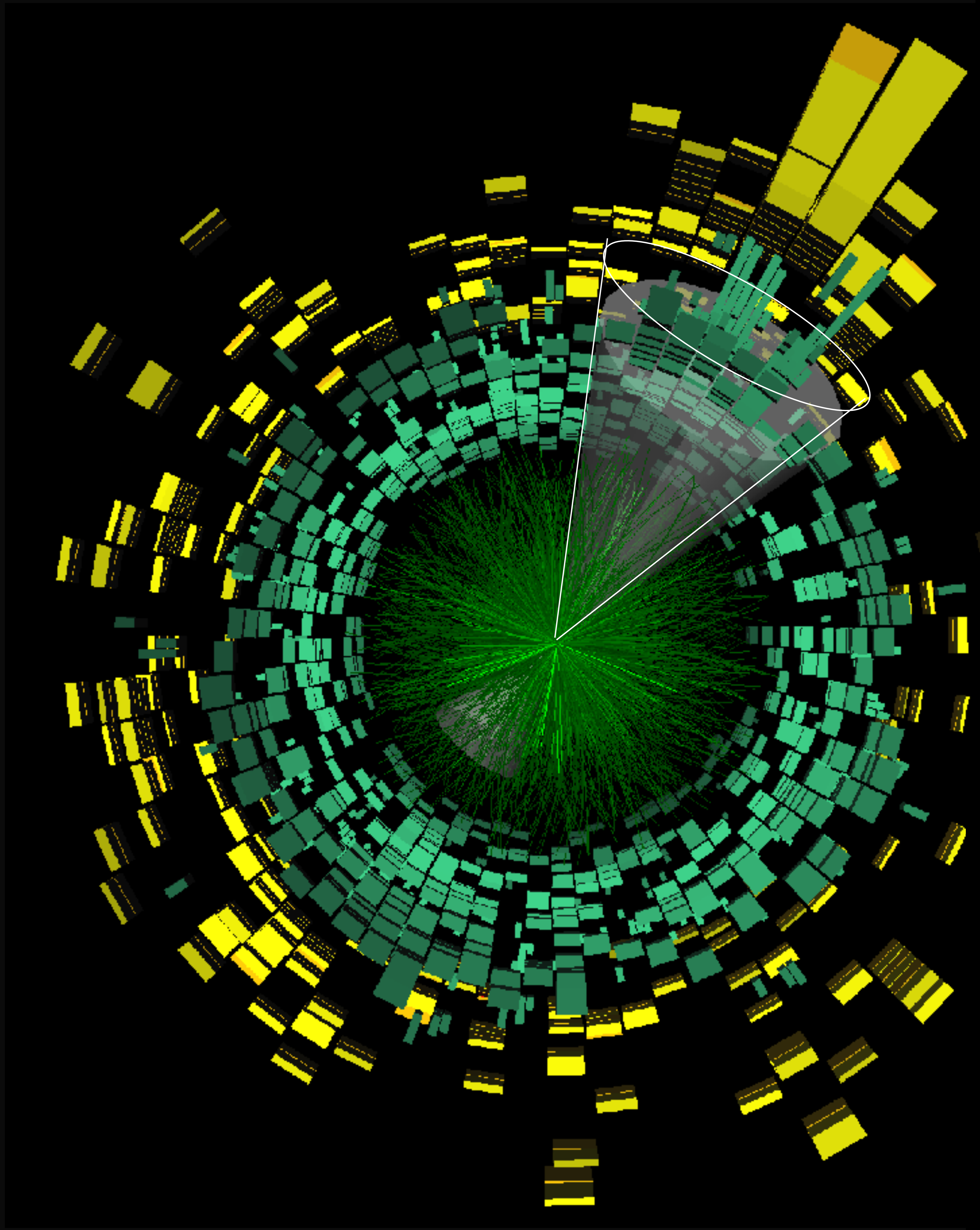
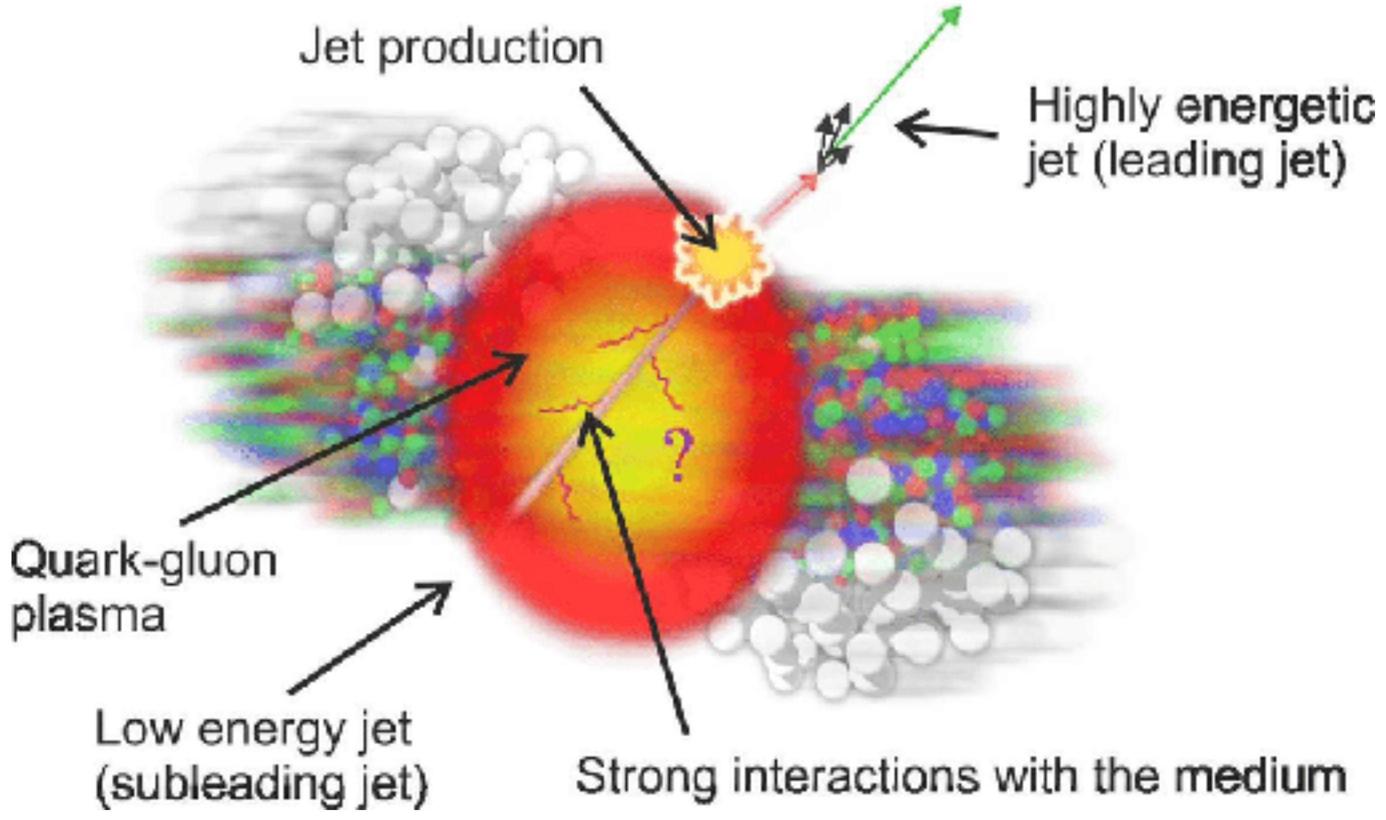
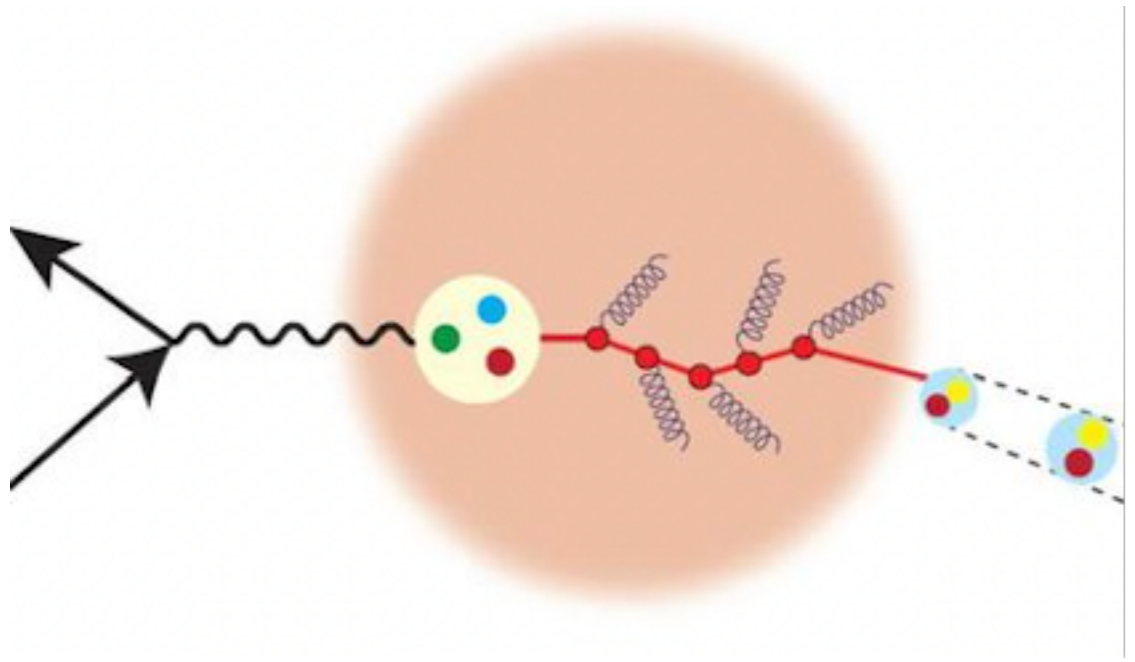
Why we need factorization for jet physics in Heavy Ion collisions

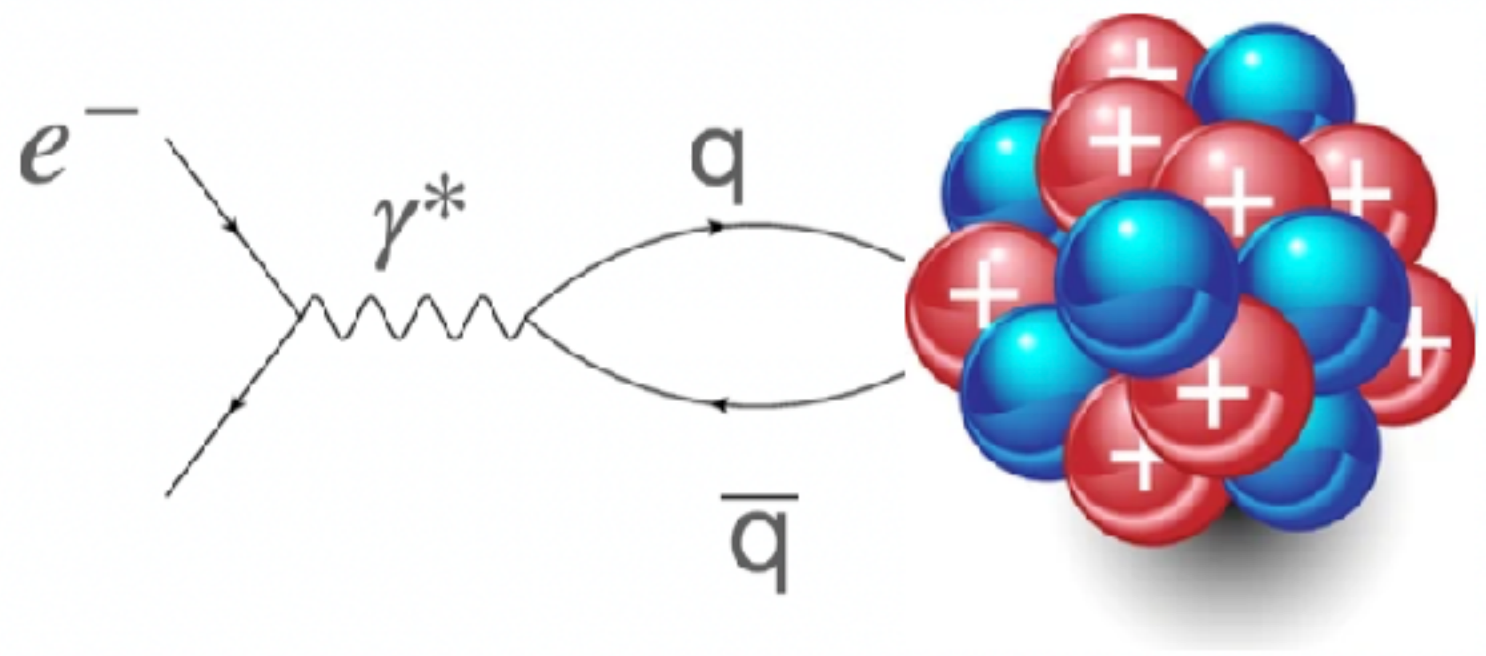
Varun Vaidya, MIT, June 30, 2022



Probing a strongly coupled medium



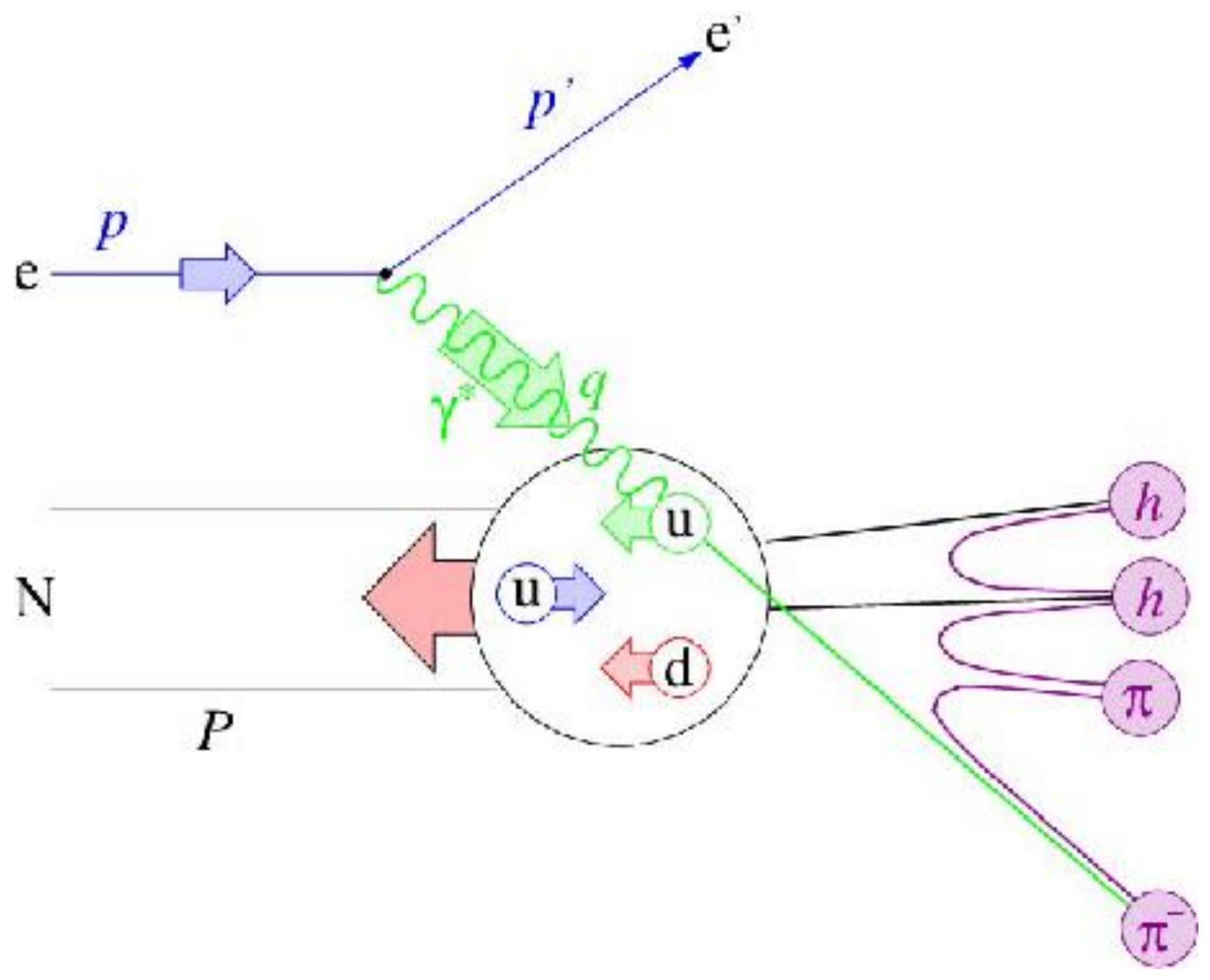


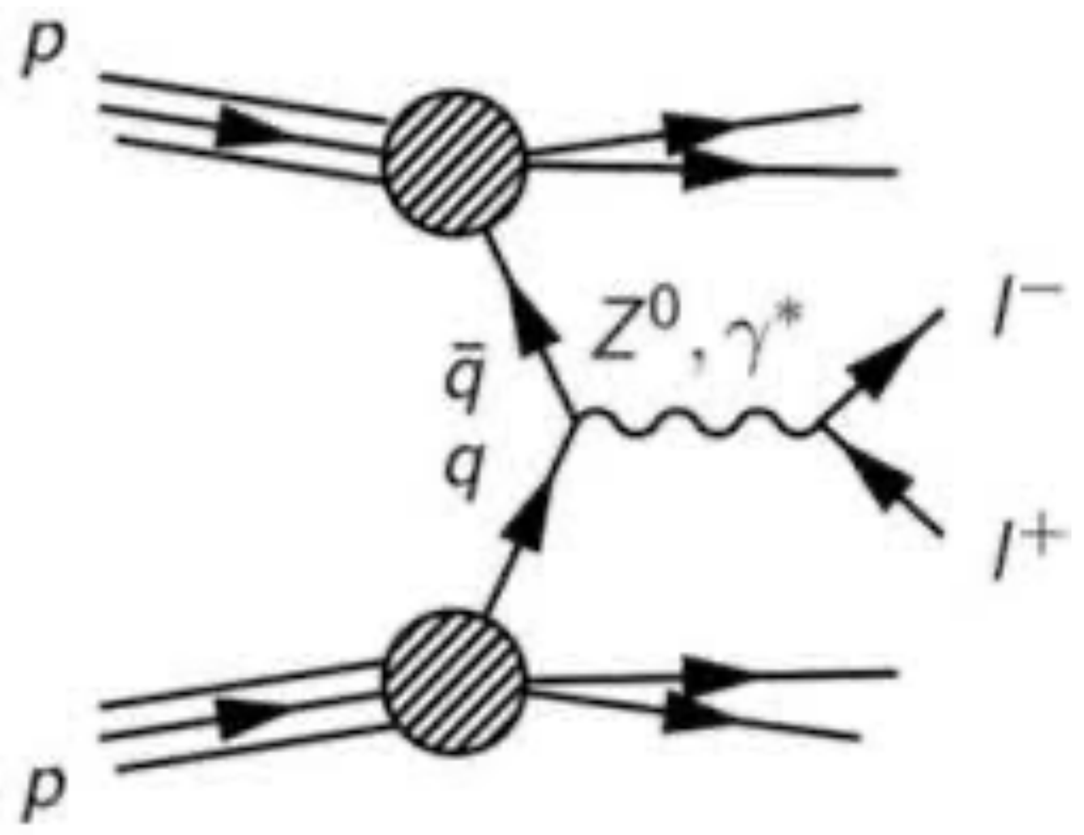


- Quark Gluon Plasma created during HIC is a strongly coupled liquid.
- Perturbation Theory is not reliable to describe the structure of the QGP.
- Hydrodynamic description works at long length scales.
- Not suitable for microscopic structure probed by high energy jets.
- Holographic methods → Qualitative results → Not systematically improvable

How can we make precise predictions for jet observables in a HIC environment?

- This is NOT a new problem in nuclear physics
- We are faced with the same issue in hadronic colliders: A strongly coupled initial state that cannot be described by perturbation theory.
- Precision predictions for ep, pp colliders still possible.

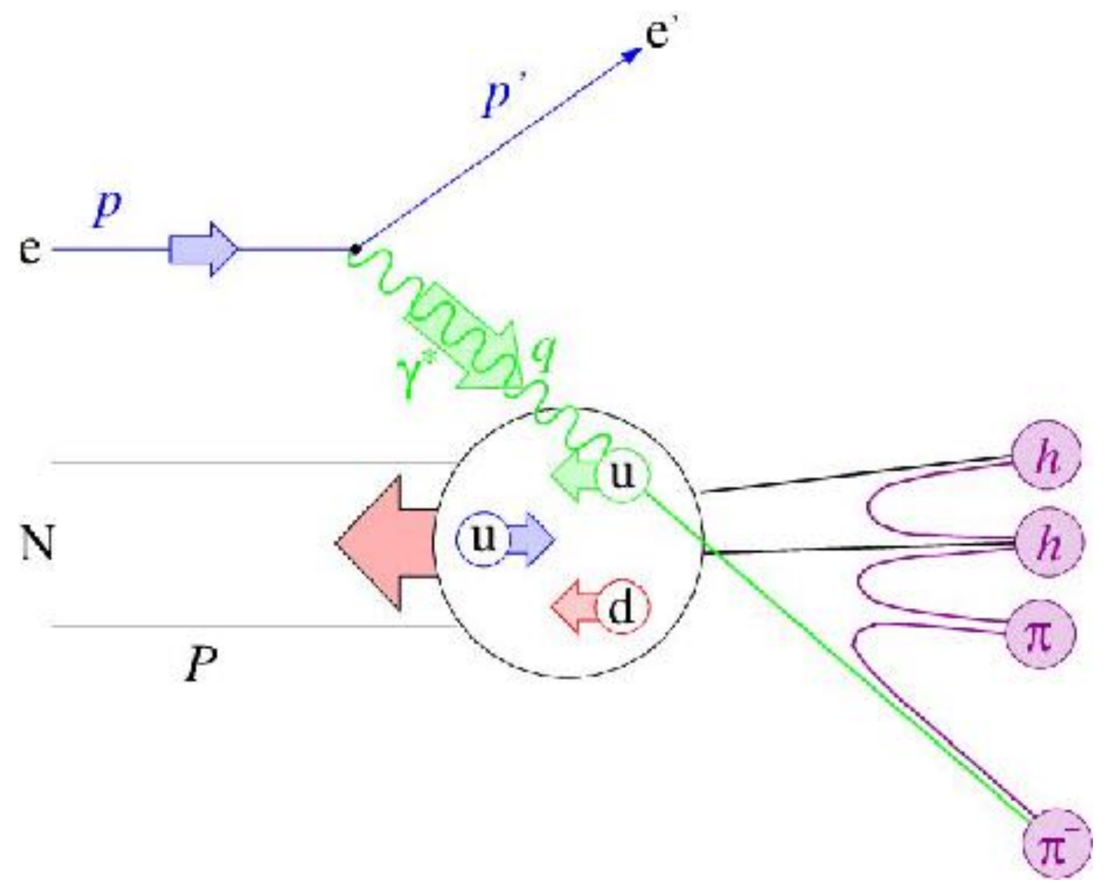




- We rely on **Factorization.**

Familiar examples

Factorization relies on separation of scales



Photon momentum Q → UV scale

Hadron Mass Λ_{QCD} → IR scale

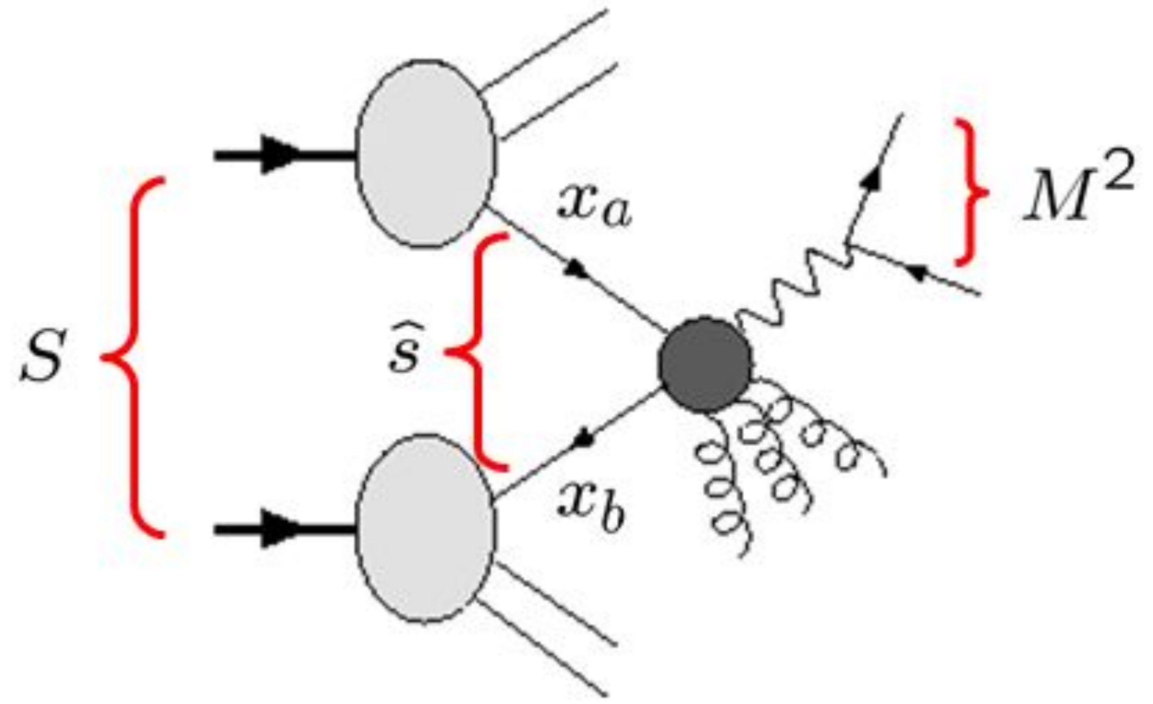
$$\frac{d\sigma}{dQ^2 dx} = H(Q^2,\mu,x) \otimes \sum_{i\in q} e_i^2 f_i(x,\mu) + O\left(\frac{\Lambda_{QCD}}{Q}\right)^2$$

Perturbatively calculable **Hard function** → Describes physics at scale Q

Non-perturbative **Parton Distribution function** → Describes physics at the scale Λ_{QCD}

expansion parameter

Factorization gives predictive power via proof of universality.



Lepton pair mass M → UV scale

Λ_{QCD} → IR scale

Perturbatively calculable

$$\frac{d\sigma}{dM^2} = \sum_{i,j} f_i(x_a,\mu) \otimes_{x_a} H(x_a, x_b, M^2, \mu) \otimes_{x_b} f_j(x_b,\mu) + O\left(\frac{\Lambda_{QCD}^2}{M^2}\right)$$

Two copies of the PDF

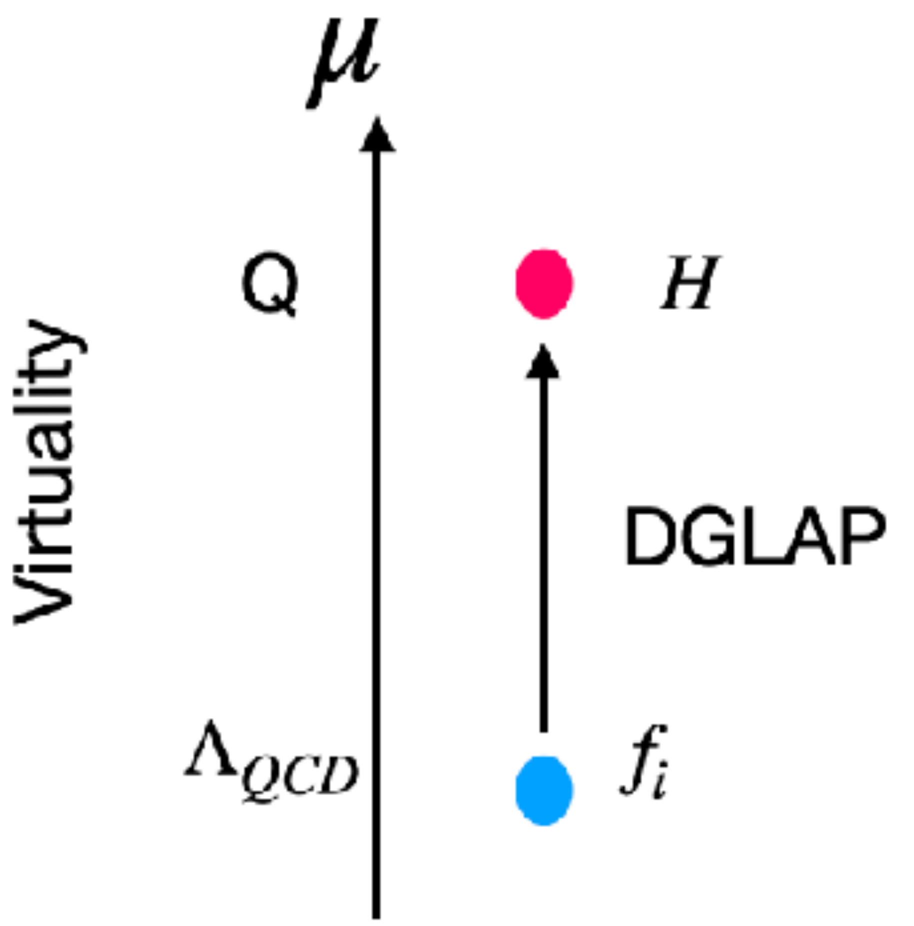
PDF isolates **universal** information about the strongly coupled initial state.

Why should we care about factorization?

- A precise gauge invariant operator definition of the universal non-perturbative physics.

$$f_q(\xi) = \int \frac{dx^+}{2\pi} e^{-ix^+\xi Q} \mathrm{Tr}\left[\bar{\chi}_q(x^+) W(x^+,0) \gamma^- \chi_q(0) \rho_H\right]$$

- **A standalone matrix element in a hadron state is much easier to compute numerically (lattice/Quantum computer) than a full simulation.**

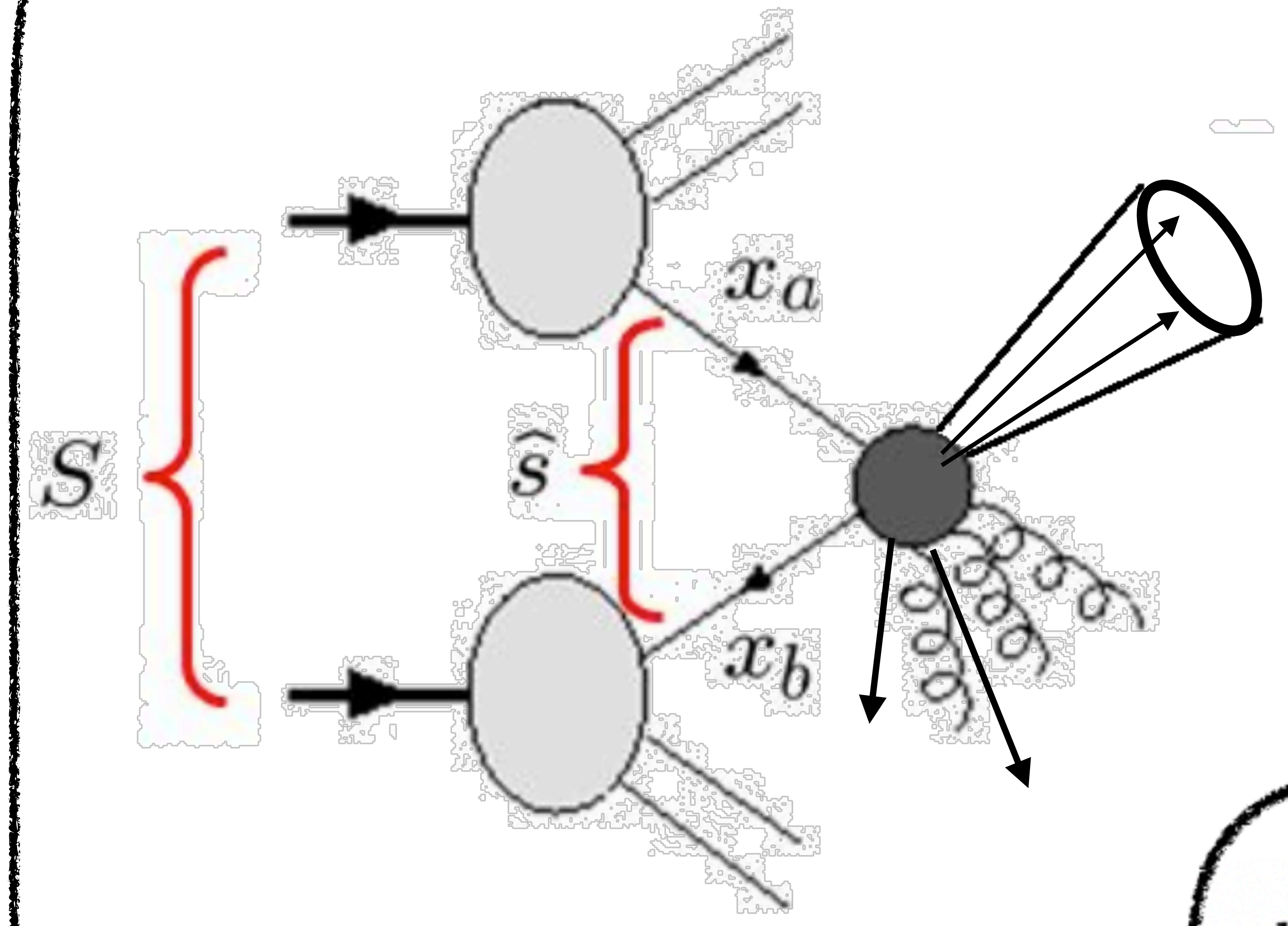


- Renormalization of the operator(**radiative corrections**) can be done **only once independent of any specific experiment** and **independent of any specific hadron :LO →DGLAP**

- Numerical predictions can be enormously improved by resumming logarithms in the expansion parameters by RG running. $(\alpha_s^n \ln^n \frac{Q}{\Lambda_{QCD}})$

- Systematically improvable →Computing perturbative functions to higher order, Including higher powers in the expansion parameter.

Factorization for Jet production



$$p + p \to jet(R, p_T) + X$$

jet $p_T \to$ UV scale

$p_T R$

Hadron Mass Λ_{QCD} $\to$ IR scale

$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s}\sum_{a,b,c}\int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a,\mu)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b,\mu) \times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2}\frac{d\hat{\sigma}^c_{ab}(\hat{s},\hat{p}_T,\hat{\eta},\mu)}{dv dz} J_c(z_c, p_T, R, \mu) + O(R^2) + O\left(\frac{\Lambda^2_{QCD}}{p_T^2}\right)$$

Hard function at p_T

Perturbatively calculable jet function at $p_T R$

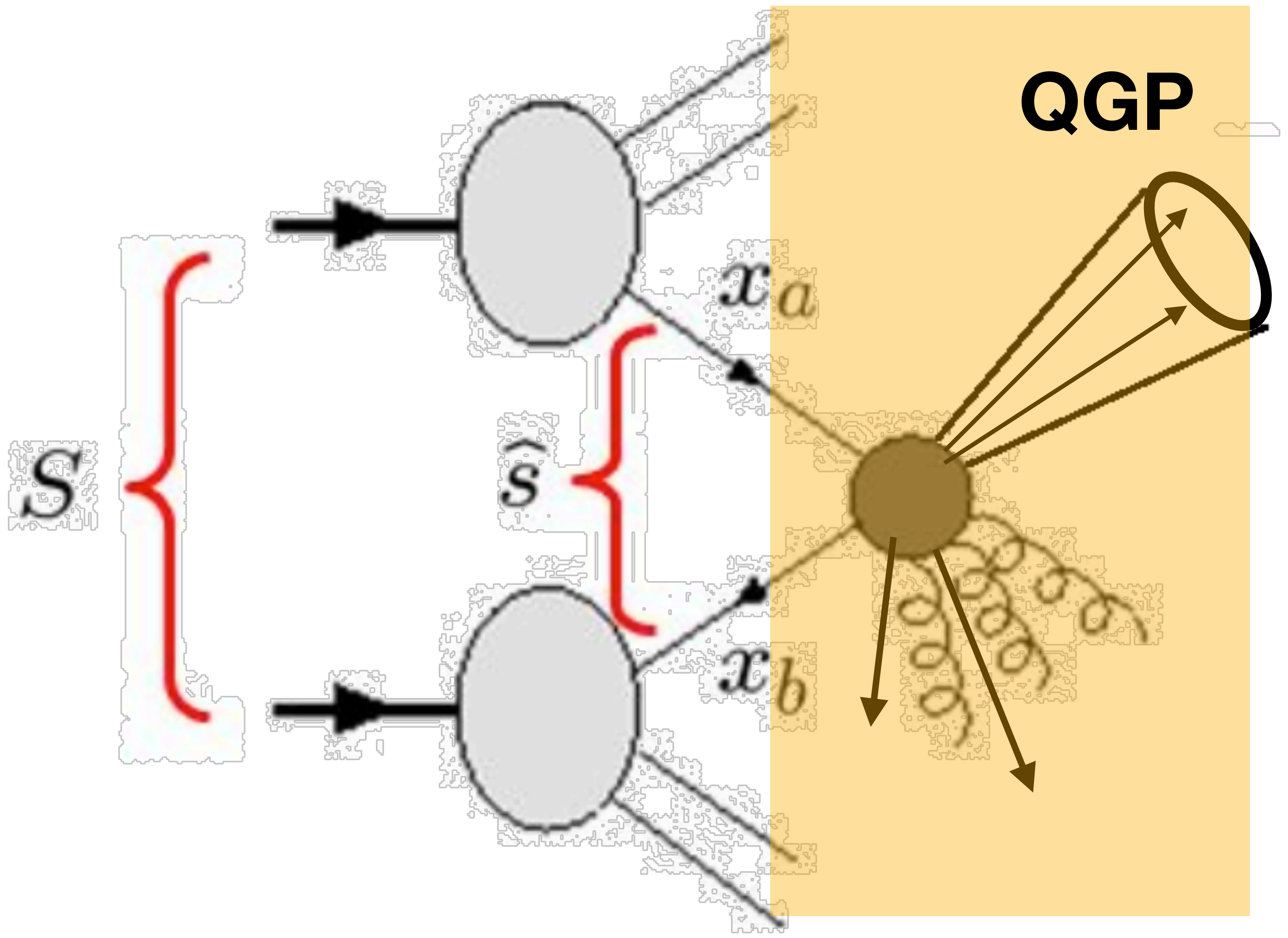
Obeys time-like DGLAP equation

The semi-inclusive jet function in SCET and small radius resummation for inclusive jet production

Zhong-bo Kang, Felix Ringer and Ivan Vitev
JHEP 10 (2016) 125

Can we do the same for jets in Heavy Ion Collisions?

Inclusive Jet production in HIC



$$A + A \to jet(R, p_T) + X$$

jet $p_T \to$ UV scale

$p_T R$

Temperature, Debye Mass, $\Lambda_{QCD} \to$ IR scale

1/ Medium size ~ 1/ L

Emergent Scales

Jet formation time

$$t_F \sim \frac{E}{q_T^2} \sim \frac{1}{p_T R^2}$$

Mean free path of the probe

$$\lambda_{mfp}(p_T, R)$$

An analogy with DIS

Define a Bjorken x for the medium

$$x = \frac{(p_T R)^2}{T p_T}$$

Two regimes

- x ~ 1 → Factorize into "Hard jet function" at scale $p_T R \sim Q$ and a "Medium PDF" at the scale T.

jet p_T

$p_T R \equiv Q$

Temperature T, Debye Mass → IR scale

In order to keep the factorization for the vacuum evolution, we need to do this "large x DIS" factorization in the target (Medium) rest frame.

Work in progress !

Factorization for Dilute Inhomogeneous Medium

- x ≪ 1 → Small x factorization → A smaller $p_T R$

jet p_T

$p_T R \equiv Q$

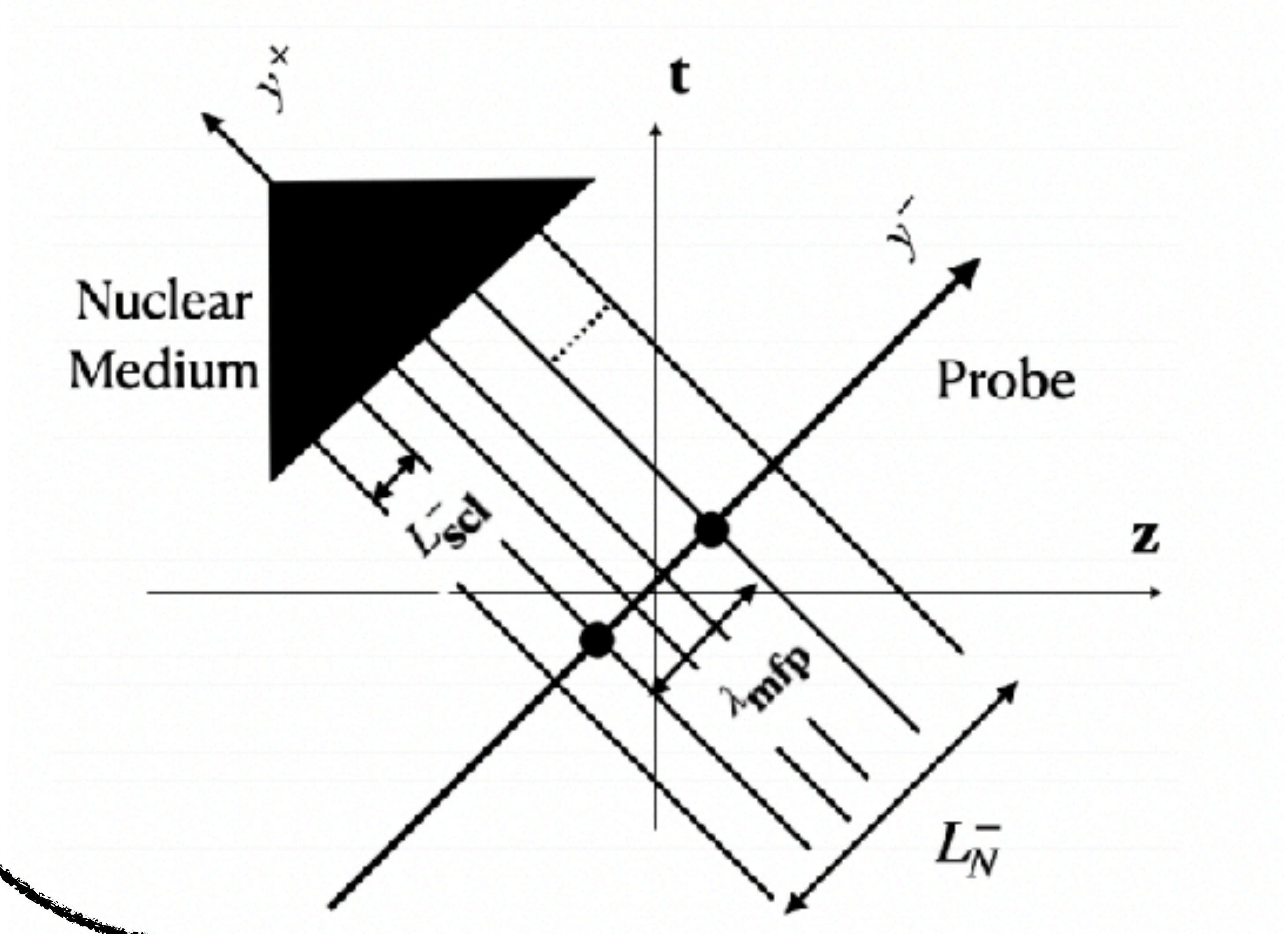
Temperature T

$$\frac{d\sigma}{d\eta dp_T} = \frac{1}{V}\int d^2y \int dy^+ \text{InvMellin}\left[\frac{d\sigma}{d\eta dp_T}(N,R,p_T)\Big|_{\text{vac}}\left(1 - \int_0^{L^-}\frac{dy^-}{\lambda_{\text{mfp}}(R,N,p_T,y)}\right)\right]$$

$y \in$ Medium

Vacuum cross section in Mellin space

Mean free path of the jet ,



A complete equation including both vacuum and medium effects.

Dilute Medium $\frac{L^-}{\lambda_{\text{mfp}}} \ll 1$ → Opacity expansion

Factorization for Dilute Inhomogeneous Medium

$\lambda^{-1}_{\text{mfp}}(R, p_T, y) \to \hat{q} =$ Jet transport parameter

The jet transport parameter depends on both the properties of the jet(via the measurements imposed on it) and the properties of the medium and hence

$\hat{q}$ **is not a direct probe of observable independent medium properties**.

Therefore we must factorize this object further to separate out the universal physics of the medium from the properties of the jet.

$$\lambda^{-1}_{\text{mfp}}(R, N, p_T, y) = H_G(p_T, \mu) \int d^2k_\perp S_{\text{med}}(k_\perp, y, \mu, \nu) J_c^{\text{med}}(R, p_T, N, k_\perp) + O(R^2)$$

Universal observable independent structure function

Medium jet function

V. Vaidya
An Effective Field Theory for jet substructure in Heavy Ion Collisions. JHEP 11, 064 (2021)

$$\hat{q}(p_T, R, y, N) = H_G(p_T, \mu) \int d^2k_\perp k_\perp^2 S_{\text{med}}(k_\perp, y, \mu, \nu) J_c^{\text{med}}(p_T, R, N, y, \mu) + O(R^2)$$

Medium Structure function

$$S_{\text{med}}(k_\perp, y) = \frac{1}{k_\perp^2}\int \frac{dk^-}{2\pi}\int d^4x e^{-ik\cdot x}\text{Tr}\left[O_S^A(x+y)O_S^A(y)\rho_{\text{Med}}\right]$$

$$O_S^A = \sum_{i\in q,g} O_S^{A,i} \qquad O_S^{A,q} = \bar{\psi}S_+T^A\frac{\gamma^+}{2}S_+^\dagger\psi$$

- Gauge Invariant operator definition for **universal** medium physics.
- Depends only on the **local** properties of the medium.

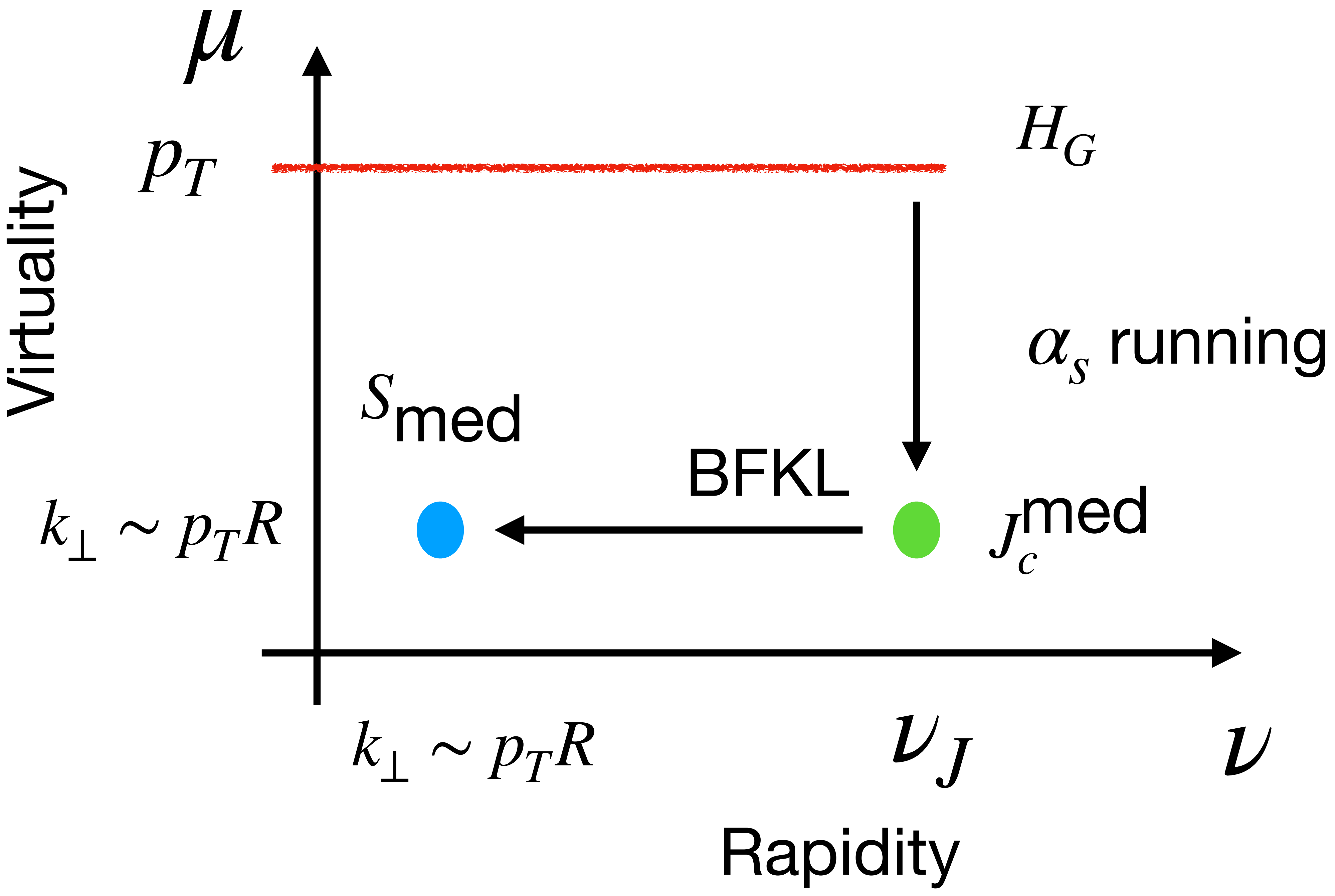
- Renormalization group Equation in rapidity is the **BFKL** equation.

V.Vaidya, Radiative corrections for factorized jet observables in heavy ion collisions, 2107.00029

- **The same operator describes the medium in small x DIS and jet propagation in EIC!**

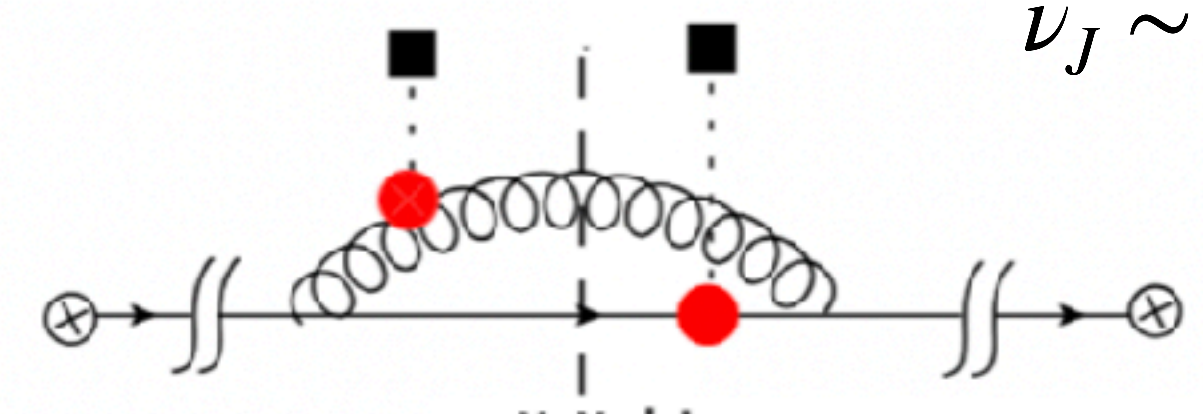
RG improved transport parameter

$$\lambda_{\text{mfp}}^{-1}(R, N, p_T, y) = H_G(p_T, \mu) \int d^2k_\perp S_{\text{med}}(k_\perp, y, \mu, \nu) J_c^{\text{med}}(R, p_T, N, k_\perp, \nu)$$



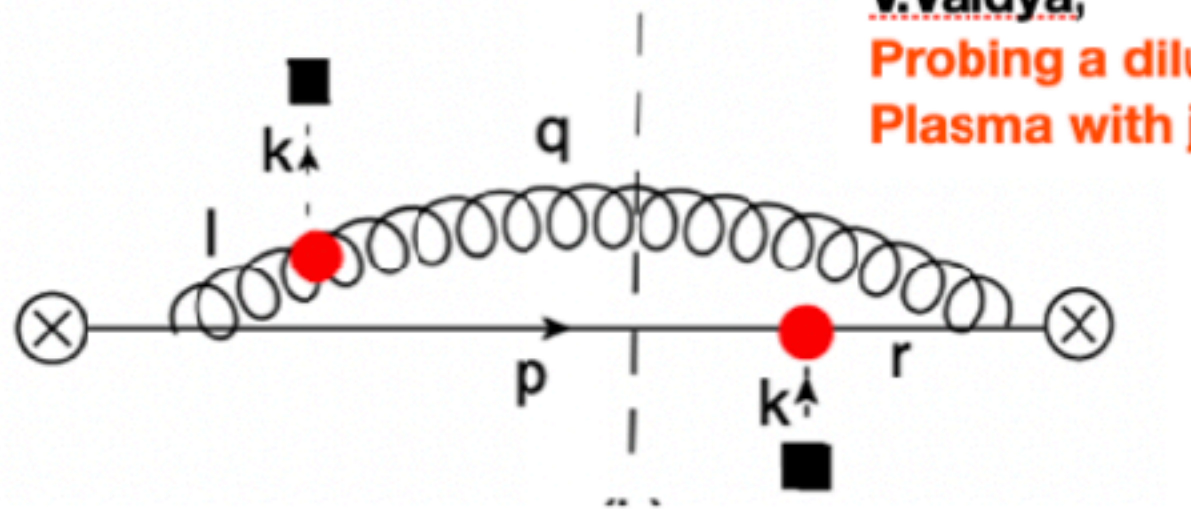
BFKL evolution resums $\alpha_s^n \ln^n \frac{\nu_J}{k_\perp}$

Long lived Dilute medium: $L^- \geq t_F \rightarrow$ medium size greater than formation time of jet $\rightarrow$ $\nu_J \sim \frac{T}{R}$



V.Vaidya, Radiative corrections for factorized jet observables in heavy ion collisions, 2107.00029

Short lived Dilute medium: $L^- \ll t_F \rightarrow$ Quantum interference $\rightarrow \nu_J \sim \quad TL^-(p_T R)$



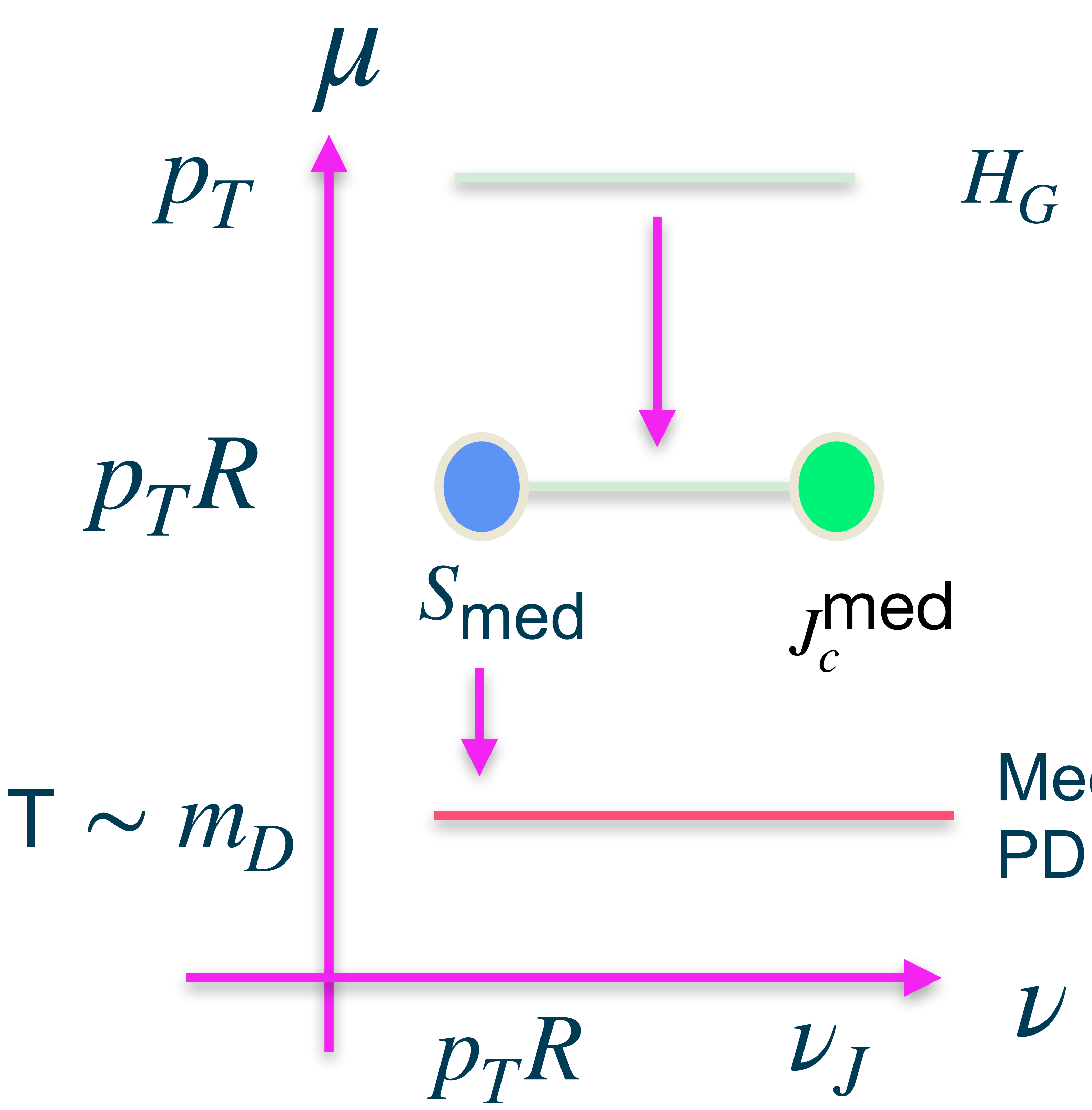
V.Vaidya, Probing a dilute short lived Quark Gluon Plasma with jets, 2109.11568

A non-perturbative medium

$$\lambda_{\text{mfp}}^{-1}(R, N, p_T, y) = H_G(p_T, \mu) \int d^2k_\perp S_{\text{med}}(k_\perp, y, \mu, \nu) J_c^{\text{med}}(R, p_T, N, k_\perp, \nu)$$

$p_T \gg p_T R \gg T$

- Jet function is perturbative and calculable.
- Medium function S_{med} can be matched onto the medium PDF at T
- Medium PDF: Non perturbative but observable independent!
- Only need to recompute the jet function for different jet substructure observables.



Expansion parameter

R

$\frac{T}{p_T R}$

Extension to a Dense Medium

$$\frac{d\sigma}{d\eta dp_T} = \frac{1}{V}\int d^2y \int dy^+ \text{InvMellin}\left[\frac{d\sigma}{d\eta dp_T}(N,R,p_T)\Big|_{\text{vac}}\left(1 - \int_0^{L^-}\frac{dy^-}{\lambda_{\text{mfp}}(R,N,p_T,y)}\right)\right]$$

Dilute Medium

Case B: Dense not so long lived medium :

$L^- \sim \lambda_{\text{mfp}} > t_F$ → Markovian limit

$$\rightarrow \sigma(N) = [\sigma(N)]_{(\text{vac})}\exp\left[-\frac{L^-}{\lambda_{\text{mfp}}(R,p_T,N)}\right]$$

Case C: Short lived Dense medium : $L^- \sim \lambda_{\text{mfp}} \ll t_F$ → Quantum Interference between successive interactions

BK/JIMWLK type equation → Also describes small x DIS physics

Case D : Long Lived Dense Medium : $L \gg t_c \geq \lambda_{\text{mfp}}$ → BDMPS -Z regime

Summary

- Factorization is crucial if we are to make quantitative predictions for jet propagation in a nonperturbative medium.
- It isolates the universal observable independent properties of a strongly coupled medium.
- A general framework to compute **any** jet observable merely by imposing different measurements on the jet function.

Outlook

This type of analysis can also be extended for the case of a Dense Medium where we need to sum multiple jet-medium interaction.

Thank You