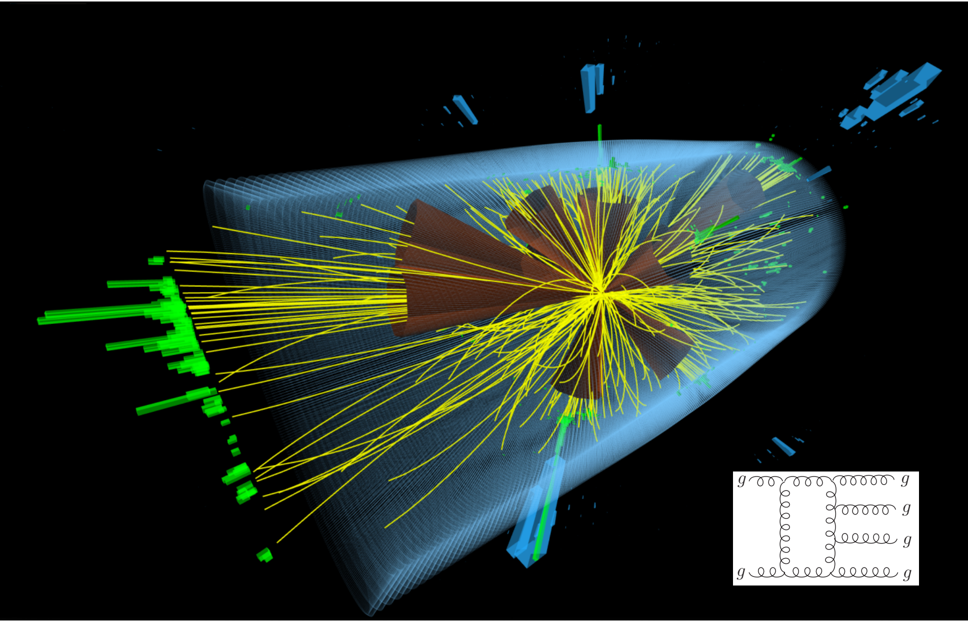
Conformal Colliders Meet the LHC

Ian Moult
Yale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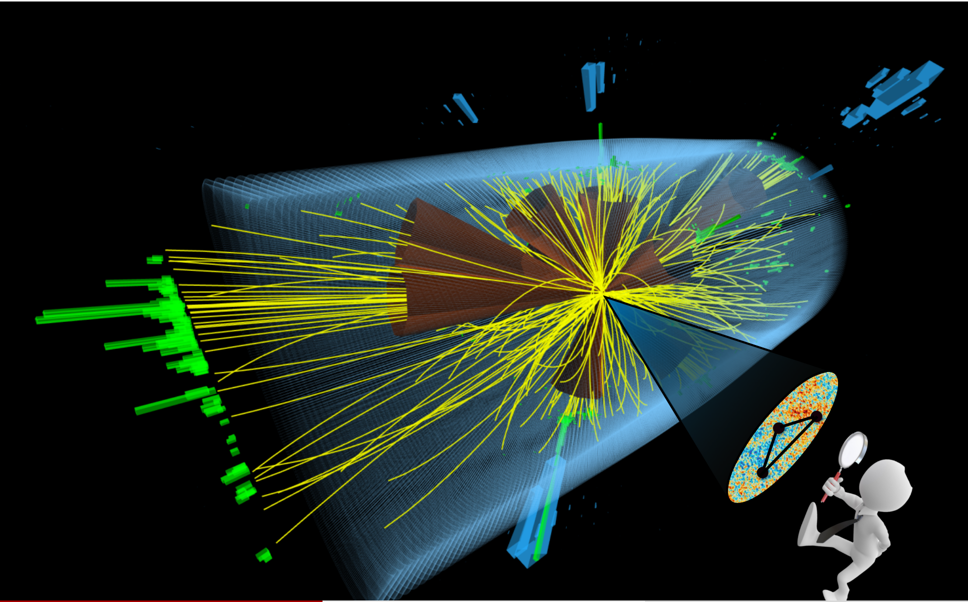
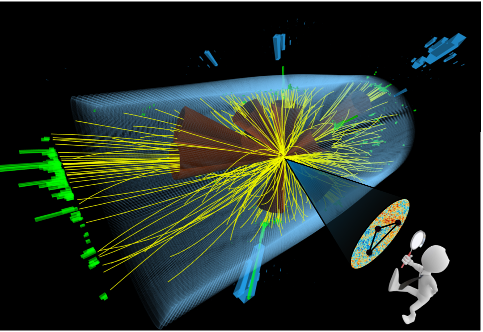
Jet Substructure!

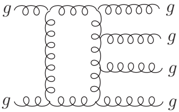


Changing the Perspective

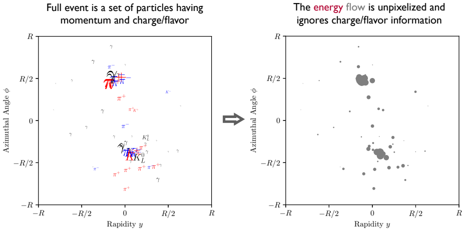
- This changes the problem from studying the production of jets (S-matrix elements) to studying the statistical properties of energy flux within jets.



S-Matrix



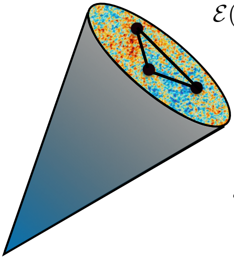
Energy Flux



- Requires the development of a new set of theoretical tools and new ways of thinking about jets.

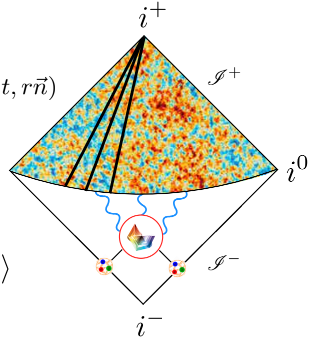
New (Very Old) Insights

- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators. [Hofman, Maldacena] [Korchemsky, Sterman] [Ore, Sterman]



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt n^i T_{0i}(t, r\vec{n})$$

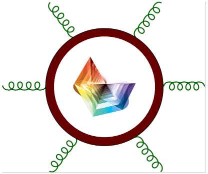
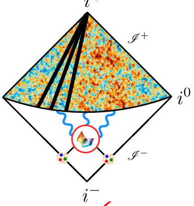
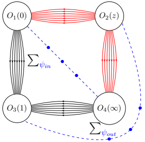
$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



- These are the fundamental building blocks for the study of jets. Play a privileged role in directly connecting to the underlying field theory.
- Any "jet shape" can be written as an infinite sum of correlators. This sum obscures the underlying physics.

Energy Correlators

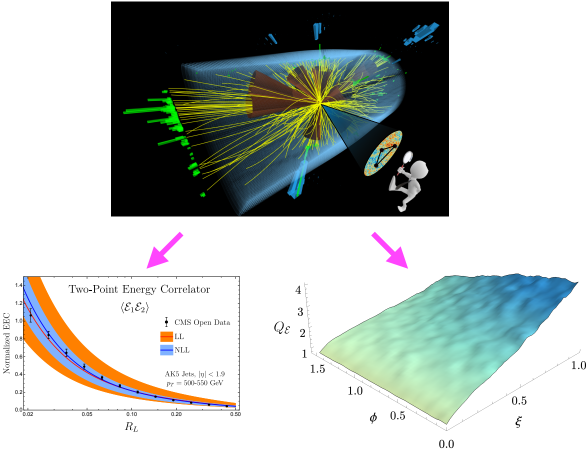
- Correlation functions of energy flow operators take an interesting intermediate position between amplitudes and correlation functions.

	Amplitudes	Energy Correlators	Correlation Functions
			
Asymptotic States	✓	✓	
IR Finite		✓	✓

- Despite their physical importance, much less explored.

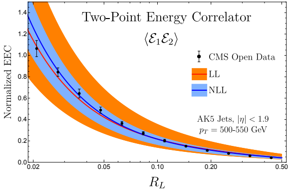
Conformal Colliders Meet the LHC

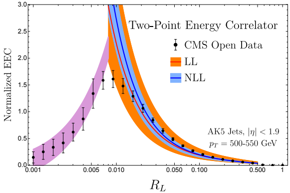
- Progress in the understanding of lightray operators allows the calculation and measurement of the shapes and scalings of multipoint correlators, inside high energy jets at the LHC.

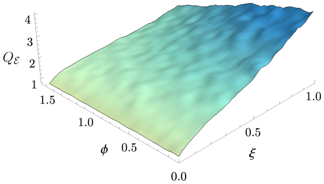


Outline

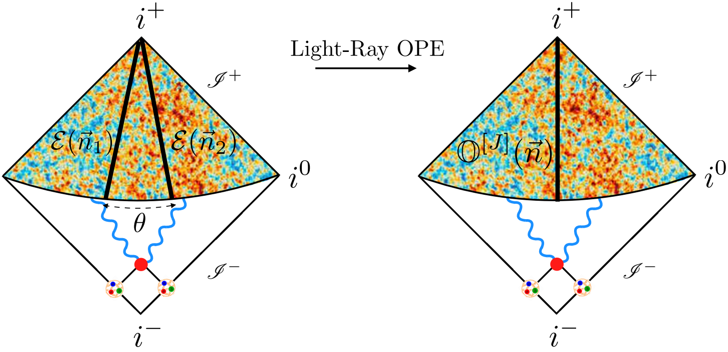
- Scaling Behavior in Jet Substructure
- Imaging the Confinement Transition
- Non-Gaussianities in Energy Flux







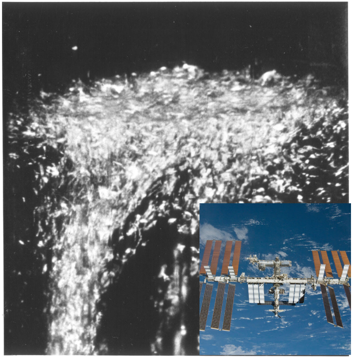
Scaling Behavior in Jet Substructure

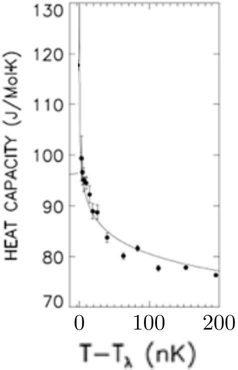


Scaling Behavior in QFT

- QFTs exhibit universal behavior as operators are brought together.
- For local operators, this is captured by the operator product expansion (OPE) $\implies$ scaling behavior!

λ-point of Helium

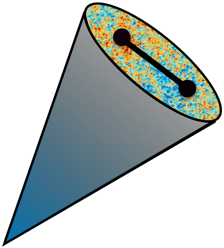


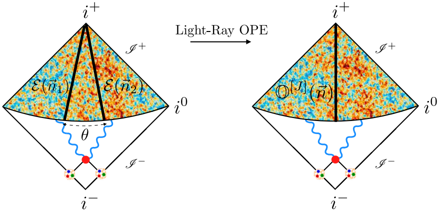


$$\boxed{\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i}$$

The OPE Limit of Lightray Operators

- Energy flow operators admit an OPE!
- Jet Substructure is the study of the OPE limit of lightray operators.





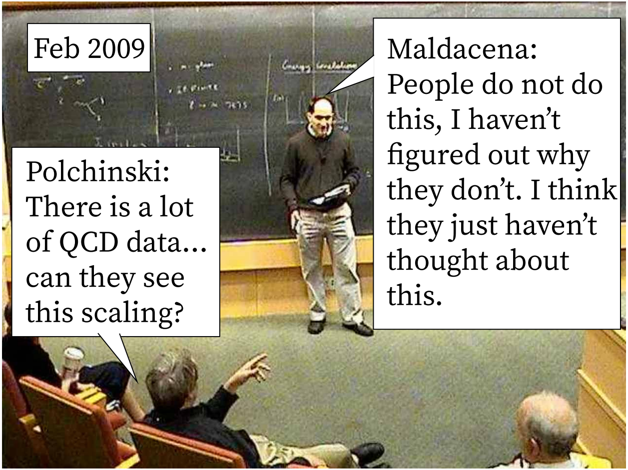
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

- Allows a completely new approach to jet substructure as the study of the symmetry and OPE structure of these operators.

Theory-Experiment Gap

- OPE scaling is the most basic prediction of QFT for jet substructure.

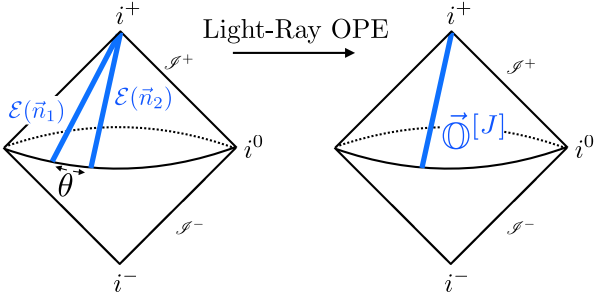


- Shockingly, still true as of 2022...

The Lightray OPE

- In CFTs, the lightray OPE is a convergent, and rigorous expansion. [Hofman, Maldacena; Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$



- To describe the leading scaling at the LHC, we can restrict to the leading term in the OPE $\implies$ twist-2 light ray operators.

The Leading Twist Lightray OPE

[Hofman, Maldacena]
[Chen, IM, Zhu]

- The twist-2 operators in QCD are characterized by a spin-J and a transverse spin $j = 0, 2$.
- These can be light-transformed to obtain a vector of twist-2 lightray operators parametrized by spin-J:

Local Operators

[Kravchuk, Simmons Duffin]

transverse spin-0

$$\begin{cases} \mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi \\ \mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+} \end{cases}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$

$$\xrightarrow{\lim\limits_{r\to\infty} r^2\int_0^\infty dt} \quad \vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

unpolarized

polarized

- The anomalous dimensions of these operators,

$$\frac{d}{d\ln\mu^2}\vec{\mathbb{O}}^{[J]}(\hat{n}_1) = \hat{\gamma}(J)\vec{\mathbb{O}}^{[J]}(\hat{n}_1)$$

determines the leading behavior of jet substructure.

Explicit Structure

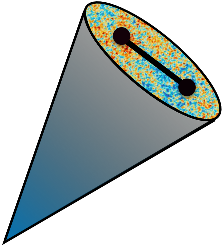
[Chen, IM, Zhu]

- The OPE coefficients can be expressed in terms of a matrix, $\hat{C}_\phi(J)$, whose entries are analytic functions of J:

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\vec{\mathcal{J}}\left[\widehat{C}_\phi(2) - \widehat{C}_\phi(3)\right]\vec{\mathbb{O}}^{[3]}(\hat{n}_1) + \cdots\,,$$

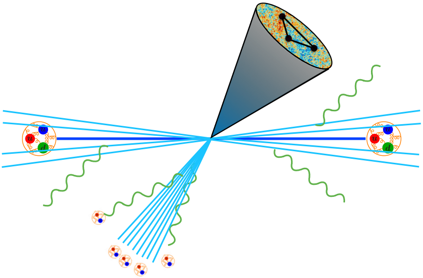
$$\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \cdots$$

- By studying asymptotic energy flow, one can directly observe scaling behavior associated with the twist-2 operators in QCD.



Factorization Theorem at Hadron Colliders

- Can derive factorization theorems at hadron colliders using the proofs of Collins-Soper-Sterman for inclusive fragmentation:



$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T\,\mathrm{d}\eta\,\mathrm{d}\{\zeta\}} = \sum_i \mathcal{H}_i\left(p_T/z,\eta,\mu\right)$$

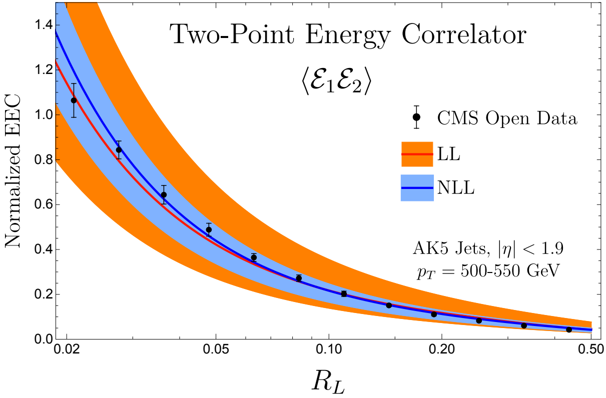
[Lee, Mecaj, Moult]

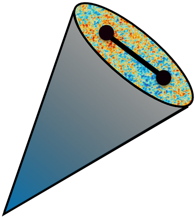
$$\otimes \int_0^1 dx\, x^N\, \mathcal{J}_{ij}(z,x,p_T R,\mu)\, J_j^{[N]}(\{\zeta\},x,\mu)\,.$$

The OPE Limit in Data

[Komiske, Moult, Thaler, Zhu]
[Dixon, Moult, Zhu]
[Lee, Mecaj, Moult]

- The $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$ OPE inside high-energy jets!

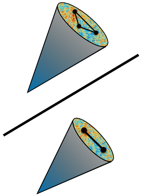




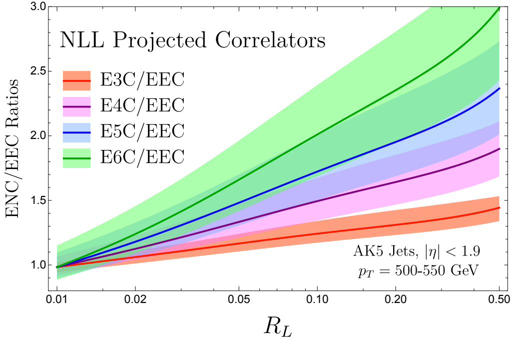
- Beautiful illustration of the universality of the OPE limit in QFT!
- Universality allows calculations in the complicated LHC environment.

Higher Point Scaling

- The light-ray OPE predicts that the N-point correlators develop an anomalous scaling that depends on N.



$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

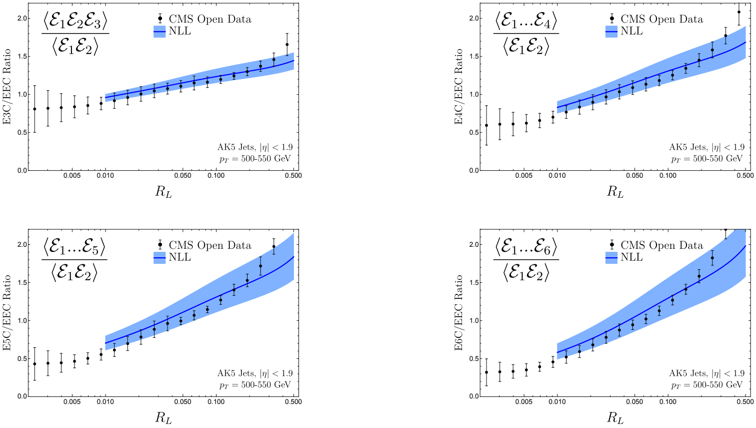


- Directly probes the spectrum of (twist-2) lightray operators in QCD using asymptotic energy flux!

Higher Point Scaling

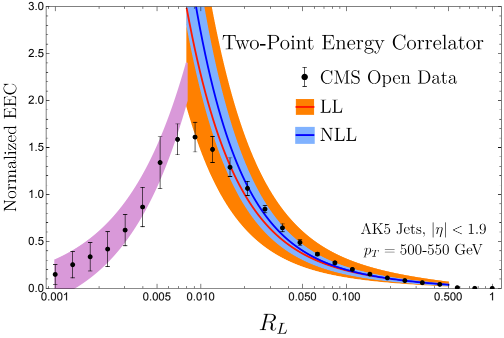
[Chen, Moult, Zhang, Zhu]
[Lee, Mecaj, Moult]

- The remarkable LHC dataset allows these scalings to be measured at the quantum level.



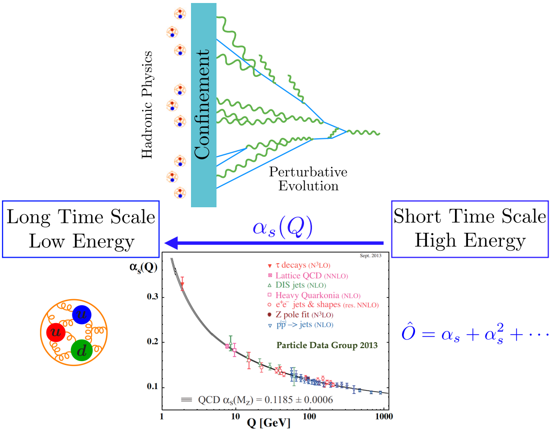
- Fundamentally new probes of jets at colliders!!

Imaging the Confinement Transition



The Confinement Transition

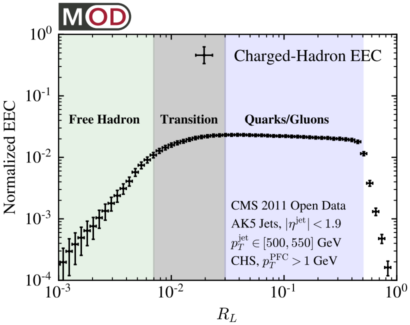
- Jets exhibit a transition from weakly coupled quarks and gluons to freely propagating hadrons.

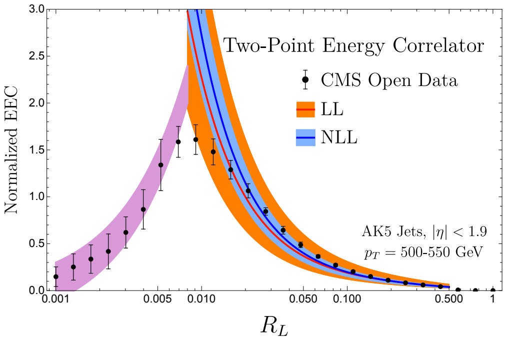


- Can it be directly imaged?

The Confinement Transition

- Distinct scalings associated with interacting quarks and gluons and free hadrons clearly visible!



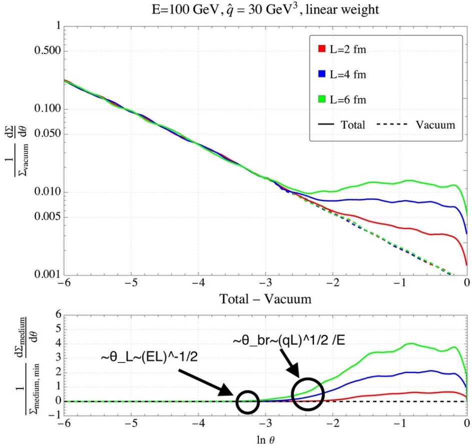


- Precision measurements of the confinement transition possible.
- https://www.youtube.com/watch?v=ORwDv1KTB5U

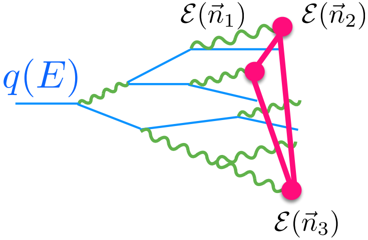
Resolving the Scales of the QGP

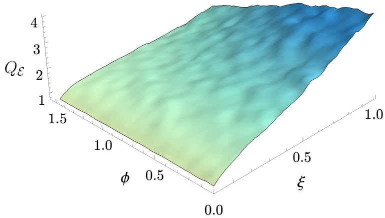
- More generally, correlators pick up on scales (unlike jet shapes!).
- This is particularly useful if you don't have robust theoretical control, but just want to identify a scale.

[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]



Non-Gaussianities in Energy Flux

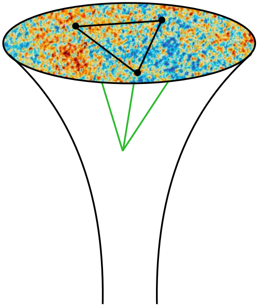


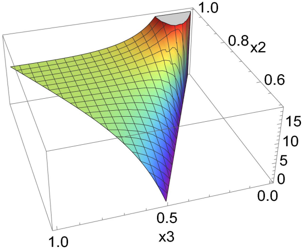


[Chen, Moult, Thaler, Zhu]

Non-Gaussianities

- Higher-point correlators probe more detailed aspects of interactions.
- e.g. Non-Gaussianities allow one to distinguish models of inflation.
- Three-point function, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$, first computed by Maldacena.





[Cabass, Pajer, Stefanysyzn, Supel]

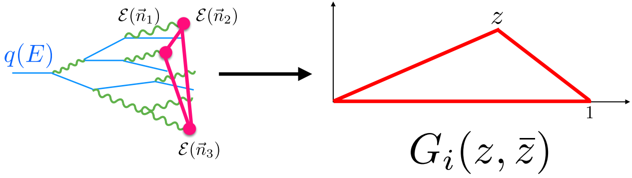
- Can we compute higher-point functions of energy flux?

Multipoint Correlators

- The only explicit results for correlators with $N > 2$ are the remarkable strong coupling results of Hofman and Maldacena:

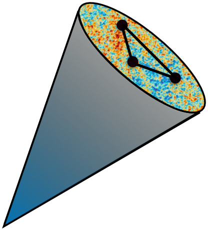
$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_n)\rangle = \left(\frac{q}{4\pi}\right)^n \left[1 + \sum_{i<j}\frac{6\pi^2}{\lambda}[(\vec{n}_i.\vec{n}_j)^2 - \frac{1}{3}] + \right.$$
$$\left. + \frac{\beta}{\lambda^{3/2}}[\sum_{i<j<k}(\vec{n}_i.\vec{n}_j)(\vec{n}_j.\vec{n}_k)(\vec{n}_i.\vec{n}_k) + \cdots] + o(\lambda^{-2})\right]$$

- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



Multi-point Correlators at Weak Coupling

- Turn out to have an elegant perturbative structure. e.g. in $\mathcal{N} = 4$



[Chen, Luo, Moult, Yang, Zhang, Zhu]

$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v)$$
$$-(1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z)$$
$$+\frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

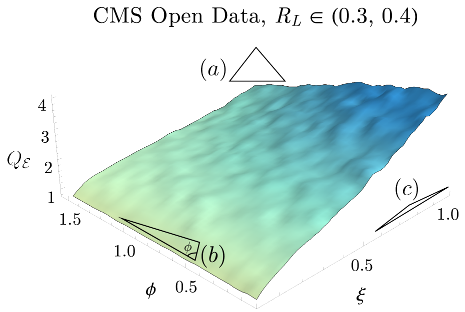
- Here Φ and D_2^+ are polylogarithmic functions

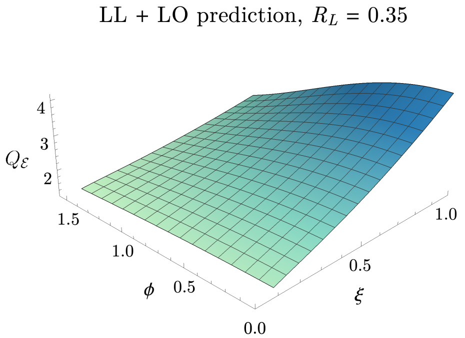
$$\Phi(z) = \frac{2}{z-\bar{z}}\left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\left(\log(1-z) - \log(1-\bar{z})\right)\log(z\bar{z})\right)$$
$$D_2^+(z) = \mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\log(|1-z|^2)\log(|z|^2)$$

- Real world QCD involves more complicated polynomials, but is otherwise similar.

Shape Dependence of Non-Gaussianity in Data

- Can directly study non-gaussianities inside high energy jets.





- Illustrates theoretical control over multi-point correlations!

Multi-Point Correlators in Data

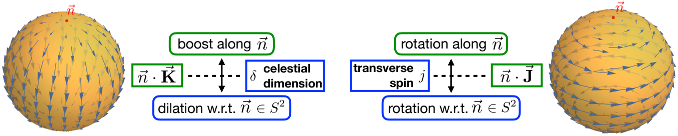
- This is an honest to goodness correlation function living on the celestial sphere!
- It has all the nice theoretical properties one could want, but in the real physical observable!
- e.g. amplitudes are beautiful, jet cross section at LHC ugly.
- One simple example to highlight this structure: Celestial Block decomposition

$$\boxed{g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})}$$

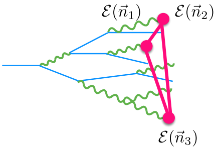
- Exactly analogous to decomposition of $2 \to 2$ scattering into partial waves with $\mathrm{SO}(3)$ quantum numbers.

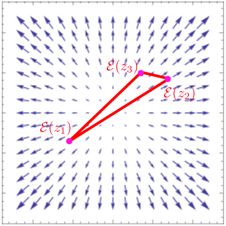
Celestial Blocks

- Are derived by studying the action of the Lorentz group on the celestial sphere:



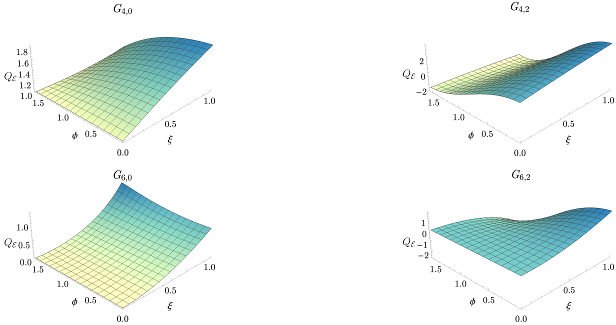
- In the "jet substructure" limit, reduces to living in the plane transverse to the jet:



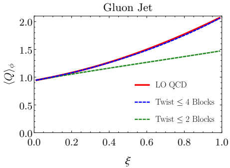


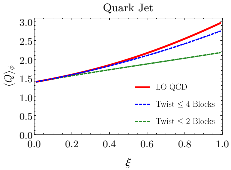
Celestial Partial Waves

- These are partial waves living on the detector:



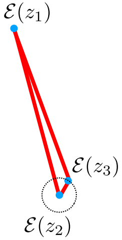
- Celestial block expansion converges rapidly.

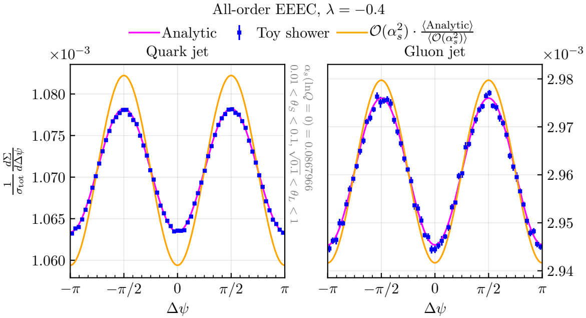




Parton Shower Development

- Illustrates complete control of three-point correlations in jets.
- Crucial for validating implementations of higher order effects in parton showers. e.g. Spin Correlations (transverse spin operators)





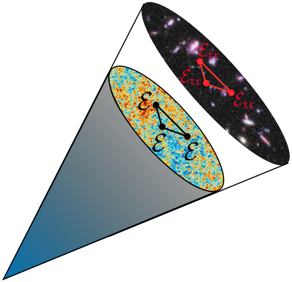
[Karlberg, Salam, Scyboz, Verheyen]

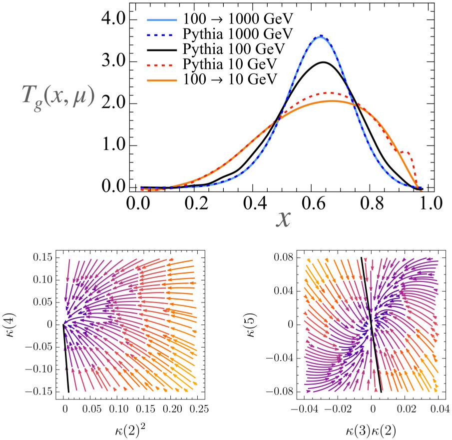
- Full incorporation of higher-point correlations in parton showers will play an important role in enhancing the LHC search program.

Track Functions

[Chang, Procura, Thaler, Waalewijn]
[Li, Moult, Van Velzen, Waalewijn, Zhu]
[Jaarsma, Li, Moult, Waalewijn, Zhu]

- A key in the ability to study higher point correlations has been the development of QFT formalisms for performing calculations on charged particles (tracks).

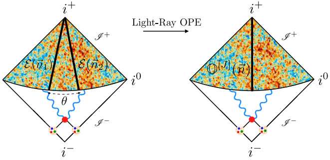




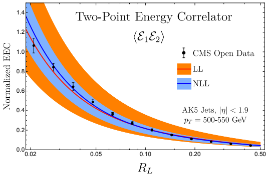
- Described by non-perturbative track functions satisfying non-linear RG evolution, encoding correlations in the hadronization process.

Summary

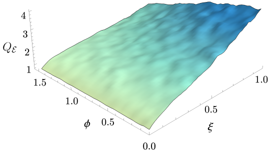
- Insights from formal theory are transforming the way we think about jet substructure.

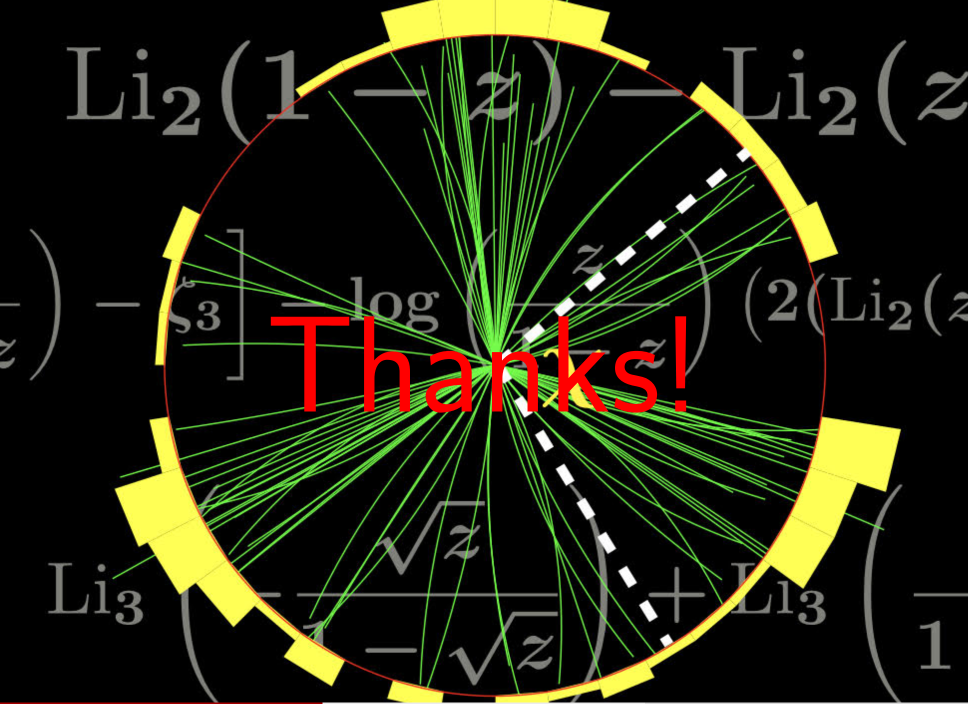


- Jet Substructure provides a physical realization of the OPE limit of lightray operators $\Longrightarrow$ direct bridge between recent field theory developments and QCD phenomenology.



- Opens the door to a precision physics program using jet substructure, and many new opportunities to learn about QCD!





Thanks!