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Single Inclusive Jet Production in pA Collisions at NLO

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Jet Physics: from RHIC/LHC to EIC

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Based on

[HL,Xie, Kang,Liu,arXiv:2204.03026]

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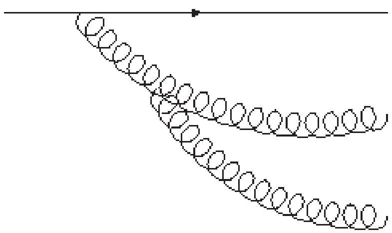
Outline

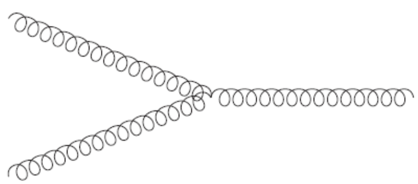
- Review of CGC effective theory
- forward jet production in pA
  - Motivation and Difficulties
  - Subtraction method
  - result
- Outlook

Gluon Saturation

The gluon density increases with Bjorken x decreases



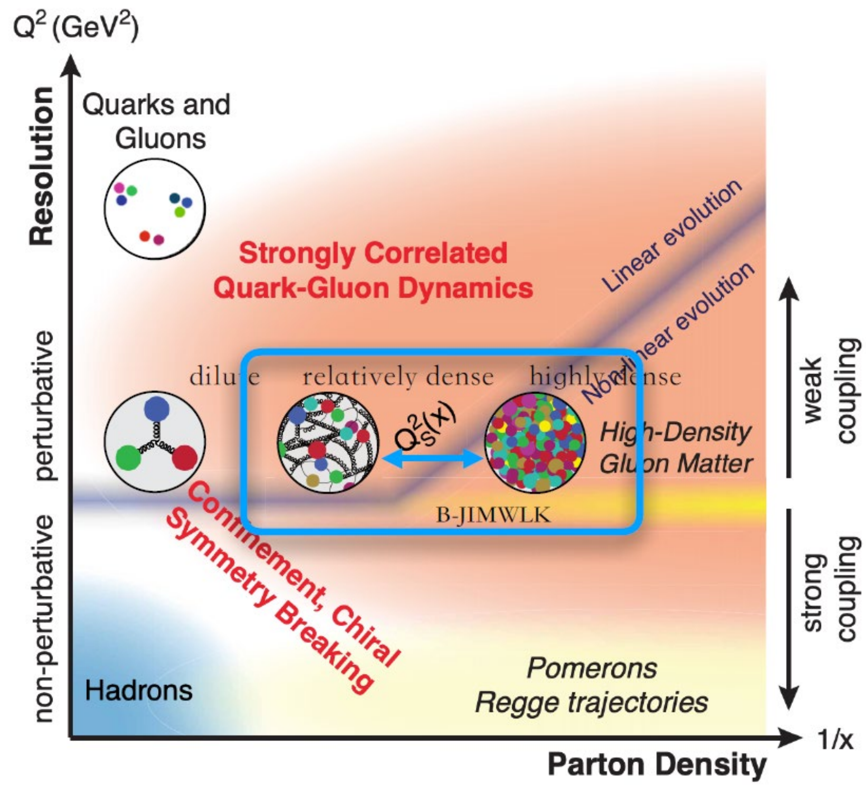
splitting



recombination

Non-linear evolution

saturation



CGC effective theory (Color Glass Condensate) is most appropriate theory for saturation

The distribution in CGC theory

Dipole amplitude $S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{b}'_\perp) = \frac{1}{N_c}\langle Tr[W(\mathbf{b}_\perp)W^\dagger(\mathbf{b}'_\perp)]\rangle_{X_f}$

X_f the factorization scale $W(\mathbf{x}_\perp)$ Wilson Line denoting multi-interaction

Balitsky-Kovchegov evolution equation [I.Balitsky ,NPB,1997] [Y.Kovchegov,PRD,2000]

What we use

$\boxed{\text{LO BK equation with running coupling}}$ [I.Balitsky ,G.Chirilli,PRD,2008] [H.Fujii,K.Watanabe,NPA,2013]

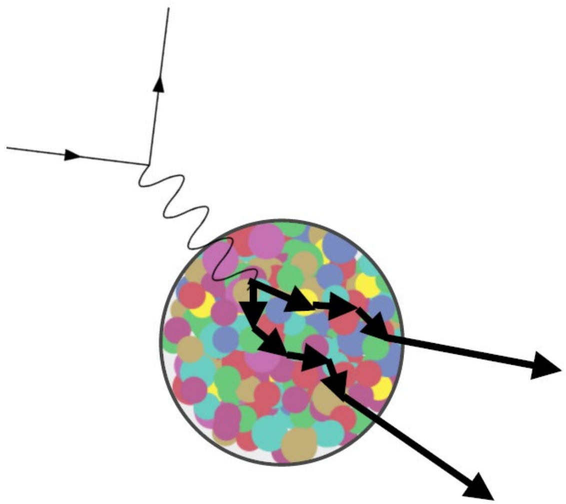
NLO BK equation with resummaiton [T. Lappi, H. Mäntysaari,PRD,2016] [G. Beuf, H. Hänninen, T. Lappi, H. Mäntysaari,PRD,2020]

Dynamic scale $Q_s \sim 2-4\ \mathrm{GeV}$ Perturbatively calculable

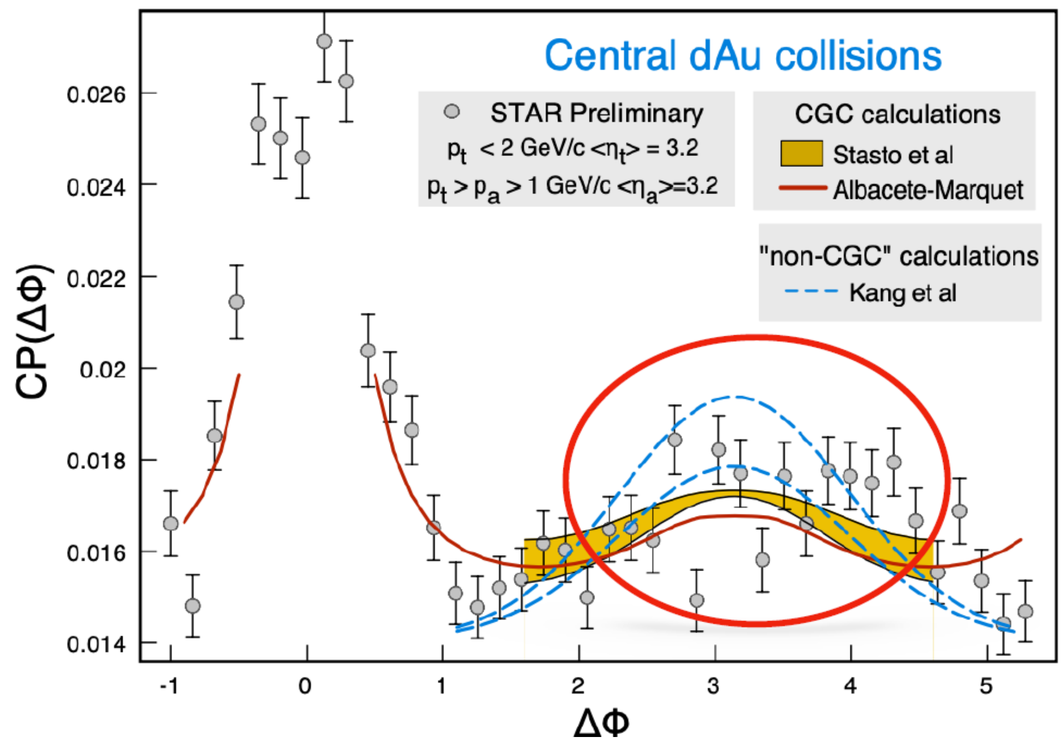
$\alpha_s(Q_s) \sim 0.2-0.3$ is not very small, higher order calculation is necessary

Searching for deterministic evidence of saturation

One of the strong hints for saturation



Away-side peak of the di-hadron correlation



Prediction based on CGC describe the data well

[E. Braidot [STAR Collaboration], NPA,2011.]
[Z. Kang, I. Vitev and H. Xing, PRD, 2012]
[A.Stasto, S.Wei, B.Xiao, F.Yuan, Phys.Lett.B 784 (2018)]
[J..Albacete, G.Giacalone, C.Marquet, M.Matas, PRD,2019]

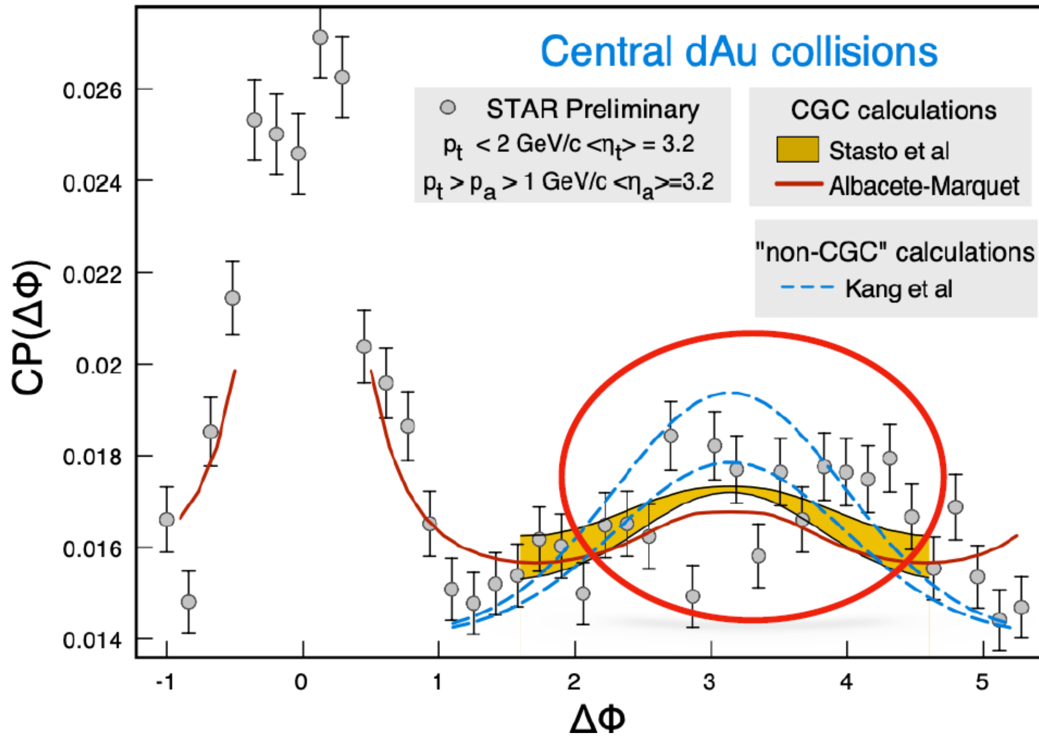
Searching for deterministic evidence of saturation

Compatible with both CGC and collinear twist calculation

How to distinguish

Reduce Error
Theory side
Higher order

More processes
Besides hadron



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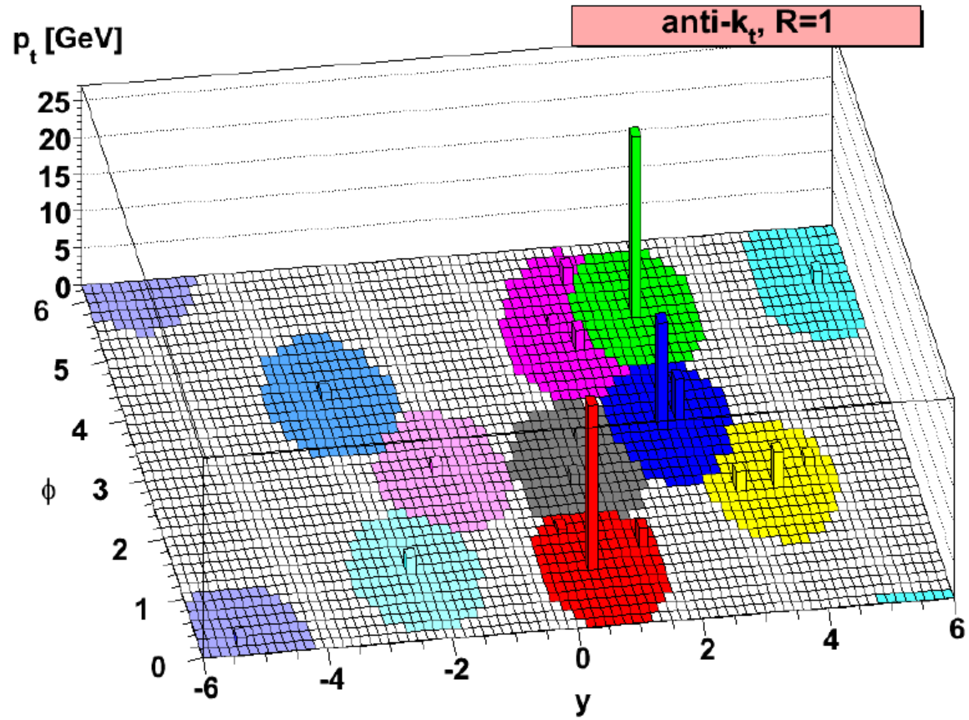
What is a jet?

- Jet is a bunch of hadrons flying nearly in the same direction in high energy collider
- More than half of the papers published by ATLAS and CMS make use of jets since 2010!



[Cacciari, Salam, and Soyez,JHEP,2008]

What is a jet?

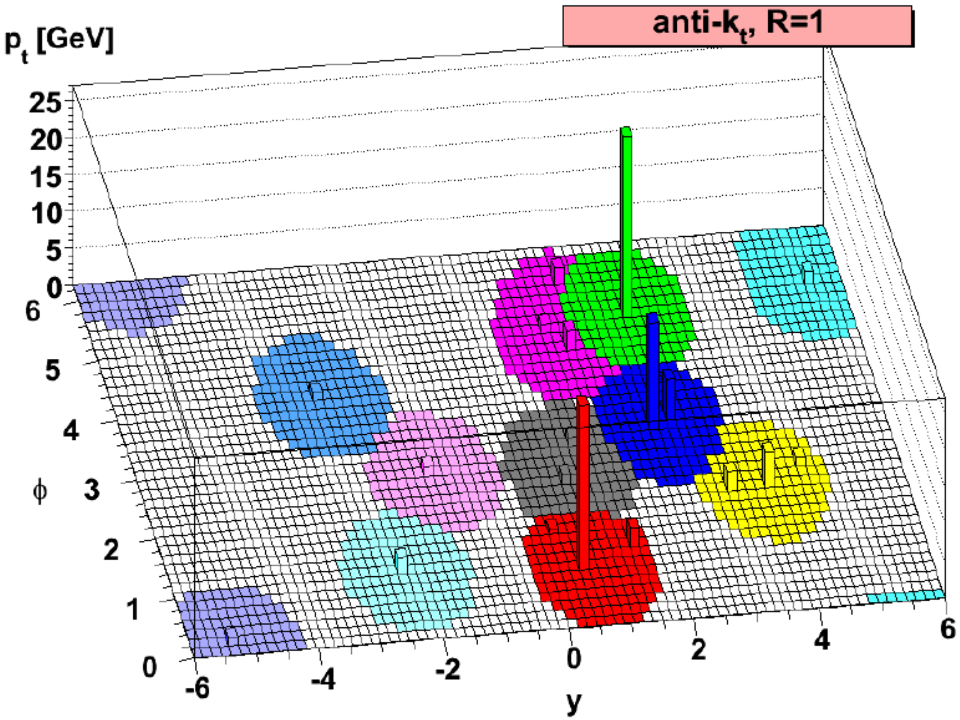
- Jet algorithms are used to classify particles into jets

$\{p_i\}$ particles

jet clustering algorithms

$\{j_k\}$ jets

R controls the extension of the jets



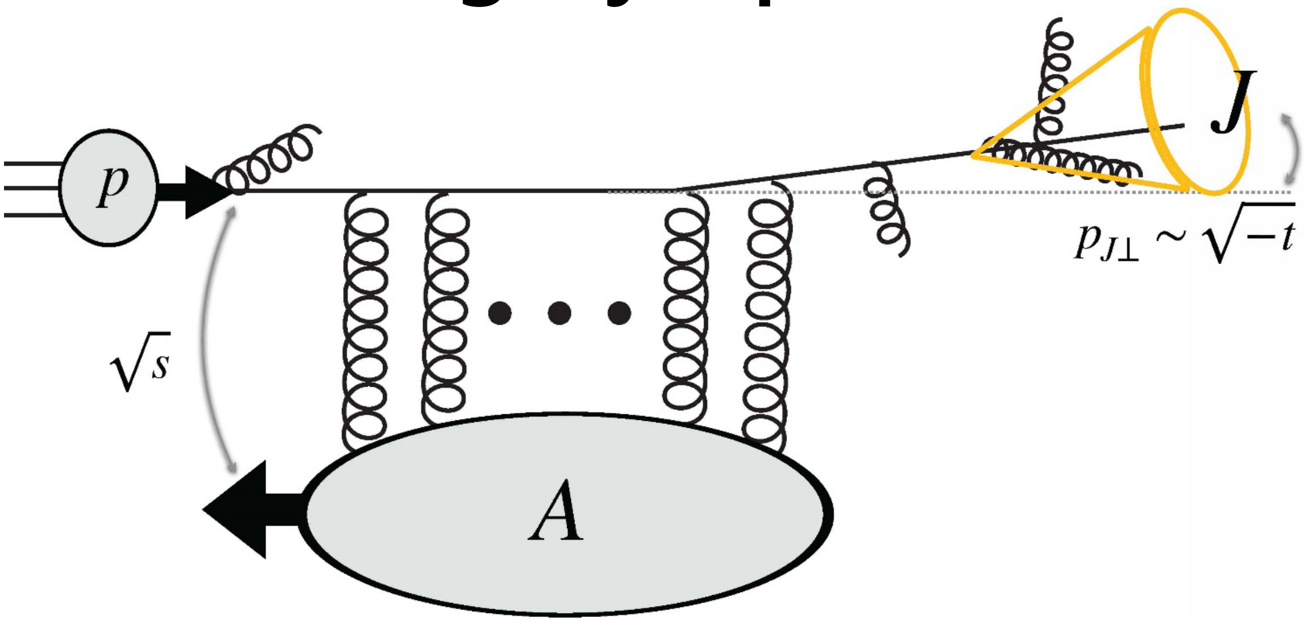
[Cacciari, Salam, and Soyez,JHEP,2008]

Motivation for NLO jet production

- Comparing with hadron, jet is cleaner, in sense that is perturbatively calculable
- Plenty of works on jets in CGC:
  [A.Dumitru, J.Jalilian-Marian. PRL, 2002]
  [A.Dumitru, T.Lappi, V.Skokov. PRL, 2015]
  [Y.Hatta, B.Xiao, F. Yuan. PRL, 2016]
  [H.Mäntysaari, H.Paukkunen. PRD, 2019]
  [R.Boussarie, H.Mäntysaari, F.Salazar, B.Schenke. JHEP, 2021]
- Higher order correction is important for $\alpha_s(Q_s)$ is not small enough

  NLO attempts in small cone approximation:
  [D. Ivanov, A.Papa. JHEP, 2012]
  [P.Caucal, F.Salazar, R.Venugopalan. JHEP, 2021]
  [E. Iancu and Y. Mulian, JHEP ,2021]
  [E. Iancu and Y. Mulian. NPA, 2019]
- An apple-to-apple comparison of the CGC theory with the experimental results, including the jet clustering procedure that strictly follows the experimental analyses

Forward single jet production



Similar to hadron production, with hadron replaced by anti-kT jet

The phase space LO and virtual are identical to hadron production

Things are different for the real correction because of the jet algorithm

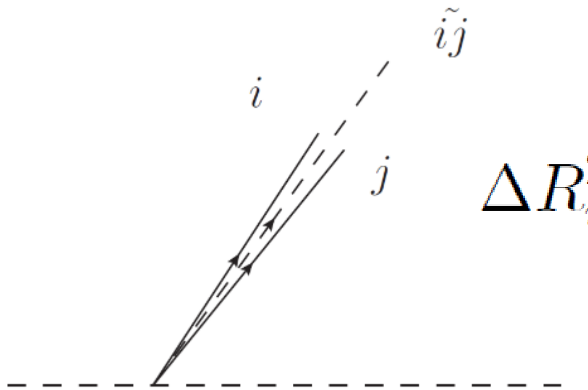
Anti-kT jet algorithm

The distances [Cacciari, Salam, and Soyez, JHEP,2008]

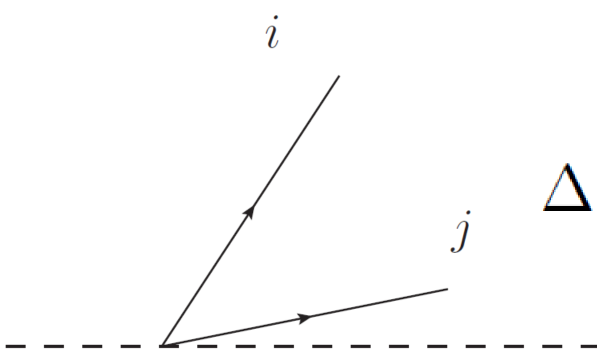
$$\rho_i = k_{T,i}^{-2} \qquad \rho_{ij} = \min(k_{T,i}^{-2}, k_{T,j}^{-2})\frac{\Delta R_{ij}^2}{R^2}$$

where $\Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2$

$k_{T,i}$ transverse momenta η_i and ϕ_i the rapidity and azimuthal angle



$$\Delta R_{ij}^2 \leq R^2$$



$$\Delta R_{ij}^2 > R^2$$

- If ρ_{ij} is the smallest, i and j will be clustered
- If ρ_i is the smallest, i will be a jet

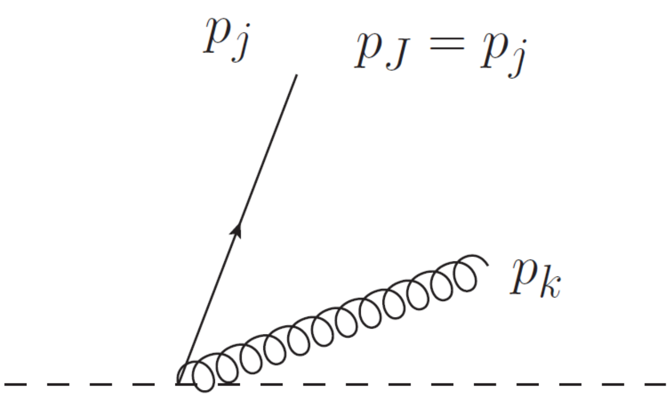
Removed from the list

Until all the particles clustered

Real correction phase space

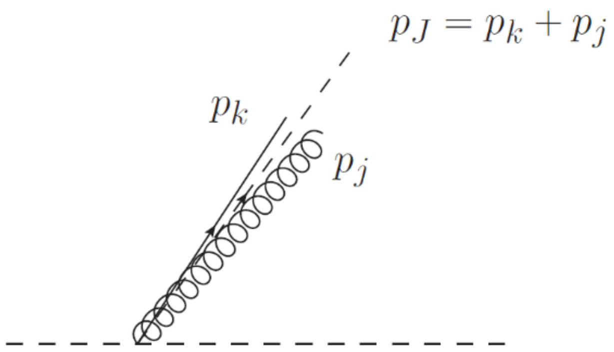
Constraint of phase space for jet $\int d\Phi \times \Theta_{alg}$

No constraint to hadron $\int d\Phi \times 1$



2 jets case

$$\Theta_2 = \Theta(\Delta R_{jk}^2 - R^2)$$



1 jet case

$$\Theta_1 = \Theta(R^2 - \Delta R_{jk}^2)$$

Difficulty for real correction for jet

Trivial example to highlight the difficulty

$$\boxed{\int_0^1 dx \frac{f(x)}{x^{1+\epsilon}}}$$ ⇒ Containing both jet algorithm dependence and divergence

Divergent ⇒ Can't calculate it numerically

Complicated ⇒ Can barely calculate it analytically

Construct subtraction term

The counter term

Can be calculated numerically

$$\int_0^1 dx \frac{f(x)}{x^{1+\epsilon}} = \int_0^1 dx \left(\frac{f(x)}{x^{1+\epsilon}} - \boxed{\frac{f(0)}{x^{1+\epsilon}}} \right) + \boxed{f(0) \int_0^1 \frac{dx}{x^{1+\epsilon}}} = \boxed{\int_0^1 \frac{f(x) - f(0)}{x}} - \frac{f(0)}{\epsilon}$$

Shares exactly the same infrared behavior as the original integrand

Added back

Finite

No jet algorithm dependence

Can be calculated analytically

An example

The square of the matrix element of the final state radiation term is

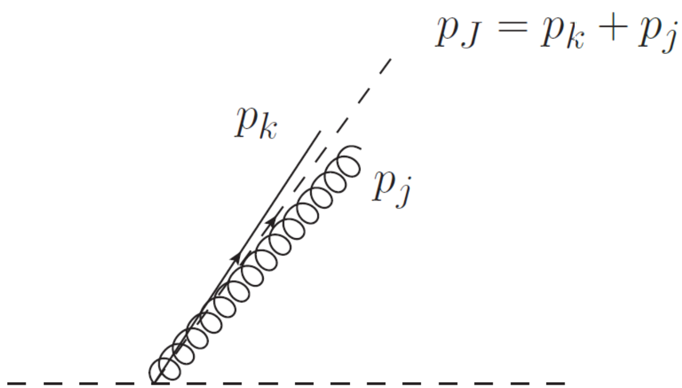
$$d\sigma_{fsr} \propto \quad xf(x)\frac{1+\xi^2}{(1-\xi)^{1+\eta}}\frac{\mathcal{F}_F(p_{k\perp}+p_{j\perp};X_f)}{[\xi p_{k\perp}-(1-\xi)p_{j\perp}]^2}(\Theta_1+\Theta_2)$$

final-final collinear limit

$\xi p_{k\perp} \to (1-\xi)p_{j\perp}$

1 jet case

$\Theta_1 = 1$ and $\Theta_2 = 0$



$$d\sigma^c_{fsr} \propto \tau f(\tau)\frac{1+\xi^2}{(1-\xi)^{1+\eta}}\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}$$

An example

Finite combination $d\sigma_{fsr} - d\sigma^c_{fsr}$

$$\frac{\alpha_s S_\perp}{2\pi^2}\frac{N_C}{2}\int_0^1 d\xi \frac{1+\xi^2}{1-\xi}\int d^2p_{k\perp}\left\{\Theta_2\, xf(x)\frac{\mathcal{F}_F(p_{k\perp}+p_{J\perp};X_f)}{[\xi p_{k\perp}-(1-\xi)p_{J\perp}]^2}\right.$$
$$\left.+\Theta_1\,\tau f(\tau)\frac{\mathcal{F}_F(p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}-\tau f(\tau)\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}\right\},$$

Free of divergence → Numerically calculable

The counter term $d\sigma^c_{fsr}$

$$\boxed{\frac{\alpha_s S_\perp}{2\pi^2}\frac{N_C}{2}\tau f(\tau)\int_0^1 d\xi\frac{1+\xi^2-\epsilon(1-\xi)^2}{(1-\xi)^{1+\eta}}\left(\frac{\nu}{p_q^+}\right)^\eta\int d^{D-2}p_{k\perp}\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}}$$

No dependence on jet algorithm → Analytically calculable

Small-R approximation

Motivation for the approximation:

Find analytical approximations for terms likes $d\sigma_{fsr} - d\sigma^{c}_{fsr}$

Test how good the approximation is

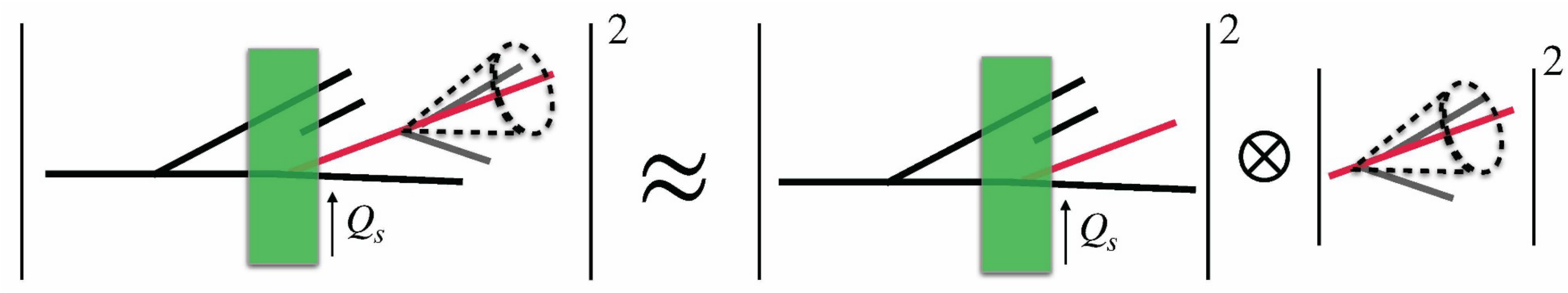
The result will be simplified by further factorized in small-R limit for the collinear factorization case [Kang, Ringer and Vitev, JHEP ,2016]

Similar factorization exists in the CGC case

Small-R approximation

Factorization under the Approximation

$$d\sigma_R = \int d\xi \frac{d\zeta}{\zeta^2} x f(x) d\hat{\sigma}_{q\to q}(\xi, p_J/\zeta)\, J_q(\zeta)$$



We can get it from the our full result

$d\hat{\sigma}_{q\to q}$ partonic single hadron production result

$J_q(\zeta)$ semi-inclusive quark jet function in the large Nc limit

Formally identical to the siJF in collinear factorization

[Kang, Ringer and Vitev, JHEP ,2016]

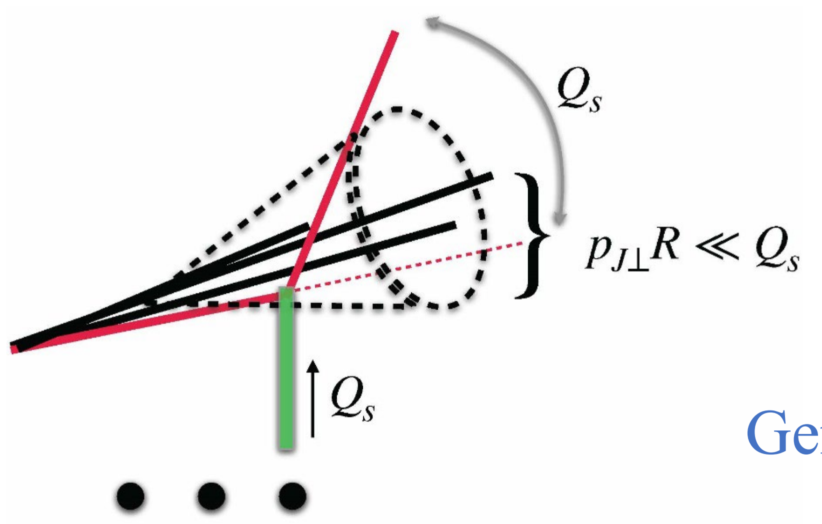
Ignorant of the existence of the CGC shock wave

Small-R approximation

The parton inside the jet has a typical transverse momentum scale

$p_{J\perp} R$ $\qquad$ $p_{J\perp} R \ll p_{J\perp} \sim Q_s$

The parton interacted with the shock wave will be knocked out to the jet because of obtaining an $p_\perp \sim Q_s$



Generalized to other jet observables

Comparison between full and small-R

$$\rho_{R,p_{J\perp}} \equiv \frac{d\sigma_{\text{full}}/dp_{J,\perp}}{d\sigma_{\text{small } R}/dp_{J,\perp}}$$

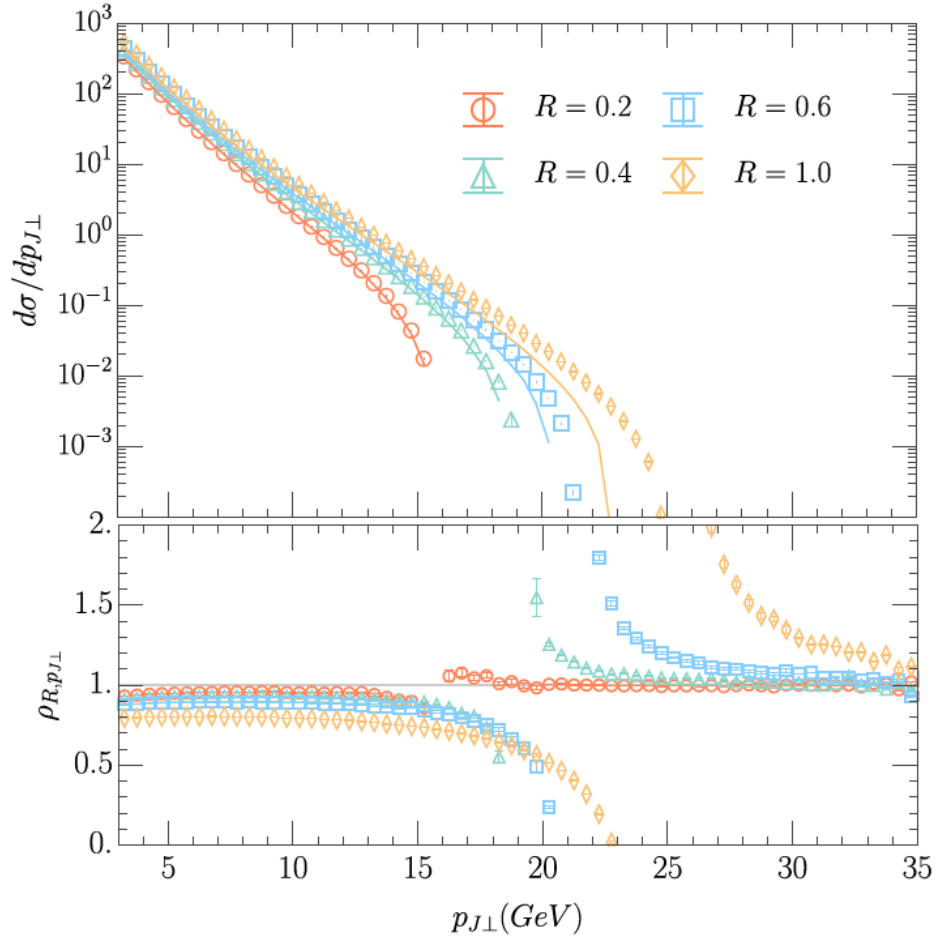
Negative cross section for large $p_{J\perp}$

Bigger R, bigger cross section

Smaller R, better approximation

>90% accuracy for R=0.2,0.4,0.6

The approximation can break down if strong cancellation exists



Comparison between full and small-R

E_J spectrum

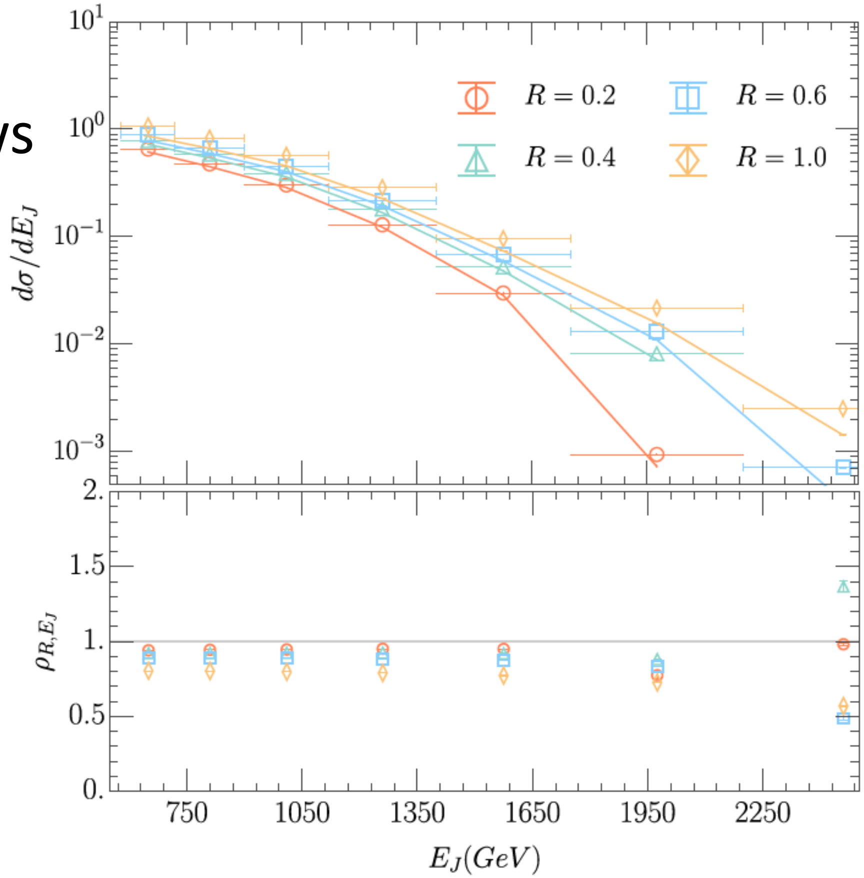
The division of the energy bins follows [CMS Collaboration,Sirunyan et al.,. JHEP,**2019**]

Similar behavior to $p_{J\perp}$ case

Negative cross section

Better approximation because the $p_{J\perp}$ for E_J is relatively small

Distribution of other observables can be generated by histogram



Comparison between full and threshold

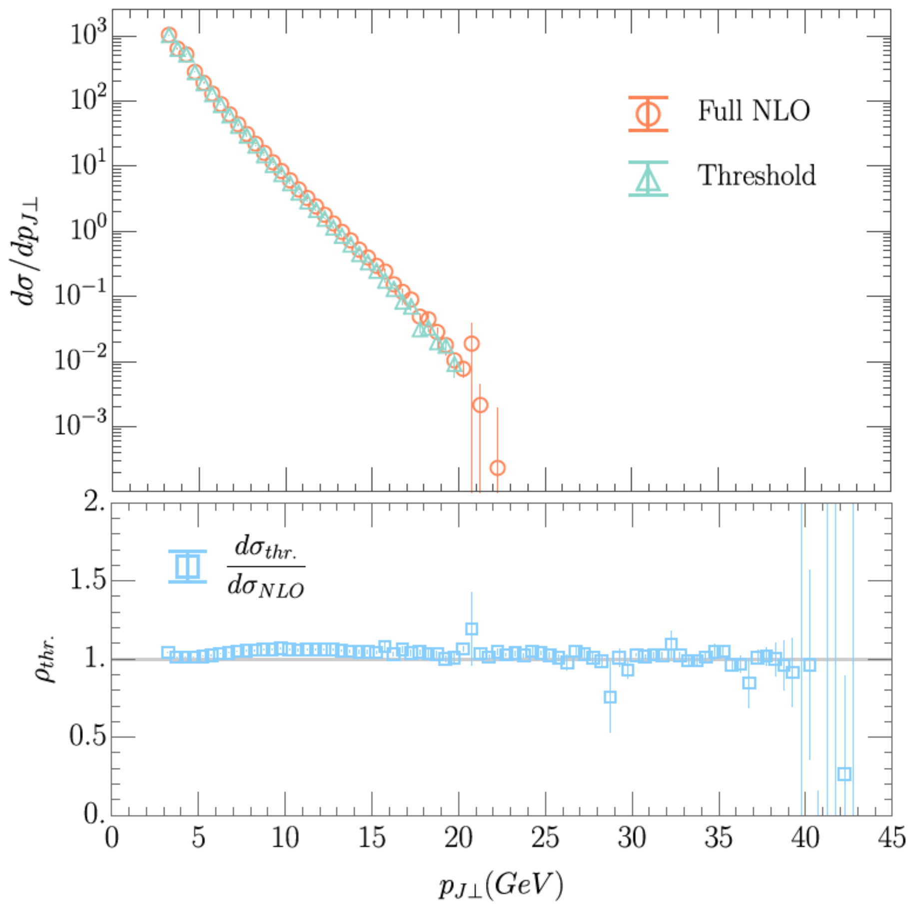
Compare the result of full NLO and threshold approximation

$$\rho_{thr.} = \frac{d\sigma_{thr.}/dp_{J\perp}}{d\sigma_{\mathrm{NLO}}/dp_{J\perp}}$$

The common $\delta(1-\xi)$ terms are Removed when calculating the ratio

The approximation is very good when $p_{J\perp}$ is large

We know how to deal with the negative cross section



Outlook

We look forward to do a full comparison to the experimental data, for instance, to [CMS Collaboration,Sirunyan et al.,. JHEP,**2019**].

The method in the small-R approximation can be generalized to other jet related observables or other processes depend on constraint, for instance, the isolated photon production [B. Duclou´e, T. Lappi, and H. M¨antysaari.PRD, 2018].

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Thank You!

The Back up

Threshold resummation

Mellin transformation $M_N(f(\xi)) = \int_0^1 d\xi \xi^{N-1} f(\xi)$

$$M_N(1) \to 0 \qquad M_N\left(\frac{1}{(1-\xi)_+}\right) \to -\ln \bar{N} \qquad M_N\left(\left[\frac{\ln(1-\xi)}{1-\xi}\right]_+\right) \to \frac{1}{2}\ln^2 \bar{N} + \frac{\pi^2}{12}$$

The small-R limit result becomes

$$d\hat{\sigma}^{(1)}_{q\to q,thr.} = \langle \mathcal{M}_0 | \frac{\alpha_s}{\pi} \left(\mathbf{T}_i^2 + \mathbf{T}_j^2\right) \ln \bar{N} \ln \frac{\mu^2}{p_{J\perp}^2}$$

$$-\frac{\alpha_s}{\pi} \int \frac{dr_\perp}{\pi} \left[-2 \ln \bar{N} \left(\frac{x_\perp \cdot y_\perp}{x_\perp^2 y_\perp^2}\right)_+ + \ln \frac{X_f}{X_A} \left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2}\right)_+ \right] \mathbf{T}_j^{a'} W_{a'a}(r_\perp) \mathbf{T}_i^a | \mathcal{M}_0 \rangle$$

$\mathbf{T}_i^a$ Catani operator

Threshold resummation

$$\frac{\alpha_s}{\pi}\left(\mathbf{T}_i^2+\mathbf{T}_j^2\right)\ln\bar{N}\ln\frac{\mu^2}{p_{J\perp}^2}$$

Terms proportional to $\mathbf{T}_j^2$ can be resummed by the techniques the Sudakov logarithms resummation

$$-\frac{\alpha_s}{\pi}\int\frac{dr_\perp}{\pi}\left[-2\ln\bar{N}\left(\frac{x_\perp\cdot y_\perp}{x_\perp^2 y_\perp^2}\right)_+ +\ln\frac{X_f}{X_A}\left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2}\right)_+\right]\mathbf{T}_j^{a'}W_{a'a}(r_\perp)\mathbf{T}_i^a$$

This term can not be resummed by the Sudakov log resummation techniques, shares the same color structure as the BK evolution.

At higher orders, every additional ISR will generate an additional Wilson line that complicates the color structures.

The factorization formula

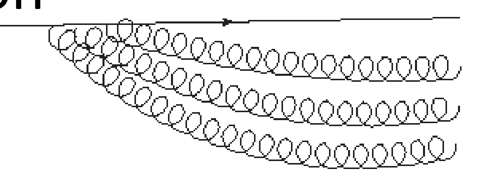
We firstly reexamine the factorization formula by power counting

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_h \mathrm{d}^2 p_{h\perp}} = \sum_{i,j=g,q} \frac{1}{4\pi^2} \int \frac{\mathrm{d}\xi}{\xi^2} \frac{\mathrm{d}x}{x} \, z x f_{i/P}(x,\mu) D_{h/j}(\xi,\mu)$$

$$\times \int \mathrm{d}^2 b_\perp \mathrm{d}^2 b'_\perp \, e^{i p'_\perp \cdot r_\perp} \Big\langle \langle \mathcal{M}_0(b'_\perp) | \, \boldsymbol{\mathcal{J}}(z,\mu,\nu,b_\perp,b'_\perp) \boldsymbol{\mathcal{S}}(\mu,\nu,b_\perp,b'_\perp) | \mathcal{M}_0(b_\perp) \rangle \Big\rangle_\nu$$

$|\mathcal{M}_0(b_\perp)\rangle$ Standard color space notation [Catani et al.NPB, 2000]

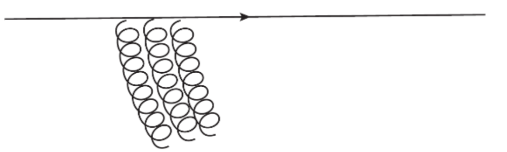
$\mathcal{J}$ Jet function Contribution from Collinear radiation



Gluon in forward direction with momentum

$$\sqrt{s}(1,\lambda^2,\lambda) \qquad \lambda \sim p_{h,\perp}/\sqrt{s} \ll 1$$

$\mathcal{S}$ Soft function Contribution from soft radiation



Gluon in central direction with momentum

$$\sqrt{s}(\lambda,\lambda,\lambda)$$

Large log and evolution

For the threshold region $z \to 1$ $\qquad \bar{n}\cdot k = \bar{n}\cdot p(1-z) \sim p'_\perp$

real emitted gluon $(\bar{n}\cdot k, n\cdot k, k_\perp) \sim \sqrt{s}(\lambda,\lambda,\lambda)$ soft

$\mathcal{J}$ Contains only virtual correction contribution

$\mathcal{S}$ Contains real correction contribution

$\mathcal{J}$ and $\mathcal{S}$ can be calculated perturbatively

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots$$

$$S^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_s}\right) + \alpha_s D_s(\nu_s) + \ldots$$

We reproduce the full fixed order results with $z \to 1$

$D_s(\nu_s)$ contains $\ln(\nu_s/\bar{n}\cdot p)$, $\ln(\nu_s/p'_\perp)$ and $\dfrac{1}{(1-z)_+}$

$\nu_J = \bar{n}\cdot p \qquad \nu_s \sim p'_\perp \qquad p'_\perp \ll \bar{n}\cdot p$

So the evolution equation is $\nu \dfrac{d}{d\nu}\mathcal{F}(\nu) = \gamma_{\mathcal{F}}\mathcal{F}(\nu) \qquad \mathcal{F} = \mathcal{J} \text{ or } \mathcal{S}$

Leading log result

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots$$

$$J^{(0)} + J^{(1)} \propto (1 + \alpha_s \ln\left(\frac{\nu}{\nu_J}\right))(1 + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots)$$

All order

Evolution kernel $\boldsymbol{U}_{\mathcal{F}}(\nu, \nu_{\mathcal{F}})$

$\boldsymbol{\mathcal{F}}(\nu_{\mathcal{F}})$ Initial condition

$$\boldsymbol{\mathcal{F}}(\nu) = \boldsymbol{U}_{\mathcal{F}}(\nu, \nu_{\mathcal{F}})\, \boldsymbol{\mathcal{F}}(\nu_{\mathcal{F}})$$

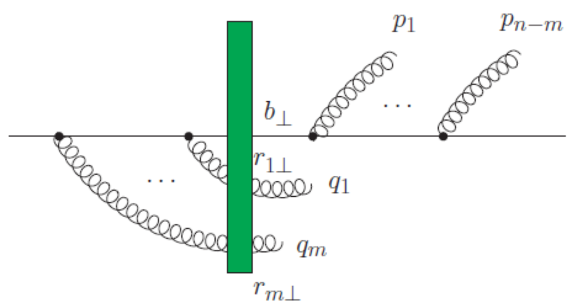
$$\boldsymbol{U}_J\, \boldsymbol{U}_S = \exp\left[-\frac{\alpha_s}{\pi}\int \frac{\mathrm{d}x_\perp}{\pi}\left(\ln\frac{\nu_S}{\nu_J} I_{BK,r} + \ln\frac{X_f}{X_A} I_{BK}\right)\mathbf{T}_i^a \mathbf{T}_j^{a'}\, W_{aa'}(x_\perp)\right]$$

Proof under strong ordering limit

For the leading log ,considering the independent n-multiple soft gluon strong ordering emission at $\mathrm{N}^{(n)}\mathrm{LO}$, in which

$$q_1^- \gg q_2^- \gg \cdots \gg q_m^- \qquad p_1^- \gg p_2^- \gg \cdots p_{n-m}^-$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left| \sum_{m=0}^{n} \right.$$

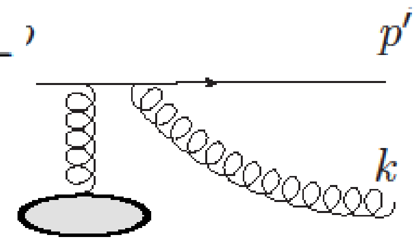


$$\left. \right|^2$$

$$= \langle \mathcal{M}_0 | \exp\left\{ -\frac{\alpha_s}{\pi} \int \frac{dr_\perp}{\pi} \left[-2\ln \bar{N} \left(\frac{x_\perp \cdot y_\perp}{x_\perp^2 y_\perp^2} \right)_+ + \ln \frac{X_f}{X_A} \left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2} \right)_+ \right] \mathbf{T}_j^{a'} W_{a'a}(r_\perp) \mathbf{T}_i^a \right\} | \mathcal{M}_0 \rangle$$

Our resummation formula hold in this limit.

Dominate terms for large $p_{h,\perp}$,



$$\frac{\mathrm{d}^2\hat{\sigma}^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^{\,2}_\perp r''^{\,2}_\perp}\right.$$

$$\left.+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^{\,2}_\perp r''^{\,2}_\perp}\right]_+\right\}W_{aa'}(x_\perp)+\ldots$$

$$\frac{\bar{n}\cdot p'}{\bar{n}\cdot p}=z \qquad \frac{\bar{n}\cdot k}{\bar{n}\cdot p}=1-z$$

$$\tilde{P}_{i\to i}(z)\to\frac{2}{(1-z)_+}$$

$$x_p = p_{h,\perp}e^{y_h}/\xi\sqrt{s}\to 1$$

$$x_p < z < 1$$

The threshold contribution is proportional to $\dfrac{f(x_p/z)-f(x_p)}{1-z}$

Because the PDF decreases rapidly when x_p is large, $f(x_p/z)\ll f(x_p)$ even when z is not far from 1

$\dfrac{f(x_p/z)-f(x_p)}{1-z}\to-\dfrac{f(x_p)}{1-z}$ and becomes a large log and is negative

Generating Histogram

With form of $d\sigma$ and information of p_j and p_k

Distribution of any observable is available by histogram

We take E_J as example, the steps are as follow

1. Divide the observable spectrum into N different bins
$$(E_{J,0}, E_{J,1}) \,,\, (E_{J,1}, E_{J,2}) \,, \ldots \,, (E_{J,i}, E_{J,i+1}) \,, \ldots \,, (E_{J,N-1}, E_{J,N})$$
2. Generate the momenta p_j and p_k out of the free variables p_J^+ and $p_{J\perp}$ according to whether it is a 1-jet or 2-jets case the event is kept is the momenta satisfies the jet clustering algorithm , otherwise vetoed
3. Get E_J by p_j and p_k according to 1-jet or 2-jets case, if $E_J \in (E_{J,i}, E_{J,i+1})$, fill the event into this bin with weight $d\sigma$
4. Repeat step 2 and 3