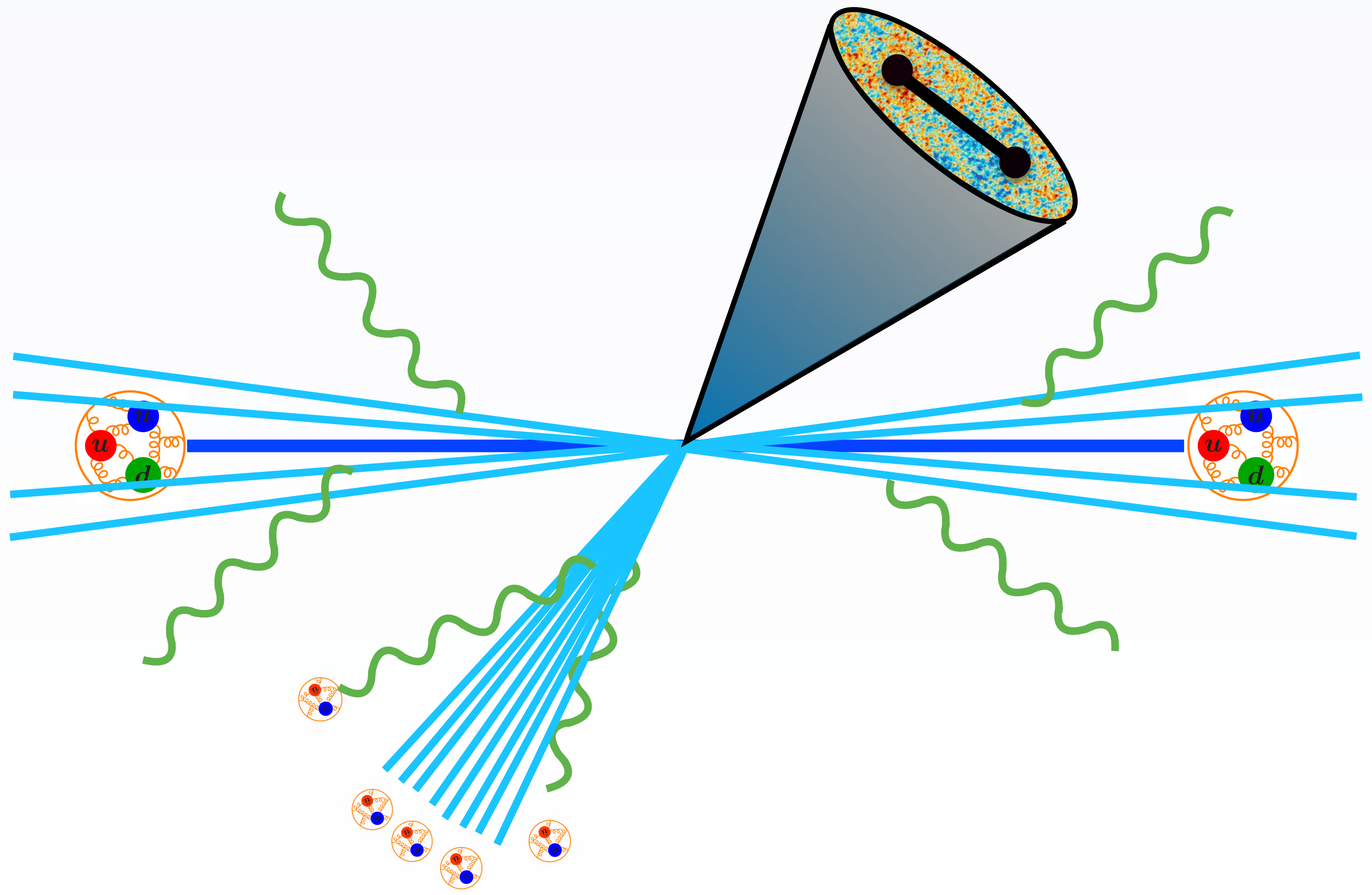
Conformal Colliders Meet the LHC with Jet Fragmentation Functions



Kyle Lee
LBNL

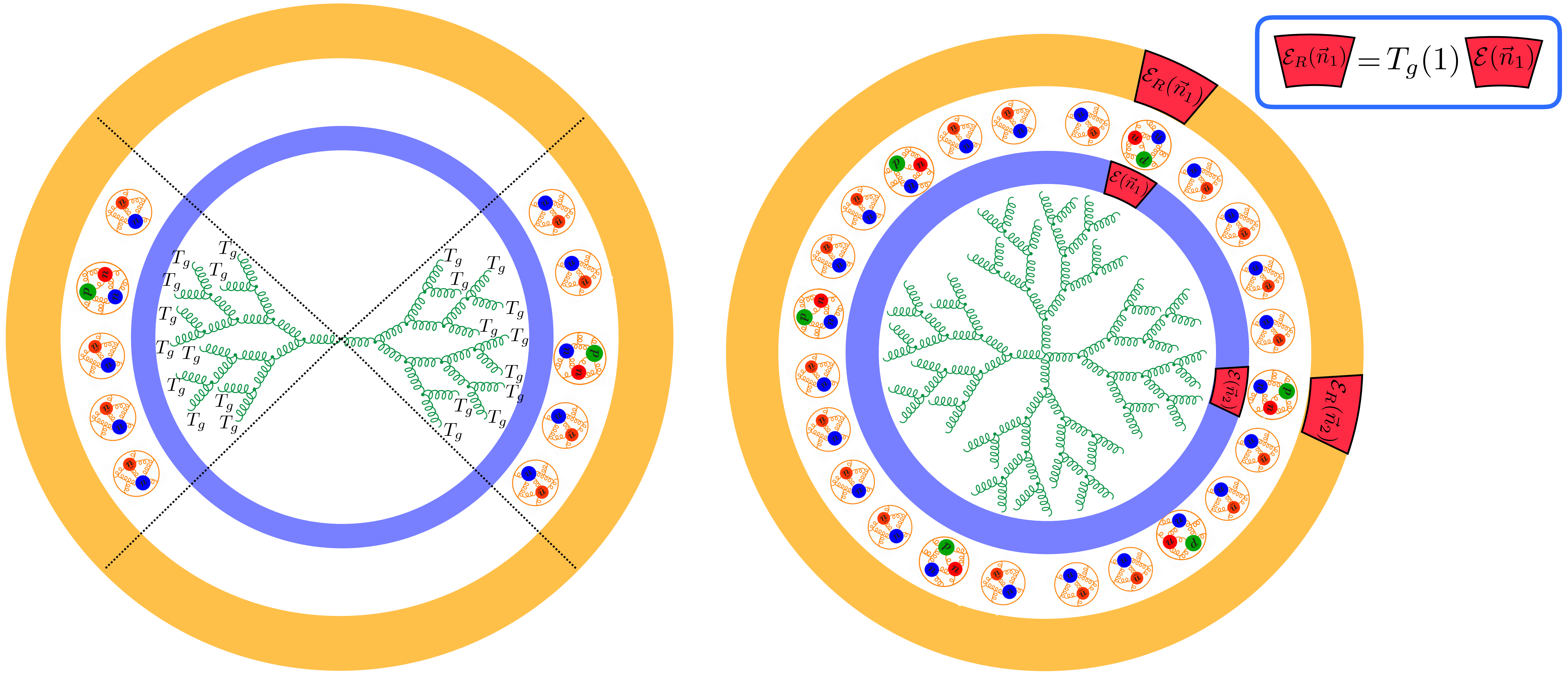
Jet Physics : From RHIC/LHC to EIC
June 29th - July 1st, 2022

Energy correlators as jet substructure

See also Ian's talk

- In the collinear limit, $z_{ij} \to 1$ (i.e. $\theta_{ij}^2 \to 0$)
  - Fixed number of detectors

space of the states **vs** space of detectors



$$\mathcal{E}(\hat{n}) \to \mathcal{E}_R(\hat{n}) = T_i(1,\mu)\mathcal{E}(\hat{n})$$

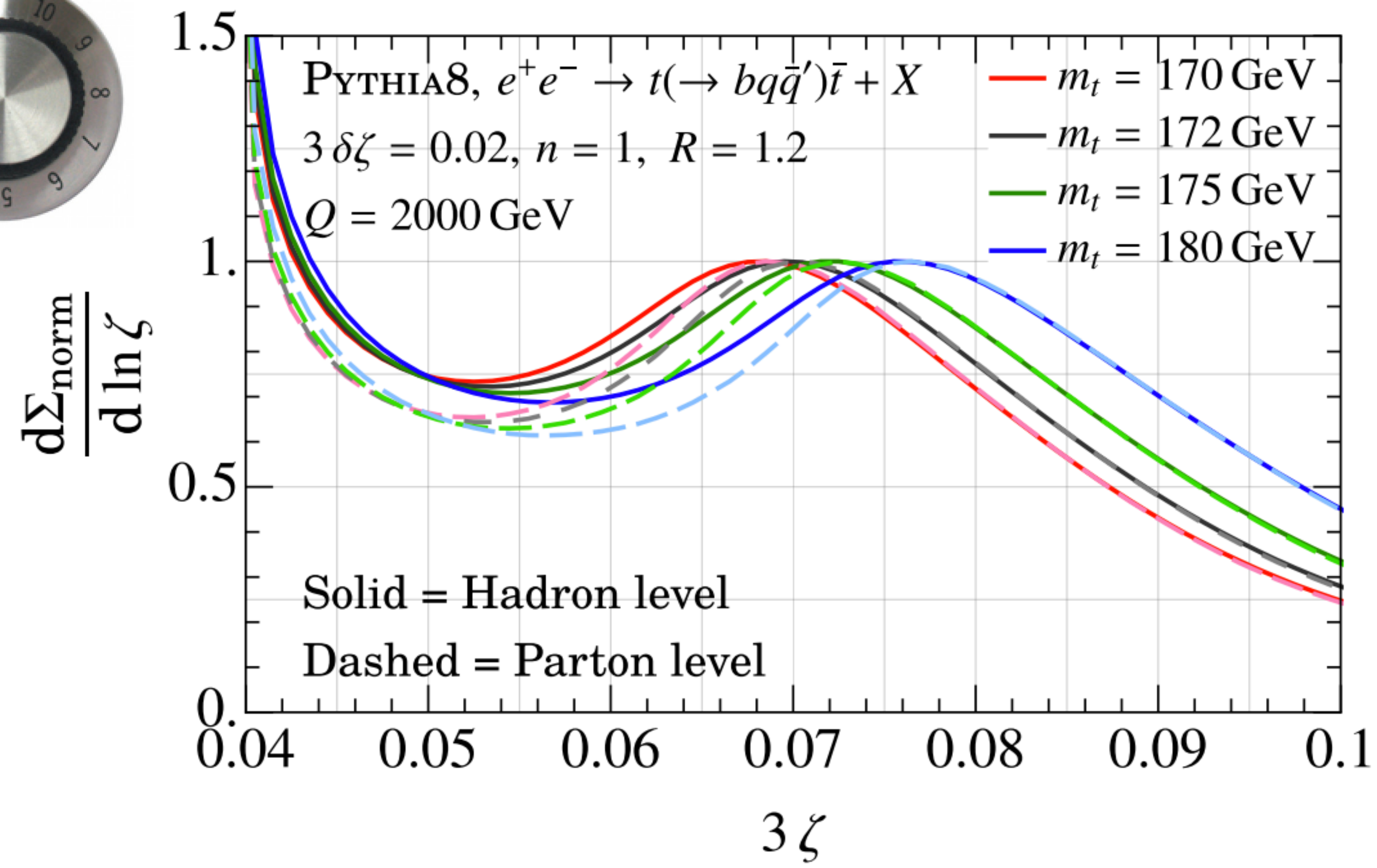
Chen, Moult, Zhang, Zhu, `20
Li, Moult, van Velzen, Waalewijn, Zhu, `21
Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- Probes fixed scale

$$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$$

Holguin, Moult, Pathak, Procura, `22

scale knob

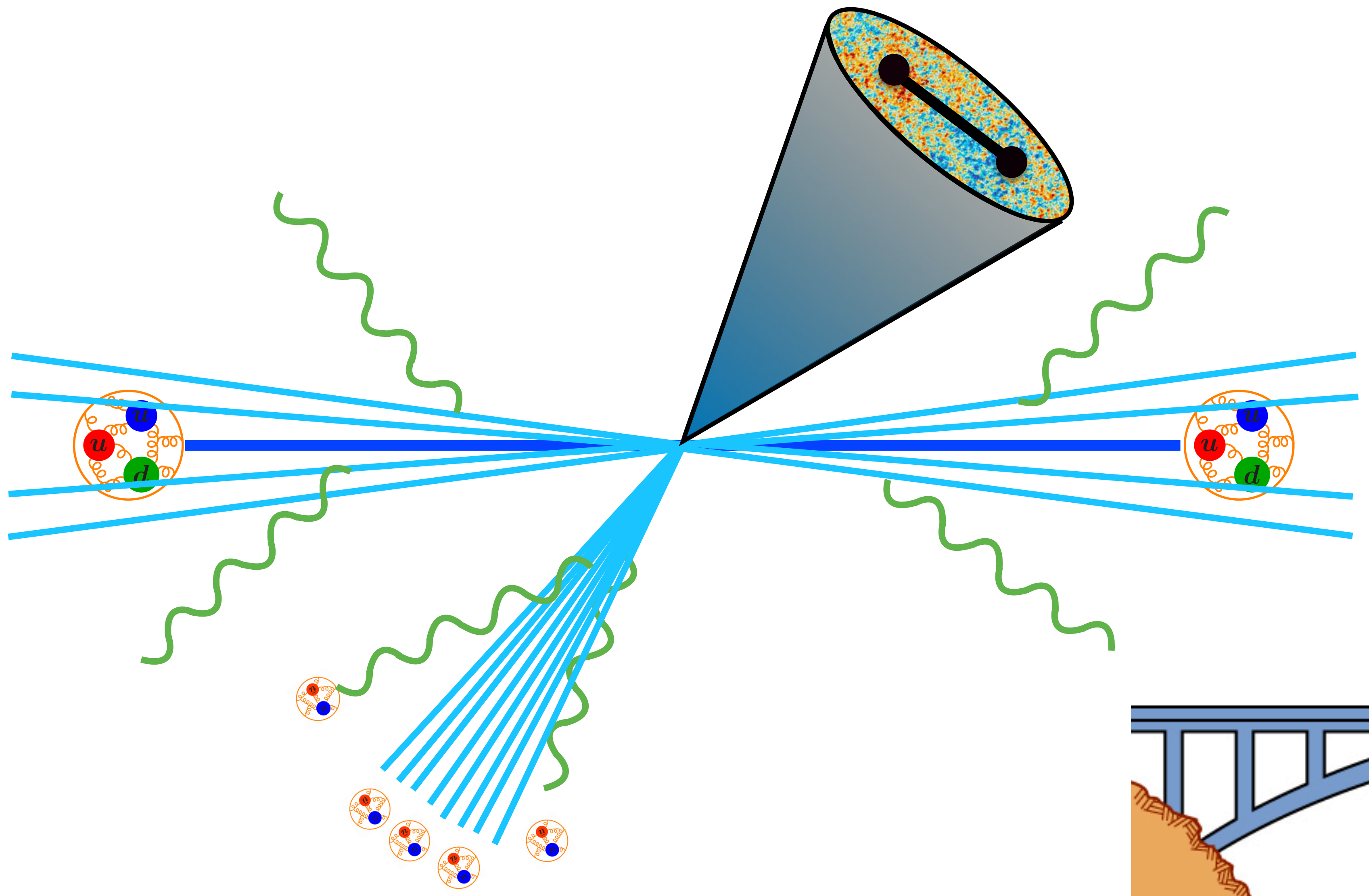


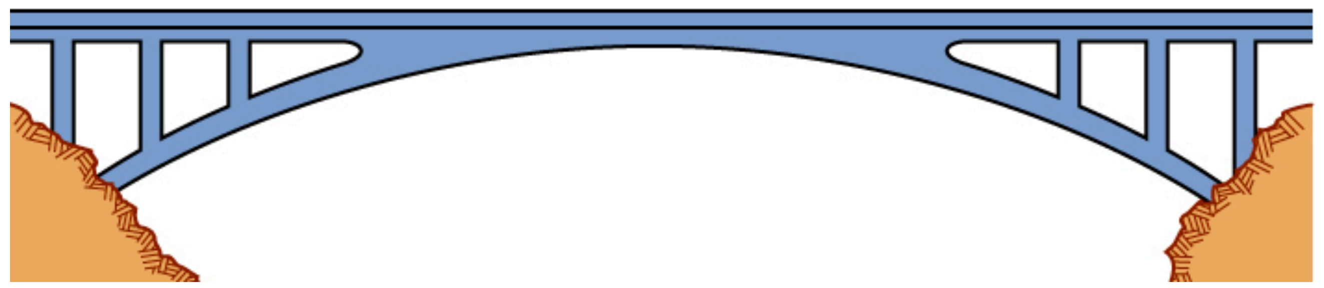
Collinear Limit of Energy Correlators

F. Ore, Sterman, `79
Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Korchemsky, Sterman, `01
Hofman, Maldacena, `08

- In the collinear limit, $z_{ij} \to 1$ (i.e. $\theta_{ij}^2 \to 0$), give rise to

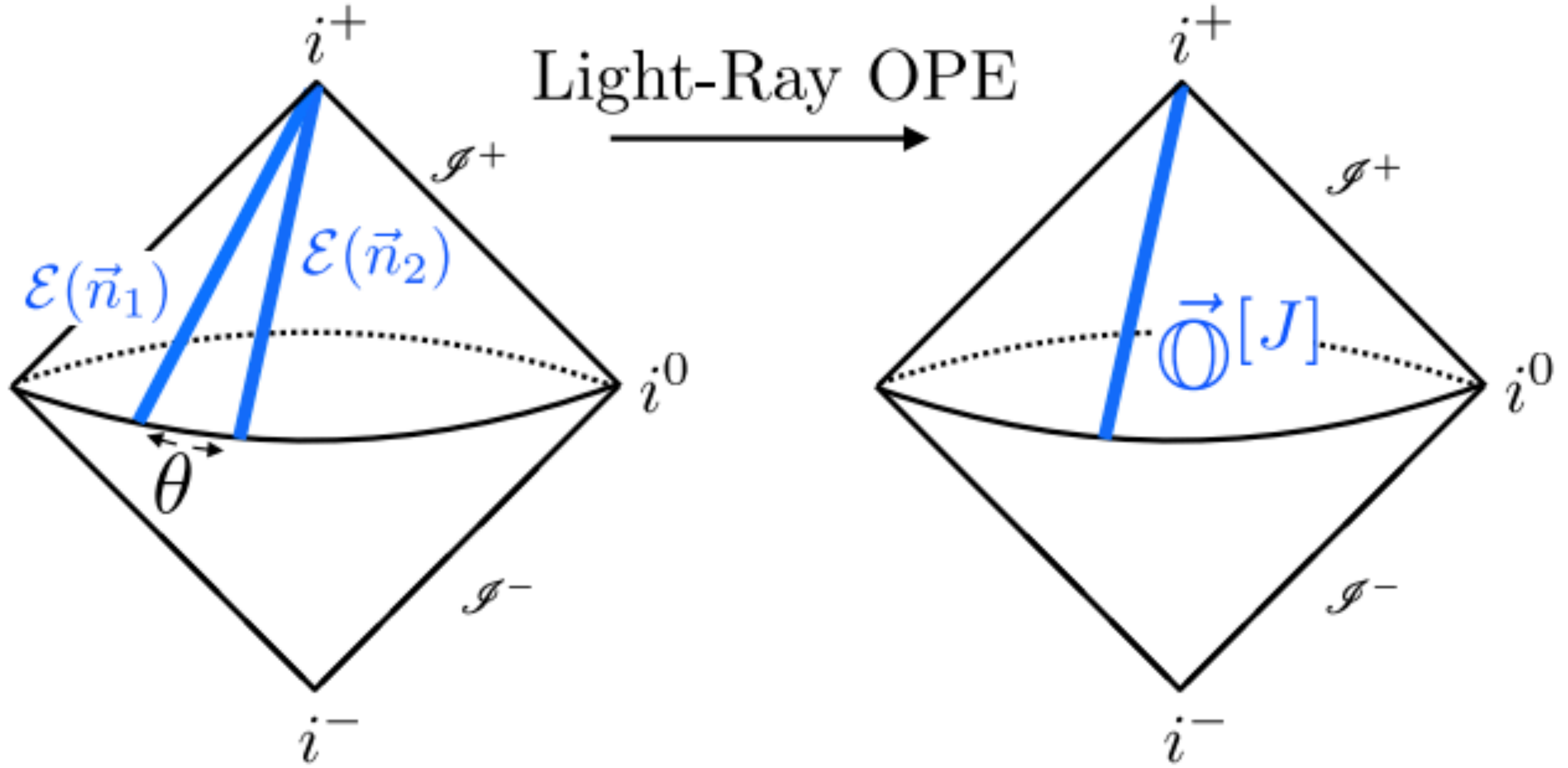
Phenomenological tools





- Jet substructure study

Theoretical tools

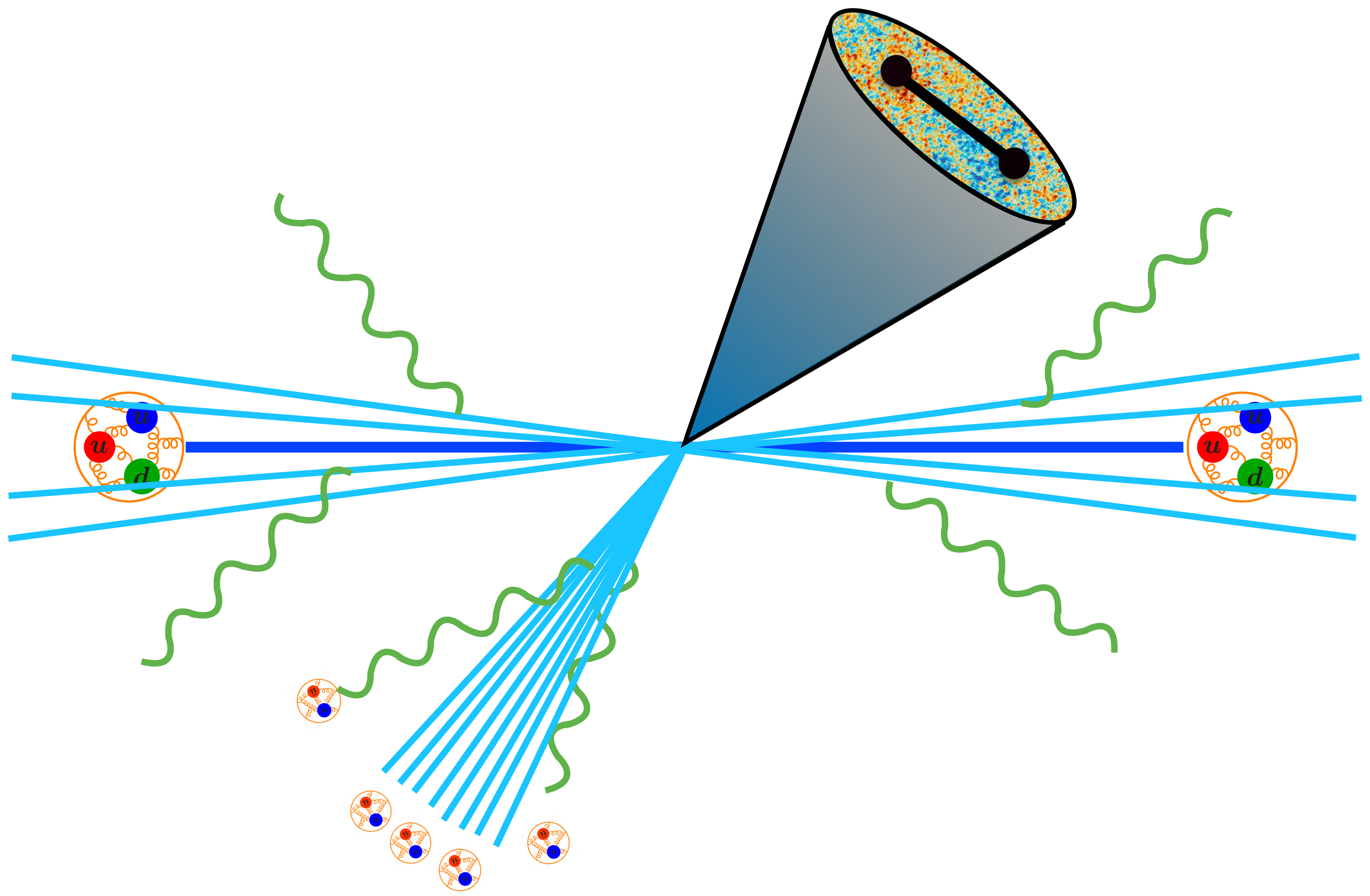


$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_N) | \Psi \rangle$$

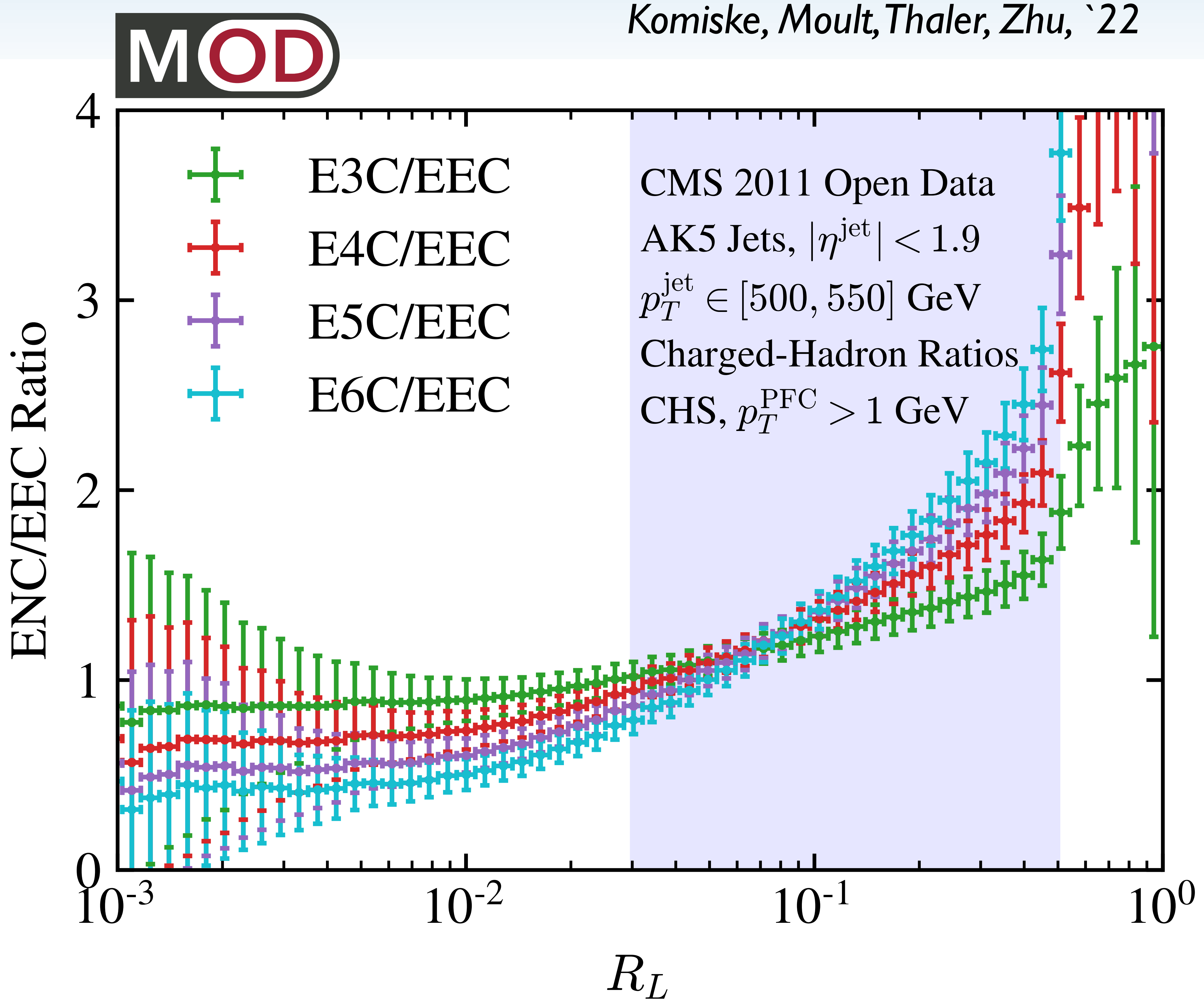
$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r \to \infty} r^2 n^i T_{0i}(t, r\hat{n})$$

- Light-ray Operator Product Expansion (OPE)

Energy correlators as jet substructure



Komiske, Moult, Thaler, Zhu, `22



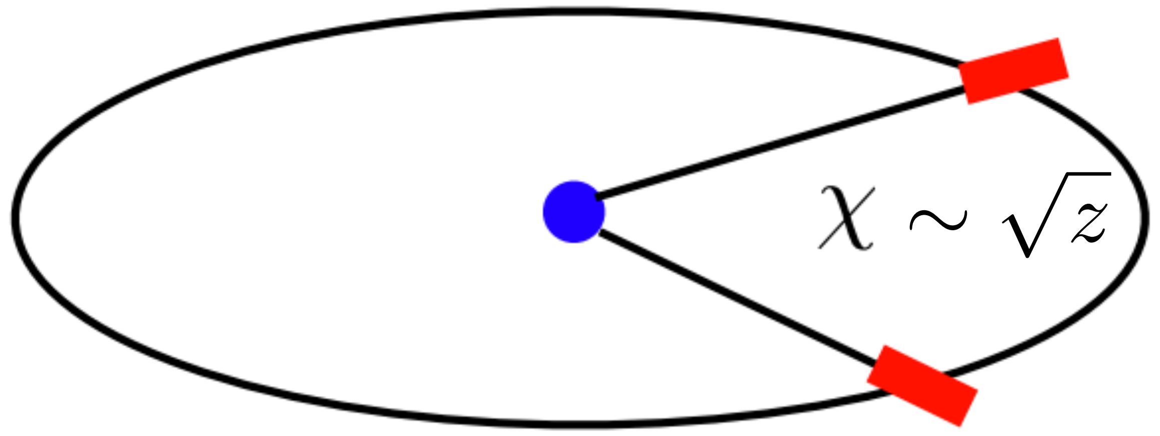
- Want to be able to extend the formalism to study energy correlators as jet substructure at the LHC!

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

For convenience, cumulant: $\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) \equiv \frac{1}{\sigma_0}\int_0^z dz' \frac{d\sigma}{dz}\left(z', \ln\frac{Q^2}{\mu^2}, \mu\right)$

$$[\ln^j z/z]_+ \to 1/(j+1) \times \ln^{j+1} z \quad \text{and} \quad \delta(z) \to 1$$

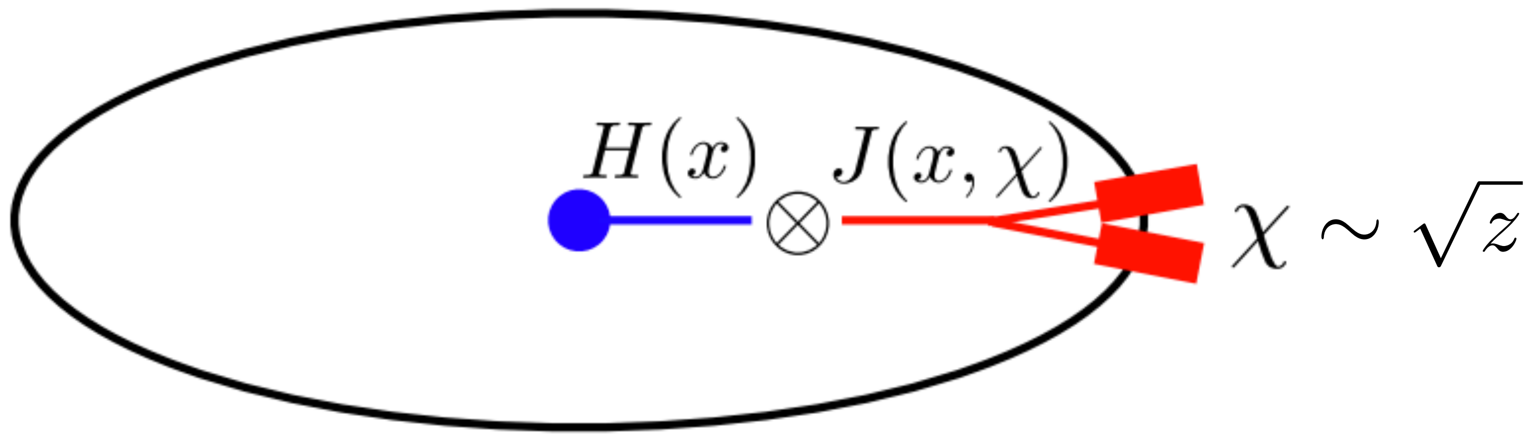


- In the collinear limit, $z \to 1$ (i.e. $\chi_{ij}^2 \to 0$), factorizes as (using SCET)

$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

$\mu_{\text{EEC}} \sim \sqrt{z}\,Q$ $\quad$ $\mu_H \sim Q$

Hard function (source)



Dixon, Moult, Zhu, `19

$$\vec{J} = \{J_q, J_g\}$$

EEC Jet function

$$J_q(z) = \sum_X \sum_{i,j\in X} \langle 0|\bar{\chi}_n|X\rangle \frac{E_i E_j}{(Q/2)^2} \Theta(\theta_{ij} < \chi) \langle X|\chi_n|0\rangle$$

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

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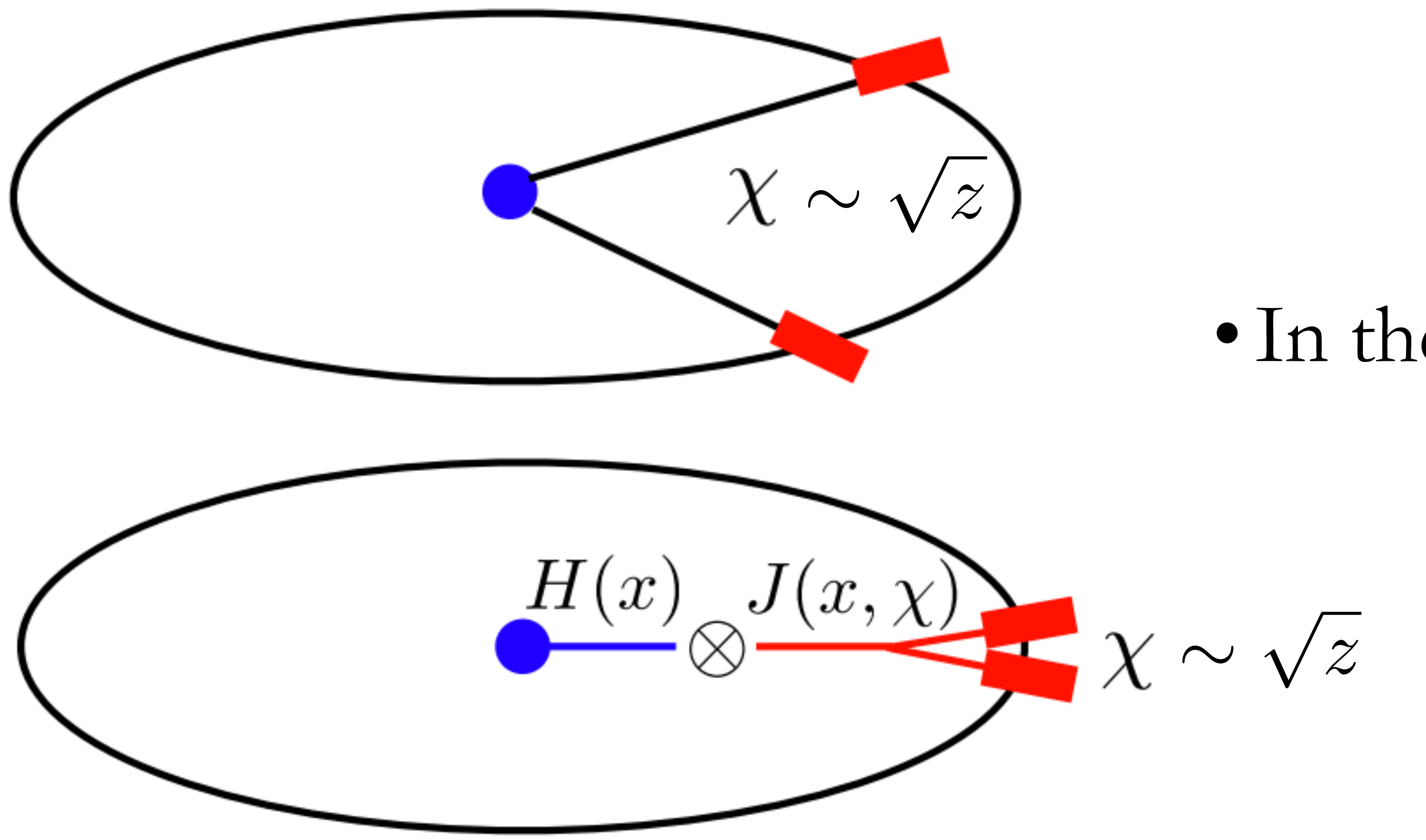

Dixon, Moult, Zhu, `19

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$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

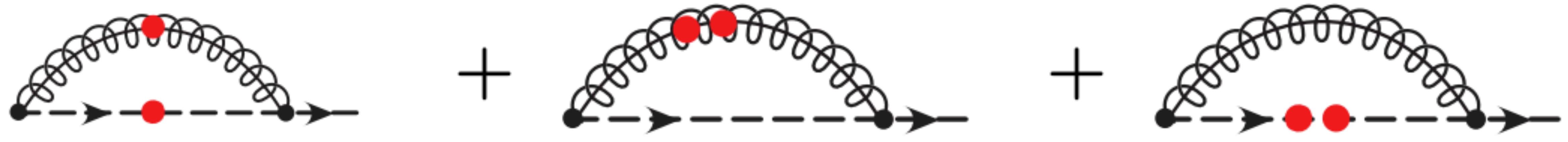
$\mu_{\rm EEC} \sim \sqrt{z}\,Q$ $\quad$ $\mu_H \sim Q$

EEC Jet function $\quad$ Hard function (source)

$\vec{J} = \{J_q, J_g\}$

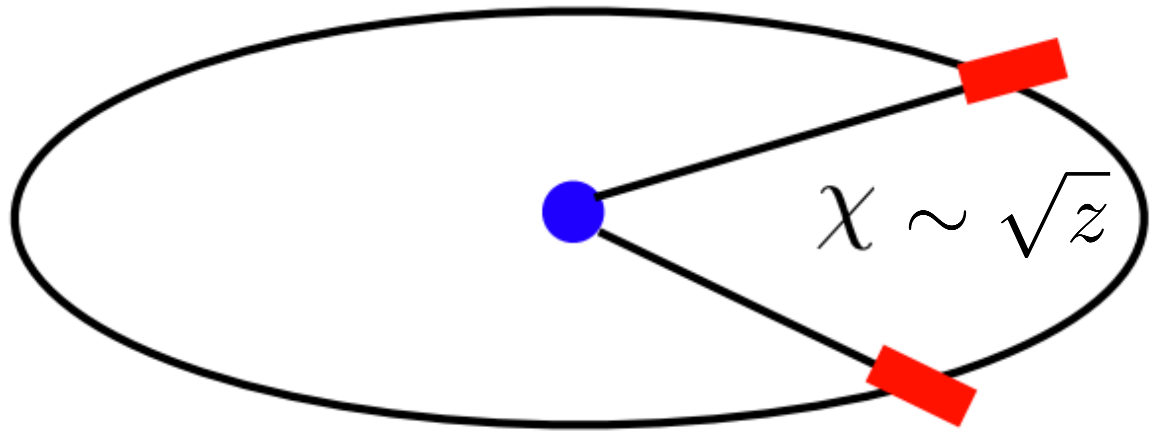
$$\frac{E_i E_j}{Q^2} \sim x^2 x_i x_j$$

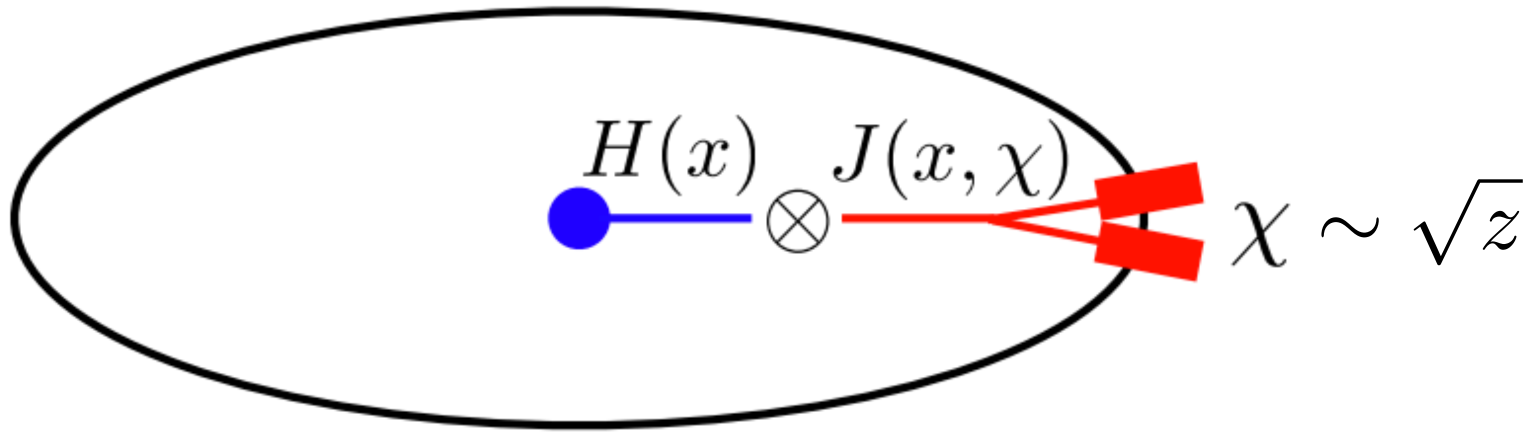
$\vec{J}$ at **NLO**



Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$





Dixon, Moult, Zhu, `19

- In CFTs,

$$\Sigma(z) = \frac{1}{2} C(\alpha_s)\, z^{\gamma_J^{\mathcal{N}=4}(\alpha_s)} \quad \Longleftrightarrow \quad \mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) = \theta^{\gamma_i} \sum \mathbb{O}_i(\hat{n}_1)$$

power-law behavior with scaling from twist-2 spin-3 anomalous dimension, related to OPE.

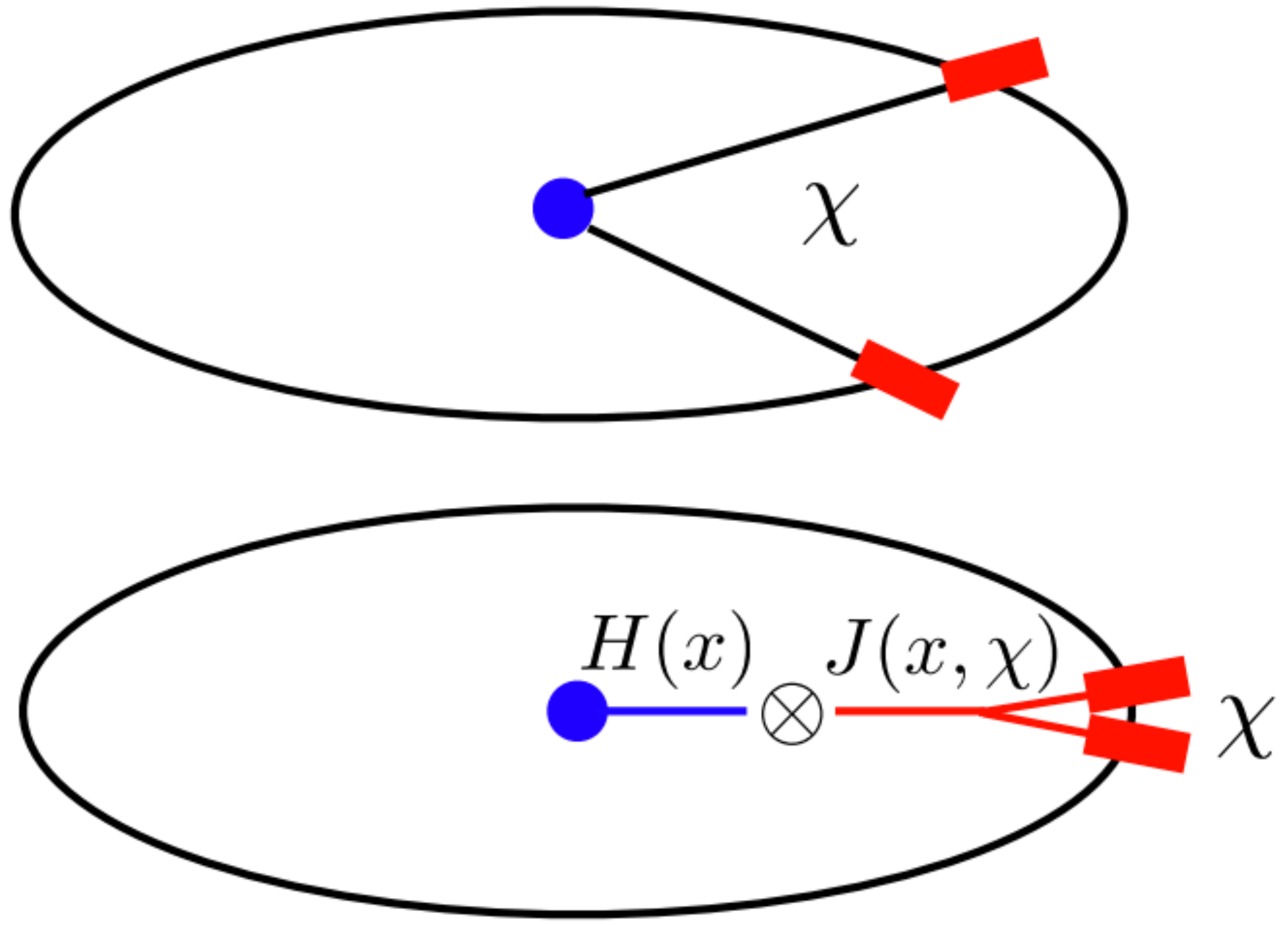
$$\gamma(3) > 0 \implies z\frac{d\sigma}{dz}|_{z\to 0} = 0$$

can be computed using OPE alone!

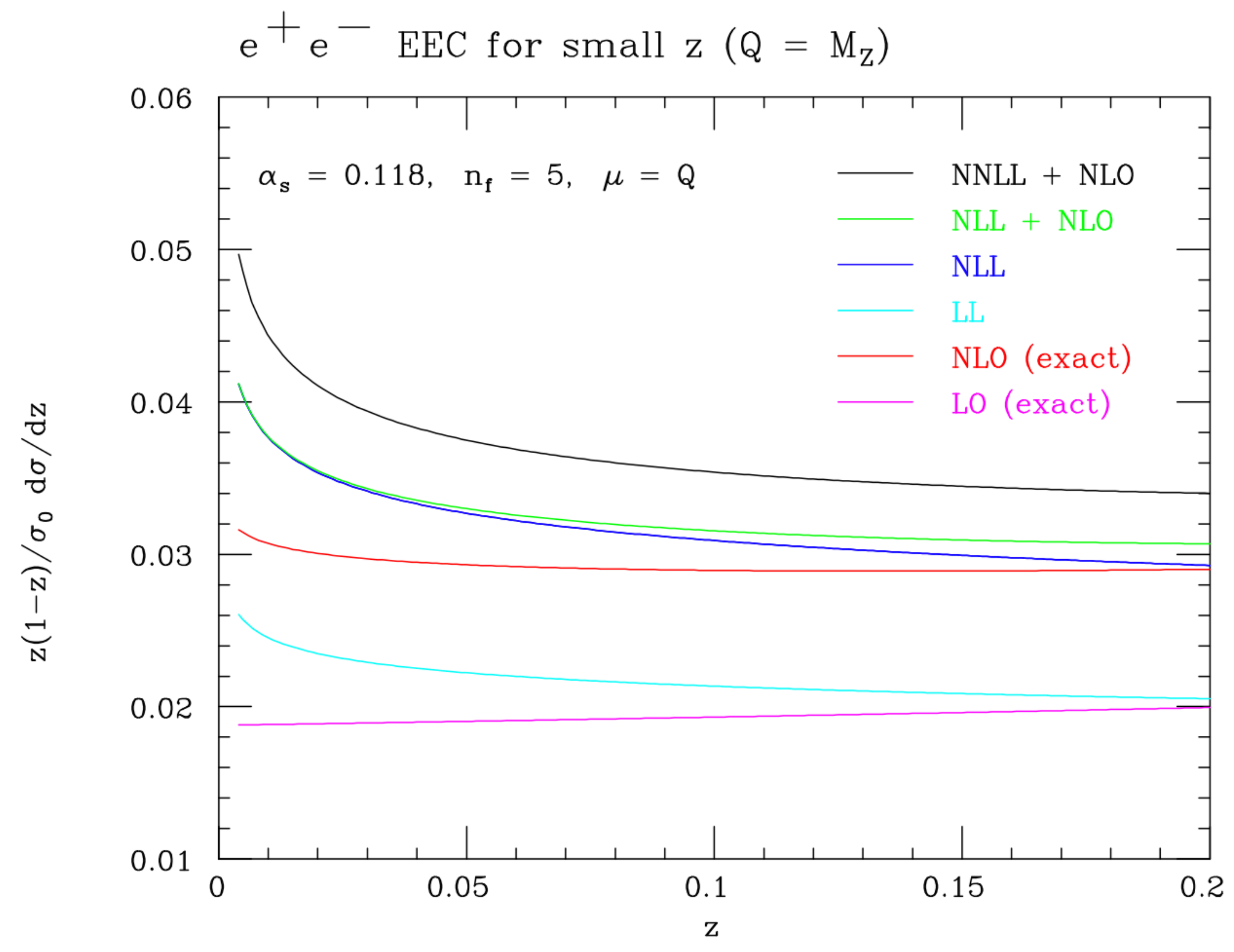
Energy correlators at e^+e^-

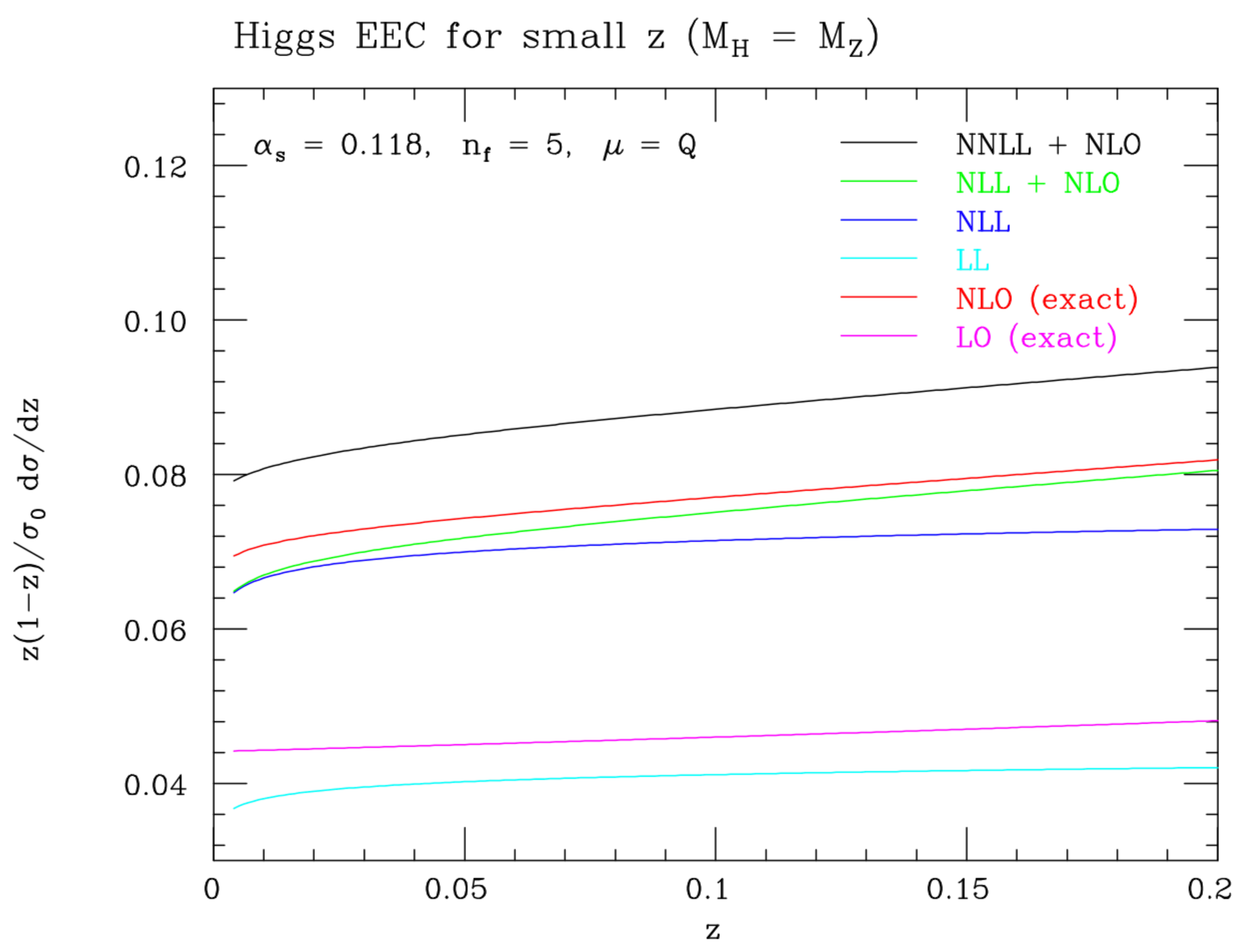
$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

- In non-CFTs (like QCD), there is competition between beta functions and twist-2 spin-3 anomalous dimension.



Dixon, Moult, Zhu, `19





- Higher scale would give larger window of region where the contribution from the twist-two anomalous dimension dominates over that of beta function, giving phenomenological connection to Light-ray OPE and other CFT techniques
- Higher energy provides more particles in jet, allowing us to study higher-point correlators
- Smaller NP corrections

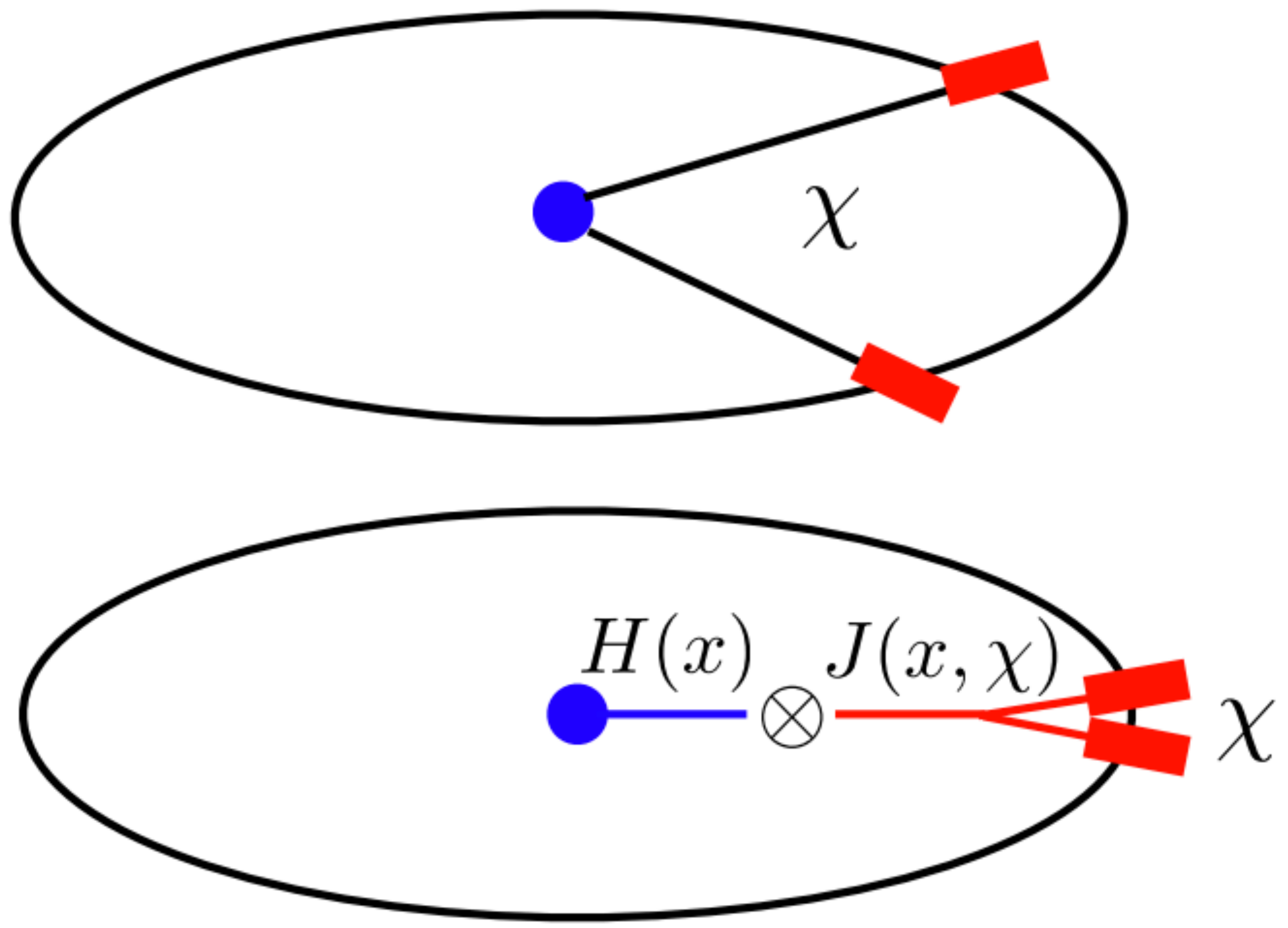
$\Longrightarrow$ **Jets at the LHC!**

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

- In non-CFTs (like QCD), there is competition between beta functions

and



Dixon, Moult, Zhu, `19

Note the similarity

EEC factorization

$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

Hadron production

$$\frac{d\sigma^h}{dz_h} = \int_{z_h}^1 \frac{dx}{x} \vec{D}^h\left(\frac{z_h}{x}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

Collinear dynamics factorize identically from the hard functions (source)

Hadron production inside jets = Jet Fragmentation Functions

- Higher scale would give larger w

dominates over that of beta func

- Higher energy provides more particles in jet, allowing us to study higher-point correlators
- Smaller NP corrections

$\Longrightarrow$ **Jets at the LHC!**

The jet fragmentation function $pp \to (\text{jet}h)X$

Factorization

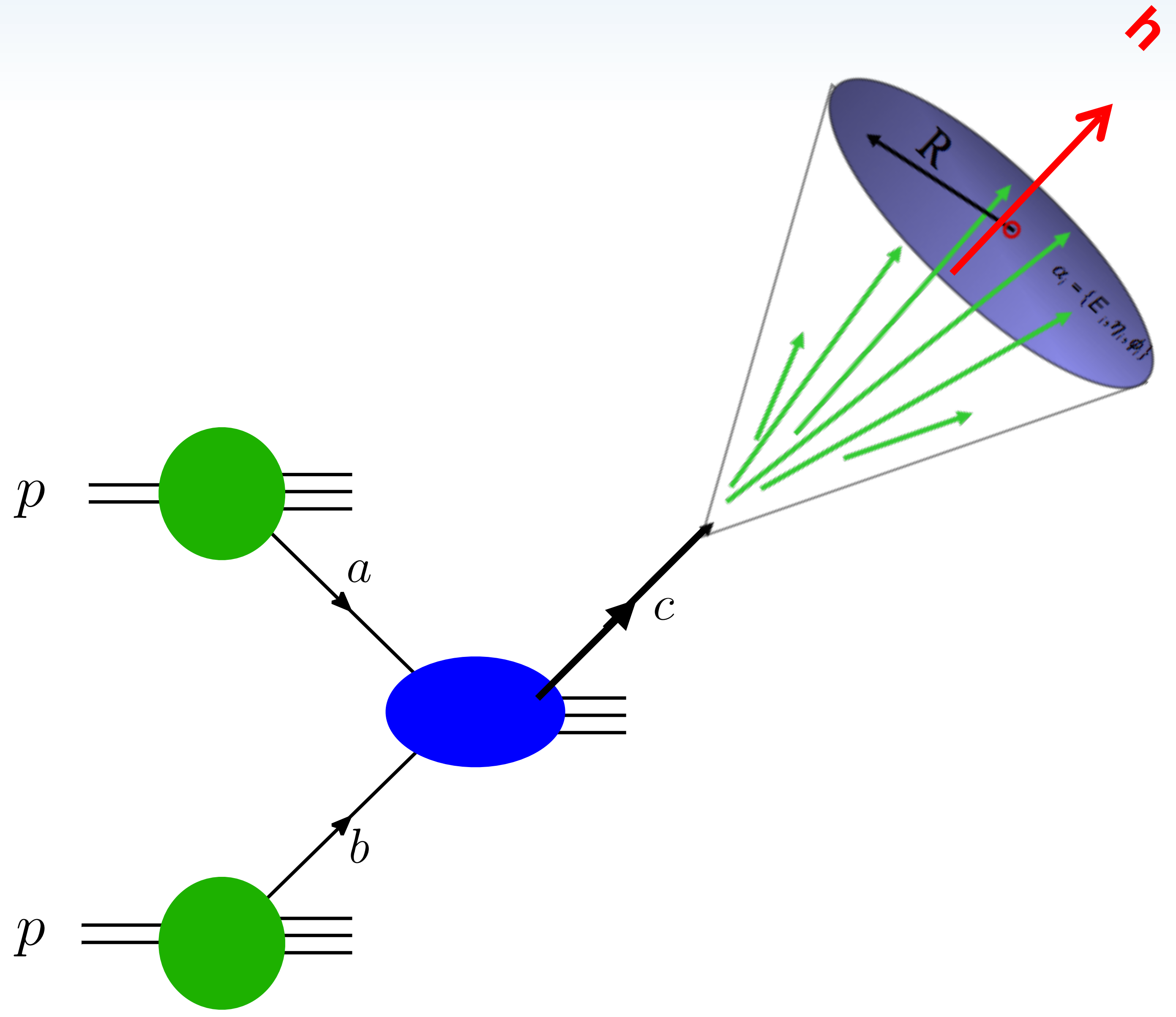
$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h)$$

$\Lambda_{\rm QCD}$ $\quad p_T \quad p_T R$

$\Lambda_{\rm QCD}$

where $\quad z_h = p_T^h/p_T$

$z = p_T/p_T^c$



- Jet dynamics factorized from the rest of the process.
- The jet function $\mathcal{G}_c^h(z_h)$ describes production of hadron h inside the jet initiated by the parton c.

IR sensitive and requires matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \int_{z_h}^1 \frac{dx}{x} \mathcal{J}_{ij}(z, x, p_T R, \mu) D_j^h\left(\frac{z_h}{x}, \mu\right)$$

$p_T R \qquad \Lambda_{\rm QCD}$

matching coefficients **collinear FFs**

Collinear JFFs can be related to collinear FFs

Procura, Stewart `10
Jain, Procura, Waalewijn, `11
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16
Kang, KL, Zhao `20

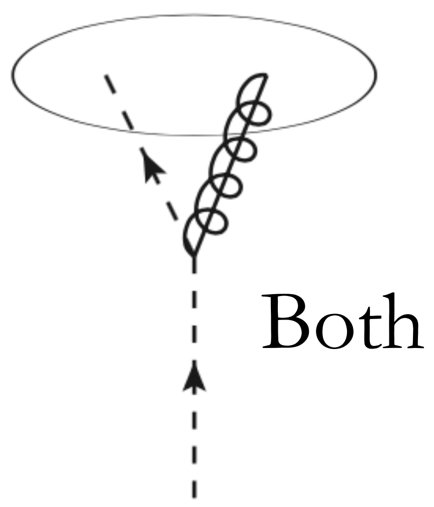
Also, Collins, Soper, Sterman `81-89
Nayak, Qiu, Sterman `05

The jet fragmentation function $pp \to (\text{jet}\,h)X$

Factorization

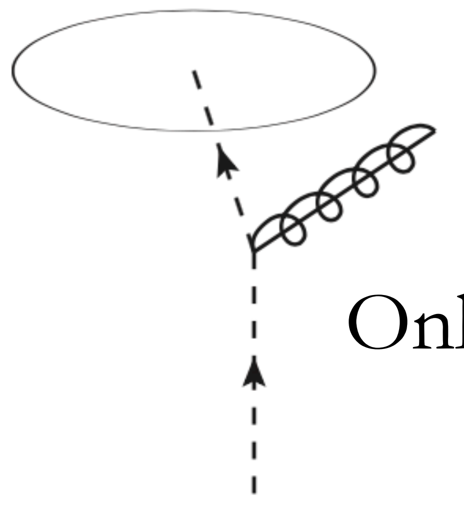
$$\mathcal{G}_c^h\left(z, z_h, p_T R, \mu\right) = \sum_j \int_{z_h}^1 \frac{dx}{x} \mathcal{J}_{ij}\left(z, x, p_T R, \mu\right) D_j^h\left(\frac{z_h}{x}, \mu\right)$$

- At NLO, diagonal part for quark case:



Both particles in jet

$\delta(1-z)$

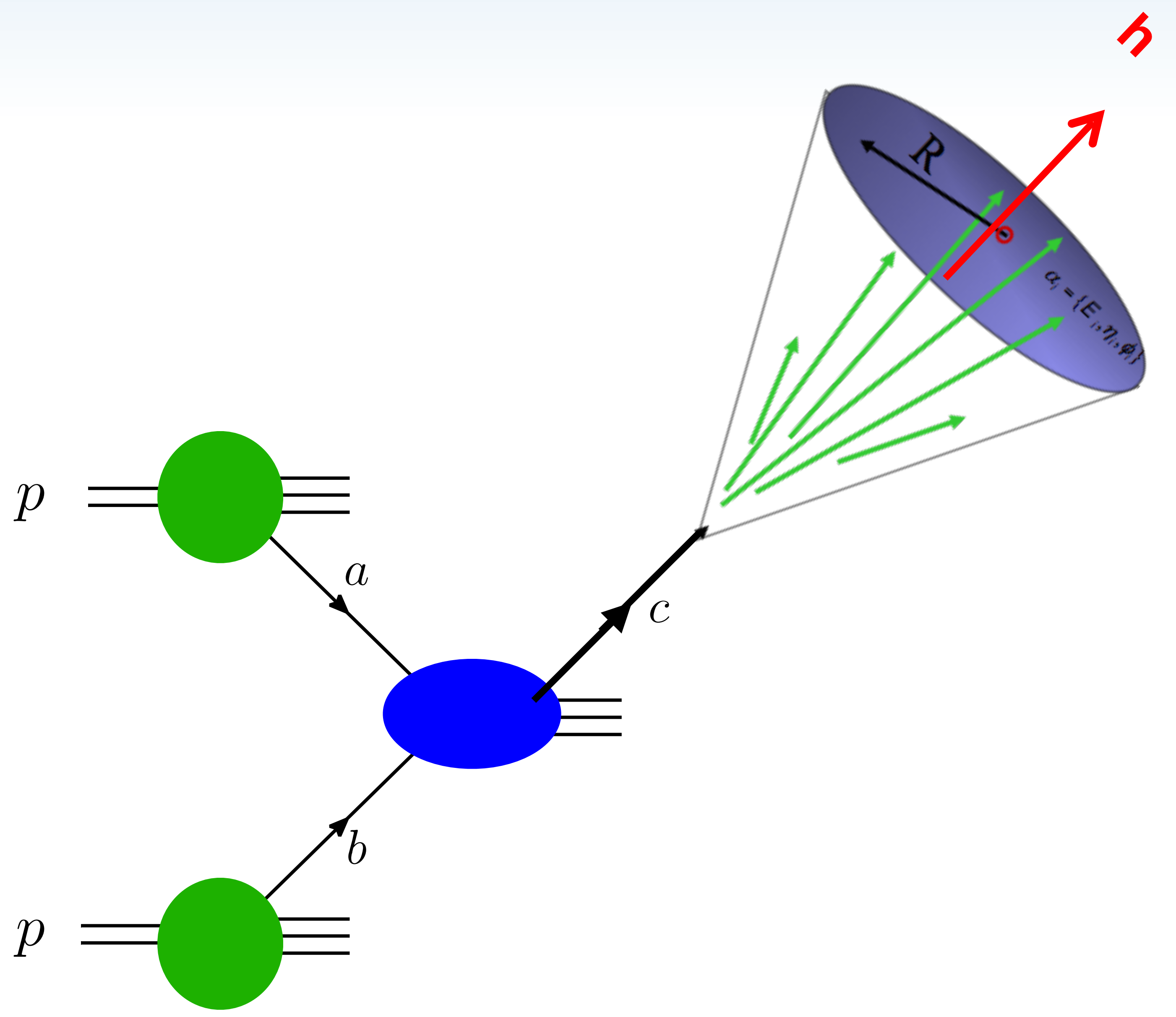


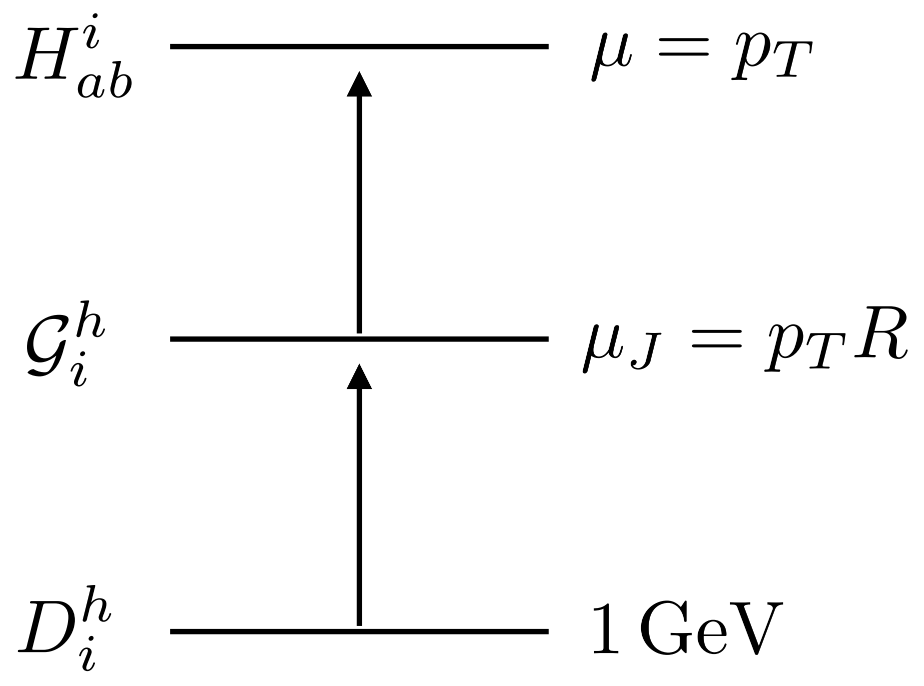
Only quark in jet

$\delta(1-z_h)$

Jet algorithm: $\Theta_{\text{anti-k}_\text{T}} = \theta\left(x(1-x)p_T R - q_T\right)$ $\quad\Theta_{\text{anti-k}_\text{T}} = \theta\left(q_T - (1-x)p_T R\right)$

$$\begin{aligned}\mathcal{J}_{qq}(z, z_h, \omega_J, \mu) =&\,\delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\Bigg\{L\left[P_{qq}(z)\delta(1-z_h) - P_{qq}(z_h)\delta(1-z)\right] \\ &+ \delta(1-z)\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + \mathcal{I}_{qq}^{\text{alg}}(z_h)\right] \\ &- \delta(1-z_h)\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]\Bigg\},\end{aligned}$$



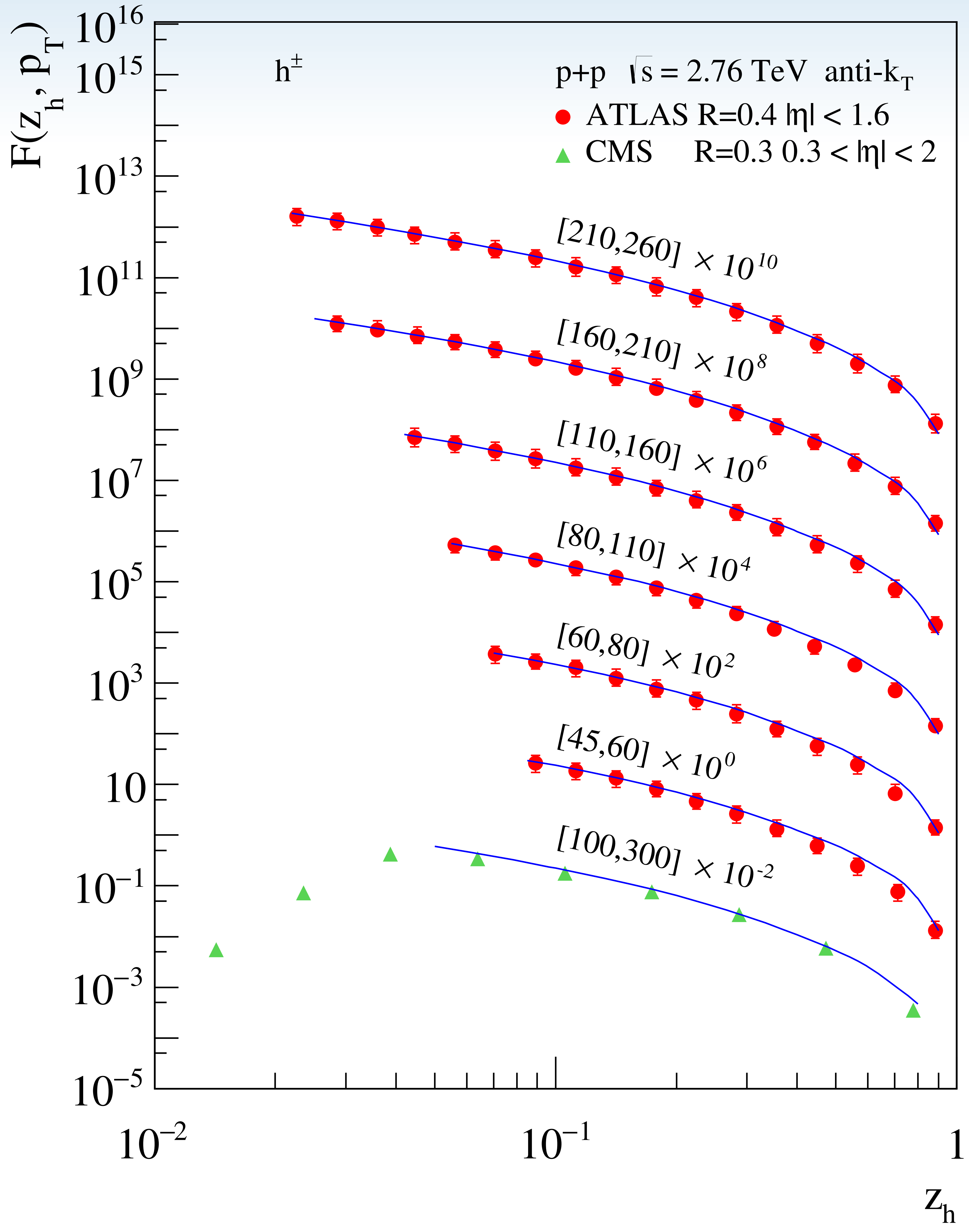


2 DGLAPs

The jet fragmentation function $pp \to (\text{jet}h)X$

- **Light charged hadrons**
  - *Arleo, Fontannaz, Guillet, Nguyen `14*
  - *Kaufmann, Mukherjee, Vogelsang `15*
  - *Kang, Ringer, Vitev `16*
  - *Neill, Scimemi, Waalewijn `16*
- **Photons**
  - *Kaufmann, Mukherjee, Vogelsang `16*
- **Heavy flavor mesons**
  - *Chien, Kang, Ringer, Vitev, Xing `15*
  - *Bain, Dai, Hornig, Leibovich, Makris, Mehen `16*
  - *Anderle, Kaufmann, Stratmann, Ringer, Vitev `17*
- **Quarkonia**
  - *Baumgart, Leibovich, Mehen, Rothstein `14*
  - *Bain, Dai, Hornig, Leibovich, Makris, Mehen `16*
  - *Kang, Qiu, Ringer, Xing, Zhang `17*
  - *Bain, Dai, Leibovich, Makris, Mehen `17*
- **Polarized hadrons**
  - *Kang, KL, Zhao `20*

$$F(z_h, p_T) = \frac{d\sigma^{pp\to(\text{jet}h)X}}{dp_T d\eta dz_h} \Bigg/ \frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta}$$

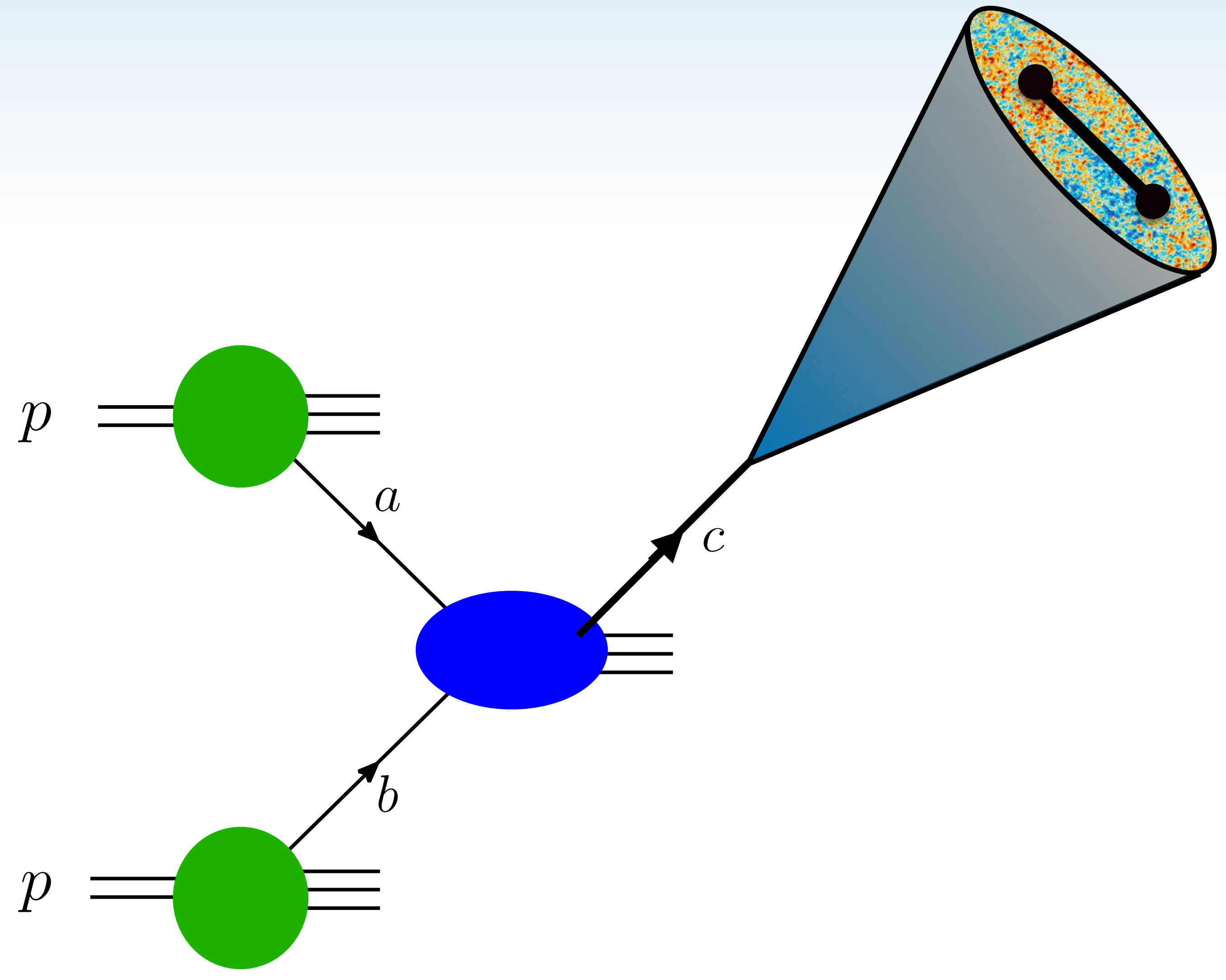


The jet fragmentation function and energy correlators

Factorization

$$\frac{d\sigma^{pp\to \text{jet(ENC)}X}}{dp_T d\eta d\{\zeta\}} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \mathcal{G}_c(\{\zeta\})$$

Λ_{QCD} $\quad p_T \quad p_T R \quad p_T\sqrt{\zeta}$



where $\{\zeta\}$ stands for the collection of angles in N-point correlators

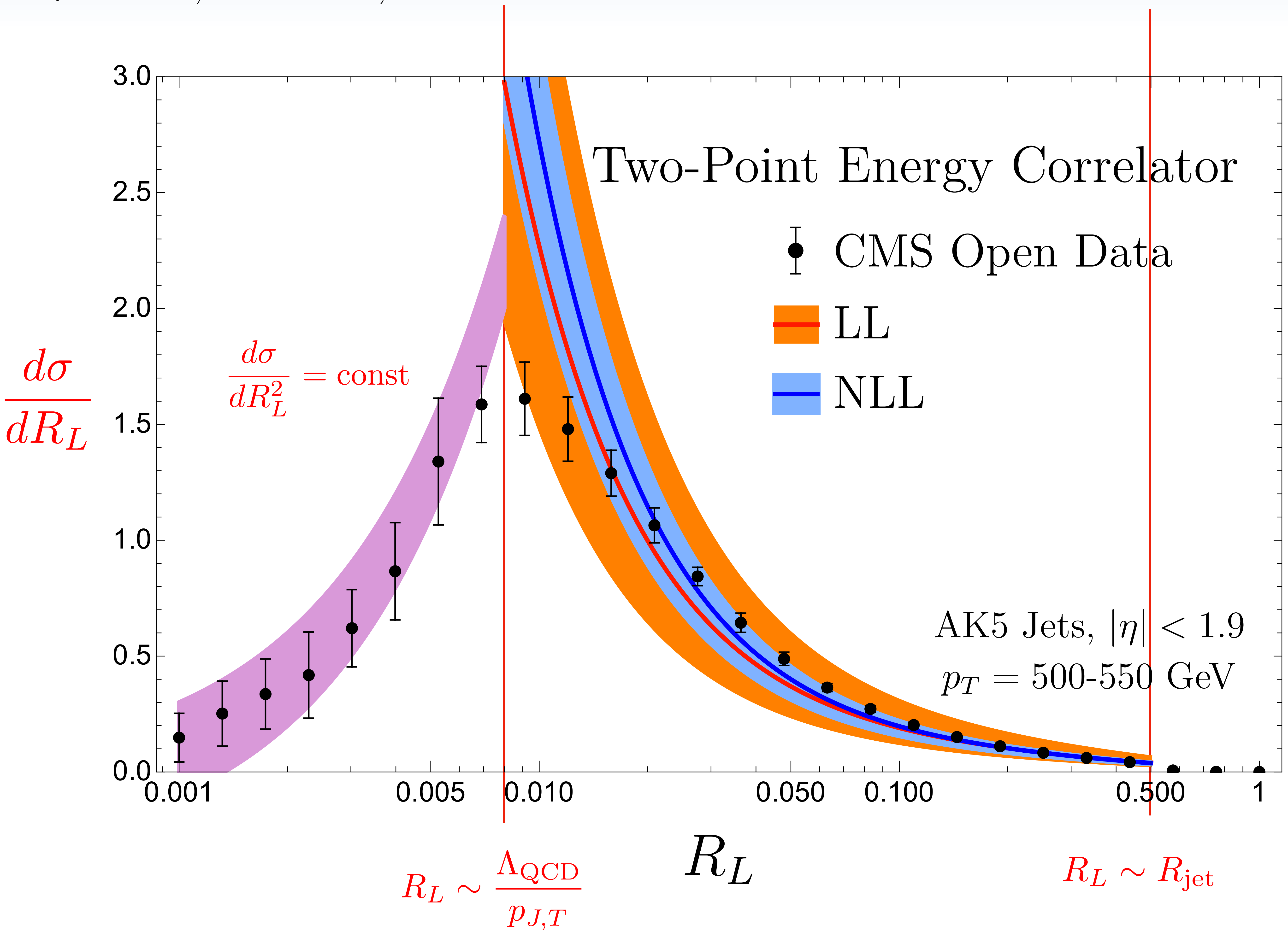
$$\mathcal{G}_c(z, \{\zeta\}, p_T R, \mu) = \sum_j \int_0^1 dx\, x^N \mathcal{J}_{ij}(z, x, p_T R, \mu)\, J_{\text{EEC}}(\{\zeta\}, x, \mu)$$

- J_{EEC} is the same EEC jet function as e^+e^- case (can use track or other cases too)
- Energy correlators are expectation values on a state $|\Psi\rangle$ $\quad \frac{d\sigma}{d\{\zeta\}} \sim \langle\Psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)|\Psi\rangle$
  In e^+e^-, the state is created by a local operator.
- As discussed, $\mathcal{G}_c$, describes how jet algorithms are used to "create" the state $|\Psi\rangle$ in which energy correlators are measured.
- More formally, $|\Psi\rangle = \sum_{\delta,j} c_{\delta,j}|\Psi_{\delta,j}\rangle$ where δ, j are the quantum numbers of the celestial sphere.

Chen, Moult, Sandor, Zhu `22
Chang, Simmons-Duffin `22

2-Point Energy correlators at the LHC

$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T} R_L$



- One can see clear transition between the perturbative and hadronization regions.
- Perturbative region agrees well with the data without any soft drop grooming, trimming, pruning, etc.
- At very small angle, the result is consistent with uniformly distributed freely propagating hadrons.

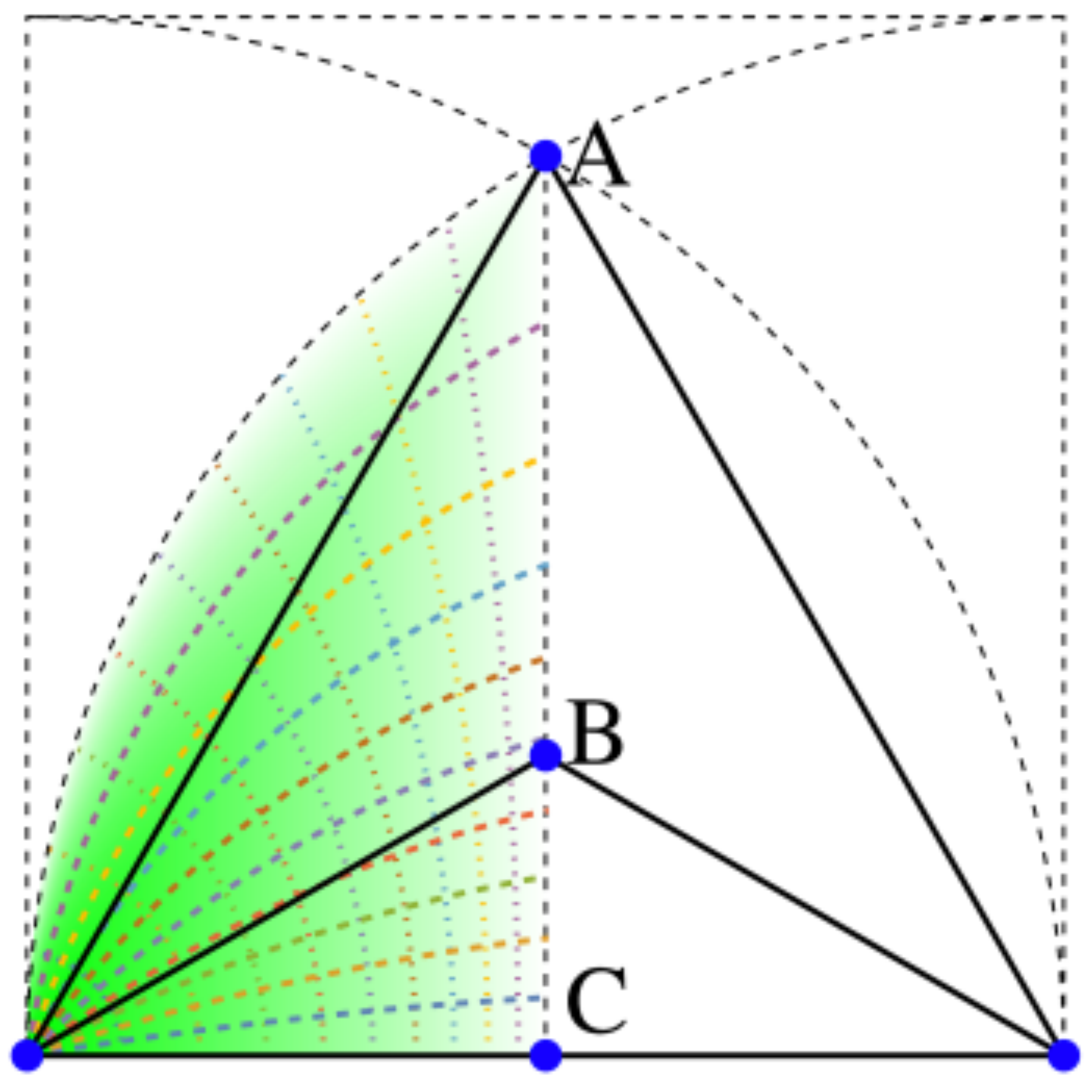
KL, Meçaj, Moult, `22

Projected Energy correlators at the LHC

$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$

$$J_{\mathrm{EEC}}^{N-\mathrm{proj}}(R_L, x, \mu) = \int d\{\zeta\}\, \delta\left(R_L - \max[\{\zeta\}]\right)\; J_{\mathrm{EEC}}^{N}(\{\zeta\}, x, \mu)$$

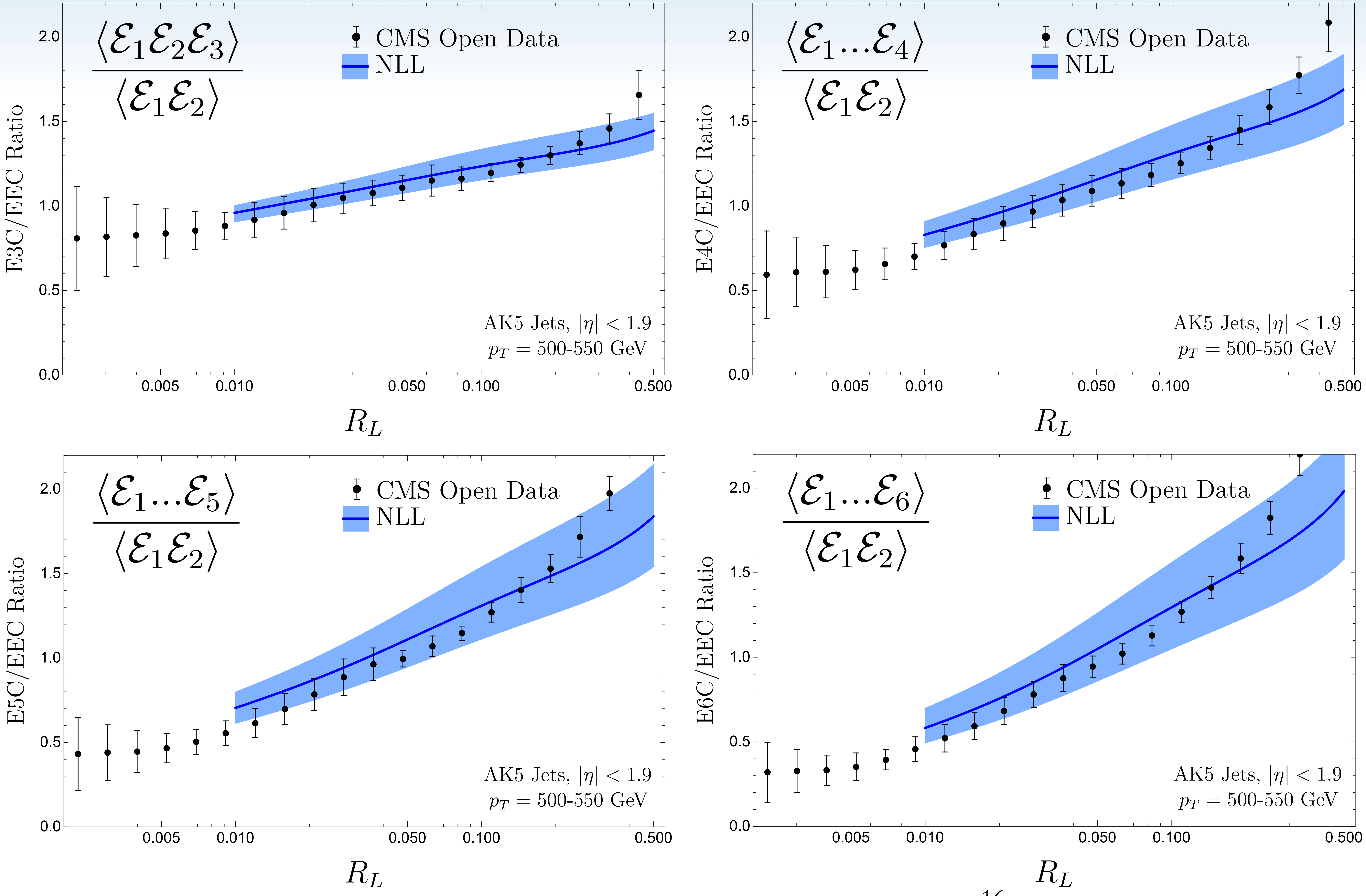
- Integrate over all shapes with fixed largest angle, R_L
- Related to the OPE limit of the N-point correlators, scales as twist-2 spin-(N+1) anomalous dimension in the conformal limit.



Space of 3-point correlator

KL, Meçaj, Moult, `22

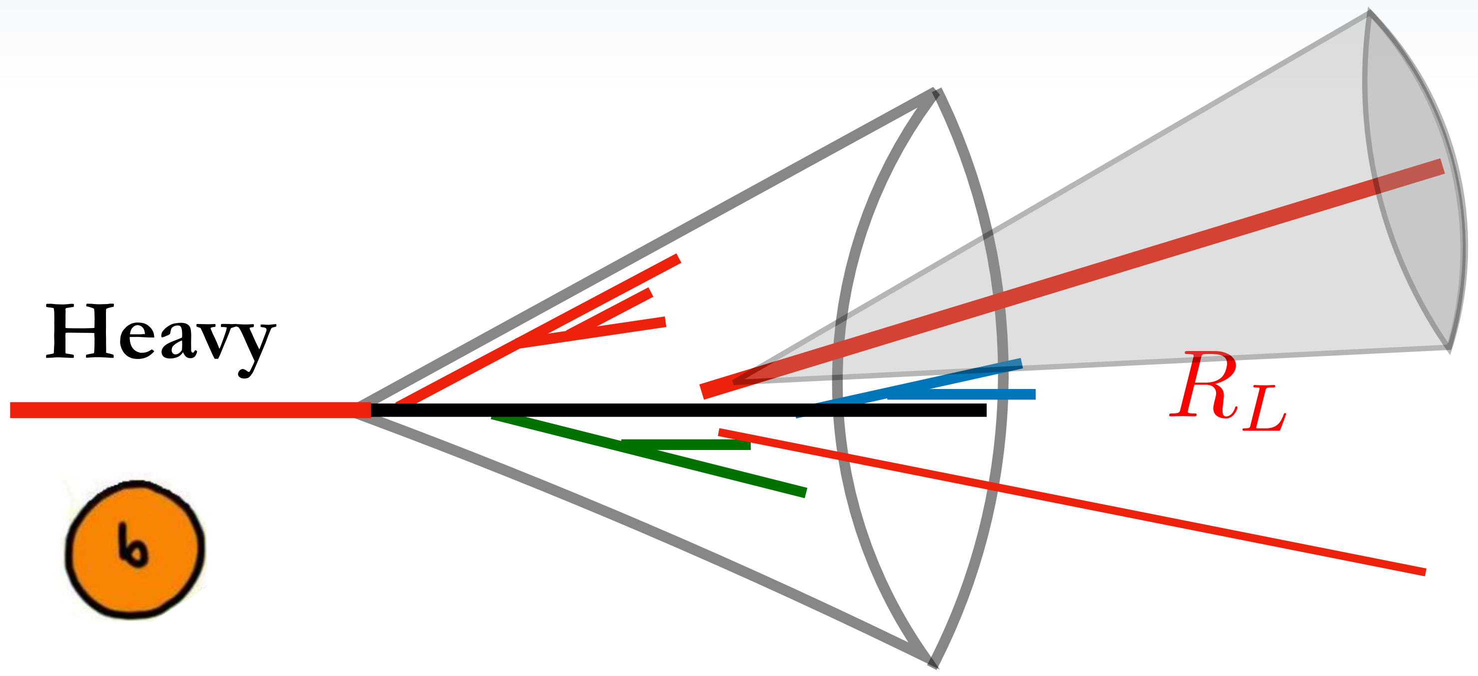
Projected Energy correlators at the LHC



- Slope increases with N as predicted by the light-ray OPEs
- Non-perturbative effects expected to cancel in ratio
- Already at competing order of accuracy as the state-of-the-art calculation of other jet substructure
- Precision calculations of α_s

KL, Meçaj, Moult, `22

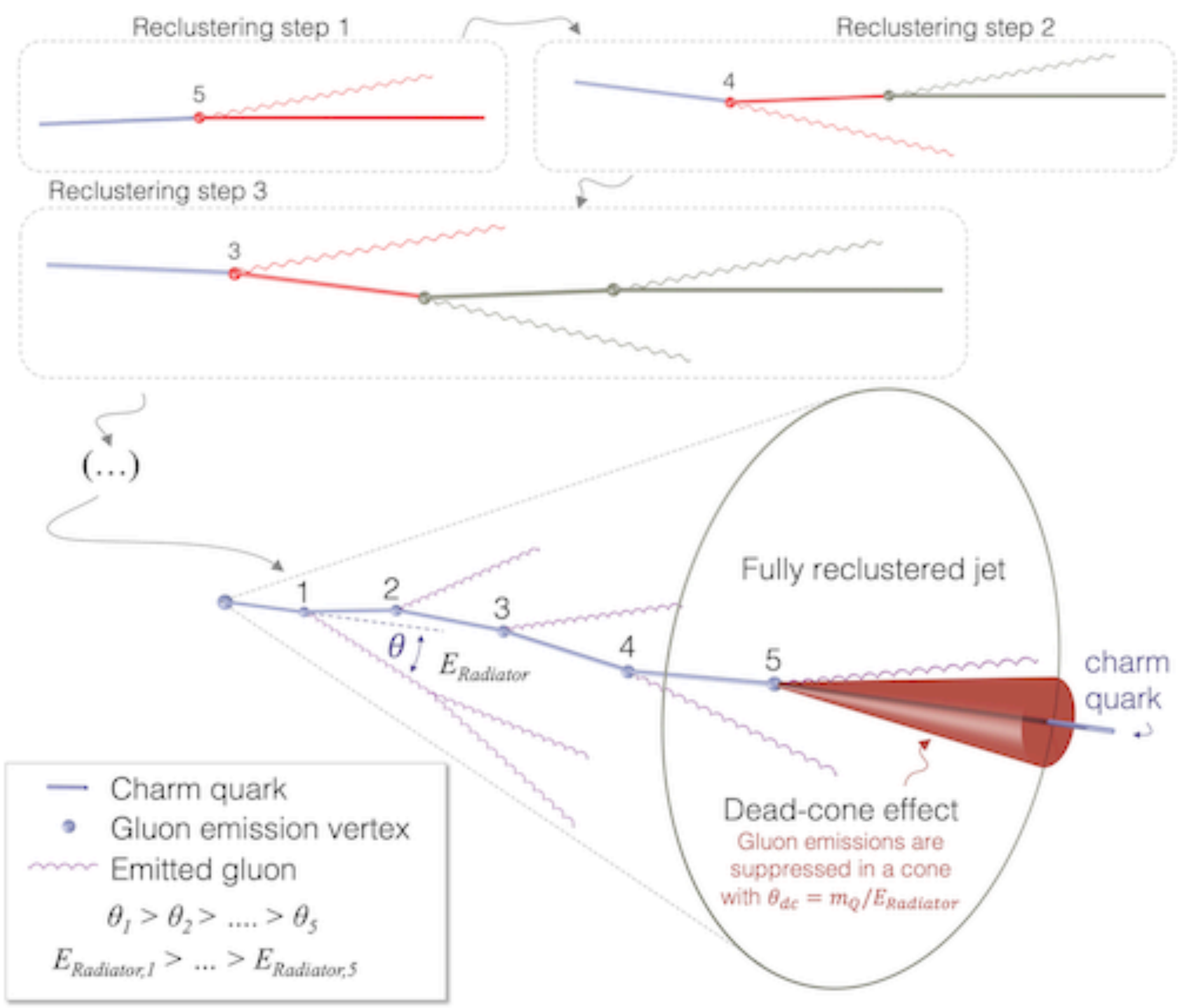
beautiful and *charming* energy correlators



- What happens if we consider energy correlators between heavy meson and other particles in a heavy jet?

- Heavy quark suppresses gluon emission around the angular region $\theta < \frac{M}{p_T}$

dead-cone



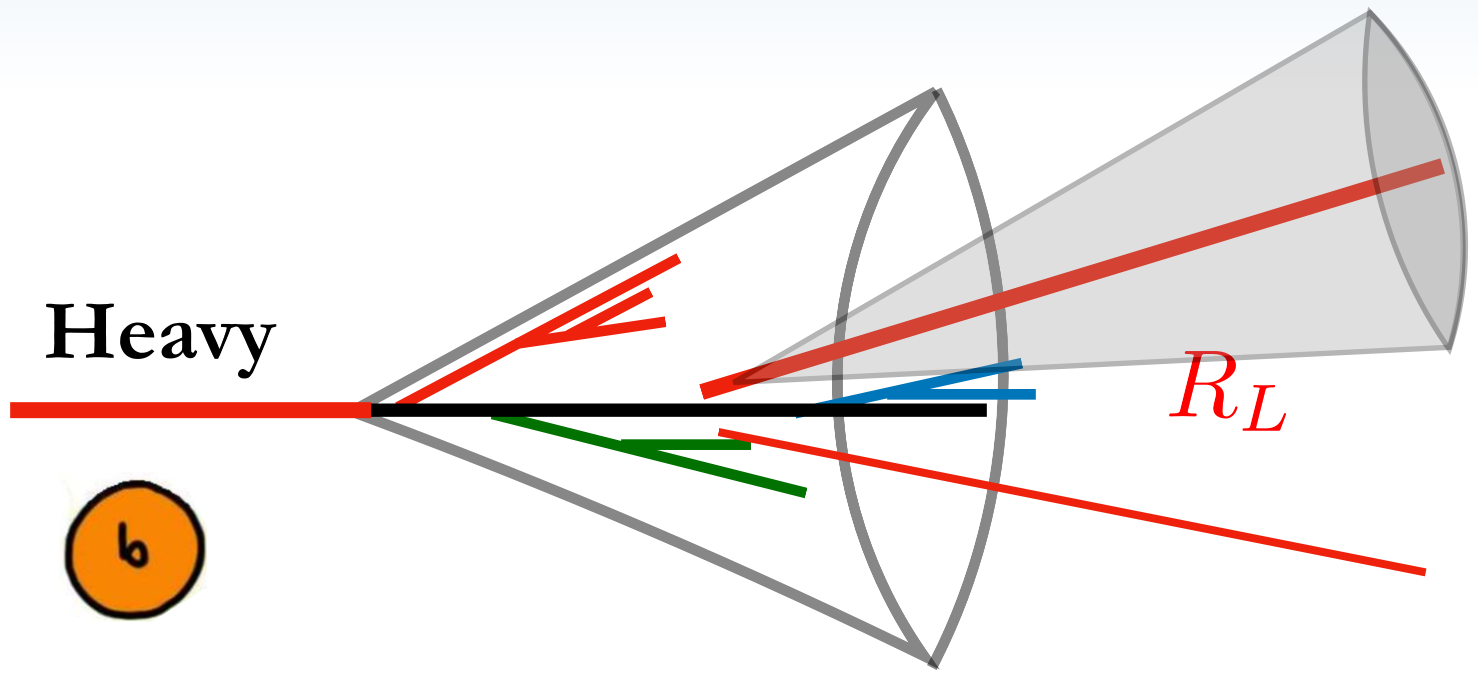
- Recently, ALICE collaboration made a direct observation of the dead-cone effect

ALICE Collaboration `22 (Nature)

- Sophisticated reclustering techniques.
  Can we observe the dead-cone effect statistically using energy correlators?

KL, Meçaj, Moult, In Progress

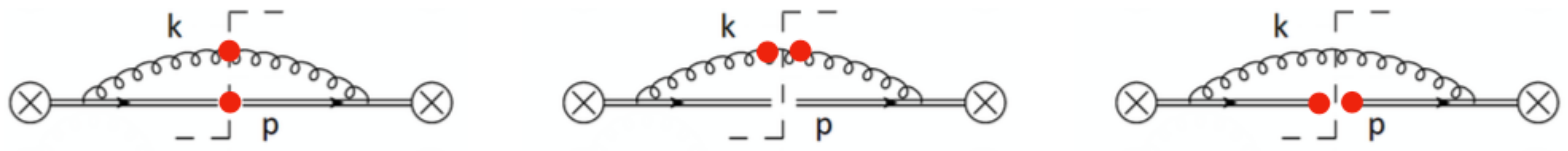
beautiful and *charming* energy correlators



- What happens if we consider energy correlators between heavy meson and other particles in a heavy jet?

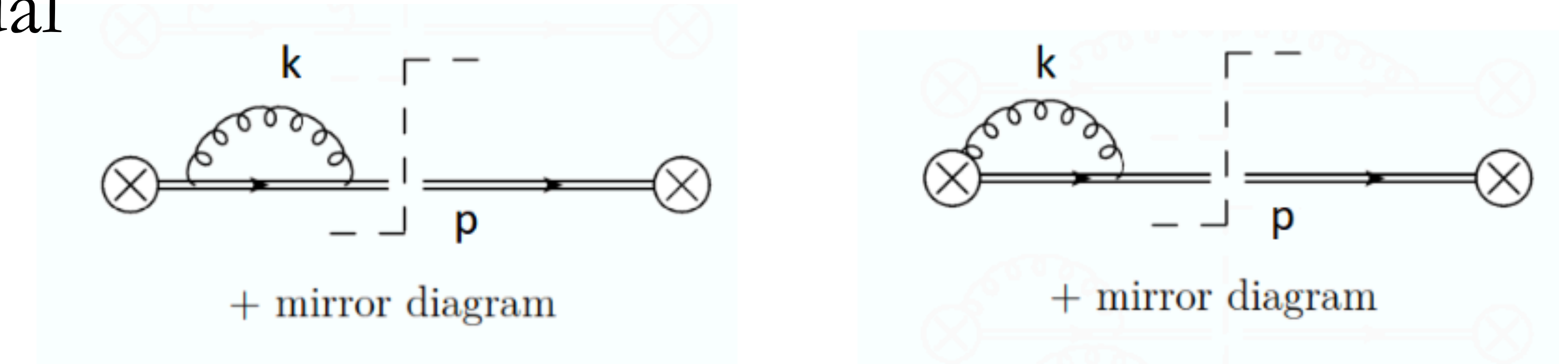
- Virtual diagrams are no longer scaleless, M acts as an IR regulator.

Real



$$J_q^{\text{bare}}(z,\mu) = \delta(z) + \frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\frac{3}{\epsilon_{\text{UV}}} - \frac{37}{3}\right) + 3\frac{Q^2}{\mu^2}\mathcal{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$
$$= \delta(z) + \frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\text{UV}}} - \frac{37}{3}\right) + 3\frac{Q^2}{\mu^2}\mathcal{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right],$$

Virtual



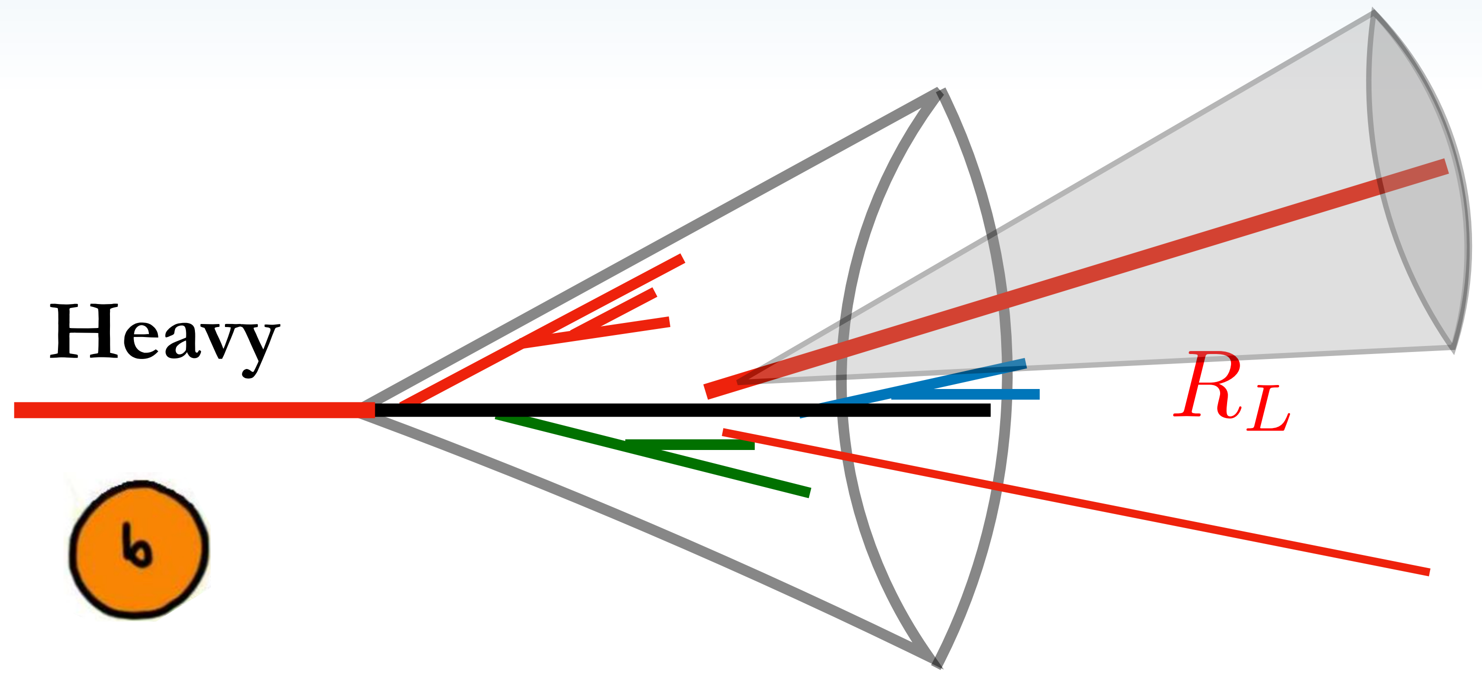
$$J_{Q\to Qg}^{\text{bare}}(z,M,\mu) = \delta(z)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\text{UV}}} + \ln\frac{\mu^2}{M^2}\right) - \frac{19}{6}\right]\right)$$
$$+\frac{\alpha_s C_F}{\pi}\frac{1}{z}\left[\frac{3}{4} - \frac{5}{2}\delta^2 - \frac{\delta^4}{1+\delta^2} + 3\delta^3\arctan\left(\frac{1}{\delta}\right) + \frac{1}{2}\delta^2(1-\delta^2)\ln\frac{\delta^2}{1+\delta^2}\right],$$

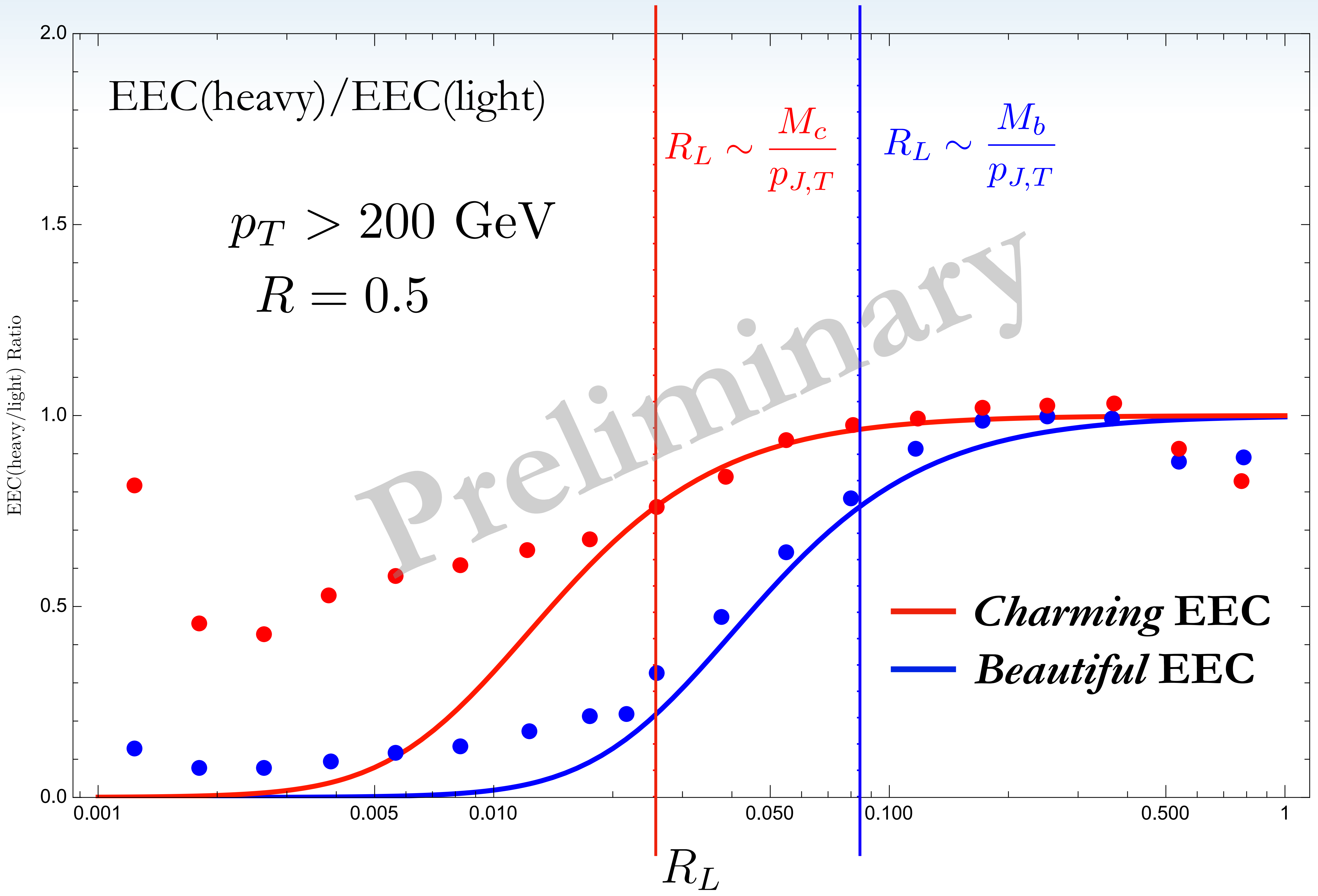
$$\text{where } \delta^2 = \frac{M^2}{Q^2 z^2} \ \& \ z \approx \frac{R_L^2}{4}$$

- UV poles match the light jet case as expected
- Can be matched to the heavy quark fragmentation functions

KL, Meçaj, Moult, In Progress

beautiful and *charming* energy correlators

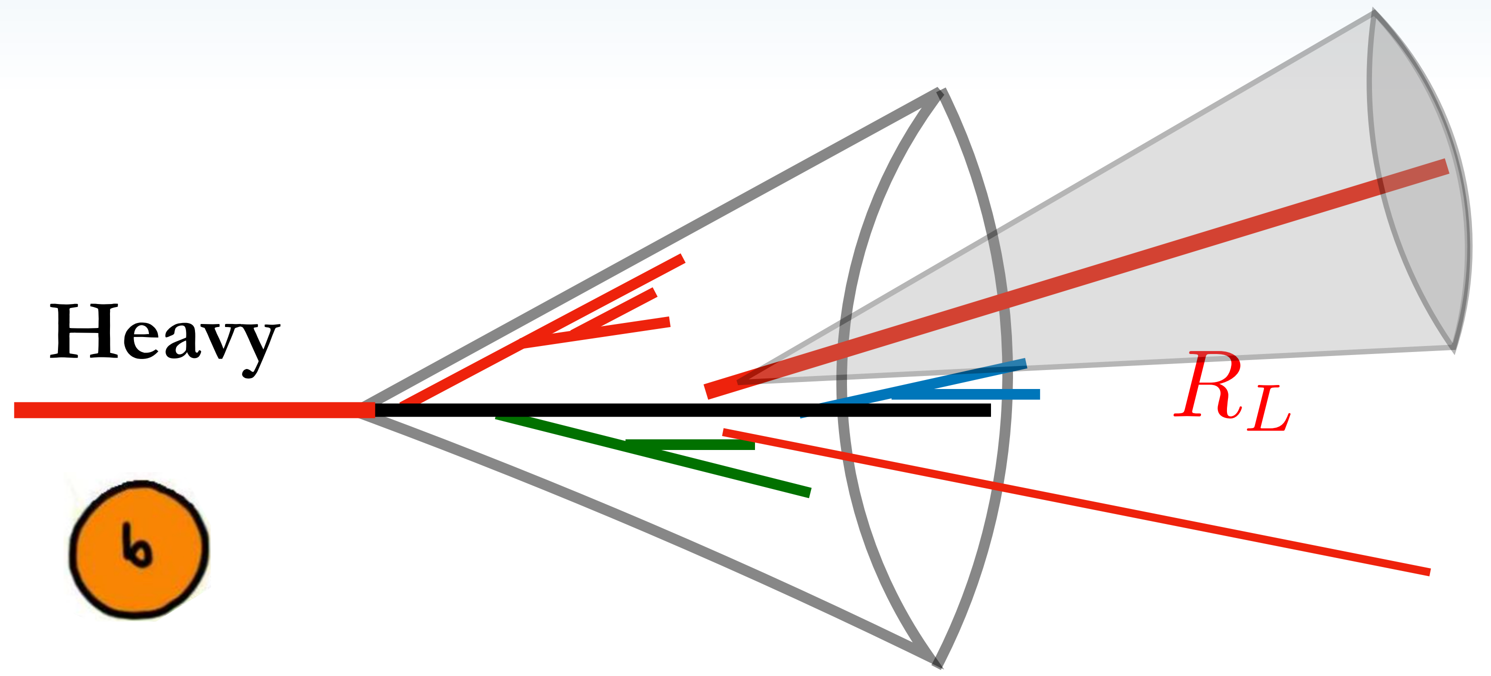


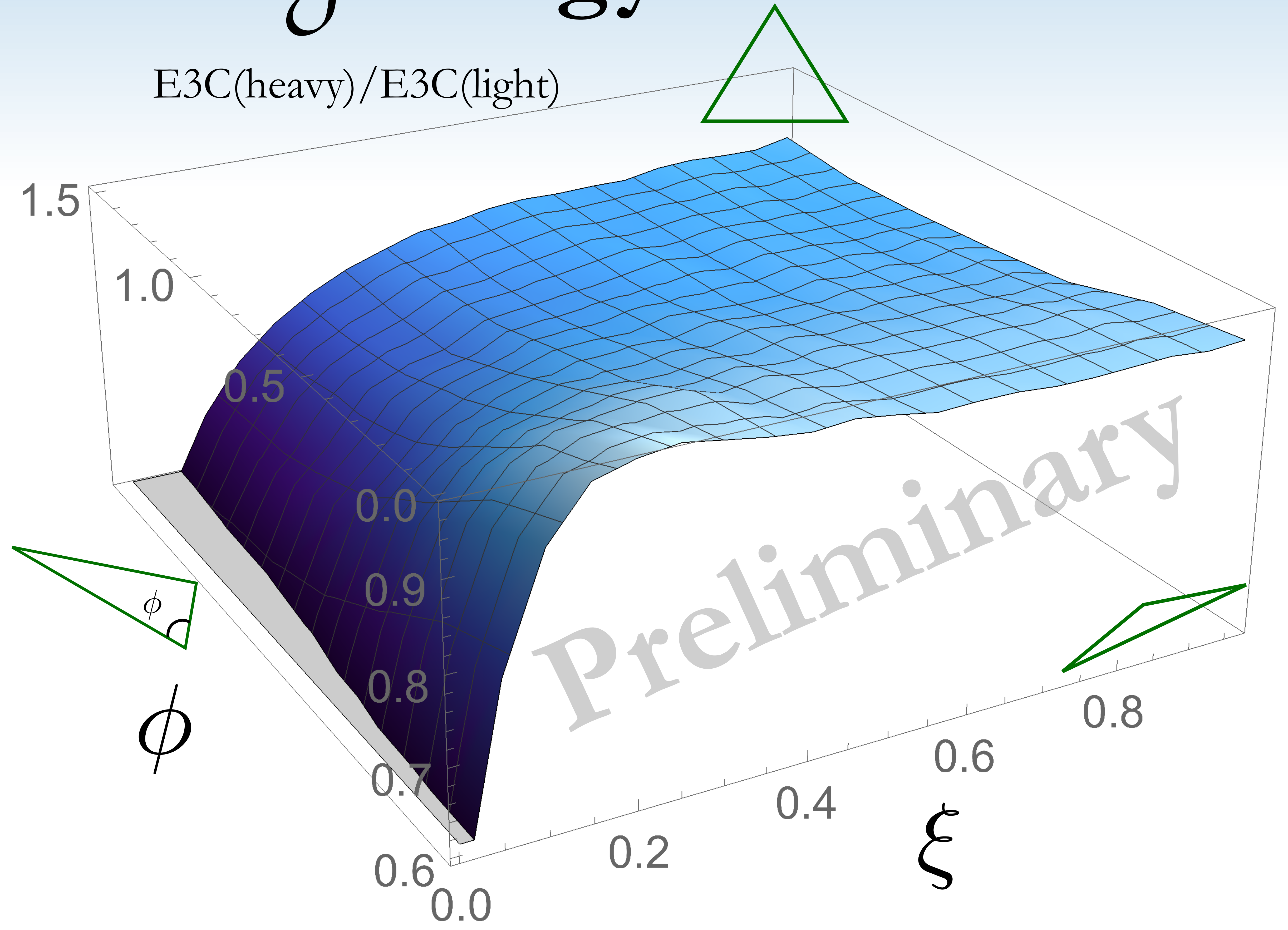


- One observes clear turning around heavy quark scale (both from Pythia and the fixed order calculation).
- Suppression at small angle can be interpreted as a direct signature of the dead-cone

KL, Meçaj, Moult, In Progress

beautiful and *charming* energy correlators





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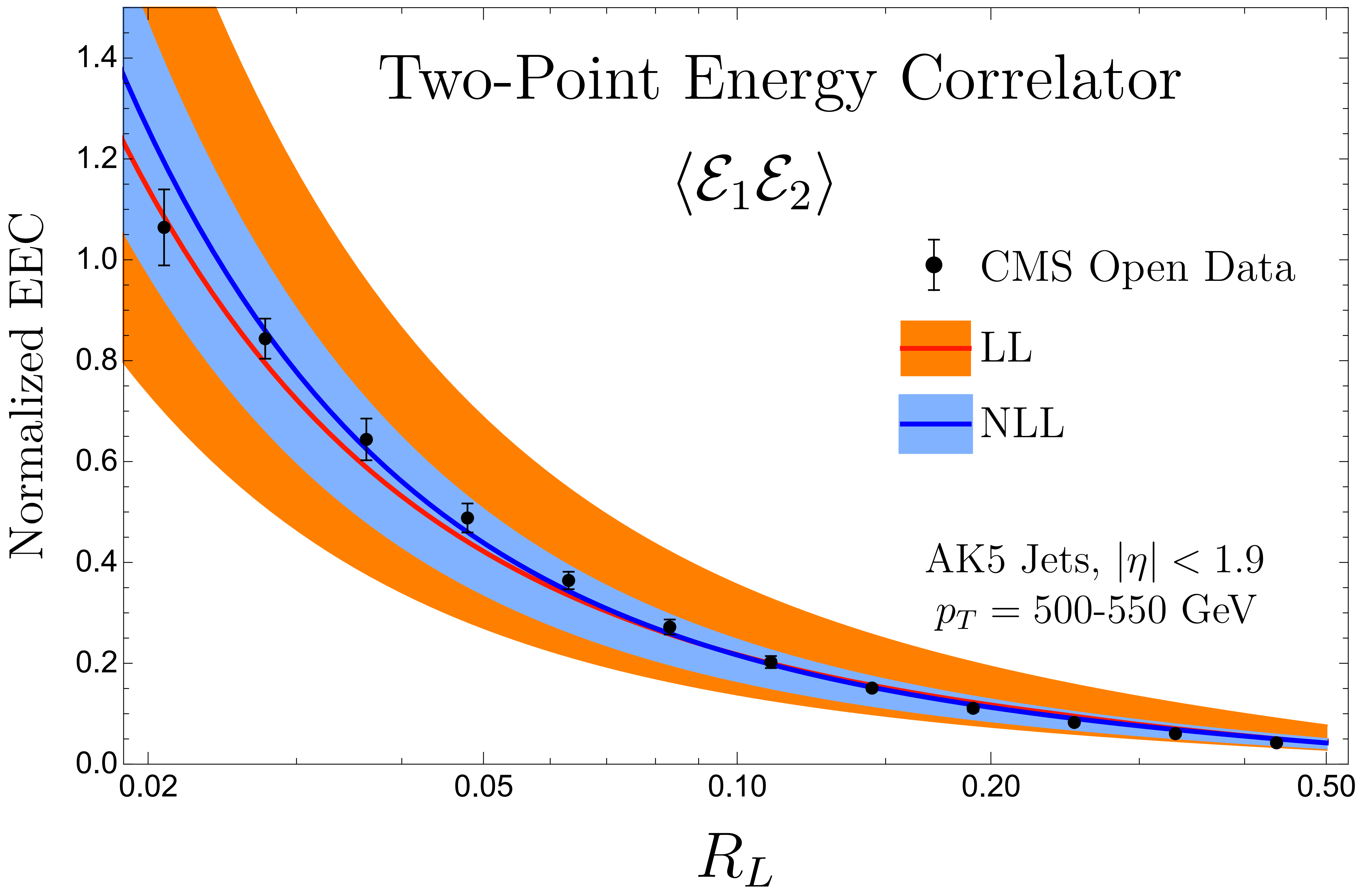
Venturing into precision calculations



Outlook

Czakon, Generet, Mitov, Poncelet `21

Partial results computed

NNLO semi-inclusive hard function ▲

$$\frac{d\sigma^{pp\to\text{jet}(\text{N}-\text{proj})X}}{dp_T d\eta dR_L} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^{\text{N}-\text{proj}}(R_L)$$

NNLO PDFs ✓

NNPDFs, CTEQ, …

$$\mathcal{G}_c^{\text{N}-\text{proj}}(z, R_L, p_T R, \mu) = \sum_j \int_0^1 dx x^N \mathcal{J}_{ij}(z, x, p_T R, \mu)\, J_{\text{EEC}}^{\text{N}-\text{proj}}(R_L, x, \mu)$$

Matching coefficients ▲

Partial results

KL, Liu, Moult, In progress

Projected ENC jet function ✓✓

Available even for the track case!

Chen, Moult, Zhang, Zhu, `20
Li, Moult, van Velzen, Waalewijn, Zhu, `21
Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- Unprecedented precision calculation of jet substructure on the horizon!