# Jet quenching in anisotropic plasmas

**1st July 2022, From RHIC/LHC to EIC**

João Barata, BNL

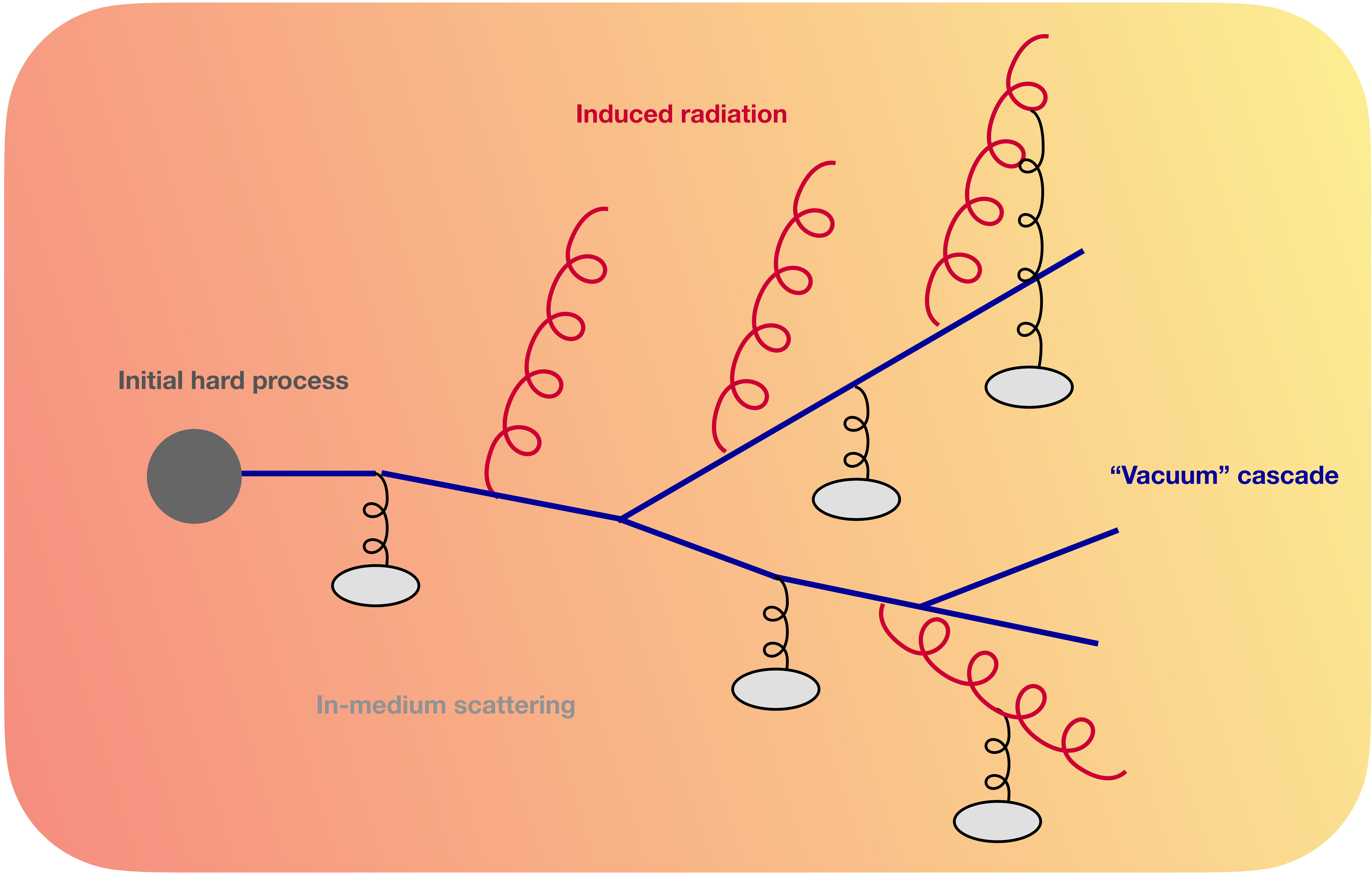
Based on work done with Xoan Mayo, Andrey Sadofyev and Carlos Salgado

2202.08847 and on going

# Jets in hot plasmas

Brookhaven National Laboratory

**An oversimplified cartoon for jet evolution in a QGP**

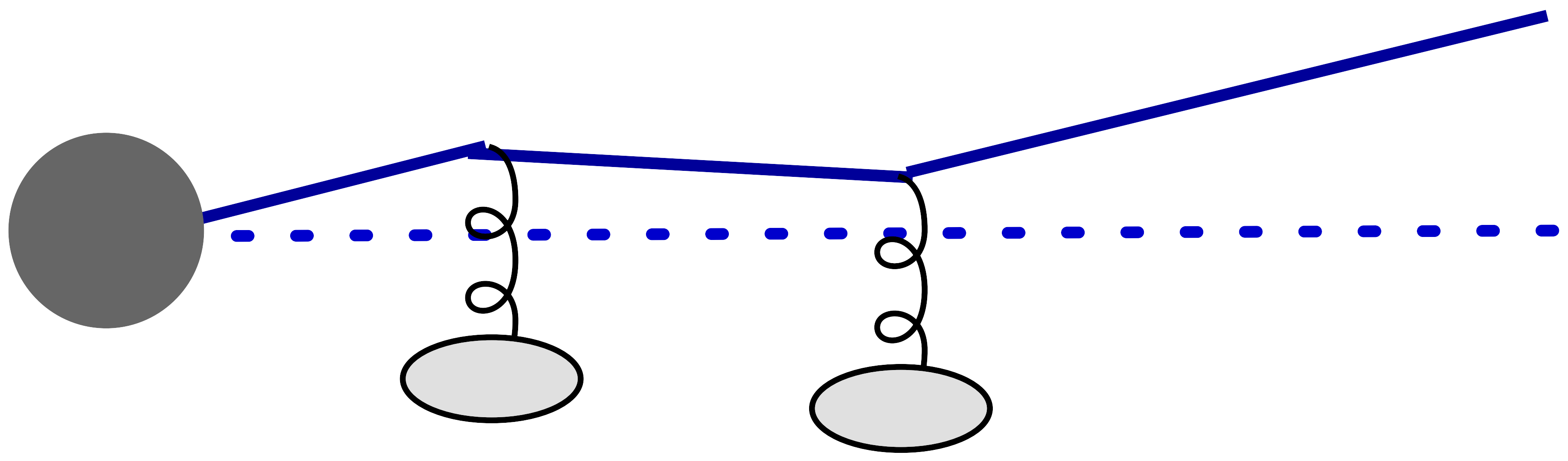


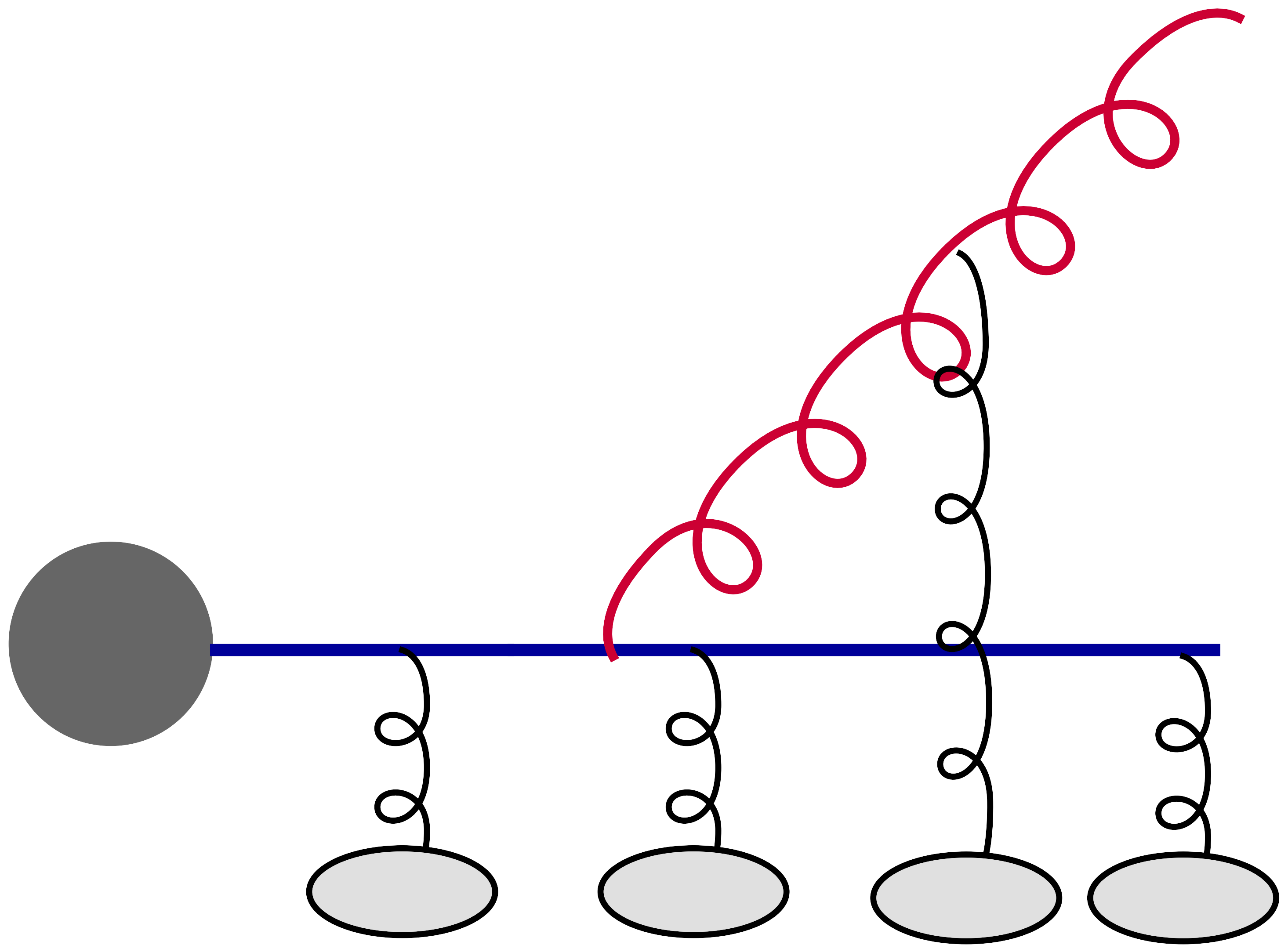
# Jets in hot plasmas

**Jet quenching pheno is based on understanding**

Medium induced radiation $\mathcal{O}(\alpha_s)$

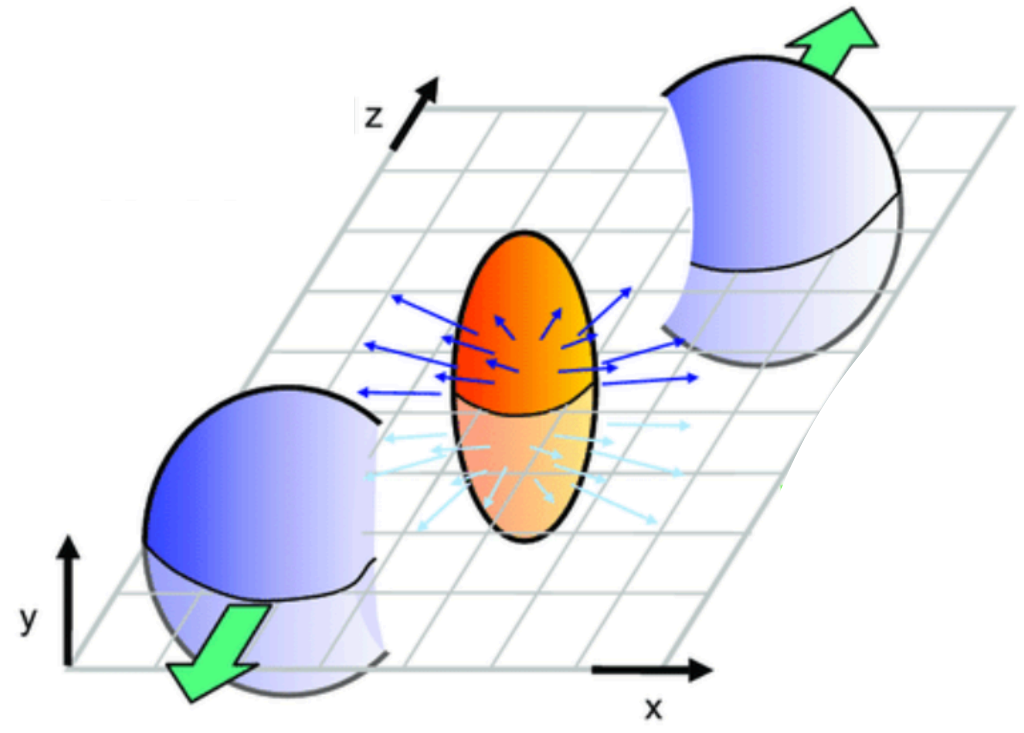
Momentum broadening $\mathcal{O}(\alpha_s^0)$





# Jets in hot plasmas

**How do we deal with these in jet quenching theory?**




Eikonal expansion with kinetic phases

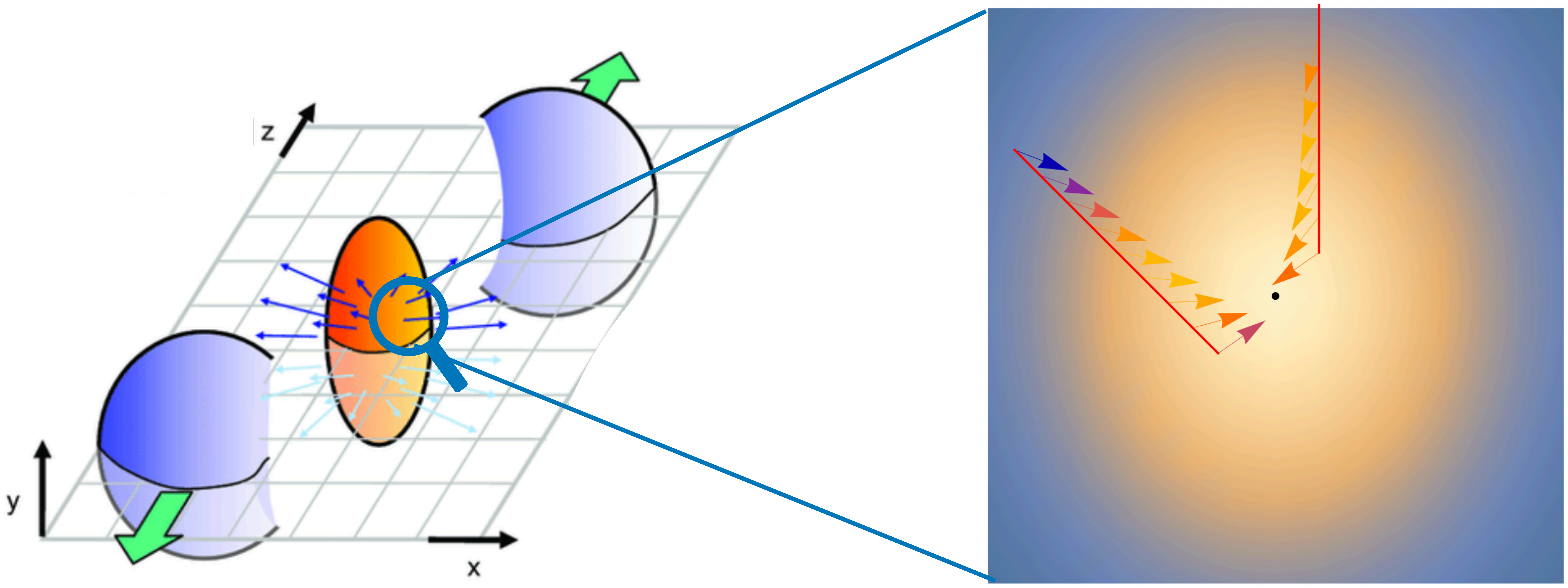
Classical background, if possible infinitely long, homogeneous, static …

✓ One can gain analytical insight into the problem: BDMPS, GLV, …

✗ Jets decouple from the plasma: what are we seeing in the plasma?

# Jets in hot plasmas

**Today: jet evolution in dense plasmas with structure (i.e. anisotropic)**




“jets” in a gradient field

# Jets in hot plasmas



## How large can these effects be?

**In the dilute regime, we can look at the leading moments** 2104.09513, A. Sadofyev, M. Sievert, I. Vitev

**sub-eikonal** **vs** **enhancing medium factor**

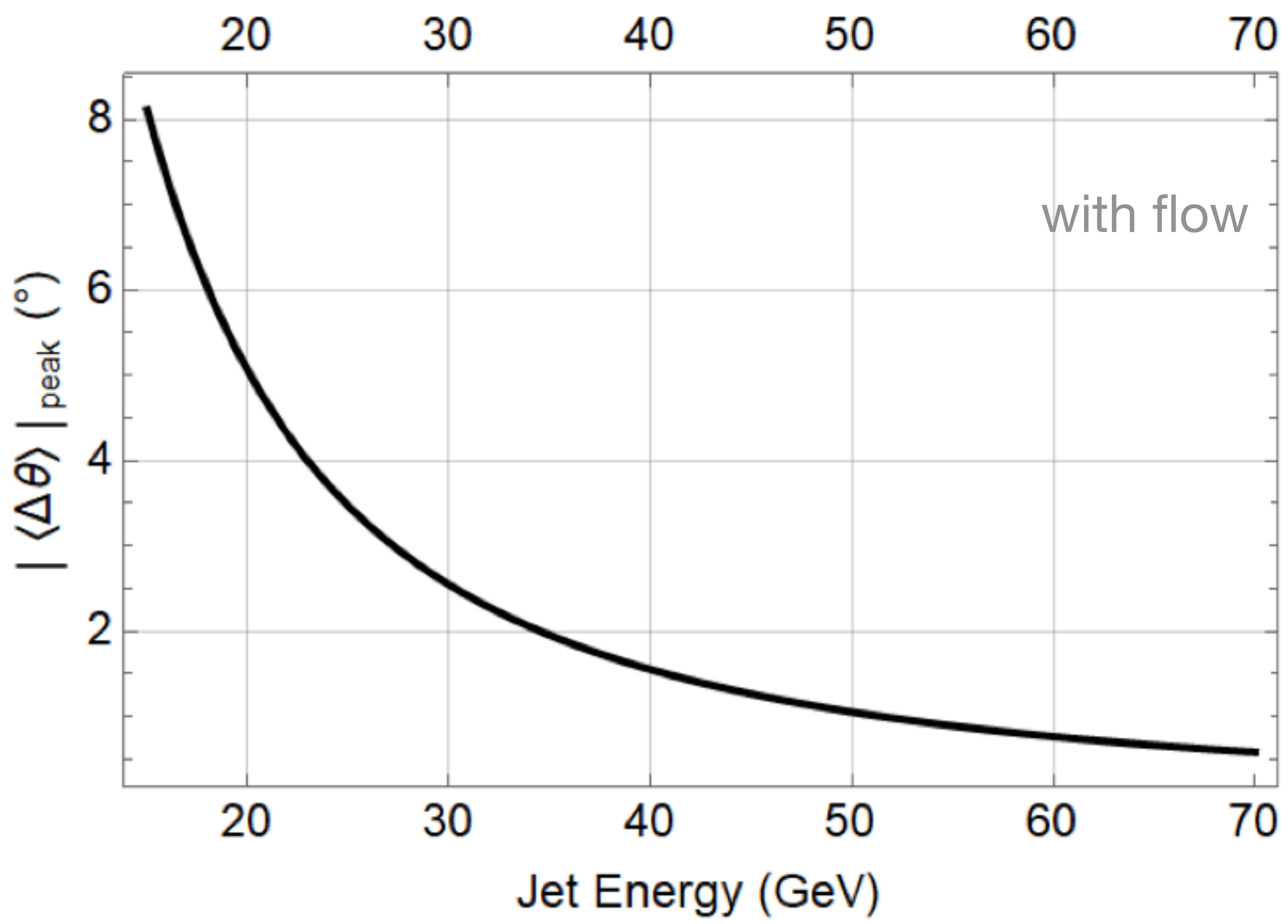
with flow

$$\langle \mathbf{p} \rangle_{\mathbf{u} \neq 0, \nabla \mathrm{T}=0} \propto \frac{u_\perp}{1 - u_z} \frac{\mu^2 L}{E\lambda}$$

with gradients

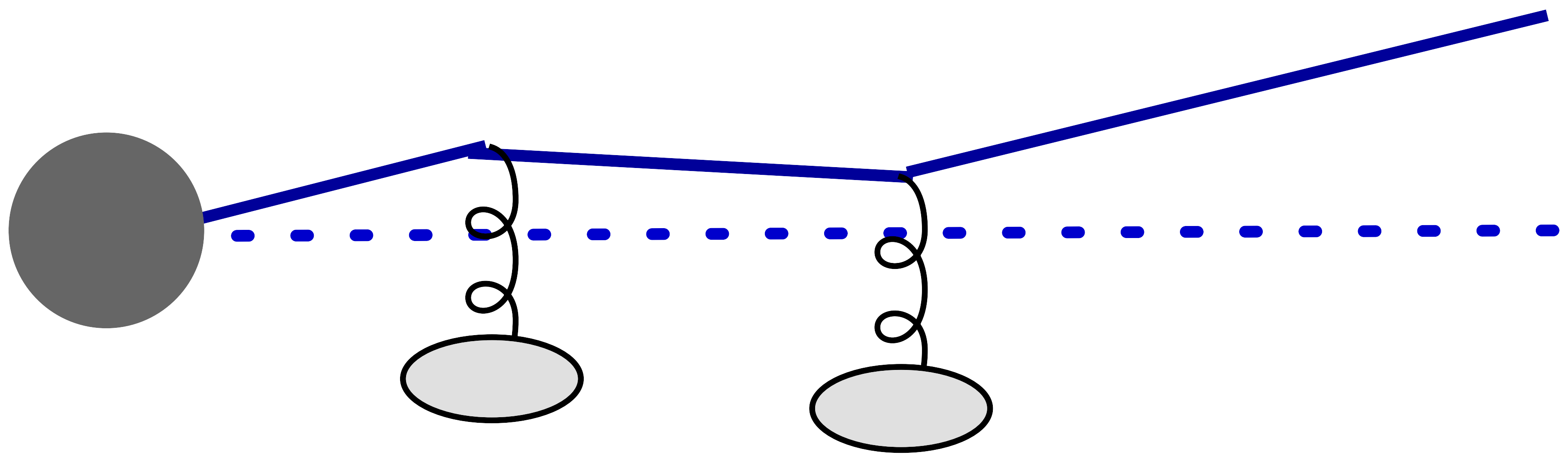
$$\langle \mathbf{p}\, \mathbf{p}^2 \rangle_{\mathbf{u}=0, \nabla \mathrm{T} \neq 0} \propto \left( \frac{\nabla T}{T} L \right) \frac{\mu^2 L}{E\lambda}$$

2110.03590, L. Antiporda et al

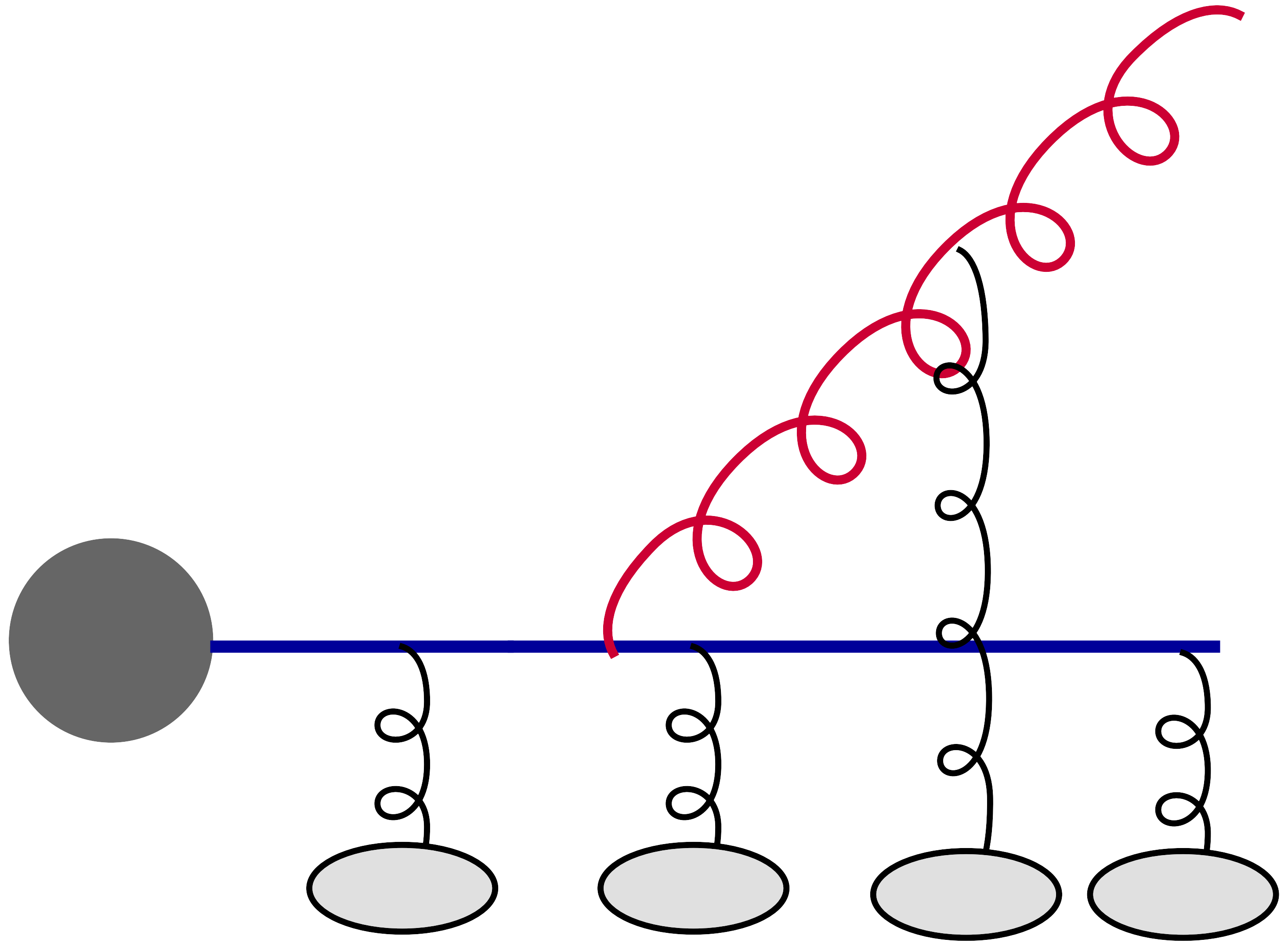


# Outline

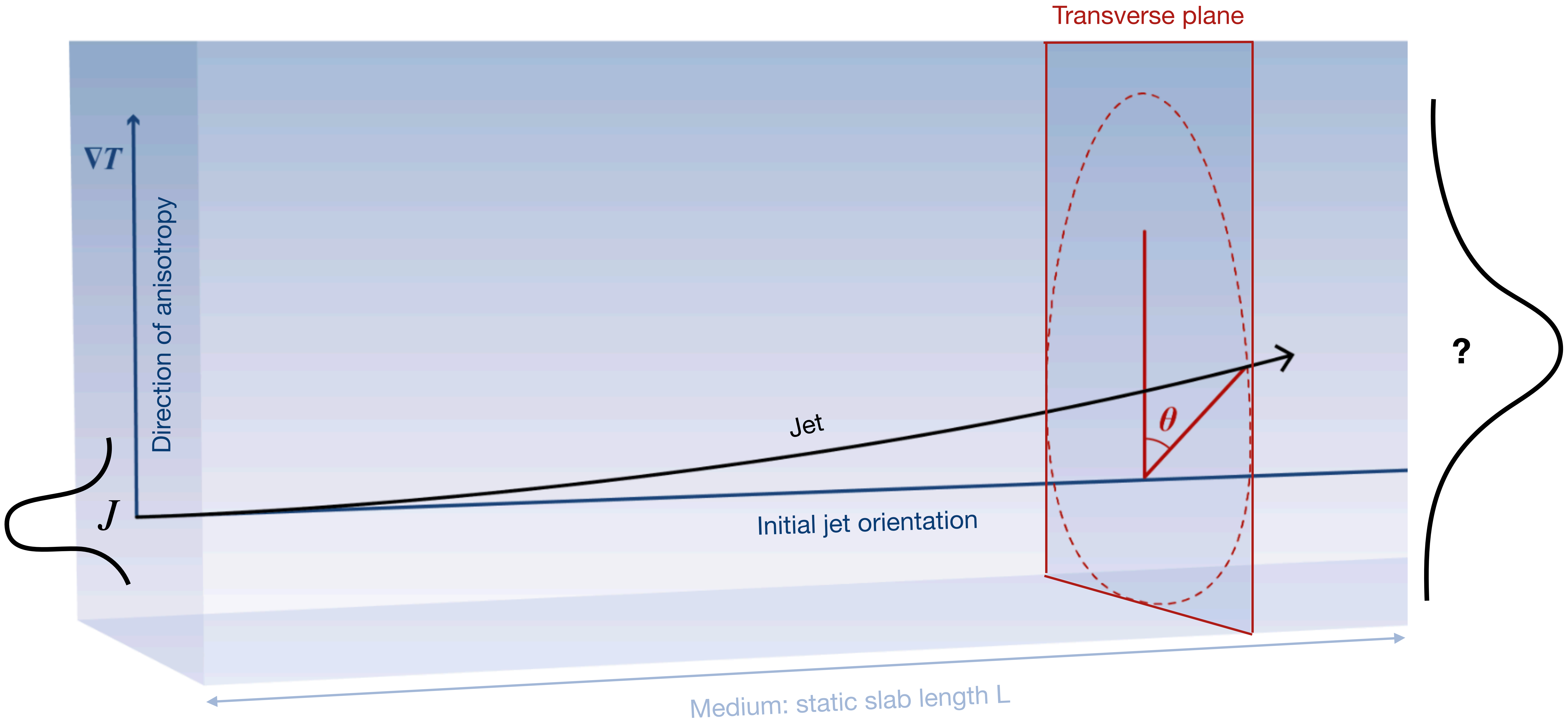
1 Momentum broadening in dense anisotropic media



2 Radiative energy loss in dense anisotropic media



# Momentum broadening in anisotropic media



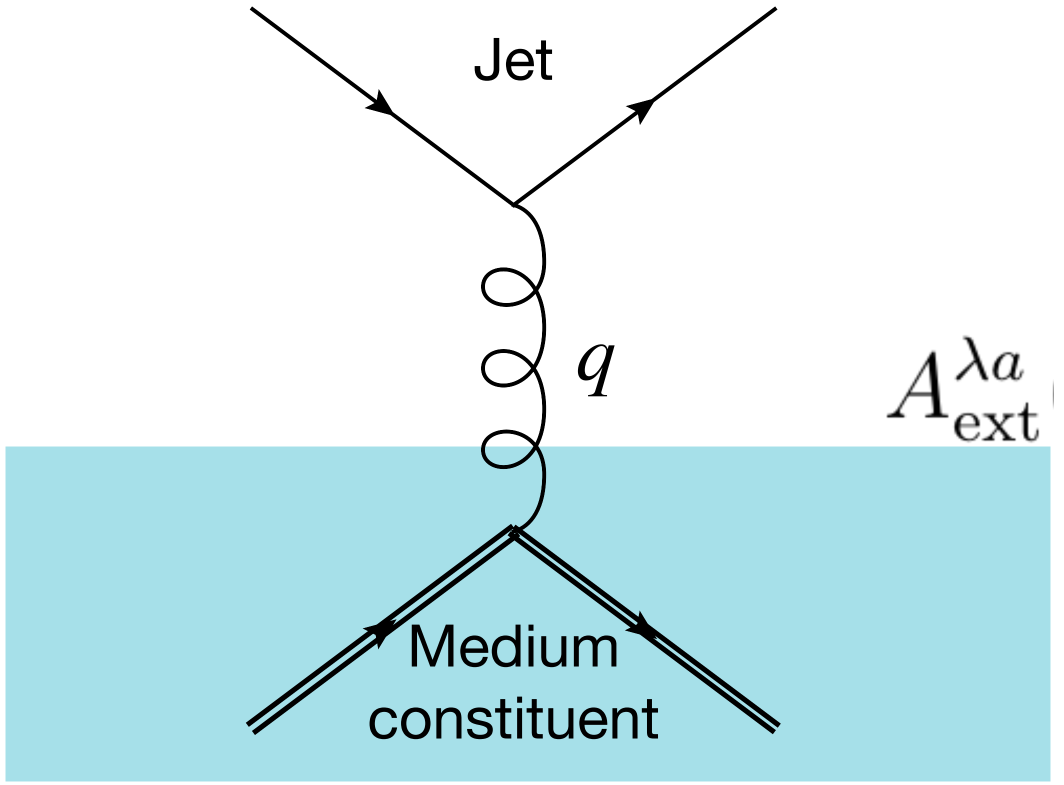
# Momentum broadening in anisotropic media

More details in for example:

1807.03799, M. Sievert, I. Vitev



## The medium is described by a classical field




Model dependent elastic scattering potential for source j

**No** energy transfer in each scattering: transverse t-channel gluon exchanges only

$$gA^{\lambda a}_{\text{ext}}(q) = -(2\pi)\, g^{\lambda 0} \sum_j e^{-i(\boldsymbol{q}\cdot\boldsymbol{x}_j + q_z z_j)}\, t^a_j\, v_j(q)\, \delta\left(q^0\right)$$

where we use the GW model

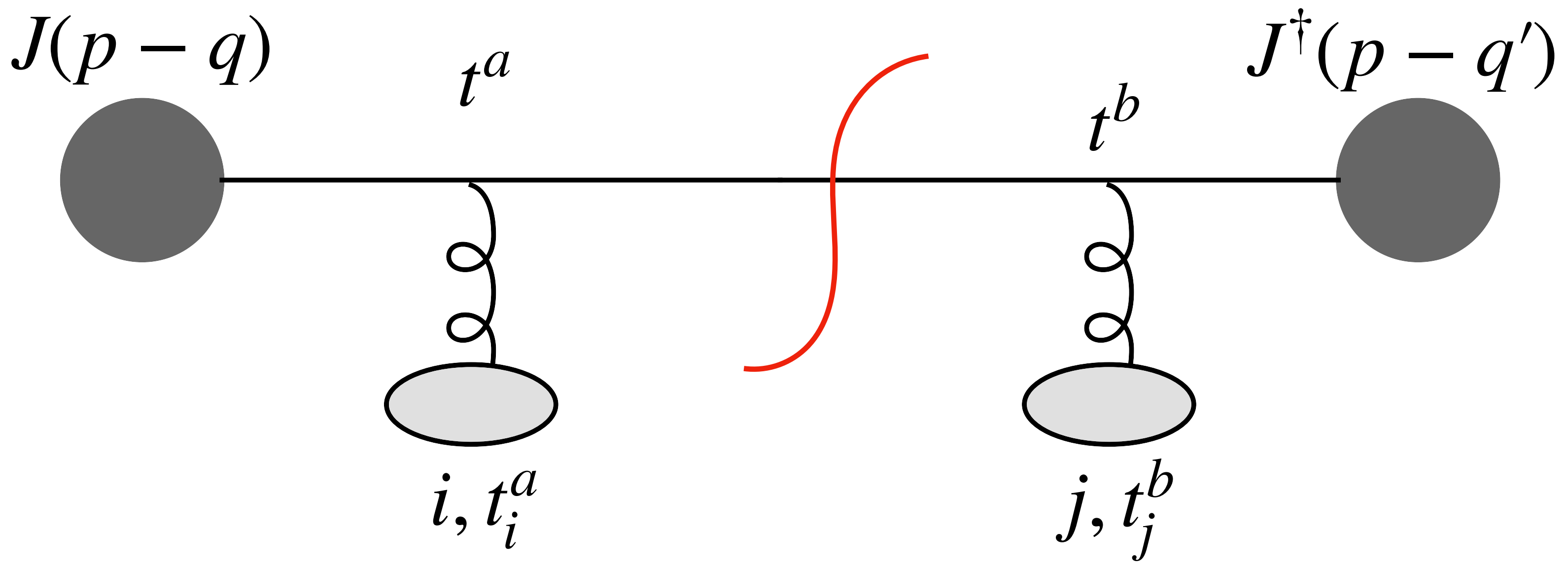
$$v_i(q) \equiv \frac{-g^2}{-q_0^2 + \boldsymbol{q}^2 + q_z^2 + \mu_i^2 - i\epsilon}$$

## Medium statistics follow from 2-gluon approximation

$$\langle t^a_i t^b_j \rangle = \frac{1}{d_{\text{tgt}}} \text{tr}\left(t^a_i t^b_j\right) = \frac{1}{2C_{\bar{R}}} \delta_{ij} \delta^{ab}$$

Only non-trivial correlator

Probe interacts with the same scattering center in amplitude and conjugate amplitude



# Momentum broadening in anisotropic media

More details in for example:

nucl-th/9306003, M. Gyulassy, X.-N. Wang

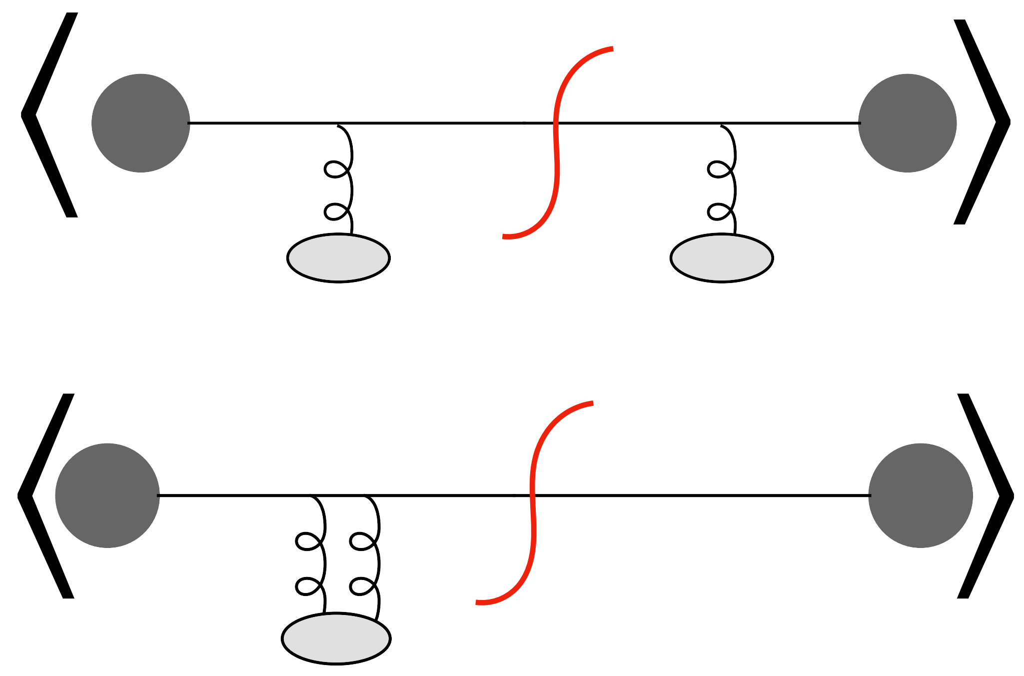
## 1 Compute all diagrams up to $2N$ field insertions

$Q_n \equiv \frac{\boldsymbol{p}_n^2 - \boldsymbol{p}_f^2}{2E}$ $\quad p_n = p_f - \sum_{m=n}^{N} q_m,\ p_{in} = p_1$

For example, the diagram with $r = N$ insertions at distinct $i_n$ reads

$$iM_r = \prod_{n=1}^{r} \left[ \sum_{i_n} \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2}\, i t^a_{\text{proj}} t^a_{i_n}\, \theta_{i_n, i_{n-1}}\, e^{-i\boldsymbol{q}_n \cdot \boldsymbol{x}_{i_n}} e^{-iQ_n\left(z_{i_n} - z_{i_{n-1}}\right)}\, v_{i_n}(q_n) \right] J(p_{in})$$

LPM phase factor



## 2 For each $N$, square and average the respective diagrams

$$\langle |M|^2 \rangle = \underbrace{\langle |M_0|^2 \rangle}_{N=0} + \underbrace{\langle |M_1|^2 \rangle + \langle M_2 M_0^* \rangle + \langle M_0 M_2^* \rangle}_{N=1} + \underbrace{\langle |M_2|^2 \rangle + \langle M_3 M_1^* \rangle + \langle M_1 M_3^* \rangle + \langle M_4 M_0^* \rangle + \langle M_0 M_4^* \rangle}_{N=2} + \ldots$$

The averaging is performed by taking the limit of continuous distribution in the medium

$$\sum_i f_i = \int d^2\boldsymbol{x}\, dz\, \rho(\boldsymbol{x}, z)\, f(\boldsymbol{x}, z) \xrightarrow[\nabla T = 0]{\rho(\boldsymbol{x}, z) \quad \mu^2(\boldsymbol{x}, z)} \int d^2\boldsymbol{x}_n\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = (2\pi)^2\, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

Density of scattering centers

# Momentum broadening in anisotropic media

More details in for example:

nucl-th/9306003, M. Gyulassy, X.-N. Wang

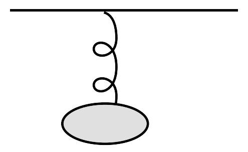
## 3 Resum the Opacity Series

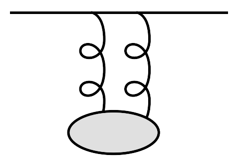
A detailed derivation shows that the square amplitude for $2N$ insertions has the form

$$\langle |M|^2 \rangle^{(N)} = \prod_{n=1}^{N} \left[ (-1) \int_0^{z_{n+1}} dz_n \int \frac{d^2 \boldsymbol{q}_n}{(2\pi)^2} \mathcal{V}(\boldsymbol{q}_n, z_n) \right] \left| J\left(E, \boldsymbol{p}_{in}\right) \right|^2$$

where we identify the effective scattering potential

$$\mathcal{V}(\boldsymbol{q}, z) \equiv -\mathcal{C}\,\rho(z) \left( \left| v(\boldsymbol{q}^2) \right|^2 - \delta^{(2)}(\boldsymbol{q}) \int d^2\boldsymbol{l}\, \left| v(\boldsymbol{l}^2) \right|^2 \right)$$





The resummation in this case gives:

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x} dE} = \sum_{N=0}^{\infty} \int \frac{d^2\boldsymbol{p}\, d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})} \frac{(-1)^N \left[\mathcal{V}(\boldsymbol{r})L\right]^N}{N!} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r} dE} = e^{-\mathcal{V}(\boldsymbol{x})L} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x} dE}$$

# Momentum broadening in anisotropic media

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2202.08847

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**When averaging in [2] we used**

$$\sum_i f_i = \int d^2\boldsymbol{x}\, dz\, \rho(\boldsymbol{x}, z)\, f(\boldsymbol{x}, z) \xrightarrow[\nabla T = 0]{} \int d^2\boldsymbol{x}_n\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = (2\pi)^2\, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

**For anisotropic media this no longer holds; hard to tackle in general**

We perform a gradient expansion for the 2 relevant parameters: $\rho$ and $\mu$ 2104.09513, A. Sadofyev, M. Sievert, I. Vitev

$$\rho(\boldsymbol{x}, z) \approx \rho(z) + \boldsymbol{\nabla}\rho(z)\cdot \boldsymbol{x} \qquad \mu^2(\boldsymbol{x}, z) \approx \mu^2(z) + \boldsymbol{\nabla}\mu^2(z)\cdot \boldsymbol{x}$$

So that when averaging instead of a momentum space Dirac delta one obtains

$$\int d^2\boldsymbol{x}_n\, x_n^\alpha\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = i\,(2\pi)^2 \frac{\partial}{\partial(q_n \pm \bar{q}_n)_\alpha}\, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

With this modification, we find that the $N$ order squared contribution now reads

$$\langle |M|^2 \rangle^{(N)} = \prod_{n=1}^{N} \left[ \int_0^{z_{n+1}} dz_n \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2} \right] \left( 1 + \frac{1}{E} \sum_{m=1}^{N} (z_m - z_{m-1}) \boldsymbol{p}_m \cdot \sum_{k=m}^{N} \left( \boldsymbol{\nabla}\rho \frac{\delta}{\delta \rho_k} + \boldsymbol{\nabla}\mu^2 \frac{\delta}{\delta \mu_k^2} \right) \right) (-1)^N \mathcal{V}_1(\boldsymbol{q}_1) \ldots \mathcal{V}_N(\boldsymbol{q}_N) |J(E, \boldsymbol{p}_{in})|^2$$

$$p_n = p_f - \sum_{m=n}^{N} q_m,\; p_{in} = p_1$$

# Momentum broadening in anisotropic media

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**Proceeding as in** 3 **we find that**

$$\mathcal{V}'(\boldsymbol{x}) \equiv \frac{\partial}{\partial\mu^2}\mathcal{V}(\boldsymbol{x})$$

$$\frac{d\mathcal{N}^{(N)}}{d^2\boldsymbol{x}dE} = \int \frac{d^2\boldsymbol{p}\,d^2\boldsymbol{r}}{(2\pi)^2}\,e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})}(-1)^N\left[\mathcal{V}(\boldsymbol{r})L\right]^N\left\{\frac{1}{N!}+\frac{L}{E(N+1)!}\times\sum_{m=1}^{N}\left[(N+1-m)\boldsymbol{p}\cdot\left(\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\boldsymbol{\nabla}\mu^2+\frac{1}{\rho}\boldsymbol{\nabla}\rho\right)+i(N+1-m)^2\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\rho\,\mathcal{V}(\boldsymbol{r})}\cdot\boldsymbol{\nabla}\rho\right.\right.$$
$$\left.\left.+\,i(N+1-m)\left(\frac{\boldsymbol{\nabla}\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}+(N-m)\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\right)\cdot\boldsymbol{\nabla}\mu^2\right]\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE}$$

**Resumming the opacity series then leads to the compact expression**

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = e^{-\mathcal{V}(\boldsymbol{x})L}\left\{\left[1-i\frac{\mathcal{V}(\boldsymbol{x})L^3}{6E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2+\frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+i\frac{\mathcal{V}(\boldsymbol{x})L^2}{2E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2+\frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

# Momentum broadening in anisotropic media

More details in for example:

1302.2579, Y. Mehtar-Tani, J. Milhano, K. Tywoniuk

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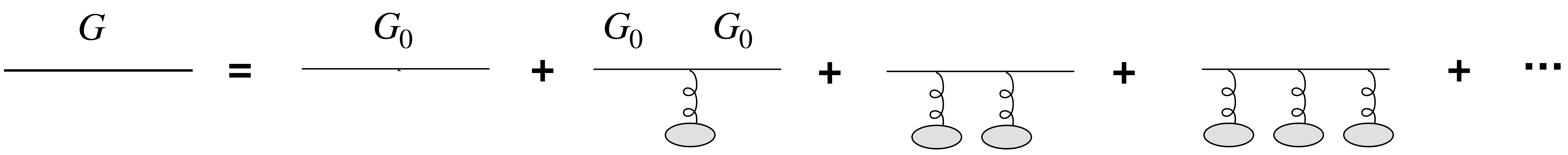
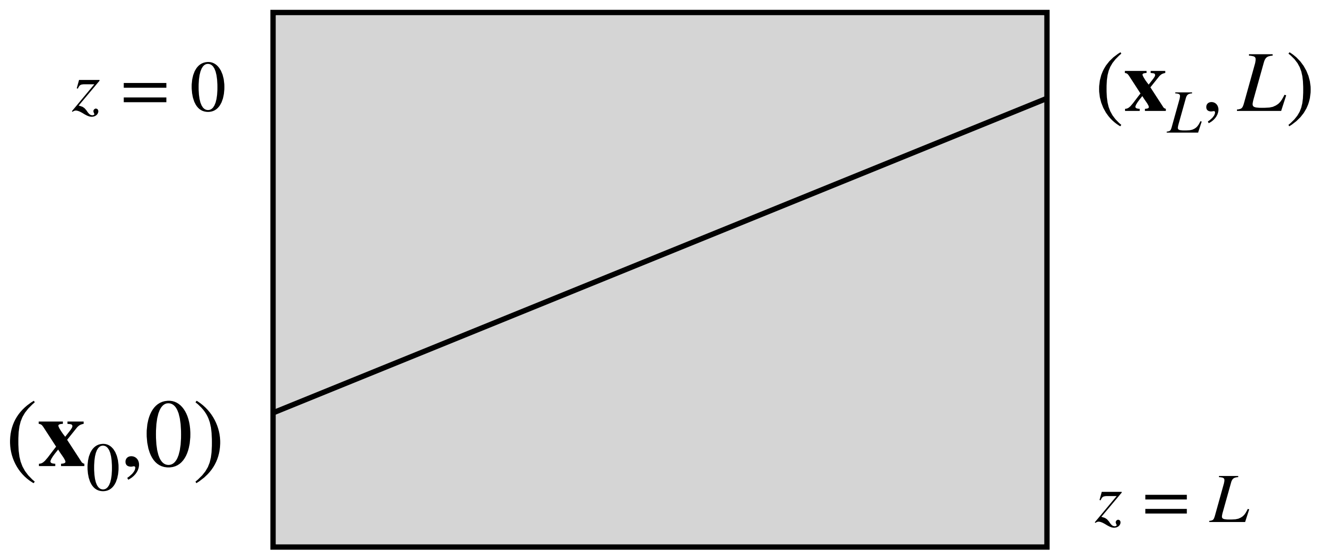
## 1 Compute an effective in-medium propagator



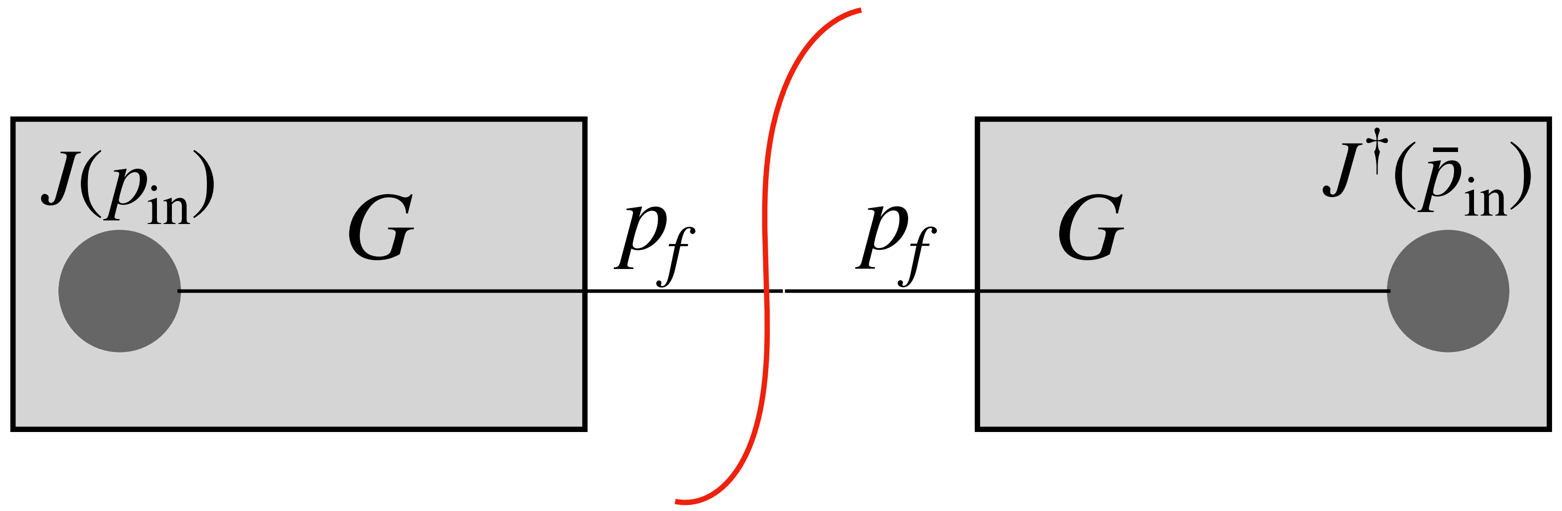

This results in an effective propagator $G$

$$G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_L} \mathcal{D}\boldsymbol{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\boldsymbol{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\boldsymbol{r}(\tau), \tau)\right)$$




## 2 Compute the relevant Feynman diagrams

$$\langle |M|^2 \rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$$




$$g A^{\mu a}_{\text{ext}}(q) = -(2\pi)\, g^{\mu 0}\, v^a(q)\, \delta\left(q^0\right) \qquad v^a(q) = \textstyle\sum_i e^{-i\vec{q}\cdot\vec{x}_i}\, t^a_i\, v_i(q)$$

# Momentum broadening in anisotropic media

More details in for example:

1302.2579, Y. Mehtar-Tani, J. Milhano, K. Tywoniuk

## 3 Solve the remaining average of dressed propagators

### Option 1) Solve first the path integrals and then average

Equivalent to Opacity Series approach

### Option 2) Perform the average before integration

In practice, by solving the remaining integrals one performs the resummation of averaged quantities directly

The **key step** is to use the fact that the color average of potential at different positions

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\overline{\boldsymbol{r}},\overline{\tau})\rangle = \mathcal{C}\, g^4 \int dz\, d^2\boldsymbol{x}\, \rho(\boldsymbol{x},\, z) \int \frac{d^2\boldsymbol{q}\, dq_z\, d^2\overline{\boldsymbol{q}}\, d\overline{q}_z}{(2\pi)^6} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{x})} e^{-i\overline{\boldsymbol{q}}\cdot(\overline{\boldsymbol{r}}-\boldsymbol{x})} e^{iq_z(\tau-z)} e^{-i\overline{q}_z(\overline{\tau}-z)}}{(\boldsymbol{q}^2+q_z^2+\mu^2(\boldsymbol{x},z))(\overline{\boldsymbol{q}}^2+\overline{q}_z^2+\mu^2(\boldsymbol{x},z))}$$

implies for $\nabla T = 0$

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right) \mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\rm proj} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}\left(\boldsymbol{r}(\tau)-\overline{\boldsymbol{r}}(\tau)\right)\right\}$$

# Momentum broadening in anisotropic media

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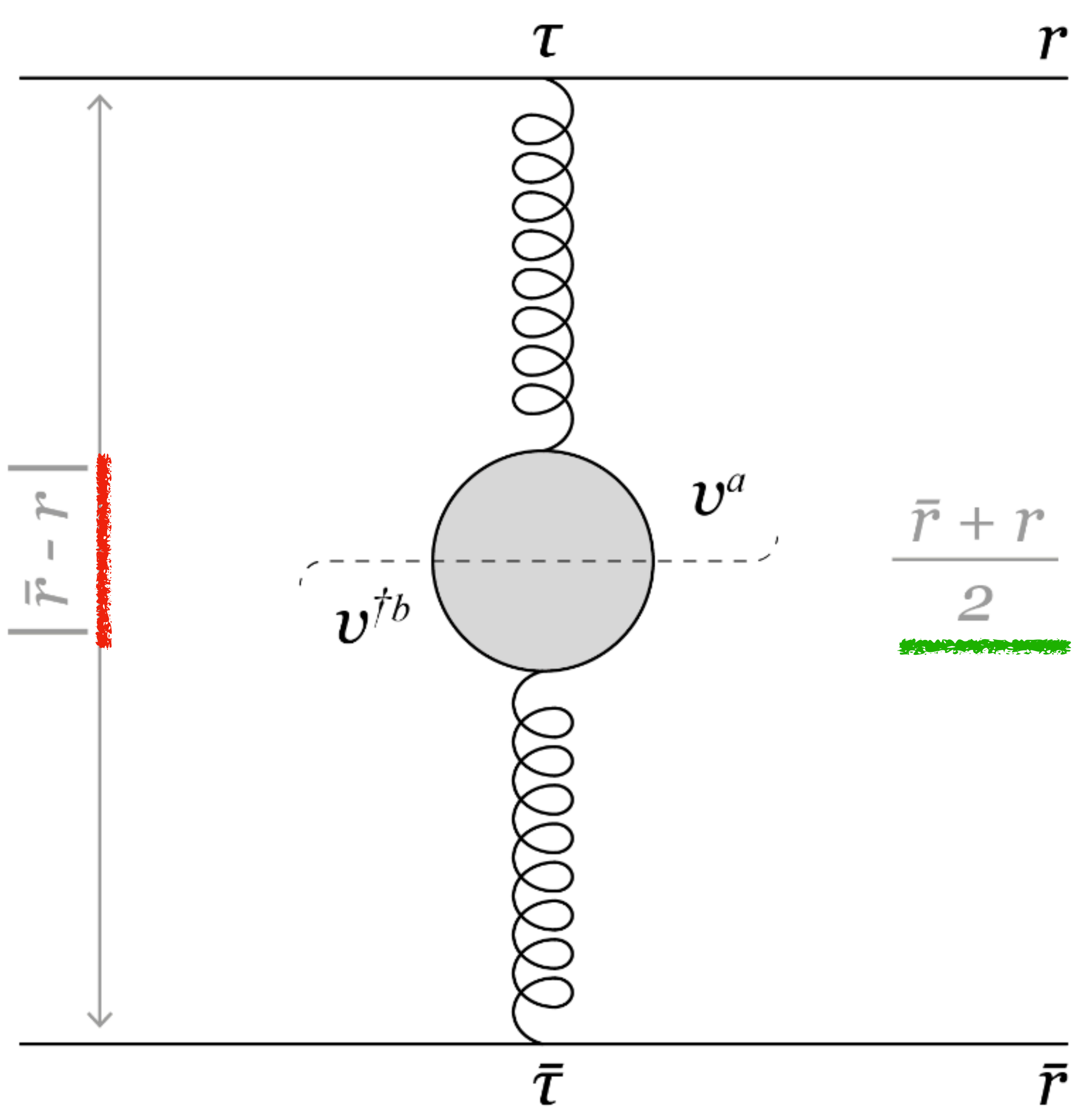
## To linear order in gradients from 3 we find now

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\bar{\boldsymbol{r}},\bar{\tau})\rangle \simeq \left(1 + \frac{\boldsymbol{r}(\tau)+\bar{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right) \mathcal{C}\,\delta(\tau-\bar{\tau})\,\rho\, g^4 \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\bar{\boldsymbol{r}})}}{(\boldsymbol{q}^2+\mu^2)^2}$$




One can still show that the 2-point correlator exponentiates

before

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\bar{\tau}\, t^b_{\rm proj} v^b(\bar{\boldsymbol{r}}(\bar{\tau}),\bar{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}\left(\boldsymbol{r}(\tau)-\bar{\boldsymbol{r}}(\tau)\right)\right\}$$

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\bar{\tau}\, t^b_{\rm proj} v^b(\bar{\boldsymbol{r}}(\bar{\tau}),\bar{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \left[1 + \frac{\boldsymbol{r}(\tau)+\bar{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right]\mathcal{V}\left(\boldsymbol{r}(\tau)-\bar{\boldsymbol{r}}(\tau)\right)\right\}$$

Center of mass of dipole — Dipole size

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**Combining all the results, computing** $\langle |M|^2\rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$

**is reduced to finding a close form for**

$$\left\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0)\right\rangle = \int_{\boldsymbol{u}_0}^{\boldsymbol{u}_L} \mathcal{D}\boldsymbol{u} \int_{\boldsymbol{w}_0}^{\boldsymbol{w}_L} \mathcal{D}\boldsymbol{w} \, \exp\left\{\int_0^L d\tau \, \left[iE\, \dot{\boldsymbol{u}} \cdot \dot{\boldsymbol{w}} - (1 + \boldsymbol{w} \cdot \hat{\boldsymbol{g}})\, \mathcal{V}\left(\boldsymbol{u}(\tau)\right)\right]\right\}$$

$$\boldsymbol{u} \equiv \boldsymbol{r} - \overline{\boldsymbol{r}}$$

$$\boldsymbol{w} \equiv \frac{\boldsymbol{r} + \overline{\boldsymbol{r}}}{2}$$

**After some algebra, the particle distribution reduces to**

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x} dE} \simeq \frac{1}{L^2} \int d^2\boldsymbol{u}_0 \, d^2\boldsymbol{u}_L \, \delta^{(2)}(\boldsymbol{x} - \boldsymbol{u}_L) \;\; \delta^{(2)}\left(\dot{\boldsymbol{u}}_c(L)\right) \exp\left\{-\int_0^L d\tau \, \mathcal{V}\left(\boldsymbol{u}_c(\tau)\right)\right\} \;\; \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{u}_0 dE}$$

↓

$$\boldsymbol{u}_c(\tau) = \boldsymbol{u}_L + \frac{i}{E}\, \hat{\boldsymbol{g}}\, \mathcal{V}\left(\boldsymbol{u}_L\right) \left\{\frac{(\tau - L)^2}{2}\right\}$$

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For real $J$ this leads to

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}\left(\boldsymbol{x}\right)L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x}\right)\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

Same result as in Opacity Expansion approach

For time dependent medium profile

$$\hat{\boldsymbol{g}}\left(\tau\right)=\left(\boldsymbol{\nabla}\rho(\tau)\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2(\tau)\frac{\delta}{\delta\mu^2}\right)$$

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\int_0^L d\tau\,\mathcal{V}\left(\boldsymbol{x},\tau\right)\right\}\left\{\left[1-\frac{i}{E}\int_0^L d\tau\,\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x},\tau\right)\cdot\left(\int_L^\tau d\zeta\int_0^\zeta d\xi+(L-\tau)\int_0^L d\xi\right)\hat{\boldsymbol{g}}(\xi)\mathcal{V}\left(\boldsymbol{x},\xi\right)\right]+\frac{i}{E}\int_0^L d\zeta\int_0^\zeta d\xi\,\hat{\boldsymbol{g}}\left(\xi\right)\mathcal{V}\left(\boldsymbol{x},\xi\right)\cdot\boldsymbol{\nabla}\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

# Momentum broadening distribution

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**The final distribution has the form**

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}(\boldsymbol{x})\,L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \mathcal{P}(\boldsymbol{x})\,\hat{\mathrm{S}}(\boldsymbol{x})\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

In the literature referred to as single particle broadening distribution (when Fourier transformed)

Usually a unit operator, but now it acts with $\nabla$ on initial distribution

Effective factorization no longer holds in general due to operator nature

Still
$$\int d^2\boldsymbol{p}\,\frac{d\mathcal{N}}{d^2\boldsymbol{p}\,dE} = \left.\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \left.\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \int d^2\boldsymbol{p}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE}$$

**We consider first the case of a source with finite width**

$$E\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE} = \frac{f(E)}{2\pi w^2}e^{-\frac{\boldsymbol{p}^2}{2w^2}}$$

It is possible to show that even though $\langle\boldsymbol{p}\rangle = 0$

higher odd moments can be generated, for example

$$\langle p^\alpha\,\boldsymbol{p}^2\rangle = \frac{w^2L^2\mu^2}{E\,\lambda}\frac{\nabla^\alpha\rho}{\rho}\ln\frac{E}{\mu} + \frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2$$

$N=1$ $\quad$ $N=2$

Higher $N$ terms dominate due to diverging potential at large momenta

Coulomb logarithm

# Momentum broadening distribution

J.B., A. Sadofyev, C. Salgado
2202.08847



**If we neglect initial state effects, then we are left with** $\chi = \frac{\mathcal{C} g^4 \rho}{4\pi\mu^2} L$ medium opacity

$$\mathcal{P}(\boldsymbol{p}) = \int d^2\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-\mathcal{V}(\boldsymbol{x})L} \left[1 - \frac{iL^3}{6E} \boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x}) \cdot \hat{\boldsymbol{g}}\mathcal{V}(\boldsymbol{x})\right]$$

where for GW model $\boldsymbol{p}^2 \gg \mu^2$

$$\frac{4L}{\chi}\mathcal{V}^{\mathrm{GW}}(\boldsymbol{x}) = \mu^2\boldsymbol{x}^2 \log\frac{4e^{1-2\gamma_E}}{\mu^2\boldsymbol{x}^2} + \mathcal{O}\left(\mu^4\boldsymbol{x}^4\right)$$

In the hard region where $\boldsymbol{p}^2 \gg \chi\mu^2$ it can be written in a closed form

$$\mathcal{P}(\boldsymbol{p}) \simeq \frac{4\pi\mu^2\chi}{\boldsymbol{p}^4} + \frac{16\pi\mu^4\chi^2}{\boldsymbol{p}^6}\left(\log\frac{\boldsymbol{p}^2}{\mu^2} - 2\right) + \frac{4\pi\mu^4\chi^2 L}{3E}\left[\frac{\boldsymbol{\nabla}\rho}{\rho}\left(\log\frac{\boldsymbol{p}^4}{\mu^4} - 4\right) - \frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right]\cdot\frac{\boldsymbol{p}}{\boldsymbol{p}^6}$$

Coulomb tail

In the complementary region where $\mu^2 \ll \boldsymbol{p}^2 \leq \chi\mu^2$ one has $\frac{4L}{\chi}\mathcal{V}^{\mathrm{GW}}(\boldsymbol{x}) \simeq \mu^2\boldsymbol{x}^2\left(\log\frac{Q^2}{\mu^2} + \log\frac{4e^{1-2\gamma_E}}{Q^2\boldsymbol{x}^2}\right)$

$$\mathcal{P}(\boldsymbol{p}) = \frac{4\pi}{\chi\mu^2\log\frac{Q^2}{\mu^2}}\left[1 + \frac{L}{6E}\frac{\boldsymbol{p}^2 - 2\chi\mu^2\log\frac{Q^2}{\mu^2}}{\chi\mu^2\log\frac{Q^2}{\mu^2}}\left(\frac{\boldsymbol{\nabla}\rho}{\rho} - \frac{1}{\log\frac{Q^2}{\mu^2}}\frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right)\cdot\boldsymbol{p}\right] e^{-\frac{\boldsymbol{p}^2}{\chi\mu^2\log\frac{Q^2}{\mu^2}}}$$

Usual Gaussian distribution

Gradient effects seem to be more relevant around the distribution average

# Momentum broadening distribution

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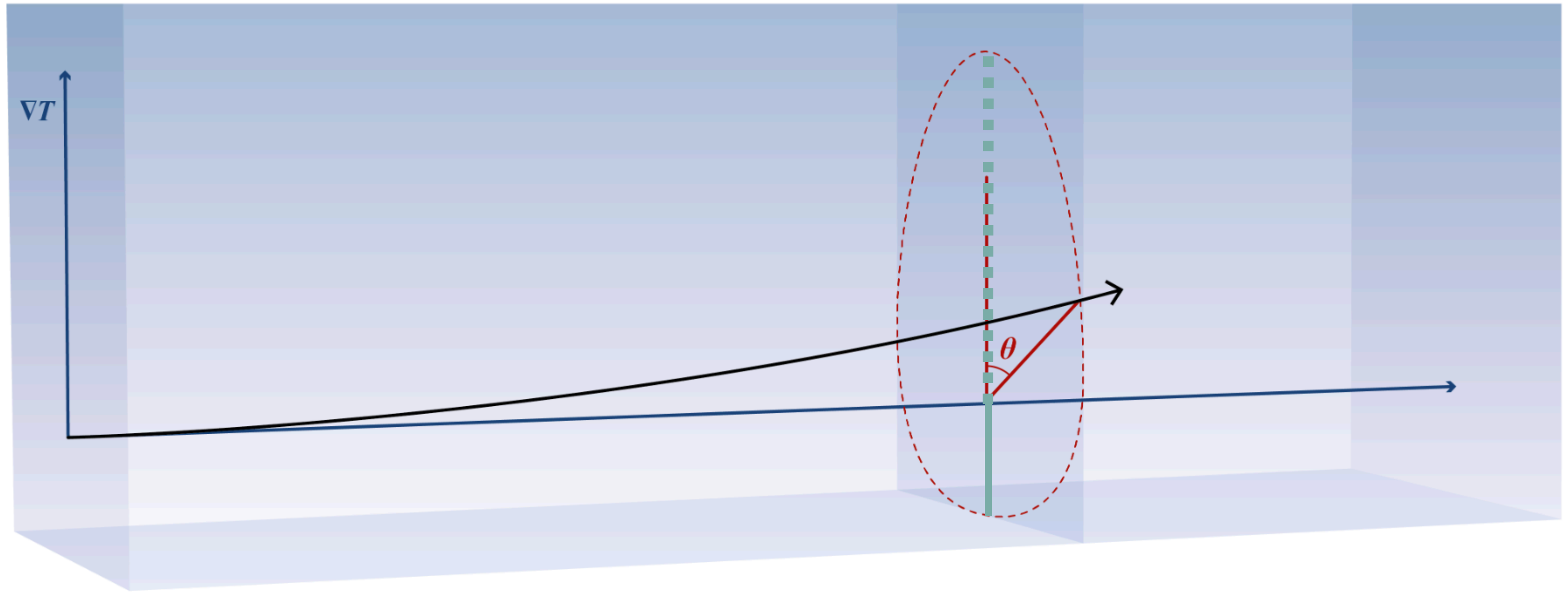
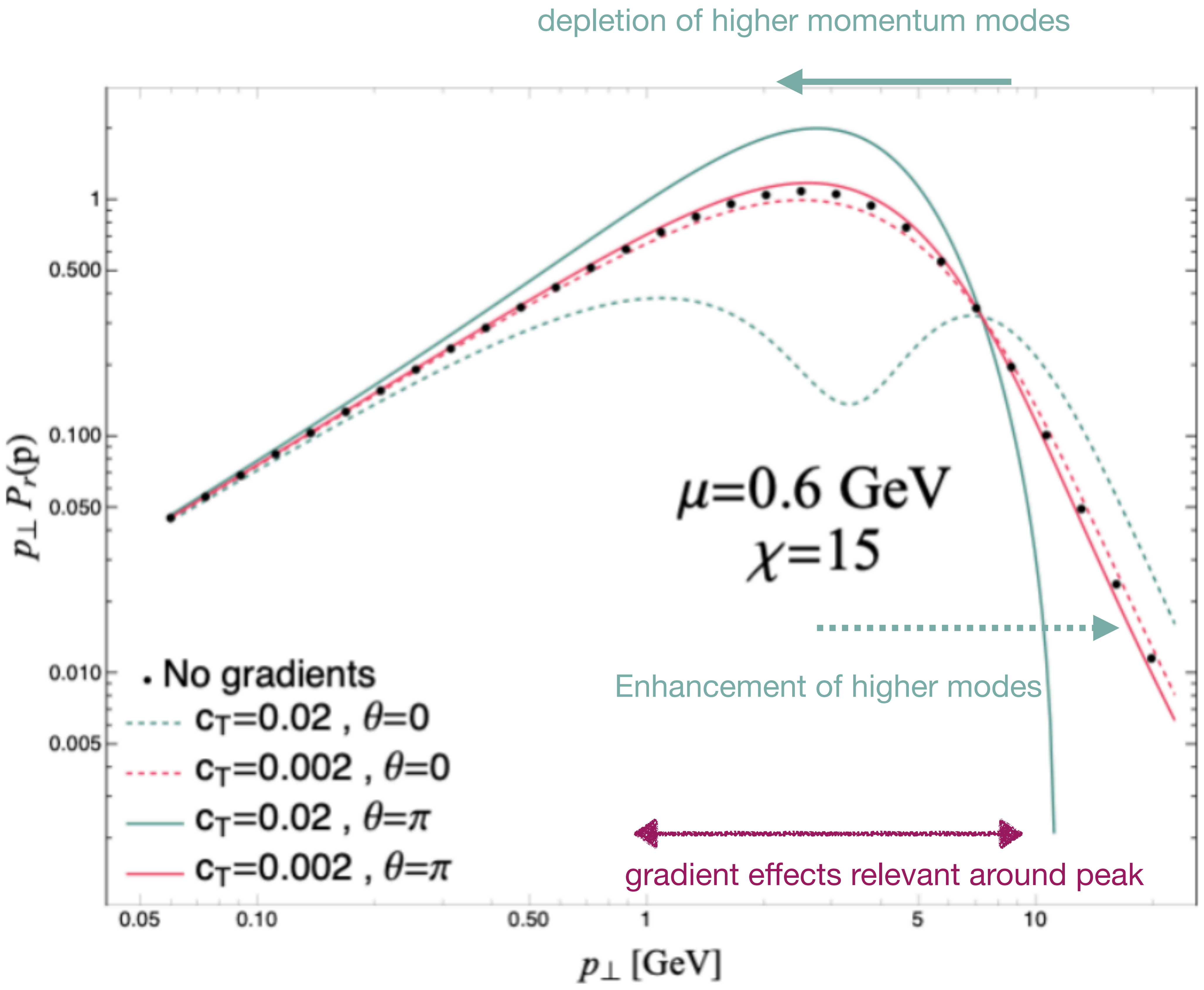
## For the full GW model we have




The full distribution is written in terms of the angle $\theta$ and parameter $c_T \equiv \left|\frac{\nabla T}{ET}\right|$.

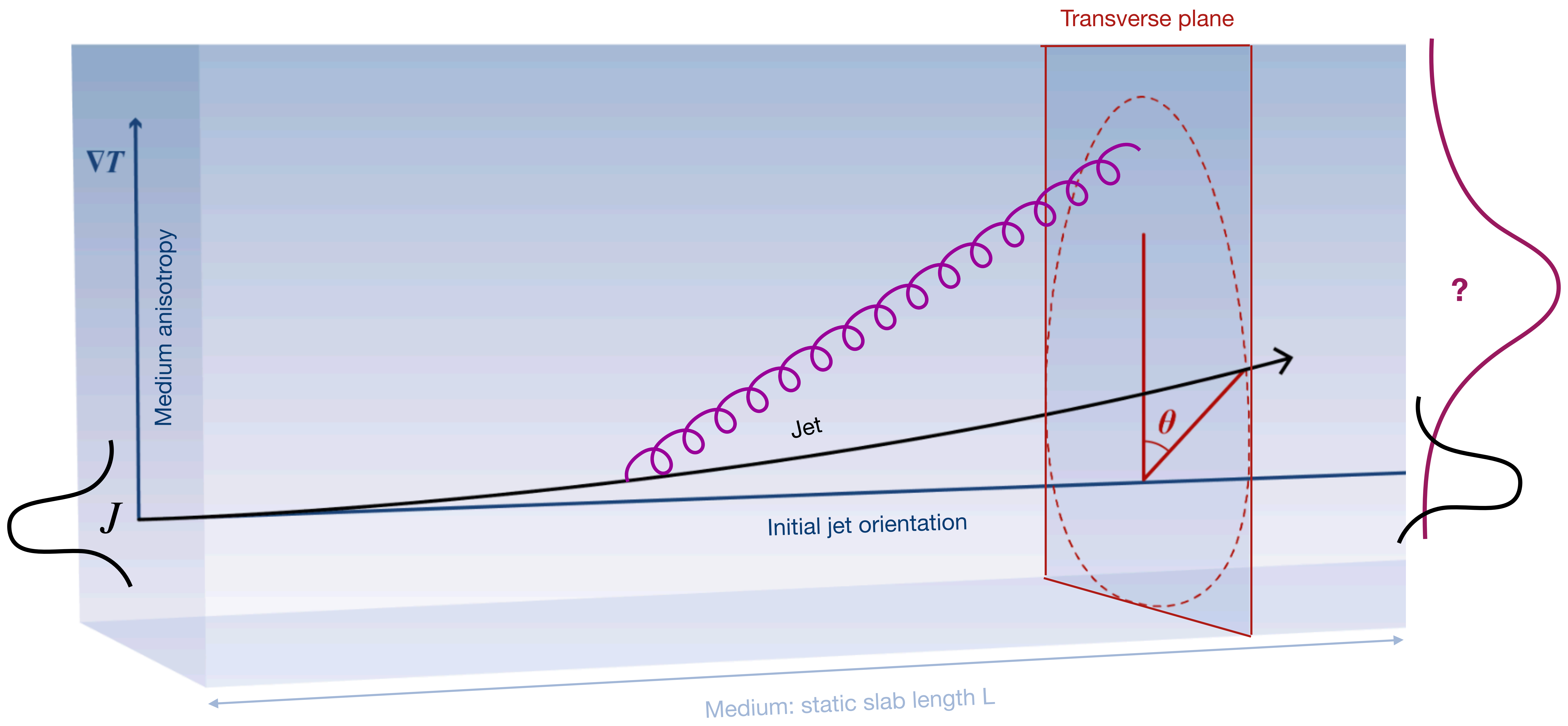
$$\mathcal{P}(\boldsymbol{p}) = 2\pi \int_0^\infty dx_\perp \, x_\perp \, e^{-\mathcal{V}^{\text{GW}}(x_\perp)L} \Bigg\{ J_0(p_\perp x_\perp) - \frac{\chi^2 \mu^2 L}{6} c_T \, x_\perp \, K_0(\mu \, x_\perp) \, J_1(p_\perp x_\perp) \times \left[1 - 3\mu \, x_\perp K_1(\mu \, x_\perp) + \mu^2 x_\perp^2 K_2(\mu \, x_\perp)\right] \cos\theta \Bigg\}$$




$$\frac{\nabla \rho}{\rho} \sim 3 \frac{\nabla T}{T}, \quad \frac{\nabla \mu^2}{\mu^2} \sim 2 \frac{\nabla T}{T}$$

# Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado



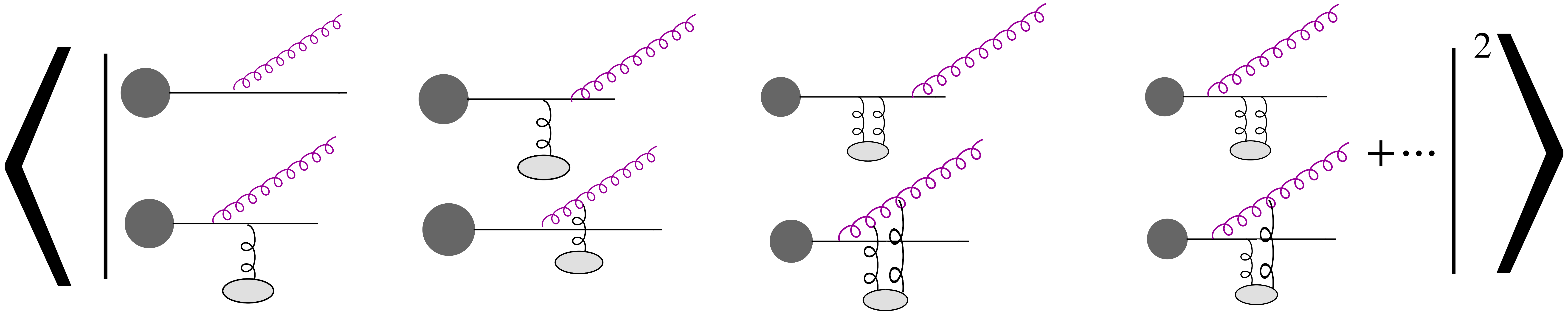
# Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado



In Opacity Expansion style calculation one needs to compute

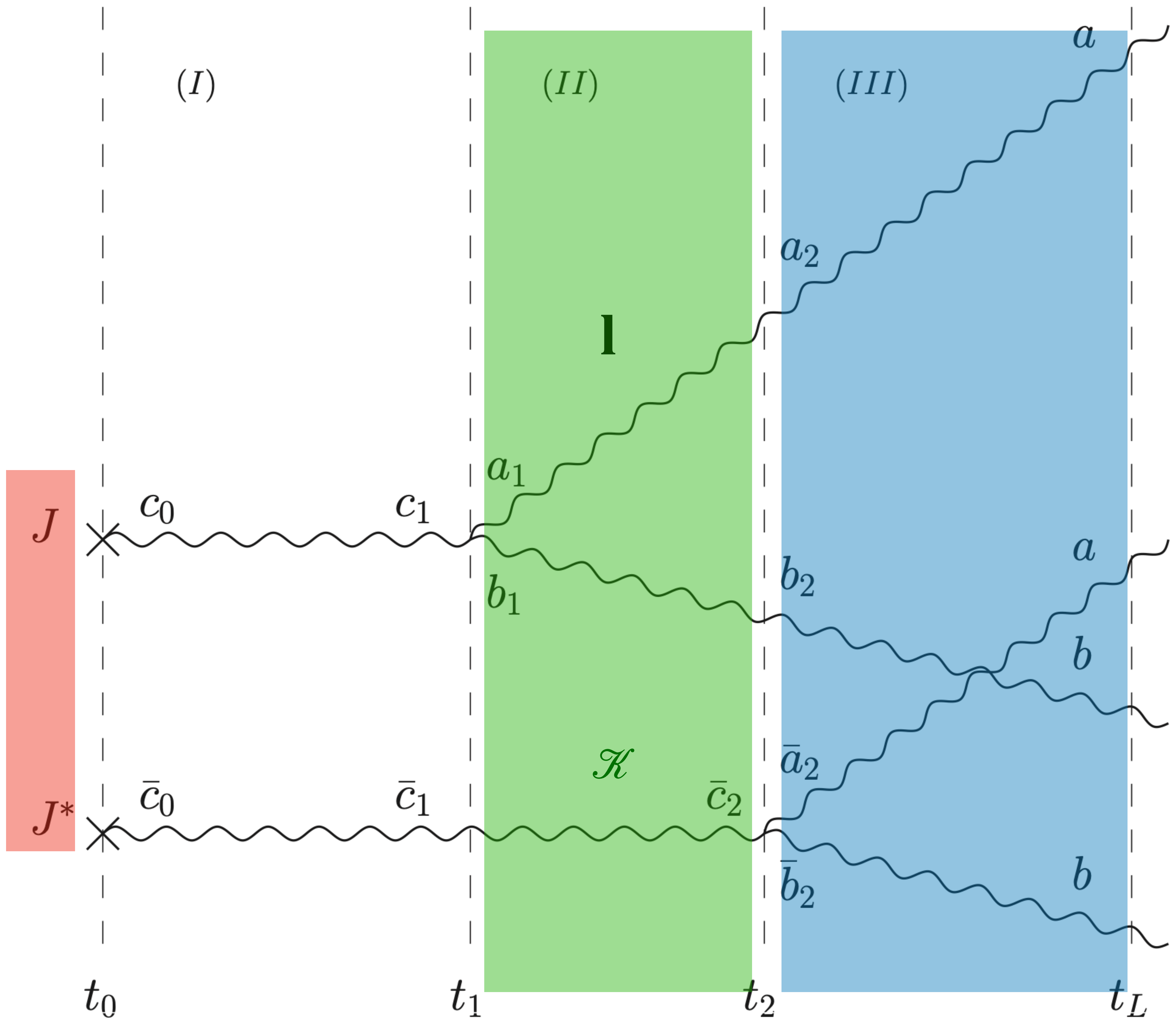
Beyond N=1, this seems to be quite hard to do



# Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado

## In the BDMPS-Z style calculation only averaging changes

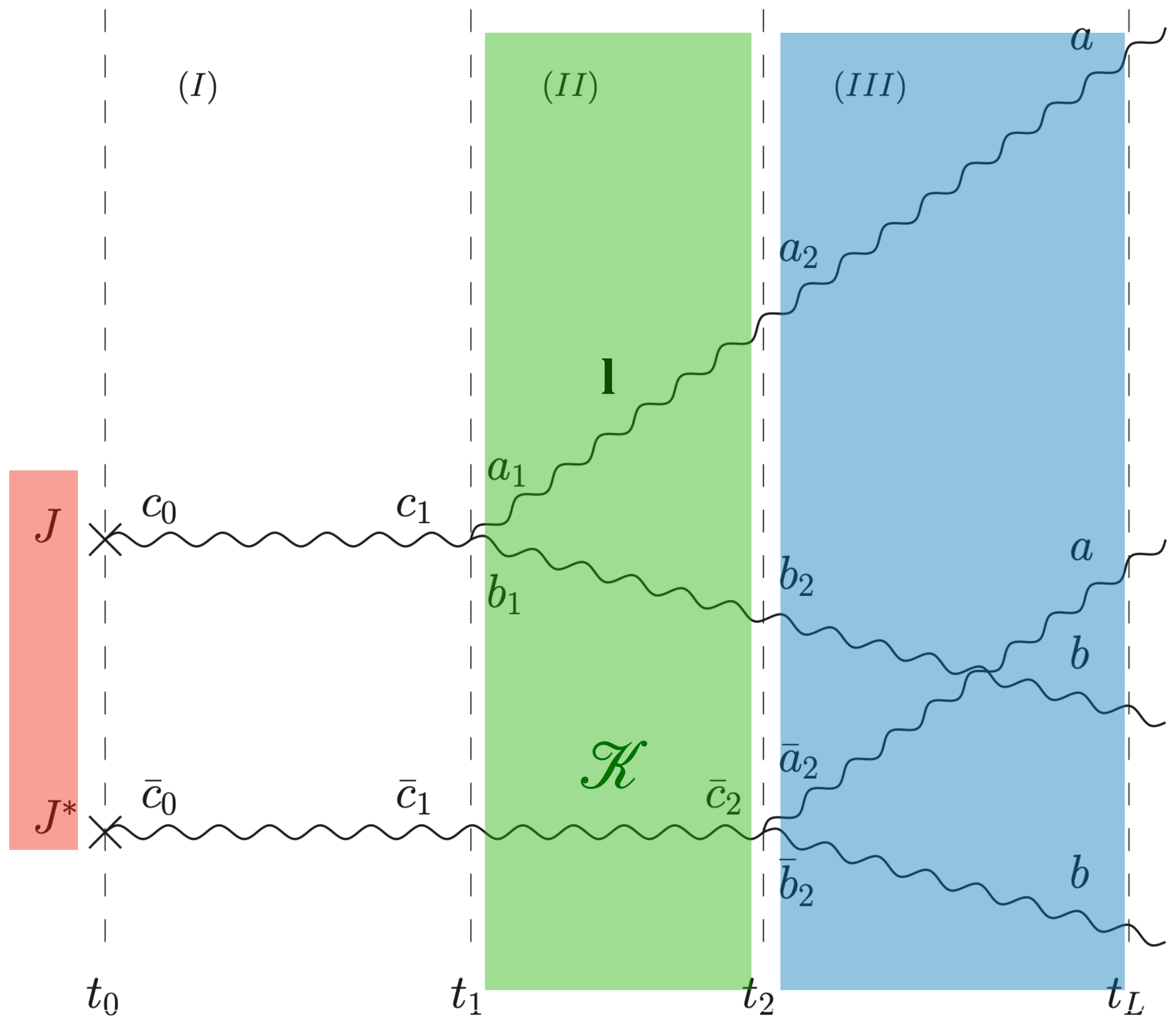



see e.g. 1209.4585, J.-P. Blaizot, F. Dominguez, E. Iancu, Y. Mehtar-Tani

# Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado






In this case, we can write the squared amplitude as

$$dN = \frac{\alpha_s C_F}{\omega^2} 2\Re \Big[ \int_{\bar{s} s \boldsymbol{z} \boldsymbol{x}_0} \partial_{\boldsymbol{y}=\boldsymbol{x}_0} \mathcal{K}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{x}_0) \cdot \partial_{\bar{\boldsymbol{y}}=\boldsymbol{x}_0} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}} e^{i\boldsymbol{q}_2\cdot\bar{\boldsymbol{y}}} S^{(2)}(\boldsymbol{k}, \boldsymbol{k}, \infty; \boldsymbol{q}_1, \boldsymbol{q}_2, \bar{s}) |J(\boldsymbol{x}_0)|^2$$

Solved!

$$\mathcal{K}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{x}_0) = \langle G_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s) W^\dagger(\boldsymbol{x}_0; \bar{s}, s)\rangle$$

For an arbitrary kernel we can always write $dN = dN^{(0)} + dN^{(1)}$

We take the splitting: $\mathcal{V}^{(0)}(\boldsymbol{x}, \boldsymbol{y}) = \frac{\hat{q}}{4}(\boldsymbol{x}-\boldsymbol{y})^2 \qquad \mathcal{V}^{(1)}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{F} \cdot \frac{\boldsymbol{x}+\boldsymbol{y}}{2} \frac{\hat{q}}{4}(\boldsymbol{x}-\boldsymbol{y})^2 \qquad \boldsymbol{F} = \nabla\rho\,\delta_\rho$

$$dN^{(0)} = \frac{\alpha_s C_F}{\omega^2} 2\Re \int_{\bar{s} s \boldsymbol{z}} e^{-i\boldsymbol{k}\cdot\boldsymbol{z}} \mathcal{P}(\boldsymbol{z}, L-\bar{s}) \times \left[ -(L-\bar{s})\mathcal{V}(\boldsymbol{z})\boldsymbol{F} + \left(1 - (L-\bar{s})\mathcal{V}(\boldsymbol{z})\boldsymbol{F}\cdot\frac{\boldsymbol{z}}{2} + \frac{i(L-\bar{s})^2}{2\omega}\mathcal{V}(\boldsymbol{z})\boldsymbol{F}\cdot\partial_{\boldsymbol{z}}\right)\partial_{\boldsymbol{z}}\right] \cdot \partial_{\boldsymbol{y}=\boldsymbol{0}} \mathcal{K}^{(0)}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{0})$$

$$dN^{(1)} = \frac{\alpha_s C_F}{\omega^2} 2\Re \int_{\bar{s} s \boldsymbol{z}} e^{-i\boldsymbol{k}\cdot\boldsymbol{z}} \mathcal{P}^{(0)}(\boldsymbol{z}, L-\bar{s})\, \partial_{\boldsymbol{z}} \cdot \partial_{\boldsymbol{y}=\boldsymbol{0}} \mathcal{K}^{(1)}_\omega(\boldsymbol{z}, \bar{s}; \boldsymbol{y}, s|\boldsymbol{0})$$

# Induced soft gluon spectrum

J.B., X. Mayo, A. Sadofyev, C. Salgado

Brookhaven National Laboratory

## One can compute the spectrum at linear order in gradients with multiple soft insertions

The final result is not particularly illuminating. In summary:

- → The same result is (seemingly) harder to recover by resuming the Opacity Series
- → Gradient effects do not affect energy loss, as expected

## For pheno it is important to have the soft gluon limit: still holds

$$dN^{(0)}_{\omega\ll\omega_c} \approx \int_0^L d\bar{s}\,\mathcal{P}(\boldsymbol{k}, L-\bar{s})\frac{\omega\, dI^{(0)}}{d\omega d\bar{s}}$$

**Very schematically:** In soft gluon limit $k_b^2 \gg k_f^2$

$$dN = \frac{\alpha_s C_F}{\omega^2} 2\Re\Big[\int_{\bar{s}s\boldsymbol{z}\boldsymbol{x}_0} \partial_{\boldsymbol{y}=\boldsymbol{x}_0}\mathcal{K}_\omega(\boldsymbol{z},\bar{s};\boldsymbol{y},s|\boldsymbol{x}_0)\cdot\partial_{\bar{\boldsymbol{y}}=\boldsymbol{x}_0}\int_{\boldsymbol{q}_1\boldsymbol{q}_2} e^{-i\boldsymbol{q}_1\cdot\boldsymbol{z}}e^{i\boldsymbol{q}_2\cdot\bar{\boldsymbol{y}}} S^{(2)}(\boldsymbol{k},\boldsymbol{k},\infty;\boldsymbol{q}_1,\boldsymbol{q}_2,\bar{s})|J(\boldsymbol{x}_0)|^2$$

Gluons typically acquire momentum $k_f^2 \sim \sqrt{\hat{q}\omega}$

Gluons typically acquire momentum $k_b^2 \sim \hat{q}L$

# Conclusion

Brookhaven National Laboratory

## 1 Momentum broadening in dense anisotropic media

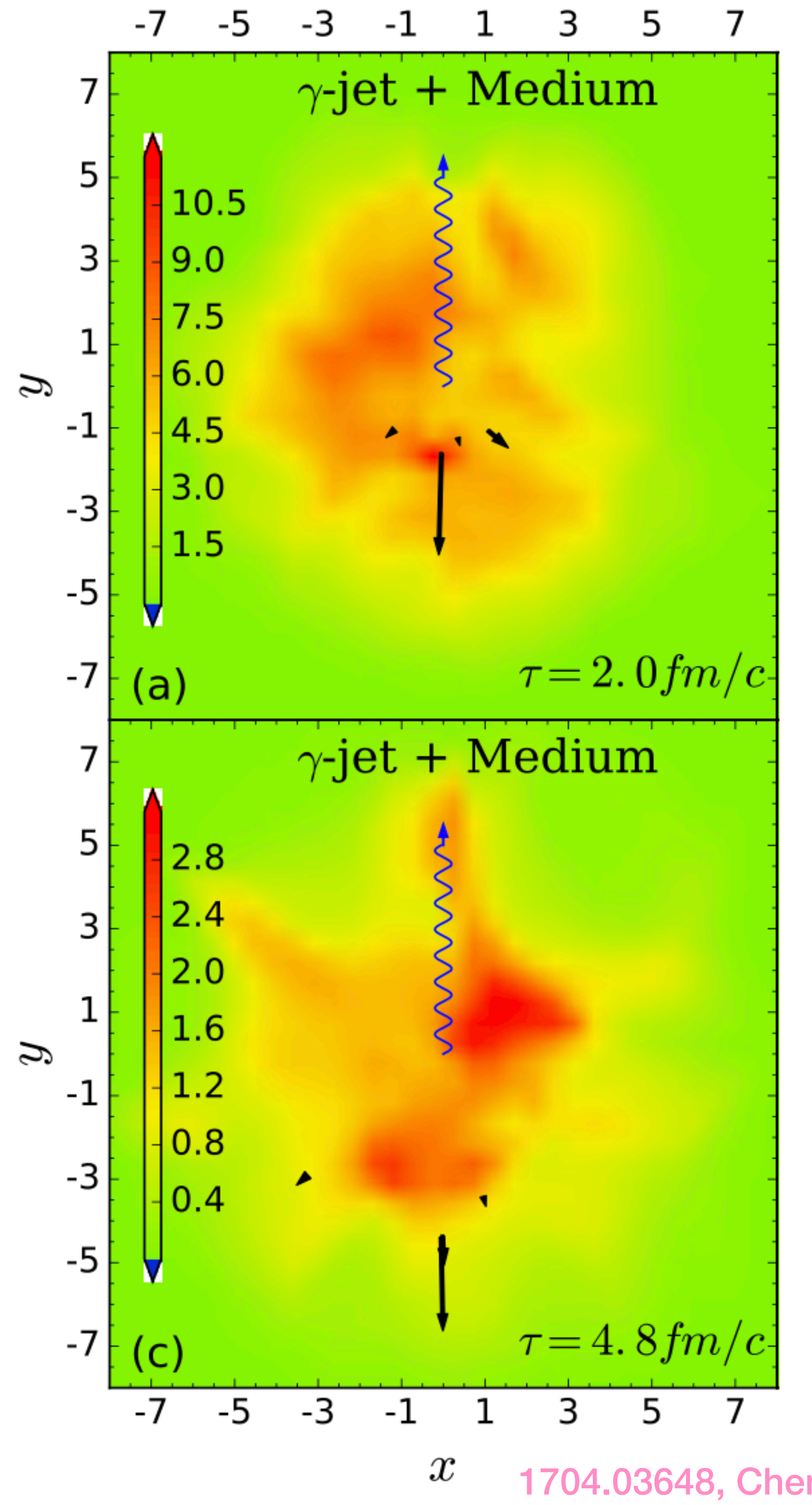
- The **broadening distribution** can be resumed for **non-flowing anisotropic** media
- Final distribution gives parametrically small corrections to the leading result. **However,** these contribute at leading order in the azimuthal distribution

## 2 Radiative energy loss in dense anisotropic media

- **BDMPS-Z** style calculation is in principle possible in the multiple scattering regime
- Pheno oriented **effective soft factorization is not broken**

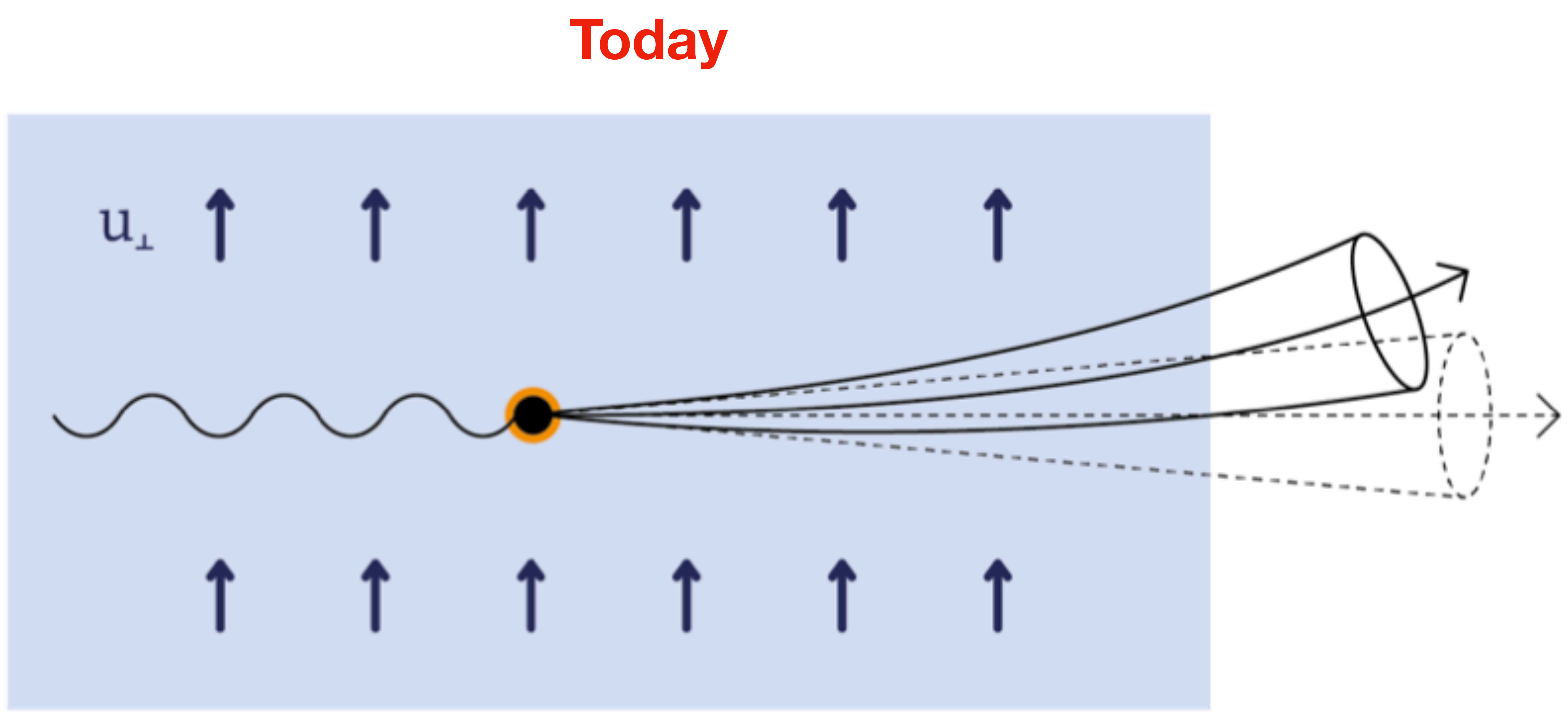
→ Further observable oriented calculations are needed to gauge and extract these effects

# Outlook




1704.03648, Chen et al

**Medium response to jets**




**Jet response to medium**

# Outlook

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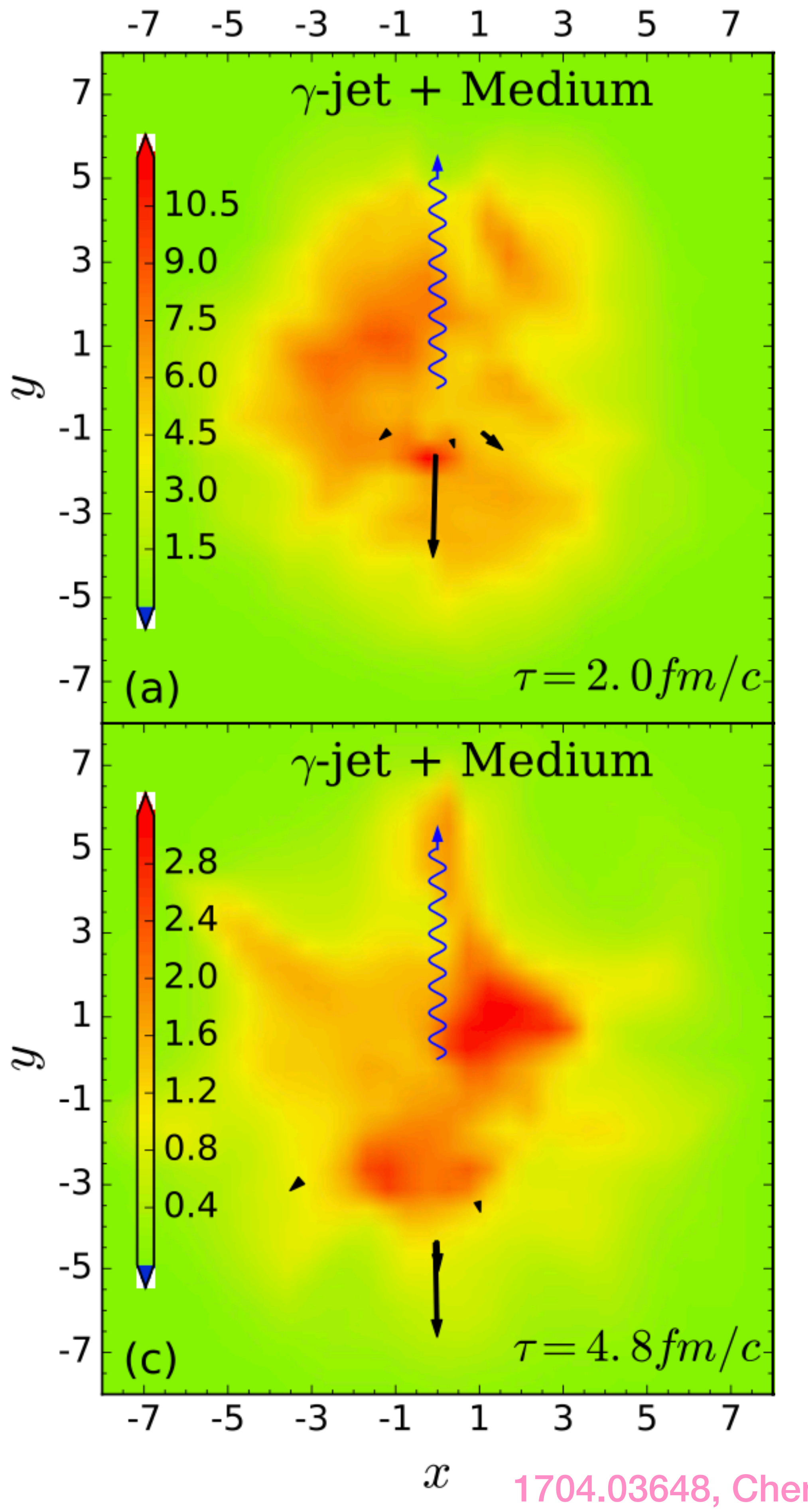
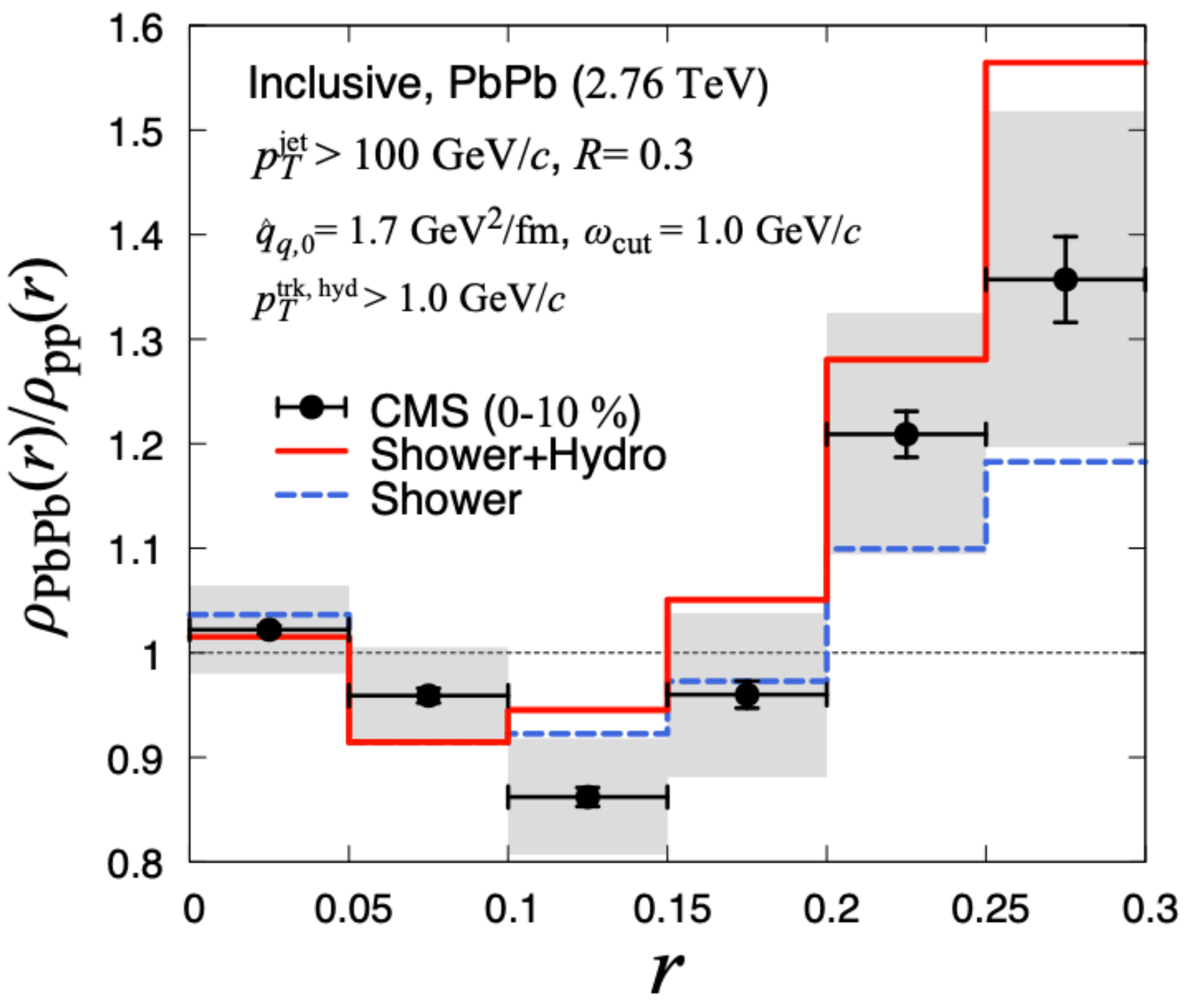



1704.03648, Chen et al

**Medium response to jets**

No "analytical" approach, but fundamental for pheno




1701.07951, Tachibana et al

**Requires more modeling**