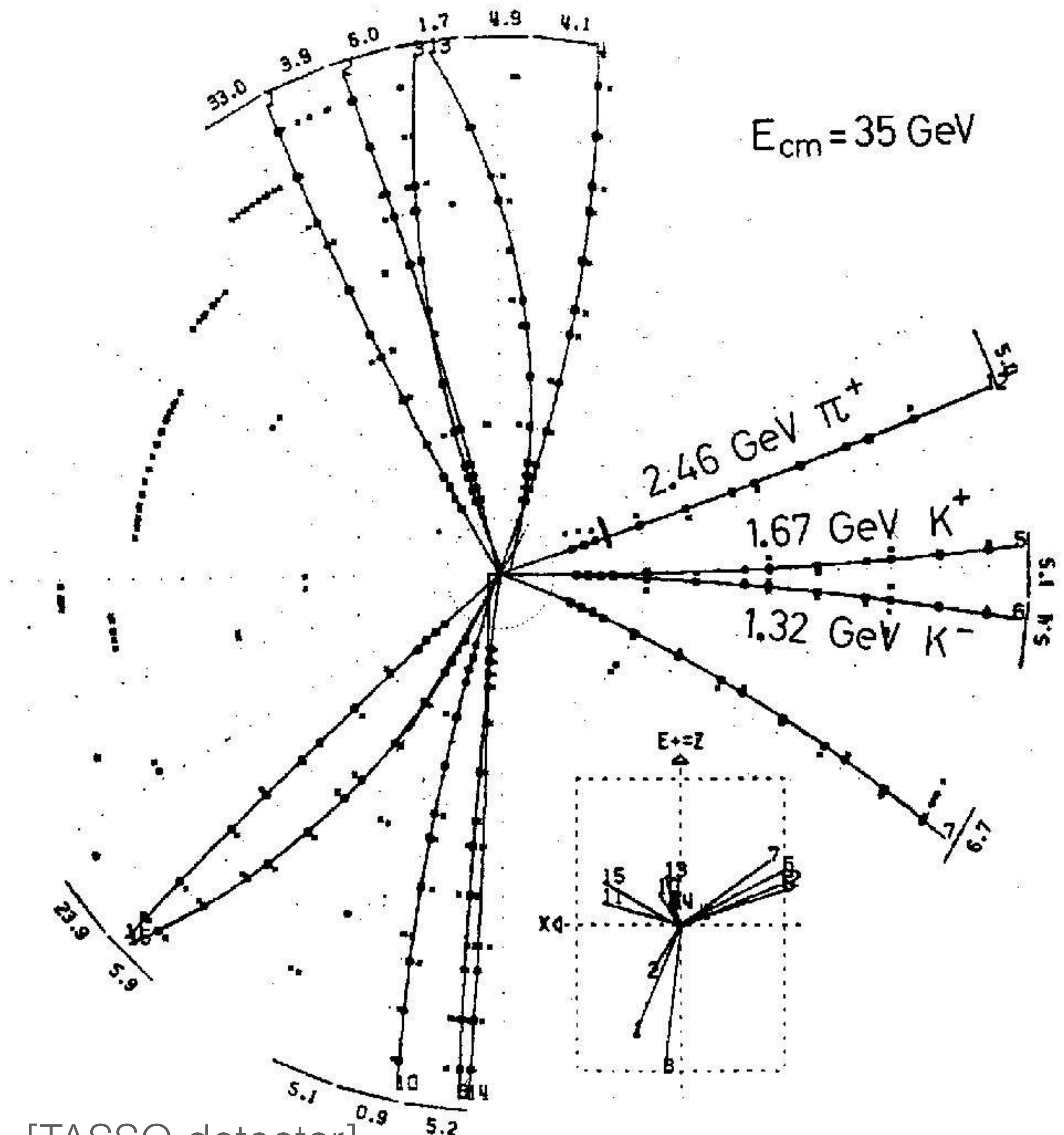
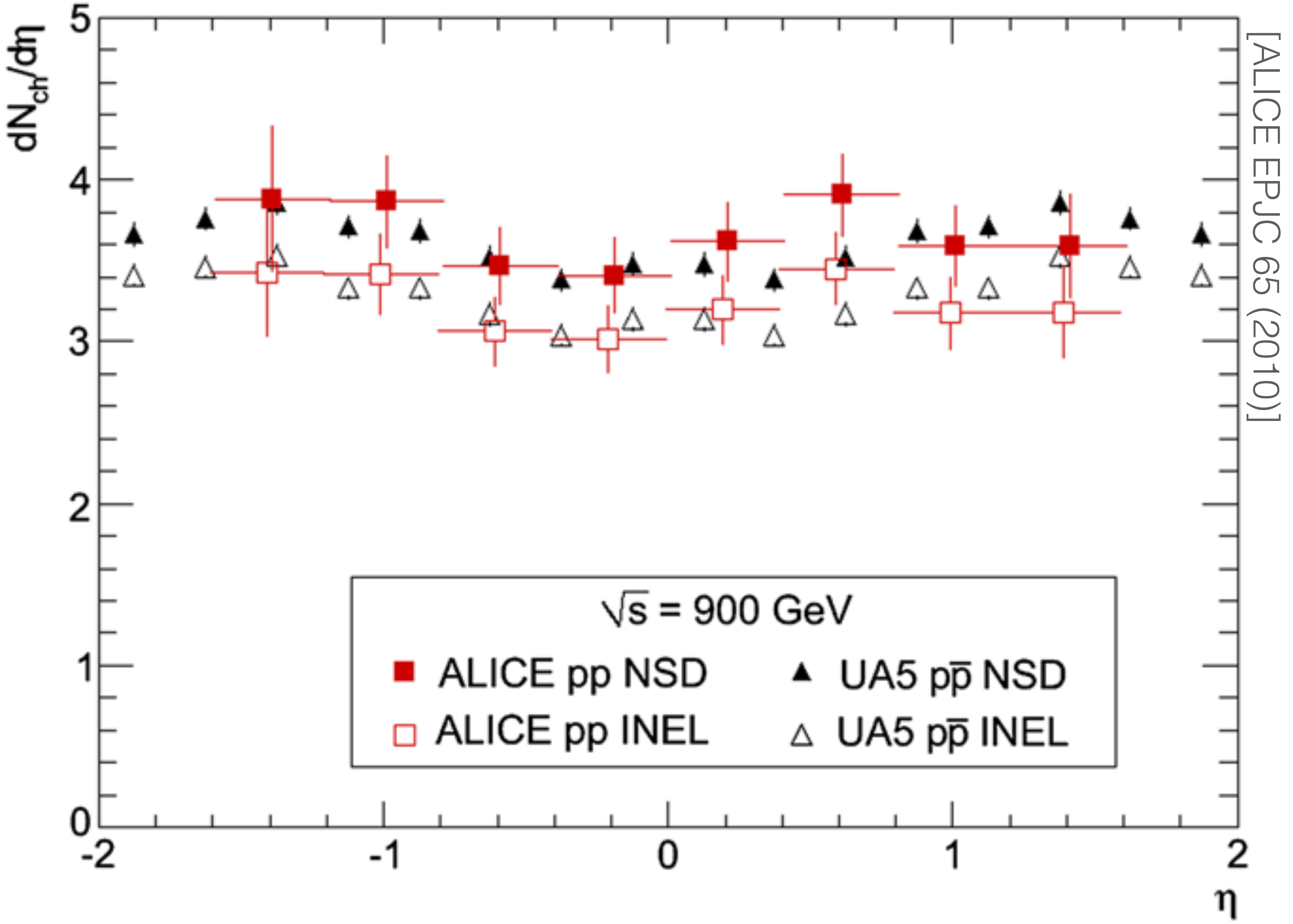
Lund multiplicity for precision physics



[TASSO detector]



[ALICE EPJC 65 (2010)]

Alba Soto-Ontoso

Jet Physics: from RHIC/LHC to EIC

Stony Brook, 30th June, 2022

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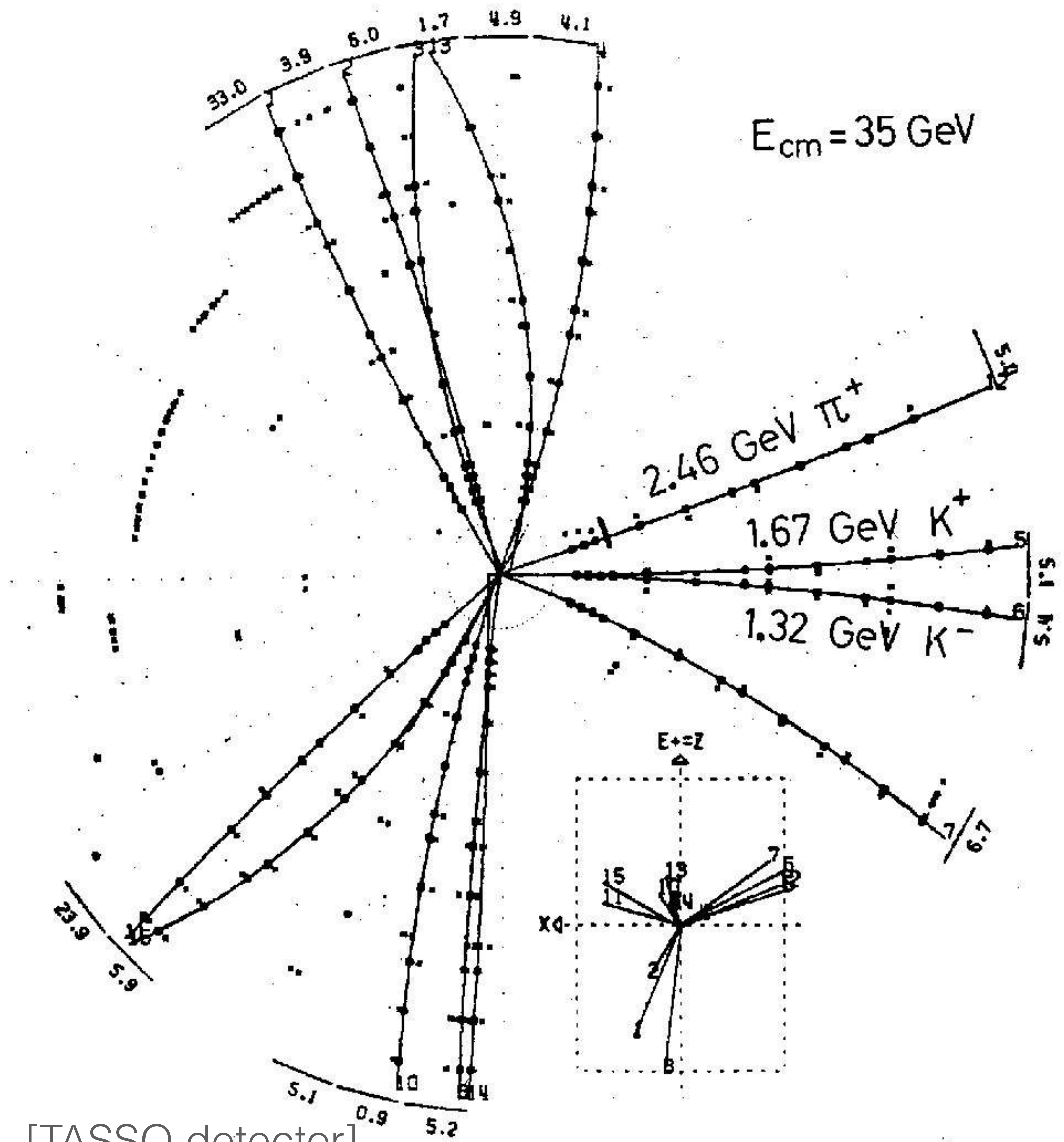

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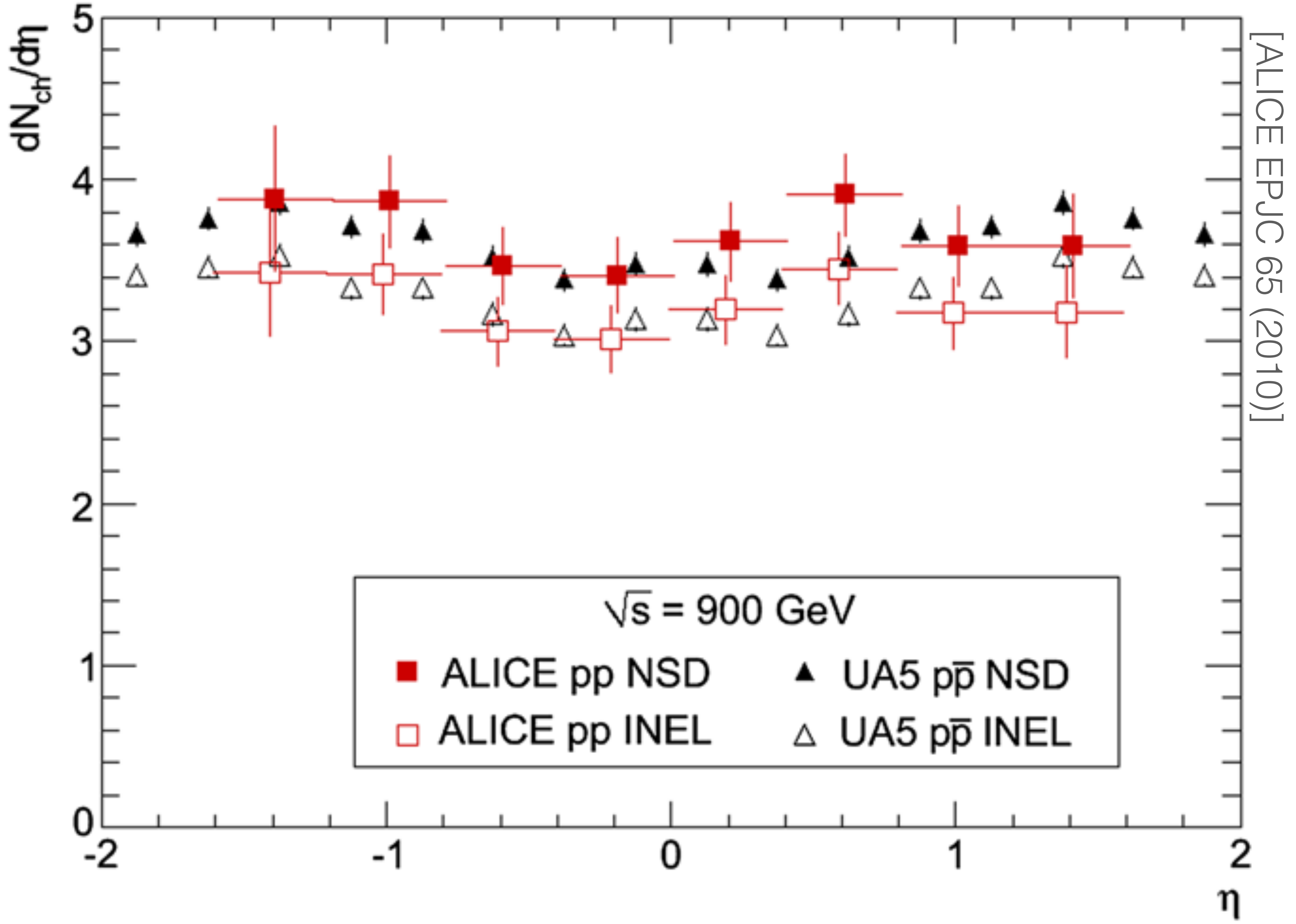

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Jet Physics: from LEP to RHIC/LHC to EIC

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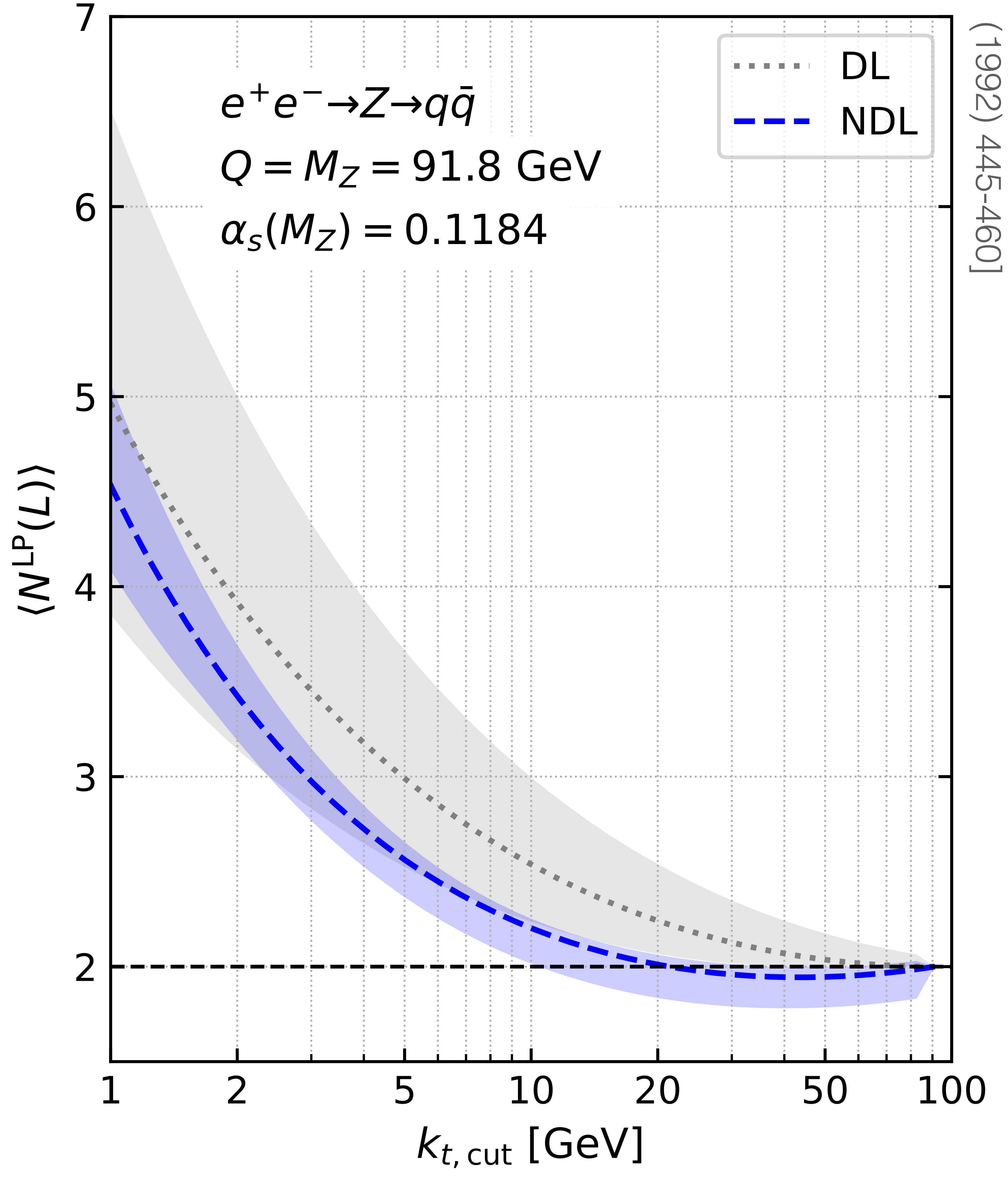
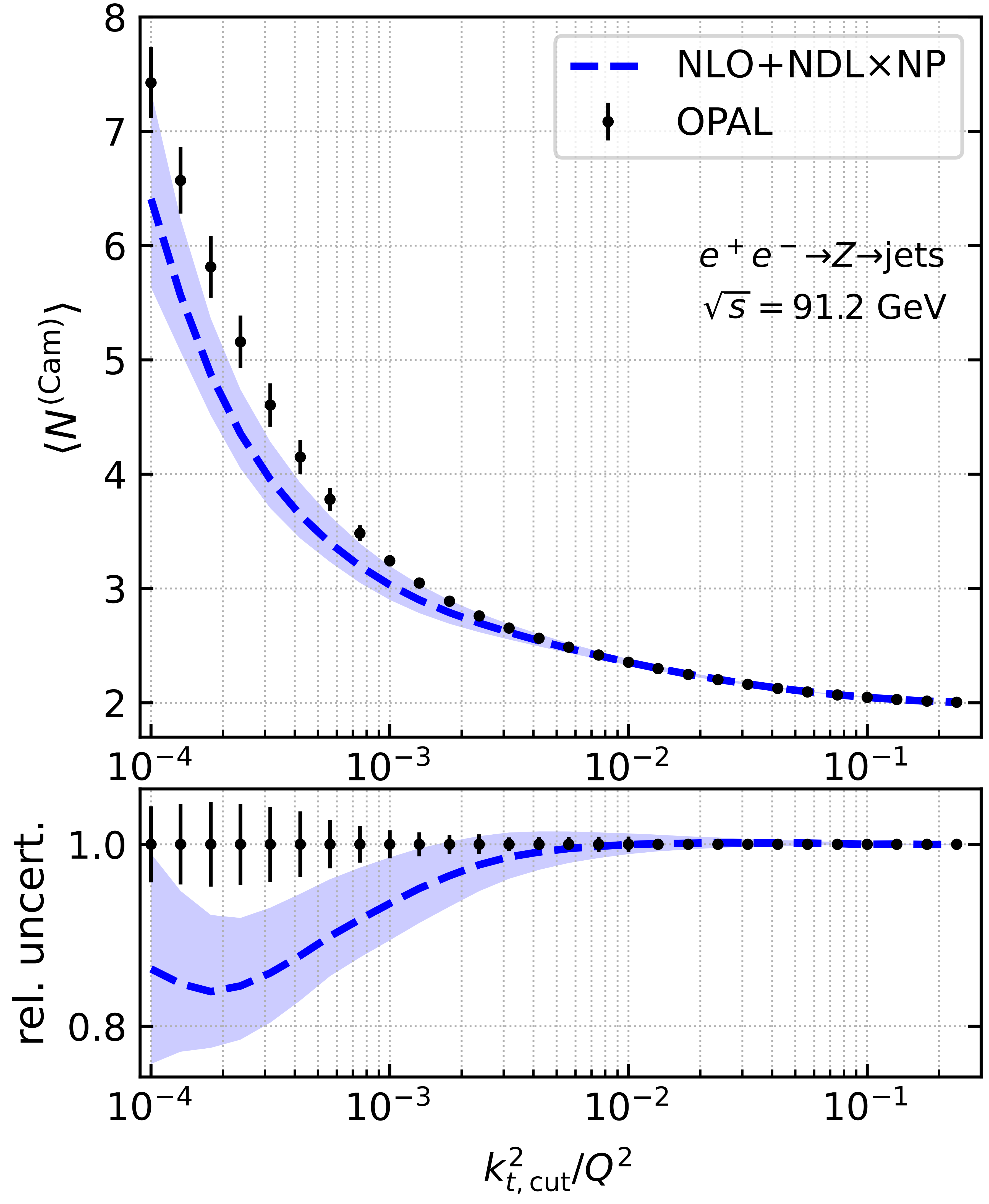
Main results of this talk

1992

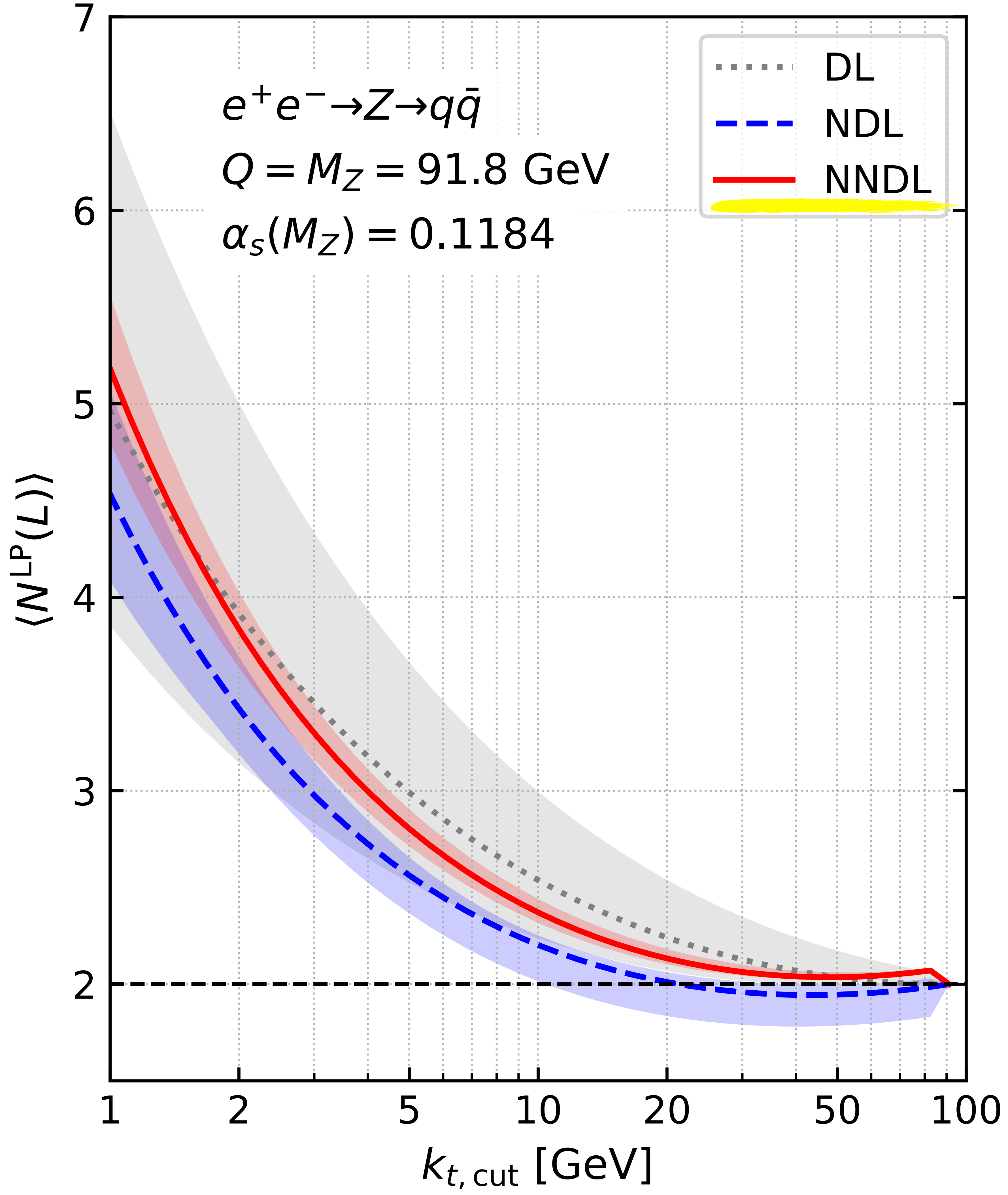


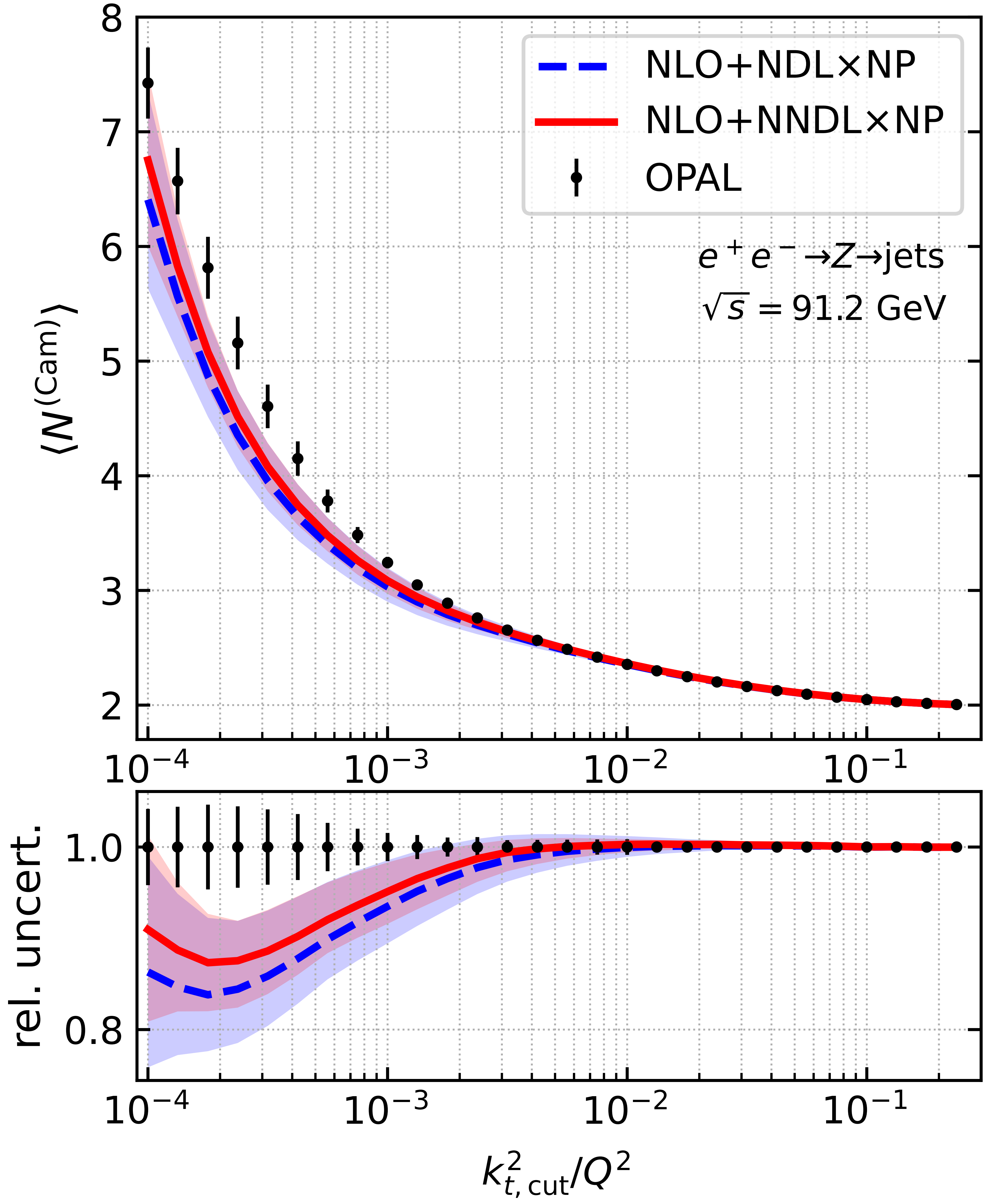
[Catani, Dokshitzer, Fiorani, Webber NPB 377 (1992) 445-460]



Main results of this talk [Medves, ASO, Soyez arXiv:2205.02861]

2022





Historic evolution of multiplicity definition

80's

Hadronic multiplicity within a jet == first moment of the fragmentation function

[Ciafaloni, Dokshitzer, Marchesini, Mueller, Petronzio, Pokorski, Webber…]

✓ Fundamental to understand the singularity structure of QCD

✗ Infrared and collinear unsafe (need to introduce ~~IR cutoff~~)

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90-95

Average number of reconstructed (sub)jets with k_t-like algorithms

[Catani, Dokshitzer, Fiorani, Webber…]

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95-00 **Average number of reconstructed (sub)jets with C/A-like algorithms**

[Catani, Dokshitzer, Forshaw, Seymour, Webber…]

✓ Reduced sensitivity to non-perturbative dynamics

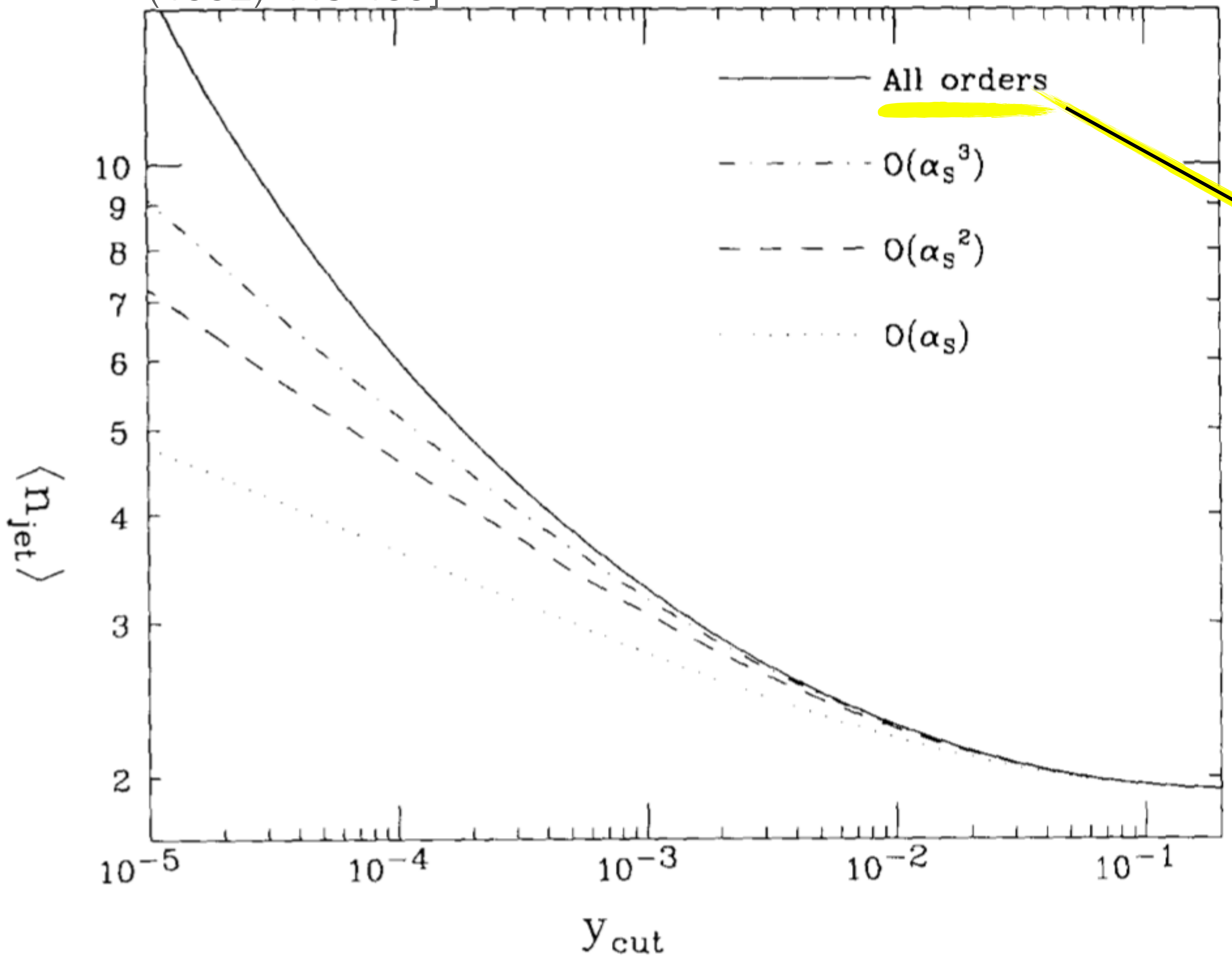
✗ *Only* $\alpha_s^n L^{2n-1}$ terms have been calculated analytically

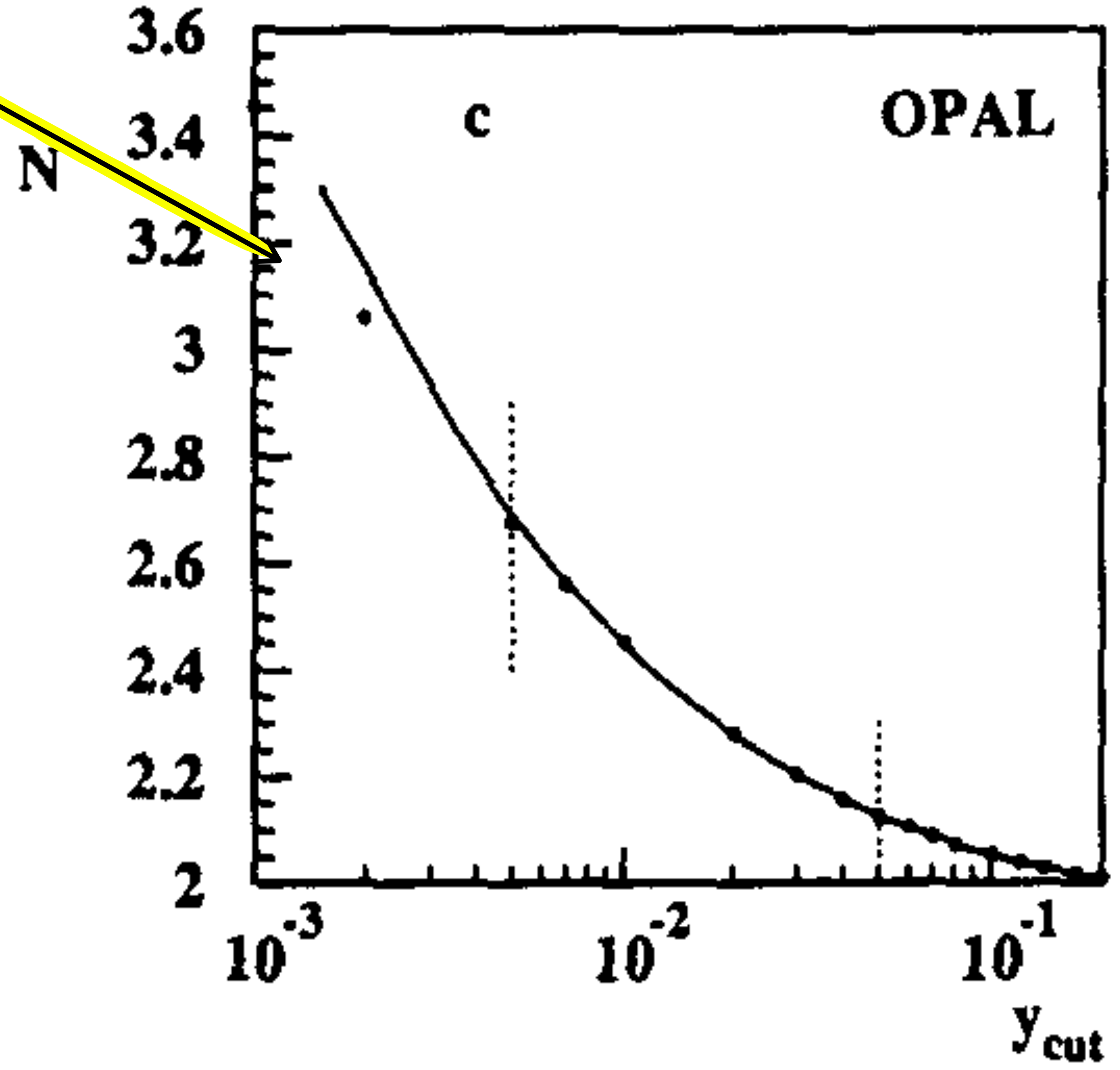
$L = \ln(k_{t,\mathrm{cut}}/Q)$

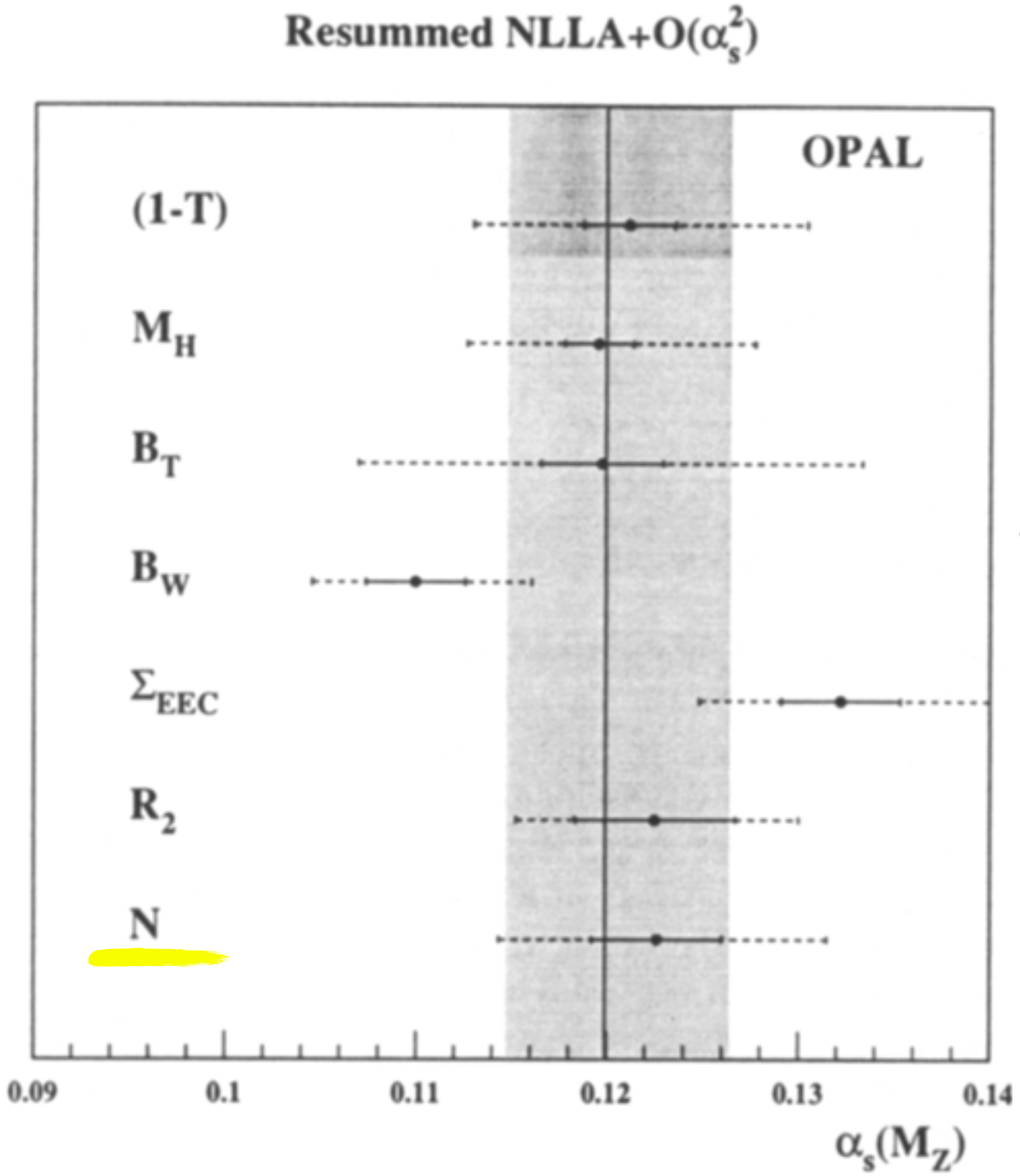
Motivation #1

No theory progress on this observable since 1992. Renewed interest in archived e^+e^- data

[Catani, Dokshitzer, Fiorani, Webber NPB 377 (1992) 445-460]



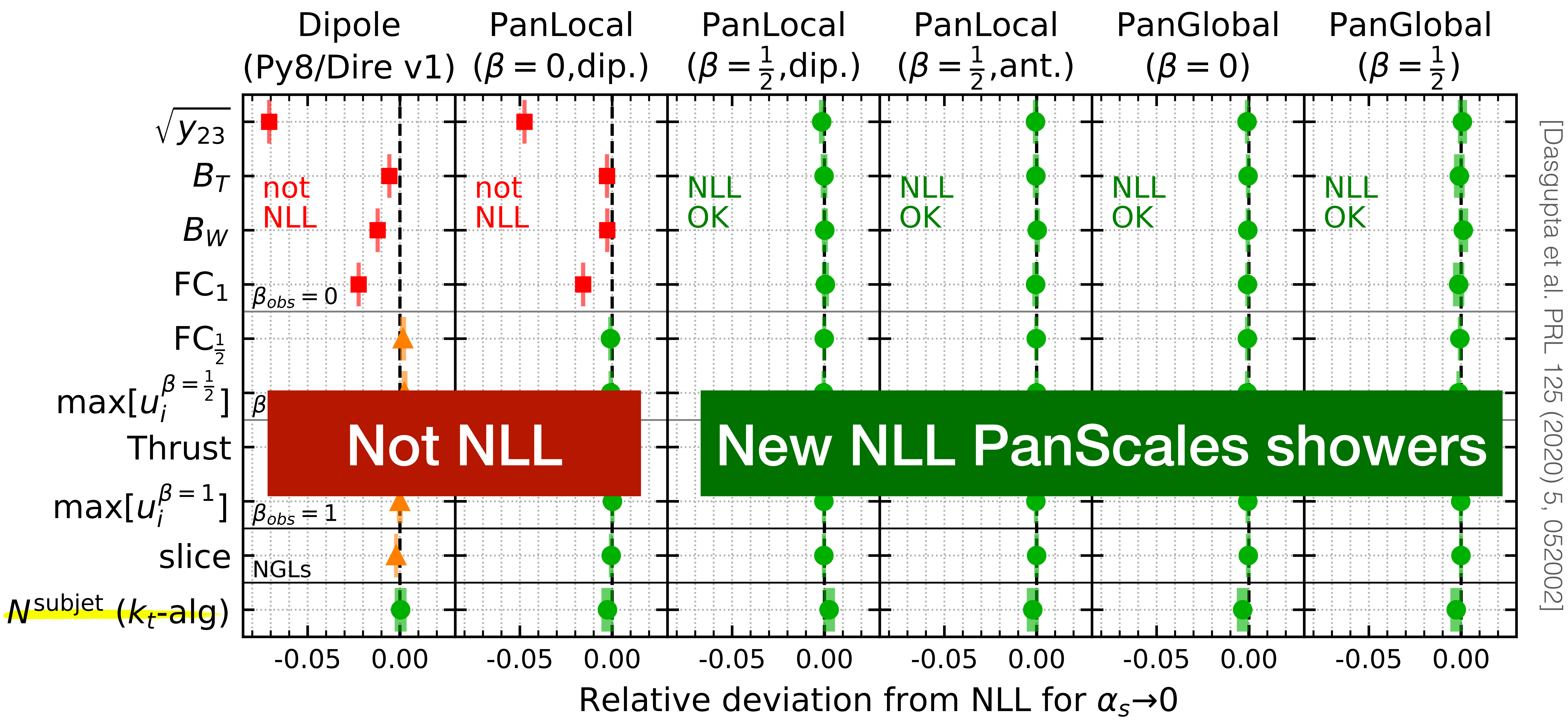




[OPAL Collab Z.Phys.C 59 (1993) 1-20]

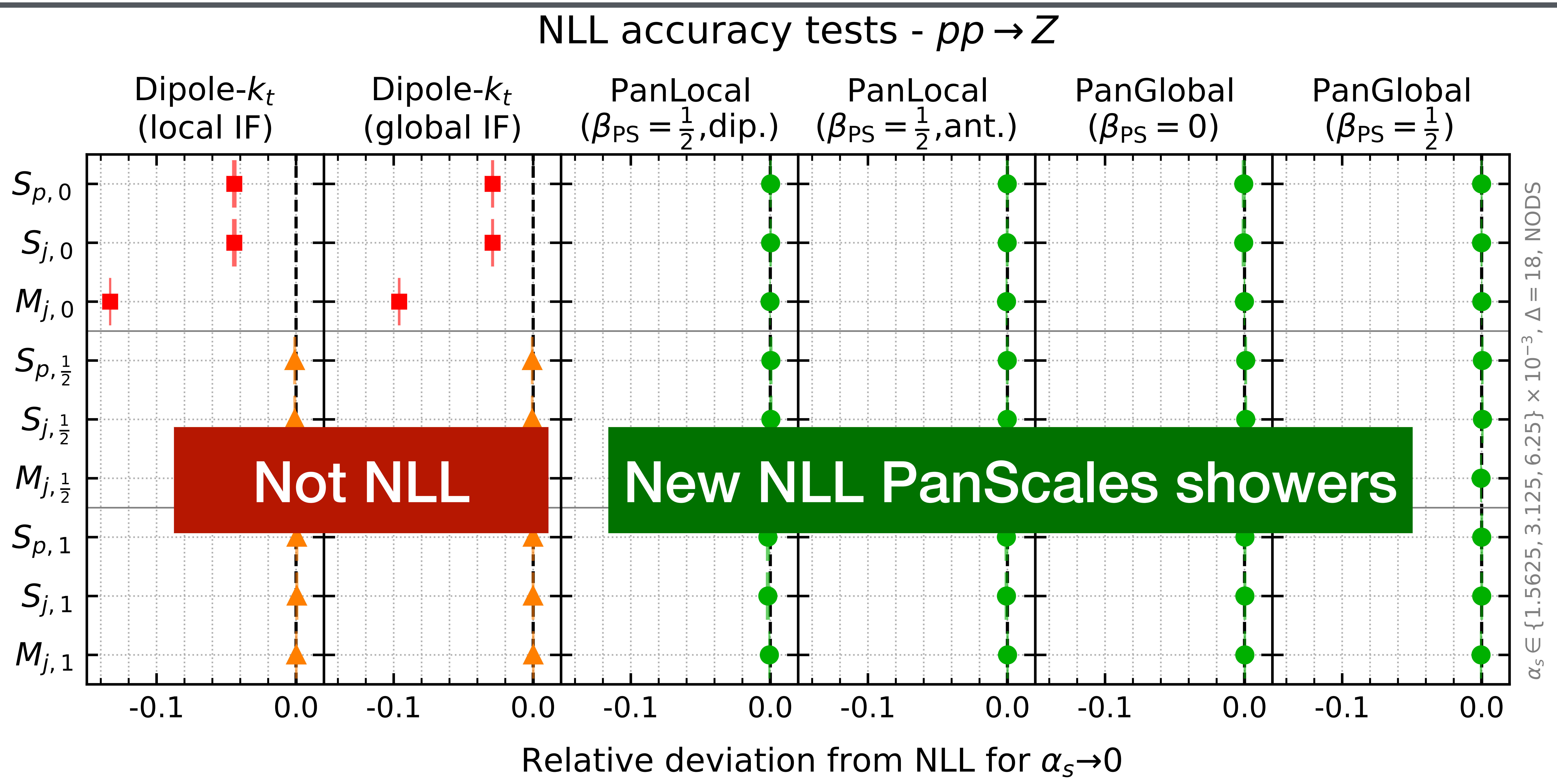
Can we improve the precision on the $\alpha_s(M_Z)$ extraction?

Motivation #2



Future generation of parton showers target NNLL accuracy.
Analytic calculations are fundamental for testing

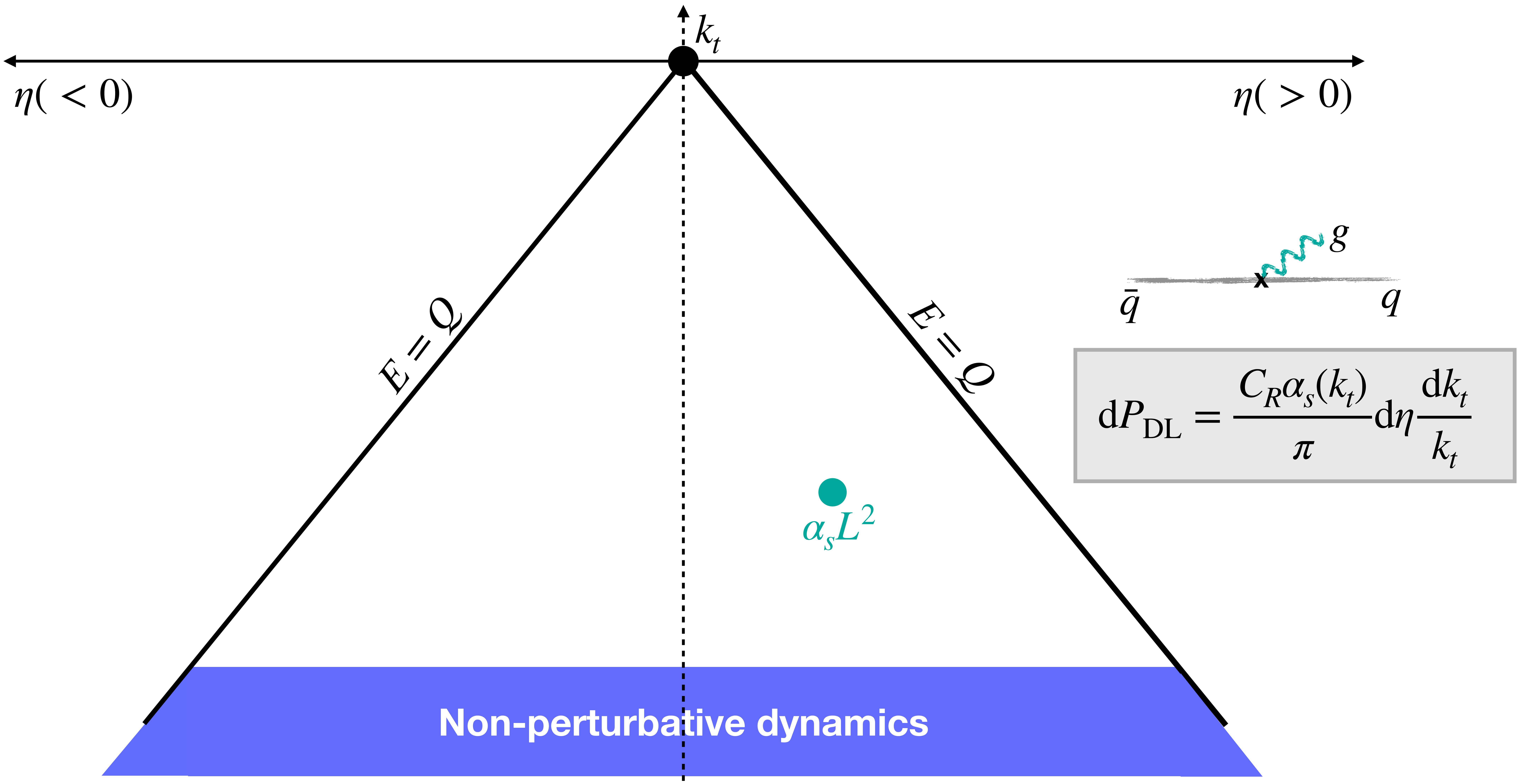
Motivation #3



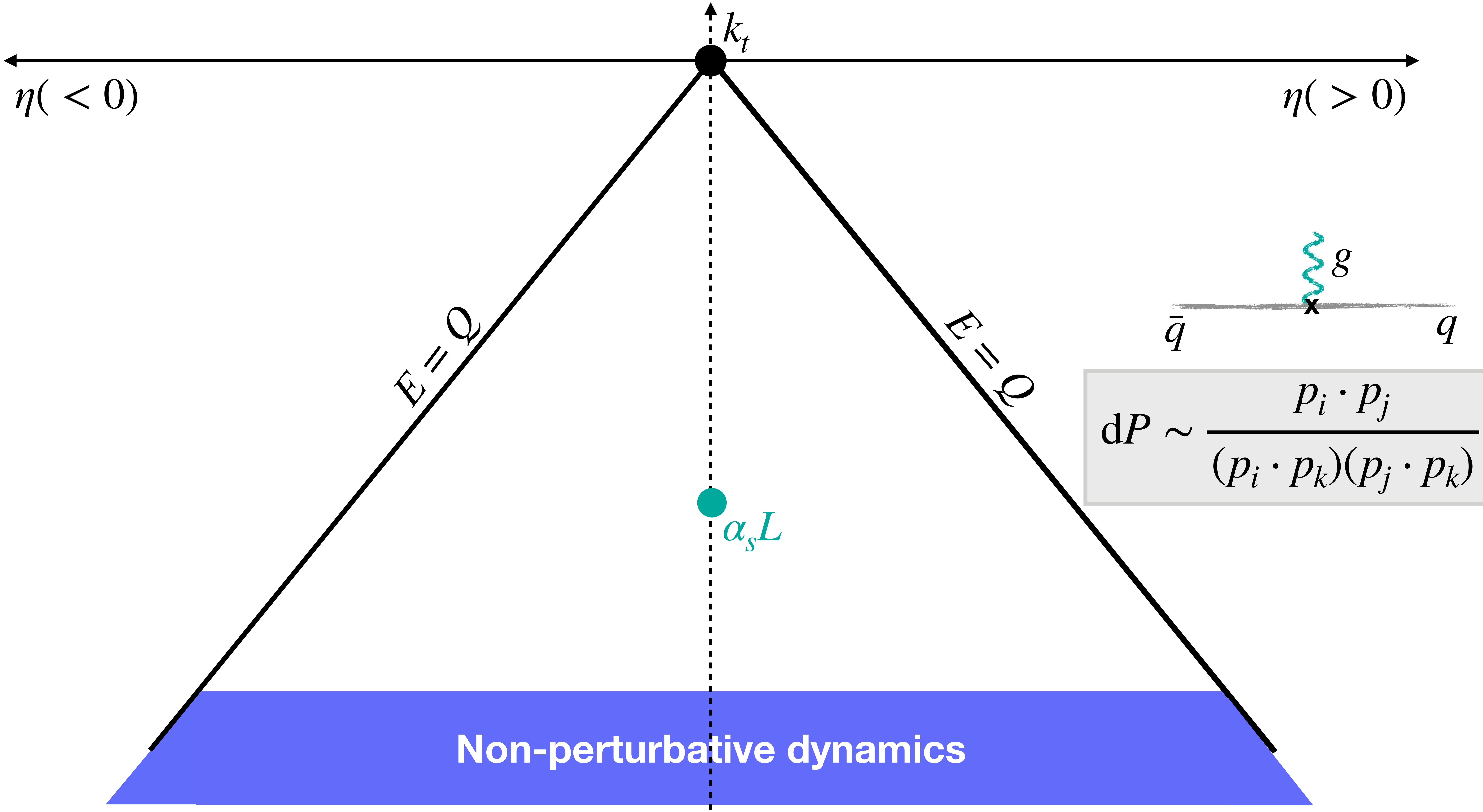
[van Beekveld et al. 2205.02237 and to appear soon]

No theoretical calculation of jet multiplicity in hadronic collisions

An interlude: the Lund jet plane

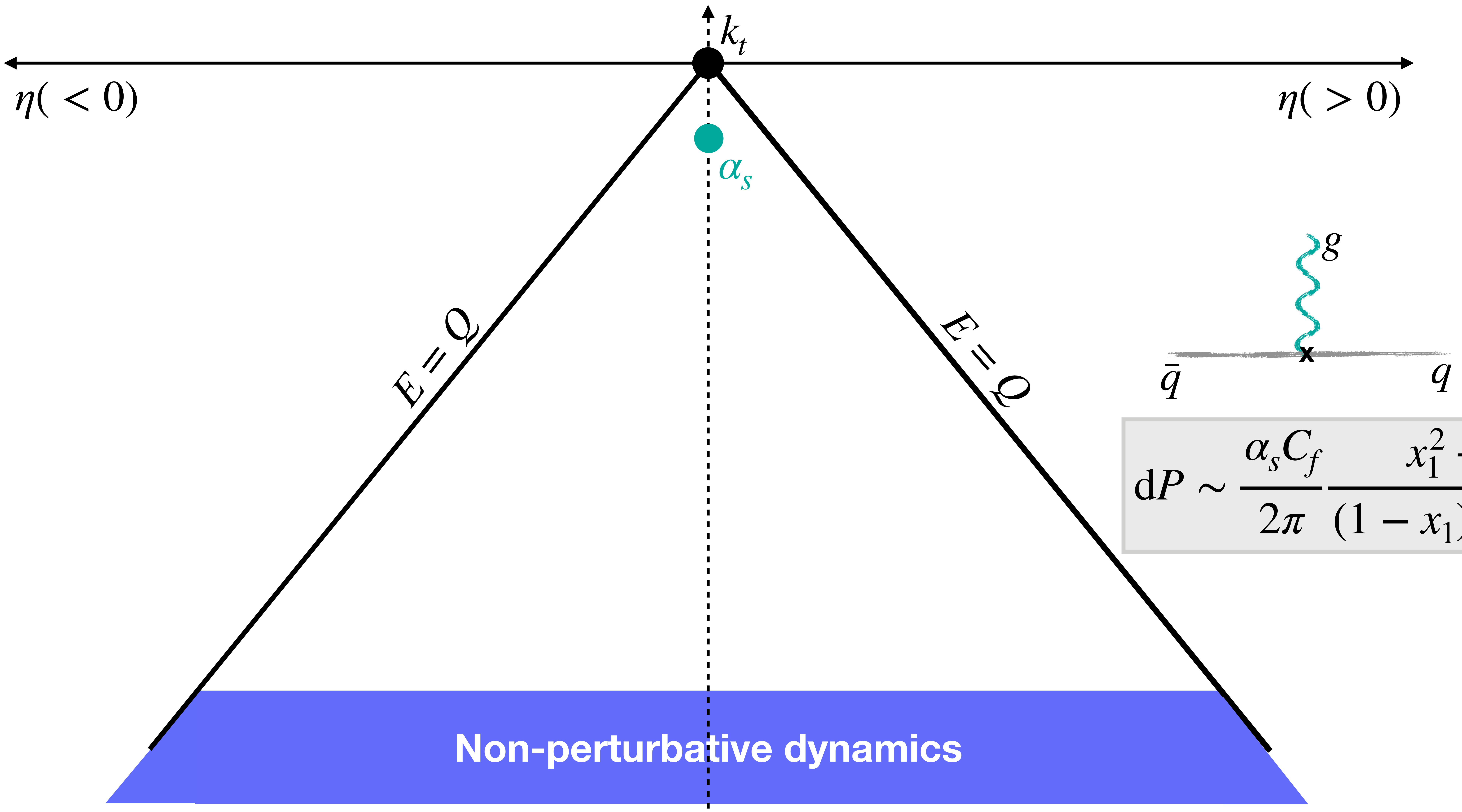


An interlude: the Lund jet plane



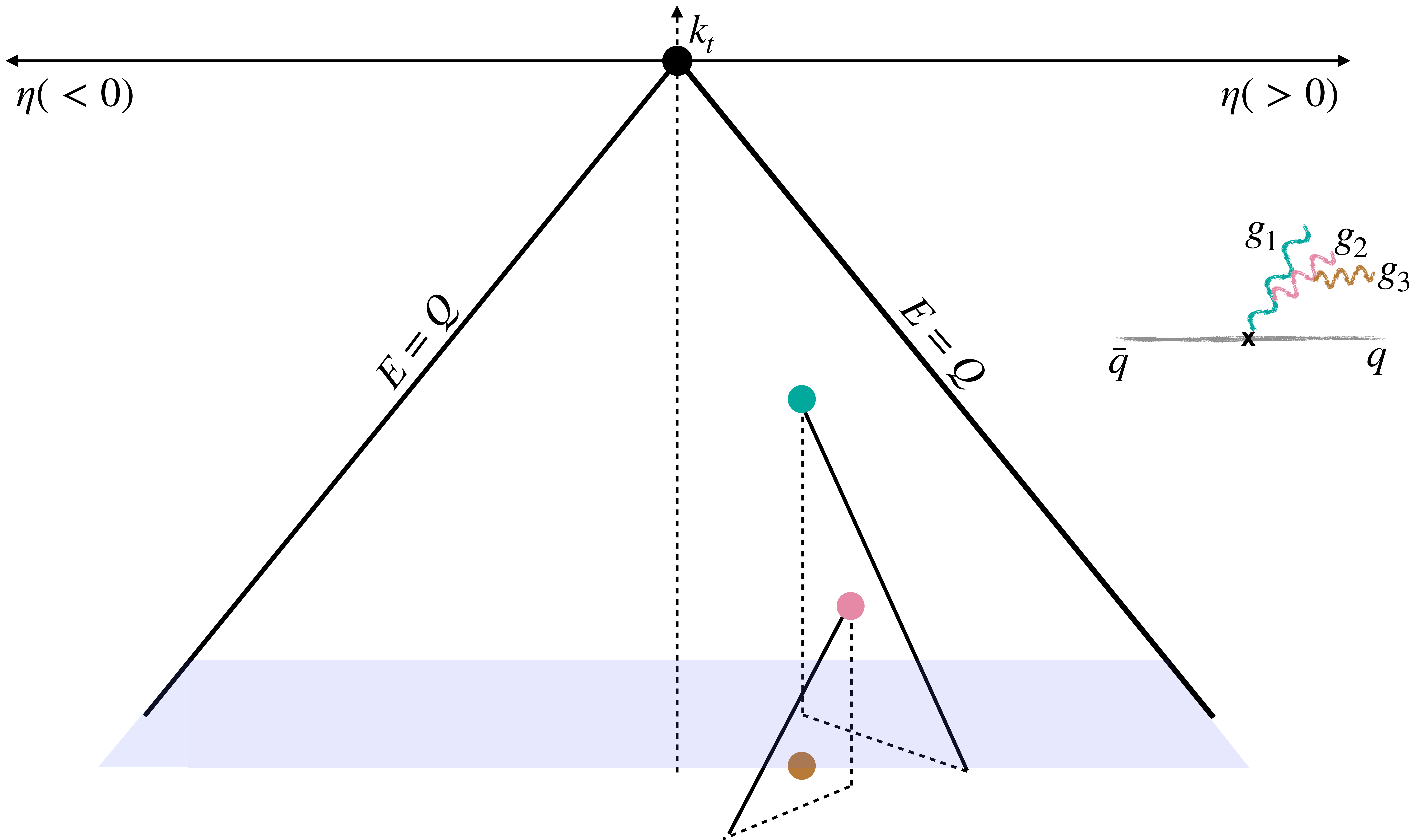
$$dP \sim \frac{p_i \cdot p_j}{(p_i \cdot p_k)(p_j \cdot p_k)} \frac{d\phi}{2\pi} d\cos\theta$$

An interlude: the Lund jet plane



$$\mathrm{d}P \sim \frac{\alpha_s C_f}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

An interlude: the Lund jet plane



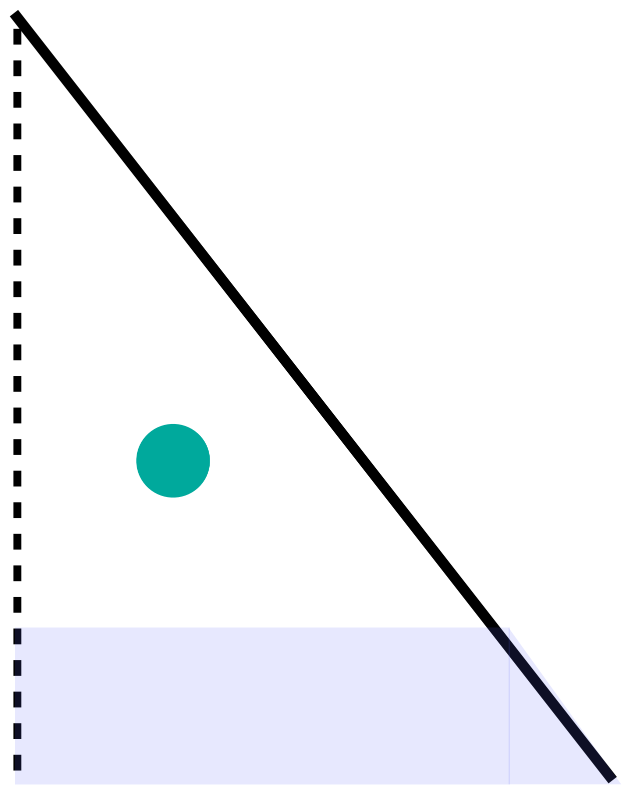
A second interlude: logarithmic counting of the multiplicity

The presence of 2 disparate scales induces large $L = \ln(Q/k_{t,\mathrm{cut}})$ that must be resummed

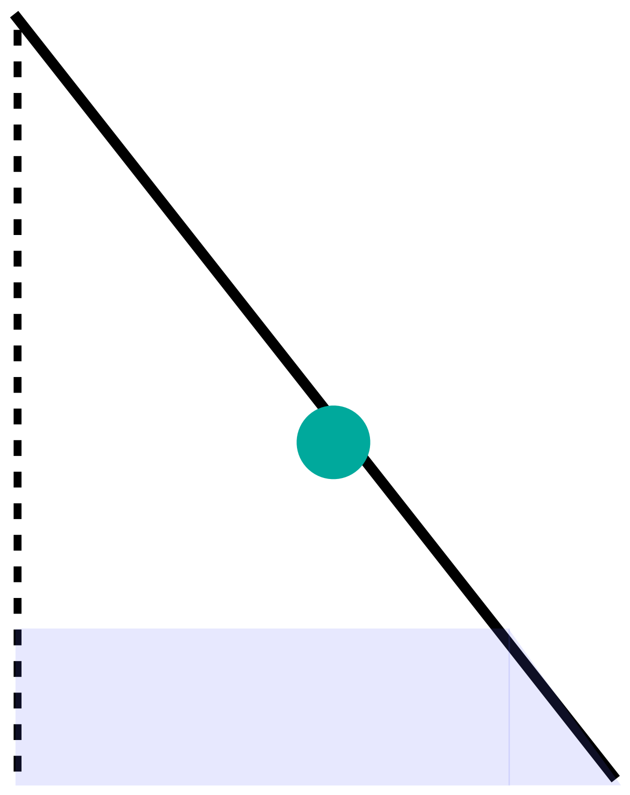
$$N(\alpha_s, L) = N(\alpha_s, 0)\left[\underbrace{h_1(\alpha_s L^2)}_{\mathrm{DL}} + \underbrace{\sqrt{\alpha_s}\, h_2(\alpha_s L^2)}_{\mathrm{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\mathrm{NNDL}} + \ldots\right] + \mathcal{O}(e^{-|L|})$$

where $\mathrm{N}^k\mathrm{DL}$ accuracy implies control over $\alpha_s^n L^{2n-k}$ terms

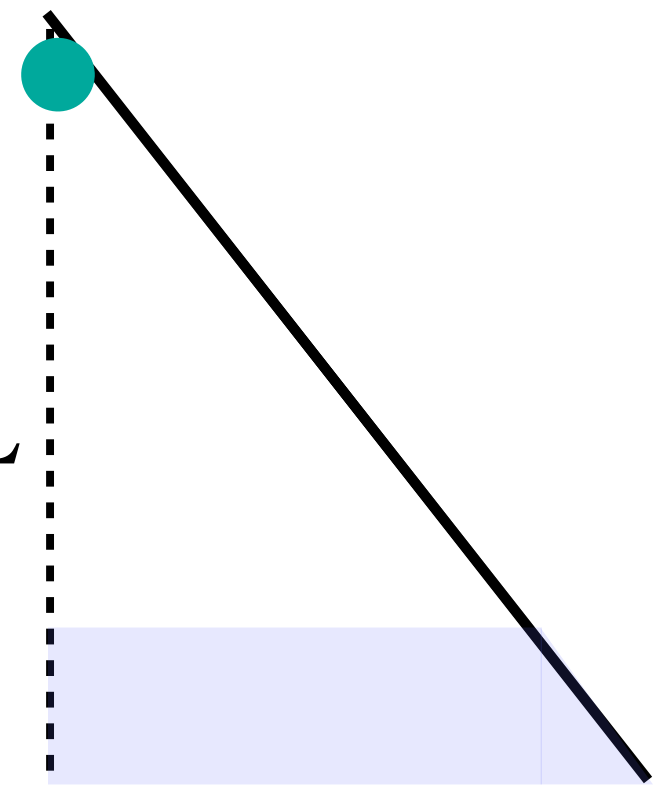
e.g. DL



NDL



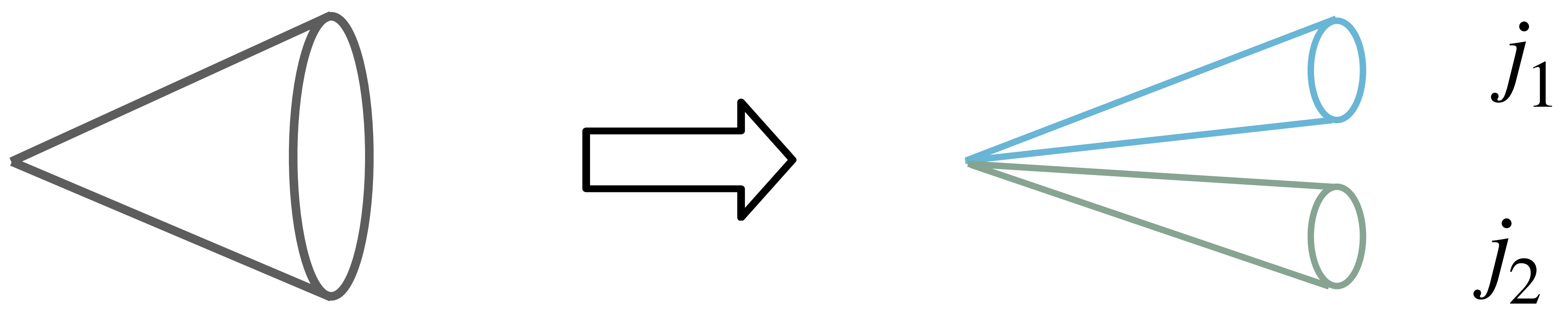
NNDL



Lund-based multiplicity

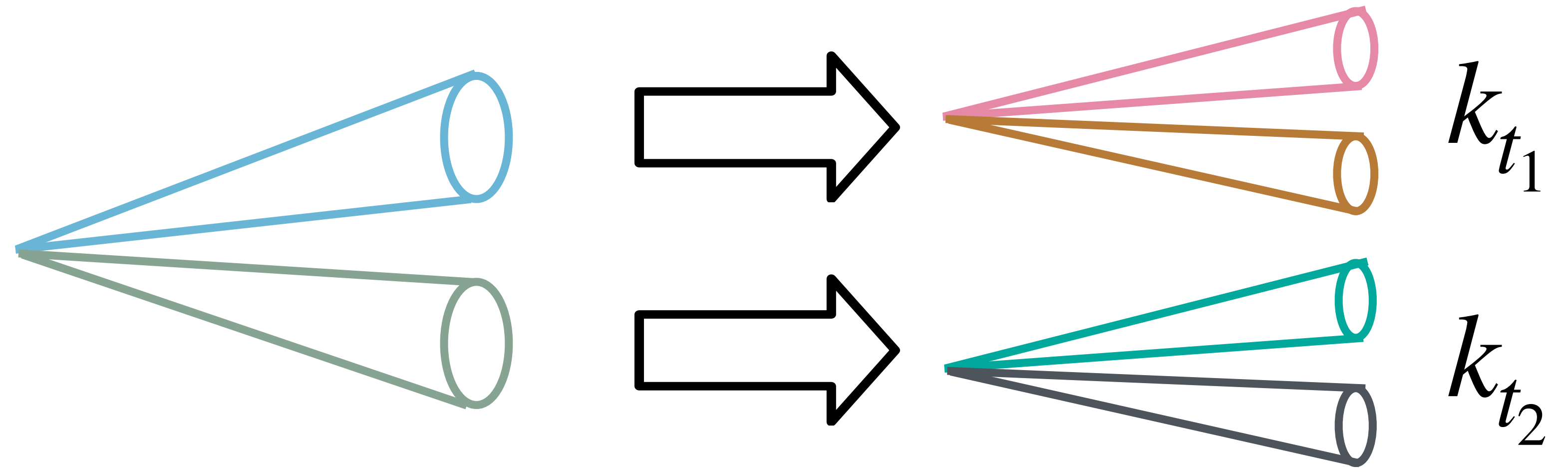
1 Cluster the event with Cambridge algorithm, i.e., using the metric $d_{ij} = 2(1 - \cos\theta_{ij})$

2 Traverse backwards the angular ordered sequence

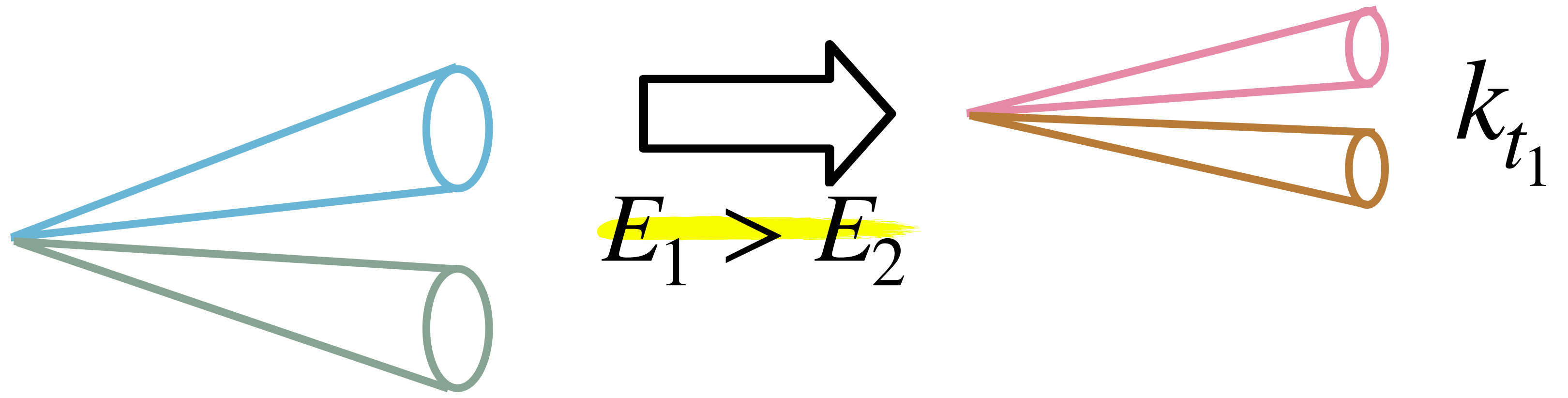


$$\boxed{k_t = \min(E_1, E_2)\ \sin\theta_{12}}$$

a If $k_t > k_{t,\text{cut}} \Longrightarrow N^{\text{LP}} = N^{\text{LP}} + 1$ and



b If $k_t < k_{t,\text{cut}} \Longrightarrow$



Relation to other multiplicities

1 Cambridge multiplicity [Dokshitzer, Leder, Moretti, Webber JHEP 08 (1997)]

✓ Counting the total number of clusterings for which $k_t > k_{t,\mathrm{cut}}$ is equivalent at NNDL to Lund multiplicity if $k_t = k_t^{\mathrm{Lund}}$

✓ If $k_t = k_t^{\mathrm{Cam}} = \min(E_1, E_2)\sqrt{2(1-\cos\theta_{12})}$, trivial extension of the calculation

Two for the price of one

Relation to other multiplicities

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Two for the price of one

2 **Primary Lund density:** corresponds to only counting on the primary branch

- ✗ More complicated resummation structure

3 **k_t-multiplicities:** choice of jet algorithm only matters from NNDL onwards.

- ✗ More complicated resummation structure

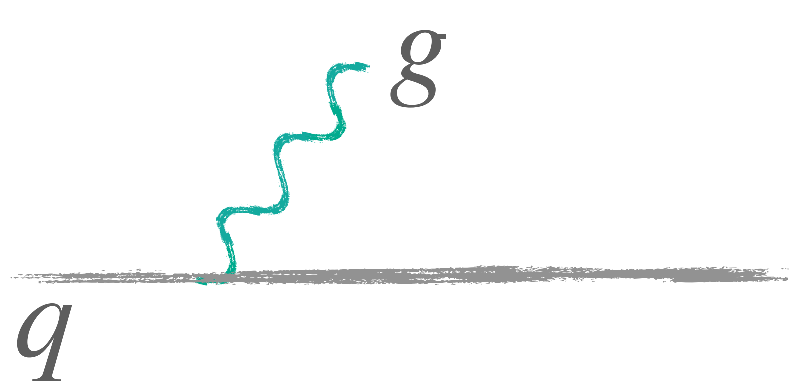
Lund multiplicity at fixed-order (and DL)

Lets focus on the Lund multiplicity of a single jet either quark or gluon initiated in e^+e^-

$\mathcal{O}(\alpha_s^0)$:

$\langle N \rangle = 1$

$\mathcal{O}(\alpha_s)$:



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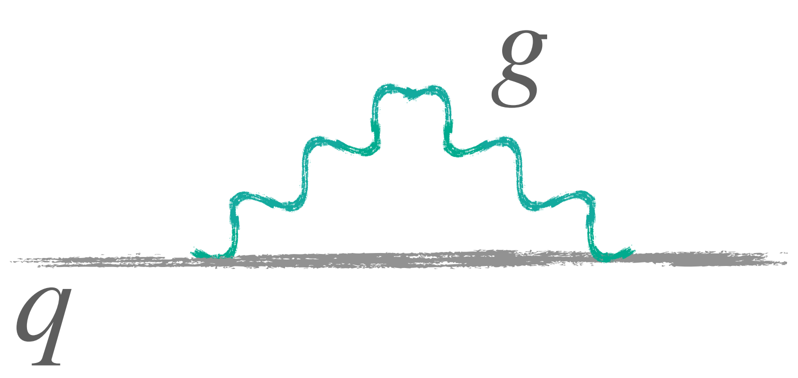

$$\langle N \rangle = \int \mathrm{d}\Phi \left| \mathcal{M}_R \right|^2 \left[1 + \Theta(k_t > k_{t,\mathrm{cut}}) \right] + \int \mathrm{d}\Phi \left| \mathcal{M}_V \right|^2 \left[1 \right]$$

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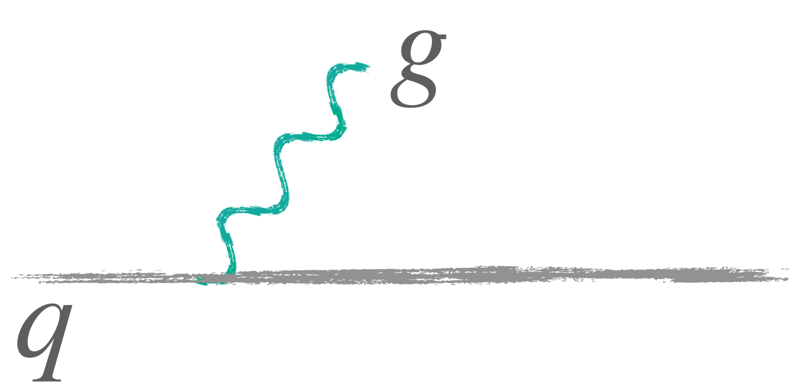

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$$\overset{\mathrm{DL}}{=} \int [\mathrm{d}k] \left| \mathcal{M}(k) \right|^2 \Theta(k_t > k_{t,\mathrm{cut}})$$

$$\overset{\mathrm{DL}}{=} \frac{2\alpha_s C_i}{\pi} \int_0^\infty \mathrm{d}\eta \int_{e^{-\eta}}^1 \frac{\mathrm{d}z}{z} = \frac{2\alpha_s C_i}{\pi} \frac{L^2}{2}$$

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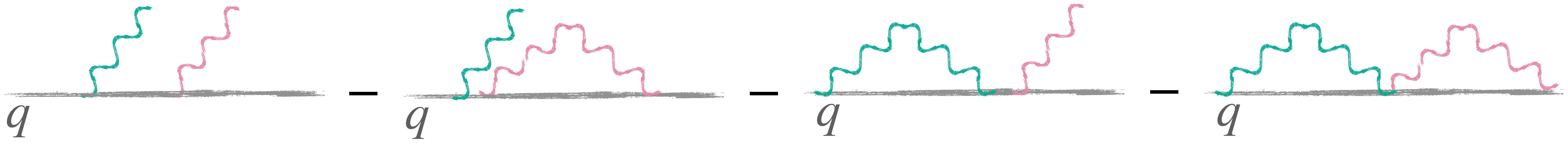
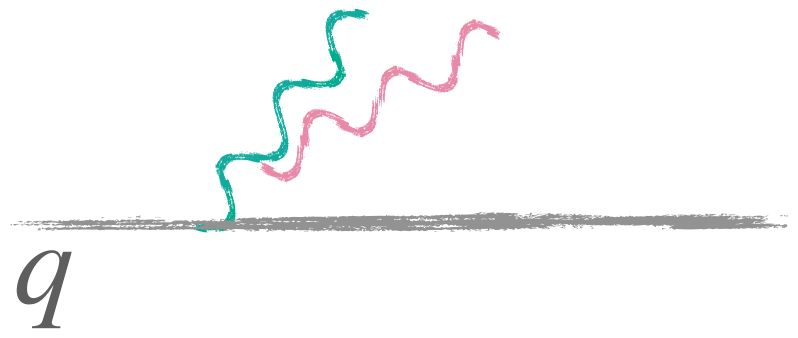


or



$$N = \alpha_s^2 C_i C_A \int_0^1 \frac{\mathrm{d}z_1}{z_1} \int_0^\infty \mathrm{d}\eta_1 \int_0^1 \frac{\mathrm{d}z_2}{z_2} \int_{\eta_1}^\infty \mathrm{d}\eta_2 \Theta(k_{t,21} > k_{t,\mathrm{cut}})$$

After real and virtual cancellations, the only configurations that survive are those of nested emissions

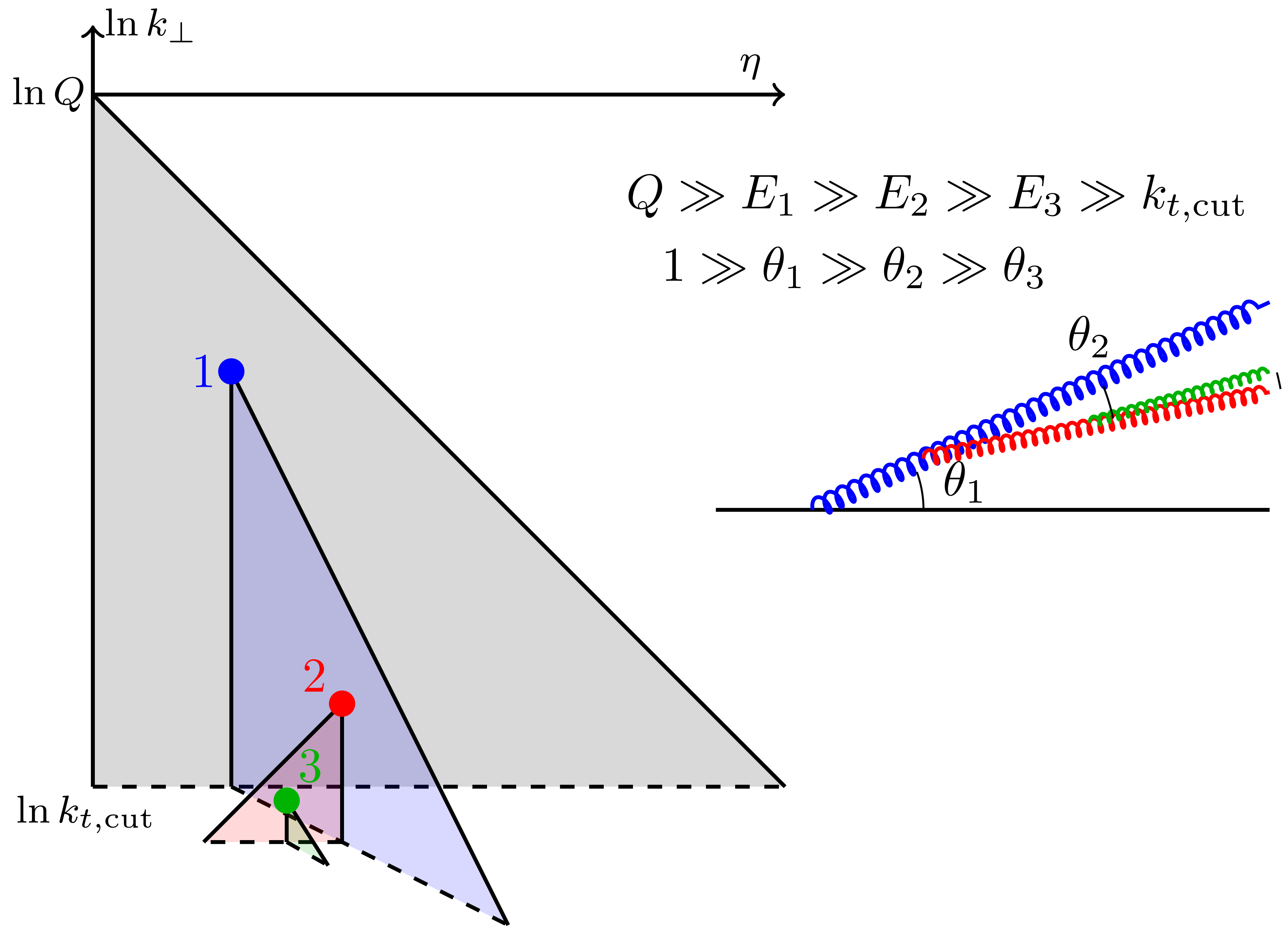
Lund multiplicity at DL: all orders

The average Lund multiplicity at double logarithmic accuracy is given by

$$\langle N\rangle_{\rm DL} = 1 + \frac{C_i}{C_A}\sum_{n=1}^{\infty}\bar{\alpha}^n \int_0^{\infty} d\eta_1 \int_{\eta_1}^{\infty} d\eta_2 \dots \int_{\eta_{n-1}}^{\infty} d\eta_n \int_0^1 \frac{dx_1}{x_1}\int_0^{x_1}\frac{dx_2}{x_2}\dots\int_0^{x_{n-1}}\frac{dx_n}{x_n}\Theta(x_n e^{-\eta_n} > e^{-L})$$

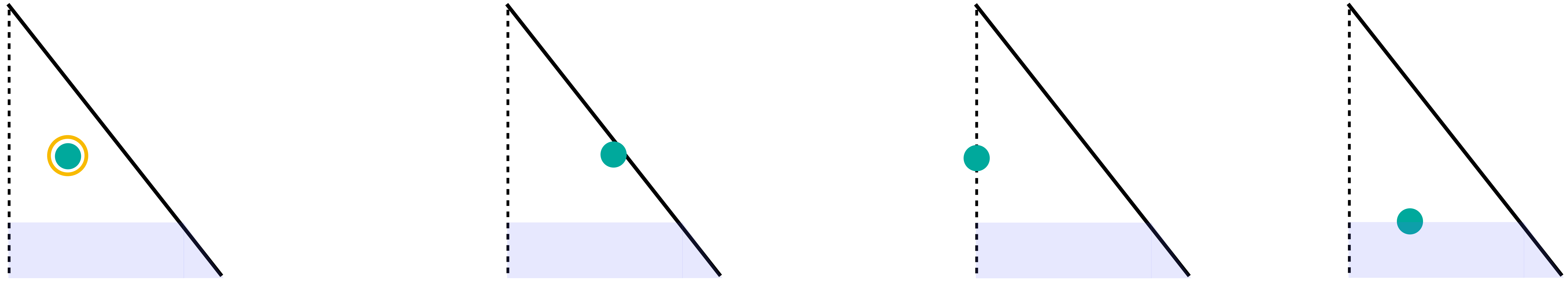
$$\boxed{\langle N\rangle_{\rm DL} = 1 + \frac{C_i}{C_A}\left[\cosh\left(\sqrt{\frac{2C_A\alpha_s L^2}{\pi}}\right) - 1\right]}$$

$$x_i = 2E_i/Q = \prod_{j\leq i} z_j$$



Lund multiplicity at NDL: configurations $\alpha_s L(\alpha_s L^2)^n$

The possible configurations that contribute to $\langle N \rangle$ at next-to-double log are



Running coupling

$$\boxed{\alpha_s \to \alpha_s - 2\alpha_s^2 \beta_0 \ell + \mathcal{O}(\alpha_s^3)}$$

with $\ell \equiv \ln(k_t/Q)$

Hard-collinear

$$\boxed{\frac{1}{z} \to C_F\left(\frac{1-z}{z} + \frac{z}{2}\right)}$$

Large-angle

$$\frac{\mathrm{d}z}{z}\mathrm{d}\eta$$

$$\downarrow$$

$$\boxed{\frac{p_i \cdot p_j}{p_i \cdot p_k p_j \cdot p_k}\frac{\mathrm{d}\phi}{2\pi}\mathrm{d}\cos\theta}$$

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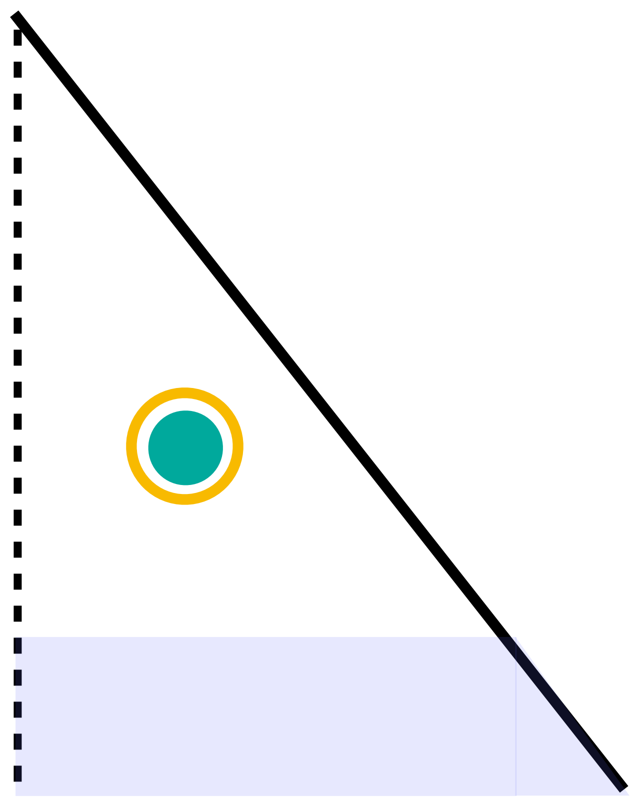

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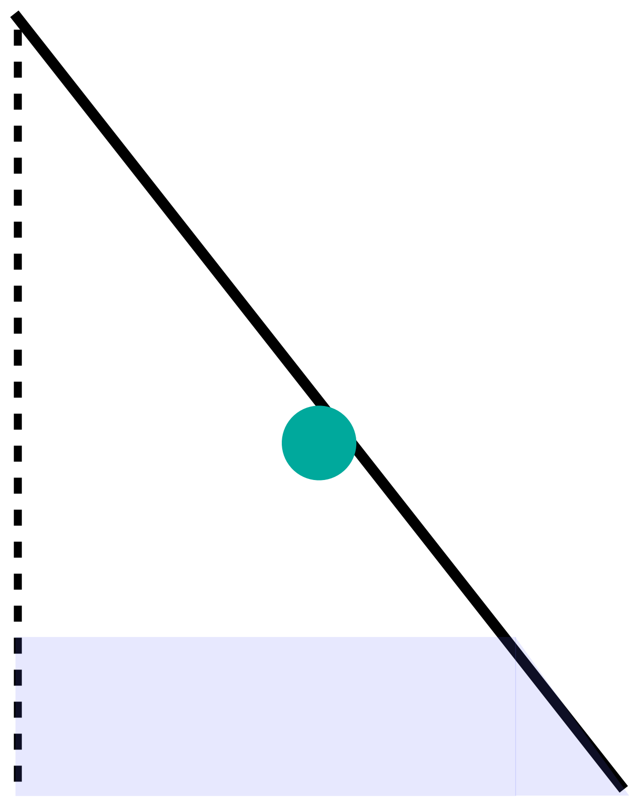

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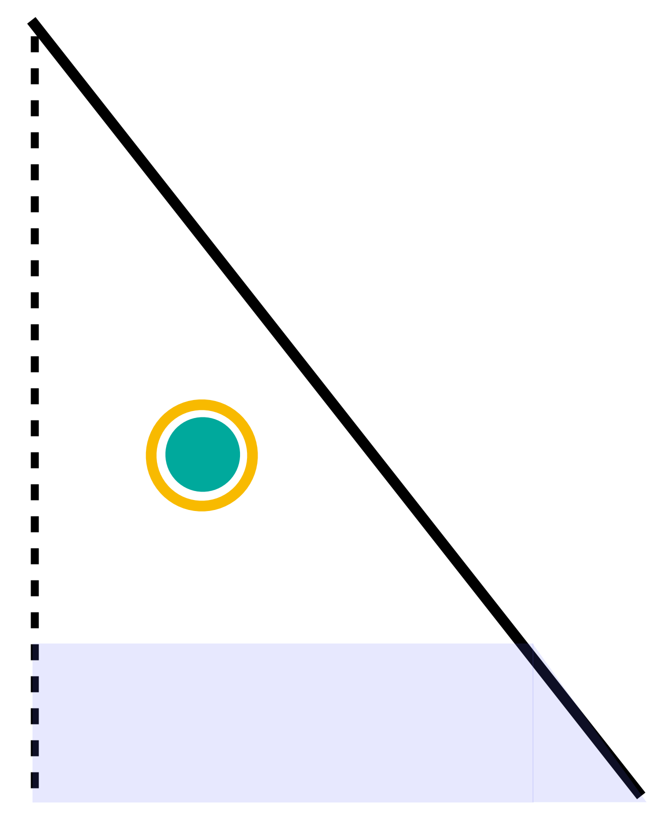
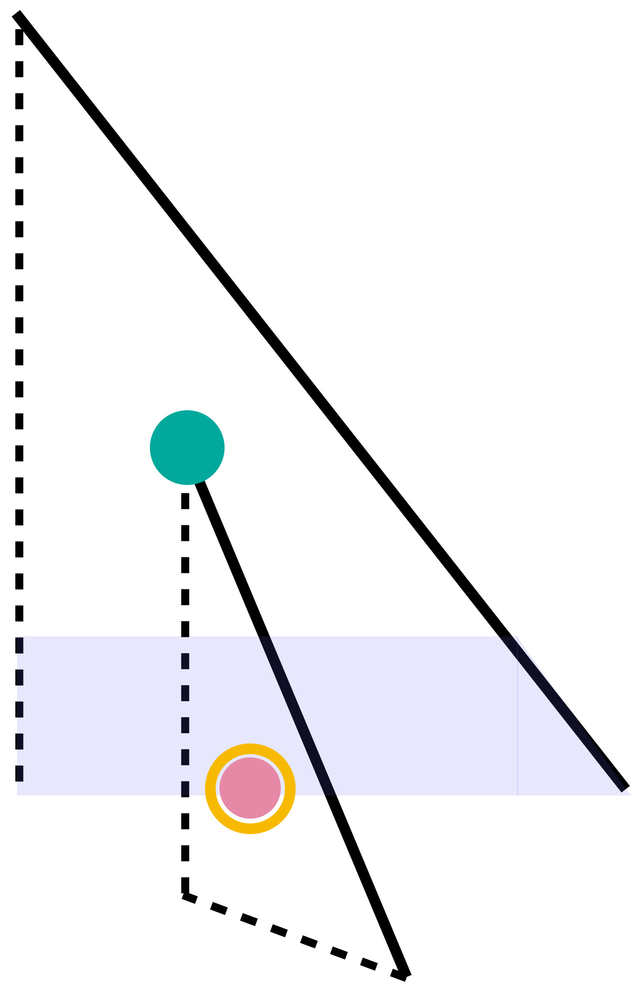
$k_t \sim k_{t,\mathrm{cut}}$

Lund multiplicity at NDL: resummation strategy

To reach NDL it is enough to have one NDL-like emission in the chain either primary or secondary, e.g. running coupling

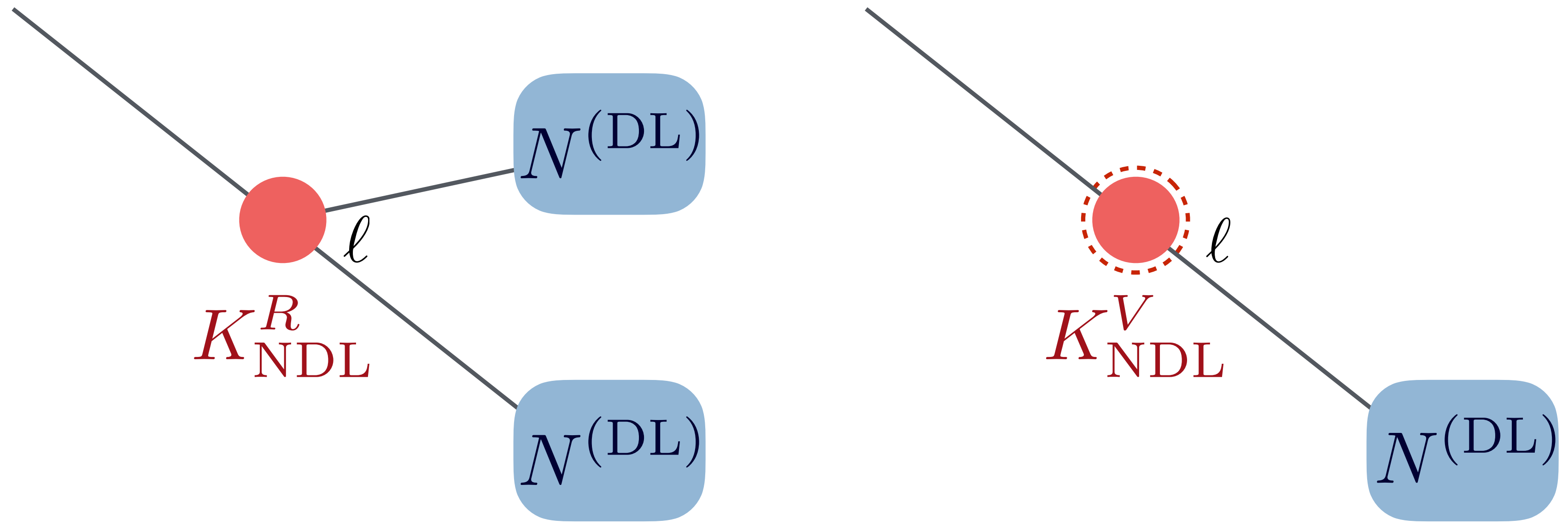
$$\langle N \rangle = 1 + \sum_{n=0}^{\infty} \int_0^{\infty} \mathrm{d}\eta_1 \mathrm{d}\eta_2 \ldots \mathrm{d}\eta_n \int_0^1 \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} \ldots \frac{\mathrm{d}x_n}{x_n} \left(\frac{2\alpha_s C_A}{\pi} \right)^n \sum_{i=1}^{n} 2\alpha_s \beta_0 (\ln x_i + \eta_i)$$





Lund multiplicity at NDL: resummation strategy

Instead of performing the sum for every single diagram, we take another approach



$$\delta N_i^{(\mathrm{NDL})} = \int_0^L \mathrm{d}\ell \left\{ K^R_{\mathrm{NDL}} \left[N^{(\mathrm{DL})}_{\mathrm{hard}}(L;\ell) + N^{(\mathrm{DL})}_{\mathrm{soft}}(L;\ell) \right] - K^V_{\mathrm{NDL}} N_i^{(\mathrm{DL})}(L;\ell) \right\}$$

Lund multiplicity at NDL: resummation strategy

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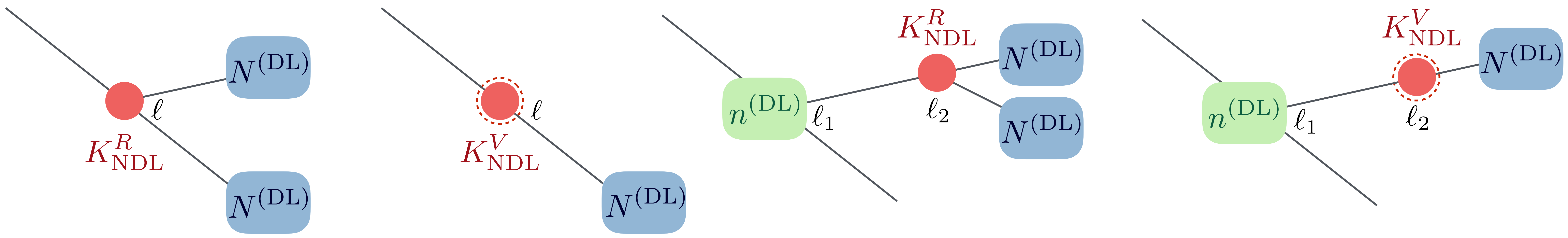

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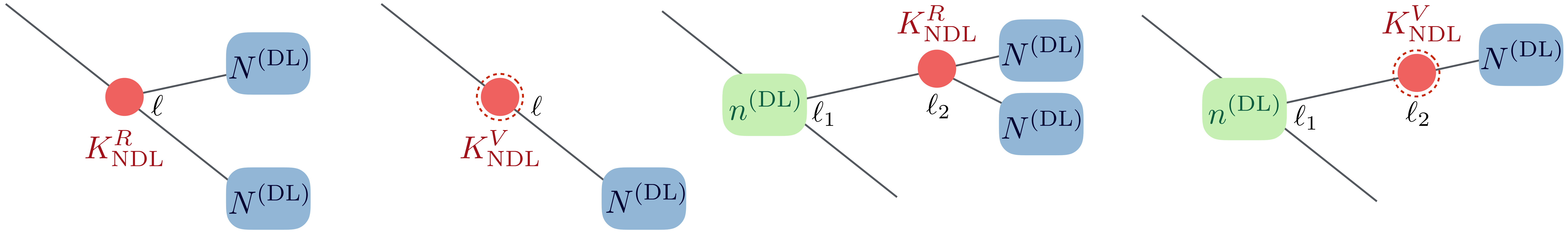
$$+ \int_0^L \mathrm{d}\ell_1 n_i^{(\mathrm{DL})}(\ell_1) \int_{\ell_1}^L \mathrm{d}\ell_2 \left\{ K^R_{\mathrm{NDL}} \left[N^{(\mathrm{DL})}_{\mathrm{hard}}(L;\ell_2) + N^{(\mathrm{DL})}_{\mathrm{soft}}(L;\ell_2) \right] - K^V_{\mathrm{NDL}} N_g^{(\mathrm{DL})}(L;\ell_2) \right\}$$

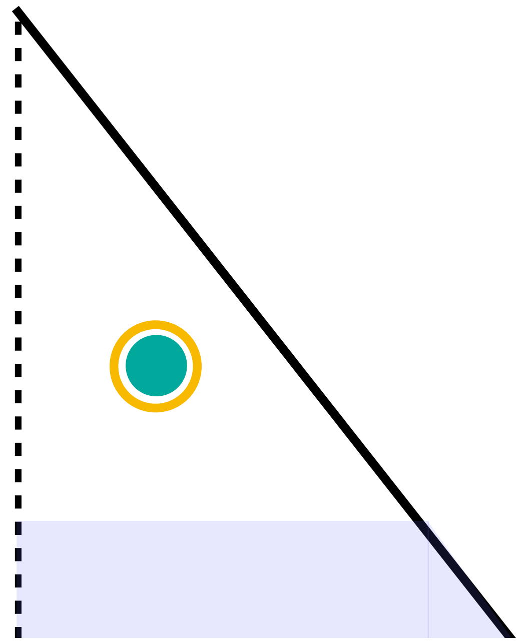
$$n_i^{(\mathrm{DL})} = \frac{\mathrm{d}N_i^{(\mathrm{DL})}}{\mathrm{d}L}$$

Lund multiplicity at NDL: resummation strategy

Instead of performing the sum for every single diagram, we take another approach

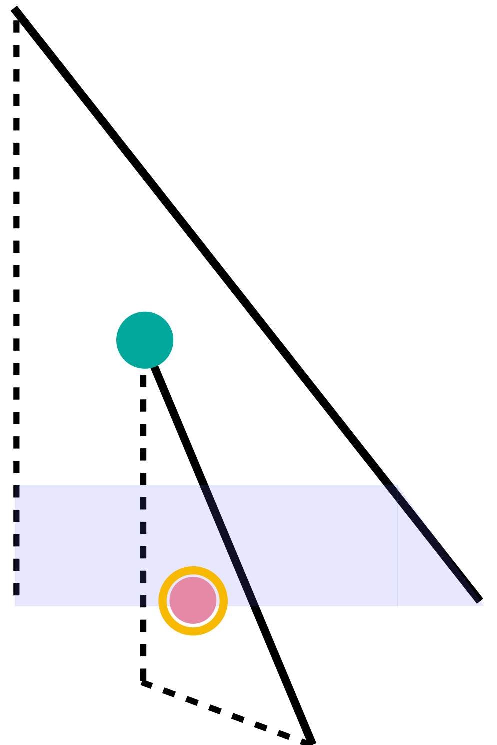


e.g. for running coupling



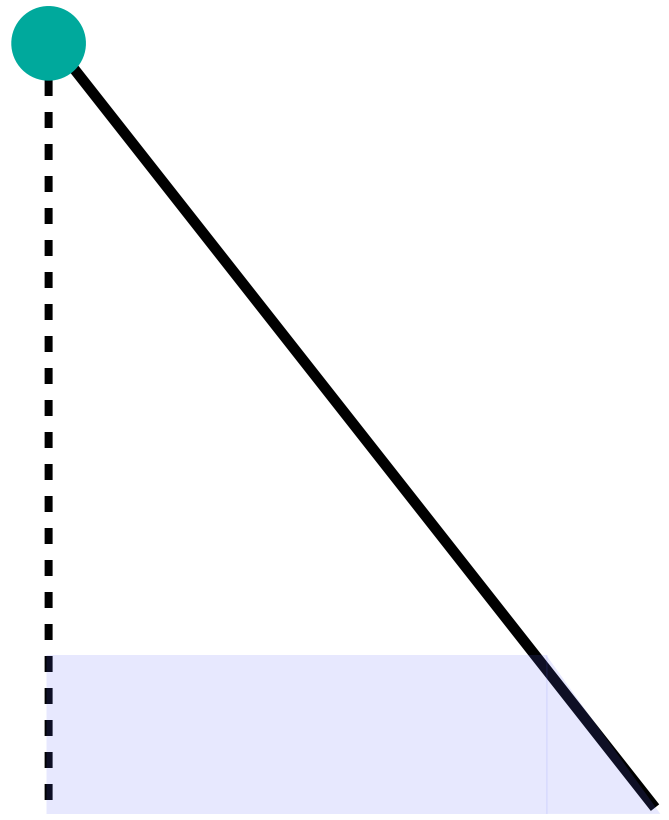
$$\delta N_{p,\beta_0} = \bar{\alpha} \int_0^L \mathrm{d}\ell \, (2\alpha_s \beta_0 \ell) \, \ell \, N_g^{(\mathrm{DL})}(L - \ell)$$

$$+$$

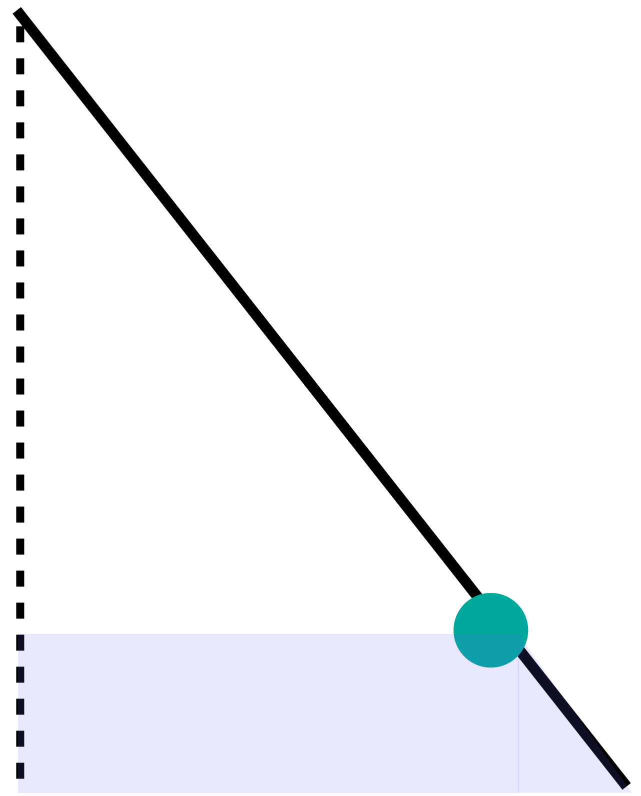


$$\delta N_{s,\beta_0} = \int_0^L \mathrm{d}\ell_1 n^{(\mathrm{DL})}(\ell_1) \, \bar{\alpha} \int_{\ell_1}^L \mathrm{d}\ell_2 (2\alpha_s \beta_0 \ell_2) \, (\ell_2 - \ell_1) \, N_g^{(\mathrm{DL})}(L - \ell_2)$$

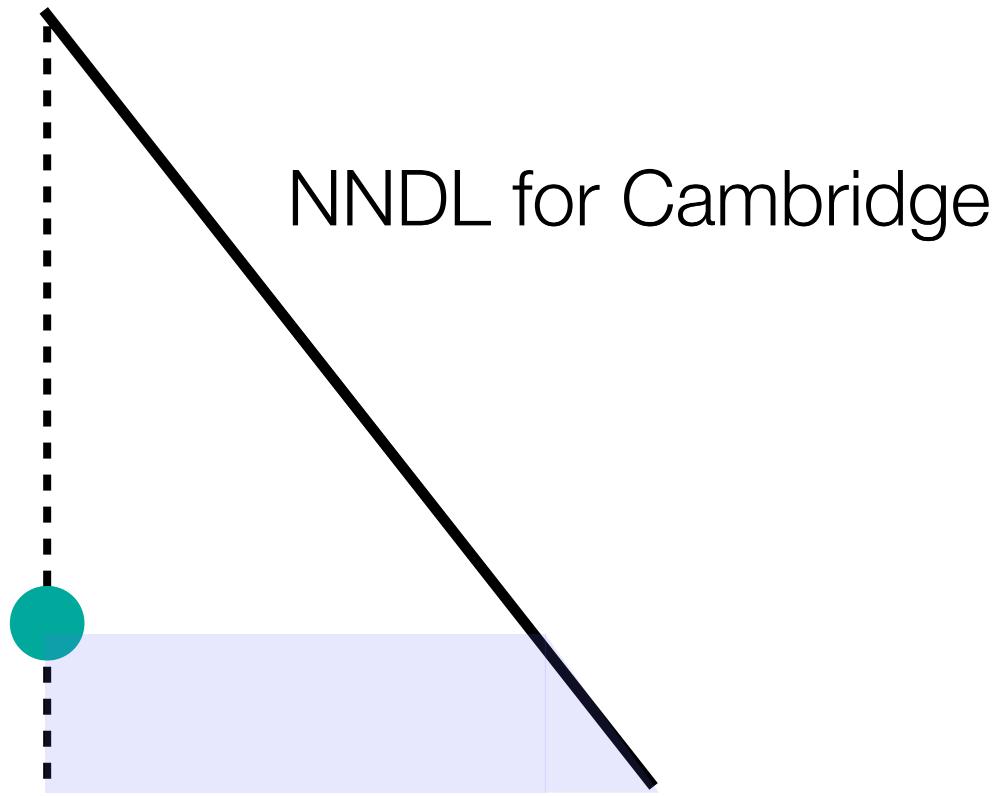
Lund multiplicity at NNDL: $\alpha_s(\alpha_s L^2)^n$



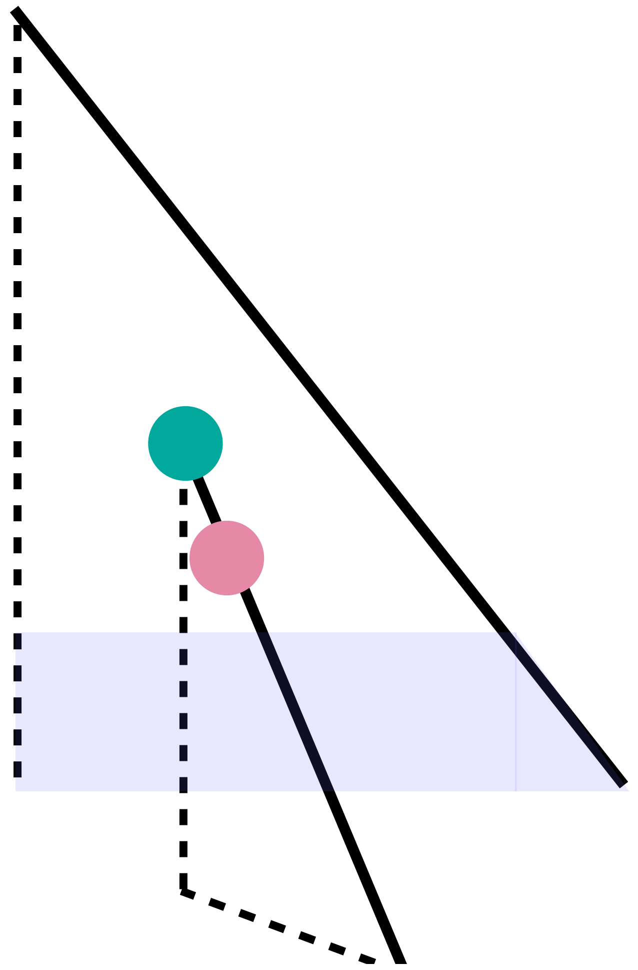
Hard ME



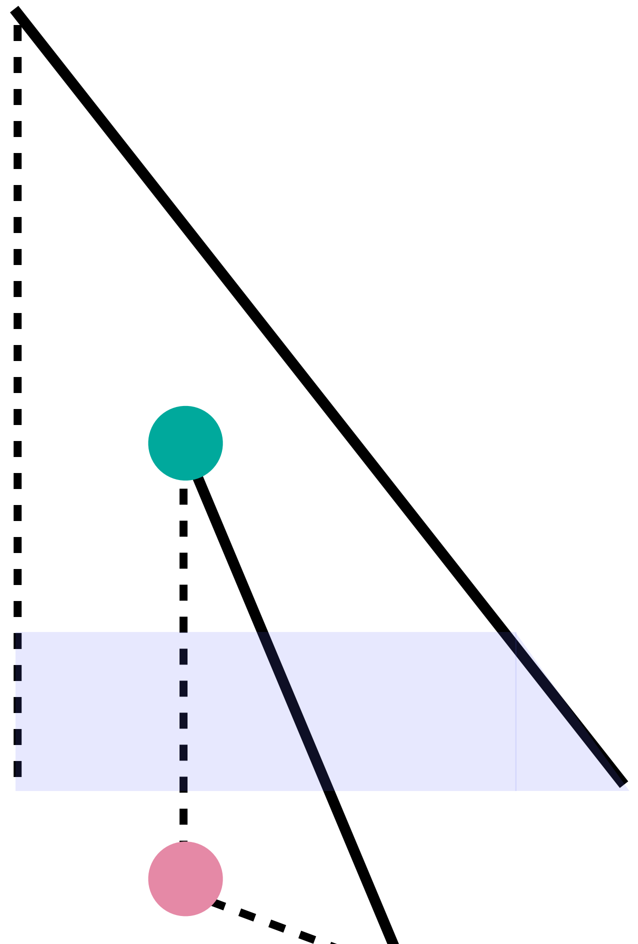
Collinear end-point



Large angle $+\, k_t \sim k_{t,\mathrm{cut}}$

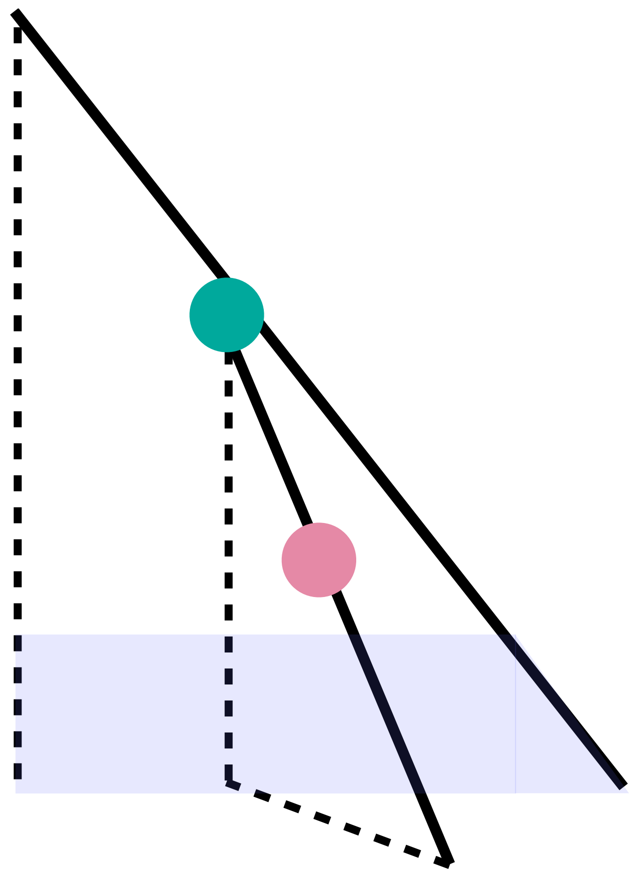


Close-by pair

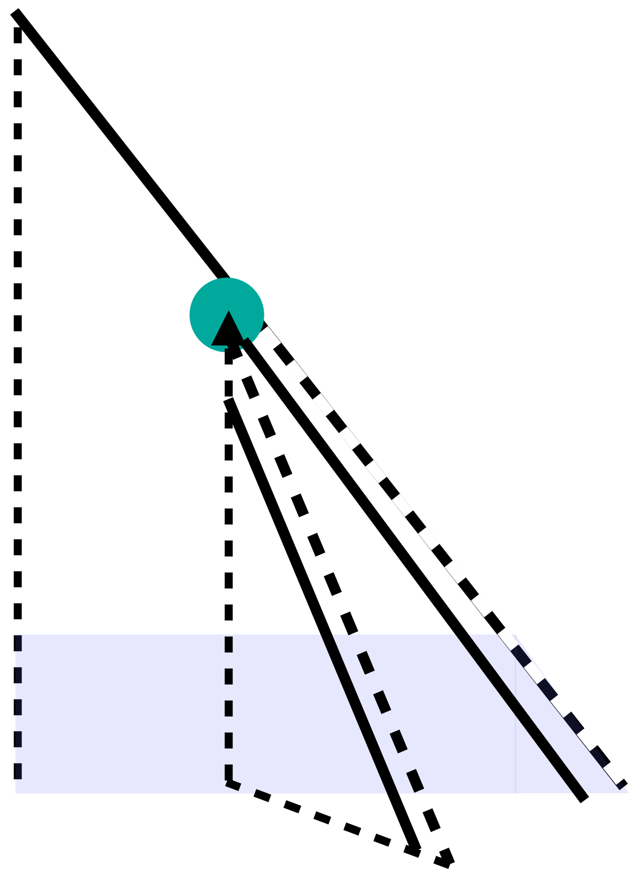


Clustering

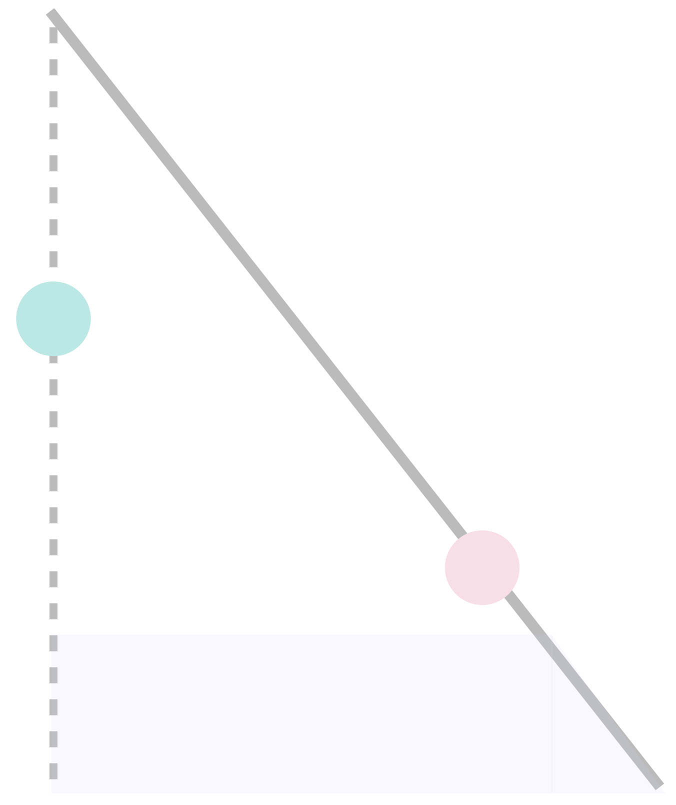
Lund multiplicity at NNDL: $(\alpha_s L)(\alpha_s L)(\alpha_s L^2)^n$



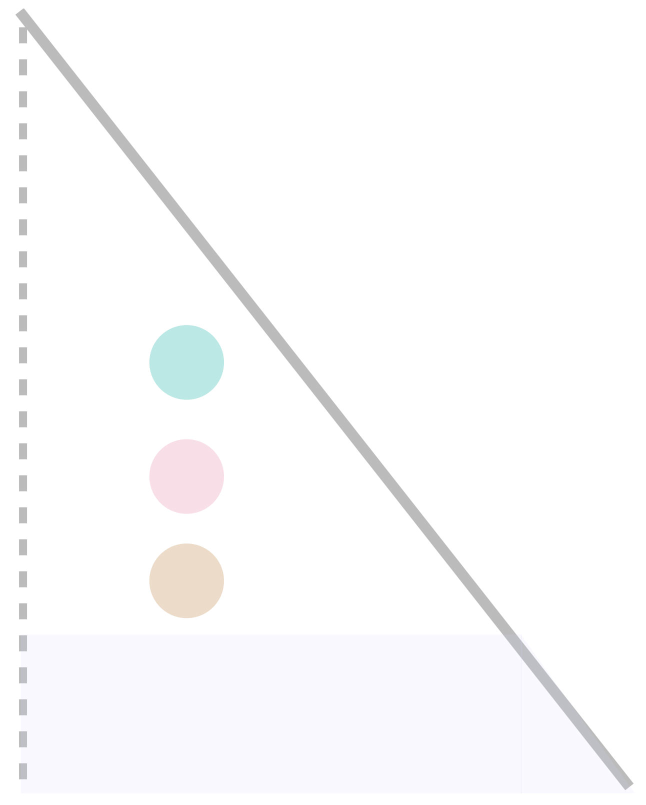
2 hard collinear



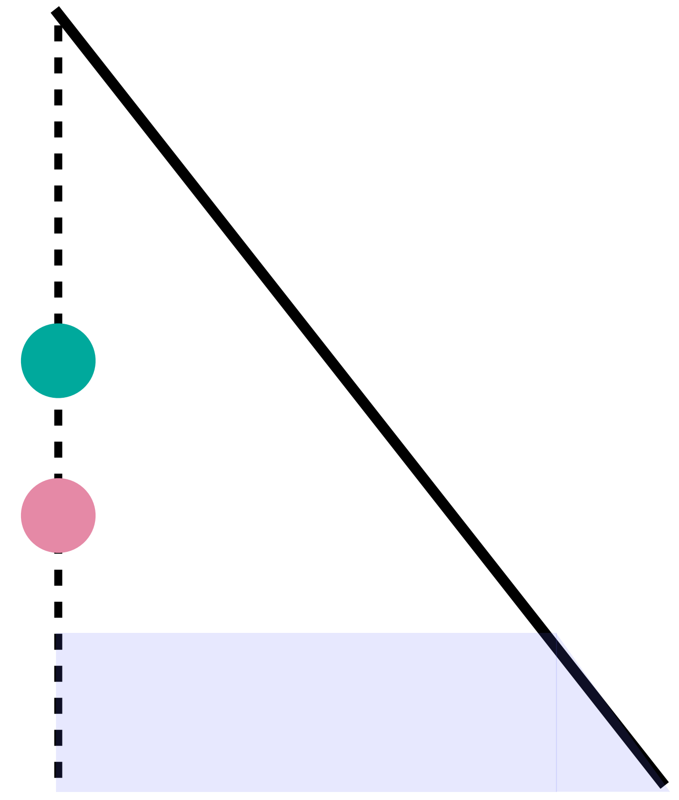
Energy loss



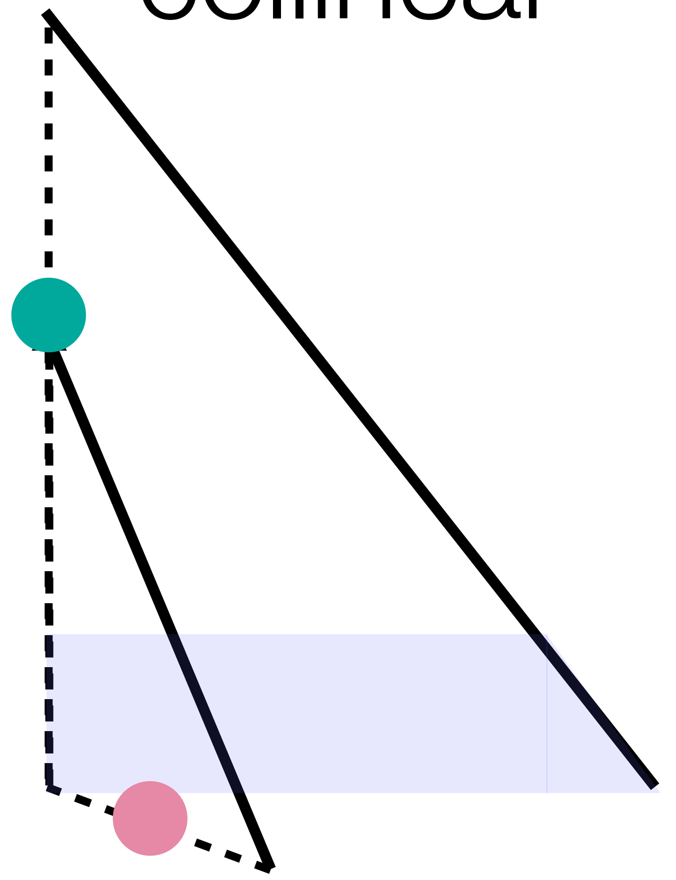
Large angle + hard-collinear



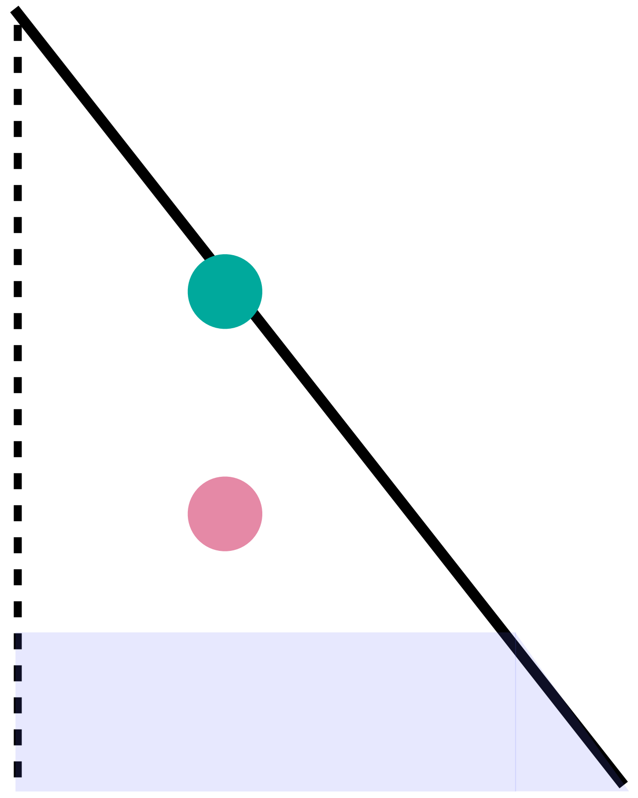
3 commensurate angles



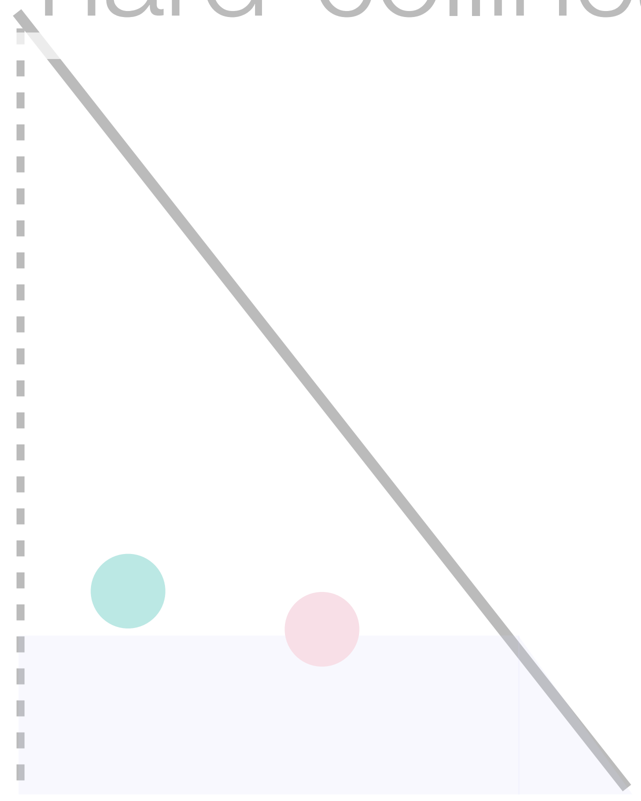
2 large angle



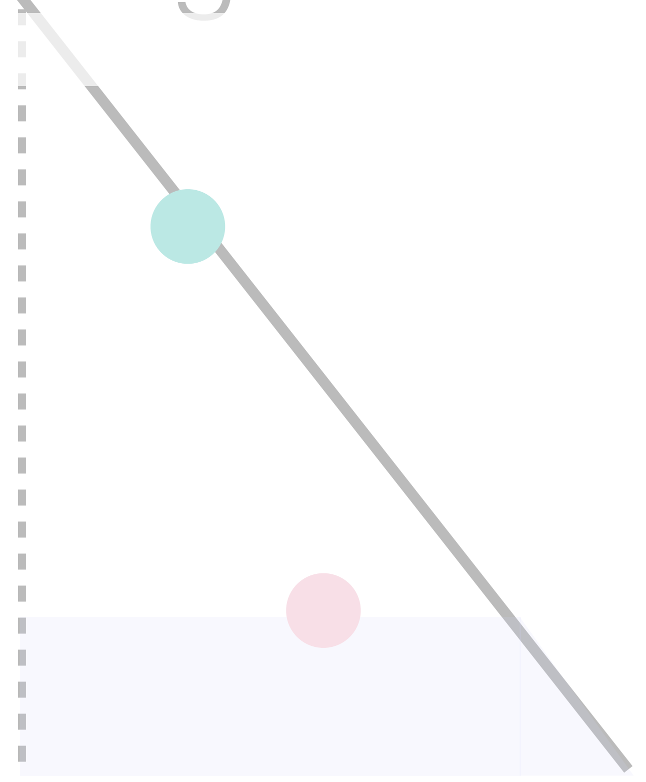
Large angle + $k_t \sim k_{t,\mathrm{cut}}$



Hard coll. + commensurate η

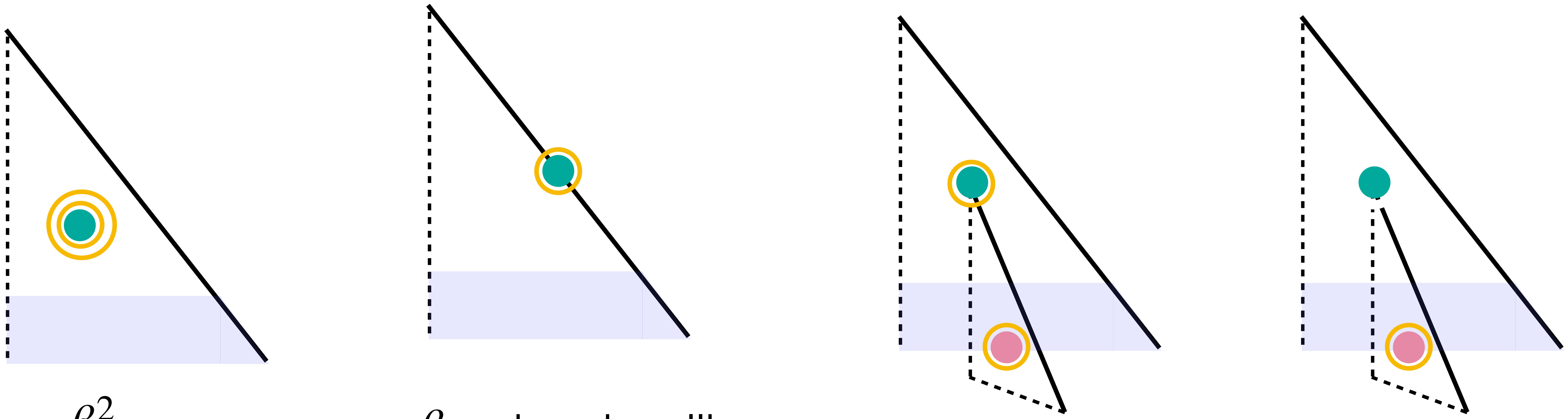


2 $k_t \sim k_{t,\mathrm{cut}}$



Hard coll. + $k_t \sim k_{t,\mathrm{cut}}$

Lund multiplicity at NNDL: running coupling



β_0^2

β_0 + hard-collinear

$\beta_0 \times \beta_0$

$\beta_0 \times$hard-collinear

Final results

$$
\begin{aligned}
2\pi h_3^{(q)} &= D_{\text{end}}^{q\to qg} + \left(D_{\text{end}}^{g\to gg} + D_{\text{end}}^{g\to q\bar q}\right)\frac{C_F}{C_A}(\cosh\nu - 1) + D_{\text{hme}}^{qqg}\cosh\nu \\
&+ \frac{C_F}{C_A}\left[(1-c_\delta)D_{\text{pair}}^{q\bar q}(\cosh\nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar q}\right)\frac{\nu}{2}\sinh\nu\right] \\
&+ C_F\left[\left(\cosh\nu - 1 - \frac{1-c_\delta}{4}\nu^2\right)D_{\text{clust}}^{(\text{prim})} + (\cosh\nu - 1)D_{\text{clust}}^{(\text{sec})}\right] \\
&+ \frac{C_F}{C_A}\left[D_{\text{e-loss}}^{g}\frac{\nu}{2}\sinh\nu + \left(D_{\text{e-loss}}^{q} - D_{\text{e-loss}}^{g}\right)(\cosh\nu - 1)\right] \\
&+ \frac{C_F}{2}\Big\{(B_{gg} + c_\delta B_{gq})^2\nu^2\cosh\nu + 8\left[2c_\delta B_{gg} - 2c_\delta B_q - (1-3c_\delta^2)B_{gq}\right]B_{gq}\cosh\nu \\
&\quad + \left[4B_q(B_{gg} + (2c_\delta + 1)B_{gq}) - (B_{gg} + c_\delta B_{gq})(B_{gg} + 9c_\delta B_{gq})\right]\nu\sinh\nu \\
&\quad + 4(1-c_\delta^2)B_{gq}^2\nu^2 + 8\left[2c_\delta B_q - 2c_\delta B_{gg} + (1-3c_\delta^2)B_{gq}\right]B_{gq}\Big\} \\
&+ \frac{C_F}{C_A}\frac{\pi\beta_0}{2}\Big\{(B_{gg} + c_\delta B_{gq})\nu^3\sinh\nu + \left[2B_q - 2B_{gg} + (6-8c_\delta)B_{gq}\right]\nu\sinh\nu \\
&\quad + 2(B_q + B_{gg} + B_{gq})\nu^2\cosh\nu - 4(1-c_\delta)B_{gq}(2\cosh\nu - 2 + \nu^2)\Big\} \\
&+ \frac{C_F}{C_A}\frac{\pi^2\beta_0^2}{8C_A}\left[3\nu(2\nu^2 - 1)\sinh\nu + (\nu^4 + 3\nu^2)\cosh\nu\right]
\end{aligned}
$$

$$
\begin{aligned}
2\pi h_3^{(g)} &= \left(D_{\text{end}}^{g\to gg} + D_{\text{end}}^{g\to q\bar q}\right)\cosh\nu + \left[D_{\text{hme}}^{ggg}\cosh\nu + D_{\text{hme}}^{gq\bar q}\left(c_\delta\cosh\nu + 1 - c_\delta\right)\right] \\
&+ \left[(1-c_\delta)D_{\text{pair}}^{q\bar q}(\cosh\nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar q}\right)\frac{\nu}{2}\sinh\nu\right] \\
&+ C_A\left(D_{\text{clust}}^{(\text{prim})} + D_{\text{clust}}^{(\text{sec})}\right)(\cosh\nu - 1) + D_{\text{e-loss}}^{g}\frac{\nu}{2}\sinh\nu \\
&+ \frac{C_A}{2}\Big\{(B_{gg} + c_\delta B_{gq})^2\nu^2\cosh\nu - 8(1-c_\delta^2)B_{gq}^2(\cosh\nu - 1) \\
&\quad + \left[(B_{gg} + c_\delta B_{gq})(3B_{gg} - 5c_\delta B_{gq}) + 4(1+c_\delta)B_{gq}B_q\right]\nu\sinh\nu\Big\} \\
&+ \frac{\pi\beta_0}{2}\Big\{(B_{gg} + c_\delta B_{gq})\nu^3\sinh\nu + 6(1-c_\delta)B_{gq}\nu\sinh\nu + 2\left[2B_{gg} + (1+c_\delta)B_{gq}\right]\nu^2\cosh\nu \\
&\quad - 8B_{gq}(1-c_\delta)(\cosh\nu - 1)\Big\} + \frac{\pi^2\beta_0^2}{8C_A}\left[3\nu(2\nu^2 - 1)\sinh\nu + (\nu^4 + 3\nu^2)\cosh\nu\right]
\end{aligned}
$$

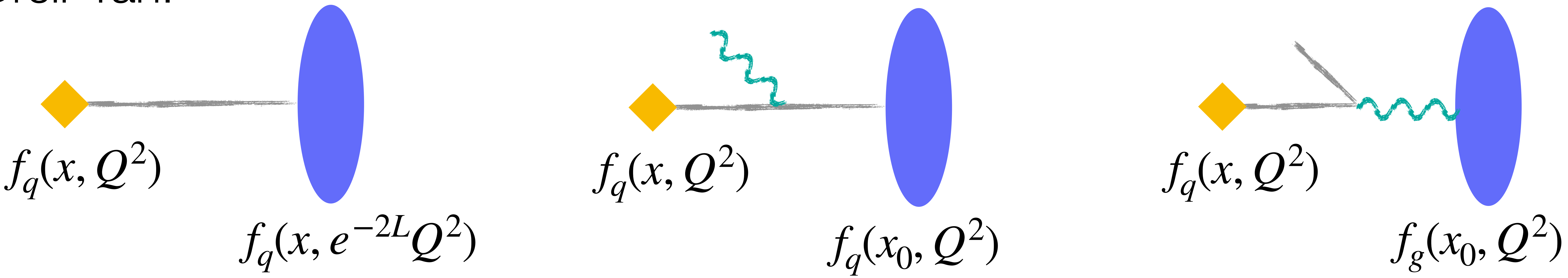
$$
\langle N^{(\text{Cam})}(\alpha_s, L)\rangle = \langle N^{(\text{Lund})}(\alpha_s, L)\rangle + \frac{\alpha_s C_i}{2\pi}\frac{\pi^2}{6}
$$

Lund multiplicity in hadronic collisions at NDL

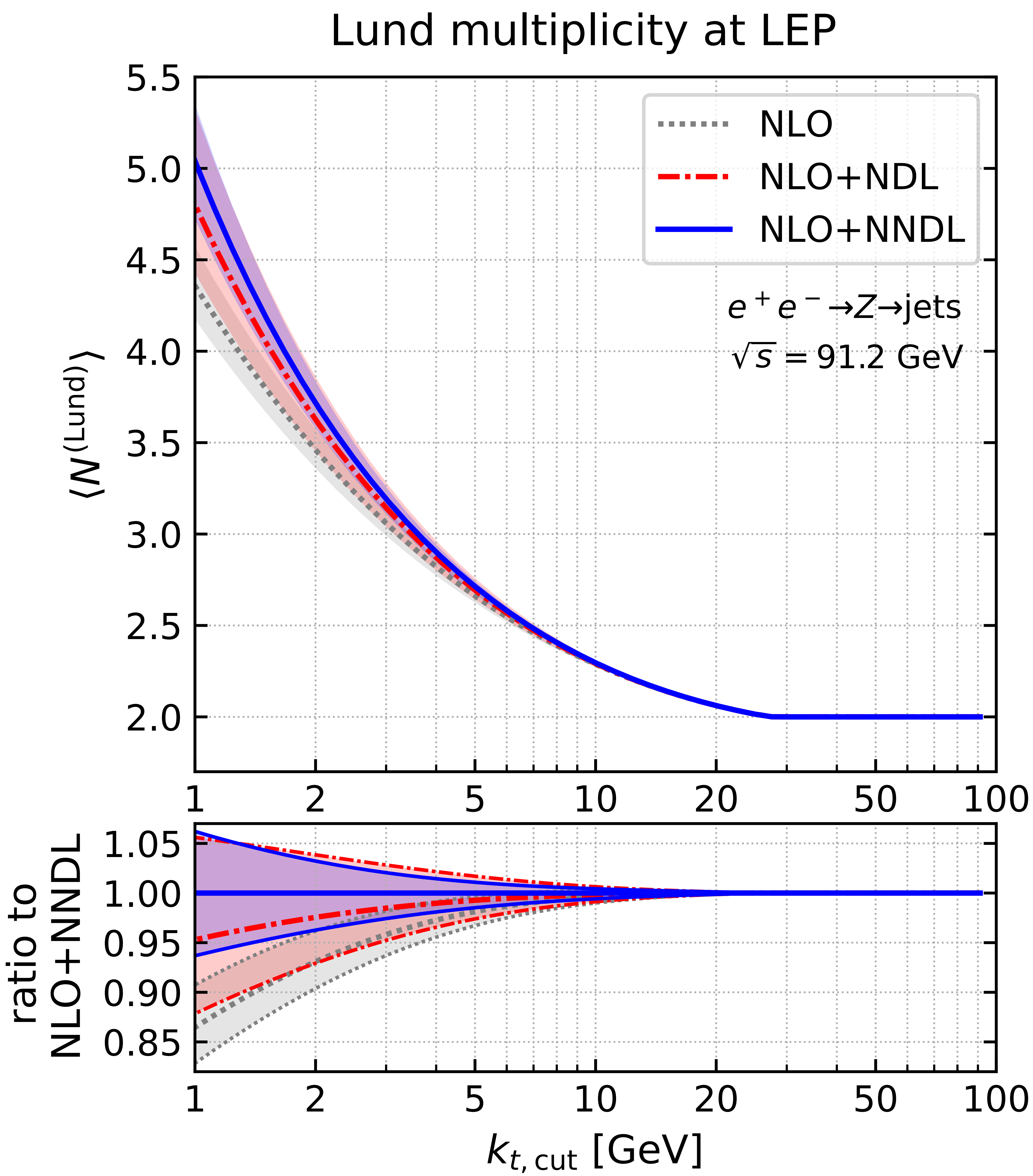
Extension to hadronic collisions requires PDFs and initial state radiation

$$N_{pp}^{\mathrm{NDL}} = N_{e^+e^-}^{\mathrm{NDL}} + \delta N$$

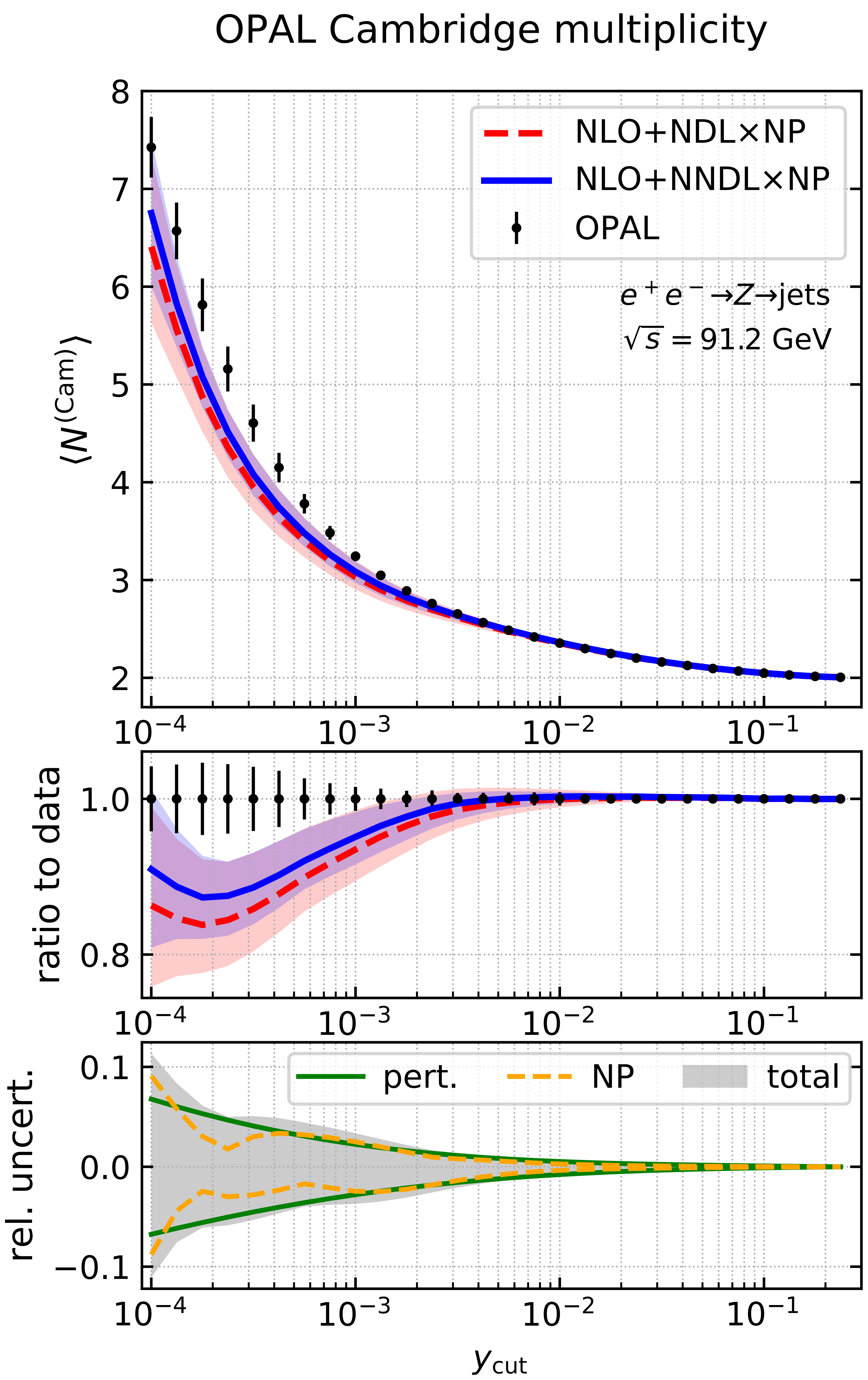
e.g. Drell-Yan:



Systematic and flexible framework: pp as a proof of concept



Lund multiplicity at LEP
NLO
NLO+NDL
NLO+NNDL
$e^+e^- \to Z \to$ jets
$\sqrt{s} = 91.2$ GeV
$\langle N^{(\mathrm{Lund})} \rangle$
ratio to NLO+NNDL
$k_{t,\mathrm{cut}}$ [GeV]



OPAL Cambridge multiplicity
NLO+NDL×NP
NLO+NNDL×NP
OPAL
$e^+e^- \to Z \to$ jets
$\sqrt{s} = 91.2$ GeV
$\langle N^{(\mathrm{Cam})} \rangle$
ratio to data
rel. uncert.
pert.
NP
total
y_{cut}