

High precision FF measurements  
at BELLE  
or experience on systematic  
limitations

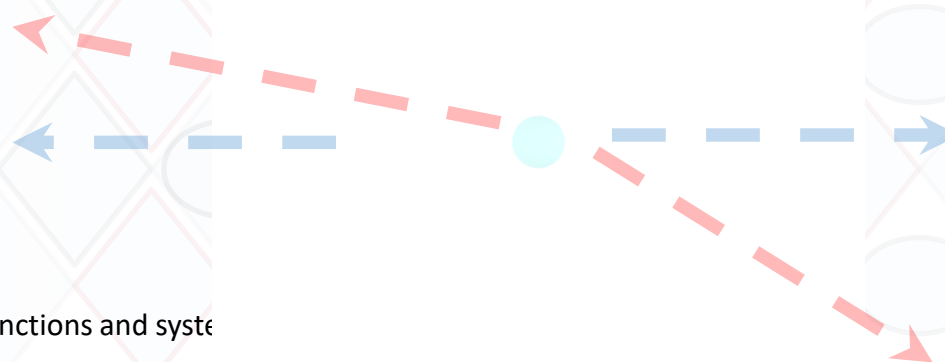
**HL EIC workshop**  
**June 23, 2022**

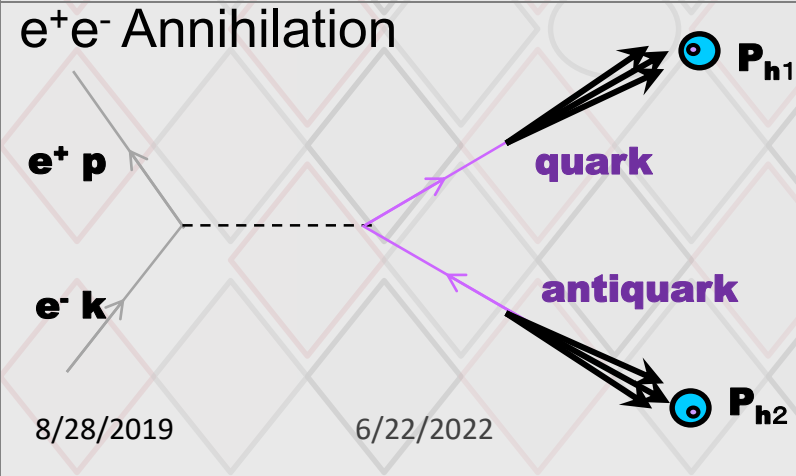
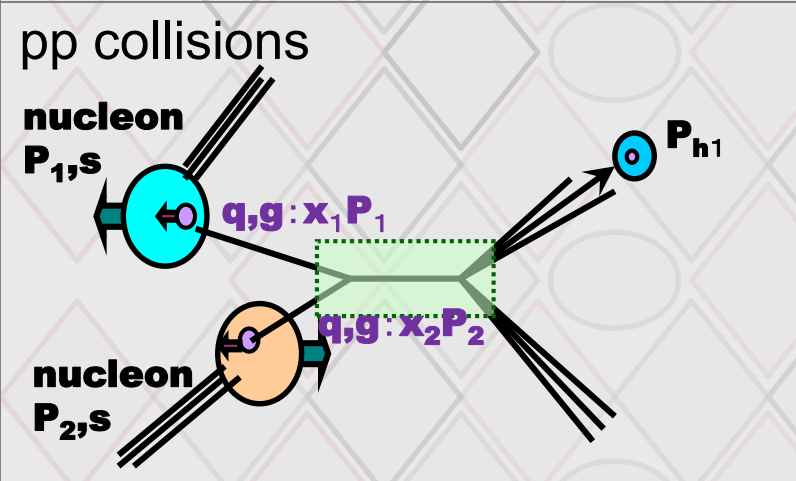
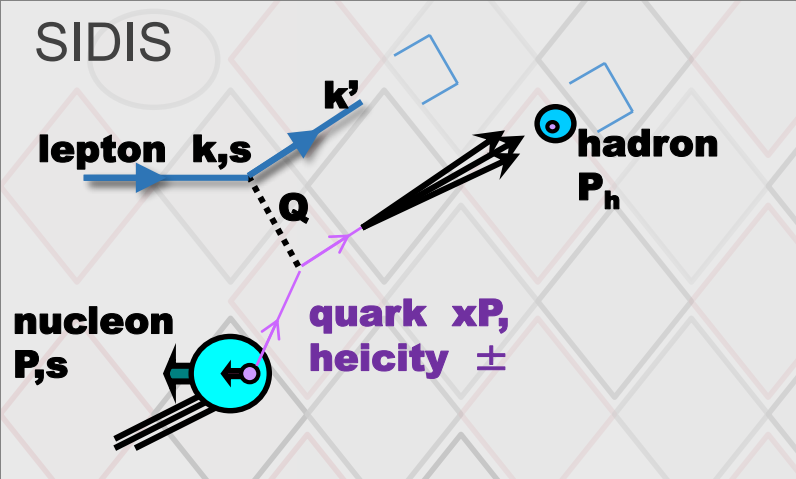
**Ralf Seidl (RIKEN)**

# Fragmentation functions and (spin) structure of the nucleon

- Unpolarized fragmentation functions:
  - Provide **flavor** information in nucleon/nuclei
  - Most apparent in SIDIS measurements related to  $\Delta q(x)$
  - But also required for all RHIC hadron asymmetries (especially pion  $A_{LL}$  charge ordering)
  - Transverse momentum dependence needed for Sivers and other TMDs

- Polarized fragmentation functions:
  - For transverse **spin** almost unique access (require two chiral-odd functions):
    - DY:  $\delta q \times \delta q$  or
    - SIDIS/RHIC:  $\delta q \times \text{Collins}$  or  $\delta q \times \text{IFF}$
  - FFs from Belle/Babar





# Access to FFs

- **SIDIS:**

$$\sigma^h(x, z, Q^2, P_{h\perp}) \propto \sum_q e_q^2 q(x, p_t, Q^2) D_{1,q}^h(z, k_t, Q^2)$$
  - Relies on unpol PDFs
  - Parton momentum known at LO
  - Flavor structure directly accessible
  - Transverse momenta convoluted between FF and PDF
- **pp:**

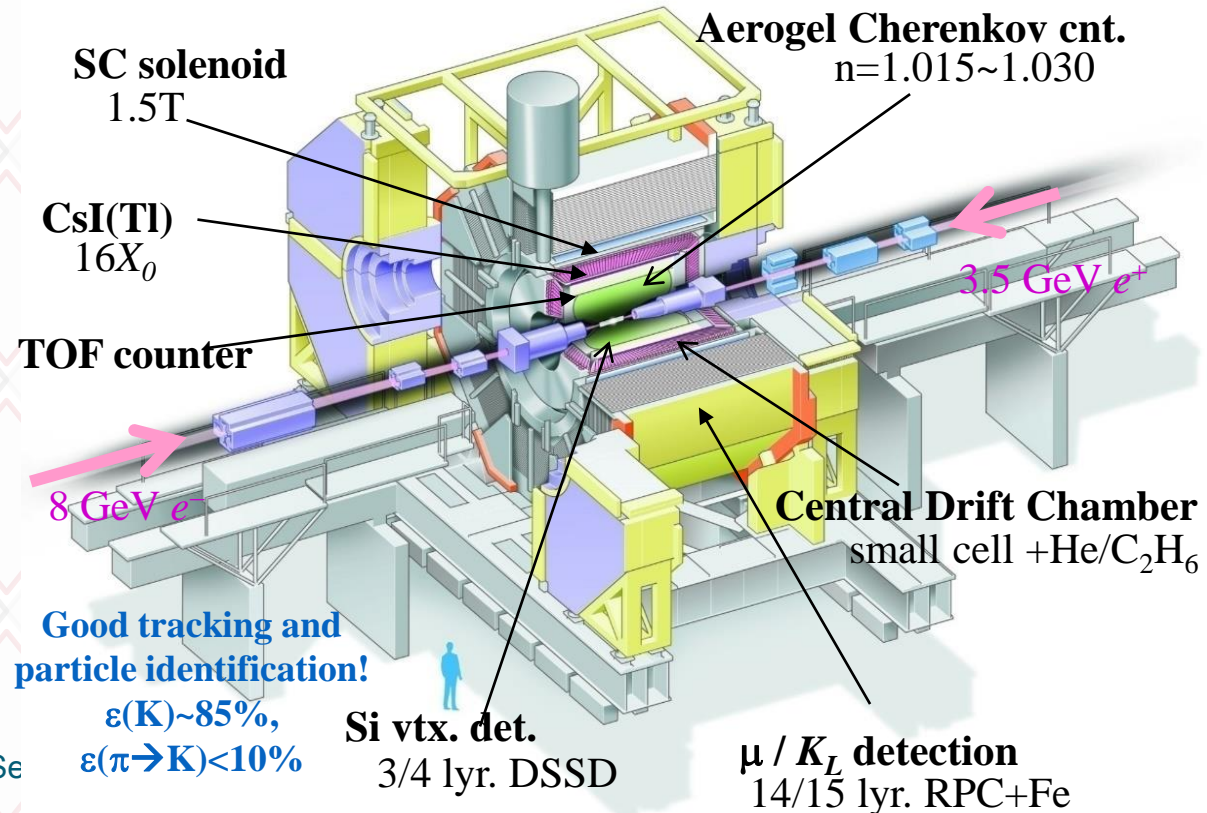
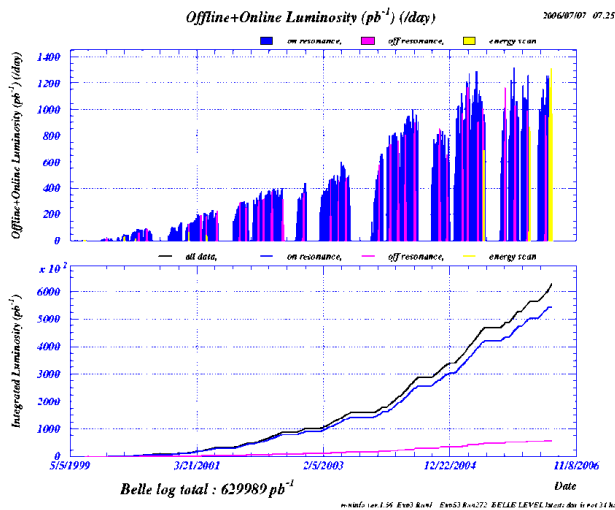
$$\sigma^h(P_T) \propto \int_{x_1, x_2, z} \sum_{a, a' \in q, g} f_a(x_1) \otimes f_{a'}(x_2) \otimes \sigma_{aa'} \otimes D_{1,q}^h(z)$$
  - Relies on unpol PDFs
  - leading access to gluon FF
  - Parton momenta not directly known
- **$e^+e^-$ :**

$$\sigma^h(z, Q^2, k_t) \propto \sum_q e_q^2 (D_{1,q}^h(z, k_t, Q^2) + D_{1,\bar{q}}^h(z, k_t, Q^2))$$
  - No PDFs necessary
  - Clean initial state, parton momentum known at LO
  - Flavor structure not directly accessible\*

# Belle Detector and KEKB



- Asymmetric collider
- $8\text{GeV } e^- + 3.5\text{GeV } e^+$
- $Y(4S)$  production  $\sqrt{s} = 10.58\text{GeV}$
- $e^+e^- \rightarrow Y(4S) \rightarrow B \bar{B}$
- Continuum production:  $\sqrt{s} = 10.52\text{ GeV}$
- $e^+e^- \rightarrow q \bar{q}$  (u,d,s,c) (also >70% at 10.58 GeV)
- Integrated Luminosity:  $>1000\text{ fb}^{-1}$  ( $>70\text{fb}^{-1}$ )



Ralf Se

6/22/2022

## Single hadron FF



Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Single hadron cross sections: $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL111 (2013) 062002</a>  <a href="#">PRD101(2020) 092004</a> </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL 96 (2006) 232002</a>  <a href="#">PRD 78 (2008) 032011</a> </div>	Unpol SIDIS, pp: $\frac{d\sigma}{dz}$ $e^+e^- \rightarrow (h)(h)X$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD92 (2015) 092007</a>  <a href="#">PRD101(2020) 092004</a> </div> and scale dependence
Transverse momentum dependent FFs: $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD 99 (2019) 112006</a> </div>	Transverse momentum dependent asymmetries $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD100 (2019) 92008</a> </div>	Polarizing $\Lambda$ fragmentation <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL 122 (2019), 042001</a> </div> $D_{1,q}^{\perp h}(z, k_T, Q^2)$



## Dihadron FF (IFF)

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Dihadron cross sections $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD96 (2017) 032005</a> </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL107 (2011) 072004</a> </div>	Unpol SIDIS, pp: $\frac{d^2\sigma}{dzdm}$

## Single hadron FF

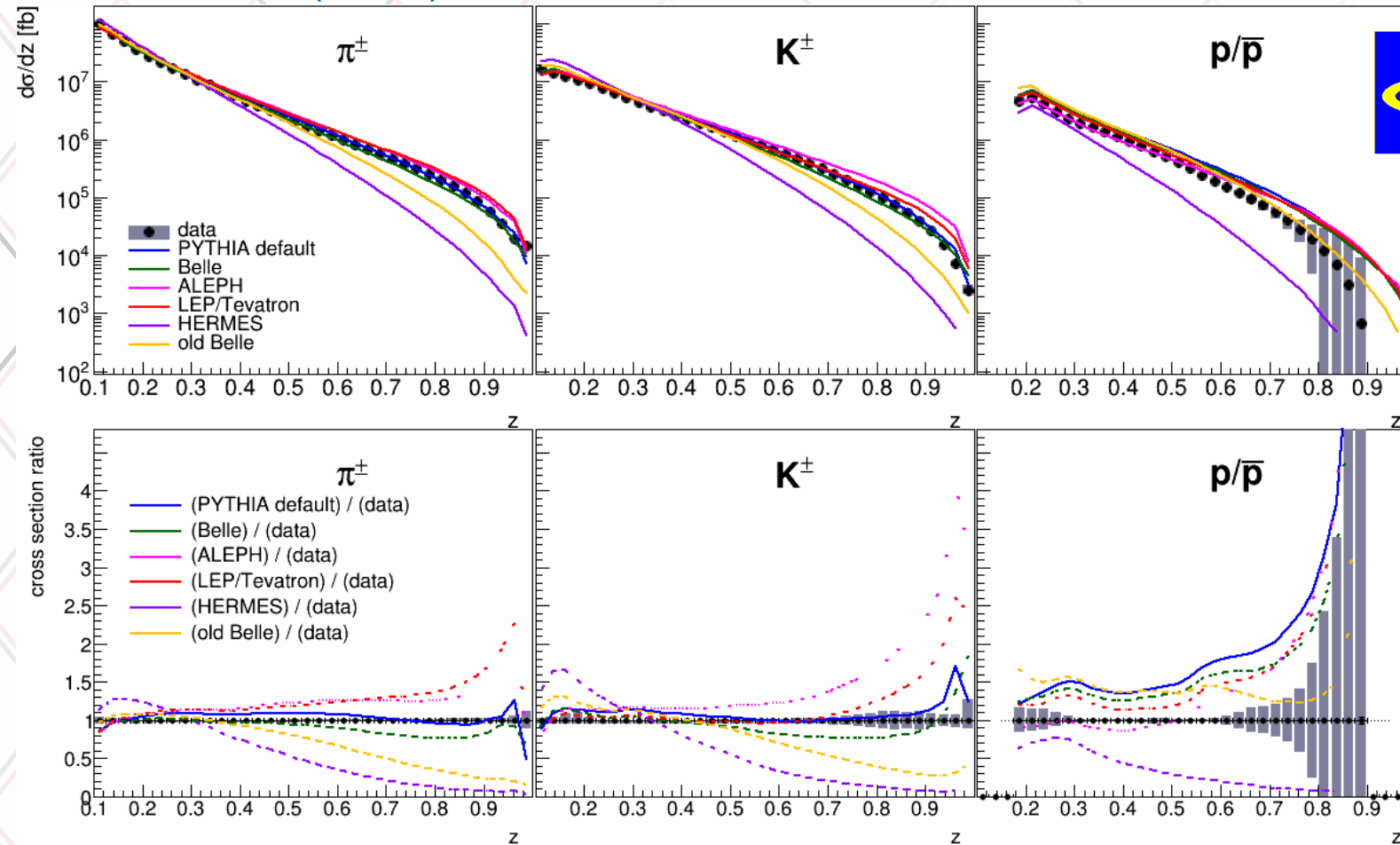
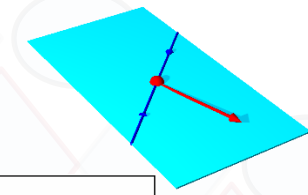
Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Single hadron cross sections: $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD 88 (2013) 032011</a>                          (Babar)                     </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD 92 (2015) 111101</a> (Babar K)  <a href="#">PRL 116 (2016) 042001</a> (BESIII)                     </div>	Unpol SIDIS, pp: $\frac{d\sigma}{dz}$ $e^+e^- \rightarrow (h)(h)X$  and scale dependence
Transverse momentum dependent FFs: $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$	Transverse momentum dependent asymmetries $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD 90 (2014) 052003</a> (Babar)                     </div>	 

## Dihadron FF (IFF)

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Dihadron cross sections $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$ 6/22/2022	Azimuthal asymmetries: $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ Ralf Seidl: Fragmentation functions and systematics	Unpol SIDIS, pp: $\frac{d^2\sigma}{dzdm}$

# Unpolarized single hadrons

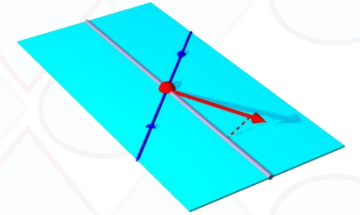
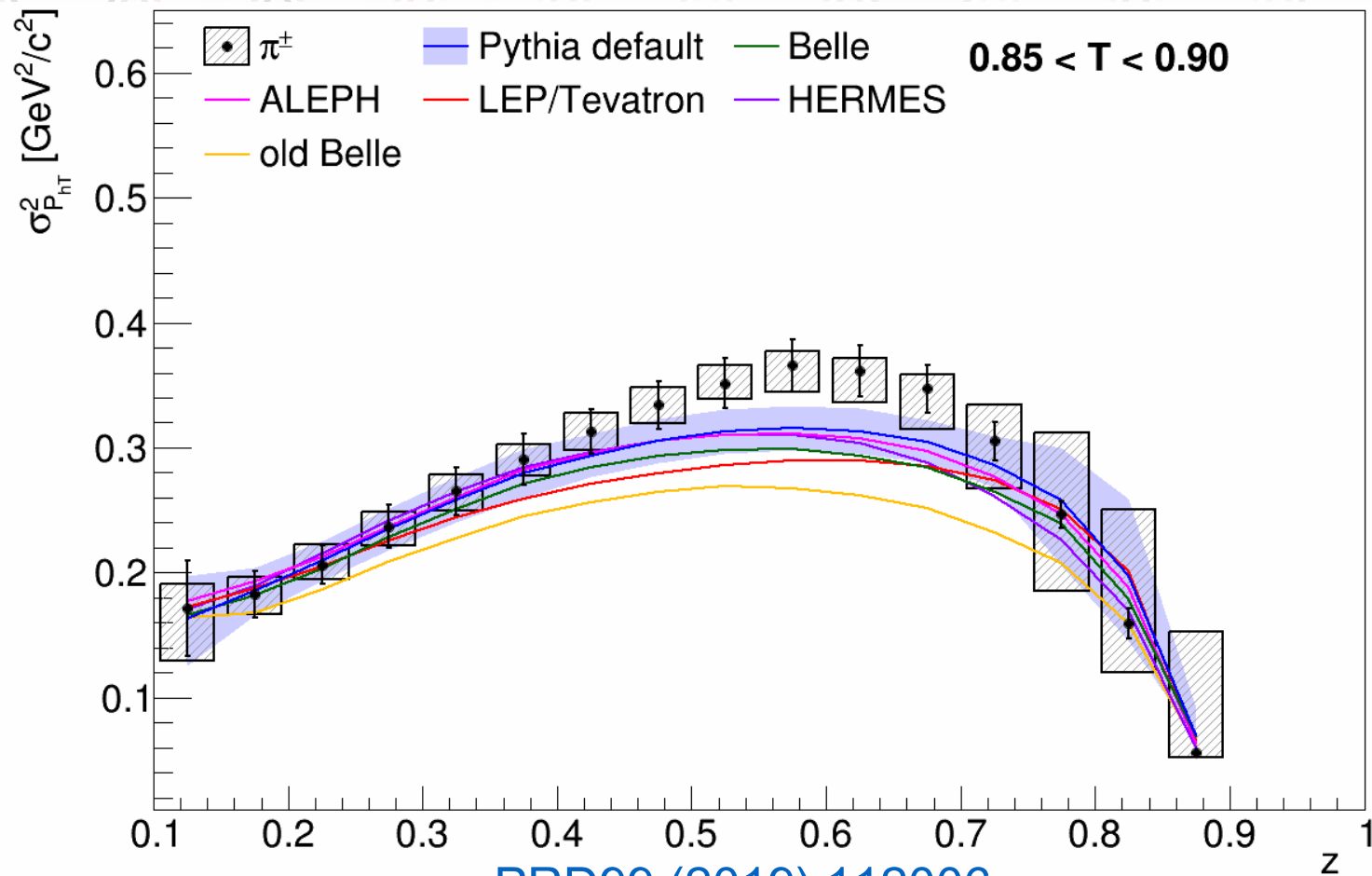
PRD 101 (2020) 092004



- Update with better ISR correction
- Correlated and uncorrelated uncertainties separated → improve global unpolarized FF fits

# Single hadron transverse momentum widths comparison to MC

first direct (no convolutions) measurement of z dependence of Gaussian widths



Pythia6  
MSTP(21):

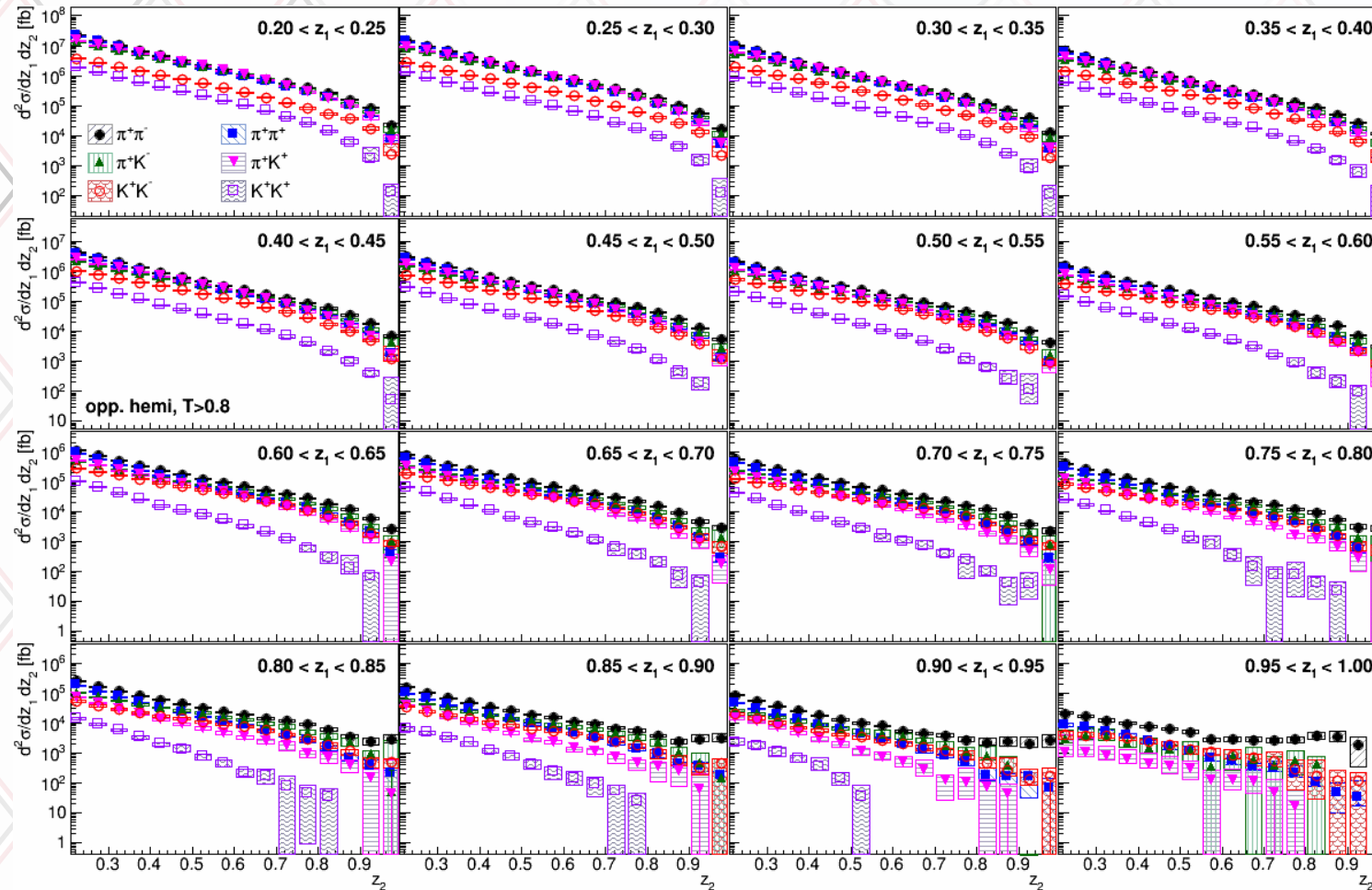
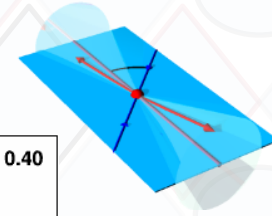
- 0.28
- 0.325
- 0.36
- 0.36
- 0.37
- 0.40

[PRD99 \(2019\) 112006](#)



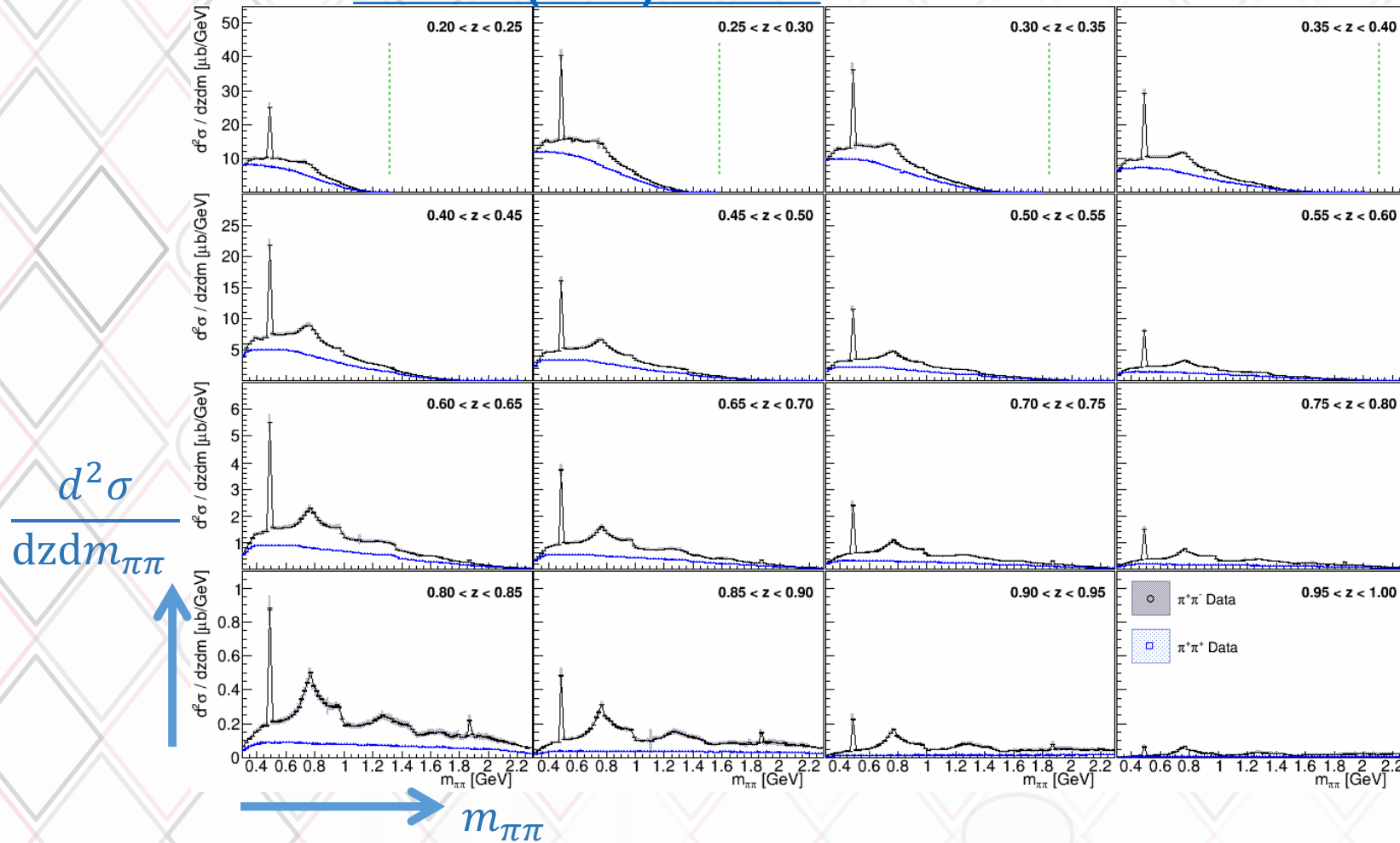
# Dihadrons in opposite hemisphere

[PRD 101 \(2020\) 092004](#)



# Di-hadron mass dependence (same hemisphere)

Belle: RS et.al. [PRD96 \(2017\) 032005](#)

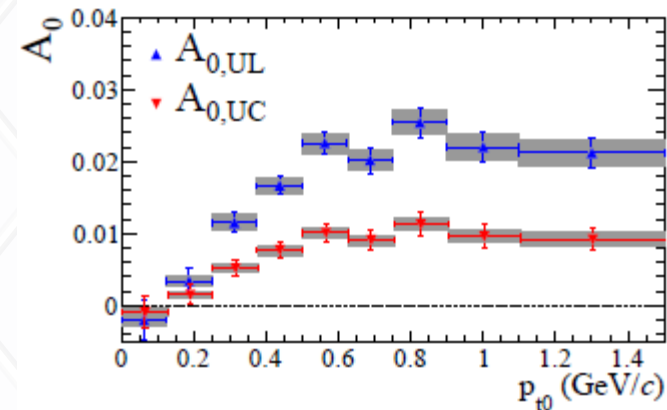
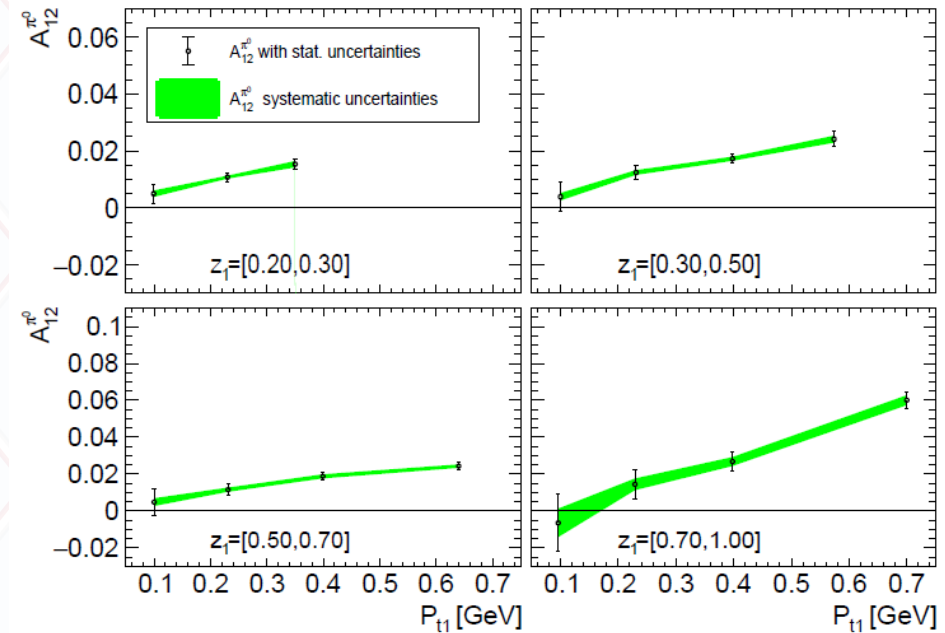


- Important input for IFF based transversity global analysis
- Individual resonances, etc quite visible; interesting for FF in itself

# Collins transverse momentum dependence

[PRD100 \(2019\) 92008](#)

- Add transverse momentum to Collins asymmetries'  $z$  dependence
- Currently only 1 or 2-dimensional extractions available ( $q_t, z_1 \times z_2, p_{t1} \times p_{t2}, z_1 \times p_{t1}$ )
- Increasing asymmetries with both  $z$  and  $p_t$ , but  $p_t$  reach limited
- Multidimensional extractions needed

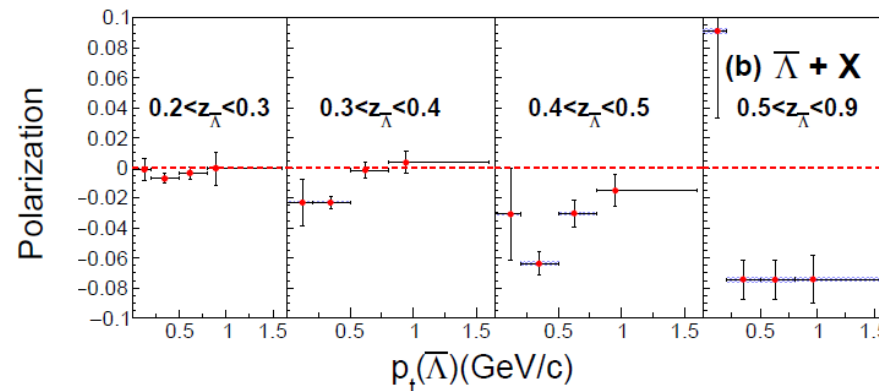
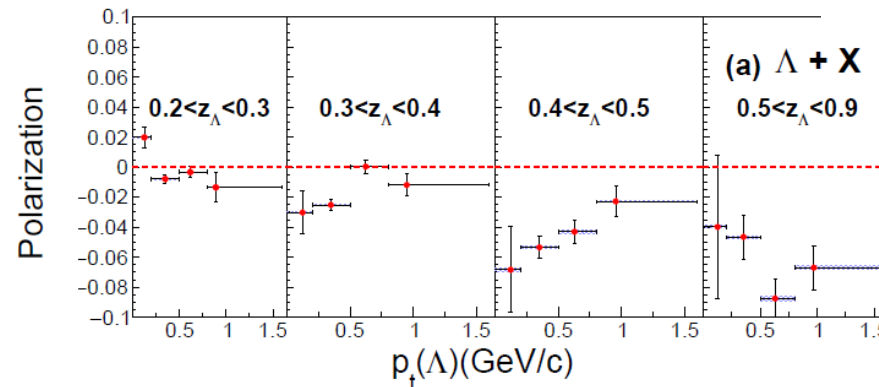
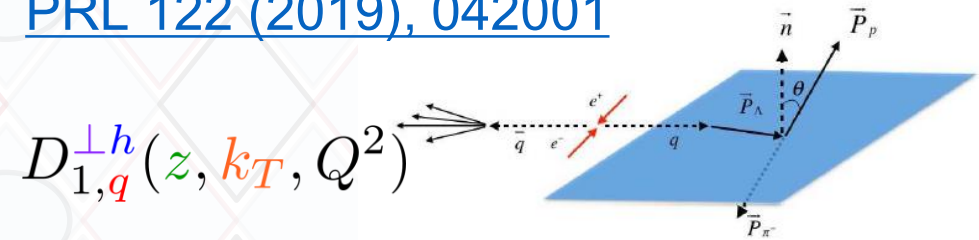


[PRD 90 \(2014\) 052003](#) (Babar)

# $\Lambda$ polarization, transverse momentum dependence

YingHui Guan (Indiana/KEK):  
[PRL 122 \(2019\), 042001](#)

- Different behavior for low and high- $z$  :
- At low  $z$  small
- At intermediate  $z$  falling Polarization with  $P_t$
- At high  $z$  increasing polarization with  $P_t$



# Systematics that limit the Belle Fragmentation measurements\*

- PID systematics
- Acceptance corrections
- ISR/FSR corrections
- Tune/MCEG dependence
  
- Multidimensional detector smearing unfolding
- Interplay of uncertainties; correlated and uncorrelated errors and combination of those, asymmetric uncertainties

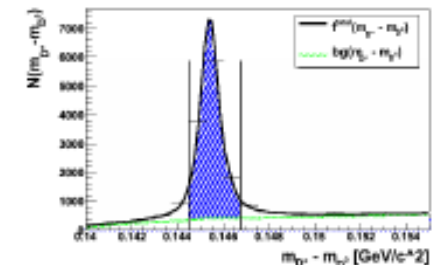
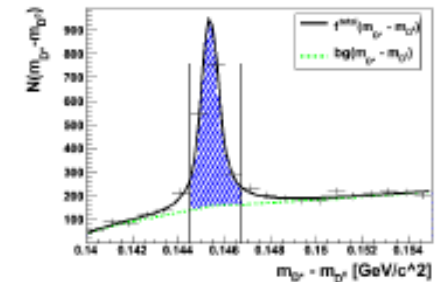
\*that are likely also relevant to EIC (SI)DIS measurements

# Correction chain for most Belle FF related xsec results

Correction	Method	Systematics
PID mis-id	PID matrices (5 x 5 types in 9 $\cos \theta_{\text{lab}}$ x 17 $p_{\text{lab}}$ bins)	MC sampling of inverted matrix element uncertainties, variation of PID correction method
Momentum smearing	MC based smearing matrices, SVD unfolding	SVD unfolding vs analytically inverted matrix, reorganized binning, MC statistics
Non-qqbar BG removal	eeuu, eess, eecc, tau MC subtraction, ( $Y(5S)$ decays)	Variation of size, MC statistics
Acceptance I (cut efficiency)	In barrel: reconstructed vs generated MC	MC statistics
Acceptance II	Various generated MCs: barrel to $4\pi$	MC statistics, variation in tunes
Weak decay removal (optional)	udcs check evt record for weak decays	Compare to other Pythia settings
ISR	ISR on vs ISR off in Pythia	Variations in tunes

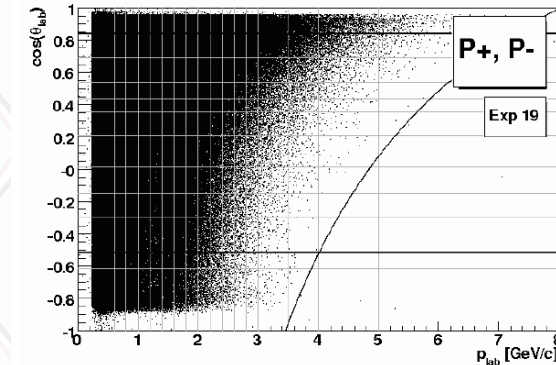
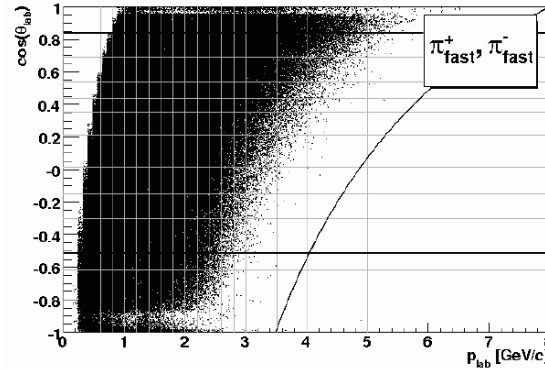
# PID correction

- Both at Belle and EIC an interplay between many PID detectors (Aerogel, TOF, CDC, KLM, EMCAL) + tracking
- description of PID efficiencies via MC limited
- Try to obtain efficiencies based on data, but will strongly depend on lab momentum and polar angle
- $17 \times 9 p_{\text{Lab}} \times \theta_{\text{Lab}}$  bins
- Electrons, muons via J/Psi decays
- Kaon/pion identification via  $D^{*\pm} \rightarrow D^0(K\pi)\pi^\pm$  (mass distribution + charge of slow pion)
- Protons via  $\Lambda$  decays
- (possible other decays that could be used  $K_S \rightarrow \pi\pi$ ,  $\phi \rightarrow KK$ , etc)
- General difficulty of combinatoric BG below mass peaks

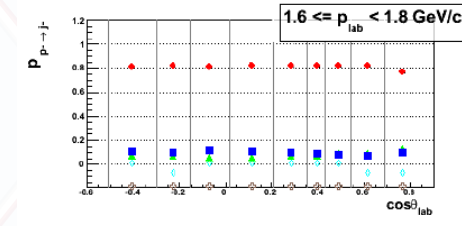
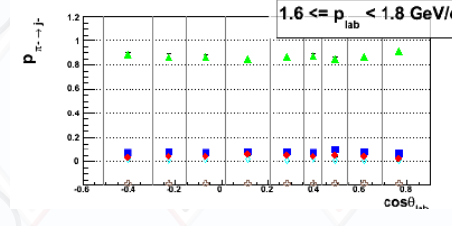
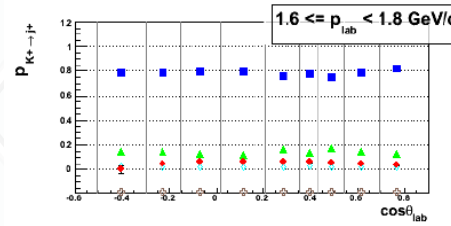
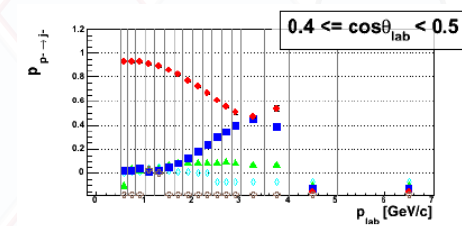
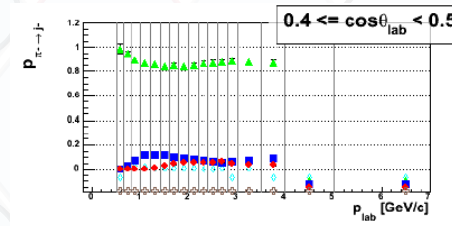
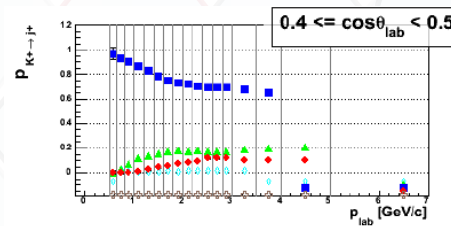


# Belle PID efficiency evaluation

- Sources of uncertainties:
  - Nonzero backgrounds below mass peaks
  - Kinematically inaccessible regions (try other particles, or fall back to MC extrapolating from accessible region)
  - Statistical uncertainties through matrix inversion (Sampling technique)



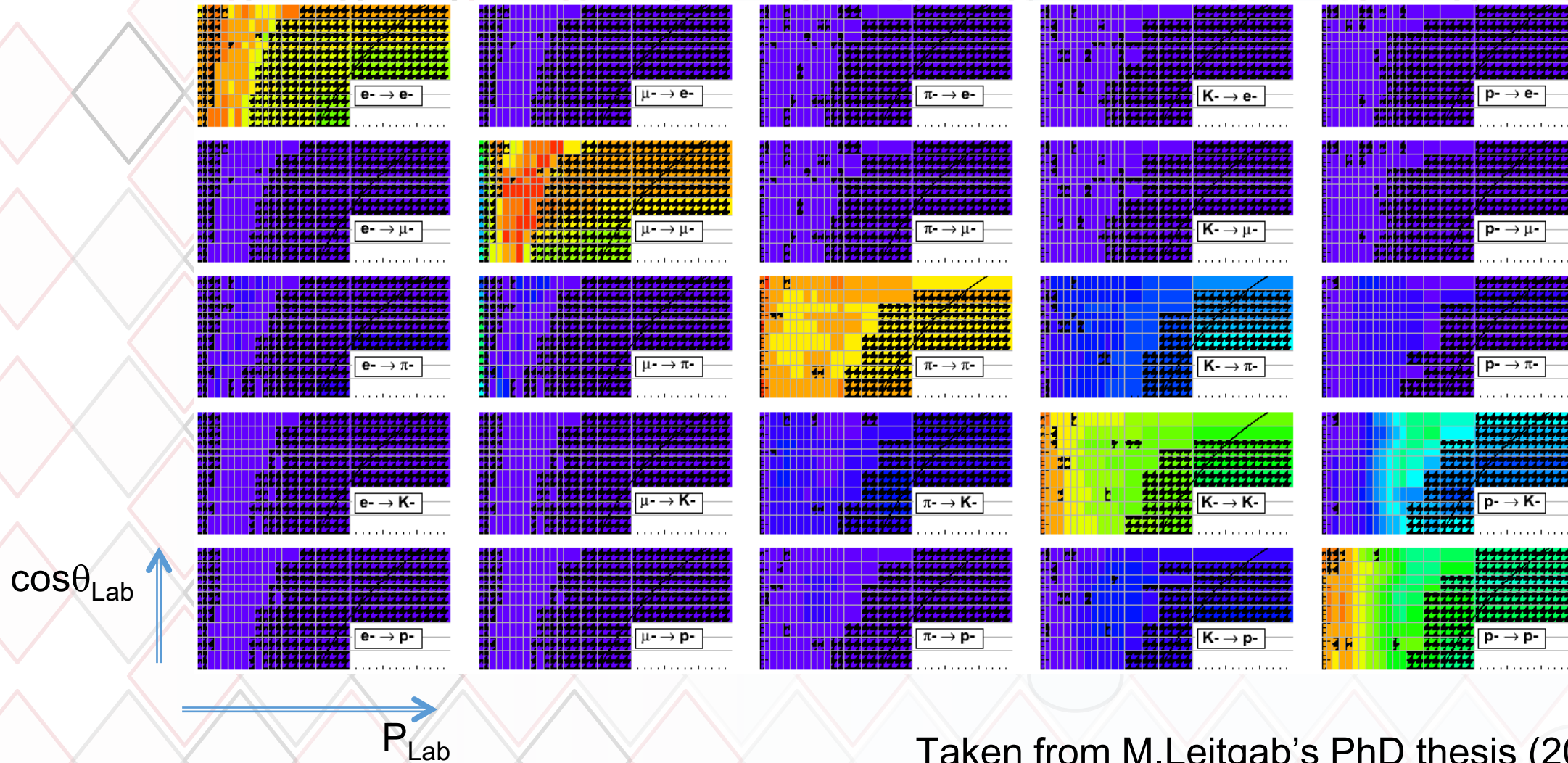
REAL Particles		$\pi$	K	p	$\mu$	e
Reconstructed particles	$\pi$					
	K					
	p					
	$\mu$					
	e					



Taken from M. Leitgab's PhD thesis (2013)



# PID efficiencies



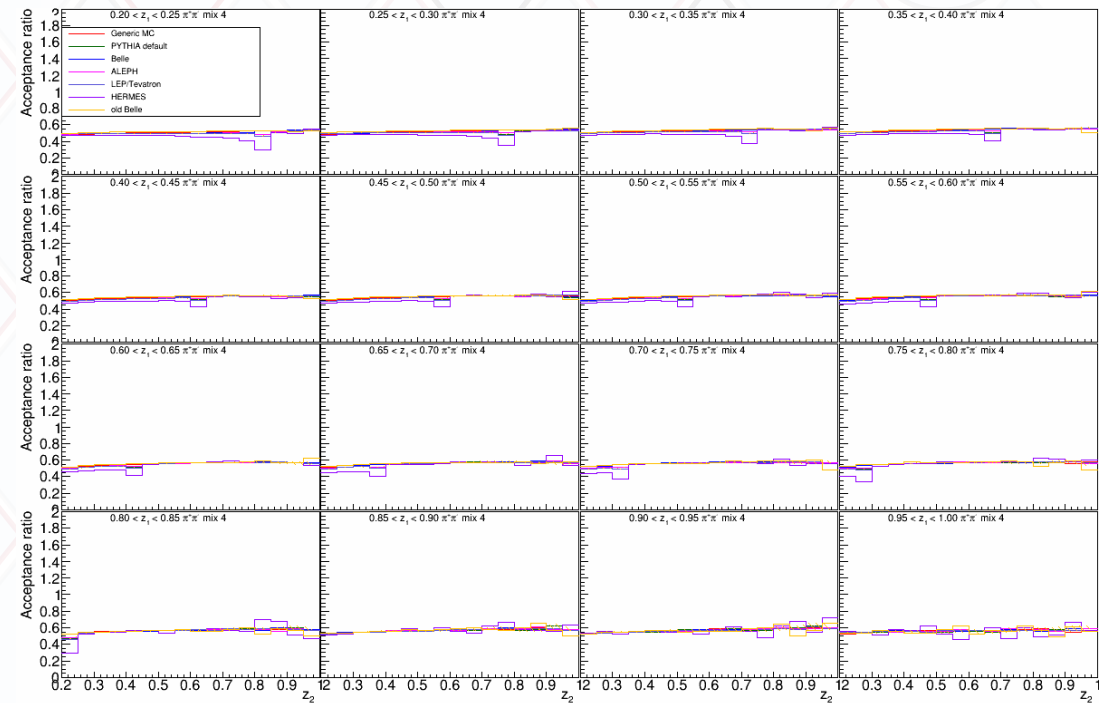
6/22/2022

Ralf Seidl: Fragmentation functions and systematics

Taken from M. Leitgab's PhD thesis (2013)

# Acceptance

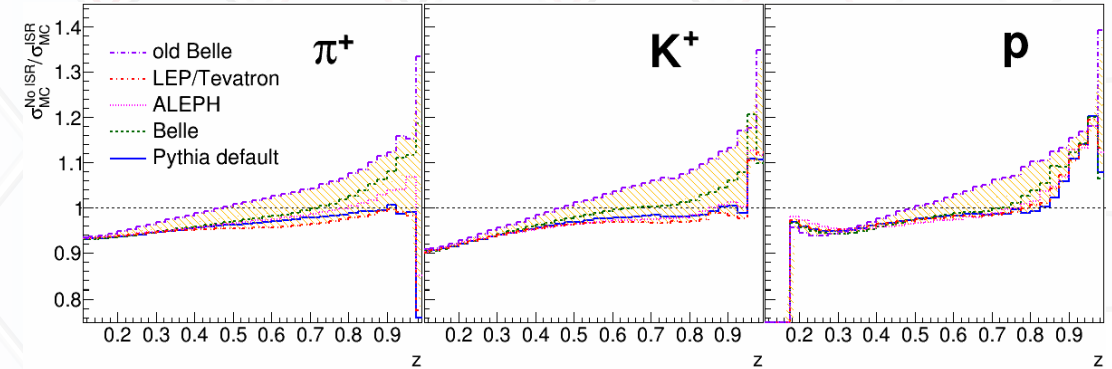
- Correcting from your detector acceptance to  $4\pi$  in principle straightforward, but:
  - Will to some extent again depend on your actual measurement, e.g. partonic  $1+\cos^2\theta$  dependence needs to be fulfilled for  $z \rightarrow 1$ , but below will depend on FFs ( $z$  dependence, strong decays, etc)
- Tune dependence



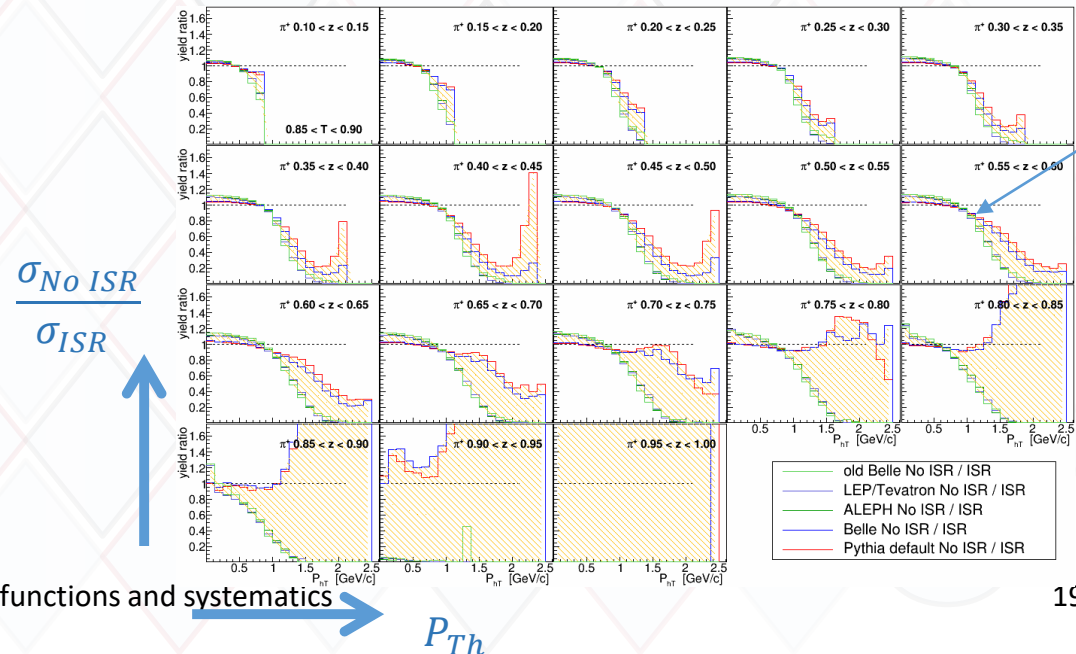
# ISR/FSR

- We correct for ISR by calculating cross section ratios in the MC with ISR switched off (MSTP(13) =0) over on (=1)
- Mostly straightforward correction in e+e-; ISR reducing qqbar system energy  $\rightarrow$  moving hadron momenta (and z) to lower values
- However, also changes the boost of the qqbar system
- $\rightarrow$  accumulation of (supposedly) high kt particles wrt thrust axis
- $\rightarrow$  accumulation of higher-mass di-hadrons from opposite (true) hemispheres
- Similar (+spin) effects need to be considered in SIDIS measurements!
- Also: see Tune dependence

recent single/dihadron update:  
[PRD 101 \(2020\) 092004](#)



Single hadron transverse momentum dependent results: [PRD99 \(2019\) 112006](#)



Node

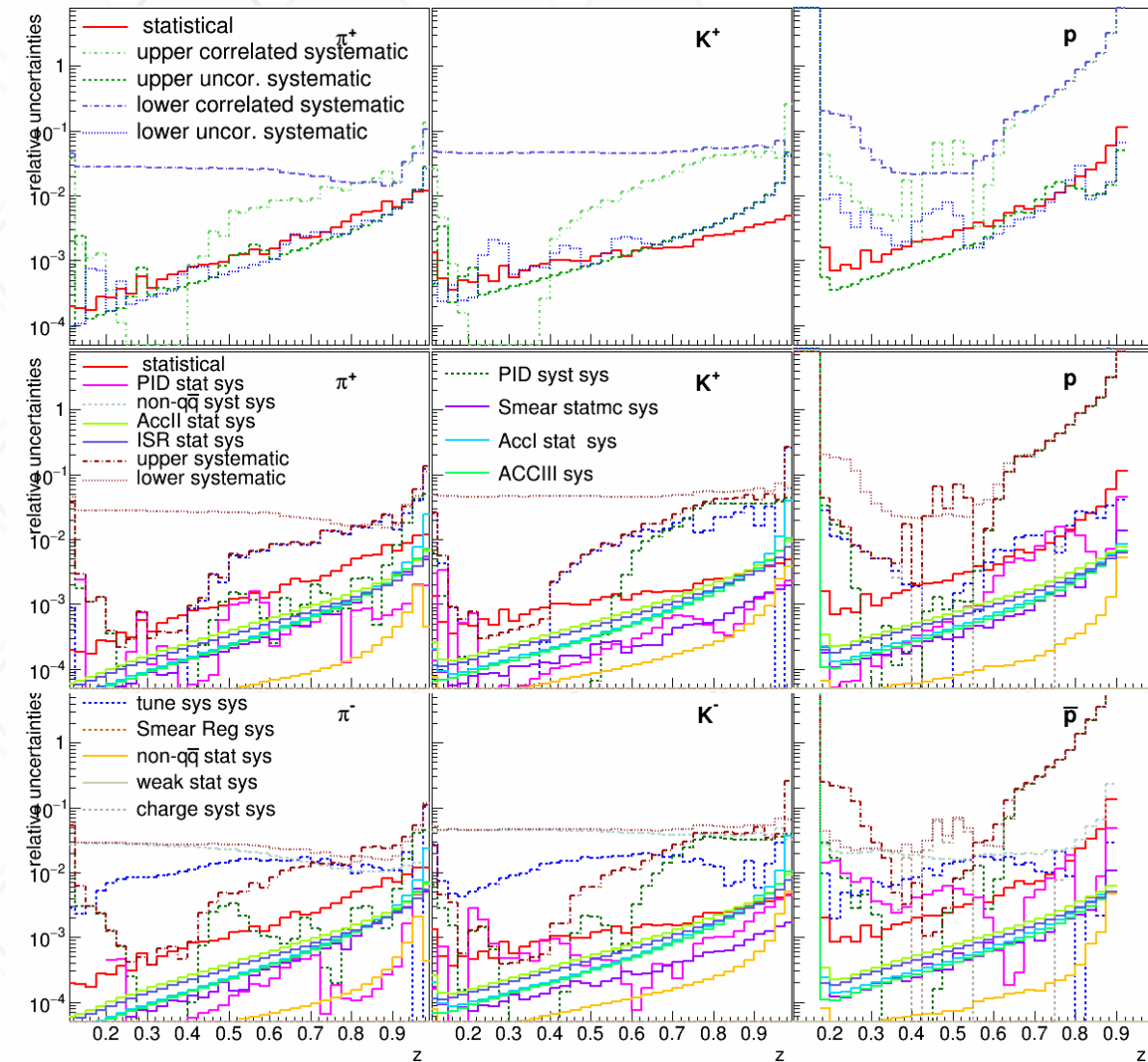


# Unfolding

- Unfolding (smearing correction) in principle straightforward, but:
  - Regularization with SVD unfolding can be tricky, need to good initial description of data yields, only artificial multi-dimensional unfolding possible → source of uncertainties
  - Iterative unfolding (eg RooUnfold's Bayes unfolding) generally more reliable, up to three dimensions directly available
- However:
    - MC needs a good description of the detector response
    - (significantly) larger amount of MC data wrt to actual data, especially important for multi-dimensional unfolding

# Tune dependence

- Some efforts to systematically improve the MC tunes for a certain cms energy (Professor  $\rightarrow$  apprentice)
- Relies heavily on the measurements itself and choice of input measurements
- Ideally will give you reasonable uncertainty ranges on all relevant MC tune parameters, so far mostly variation of some tunes that “kind of work” (somewhere)
- Requires many simulations of each reasonable parameter set, preferably including full detector description



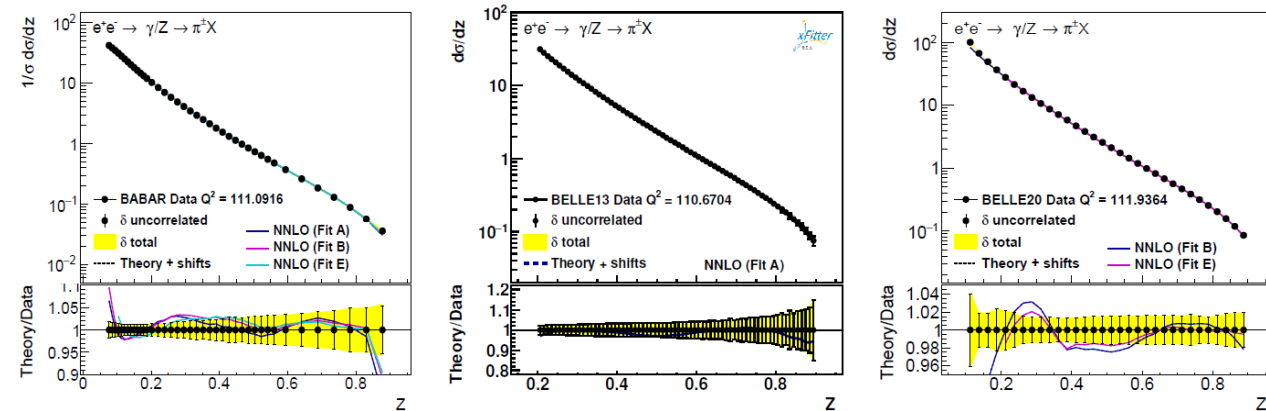
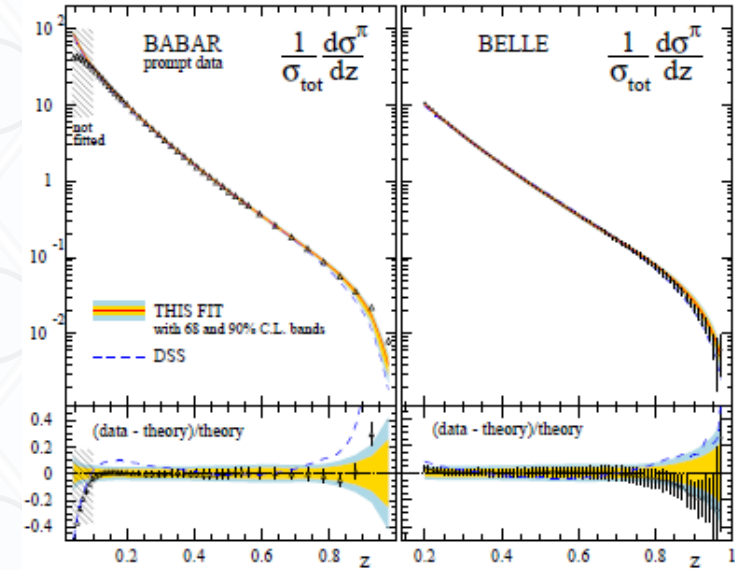
Total, individual upper and individual lower uncertainty budgets for single hadron cross sections: [PRD101\(2020\) 092004](#)

# From “your errors are too conservative” to “your errors are too precise”

Phys.Rev. D91 (2015) 1, 014035

*One group: “However we do not consider it because of a poor control of the degree of correlation of systematic uncertainties”*

- Initial single hadron cross section measurement in very fine  $z$  binning, thus large bin-to-bin migration. Unfolding performed but assigned very conservative uncertainties  $\rightarrow$  global fit's  $\chi^2$  generally too low for our data set
- Recent update ('20) with more realistic binning, much better understanding of all systematic uncertainty sources, correlated and uncorrelated uncertainties provided separately
- However, fitters would prefer:
  - all systematics separately,
  - all systematics symmetric (INCORRECT!)



# Future work at Belle, BelleII

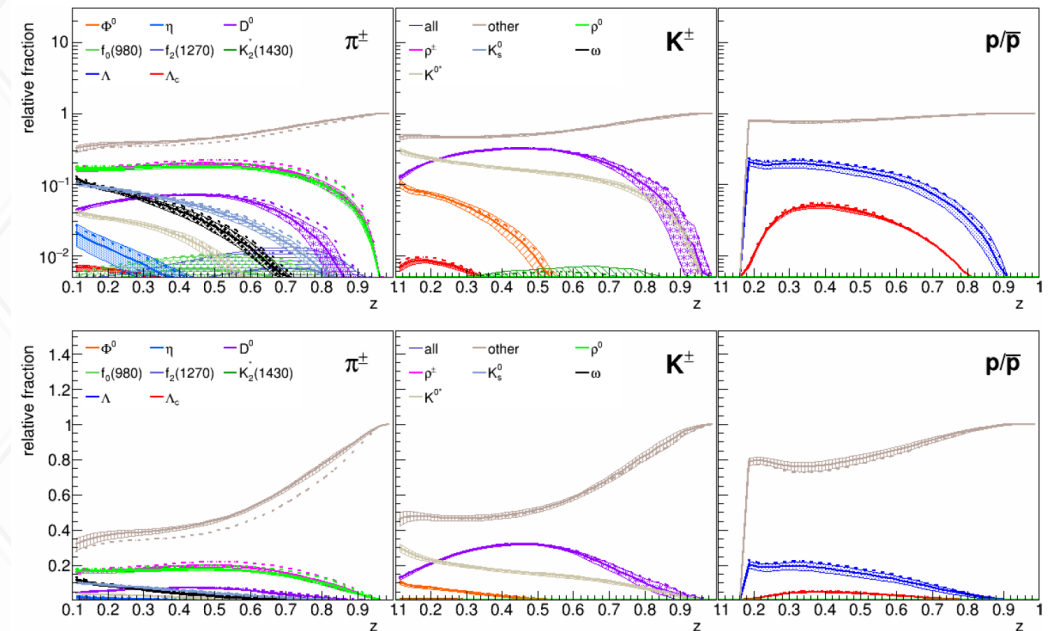
See also Snowmass white paper: <https://arxiv.org/abs/2204.02280>

- Improve/Tune the MC description of fragmentation
  - "Traditional" event shapes like thrust and (linear) sphericity
  - Identified particle spectra including  $\pi$ ,  $K$ ,  $p$ ,  $\gamma$ ,  $\pi^0$ ,  $\eta$ ; multi-dimensionally
  - Resonance (especially Vector-mesons) and heavier baryon cross sections
  - Tests of charge, strangeness, baryon number conservation, especially along thrust direction/hemispheres
  - Jet measurements
  - Make all information available to Theorists/MCEGs  $\rightarrow$  Hepdata/RIVET
- Comparisons of MC description of polarized fragmentation
- Jet-based fragmentation measurements

# Not TMD(yet) but indirectly related: Weak and strong decay feed-down

- Hadrons from Weak decays technically not part of FF definition, but often included
- Strong decays part of total sum over hadronic final state
- Both can affect the  $z$  (and transverse momentum) dependence of the detected hadrons:
  - naturally included in unpolarized MC,
  - in part added to polarized generators
  - How does PHENO handle this (additional parameters?)

Decaying hadron fractions in light hadrons at  $\sqrt{s} = 10.58$  GeV (PYTHIA6):



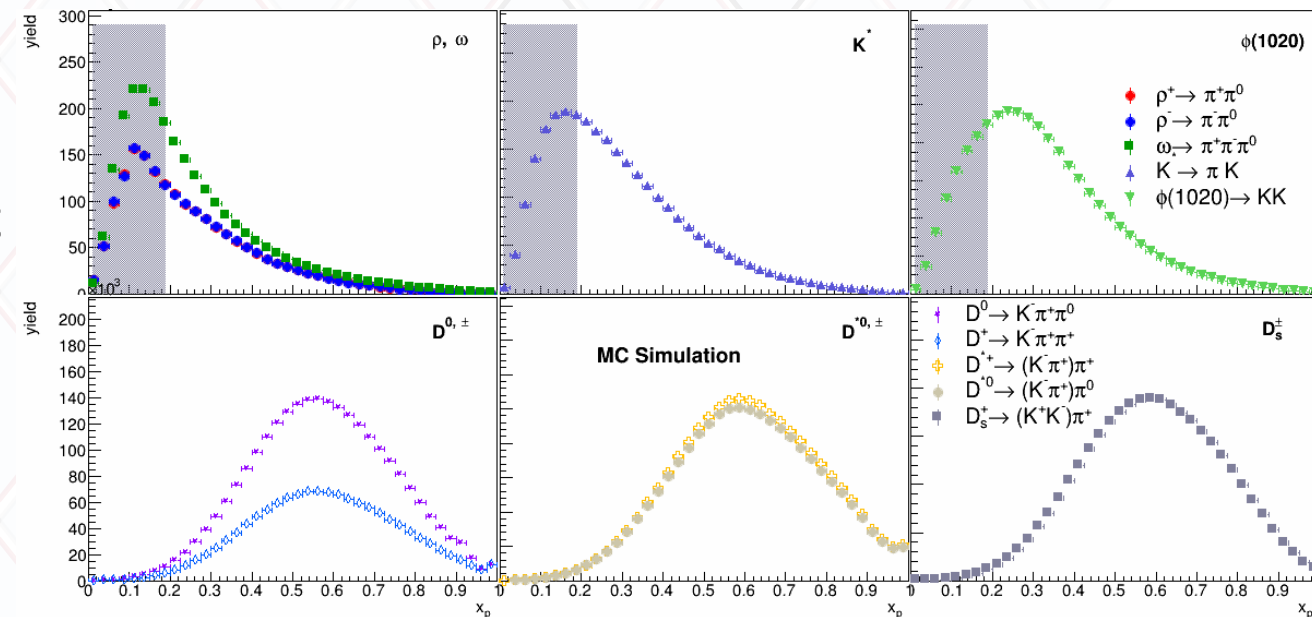
Bands: various Pythia tunes, including PARJ(11 VM to PS ratio) range from 0.3-0.55  
 Dashed lines: default, but PARJ(11) = 0.6



# Ongoing: Decaying particle FFs

- Study the explicit differential cross sections for VMs, D mesons as a function of  $x_p$
- Mostly mass distributions and fits well-behaved, except for  $\rho-\omega$  (interference) and more exotic resonances
- Also of interest for ultra high-energetic cosmic ray air shower research (muon problem)

- Example from MC at Belle energies (for  $4\pi$  acceptance):



# Summary

- Many Belle/Babar/BES3 fragmentation related measurements available that form the basis for semi-inclusive DIS measurements:
  - Unpolarized single hadrons, and transverse momentum dependence wrt thrust axis
  - Collins asymmetries ( $k_t$  dependent, kaons)
  - Polarizing  $\Lambda$  fragmentation
  - Detailed di-hadron cross section and asymmetry measurements
- Most of these measurements are systematics limited with the main uncertainties from MC tune-dependence, PID corrections, kinematic unfolding, treatment of ISR/FSR
- Need to reduce systematics and provide as much information (individual contributions, type of uncertainty) to fitters
- More measurements on going for:
  - Other venues for unpolarized TMDs (opposite hemisphere dihadrons, hadron in jet)
  - Multi-dimensional extractions of Collins asymmetries
  - Other studies ongoing on the fragmentation of VMs and Ds
  - More to come from Belle and BelleII