Back-to-back inclusive dijets in DIS at small x: NLO results

Paul Caucal

with Farid Salazar, Bjoern Schenke and Raju Venugopalan

Brookhaven National Laboratory

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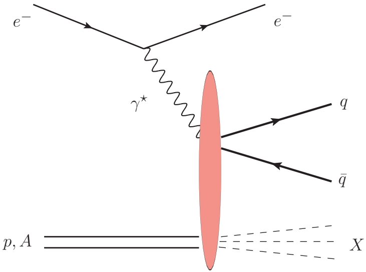
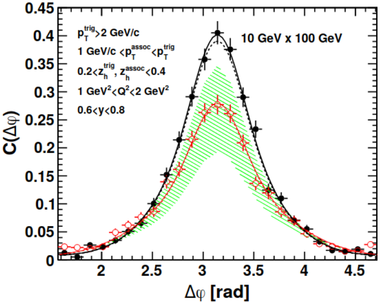
Inclusive dijet production in DIS at small-x

$\Rightarrow$ probe of the saturated regime of QCD

$\Rightarrow$ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.

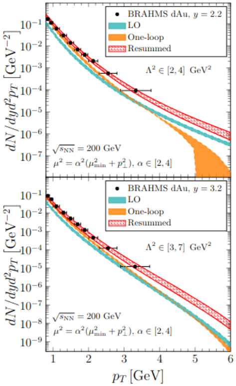




Zheng, Aschenauer, Lee, Xiao, 1403.2413

Precision physics at small-x: many recent progresses!

- NLO JIMWLK evolution for massive quarks.
  Dai, Lublinsky, 2203.13695
- One-loop light cone wave functions with massive quarks.
  Beuf, Lappi, Paatelainen, 2112.03158
- Forward hadron production in pp/pA at NLO.
  Shi, Wang, Wei, Xiao, 2112.06975
- Forward **jet** production in pA at NLO.
  Liu, Xie, Kang, Liu, 2204.03026
  See talk by Hao-yu tomorrow



In this talk: NLO impact factor for inclusive dijet production in DIS

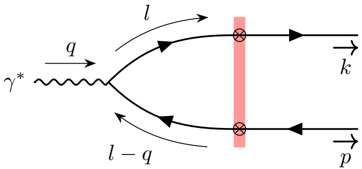
- Reliable QCD prediction requires to account for NLO corrections.
- Systematic determination of the theoretical uncertainties.

Outline

- Brief overview of the computation.
- Divergences
- Back-to-back limit at NLO: Sudakov logarithms and connection with TMD factorization.

Dipole picture, CGC EFT, covariant perturbation theory

- We work in the dipole picture of DIS, large q^-.



- Covariant perturbation theory.

- CGC effective vertex:
  $= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$

  $\Rightarrow$ multiple gluon interactions with the target resummed via Wilson lines $V(\boldsymbol{x}_\perp)$

LO cross-section

- Differential cross-section at leading order:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp\mathrm{d}^2\boldsymbol{p}_\perp\mathrm{d}\eta_q\mathrm{d}\eta_{\bar{q}}}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\int\mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_\perp\boldsymbol{r}_{xx'}}e^{-i\boldsymbol{p}_\perp\boldsymbol{r}_{yy'}}\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}'_{xy})$$

- **Factorization** between perturbative factor describing the $\gamma^\star\to q\bar{q}$ splitting...

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}'_{xy}) = 8z_q^3z_{\bar{q}}^3Q^2K_0(\bar{Q}r_{xy})K_0(\bar{Q}r_{xy'})$$

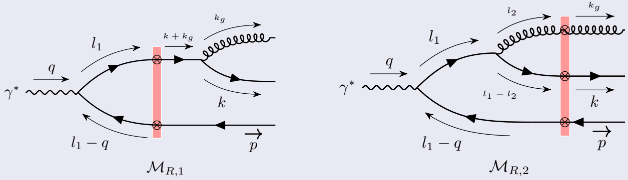
- ... and a color structure describing the interaction of $q\bar{q}$ with the dense target

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) = \left\langle \underbrace{Q(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)}_{quadrupole} - D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) - \underbrace{D(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)}_{dipole} + 1\right\rangle_Y$$

$$\text{Dipole:}\qquad D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\langle\mathrm{Tr}(V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp))$$

NLO computation: real amplitudes

Real diagrams

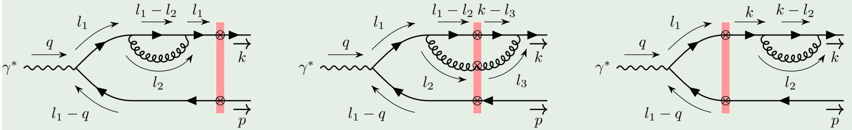


$+(q \longleftrightarrow \bar{q})$ $+(q \longleftrightarrow \bar{q})$

- Already computed by Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 1701.07143 using spinor helicities techniques.
- We recover their results.

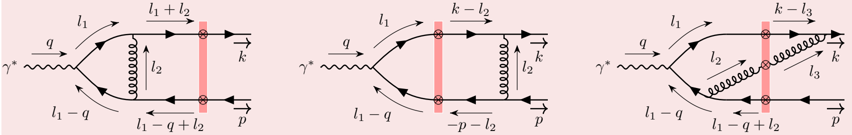
NLO computation: virtual amplitudes

Self-energies



$+(q \longleftrightarrow \bar{q})$ $+(q \longleftrightarrow \bar{q})$ $+(q \longleftrightarrow \bar{q})$

Vertex corrections



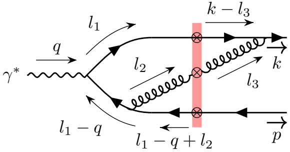
$+(q \leftrightarrow \bar{q})$

See also:

- Beuf, 1606.00777 (LCPT)
- Hänninen, Lappi, and Paatelainen 1711.08207 (LCPT)
- Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371 (exclusive dijet)
- Taels, Altinoluk, Beuf, Marquet, 2204.11650 (LCPT, $Q^2 = 0$)

Reducing the number of integrals

- Example: the dressed vertex correction for longitudinally polarized $\gamma^\star$.



$$= \frac{e e_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} [t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a]$$

$$\times \frac{\alpha_s}{\pi^2} 2(z_q z_{\bar{q}})^{3/2} Q \delta_{\sigma,-\bar{\sigma}} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} e^{-i z_g \boldsymbol{k}_\perp / z_q \cdot \boldsymbol{r}_{zx}} \left(1 + \frac{z_g}{z_{\bar{q}}}\right) \left(1 - \frac{z_g}{z_q}\right) K_0(QX_V)$$

$$X_V^2 = z_{\bar{q}}(z_q - z_g)\boldsymbol{r}_{xy}^2 + z_g(z_q - z_g)\boldsymbol{r}_{zx}^2 + z_g z_{\bar{q}} \boldsymbol{r}_{zy}^2$$

$$\times \left\{ \left[1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + i\sigma \left[\frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \times \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} \right\}$$

Take home message

- Compact expression!
- Hopefully suitable for numerical evaluation.

Divergences

Divergences

What kind of divergence do we get?

- UV (short distance) divergences
  - internal momentum goes to ∞ or $|\boldsymbol{z}_\perp - \boldsymbol{x}_\perp| \to 0$.
  - we use dim. reg. in the transverse plane to extract the UV pole of each diagram if any.
- Rapidity divergence, "slow gluon" when $k_g^- \to 0$.
- Soft divergence $k_g^\mu \to 0$.
- Collinear divergence, $z_q \boldsymbol{k}_{\perp g} - z_g \boldsymbol{k}_\perp \to 0$ or $z_{\bar{q}} \boldsymbol{k}_{\perp g} - z_g \boldsymbol{p}_\perp \to 0$.

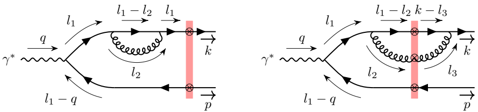
Our regularization scheme

Dim. reg. in the transverse plane + lower cut-off Λ^- in the longitudinal direction:

$$\int_{\Lambda^-}^{\infty} \frac{\mathrm{d}k_g^-}{k_g^-} \mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon} \boldsymbol{k}_{g\perp}}{(2\pi)^{2-\varepsilon}} f(k_g^-, \boldsymbol{k}_{g\perp})$$

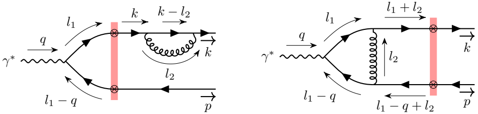
Cancellation of UV and IR poles

- Massless quark + universality of quark electric charge ⇒ **no need for UV renormalization**
- UV divergence cancels between free self-energy before shock-wave and dressed self energy



- The free self-energies after SW turn UV divergence of the free vertex correction before shock-wave into IR

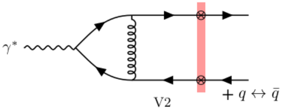
Remaining $\frac{2}{\varepsilon_{\mathrm{IR}}}$ pole canceled by the real corrections for IRC safe cross-section ⇒ jets

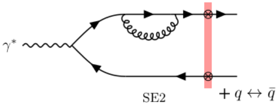


$$\propto \left(\frac{2}{\varepsilon_{\mathrm{IR}}} - \frac{2}{\varepsilon_{\mathrm{UV}}}\right) \qquad \propto \frac{2}{\varepsilon_{\mathrm{UV}}}$$

Cancellation of soft divergences

- Soft divergences: double log of the Λ^- cut-off, $\ln^2(\Lambda^-/q^-)$.
- Amplitude-level factorization of soft gluons: $\propto$ to the LO color structure or the cross-diagram color structure.
- For the LO color structure, cancel separately among the virtual diagrams and among the real (between in-cone and out-cone terms)





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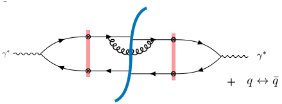
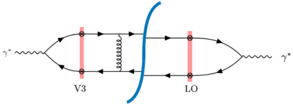
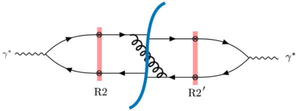


- For the cross color structure, cancel between real and virtual:





Cancellation of rapidity divergences

- Rapidity divergence is regularized with a **longitudinal momentum cut-off** Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.

- We have found:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_q \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}\eta_{\bar{q}}}\right|_{\mathrm{slow}} &= \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}})\ln\left(\frac{z_f}{z_0}\right)\frac{\alpha_s N_c}{4\pi^2}\int \mathrm{d}\Pi_{\mathrm{LO}}\mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \\
\times\Bigg\langle\int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ & \frac{\boldsymbol{r}_{xy}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}(2D_{xy}-2D_{xz}D_{zy}+D_{zy}Q_{y'x',xz}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xy}D_{y'x'}) \\
&+\frac{\boldsymbol{r}_{x'y'}^2}{\boldsymbol{r}_{zx'}^2\boldsymbol{r}_{zy'}^2}(2D_{y'x'}-2D_{y'z}D_{zx'}+D_{zx'}Q_{xy,y'z}+D_{y'z}Q_{xy,zx'}-Q_{xy,y'x'}-D_{xy}D_{y'x'}) \\
&+\frac{\boldsymbol{r}_{xx'}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zx'}^2}(D_{zx'}Q_{xy,y'z}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xx'}D_{y'y}) \\
&+\frac{\boldsymbol{r}_{yy'}^2}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{zy'}^2}(D_{y'z}Q_{xy,zx'}+D_{zy}Q_{y'x',xz}-Q_{xy,y'x'}-D_{xx'}D_{y'y}) \\
&+\frac{\boldsymbol{r}_{xy'}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy'}^2}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{zx'}Q_{xy,y'z}-D_{zy}Q_{y'x',xz}) \\
&+\frac{\boldsymbol{r}_{x'y}^2}{\boldsymbol{r}_{zx'}^2\boldsymbol{r}_{zy}^2}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{y'z}Q_{xy,zx'}-D_{xz}Q_{y'x',zy})\Bigg\}\Bigg\rangle_Y . \qquad (6.40)
\end{aligned}
$$

Cancellation of rapidity divergences

- Rapidity divergence is regularized with a **longitudinal momentum cut-off** Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.

- We have proven:

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star\to q\bar{q}+X} = \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \underbrace{\mathcal{K}_{\mathrm{LL}} \otimes \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^\star\to q\bar{q}+X}}_{\text{action of LL JIMWLK on the LO x-section}} + \overbrace{\text{finite}}^{\Lambda^-\to 0}$$

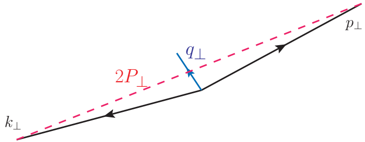
- Thus, the Λ^- dependence of the NLO impact factor is canceled by the JIMWLK evolution of the LO cross-section from Λ^- to k_f^-.

The back-to-back limit

Back-to-back limit

The back-to-back limit

- Def: $|\boldsymbol{P}_\perp| = |z_{\bar{q}}\boldsymbol{k}_\perp - z_q \boldsymbol{p}_\perp| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_\perp + \boldsymbol{p}_\perp|$



- LO: TMD factorization Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)} \underbrace{G_{\mathrm{WW}}(\boldsymbol{b}_\perp,\boldsymbol{b}'_\perp)}_{\searrow} + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right)$$

$$\left\langle \frac{1}{N_c}\mathrm{Tr}\left[\partial_i V^\dagger(\boldsymbol{b}_\perp)V(\boldsymbol{b}'_\perp)\partial_j V^\dagger(\boldsymbol{b}'_\perp)V(\boldsymbol{b}_\perp)\right]\right\rangle_Y$$

- NLO: large Sudakov logarithms vs small-x logarithm.

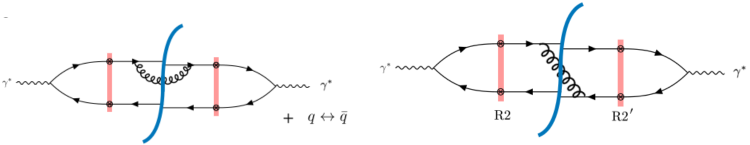
$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{NLO}} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}$$

$$\times\left[1 - \frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \ln\left(\frac{1}{x_{\mathrm{Bj}}}\right)\mathcal{K}_{\mathrm{LL}}\otimes\right] G_{\mathrm{WW}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

Conjectured in Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA.

Sudakov logarithms in our computation

- Real diagrams with soft divergences.



- However: the integration over the soft gluon gives the Sudakov with a positive sign!

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)}$$
$$\times \left[1 + \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \mathcal{K}_{\mathrm{LL}} \otimes\right] G_{\mathrm{WW}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

- Problem: overlapping phase space between soft gluons and slow gluons included in $\mathcal{K}_{\mathrm{LL}}$.

Solution: collinearly improved JIMWLK evolution

- Kinematic improvement: impose both k_g^- and k_g^+ ordering (lifetime ordering).

  $\Longrightarrow$ Resum large transverse double logarithms to all orders.

  $\Longrightarrow$ Solve the instability of NLO B-JIMWLK evolution.

  Beuf, 1401.0313, Taels, Altinoluk, Beuf, Marquet, 2204.11650

- In practice, add an additional constraint in the LL evolution kernel

$$k_g^+ \geq k_f^+ \Longrightarrow k_g^- \leq \frac{\boldsymbol{k}_{g\perp}^2}{Q_f^2} k_f^-$$

  with $Q_f^2 \sim Q^2 \sim \boldsymbol{P}_\perp^2$.

- With this modification $\mathcal{K}_{\rm LL} \to \mathcal{K}_{\rm LL,coll}$, one recovers the expected double logarithm.

$$\mathrm{d}\sigma_{\rm NLO}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)}$$
$$\times \left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \mathcal{K}_{\rm LL,coll} \otimes\right] G_{\rm WW}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

Sudakov resummation at single log accuracy

- Exponentiation of the Sudakov logarithms $G_{\mathrm{WW}}(\boldsymbol{r}_{bb'}) \to G_{\mathrm{WW}}(\boldsymbol{r}_{bb'})\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$

$$\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) = \exp\left(-\int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\boldsymbol{P}_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2}\frac{\alpha_s(\mu^2)N_c}{\pi}\left[\frac{1}{2}\ln\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) + \frac{C_F}{N_c}s_0 - s_f\right]\right)$$

- Double and **single** Sudakov logarithms with **exact** N_c dependence:
- Dijet geometry single log s_0

$$s_0 = \ln\left(\frac{2(1+\cosh(\Delta Y_{12}))}{R^2}\right) + \mathcal{O}(R^2)$$

See also Hatta, Xiao, Yuan, Zhou, 2106.05307

- Single log from the interplay between small-x and Sudakov resummation:

$$s_f = \ln\left(\frac{\boldsymbol{P}_\perp^2 x_{\mathrm{Bj}}}{z_1 z_2 Q^2 c_0^2 x_f}\right)$$

$\Rightarrow$ Dependence on the rapidity factorization scale x_f!

Finite terms in α_s in the back-to-back limit

Azimuthal anisotropies from soft gluon radiations

- We can also access pure α_s (and non power suppressed) corrections.
- The dominant one are coming from soft gluon radiations. Hatta, Xiao, Yuan, Zhou, 2010.10774
- Azimuthally averaged x-section sensitive to the linearly polarized gluon TMD at NLO!

$$\langle \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$$
$$\times \frac{\alpha_s}{\pi} \left\{ \frac{N_c}{2} + C_F \ln(R^2) - \frac{1}{2N_c} \ln(z_1 z_2) \right\}$$

- The $\cos(2\phi)$ anisotropy is also sensitive to the unpolarized gluon TMD.

$$\langle \cos(2\phi) \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \cos(2\theta) \hat{G}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$$
$$\times \frac{\alpha_s}{\pi} \left\{ N_c + 2C_F \ln(R^2) - \frac{1}{N_c} \ln(z_1 z_2) \right\}$$

See also Yang-Ting's talk today in $pp \to Z + \text{jet}$.

TMD factorization at NLO at small-x

- Can we put the full result in a factorized TMD form?
- At NLO, non-trivial color correlators, e.g.

$$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'} D_{xz} - D_{xz} D_{zy}\rangle_Y$$

which does not reduce to the WW gluon TMD, unless $Q_s^2 \ll \boldsymbol{k}_{g\perp}^2$.

- For the numerics, one may first focus on contributions which are naturally proportional to S_{WW} including
  - Sudakov double and single logs,
  - $\mathcal{O}(\alpha_s)$ finite term.
- Requires numerical solution of collinearly improved JIMWLK!
Hatta, Iancu, 1606.03269, Korcyl (talk at DIS 2022)

Summary and outlook

- **Proof of UV and IR finiteness** of the dijet cross-section in the CGC.

- **Proof of JIMWLK factorization** of the rapidity divergence for a process with non-trivial final state.

- We have obtained a **numerically tractable NLO impact factor** $\Rightarrow$ reach $\alpha_s^3 \ln(1/x)$ accuracy when combined with extant results for NLO BK-JIMWLK.

- Back-to-back limit: Sudakov double and single log at exact N_c, and impact factor.

- Necessity to use a collinearly improved small-x evolution to find the correct Sudakov double log.

- Towards a numerical evaluation of the impact factor with saturation corrections: **very challenging!** (9 to 11-dimensional integrals + Fourier phases...)