Resurrecting Parton-Hadron Duality Systematically

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June 30, 2022

Outline

arXiv:2003.02275 w/ Felix Ringer
arXiv:2010.02934

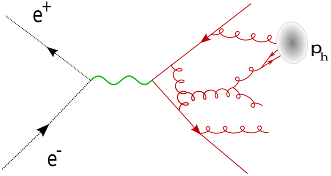
- Traditional approach to Fragmentation
- Angular Ordering
- Angular Ordering Revisited
- Plots and Data
- SIDIS?

Question

How does QCD make hadrons after a hard interaction?

Question Narrowed

How does QCD divide the total energy of an event into its asymptotic states (hadrons)?
For instance, e^+e^-,



q momentum of photon (green wave), $Q^2 = |q^2|$.

- Fragmentation: $D(x, Q^2)$: number of hadrons carrying energy fraction x.

The Answer

$$FF: \ D(x, Q^2) = \sum_i \int_x^1 \frac{dz}{z} d_{h/i}\Big(\frac{x}{z}, \mu^2\Big) C_i^T\Big(z, \mu^2, Q^2\Big), \quad x = \frac{2P_h \cdot q}{Q^2}$$

- C^T encode the quantum fluctuations of photon interacting with QCD.
- d : randomly break a parton into hadrons.
- x is the momentum fraction.
- True to all orders in perturbation theory.
- C^T is *understood* very well.
- Extension to SIDIS *straightforward.*

The Answer Explained

$$FF: \; D(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} d_{h/i}\Big(\frac{x}{z},\mu^2\Big) C_i^T\Big(z,\mu^2,Q^2\Big), \quad x = \frac{2P_h \cdot q}{Q^2}$$

- d universal and non-perturbative,
  sensitive to quantum fluctuations at $\Lambda_{QCD} \sim 1$ GeV.
- C^T process dependent and perturbative,[1]
  sensitive to quantum fluctuations at $Q^2 \gg \Lambda_{QCD}^2$.
- μ defines boundary between these scales,
  controlled renormalization group equations,
  giving scaling properties of D.

[1] Due to asymptotic freedom.

Aside: Moments

Often we will switch to moment space:

$$\bar{g}(n) = \int_0^1 dx x^n g(x),$$

$$xg(x) = \int_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} \bar{g}(n)$$

$x \to 0$ means $n \to 0$

$$\frac{1}{n^\ell} \leftrightarrow \frac{1}{x} \ln^\ell x$$

Factorization Theorem Approach to FF

Usual procedure to tackle FF.

- Perturbation theory gives sum over interaction histories, given as sums over "loop integrals."
- Each integral a summation over all possible momentum regions for a given interaction history.
- These are *infra-red* divergent.
- Regulate IR in **dimensional regularization.** $\int d^4p \rightarrow \int d^dp$.
- Bare non-perturbative functions corrected by *scaleless integrals* which are zero.
- Renormalized non-perturbative functions are the IR singularities of perturbation theory.
- Integration of anomalous dimensions give IR singularities.

Factorization Theorem Approach to FFs

This yields:

$$xD\Big(x, Q^2, \Lambda^2\Big) = \int\limits_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} \exp\Big(\int\limits_{0}^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\alpha,\epsilon)} \gamma^T(\alpha, n)\Big) \bar{C}^T\Big(n, Q^2, \mu^2\Big),$$

γ^T are time-like anomalous dimensions of QCD.
$\beta(\alpha,\epsilon)$ gives the renormalization group equation of the QCD coupling constant.

μ and $d = 4 - 2\epsilon$ dimensional regularization parameters.

Last Step in Factorization Approach to FF

- IR divergences of perturbation theory, i.e., $\exp\Big(\int_0^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\alpha,\epsilon)}\gamma^T(\alpha,n)\Big)$, are not physically real.
- But γ^T and β determine the Q^2 dependence of D.

Final Step: discard singularities for some generic function for d, and fit to data.

Why Dimensional Regularization?

Dimensional regularization is nice!

- Minimal subtraction schemes.
- No power-law divergences.
- Mass independent renormalization group eqs.
- Gauge invariant.
- Can import advance math (algebraic geometry, number theory, complex analysis) to perform loop integrals analytically.
- Ease of calculations, high orders in perturbation theory.

$\gamma^T(n)$ known to 3 loops [**Chen, Yang, Zhu, Zhu arXiv:2006.10534**], β at 5.

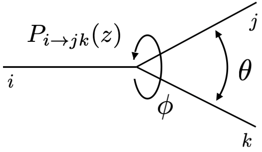
Limits of Tradition

- Pro: Solid grounding in Field Theory.
- Con: No insight into the physics of $d_{h/i}$, the fragmentation function.
- Con: Restricts itself to $0 \ll x < 1$, as $x \to 0$ live DRAGONS.[2]

[2] One exception: [**Anderle, Kaufmann, Stratmann, Ringer 1611.03371**] see also [**Kom,Vogt,Yeats 1207.5631**]

Alternative: "Angular-Ordered DGLAP"

Embrace $x \to 0$.



- Emissions are collinear when they are close in *angle* on celestial sphere.
- IR divergences in fragmentation from $\theta \to 0$.
- Use an *angular ordered evolution* with a *relative transverse momentum cutoff*.
- Hadronization: Cutoff transverse momentum fluctuations below $\sim 200\text{MeV}$.
- Resums $\ln x$ as $x \to 0$ at leading logarithm.

Angular Ordering + Parton-Hadron Duality Hypothesis

In fragmentation: This is old idea (1980$\sim$ 1991):
Mueller, Dokshitzer, Marchesini, ...
But no systematics, beyond leading log,

- What is being resummed relative to QCD factorization?
- What is the corresponding fragmentation function and coefficient function?

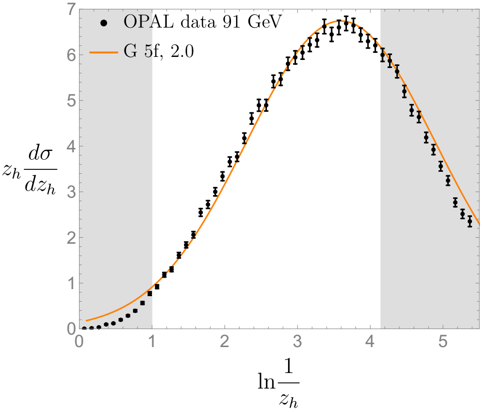
- Y. L. Dokshitzer, V. A. Khoze, and S. Troian, Inclusive particle spectra from QCD cascades, Int. J. Mod. Phys. A 7 (1992) 1875-1906

- C. Fong and B. Webber, One and two particle distributions at small x in QCD jets, Nucl. Phys. B 355 (1991) 54-8

Angular Ordering + Parton-Hadron Duality Hypothesis

However, it is predictive:

- As $Q \to \infty$, $xD(x, Q^2)$ is gaussian in $\ln\frac{1}{x}$.



[Opal Collab., 1990, "A study of coherence of Soft Gluons in Hadron Jets"]

Angular Ordering Hypothesis Revisited

arXiv:2003.02275 D.N., Felix Ringer

- Can we unite the phenomological success of angular-ordering with traditional factorization theorems?
- Can we elucidate the mapping between space-like and time-like processes? (e.g., local twist $\leftrightarrow$ light-ray twist OPE's, see Ian Moult's talk)

Angular Regularization in fragmentation

- Introduce Fragmentation Cross-section with minimum branching angle R_{ir} to observed hadron:

$$D_{\triangleleft}\Big(x, R_{ir}^2, \frac{\mu^2}{Q^2}\Big)$$

- x energy fraction of observed hadron.
- Always work in $d = 4 - 2\epsilon$ dimensions
- μ mass scale of dimensional regularization.

Angular Evolution in fragmentation

- Introduce Fragmentation Cross-section with minimum branching angle R_{ir} to observed hadron:

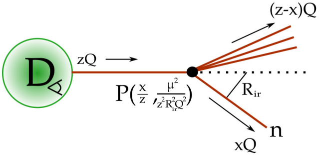
$$D_{\triangleleft}\left(x, R_{ir}^2, \frac{\mu^2}{Q^2}\right)$$

What happens as $R_{ir} \to 0$?

Angular Evolution: [D.N., Felix Ringer]

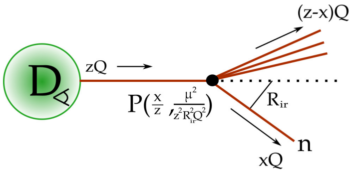
Changing cutoff R_{ir} (pure Yang-Mills):

$$R_{ir}^2 \frac{d}{dR_{ir}^2} x^{1+2\epsilon} D_{\sphericalangle}\left(x, R_{ir}^2, \frac{\mu^2}{Q^2}\right) = \rho\left(\frac{\mu^2}{R_{ir}^2 Q^2}\right) x^{1+2\epsilon} D_{\sphericalangle}\left(x, R_{ir}^2, \frac{\mu^2}{Q^2}\right)$$
$$+ \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}; \frac{\mu^2}{z^2 R_{ir}^2 Q^2}\right) z^{1+2\epsilon} D_{\sphericalangle}\left(z, R_{ir}^2, \frac{\mu^2}{Q^2}\right).$$



- $zR_{ir}Q$ is transverse momentum of *parent parton* WRT fragmented hadron's direction.
- "Divined" from a celestial "BFKL" equation.

Details of Kernel



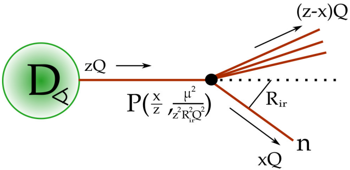
Kernels P, ρ have expansions:

$$P\left(\frac{x}{z}; \frac{\mu^2}{z^2 R_{ir}^2 Q^2}\right) = \sum_{\ell=0}^{\infty} p^{(\ell)}\left(\frac{x}{z}, \alpha_s, \epsilon\right) \times \left(\frac{\mu^2}{z^2 R_{ir}^2 Q^2}\right)^{(\ell+1)\epsilon}$$

$$\rho\left(\frac{\mu^2}{R_{ir}^2 Q^2}\right) = -\sum_{\ell=0}^{\infty} \beta_i \left(\frac{C_A \alpha_s}{\pi}\right)^{\ell+1} \left(\frac{\mu^2}{R_{ir}^2 Q^2}\right)^{(\ell+1)\epsilon}$$

β_i are the coefficients of the QCD beta function. $p^{(\ell)}$ has expansions in both ϵ and α_s.

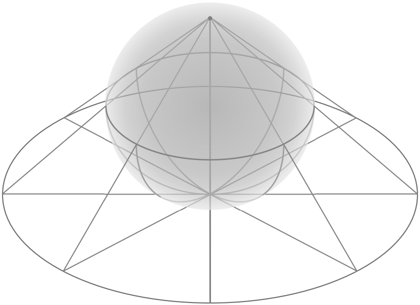
Details of Kernel: Reciprocity



Space-like/Time-like Duality: P **is determined from the anomalous dimensions for *parton distribution function.***
[Basso, Korchemsky hep-th/0612247], [Dokshitzer et al. hep-ph/0511302]

Geometrical Realization of Space-Time Reciprocity

Stereographic projection of $S^2 \to \mathbb{R}^2$



Final state radiation distributing energy over the celestial sphere about a hard scattering interaction.

$\leftrightarrow$

Initial state constituents moving along the nucleon's direction, distributed in the transverse plane with respect to the nucleon's momentum.

[Hatta 0810.0889]

What does this accomplish?

Solve evolution equation,
Evolve to $R_{ir} = 0$,
Get Dim. Reg. fragmentation cross-section:

$$D(x,Q^2) = \lim_{R_{ir}\to 0}\int_x^1 \frac{dz}{z} d_{\triangleleft}\Big(\frac{x}{z}, R_{ir}^2, R_{uv}^2, \frac{\mu^2}{z^2Q^2}\Big) D_{\triangleleft}\Big(z, R_{uv}^2, \frac{\mu^2}{Q^2}\Big)$$

$$= \int\limits_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} \exp\Big(\int\limits_0^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\alpha,\epsilon)} \gamma^T(\alpha,n)\Big) \bar{C}^T\Big(n, Q^2, \mu^2\Big)$$

- Calculates: γ^T from γ^S, as $\gamma^S(n + 2\gamma^T(n)) = \gamma^T(n)$
- Resums Soft Region: $d_{\triangleleft}$ contains **all** logarithmically enhanced terms as $x \to 0$ in both γ^T and C^T.

Checks: Does it work?

As $x \to 0$:

$$C^T(x) \sim \frac{1}{x}\Big(c_0^{LL}\alpha\ln x + c_1^{LL}\alpha^2\ln^3 x + c_3^{LL}\alpha^3\ln^5 x + ...$$
$$+ c_0^{NLL}\alpha + c_1^{NLL}\alpha^2\ln^2 x + c_3^{NLL}\alpha^3\ln^4 x + ...$$
$$\gamma^T(x) \sim \frac{1}{x}\Big(\gamma_0^{LL}\alpha + \gamma_1^{LL}\alpha^2\ln^2 x + \gamma_3^{LL}\alpha^3\ln^4 x + ...$$
$$+ \gamma_1^{NLL}\alpha^2\ln x + \gamma_3^{NLL}\alpha^3\ln^3 x + ...$$

- Predicts resummed terms to N^2LL in C^T, N^3LL in γ^T, reproducing perturbative expansion up-to and at 3 loops. (see also [**Kom, Vogt, Yeats 1207.5631**]).
- Relates the time-like fragmentation process to quantities defined by the space-like process (FF↔PDF).
- Generalizes to evolution of largest eigenvalue in singlet DGLAP matrix.

Recall *rigorous* factorization theorem:

$$D(x,Q^2)=\sum_i\int_x^1\frac{dz}{z}d_{h/i}\Big(\frac{x}{z},\mu^2\Big)C_i^T\Big(z,\mu^2,Q^2\Big),\quad x=\frac{2P_h\cdot q}{Q^2}$$

- Parton-Hadron Duality: $D(x,Q^2)\propto\delta(1-x)$ at $Q^2\sim 1$ GeV.
- *The fragmentation function is the functional reciprocal of the coefficient function!*[3]

[3] Exact reciprocal in mellin-space.

Modeling Fragmentation From Resummed Perturbation Theory

The fragmentation function is the functional reciprocal of the coefficient function???

- This manifestly does not work in fixed order perturbation theory.
- But we can resum the perturbation series.
- Possible field theory origin: evolving the cross-section using resummed *"physical"* anomalous dimension scheme, not $\overline{MS}$.

Model for Fragmentation Functions from Perturbation Theory

Take:

$$\left(\bar{C}^T\left(n, \Lambda_{QCD}^2, \mu^2\right)\right)^{-1}.$$

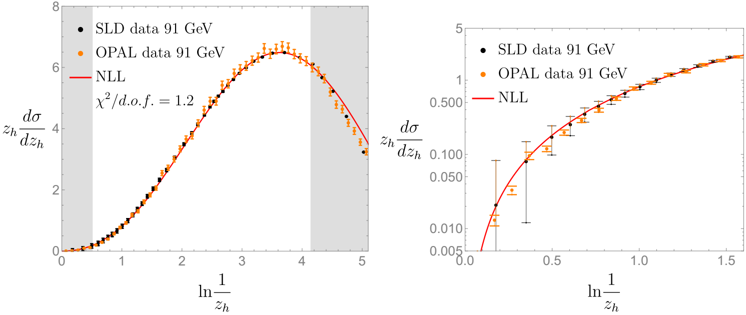
as mellin-space fragmentation function.

- Run down to $\mu \sim \Lambda_{QCD}$, fit for normalization[4] and Λ_{QCD}.
- Use $x \to 0$ resummed Coef., FF, γ^T. (Derived from angular-ordered evolution).

[4](normalization not predicted, as data is on charged hadrons only.)

Model for FF from Perturbation Theory

Even though for $x \to 0$, works almost everywhere:

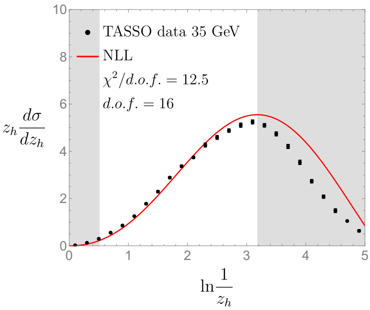


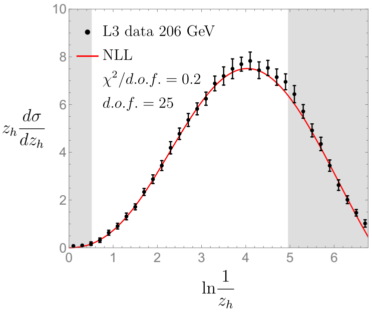
$$e^+e^- \to h^\pm + X$$

Excluded Gray Regions: hadron mass corrections to energy fraction, recall assumption $p_h^2 \approx 0$, threshold region.

Model for FF from Perturbation Theory

Evolution:



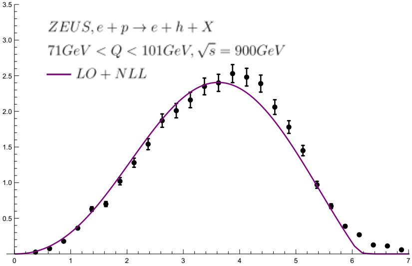


$$e^+e^- \to h^\pm + X$$

Excluded Gray Regions: hadron mass corrections to energy fraction, recall assumption $p_h^2 \approx 0$, threshold region. Lower Q data, hadron mass corrections?

SIDIS?

- Fragmentation in SIDIS: *Preliminary.*
- Using same methods from e^+e^-, in Briet Frame.



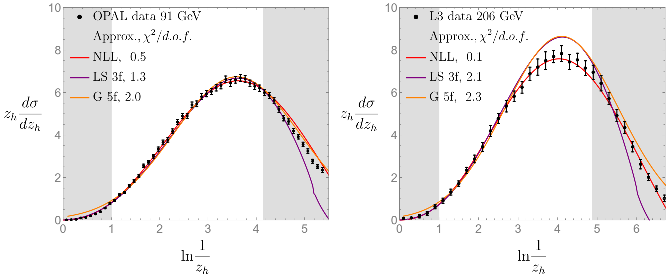
Data: [**ZEUS collab. 1001.4026**]

Conclusions:

- We can resurrect ancient angular-ordering methods, now connecting directly to traditional factorizations.
- Why does $x \to 0$ work everywhere?

MLLA

Angular ordering is old, used to derive "mixed-leading log" approximation (LS→Limiting Spectrum and G→Gaussian curves below), with unclear systematics.



Old formulation of angular ordering does not evolve nicely above $Q \sim 91$GeV. **LS**- Y. L. Dokshitzer, V. A. Khoze, and S. Troian, Inclusive particle spectra from QCD cascades, Int. J. Mod. Phys. A 7 (1992) 1875-1906

G- C. Fong and B. Webber, One and two particle distributions at small x in QCD jets, Nucl. Phys. B 355 (1991) 54-8

Mellin Space Angular Evolution

$$R_{ir}^2\frac{d}{dR_{ir}^2}\bar{d}_{\sphericalangle}(n,R_{ir}^2,R_{uv}^2,\mu^2,Q^2)=\sum_{\ell=1}^{\infty}\rho^{(\ell-1)}(\alpha_s;\epsilon)\Big(\frac{\mu^2}{R_{ir}^2Q^2}\Big)^{\ell\epsilon}\bar{d}_{\sphericalangle}\Big(n,R_{ir}^2,R_{uv}^2,\mu^2,Q^2\Big)$$
$$+\sum_{\ell=1}^{\infty}\bar{P}^{(\ell-1)}\Big(n-2\epsilon;\alpha_s;\epsilon\Big)\Big(\frac{\mu^2}{R_{ir}^2Q^2}\Big)^{\ell\epsilon}\bar{d}_{\sphericalangle}\Big(n-2\ell\epsilon,R_{ir}^2,\mu^2,Q^2\Big)\,.$$

Mellin Space Angular Evolution II

Setting $\mu = Q$, and writing:

$$\bar{d}_{\triangleleft}(n, R_{ir}^2, R_{uv}^2, \mu^2, \mu^2) = \bar{d}_{\triangleleft}(n, R_{ir}^2, R_{uv}^2)\,,$$

$$I\Big(\ell_1; n; R_{ir}, R_{uv}\Big) = 2\int_{R_{ir}}^{R_{uv}} \frac{d\theta_1}{\theta_1^{1+2\ell_1\epsilon}}\,\bar{P}^{(\ell_1-1)}(n-2\epsilon),$$

$$I\Big(\ell_1,\ldots,\ell_k; n; R_{ir}, R_{uv}\Big) = 2^k\int_{R_{ir}}^{R_{uv}} \frac{d\theta_1}{\theta_1^{1+2\ell_1\epsilon}}\,\bar{P}^{(\ell_1-1)}(n-2\epsilon)$$

$$\times\int_{R_{ir}}^{R_{uv}} \prod_{i=2}^{k}\frac{d\theta_i}{\theta_i^{1+2\ell_i\epsilon}}\,\Theta(\theta_i-\theta_{i-1})\,\bar{P}^{(\ell_i-1)}\Big(n-2\epsilon(1+\sum_{j=1}^{i-1}\ell_j)\Big)\,,\text{ if } k>1\,.$$

Then the iterative expansion is obtained to be:

$$\bar{d}_{\triangleleft}\Big(n, R_{ir}^2, R_{uv}^2\Big) = \exp\Big(-\int_{R_{ir}^2}^{R_{uv}^2}\frac{d\theta^2}{\theta^2}\,\rho(\theta^{-2})\Big)\Big(1+\sum_{\ell_1} I(\ell_1; n; R_{ir}, R_{uv})$$

$$+\sum_{\ell_1,\ell_2} I(\ell_1,\ell_2; n; R_{ir}, R_{uv}) + \sum_{\ell_1,\ell_2,\ell_3} I(\ell_1,\ell_2,\ell_3; n; R_{ir}, R_{uv}) + \ldots\Big)\,.$$

“Celestial” BFKL Equation

On “angular” space:

$$x\frac{d}{dx}\mathcal{D}(x,n_a\cdot n_b)$$
$$=-(1+2\epsilon)\mathcal{D}(x,n_a\cdot n_b)$$
$$-\Big(\frac{\mu e^{\frac{\gamma_E}{2}}}{xQ}\Big)^{2\epsilon}\frac{\alpha_s C_A}{\pi}\int\frac{d^{2-2\epsilon}\Omega_{\hat{j}}}{4\pi^{1-\epsilon}}\frac{n_a\cdot n_b}{n_a\cdot n_j\, n_j\cdot n_b}\Big(\mathcal{D}(x,n_a\cdot n_j)+\mathcal{D}(x,n_b\cdot n_j)-\mathcal{D}(x,n_a\cdot n_b)\Big)\,,$$

Null vectors: $n=(1,\hat{n})$. n_a,n_b,n_j are null vectors parametrizing eikonal lines in a large N_c-dipole.

Small angles:

$$x\frac{d}{dx}\mathcal{D}(x,\vec{\theta}^{\,2}_{ab})=-(1+2\epsilon)\mathcal{D}(x,\vec{\theta}^{\,2}_{ab})$$
$$-\Big(\frac{\mu e^{\frac{\gamma_E}{2}}}{xQ}\Big)^{2\epsilon}\frac{\alpha_s C_A}{\pi}\int\frac{d^{2-2\epsilon}\vec{\theta}_j}{2\pi^{1-\epsilon}}\frac{\vec{\theta}^{\,2}_{ab}}{\vec{\theta}^{\,2}_{aj}\vec{\theta}^{\,2}_{jb}}\Big(\mathcal{D}(x,\vec{\theta}^{\,2}_{aj})+\mathcal{D}(x,\vec{\theta}^{\,2}_{jb})-\mathcal{D}(x,\vec{\theta}^{\,2}_{ab})\Big)$$

$\vec{\theta}_{ab}$ tangent vector on sphere, pointing from n_b to n_a.