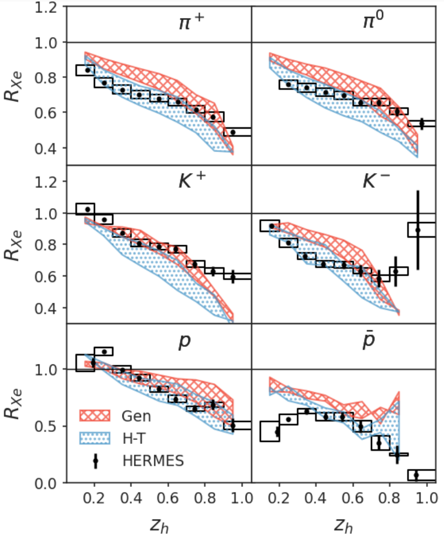
The net broadening $\Delta\langle p_T^2\rangle = \langle p_T^2\rangle_{eA} - \langle p_T^2\rangle_{ed}$



[HERMES, PLB 684 (2010) 114-118]

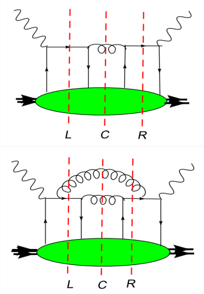
- Qualitatively similar z-dependence from simulation.
- Data drop more abruptly for $z_h > 0.7$. This region shrinks at higher colliding energies.

Hadron specie dependence: $\pi^{\pm}$, π^0, $K^{\pm}$, p, $\bar{p}$

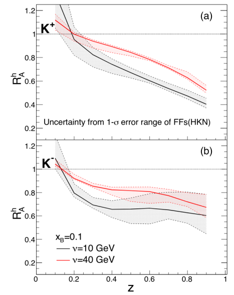


[HERMES, Nuclear Physics B 780, 24 (2007)]

- Notable difference between $R_A(K^+)$ vs $R_A(K^-)$, and $R_A(p)$ vs $R_A(\bar{p})$.
- Importance of medium-induced conversion of $g \to q$ and hadronic transport for future.



[BW Zhang, XN Wang, A Schaefer]



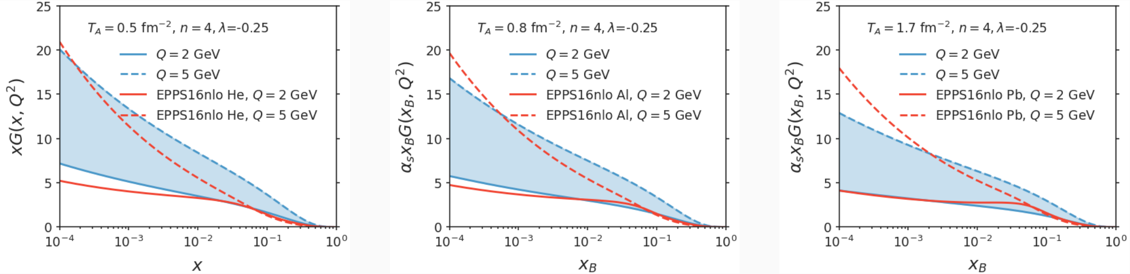
[NB Chang, WT Deng, XN Wang PRC 92 055207]

Summary and outlook

- Final-state jet-nucleus interactions lead to momentum broadening, modified fragmentation & hadron chemistry.
- Measurements of z & p_T dependent fragmentation function in e-A collisions provide strong constraints to the in-medium splitting function & hadronization models.
- eHIJING with multiple collisions and twist-4 in-medium QCD splitting functions provide good description of CLAS and HERMES data.
- Radiative processes are found to be responsible for a large fraction of momentum broadening.
- To incorporate the evolution of the Glauber gluon, important for predictive power at higher $\sqrt{s}$.

Questions?

$$xG(x, Q^2) = \int_0^{Q^2} \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \phi_g(x, \mathbf{k}_\perp^2)$$



Choice of parameters result in similar $xG(x, Q)$ at low Q^2.

Stochastic version of the medium-modified splitting functions

$$D_{2h} = \frac{d^2 N_h}{dz_1 dz_2}, z_1 > z_2; \qquad R_{2h} = \frac{D_{2h}^{eA}/D^{eA}}{D_{2h}^{ed}/D^{ed}}$$

With a large fluctuation in the number of collisions, we constructed fluctuating in-medium splitting functions

$$P_{qq}(x, \mathbf{l}_\perp) = \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{l_\perp^2} \left\{ 1 + \int dL \rho(L) \int_{\mathbf{k}_\perp} \frac{C_F}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2} \frac{C_A}{C_F} \frac{2\mathbf{k}_\perp \cdot \mathbf{l}_\perp}{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2} \left[1 - \cos \frac{L}{\tau_f} \right] \right\}$$

$$\Longrightarrow \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{l_\perp^2} \left\{ 1 + \sum_i \frac{C_A}{C_F} \frac{2(\mathbf{k}_\perp)_i \cdot \mathbf{l}_\perp}{[\mathbf{l}_\perp - (\mathbf{k}_\perp)_i]^2} \left[1 - \cos \frac{L_i}{(\tau_f)_i} \right] \right\}$$

The usual average over the medium sources are replaced by the summation over the multiple collisions of the shower parton.