EW *correction* to the factorization formalism

- Electron-Ion Collider at High Luminosity
- Lepton-Hadron Deep Inelastic Scattering (DIS)
- The "Breit" frame
  – A *natural* frame to describe hard scattering process in DIS (EIC Yellow Report)
- The "virtual" Breit frame
- Factorization formalism beyond QCD
- Summary and outlook

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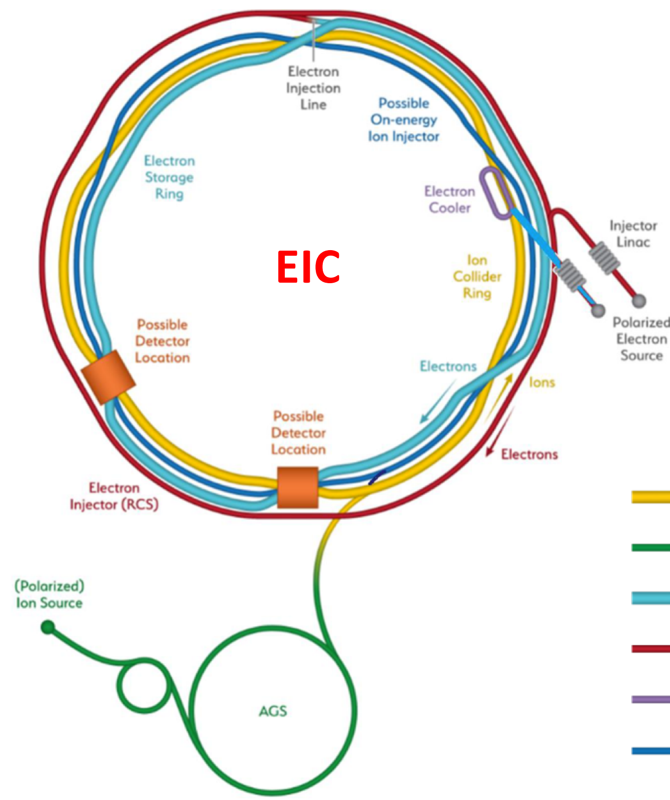
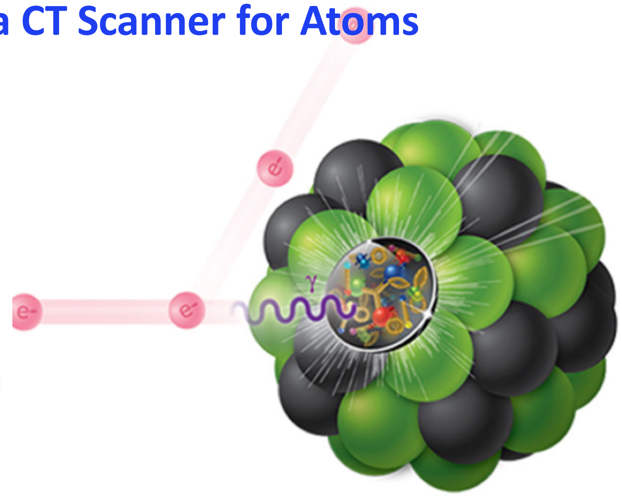
U.S. - based Electron-Ion Collider

https://www.bnl.gov/eic/

A machine that will unlock the secrets of the strongest force in Nature

Like a CT Scanner for Atoms





Basic Tech Requirements

- **Center of Mass Energies:**
  20 GeV – 141 GeV
- **Required Luminosity:**
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- **Hadron Beam Polarization:**
  80%
- **Electron Beam Polarization:**
  80%
- **Ion Species Range:**
  p to Uranium
- **Number of interaction regions:**
  up to two

High Luminosity (Phase-II): ***10^{35} $cm^{-2}s^{-1}$***

U.S. - based Electron-Ion Collider

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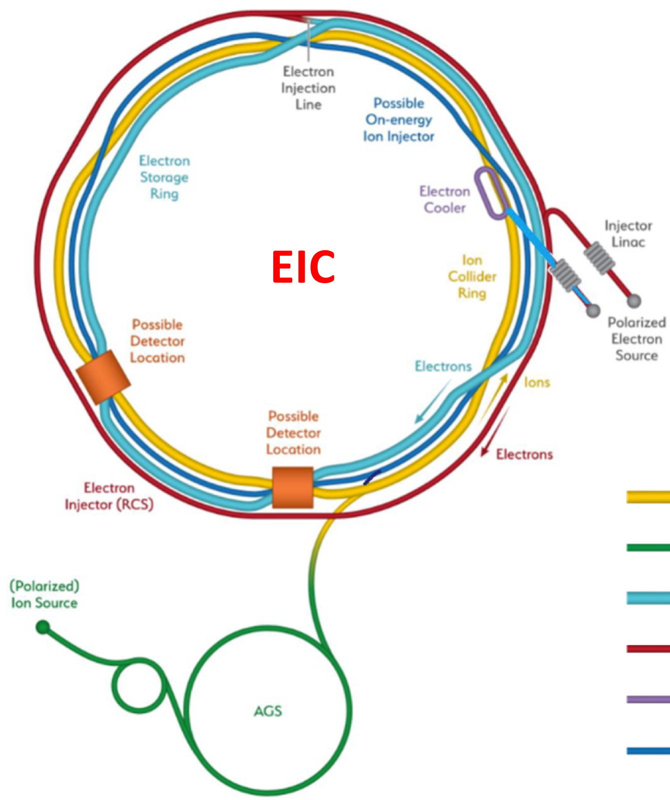
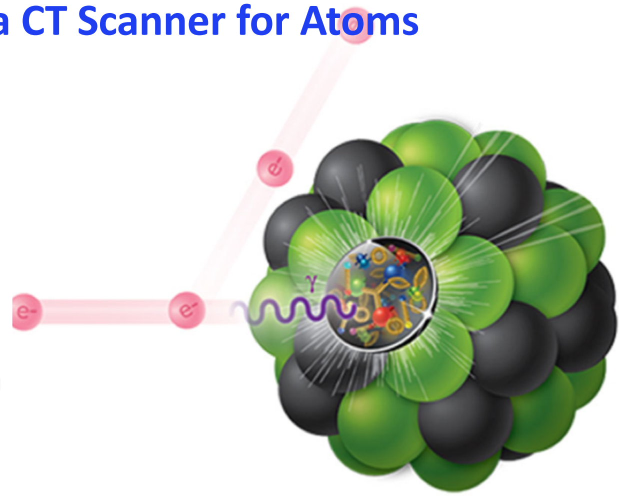




Hadron Storage Ring

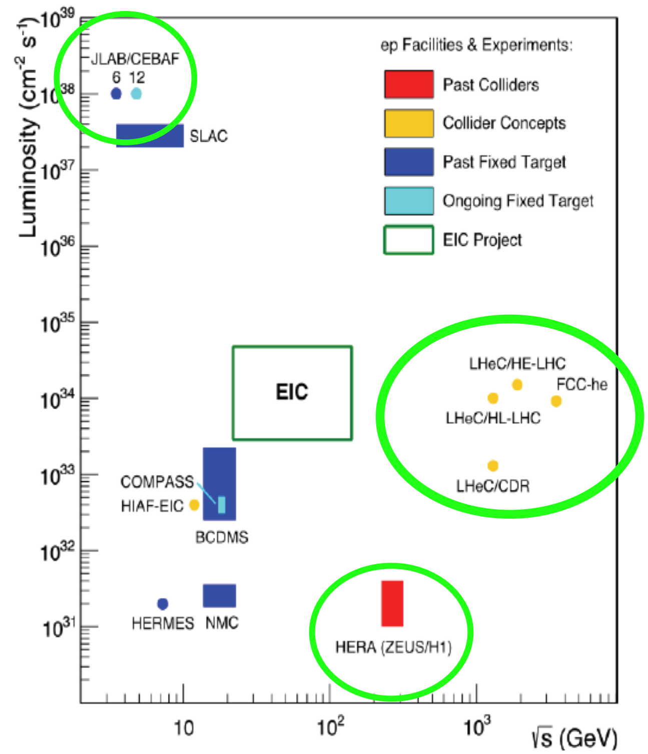
Hadron Injector Complex

Electron Storage Ring

Electron Injector Synchrotron

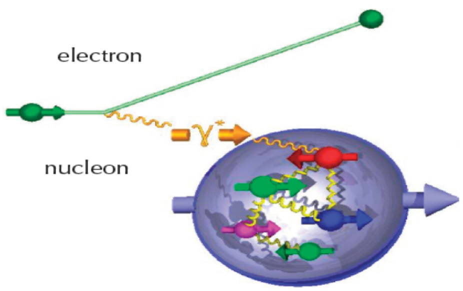
Electron Cooler

Possible On-energy Hadron Injector Ring



High Energy Lepton-Hadron Scattering

- **The new generation of "Rutherford" experiments for probing hadron structure:**



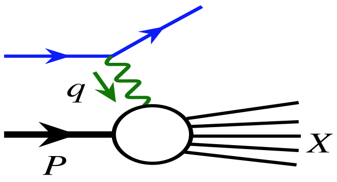
- **A controlled "probe" – the virtual photon (x_B, Q^2)**
- **Can either break *or not break* the hadron**

Mary high-energy lepton-hadron facilities have been built, or to be built, at SLAC, CERN, FNAL, DESY, JLab, BNL, …

- **Inclusive events**: e+p/A → e'+X

  Detect only the scattered lepton in the detector

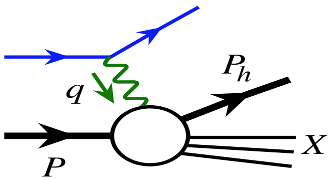
  (Modern Rutherford experiment!)



- **Semi-Inclusive events**: e+p/A → e'+h(p,K,p,jet)+X

  Detect the scattered lepton in coincidence with identified hadrons/jets

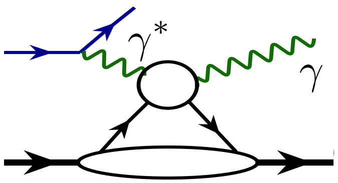
  (Initial hadron is broken – confined motion! – cleaner than h-h collisions)



- **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

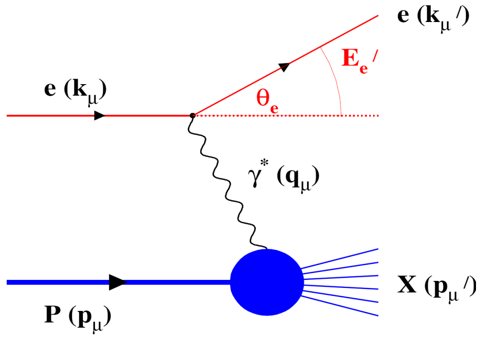
  Detect every things including scattered proton/nucleus (or its fragments)

  (Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)



Lepton-Hadron Inclusive Deep Inelastic Scattering

❑ **Approximation of one-photon exchange:**



$Q^2 = -(k-k')^2$ → **Measure of the resolution**

$y = P.(k-k')/P.k$ → **Measure of inelasticity**

$x_B = Q^2/2P.(k-k')$ → **Measure of momentum fraction of the struck quark in a proton**

$Q^2 = S\, x_B\, y$

$$E' \frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s} \frac{1}{Q^4} L^{\mu\nu}(k,k;q) W_{\mu\nu}(q,P)$$

$$L^{\mu\nu}(k,k;q) = 2(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}) + \text{spin...}$$

❑ **Deep inelastic scattering (DIS) structure functions:**

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(P+q-X) \langle P|J_\mu(0)|X\rangle \langle X|J_\nu(0)|P\rangle + \text{spin...}$$

$$= -\widetilde{g}_{\mu\nu} F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P\cdot q} F_2(x_B,Q^2) + \text{spin...}$$

$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu} P^\nu$$

QCD Factorization - Approximation

$$F_i(x_B,Q^2) \approx \sum_f C_{if}(x_B,Q^2;x,\mu^2) \otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

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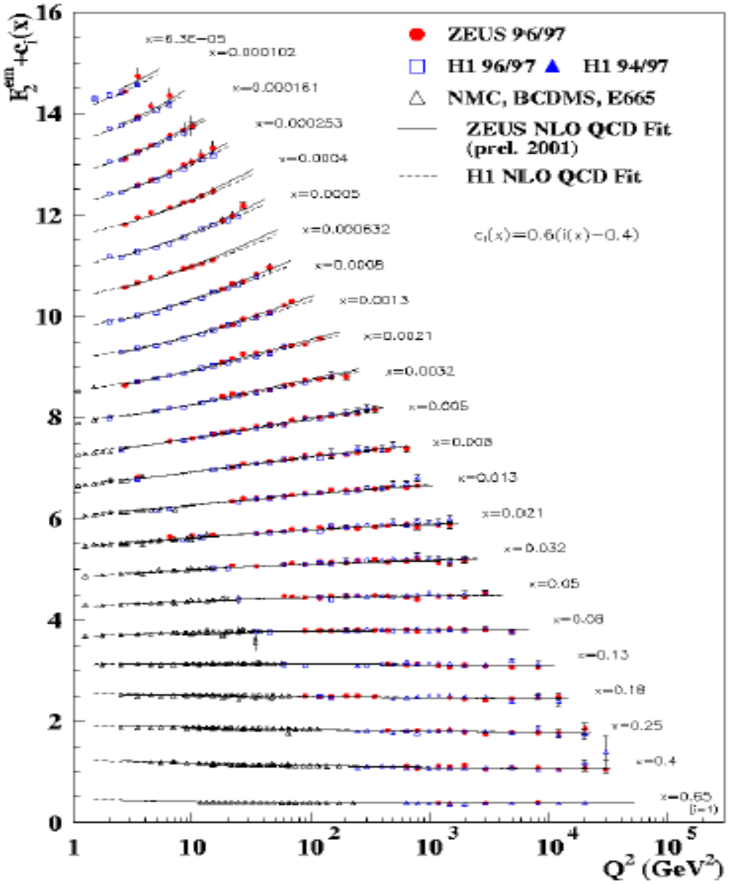
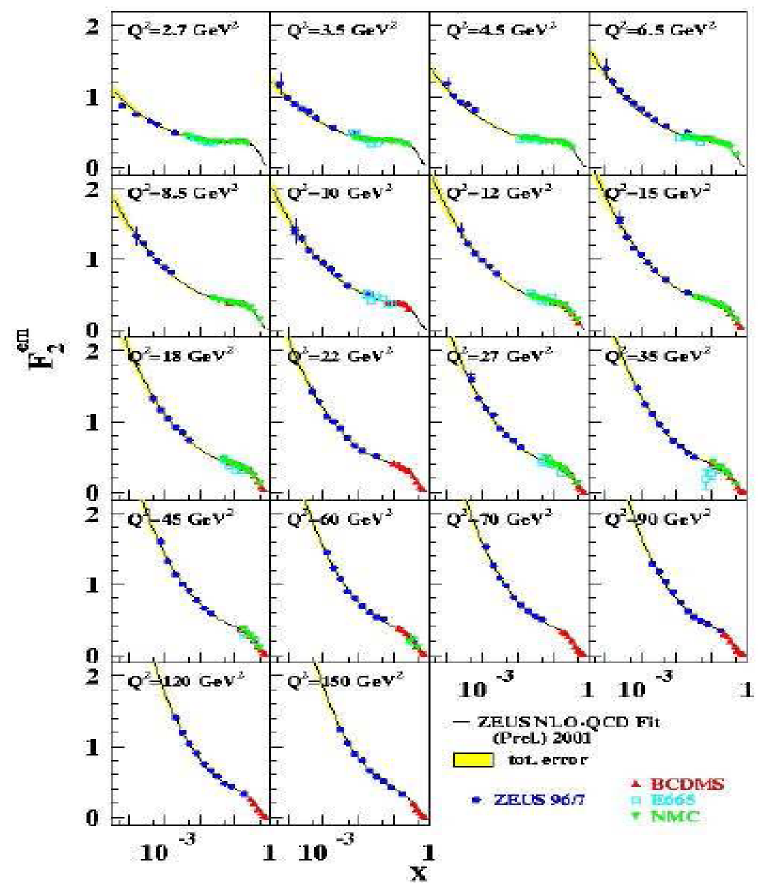
Lepton-Hadron Inclusive Deep Inelastic Scattering



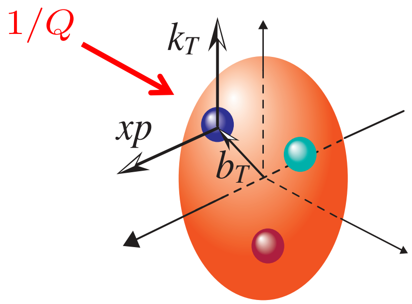


A very successful story of QCD, QCD Factorization, and QCD evolution!

Extraction of Parton Distribution Functions (PDFs) – 1D hadron structure

3D Hadron Structure – Need probes with two scales

- **Single-scale hard probe is too "localized":**



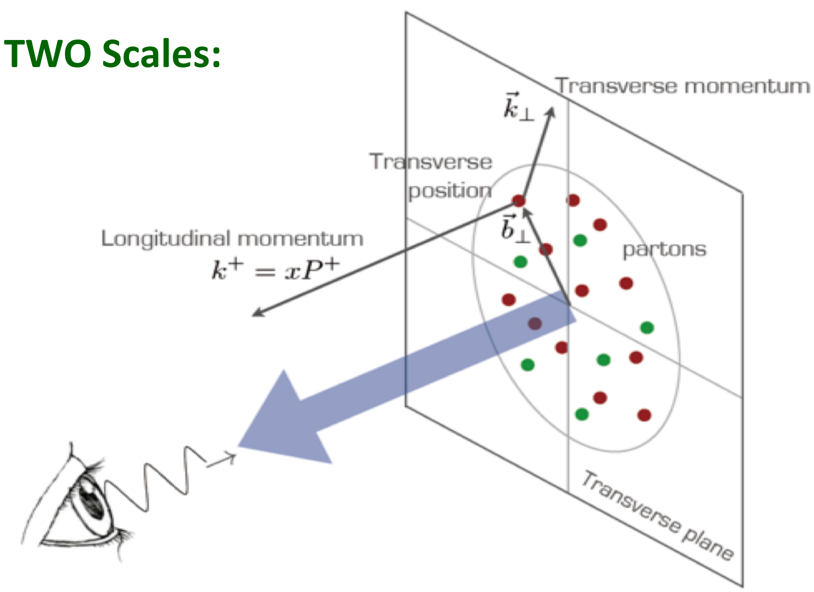
- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Confined transverse motion: $k_T \sim 1/\text{fm} \ll Q$
- Transverse spatial position: $b_T \sim \text{fm} \gg 1/Q$

- **Need new type of "Hard Probes" – Physical observables with TWO Scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\text{QCD}}$$

Hard scale: Q_1 To localize the probe – factorization particle nature of quarks/gluons

"Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm



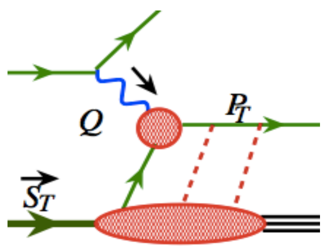
- **New challenge:**

QCD Factorization for observables with two scales!

"See" 3D Hadron Structure at a Lepton-Hadron Facility

- **Two-scale observables are natural in lepton-hadron collisions:**

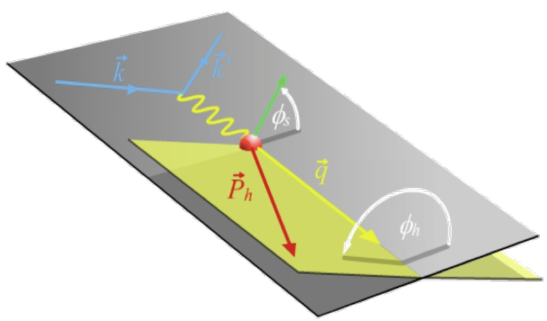
  - **Semi-inclusive DIS:**



+ …

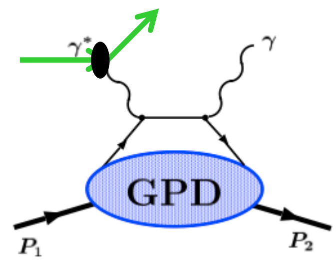
SIDIS: $Q \gg P_T$

Parton's confined motion encoded into TMDs (too many of them?)



Two scales, two planes, Angular modulation, …

  - **Exclusive DIS (without breaking the hadron):**

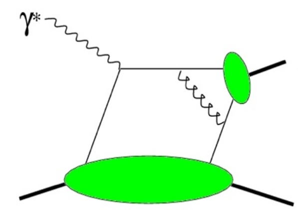


+ …

$t=(p_1-p_2)^2$

DVCS: $Q^2 \gg |t|$

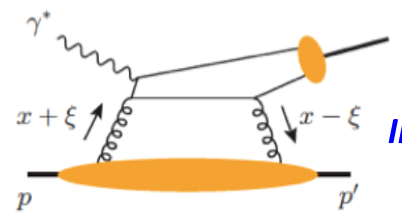
Parton's spatial imaging from Fourier transform of GPDs' t-dependence



+ …

DVEM: $Q^2 \gg |t|$

Imaging quarks



J/Ψ, Φ, …

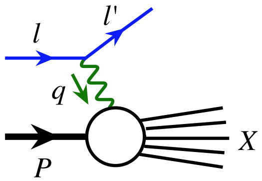
Imaging gluons

Heavy quarkonium: $Q^2+M^2 \gg |t|$

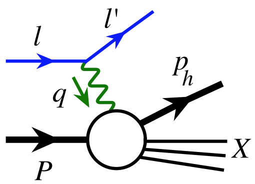
Imaging the glue only at EIC

None of these processes is sensitive to x-dependence of GPDs!
Proposed a new type of processes
(Qiu & Yu, 2022)

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

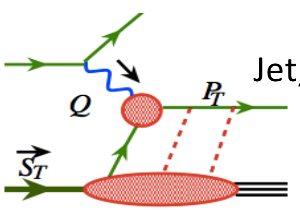


Scale: Q^2 **- PDFs**



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

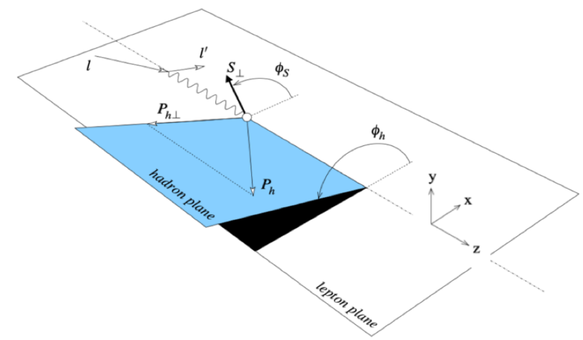


Jet, π, J/ψ, …

$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =\; & \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& +S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big] \\
& +S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] \\
& +|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& +|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$



18 SIDIS Structure Functions

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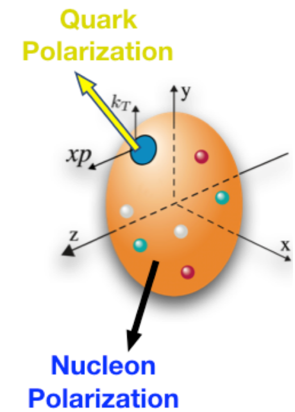
Transverse Momentum Dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

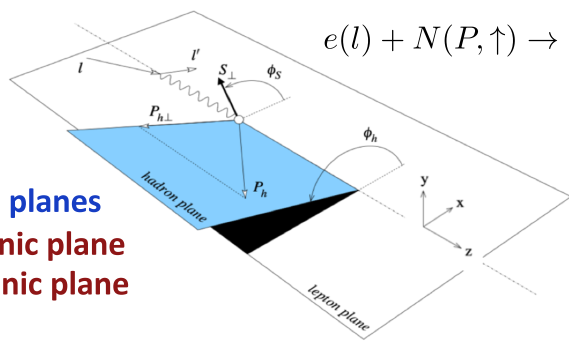
Nucleon Polarization	Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ – *Boer-Mulders*
L		$g_1(x,k_T^2)$ – *Helicity*	$h_{1L}^\perp(x,k_T^2)$ – *Long-Transversity*
T	$f_1^\perp(x,k_T^2)$ – *Sivers*	$g_{1T}(x,k_T^2)$ – *Trans-Helicity*	$h_1(x,k_T^2)$ – *Transversity* $h_{1T}^\perp(x,k_T^2)$ – *Pretzelosity*



Analogous tables for:
- **Gluons** $f_1 \to f_1^g$ etc
- **Fragmentation functions**
- **Nuclear targets** $S \neq \frac{1}{2}$

❑ Polarized SIDIS:

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

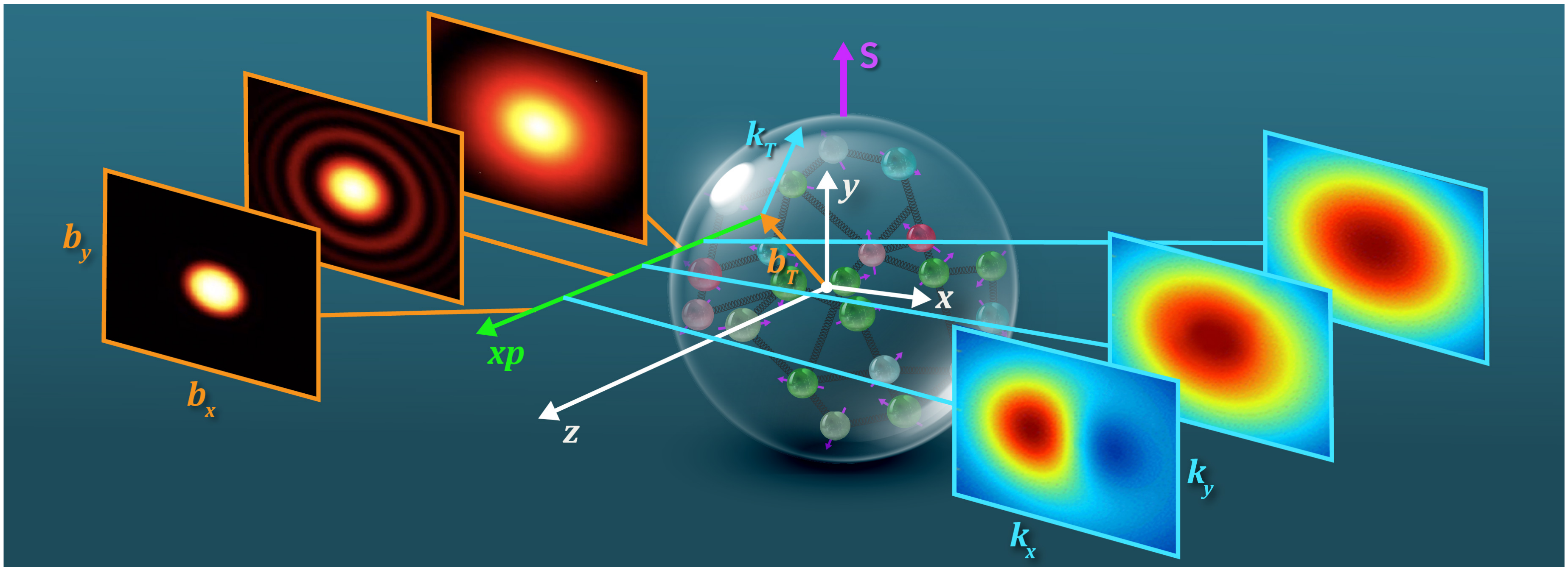
$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Angular modulation provides the best way to separate TMDs

3D Hadron Structure

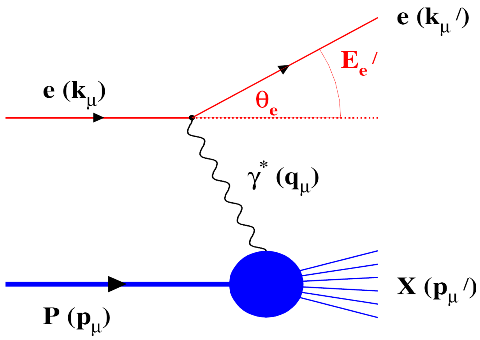
- ❑ **Tomographic images in both position and momentum space:**

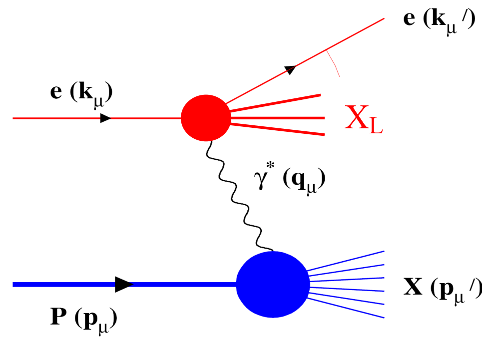


Tomographic images in slices of momentum fraction "x" for an active parton inside a hadron – controlled by the scale of the hard probe: ***Q ~ xp***

Collision Induced QED Radiation

❑ **Uncertainty in determining the photon-hadron frame:**





$$q_\mu \to \hat{q}_\mu$$

$$Q^2 = -q^2 \to \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \to \hat{x}_B = \frac{\widehat{Q}^2}{2P \cdot \hat{q}}$$

❑ **Under the "one-photon" approximation:**

$$E' \frac{d\sigma}{d^3 k'} = \frac{2\alpha_{\rm EM}^2}{s} \frac{1}{Q^4} L^{\mu\nu}(k,k;q) W_{\mu\nu}(q,P) \quad \Longrightarrow \quad E' \frac{d\sigma}{d^3 k'} = \frac{2\alpha_{\rm EM}^2}{s} \int d^4\hat{q} \left(\frac{1}{\hat{q}^2}\right)^2 \widetilde{L}^{\mu\nu}(k,k;\hat{q}) W_{\mu\nu}(\hat{q},P)$$

$$\widetilde{L}^{\mu\nu}(k,k;\hat{q}) = \sum_{X_L} \int \prod_{i\in X_L} \frac{d^3 k_i}{(2\pi)^3 2E_i} \delta^{(4)}\Big(k - k' - \hat{q} - \sum_{i\in X_L} k_i\Big) \langle k | j^\mu(0) | k' X_L \rangle \langle k' X_L | j^\nu(0) | k \rangle$$

$$= -\widetilde{g}^{\mu\nu}(\hat{q})\, L_1 + \frac{\widetilde{k}^\mu \widetilde{k}^\nu}{k\cdot k'} L_2 + \frac{\widetilde{k'}^\mu \widetilde{k'}^\nu}{k\cdot k'} L_3 + \frac{\widetilde{k}^\mu \widetilde{k'}^\nu + \widetilde{k'}^\mu \widetilde{k}^\nu}{2k\cdot k'} L_4$$

$$\to 2\big(k^\mu k'^\nu + k'^\mu k^\nu - k\cdot k' g^{\mu\nu}\big)\, \delta^{(4)}(k - k' - \hat{q})$$

$$\widetilde{g}^{\mu\nu}(\hat{q}) = g^{\mu\nu} - \frac{\hat{q}^\mu \hat{q}^\nu}{\hat{q}^2}$$

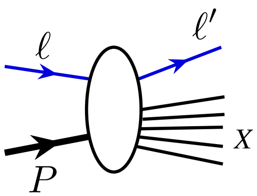
$$\widetilde{k}^\mu = \widetilde{g}^{\mu\nu}(\hat{q}) k_\nu$$

$$\widetilde{k'}^\mu = \widetilde{g}^{\mu\nu}(\hat{q}) k'_\nu$$

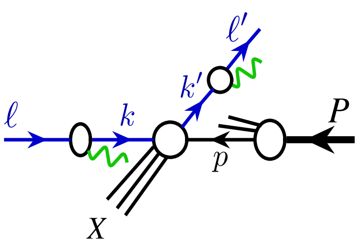
4 Lepton structure functions! Small α leads to $\hat{q}_T^2 \ll \widehat{Q}^2$ in lepton back-to-back frame!

Inclusive Lepton-Hadron DIS

- **Inclusive production of single high p_T lepton in lepton-hadron collision:** $e(\ell,\lambda_\ell)+N(P,S)\to e(\ell')+X$



$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}=\frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$



$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'}\approx\frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2)\;+\;\cdots$$

Lepton distribution functions (LDFs): $f_{i/e}(\xi,\mu^2)$

Lepton fragmentation functions (LFFs): $D_{e/j}(\zeta,\mu^2)$ $\quad i.j=e,\gamma,\bar{e},...,q,g,...$

Parton distribution functions (PDFs): $f_{a/N}(x,\mu^2)$ $\quad a=q,g,\bar{q},e,\gamma,\bar{e},...$

Short-distance hard coefficients: $\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2)$

$$\approx\widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell,xP,\ell/\zeta,\mu^2)\approx\mathcal{O}(\alpha^m\alpha_s^n)$$

- **No DIS "Structure Functions"!**
  Concept of one-photon exchange
- **QED & QCD contribution are factorized at the same scale: μ**
  $(x_B,Q^2)\;\to\;(y,\ell'_T)$
- **Corrections suppressed by power**
  $(1/\ell'_T)^\alpha$

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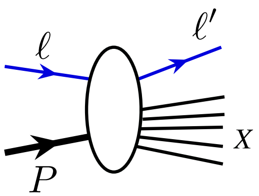
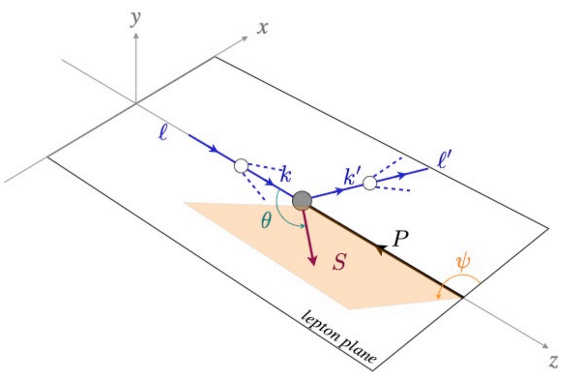
Inclusive Lepton-Hadron DIS

- Inclusive production of single high p_T lepton in lepton-hadron collision: $e(\ell,\lambda_\ell)+N(P,S)\to e(\ell')+X$



$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}=\frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$

- Recover the concept of structure functions?



$$E_{\ell'}\frac{d^3\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}}{d^3\ell'}\approx\sum_{\lambda_k}\int_{\zeta_{\min}}^1\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^1 d\xi f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2)\times\left[E_{k'}\frac{d^3\hat\sigma_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\right]_{k=\xi\ell,\,k'=\ell'/\zeta},$$

$$E_{k'}\frac{d^3\hat\sigma_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\approx\frac{2\alpha^2}{\hat s\,\widehat Q^4}L^{(0)}_{\mu\nu}(k,k',\lambda_k)\,W^{\mu\nu}(\hat q,P,S)$$

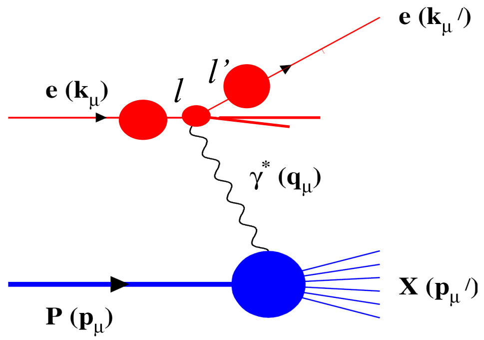
$$W^{\mu\nu}(\hat q,P,S)=-\widetilde g^{\mu\nu}(\hat q)\,F_1(\hat x_B,\widehat Q^2)+\frac{1}{P\cdot\hat q}\widetilde P^\mu(\hat q)\widetilde P^\nu(\hat q)F_2(\hat x_B,\widehat Q^2)+\dots$$

Structure functions are evaluated at $(\hat x_B,\widehat Q^2)$ instead of (x_B,Q^2)!

Collinear Factorization for QED Radiative Conntribution

- Collinear factorization with the "one-photon" approximation:

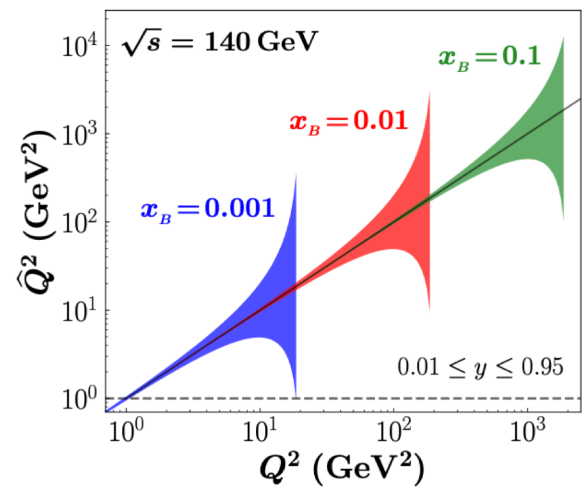
Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1 d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$
$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is IR sensitive as $m_e/Q \to 0$, into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$

$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)} \qquad \widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$

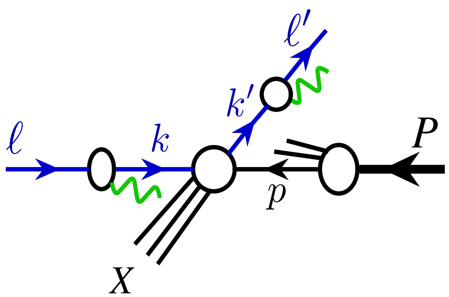


Collinear Factorization for QED Radiative Conntribution

- **Without the "one-photon" approximation:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

~ Inclusive single lepton production at high transverse momentum

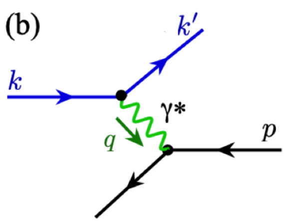


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) \;+\;\cdots$$

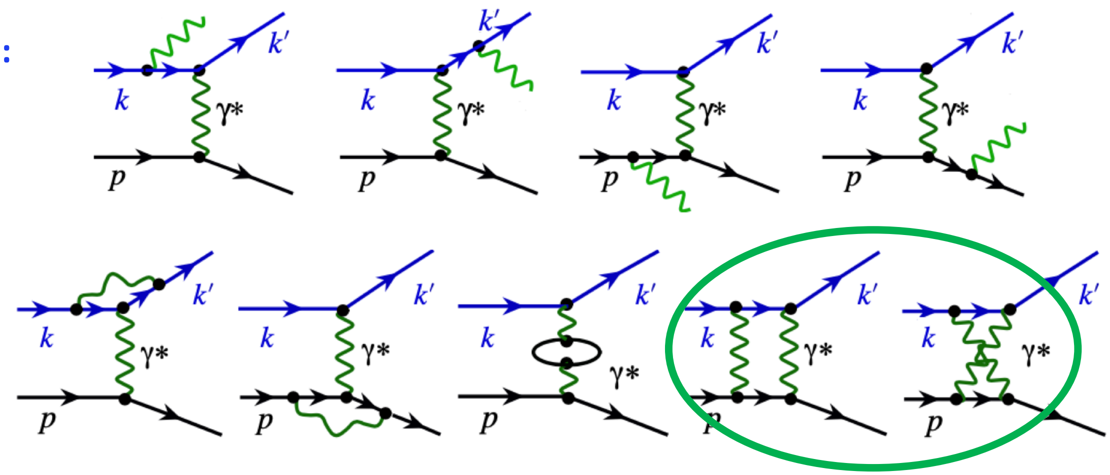
No structure functions, but have PDFs, LDFs, LFFs, …

- **Calculated hard parts in power of** $\alpha^m\alpha_s^n$ **:**

LO



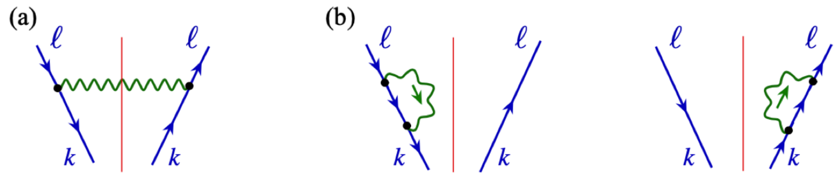
NLO:



Beyond one-photon exchange

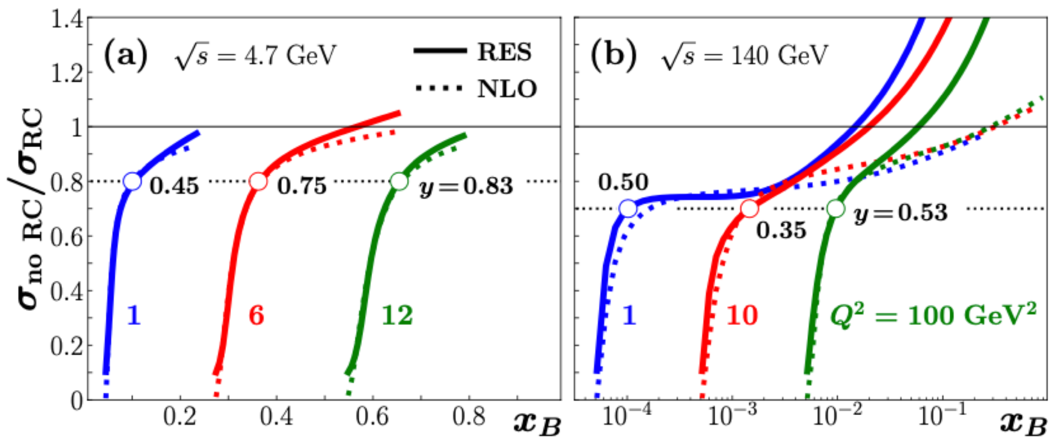
QED Radiative Corrections vs Radiative Contributions

- **Lepton distribution function:**



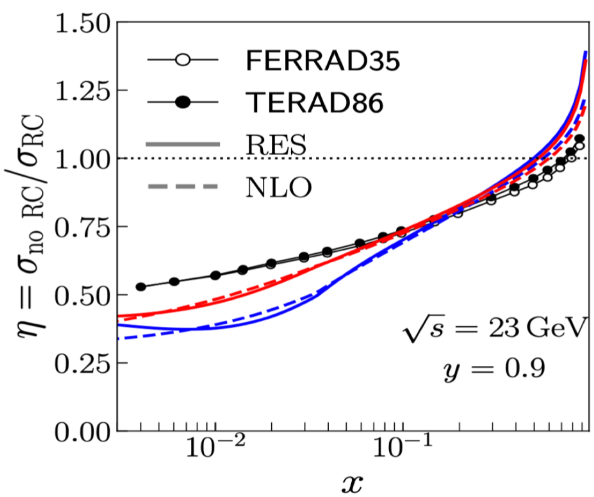
$$f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2\, m_e^2}\right]_+$$

+ nonperturbative contributions …



- **Lepton evolution – e.g., valence:**

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2} f_{e/e}(\xi,\mu^2) = \int_\xi^1 \frac{\mathrm{d}\xi'}{\xi'}\, P_{ee}\!\left(\frac{\xi}{\xi'},\alpha\right) f_{e/e}(\xi',\mu^2).$$



- **Lepton fragmentation function:**

$$D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2\, m_e^2}\right]_+$$

+ nonperturbative contributions …

QED Radiative Corrections vs Radiative Contributions

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

- **QED radiative corrections:**

$$\sigma_{\rm obs}(x_B,Q^2) \overset{?}{=} R_{\rm QED}(x_B,Q^2;x_{B,{\rm true}},Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,{\rm true}},Q^2_{\rm true}) + \sigma_X(x_B,Q^2)$$

  - The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision;
  - The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime.
  - Extraction of $\sigma_{\rm Born}$ is an inverse problem

- **QED radiative contributions:**

$$\sigma_{\rm obs}(x_B,Q^2) = \sigma^{\rm univ}_{\rm lep}(\mu^2;m_e^2) \otimes \sigma^{\rm univ}_{\rm had}(\mu^2;\Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\text{IR-safe}}(\hat{x}_B,\widehat{Q}^2,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2},\frac{m_e^2}{Q^2}\right)$$

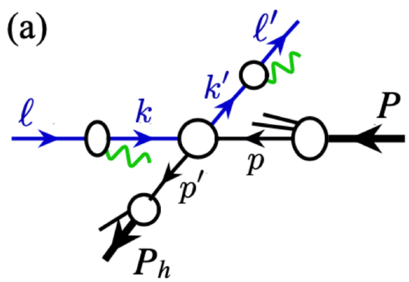
  - Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
  - Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
  - Power suppressed contributions as $m_e/Q \to 0$, are neglected

Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **Inclusive production of a lepton and a hadron:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



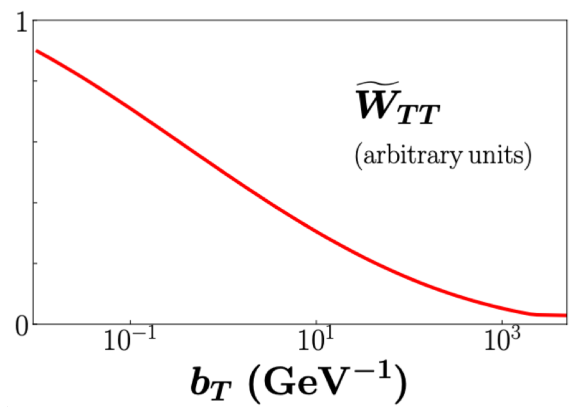
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

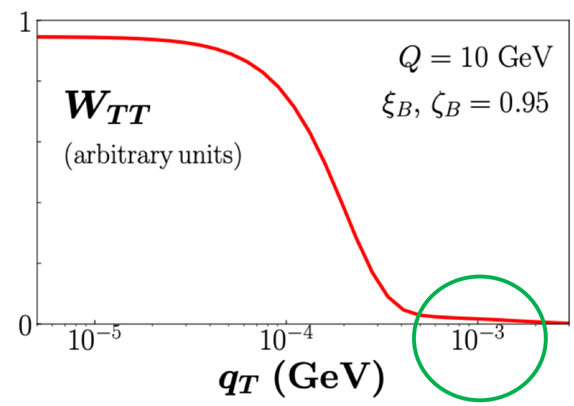
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\mathrm{QCD}}^2 + \langle k_T^2 \rangle_{\text{generated by QCD shower}}$

- **Estimate of lepton transverse momentum generated by QED shower:**

Resummation to lepton TMD





QED broadening for lepton is so much smaller than typical parton kT!

Collinear factorization for high order QED contributions

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

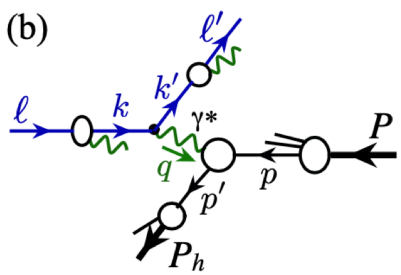
❑ QED factorization of collision induced radiation – collinear:

$$E_{\ell'}E_{P_h}\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{d^3\ell'\,d^3P_h} \approx \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\,D_{e/j}(\zeta)\int_{\xi_{\min}}^1 d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\left[E_{k'}E_{P_h}\frac{d^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{d^3k'\,d^3P_h}\right]_{k=\xi\ell,\,k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

❑ "One photon"-approximation:

(b)



$$\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{hT}^2} = \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1\frac{d\xi}{\xi}\,f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\,D_{e/j}(\zeta)$$

$$\times\,\frac{\hat{x}_B}{x_B\xi\zeta}\underbrace{\left[\frac{\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\,\frac{\hat{y}^2}{2(1-\hat{\varepsilon})}\left(1+\frac{\hat{\gamma}^2}{2\hat{x}_B}\right)\sum_n \hat{w}_n F_n^h(\hat{x}_B,\widehat{Q}^2,\hat{z}_h,\widehat{P}_{hT}^2)\right]}_{\text{Evaluated in a "virtual photon-hadron" frame}}$$

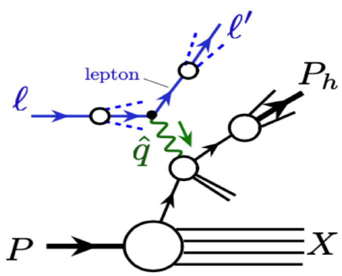
Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat{q}, P, \widehat{P}_h\} \xrightarrow[(\xi,\zeta)]{} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **Two-step approach to SIDIS:**



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi,\zeta) - p$

- **TMD factorization when** $\widehat{P}_T^2 \ll \widehat{Q}^2$
- **CO factorization when** $\widehat{P}_T^2 \sim \widehat{Q}^2$
- **Matching to get the** $\widehat{P}_T$**-distribution**

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame – lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

- **Case study F_{UU}:**

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{\boxed{F_{UU,T}}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$
$$+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **Case study F_{UU}:**

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\min}}^1 d\zeta \int_{\xi_{\min(\zeta)}}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \underbrace{\left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right] \left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B \hat{y}\, \widehat{Q}^2}\,\frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]}_{\text{Evaluated in a “virtual photon-hadron” frame}}$$

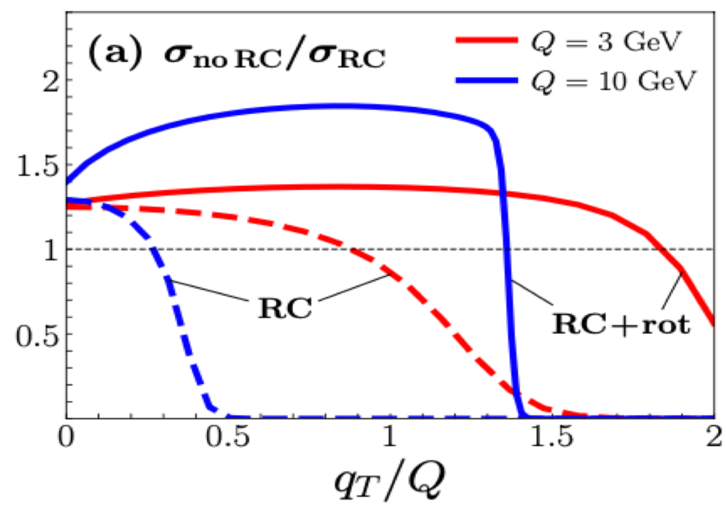
Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ, ζ) **- Dependent Lorentz transformation**

Effectively, a rotation in hadron-rest frame

Solid – with rotation
Dashed – without rotation



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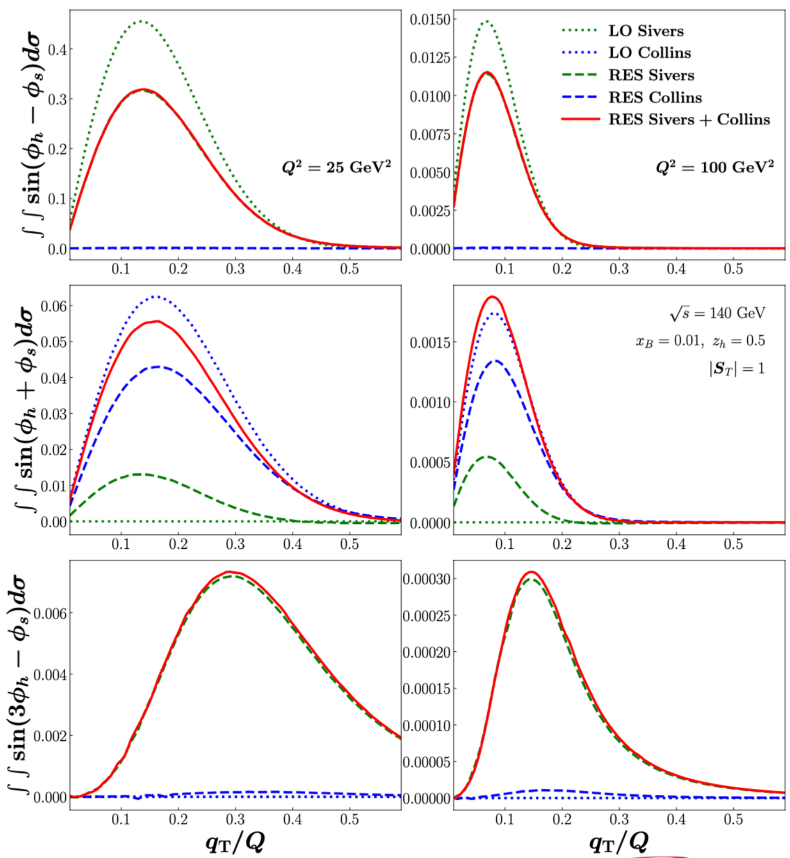
Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

❑ Case study – single transverse spin asymmetry:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} \\
&\quad+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} \\
&\quad+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right] \\
&\quad+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$



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Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **QED radiative corrections:**

Radiative Effects in the Processes of Hadron Electroproduction

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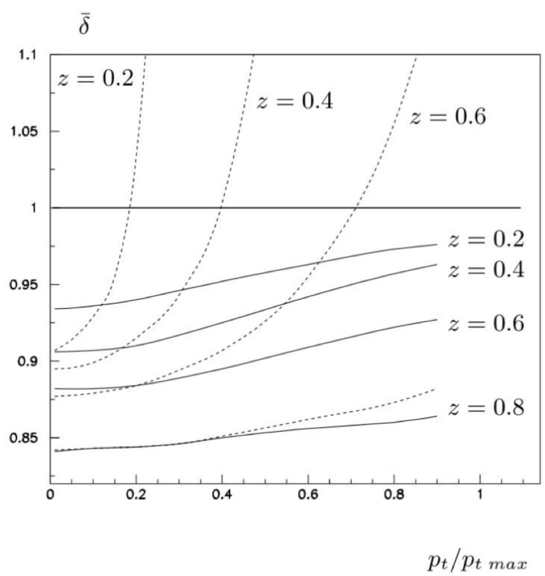
Received: date / Revised version: date

Abstract. An approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. An explicit formulae for the lowest order QED radiative correction are presented. Detailed numerical analysis is performed for the kinematics of experiments at the fixed targets.



Similar trends. Eg. RCs depends on hadronic input

- **Our formalism is different. It does not depend on hadronic input, which is what we want to probe!**
- **Our formalism is organized in terms of IR safe quantities and universal functions – advantage of factorization**

Summary and Outlook

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- Factorization approach to include both QCD and QED radiative contributions (and shower of weak particles at the LHC energies) provides a consistent and controllable approximation
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Thank you!

Special thanks to experimental colleagues at JLab for helpful discussions!