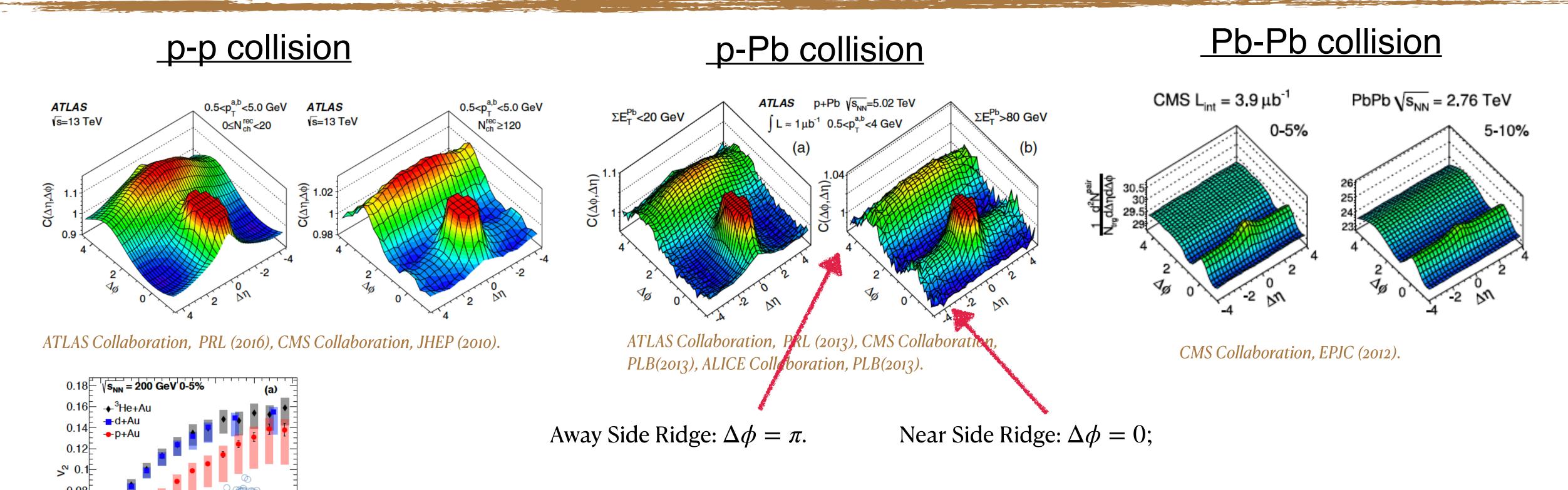
Probing Gluon Bose Correlation Through Trijet Production in Deep Inelastic Scattering

Ming Li North Carolina State University, Raleigh

Jet Physics: From RHIC/LHC to EIC Workshop @ CFNS, 07/01/2022

Alex Kovner, Ming Li and Vladimir Skokov, PRL, 128. 182003 (2022); work in progress.

Two-particle long range rapidity correlations (ridge structure, high multiplicity events)



$$C(\Delta\phi, \Delta\eta) \equiv \frac{1}{N_{\mathrm{trk}}} \frac{dN^{\mathrm{pair}}}{d\Delta\phi d\Delta\eta}$$

Two particles are more likely to be emitted with same azimuthal angles even when they have large difference in rapidities.

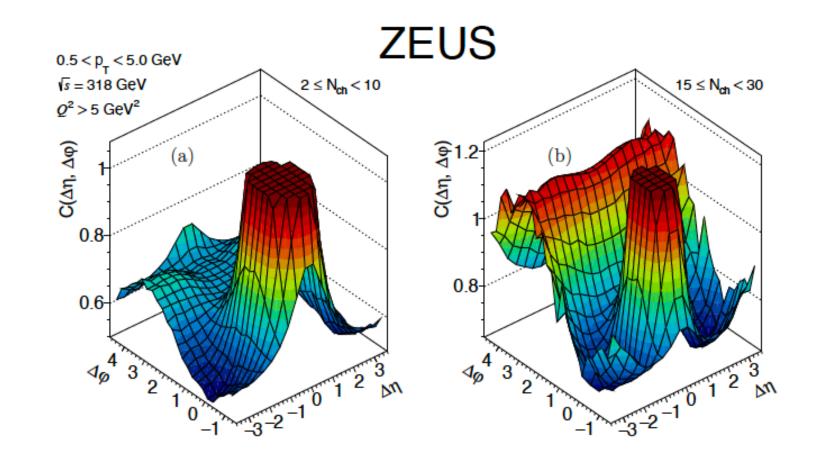
Is Quark-Gluon Plasma formed in high multiplicity p-Pb collisions and p-p collisions?
Final State Interaction: Initial State Interaction Geometry and Collective motion of particles in the fluid velocity field of QGP.

p_{_}(GeV/c)

Ridge Structure in Even Smaller Collision Systems?

e-p collision

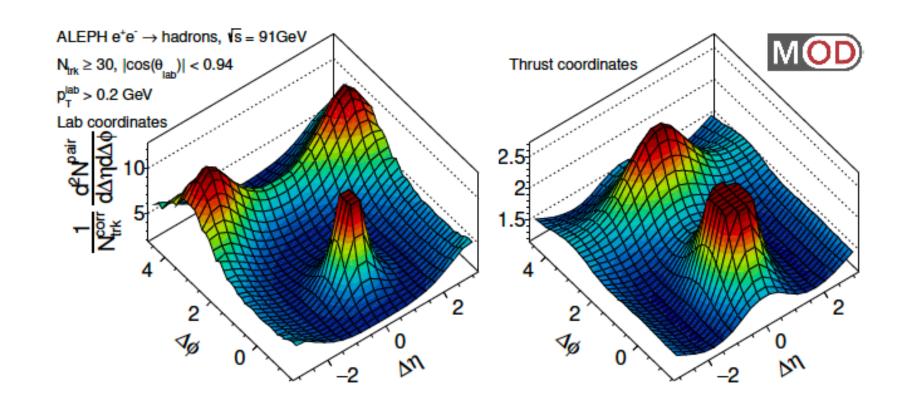
The ZEUS Collaboration, JHEP (2020)



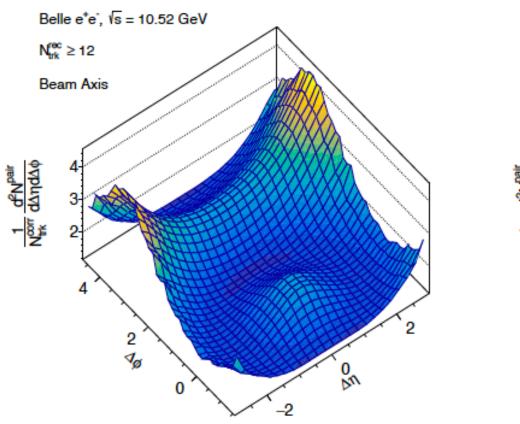
Near-Side Ridge Not Observed!

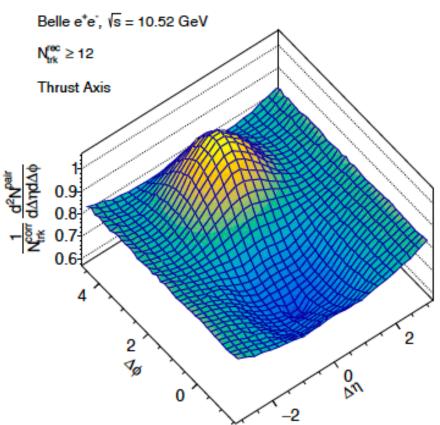
e-e collision

Badea, et al. PRL (2019)



The Belle Collaboration, PRL (2022)



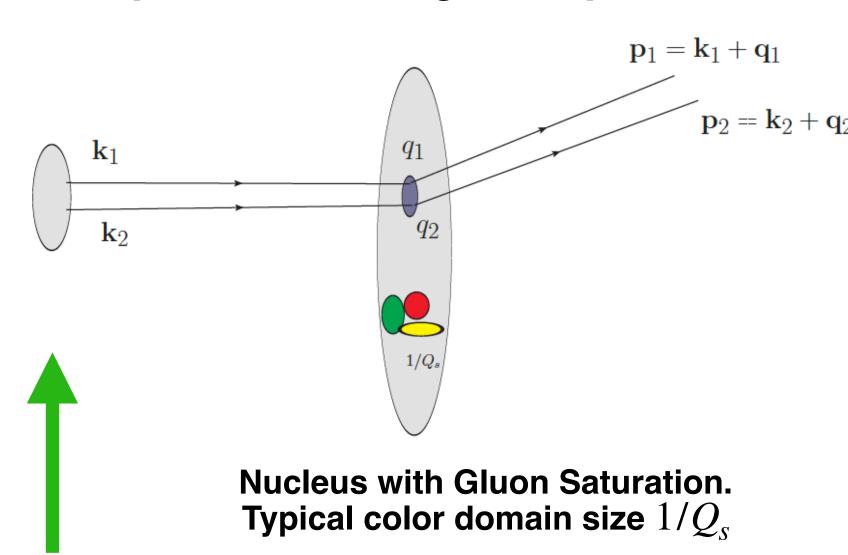


Ridge Structure from Color Glass Condensate Approach

- 1. Long Range Correlation in Rapidity: Universal Property of High Energy Collisions, Boost Invariance.
- 2. Near Side and Away Side Azimuthal Correlation.

Two partons scatter off the same color domain of the nucleus receive similar transverse momentum kicks $|\mathbf{q}_1| \sim |\mathbf{q}_2| \gg |\mathbf{k}_1|, |\mathbf{k}_2|$

A simplified fixed-configuration picture



In dilute target limit (AKA two-gluon exchange approximation)

$$\frac{dN}{d^2\mathbf{p}_1d^2\mathbf{p}_2} = \int_{\mathbf{q}_1,\mathbf{q}_2} \tilde{F}(\mathbf{p}_1,\mathbf{q}_1)\tilde{F}(\mathbf{p}_2,\mathbf{q}_2) \left(1 + \frac{(2\pi)^2}{(N_c^2 - 1)S_{\perp}} \left[\delta(\mathbf{q}_1 + \mathbf{q}_2) + \delta(\mathbf{q}_1 - \mathbf{q}_2)\right]\right)$$

Bose correlation ——— near-side ridge

Assuming the two partons from the projectile are uncorrelated in this talk.

We are interested in Bose Correlation in the Target!

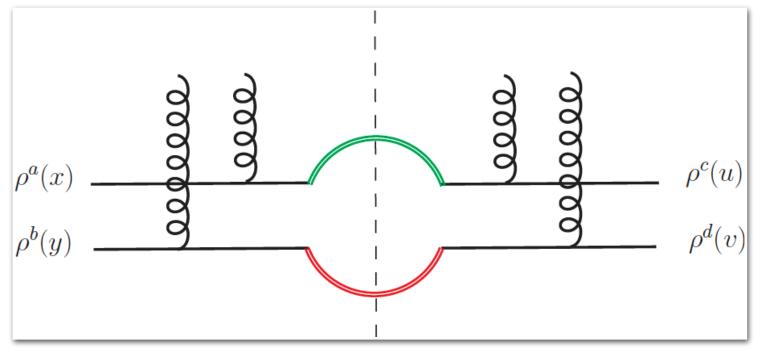
Not HBT!
$$\frac{dN}{d^2\mathbf{p}_1d^2\mathbf{p}_2} \sim \delta(\mathbf{p}_1 + \mathbf{p}_2) + \delta(\mathbf{p}_1 - \mathbf{p}_2)$$

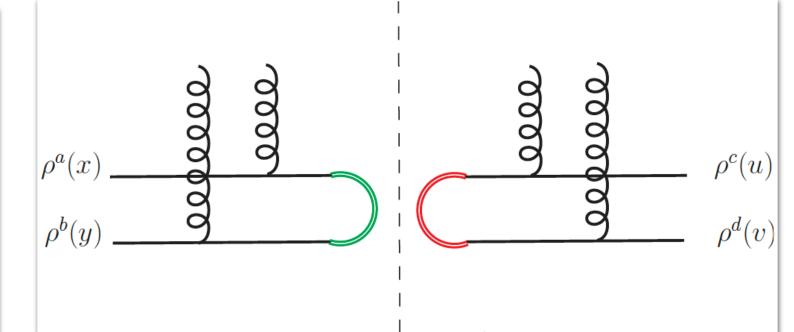
Ridge Structure from Color Glass Condensate Approach

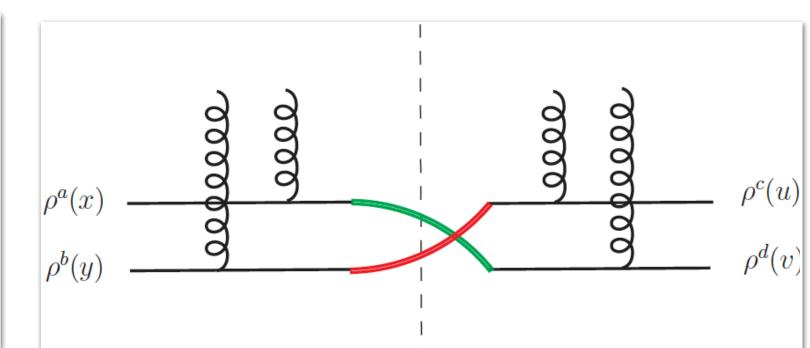
- 1. In Color Glass Condensate framework, the fundamental degrees of freedom are the color charge densities $\rho^a(x^-, \mathbf{x}_\perp)$.
- 2. In McLerran-Venugopalan model, the color charge densities are assumed to follow Gaussian distribution.

$$\langle \rho^a(\mathbf{x}^-, \mathbf{x}_\perp) \rho^b(\mathbf{y}^-, \mathbf{y}_\perp) \rangle = \delta^{ab} \delta(\mathbf{x}^- - \mathbf{y}^-) \delta^{(2)}(\mathbf{x} - \mathbf{y}) g^2 \mu^2(\mathbf{x}^-, \mathbf{x}_\perp)$$

$$\left\langle \rho^a(x)\rho^b(y)\rho^c(u)\rho^d(v)\right\rangle = \left\langle \rho^a(x)\rho^c(u)\right\rangle \left\langle \rho^b(y)\rho^d(v)\right\rangle + \left\langle \rho^a(x)\rho^b(y)\right\rangle \left\langle \rho^c(u)\rho^d(v)\right\rangle + \left\langle \rho^a(x)\rho^d(v)\right\rangle \left\langle \rho^b(y)\rho^c(u)\right\rangle$$







$$\frac{dN}{d^2\mathbf{p}_1d^2\mathbf{p}_2} = \int_{\mathbf{q}_1,\mathbf{q}_2} \tilde{F}(\mathbf{p}_1,\mathbf{q}_1)\tilde{F}(\mathbf{p}_2,\mathbf{q}_2) \left[1 + \frac{(2\pi)^2}{(N_c^2 - 1)S_\perp} \left[\delta(\mathbf{q}_1 + \mathbf{q}_2) + \delta(\mathbf{q}_1 - \mathbf{q}_2) \right] \right]$$

Multi-Parton Correlations in Nuclear Wavefunction

Proton/Nucleus is a many-parton system, how to probe multi-parton correlations in the nuclear wavefunction?

Multi-Dimensional Imaging of Single Parton Distribution in the Nucleon

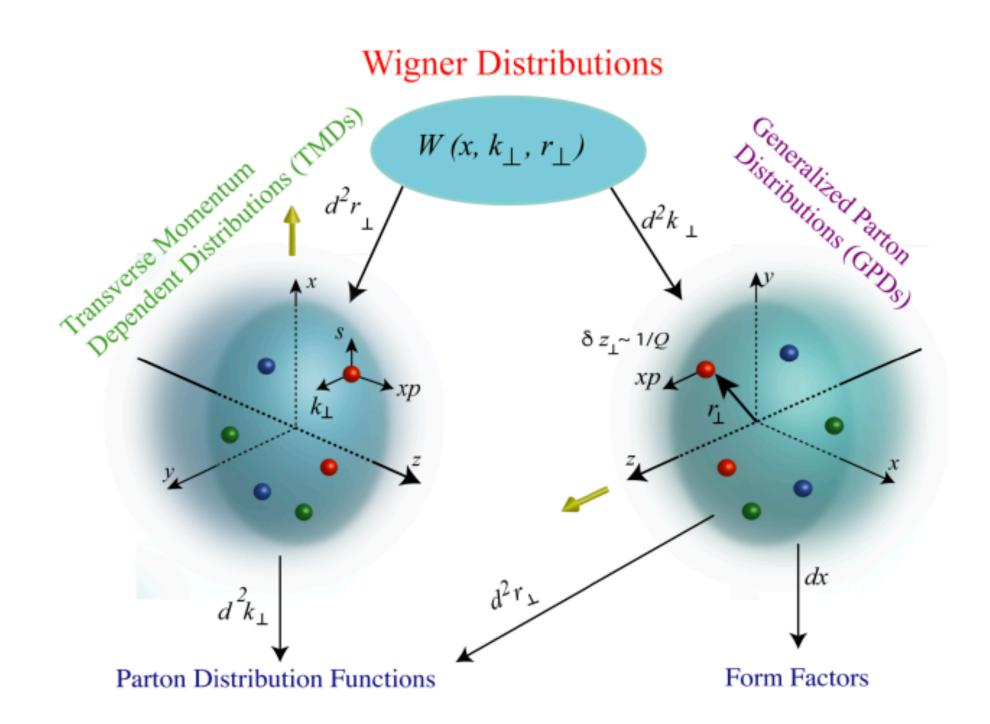


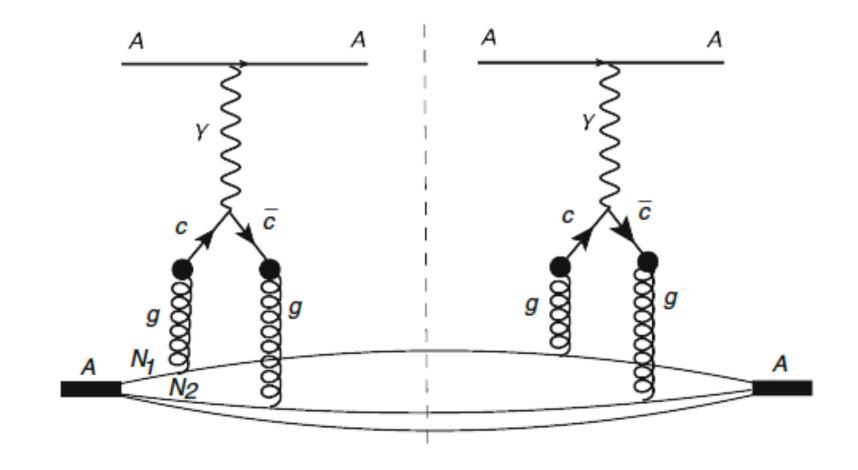
Figure from INT Program INT-17-3

B. Blok and M. Strikman, EPJC (2014)

B. Blok, Yu. Dokshitzer, L. Frankfurt and M. Strikman, EPJC (2012)

Four jets production through two hard processes to probe two-parton TMD: $_2D(x_1, x_2, p_{1t}^2, p_{2t}^2)$

$$\gamma + p/A \longrightarrow c + \bar{c} + g_1 + g_2 + X$$

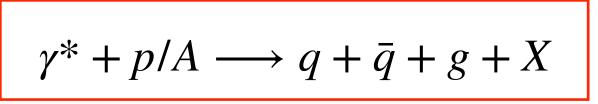


Event rate: Four-Jet/ Dijet $\lesssim 0.05\%$ with jet $p_t > 5 \,\mathrm{GeV}$

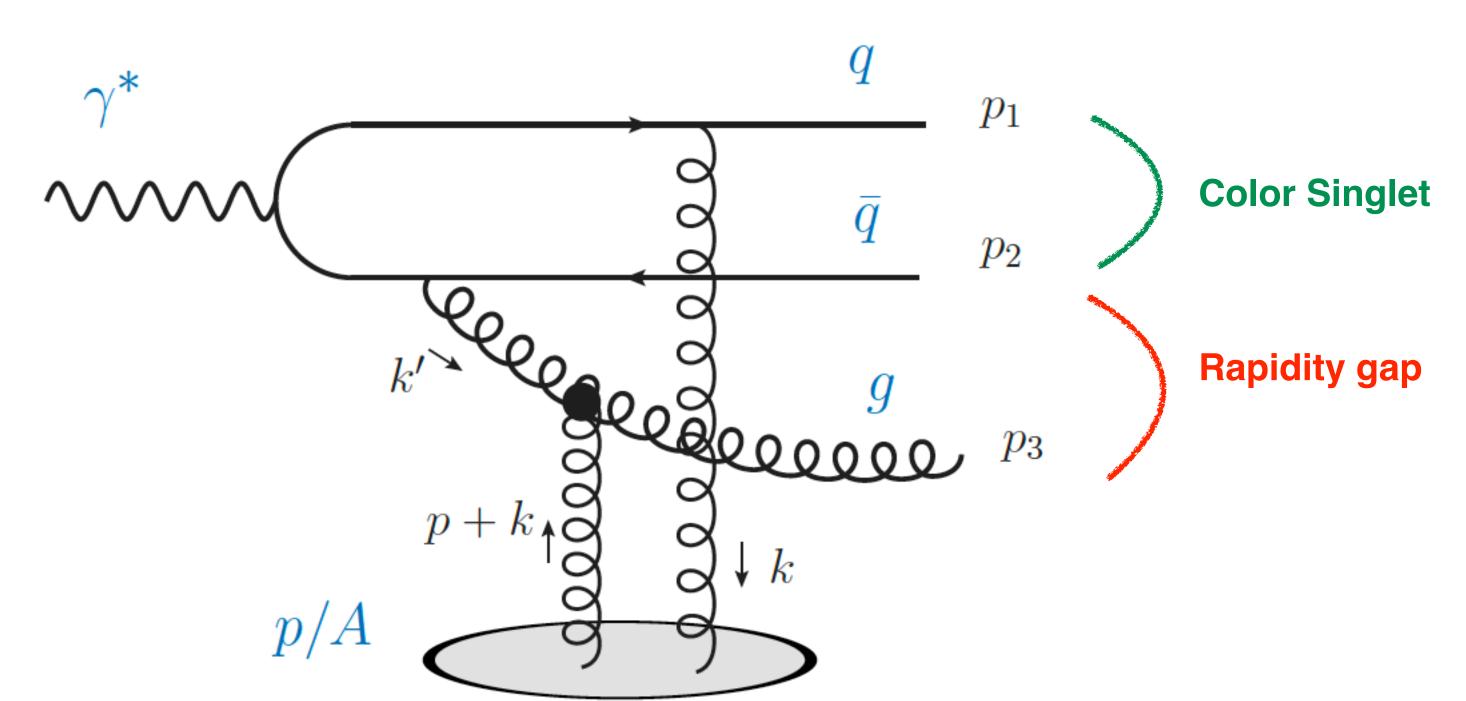
Trijet Production in Deep Inelastic Scatterirng

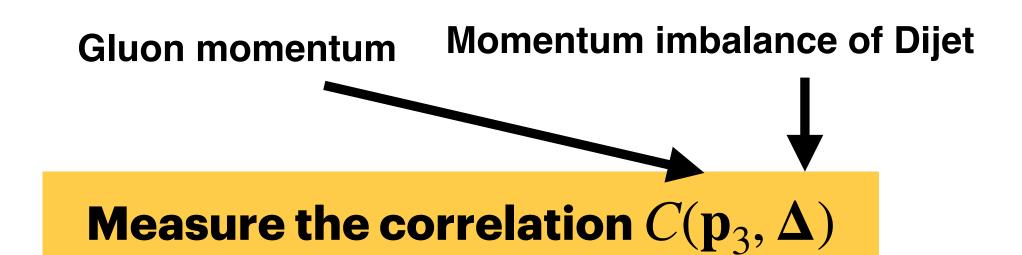
Goal: find some observable that is

- 1. Sensitive to two particle correlation in the nuclear wave function
- 2. Manifests itself in near-side ridge correlation.



(a) Quark and antiquark are in color singlet state and locate in the forward rapidity region;(b) The gluon jet locates in the central rapidity region.





Transverse Momentum Conservation

$$\mathbf{k}' + \mathbf{k} + \mathbf{p}_1 + \mathbf{p}_2 = 0$$

Notation: $\mathbf{p} \equiv \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$, $\Delta \equiv \mathbf{p}_1 + \mathbf{p}_2$.

For phase space region $|\mathbf{k}'| \ll |\mathbf{p}_3|$

$$\mathbf{p} + \mathbf{k} \sim \mathbf{p}_3, \quad -\mathbf{k} \sim \mathbf{\Delta}$$

Diffractive Quark Antiquark Dijet + Gluon Jet in DIS

Trijet Production

$$\frac{d^3N}{d^3p_1d^3p_2d^3p_3} = \sum_{\alpha_1,\alpha_2,\alpha_3} \langle \psi_F | \hat{d}_{\alpha_1}^{\dagger}(p_1)\hat{d}_{\alpha_1}(p_1)\hat{b}_{\alpha_2}^{\dagger}(p_2)\hat{b}_{\alpha_2}(p_2)\hat{a}_{\alpha_3}^{\dagger}(p_3)\hat{a}_{\alpha_3}(p_3) | \psi_F \rangle$$

Final state wavefunction

$$|\psi_F\rangle = \hat{C}^{\dagger}\hat{S}\hat{C}|\gamma^*\rangle \otimes |N\rangle.$$

Coherent operator (Soft gluon radiation operator)

$$\hat{C} = \exp\left\{i\int d^2\mathbf{x}\,b_i^a(\mathbf{x})\int_{\Lambda^-e^{\Delta y}}^{\Lambda^-} \frac{dk^-}{\sqrt{2\pi}|k^-|} \left(\hat{a}_i^{a\dagger}(k^-,\mathbf{x}) + \hat{a}_i^a(k^-,\mathbf{x})\right)\right\} \quad \stackrel{\gamma^*}{\longrightarrow}$$

S-Matrix operator

$$\hat{S} = \exp \left\{ i \int d^2 \mathbf{x} \, \hat{j}(\mathbf{x}) \, \alpha_T(\mathbf{x}) \right\} .$$

Virtual Photon Fock Space Approximation

$$|\gamma^*\rangle\simeq\sum_{qar{q}}\Psi_{qar{q}}|qar{q}
angle$$

Test NLO QCD and measure $\alpha_{\rm c}$, single particle PDF

Z. Nagy and Z. Trocsanyi, PRL(2001) ZEUS Collaboration, PLB (1998), (2001) H1 Collaboration, PLB (2001)

....

Trijet and small-x in the literature:

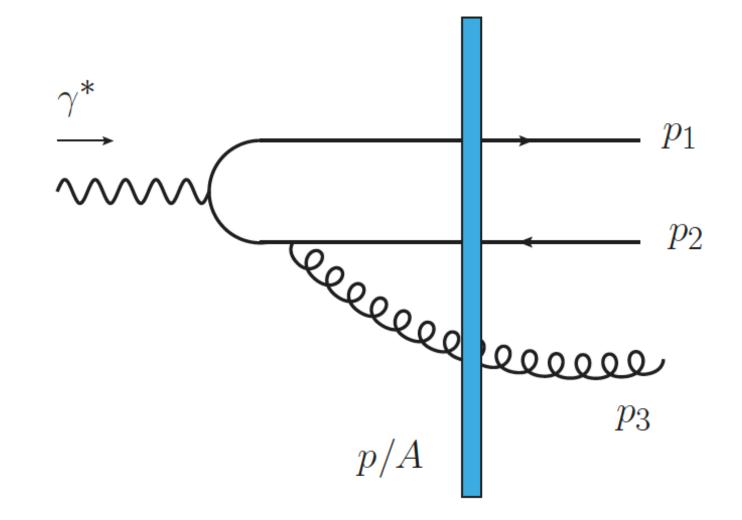
A. Kovner and U. A. Wiedemann, PRD (2001)

A. Ayala, M. Hentschinski, J. Jalilian-Marian and M.E.Tejeda-Yeomans, PLB (2016)

R. Boussarie, A.V. Grabovsky, L. Szymanowski and S. Wallon, JHEP (2016)

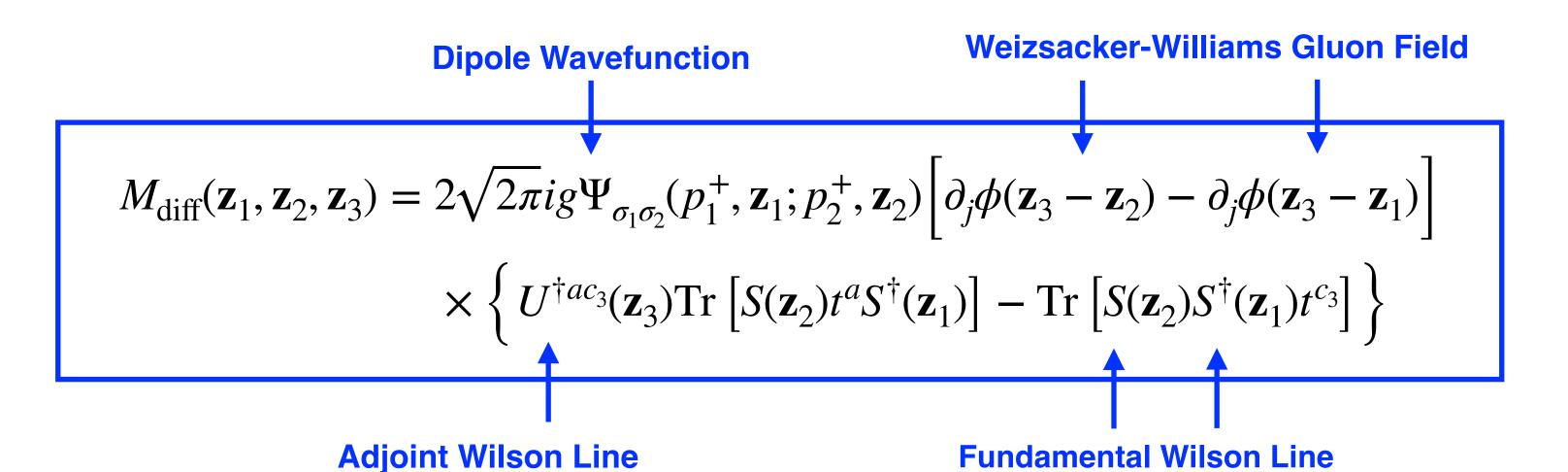
T. Altinoluk, R. Boussarie, C. Marquet and P. Taels, JHEP(2020)

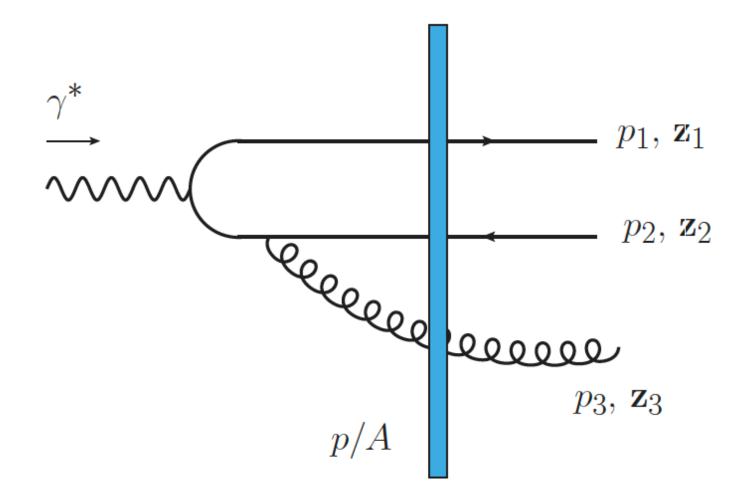
E. lancu, A. H. Mueller and D.N. Triantafyllopoulos, PRL (2022)



Diffractive Quark Antiquark Dijet + Gluon Jet in DIS

Trijet Production
$$\frac{d^3N}{d^3p_1d^3p_2d^3p_3} = \left| M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2$$





$$\left| M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2 = \int_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{l}_1, \mathbf{l}_2} G_j(\mathbf{k}_1, \mathbf{k}_2; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) G_j^*(\mathbf{l}_1, \mathbf{l}_2; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

$$\times \left\langle U^{c_3 a}(\mathbf{p} - \mathbf{k}_2 + \mathbf{k}_1) U^{c_3 b}(-\mathbf{p} + \mathbf{l}_2 - \mathbf{l}_1) \operatorname{Tr} \left[S(\mathbf{k}_2) t^a S^{\dagger}(\mathbf{k}_1) \right] \operatorname{Tr} \left[S(\mathbf{l}_1) t^b S^{\dagger}(\mathbf{l}_2) \right] \right\rangle$$

Dipole correlator

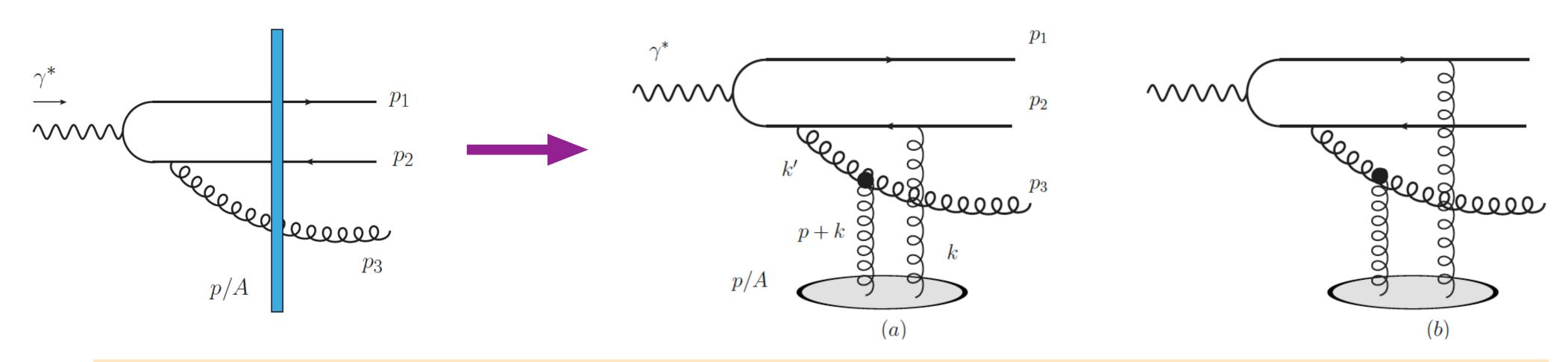
WW field correlator in the correlation limit

The Two-Gluon Exchange Approximation (Dilute Limit)

Trijet Production
$$\frac{a}{d^3n}$$

$$\frac{d^3N}{d^3p_1d^3p_2d^3p_3} = \left| M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2$$

At least two gluons have to be exchanged for the quark + antiquark to be in a color singlet state.

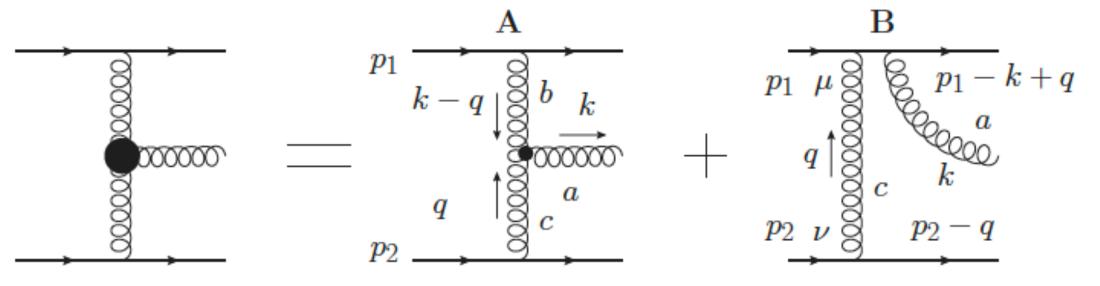


$$M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = g^3 \sqrt{2\pi} T_{ab}^{c_3} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \alpha_T^a(\mathbf{p} + \mathbf{k}) \alpha_T^b(-\mathbf{k}) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) \psi_{\sigma_1 \sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k})$$

$$\psi_{\sigma_{1}\sigma_{2}}^{D}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{k}) = \left[\Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},\mathbf{p}_{1};p_{2}^{+},-\mathbf{p}_{1}) - \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},\mathbf{p}_{1}+\mathbf{k};p_{2}^{+},-\mathbf{p}_{1}-\mathbf{k}) - \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},-\mathbf{p}_{2}-\mathbf{k};p_{2}^{+},\mathbf{p}_{2}+\mathbf{k}) + \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},-\mathbf{p}_{2};p_{2}^{+},\mathbf{p}_{2})\right]$$

Digression: The Lipatov Vertex

Gluon is emitted by the projectile quark and scattered on the target quark.



$$L_j(\mathbf{k} - \mathbf{q}, \mathbf{k}) = \frac{(\mathbf{k} - \mathbf{q})_j}{(\mathbf{k} - \mathbf{q})^2} - \frac{\mathbf{k}_j}{\mathbf{k}^2}$$

Gluon is emitted by the target quark and scattered on the projectile quark.

$$L_j(\mathbf{q}, \mathbf{k}) = \frac{\mathbf{q}_j}{\mathbf{q}^2} - \frac{\mathbf{k}_j}{\mathbf{k}^2}$$

$$\frac{L_{j}(\mathbf{q}, \mathbf{k}) L_{j}(\mathbf{q}', -\mathbf{k})}{|\mathbf{k} - \mathbf{q}|^{2} |\mathbf{k} + \mathbf{q}'|^{2}} = \frac{L_{j}(\mathbf{k} - \mathbf{q}, \mathbf{k}) L_{j}(-\mathbf{k} - \mathbf{q}', -\mathbf{k})}{|\mathbf{q}|^{2} |\mathbf{q}'| 2}$$

Fig. 3.6. The effective real-gluon emission vertex (the Lipatov vertex), defined as the sum of all gluon emission diagrams. The triple gluon vertex is denoted by the smaller solid circle while the Lipatov vertex is shown by the larger solid circle.

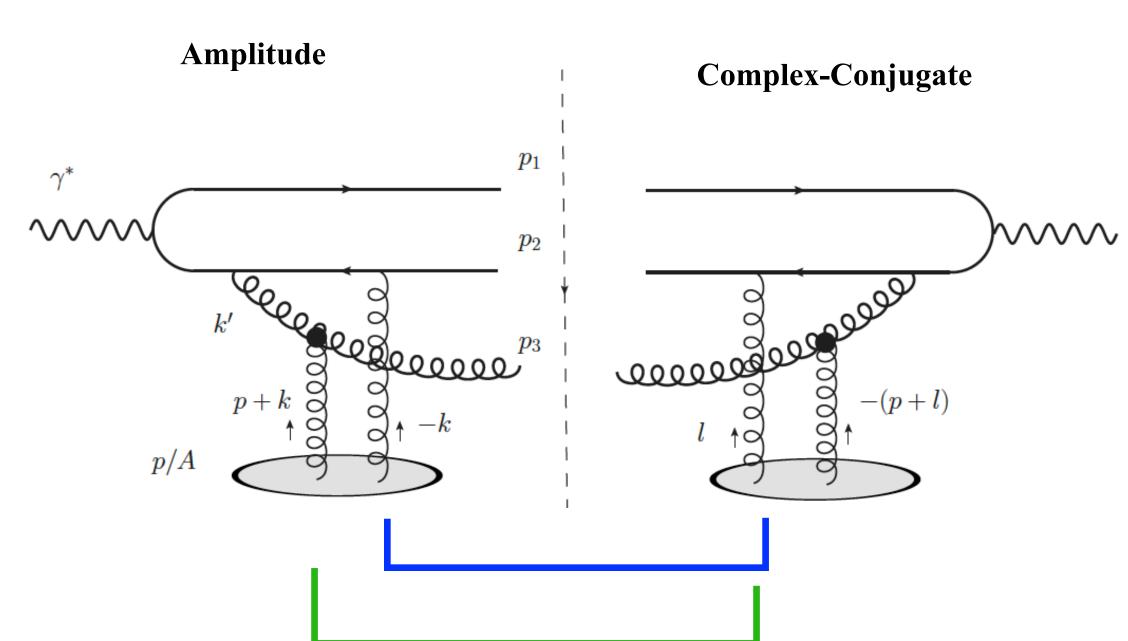
One cannot tell whether the gluon comes from the projectile quark or the target quark.

"Quantum Chromodynamics at High Energy" by Y. Kovchegov and E. Levin (2012)

The Two-Gluon Exchange Approximation (Dilute Limit)

Trijet Production

$$\frac{d^{3}N}{d^{3}p_{1}d^{3}p_{2}d^{3}p_{3}} = -(2\pi)g^{6} \int \frac{d^{2}\mathbf{k}}{(2\pi)^{2}} \frac{d^{2}\mathbf{l}}{(2\pi)^{2}} L_{j}(-(\mathbf{p}_{1} + \mathbf{p}_{2} + \mathbf{k}), \mathbf{p}_{3}) L_{j}(-(\mathbf{p}_{1} + \mathbf{p}_{2} + \mathbf{l}), \mathbf{p}_{3}) \left[\psi_{\sigma_{1}\sigma_{2}}^{D}(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{k}) \psi_{\sigma_{1}\sigma_{2}}^{D*}(\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{l}) \right] \times T_{ab}^{c_{3}} T_{cd}^{c_{3}} \left\langle \alpha_{T}^{a}(\mathbf{p} + \mathbf{k}) \alpha_{T}^{b}(-\mathbf{k}) \alpha_{T}^{c}(-\mathbf{p} - \mathbf{l}) \alpha_{T}^{d}(\mathbf{l}) \right\rangle$$



Only two non vanishing contractions of color charge densities due to the color singlet state requirement for the quark and the antiquark.

$$T_{ab}^{c_3} T_{cd}^{c_3} \left\langle \rho_T^a(\mathbf{p} + \mathbf{k}) \rho_T^b(-\mathbf{k}) \rho_T^c(-\mathbf{p} - \mathbf{l}) \rho_T^d(\mathbf{l}) \right\rangle$$

$$= -N_c (N_c^2 - 1) g^4 \mu_T^4 S_{\perp} \left[(2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{l}) - (2\pi)^2 \delta^{(2)}(\mathbf{p} + \mathbf{k} + \mathbf{l}) \right]$$

Dijet and Gluon jet uncorrelated production

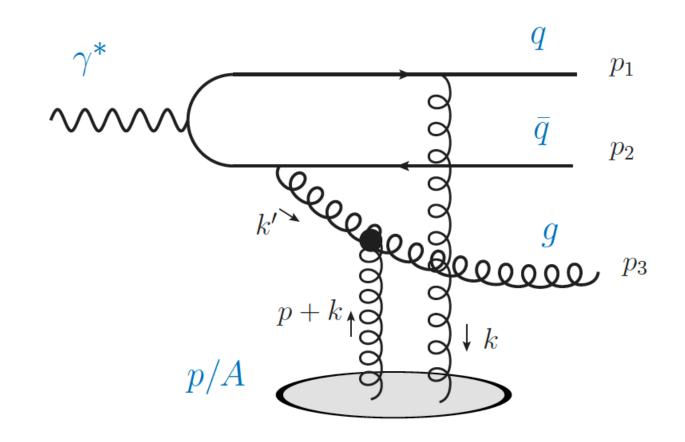
Dijet and Gluon jet near-side ridge correlation

Opposite sign and no color or area suppressions.

Beyond the Correlation Limit: Charge Neutrality of Dipole Wavefunction

Trijet Production
$$\frac{d^3N}{d^3p_1d^3p_2d^3p_3} = -(2\pi)g^6 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{l}), \mathbf{p}_3) \Big[\psi^D_{\sigma_1\sigma_2}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \psi^{D*}_{\sigma_1\sigma_2}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{l}) \Big] \\ \times T^{c_3}_{ab} T^{c_3}_{cd} \Big\langle \alpha^a_T(\mathbf{p} + \mathbf{k}) \alpha^b_T(-\mathbf{k}) \alpha^c_T(-\mathbf{p} - \mathbf{l}) \alpha^d_T(\mathbf{l}) \Big\rangle$$

Amplitude



$$\psi_{\sigma_{1}\sigma_{2}}^{D}(\mathbf{p}_{1},\mathbf{p}_{2},\mathbf{k}) = \left[\Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},\mathbf{p}_{1};p_{2}^{+},-\mathbf{p}_{1}) - \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},\mathbf{p}_{1}+\mathbf{k};p_{2}^{+},-\mathbf{p}_{1}-\mathbf{k}) - \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},-\mathbf{p}_{2}-\mathbf{k};p_{2}^{+},\mathbf{p}_{2}+\mathbf{k}) + \Psi_{\sigma_{1}\sigma_{2}}(p_{1}^{+},-\mathbf{p}_{2};p_{2}^{+},\mathbf{p}_{2}) \right]$$

$$\psi^D_{\sigma_1\sigma_2}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}) \Longrightarrow 0$$
 when $\mathbf{k}=0$ or $-\mathbf{k}'=\mathbf{k}+\mathbf{p}_1+\mathbf{p}_2=0$.

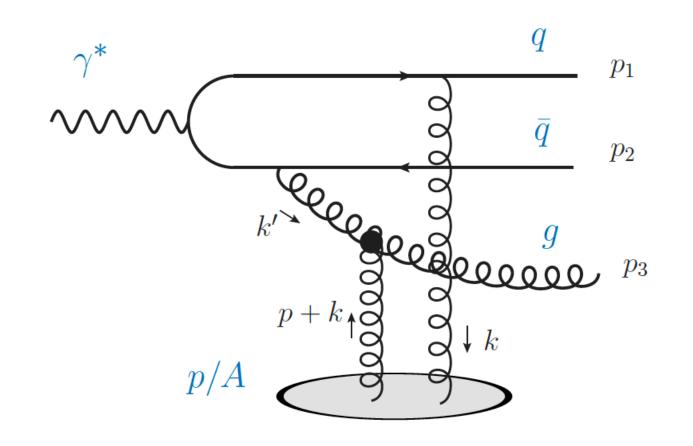
Physically speaking $|\mathbf{k}'|$, $|\mathbf{k}| \ll |\mathbf{p}_1| \sim |\mathbf{p}_2| \sim Q$, The CORRELATION LIMIT

The exchanged gluon has much smaller momentum, Thus cannot resolve the finer structure of the dipole.

Event Selection

$$C(|\mathbf{p}_3|, |\mathbf{\Delta}|, \theta; |\mathbf{p}_1|, |\mathbf{p}_2|) \equiv \frac{1}{\mathcal{N}} \int d\beta_3 \frac{d^3N}{d^3p_1d^3p_2d^3p_3},$$

Amplitude



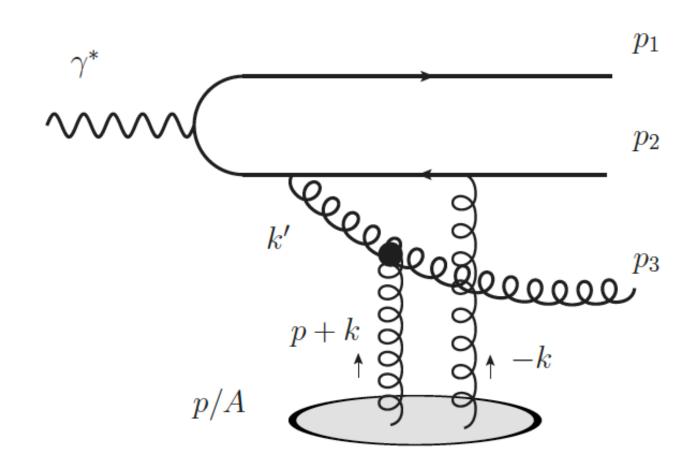
$$\cos \theta = \frac{\mathbf{p}_3 \cdot \mathbf{\Delta}}{|\mathbf{p}_3| |\mathbf{\Delta}|}$$
 β_3 : azimuthal angle of \mathbf{p}_3 .

The trijet production depends on three external momenta p_1, p_2, p_3 . These six variables characterize all the possible trijet events. We explore events $p_3 \approx \Delta = p_1 + p_2$ to see if there are enhancement.

The correlation limit:
$$|P_{\perp}|\gg |\Delta|$$
 with $P_{\perp}=\frac{1}{2}({\bf p}_1-{\bf p}_2)$

To go beyond the correlation limit, we simply specify $|\mathbf{p}_1| \gg |\mathbf{p}_2|$. Do not integrate over $|\mathbf{p}_1|$, $|\mathbf{p}_2|$.

Numerical Results: Zero-Angle Peak



Input parameters:

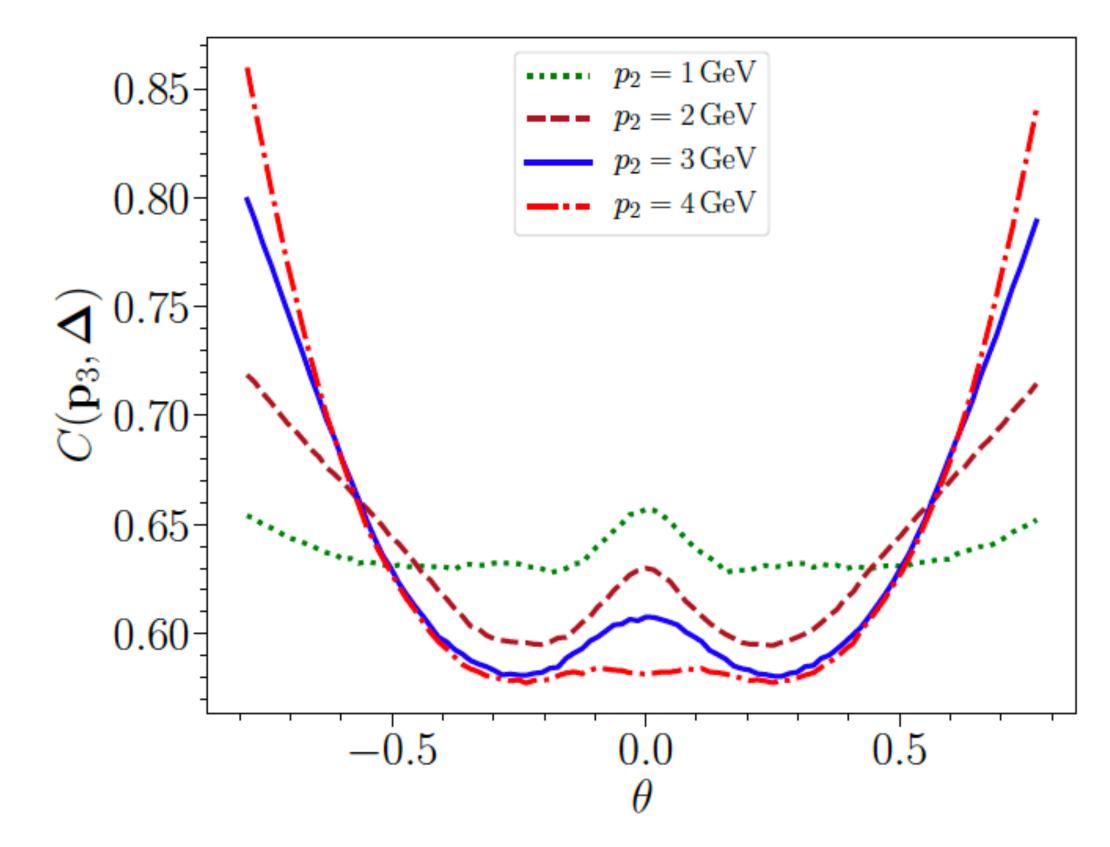
$$|\mathbf{p}_3| = |\Delta| = 10 \,\text{GeV}; \ |\mathbf{p}_1| = 10 \,\text{GeV};$$

Photon virtuality: Q = 1 GeV;

Gluon saturation scale: $Q_s = 2 \,\text{GeV}$

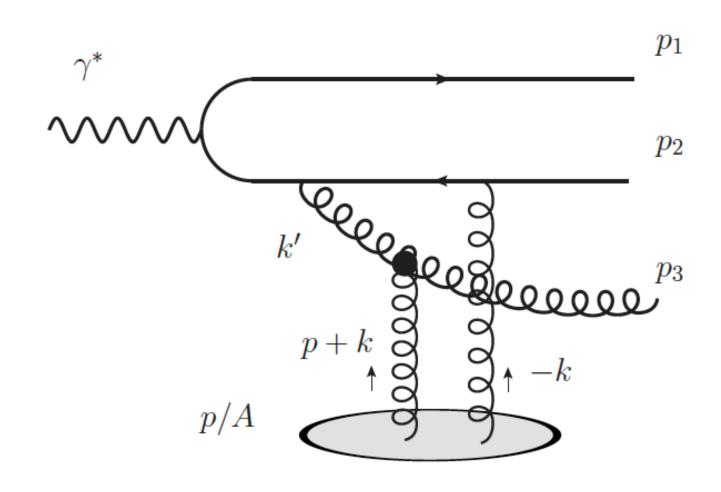
Azimuthal Angle Range: $-\pi/4 < \theta < \pi/4$

The zero-angle peak at $\mathbf{p}_3 = \mathbf{\Delta}$ is the salient feature of Bose correlation.



One can vary values of $|\mathbf{p}_3| = |\Delta|$ from $5 \, \text{GeV} - 15 \, \text{GeV}$, as well of varying the corresponding values of $|\mathbf{p}_1|$, $|\mathbf{p}_2|$ as long as $|\mathbf{p}_1| \gg |\mathbf{p}_2|$, the zero-angle peaks persist.

Zero-Angle Peak Comes from Gluon Bose Correlation



Input parameters:

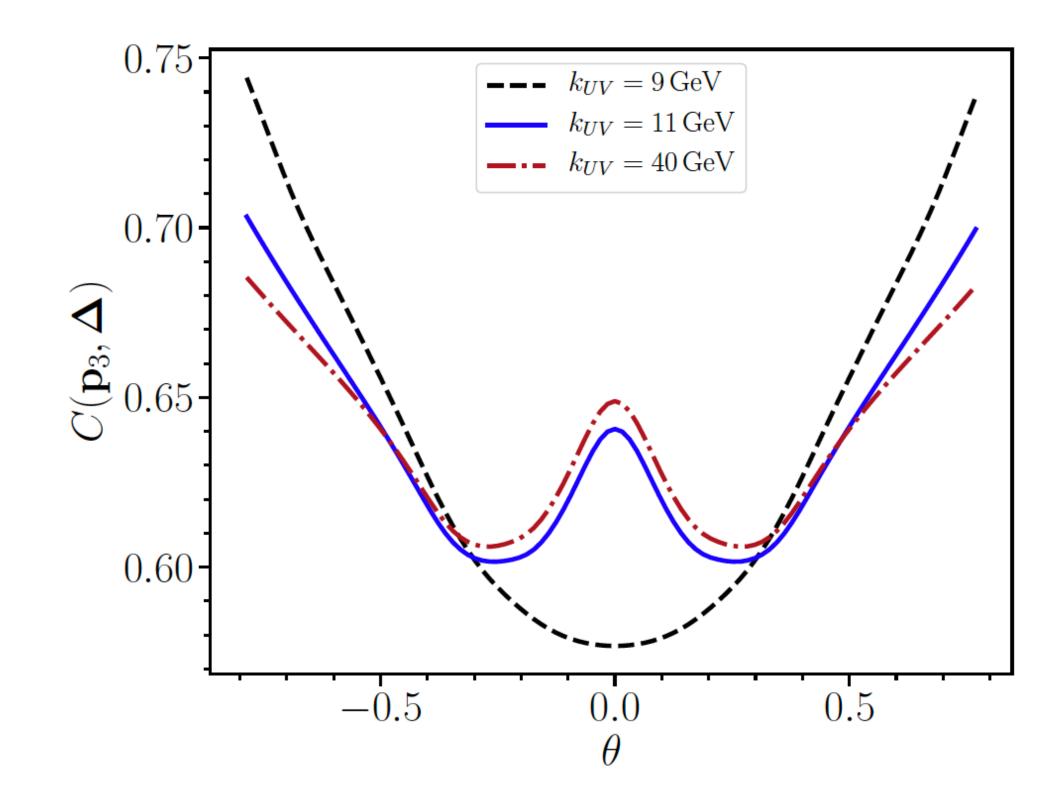
 $|\mathbf{p}_3| = |\mathbf{\Delta}| = 10 \,\text{GeV}; \ |\mathbf{p}_1| = 10 \,\text{GeV};$

Photon virtuality: Q = 1 GeV;

Gluon saturation scale: $Q_s = 2 \,\mathrm{GeV}$

Azimuthal Angle Range: $-\pi/4 < \theta < \pi/4$

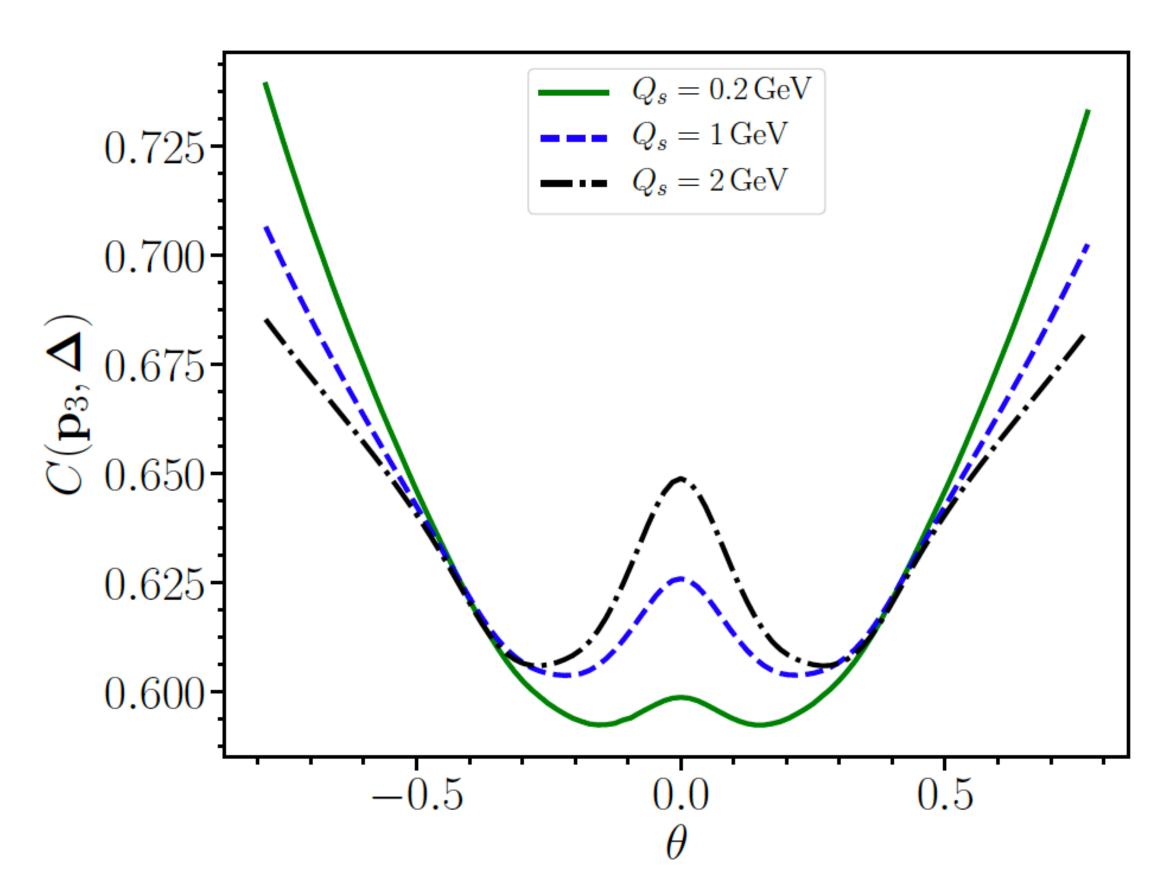
The zero-angle peak mainly comes from the momentum region $|\mathbf{k}| \sim |\Delta|$, In this situation, $-\mathbf{k} \approx \Delta$, $p+\mathbf{k} \approx p_3$, corresponding to the Bose correlation of the two gluons from the target.



Note that the other possibility $\mathbf{k} \approx \Delta$ is extremely unlikely as then $|\mathbf{k}'| \approx 2 |\Delta| \gg Q$, a quark (or antiquark) with momentum of magnitude Q can not radiate a gluon whose momentum is \mathbf{k}' .

Numerical Results: Dependence on Gluon Saturation Scale

Bose enhancement peaks become more prominent with increasing gluon saturation scale?



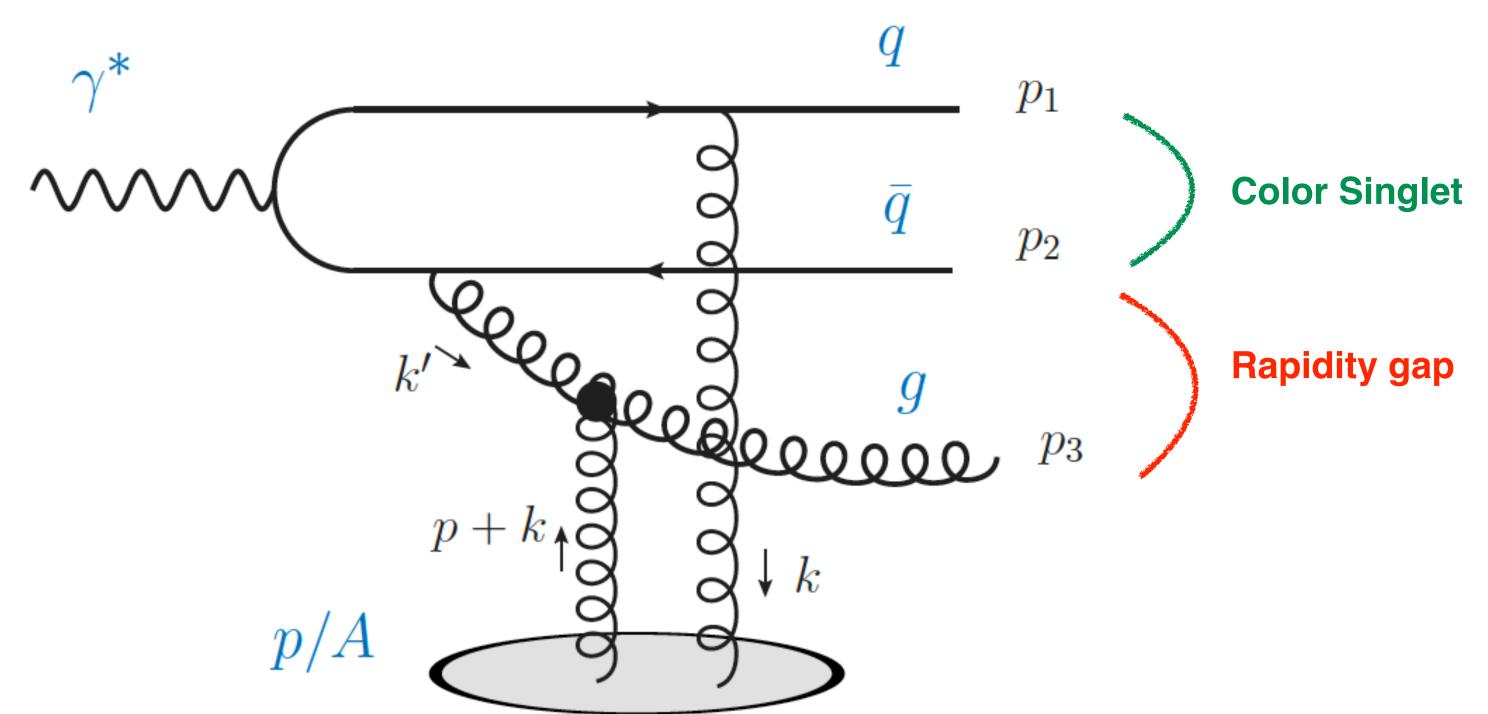
E. lancu, K. Itakura and L. McLerran, NPA (2003)

Effective gaussian model

$$\langle \rho^{a}(\mathbf{k})\rho^{b}(\mathbf{l}) \rangle = \delta^{ab}g^{2}\mu^{2}(\mathbf{k})\delta^{(2)}(\mathbf{k} + \mathbf{l})$$
$$\mu^{2}(\mathbf{k}) = \frac{k^{2}}{k^{2} + Q_{s}^{2}}$$

We know that increasing $Q_{\rm s}$ suppresses away-side correlations.

To Summarize



$$\frac{d^3N}{d^3p_1d^3p_2d^3p_3} = \left| M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2$$

- Small virtuality $Q \ll Q_{s}$,
- Large final momenta $|\mathbf{p}_1| \sim |\mathbf{p}_3| \gg |\mathbf{p}_2| \gg Q_s$,
- Enhancement when $\mathbf{p}_3 \approx \mathbf{\Delta} = \mathbf{p}_1 + \mathbf{p}_2$,
- Dominant phase space region $\, {\bf k}' \sim 0$.
- $^{\bullet}$ The two gluons exchanged with the target have momenta $p_3 \approx \Delta$

Summary and Outlook

- We proposed and demonstrated that diffractive quark antiquark dijet + gluon jet production in DIS has near-side ridge correlation which originates from the Bose correlations in the nuclear wave function.
- The correlation signals are prominent for events outside the conventional correlation limit.
- The numerical computations are done in the two-gluon exchange approximation (dilute limit). It would be very interesting, although challenging, to analyze the general case of dense nuclear target.
- Study similar process in ultra-peripheral heavy-ion collisions at high energy.