Quark structure of pion from lattice QCD

CFNS Workshop: High Luminosity-EIC

June 23, 2022

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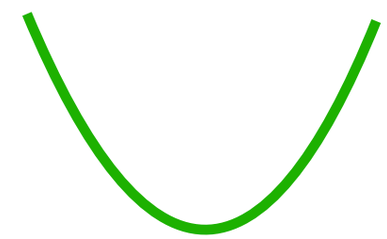
Jefferson Lab - William & Mary

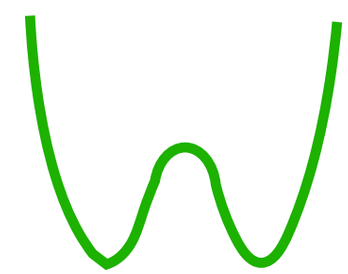
Understanding the internal structure of Nambu-Goldstone boson

- String tension in pure-gauge theory, massive proton even in massless limit,…
- Spontaneous Chiral symmetry breaking in QCD coupled to massless quarks

$$\mathrm{SU}_L(N_f) \times \mathrm{SU}_R(N_f) \times U_V(1)$$

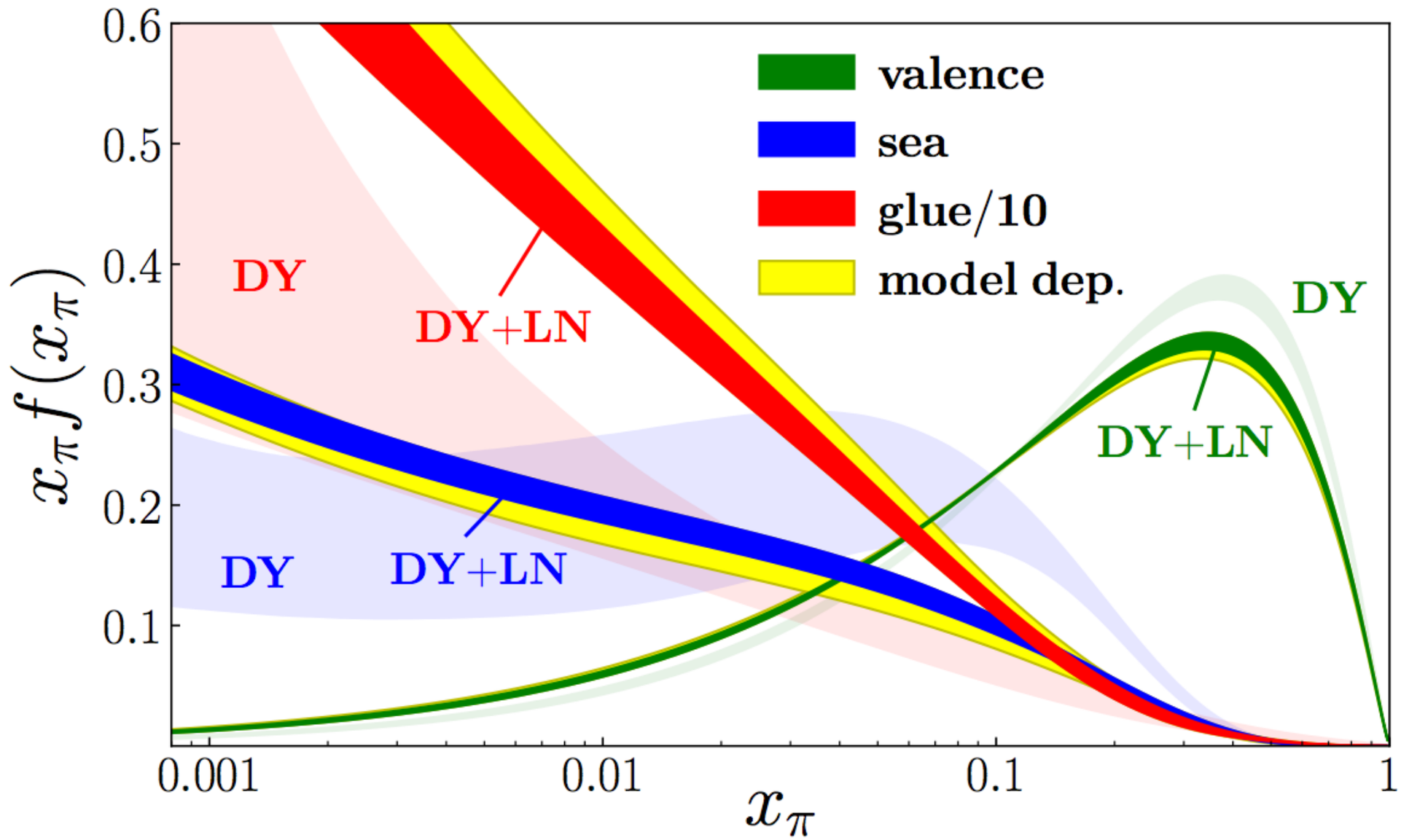


$$\mathrm{SU}_V(N_f) \times U_V(1)$$

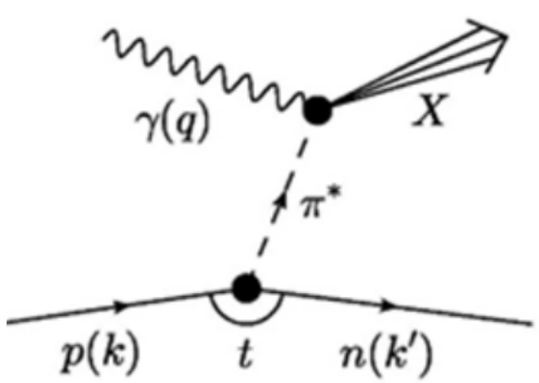


- $\Sigma = \langle \bar{\psi}_i \psi_i \rangle \neq 0$ Chiral-condensate
- $N_f^2 - 1$ Massless Nambu-Goldstone bosons (“Pions”)
- $\langle 0 | J_{5\mu}(0) | \pi \rangle = F_\pi p_\mu$ Decay constant

Valence PDF of $\pi^+(u\bar{d})$



JAM18, P. C. Barry et al, 2018



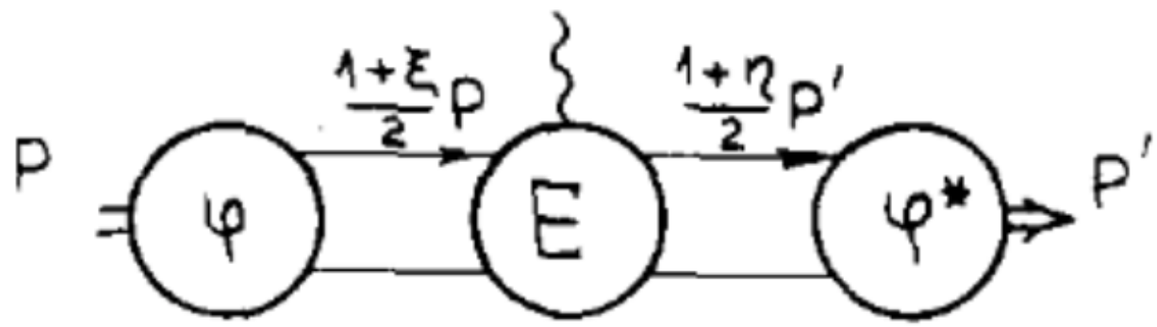
No pion targets to perform DIS on.
PDF determination via indirect processes and assumptions

Direct determination on the lattice

Factorization of form-factors: Distribution amplitude

Lepage, Brodsky '80

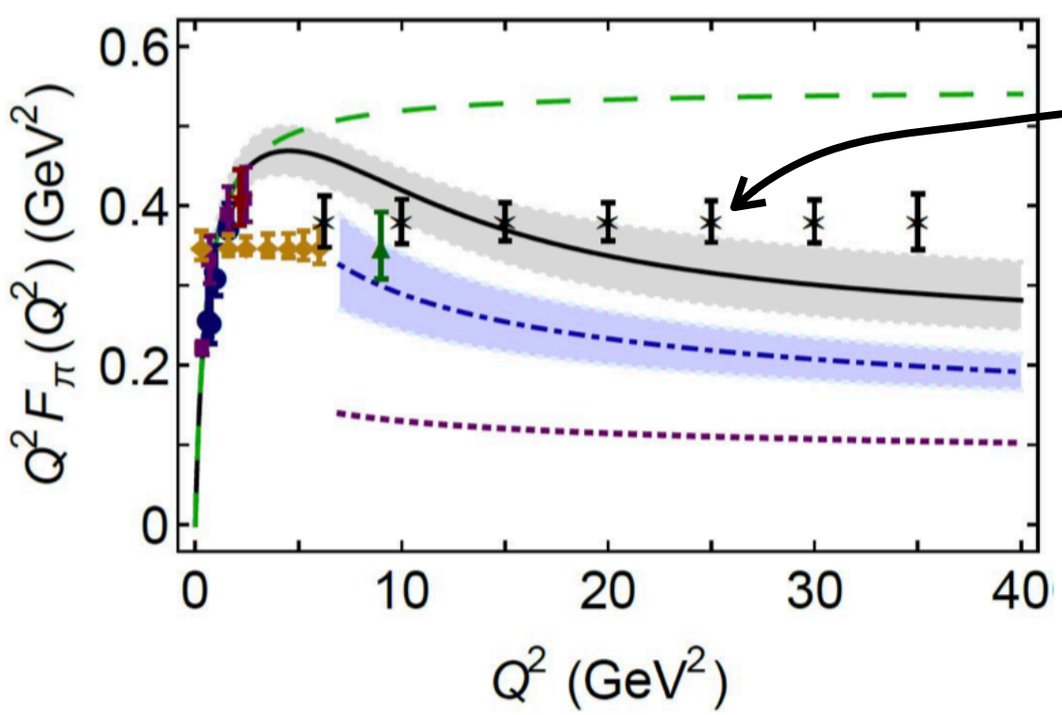
Efremov, Radyushkin '80



$$\langle P'|\mathcal{O}(Q)|P\rangle \sim \int_0^1 \int_0^1 dx dy\ \phi(x) E_{\mathcal{O}}(x,y)\phi(y)$$

EM current, stress tensor,…

Beyond what values of Q^2 do FFs approach their partonic description?



Projection for EIC
(20fb^{-1}, 5 GeV e$^-$, 100 GeV p)

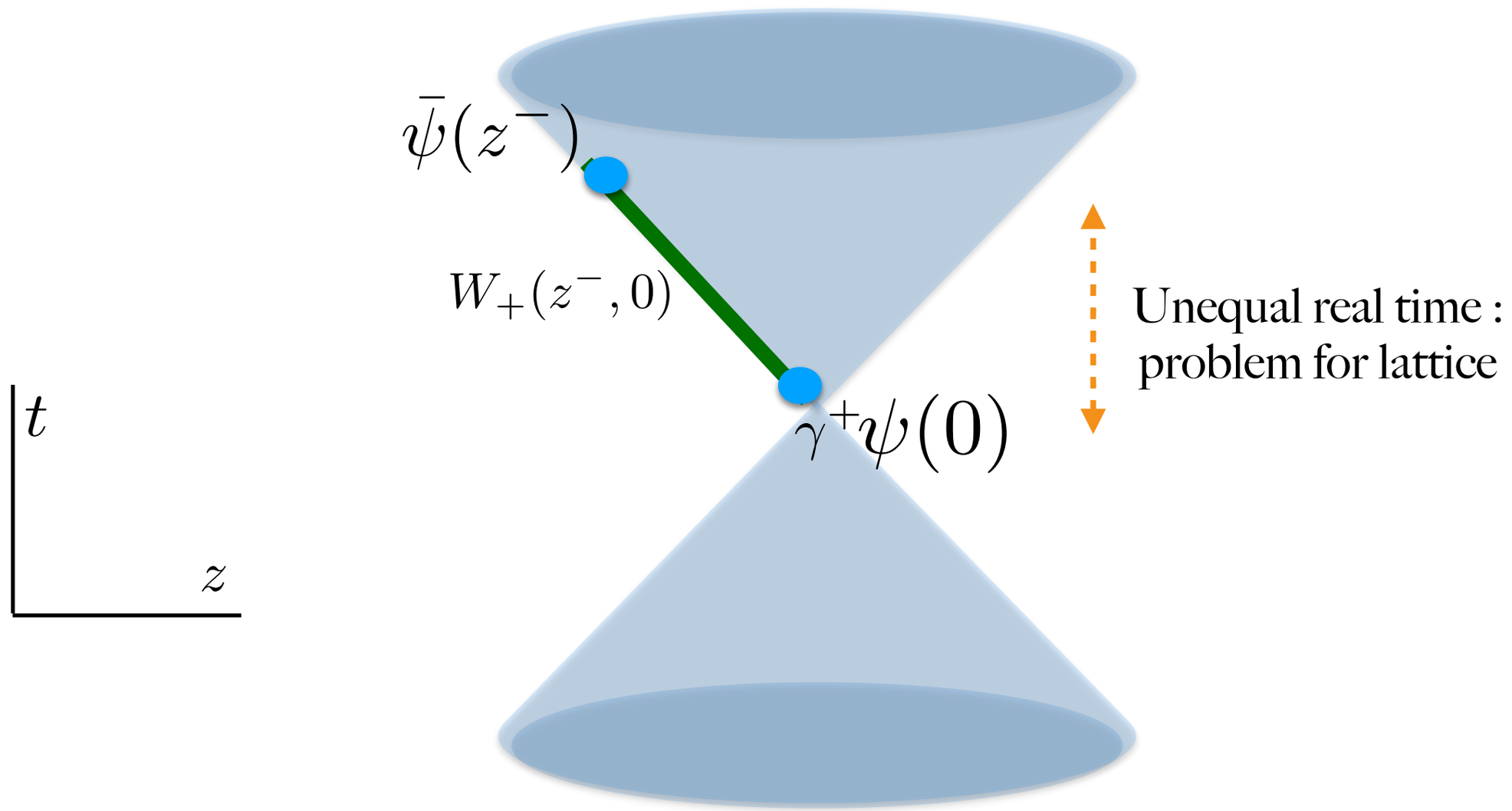
Aguilar et al, EPJA 55 (2019) 10, 190

What do we talk about when we talk about "*x*-dependent PDF from lattice"

I will restrict myself to one particular approach in this talk.

PDF as light-like separated q-$\bar{q}$ correlation

$$\mathcal{M}_{\rm LF}(z^- P^+, \mu) = \frac{1}{2P^+}\langle H(P)|\ \bar{\psi}(z^-)W_+(z^-,0)\gamma^+\psi(0)\ |H(P)\rangle$$

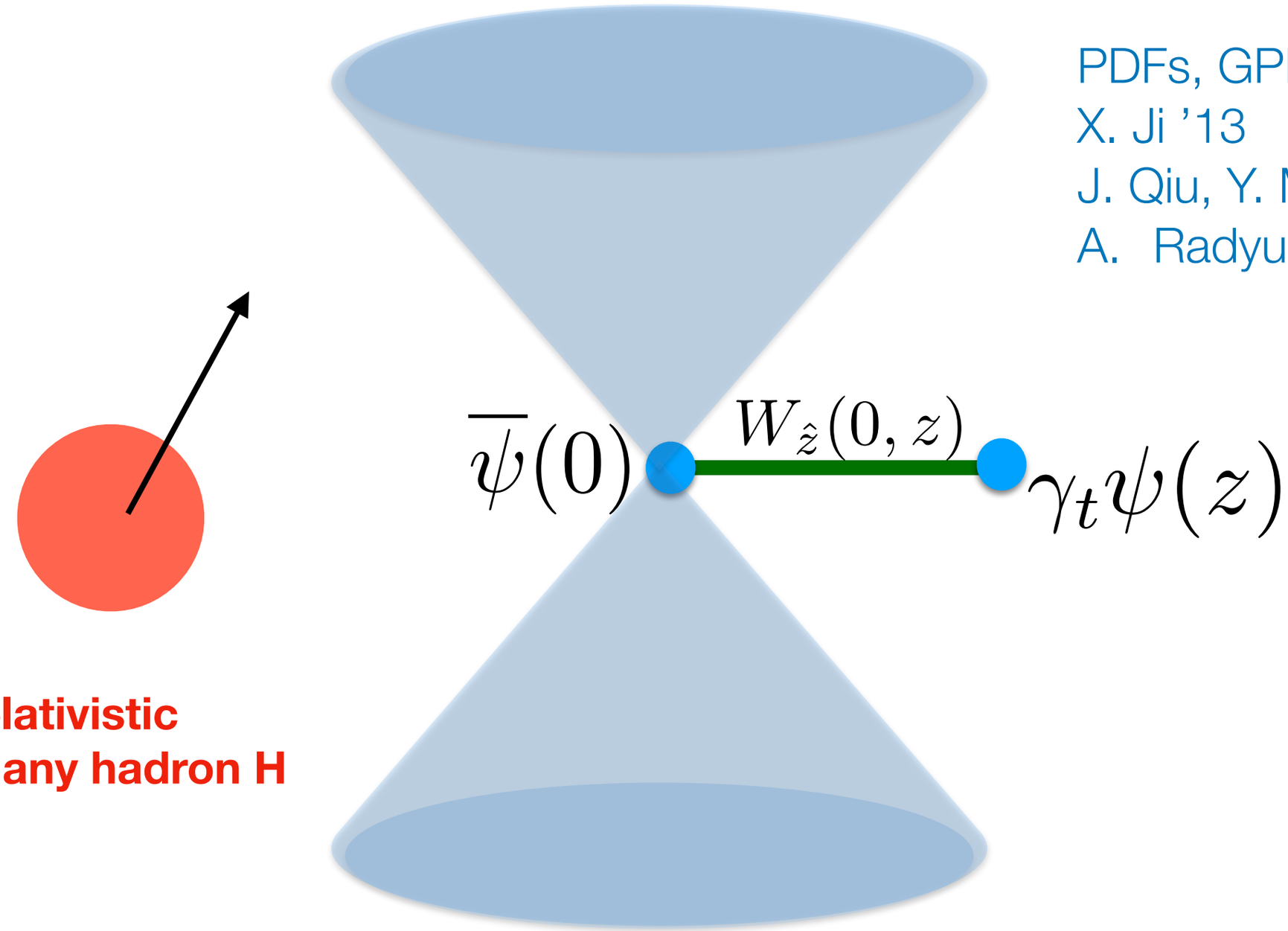


$$f(x,\mu) = \int \frac{d\nu}{2\pi} e^{-ix\nu}\mathcal{M}_{\rm LF}(\nu,\mu)$$

Light-Front physics from equal-time matrix elements

(conformal OPE for DA)
V. Braun, D. Muller '08

PDFs, GPDs, TMDs,..
X. Ji '13
J. Qiu, Y. Ma '14
A. Radyushkin '17



Ultra-relativistic
Pion or any hadron H

$$\mathcal{M}(zP_z, z^2) \overset{\textbf{Def}}{=} \langle H(\vec{P})| \; z \; |H(\vec{P})\rangle$$

Twist expansion as the unifying concept

Distill leading-twist terms from Euclidean matrix elements at large-hadron momentum and/or short-distances

(Renormalized quantity from LQCD e.g: ratios of qPDF matrix elements)

(IR-safe Wilson coeff.)

($\overline{\text{MS}}$ PDF moments)

$$\mathcal{M}(zP_z, z^2) = \sum_n c_n(z^2\mu^2)\frac{(-izP_z)^n}{n!}\langle x^n\rangle(\mu)$$

$$+ \text{ twist-2 trace } t \quad M^k P_z^{n-k} z^n$$

$$+ \text{ Higher twist } t \quad \Lambda_{\text{QCD}}^{2k} P_z^{2n-2k} z^{2n}$$

Suppress

* $\langle x^n\rangle = \int_0^1 x^n f(x)dx$

** **"Twist" = [Dimension] — [Spin]**

$$h(z, P_z) = \langle P_z | \overline{\psi}(0) \, W(0,z) \, \gamma_t \psi(z) | P_z \rangle$$

Renormalize

$$\mathcal{M}(zP_z, z^2) = \frac{h(z, P_z)}{h(z, 0)}$$

Fourier Transform at fixed P_z

$$q(x, P_z)$$

Twist-2 OPE $\sum_n \langle x^n \rangle c_n(\mu^2 z^2) \frac{(-iP_z z)^n}{n!}$

$$\int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f(y, \mu)$$

Compare in a range of zP_z

$$f(x, \mu^2)$$

An experimental determination

$$\langle x^n \rangle$$

Access moments directly

Fit Ansätze

$$x^\alpha (1-x)^\beta \mathcal{G}(x)$$

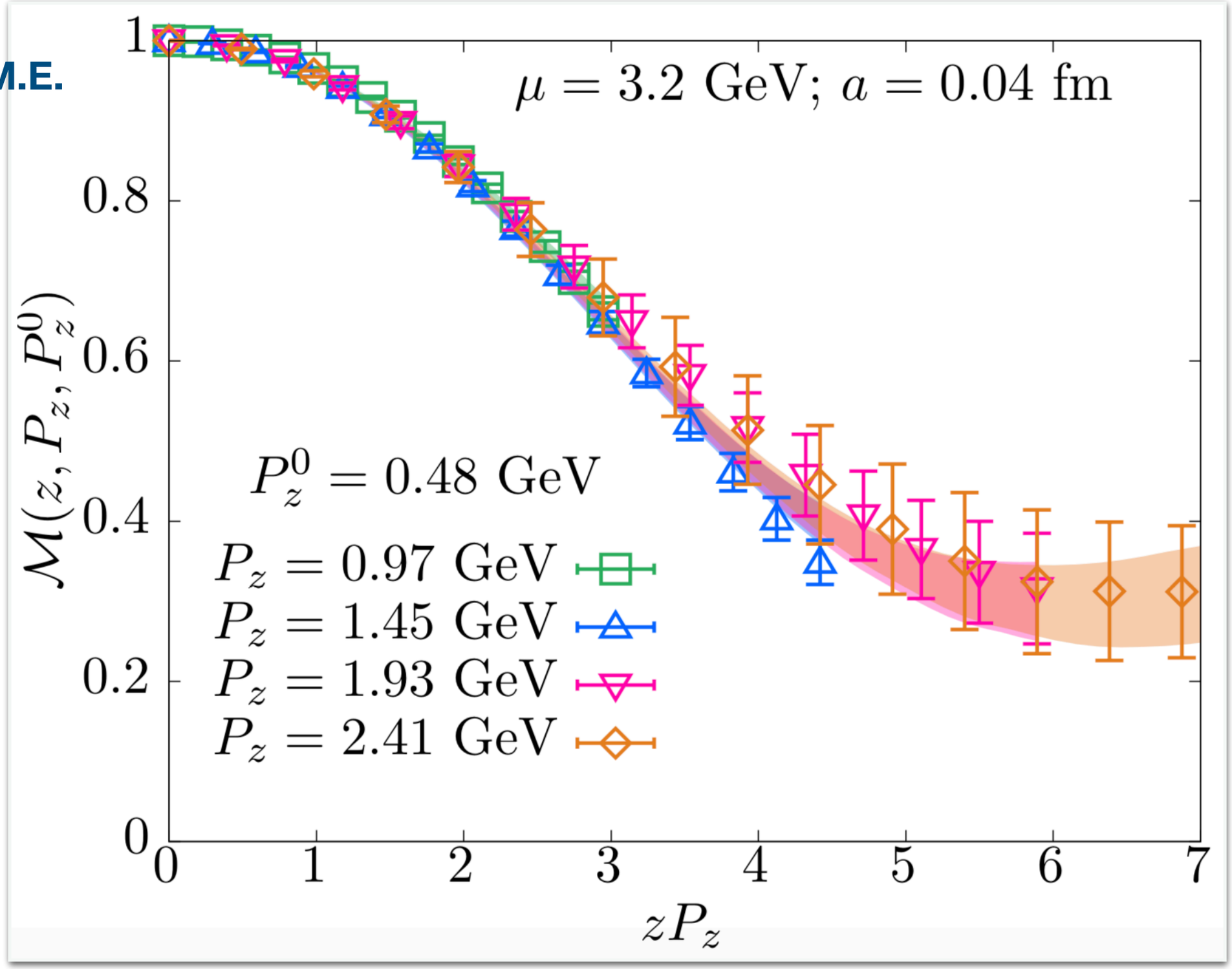
x-space convolution

$$f(x, \mu^2)$$

Valence PDF of pion: multiple perspectives on analysis

The basic object that lattice QCD measures:

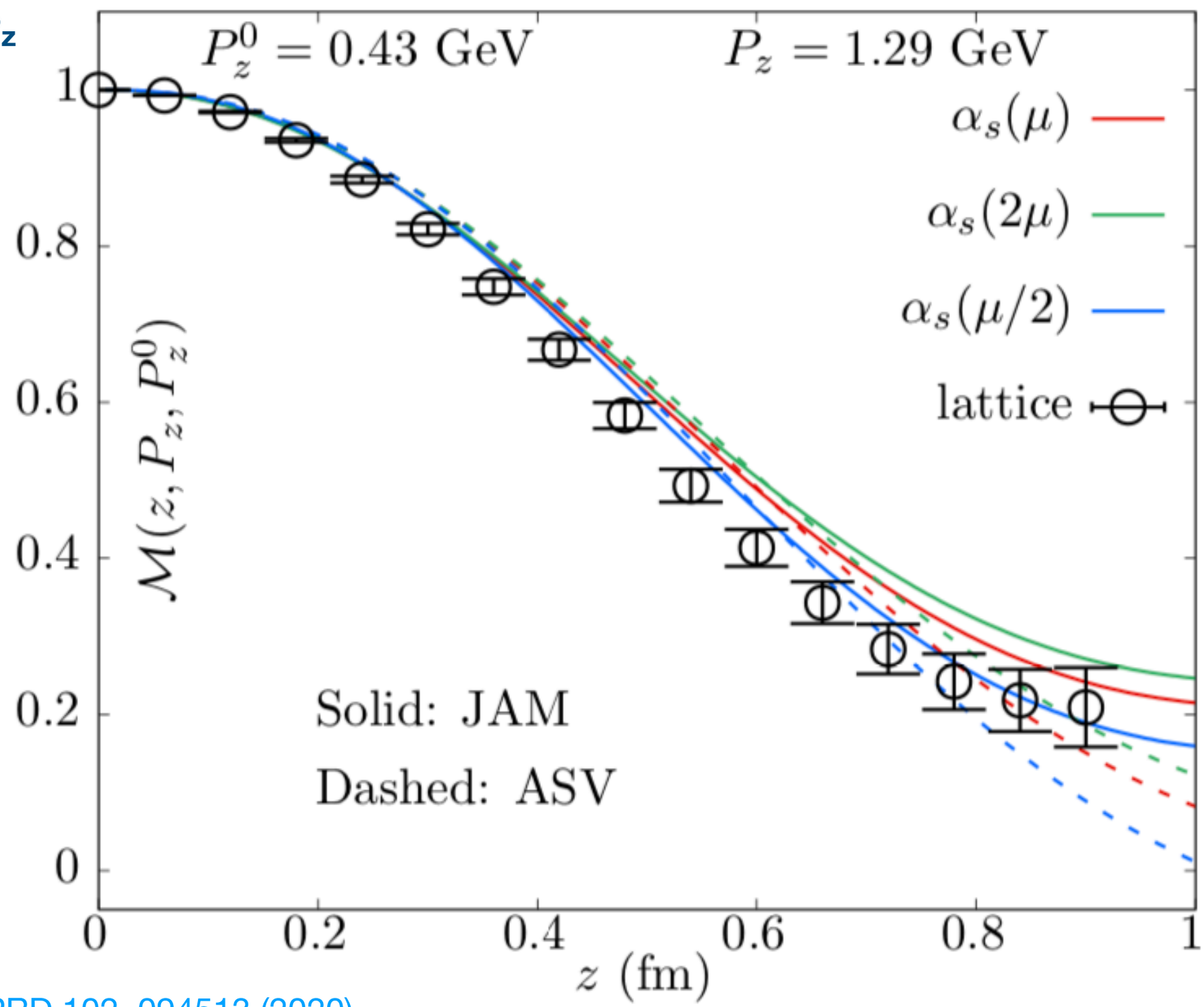
Ratio of qPDF M.E.



(Variable that is Fourier conjugate to Bjorken x)

Comparing with experimental results <u>without</u> data processing

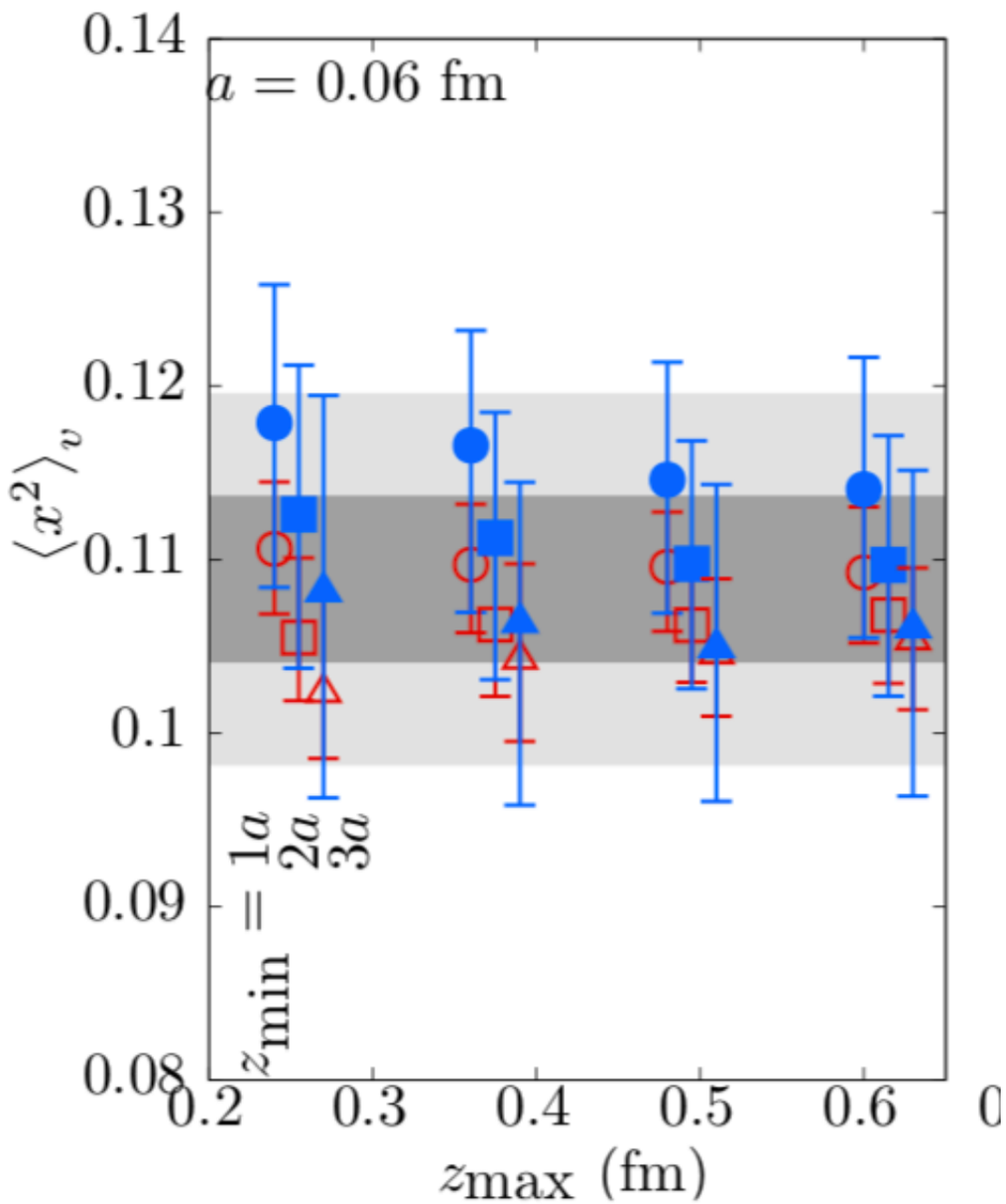
Pion matrix element at momentum P_z

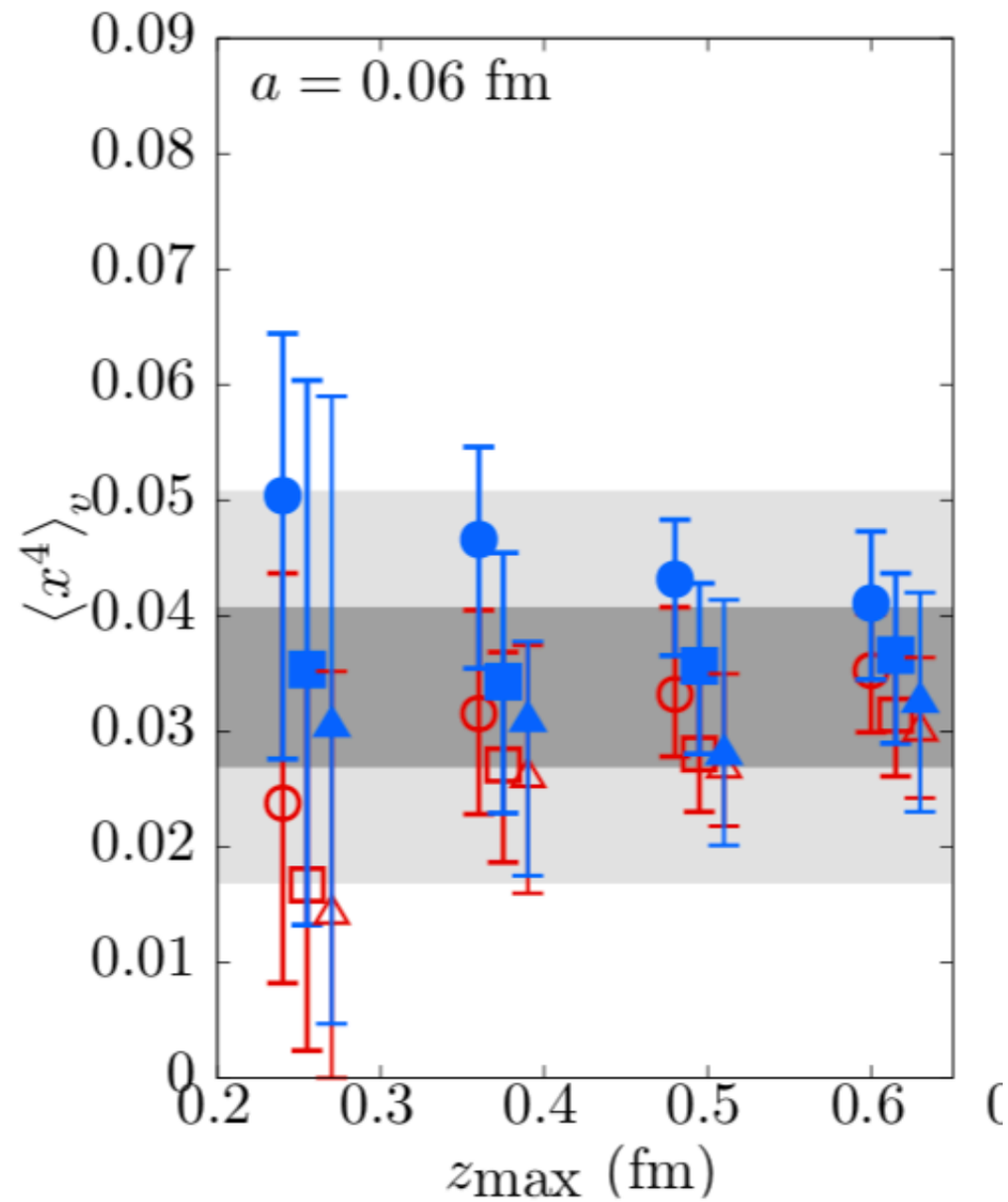


X. Gao, NK et al, PRD 102, 094513 (2020)

Extracting PDF data with minimal modeling: Mellin moments

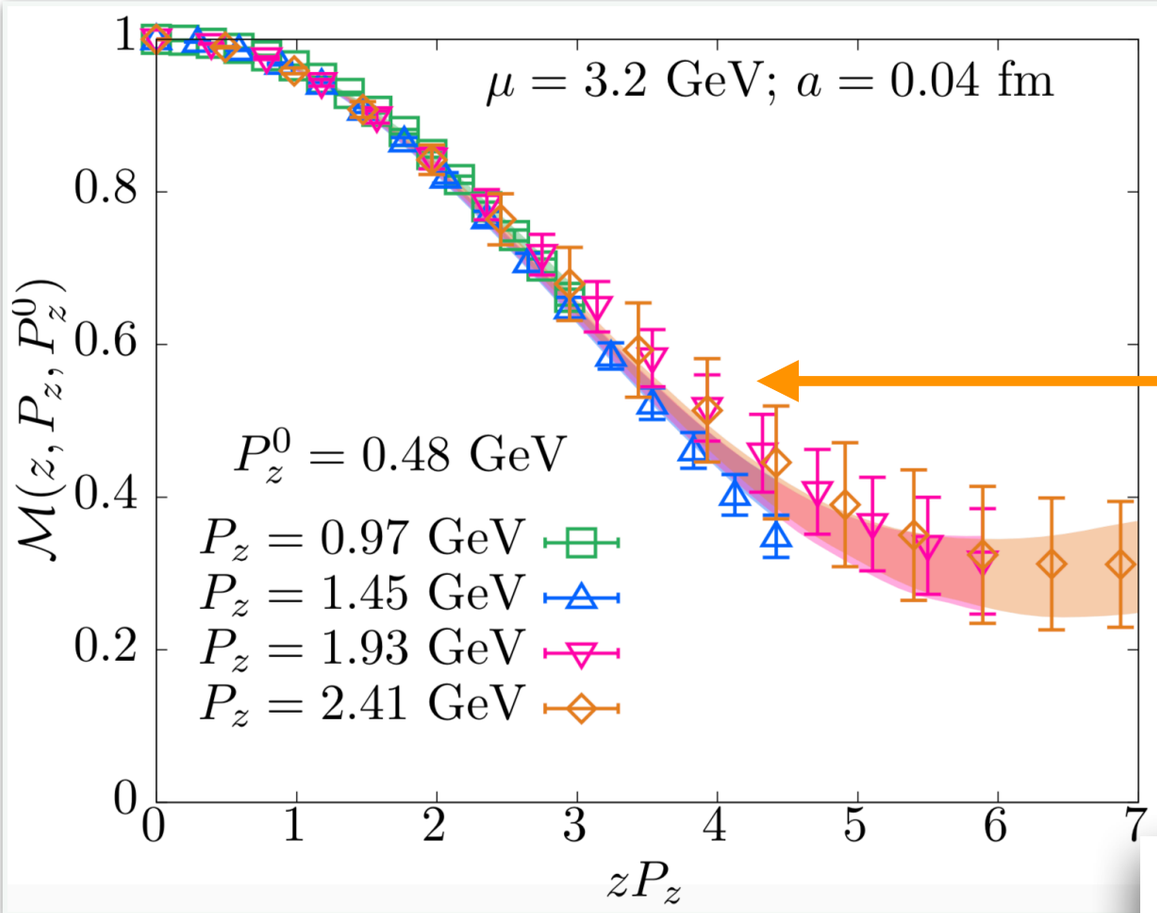
Fit PDF moments as free parameters without modeling the x-dependence





(maximum of z-range used)

X. Gao, NK et al, PRD 102, 094513 (2020)

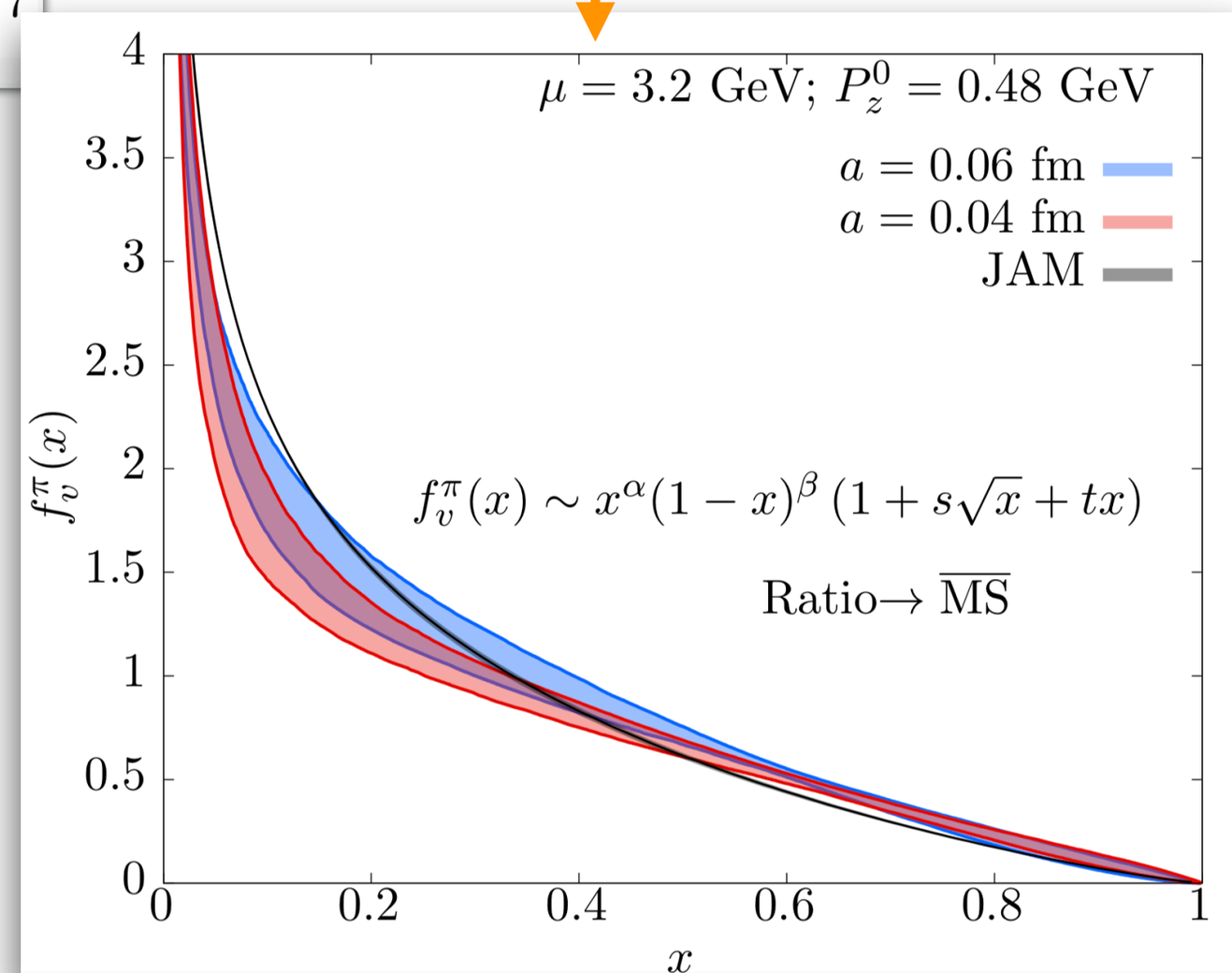


$$\mathcal{M}^{\mathrm{tw2}}(zP_z, z^2) = \sum_n \frac{(-izP_z)^n}{n!} c_n(\mu^2 z^2)\langle x^n \rangle$$

Fit z & zP_z dependencies to model PDFs

$$f_v^\pi(x,\mu) = \mathcal{N} x^\alpha (1-x)^\beta \left(1 + s\sqrt{x} + tx\right)$$

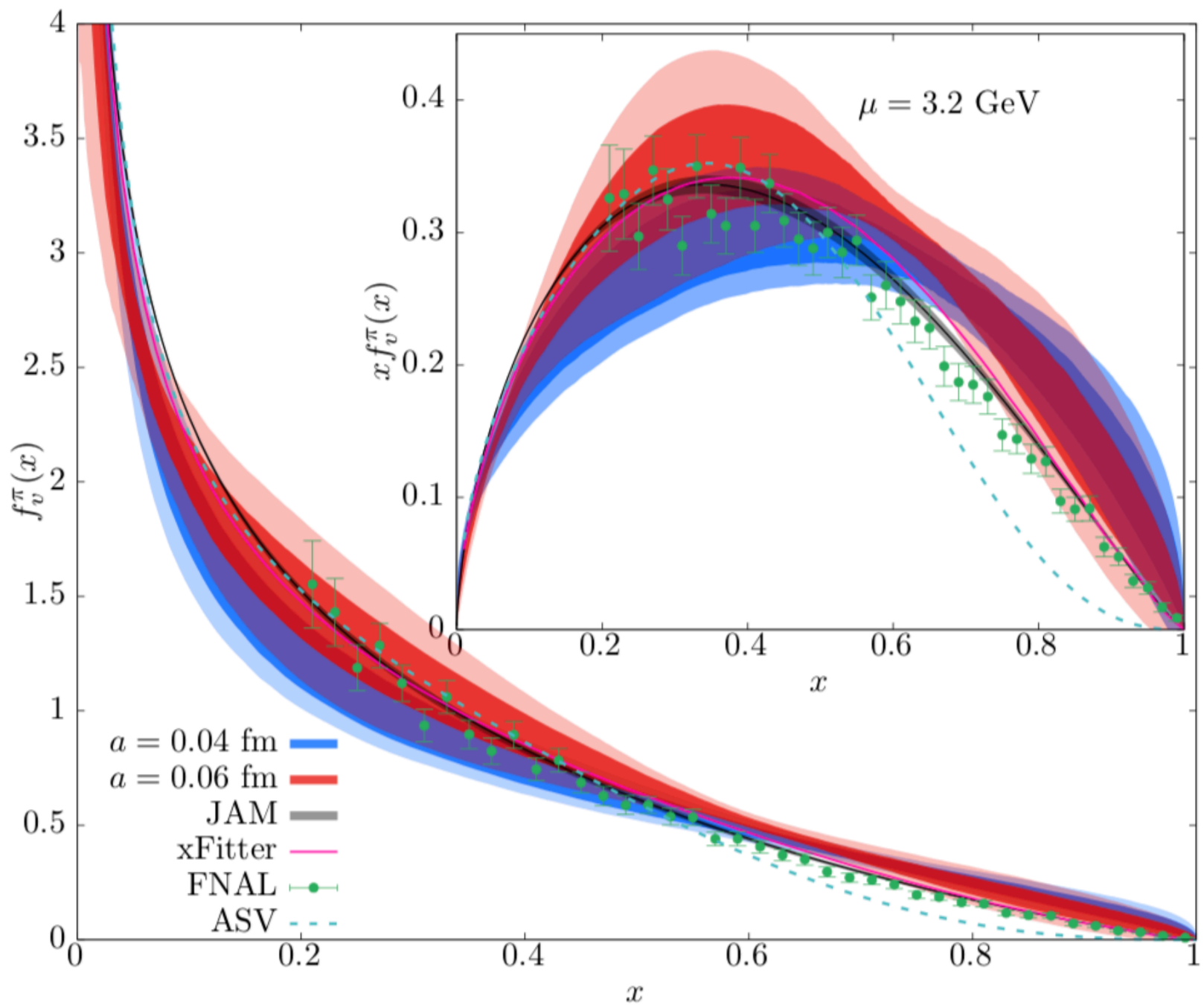
Corresponding PDF



Extracting x-dependent PDF Using Ansätze

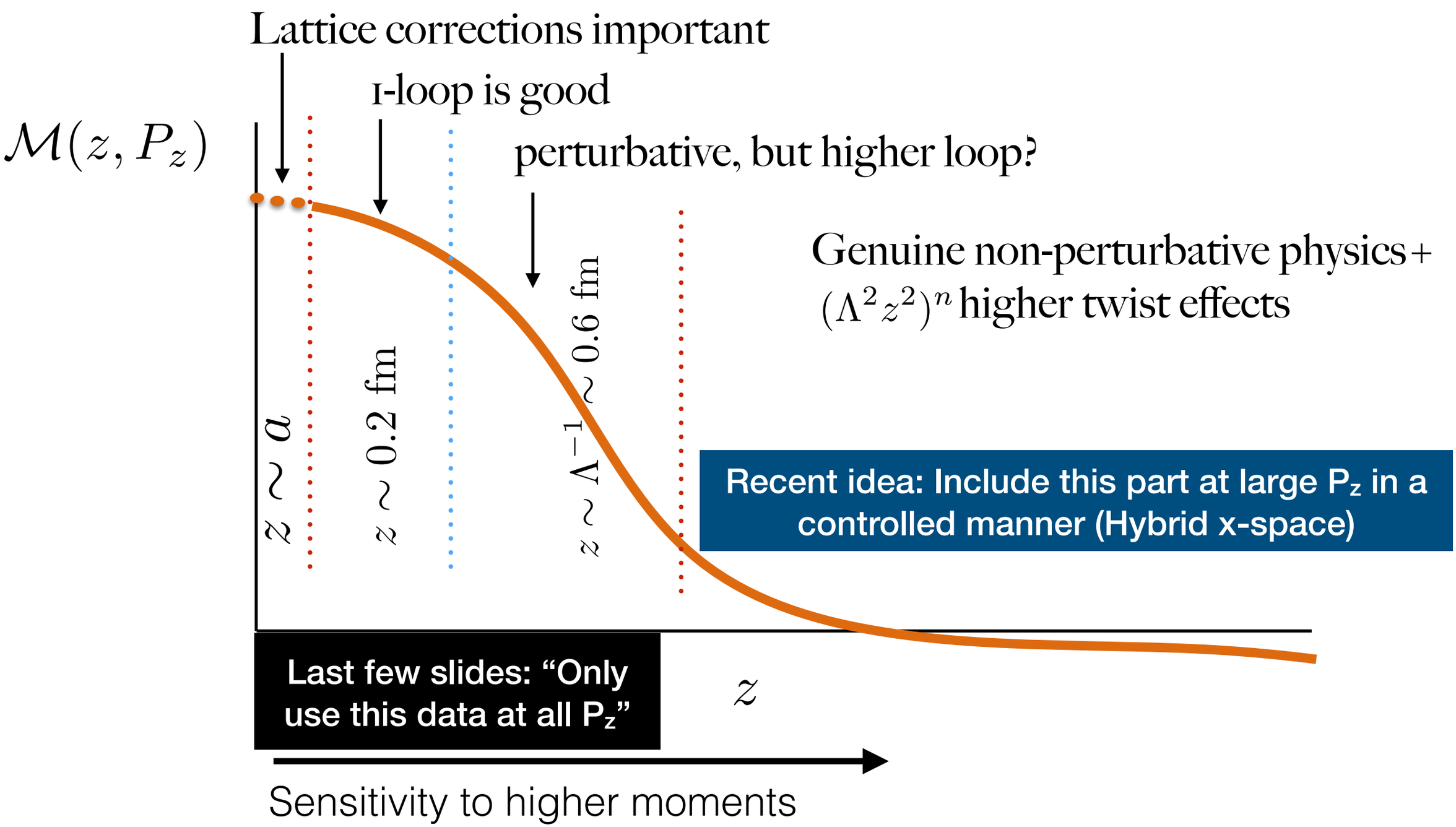
Towards continuum limit

X. Gao, NK et al, PRD 102, 094513 (2020)
T. Izubuchi, NK et al, PRD 100, 034516 (2019)

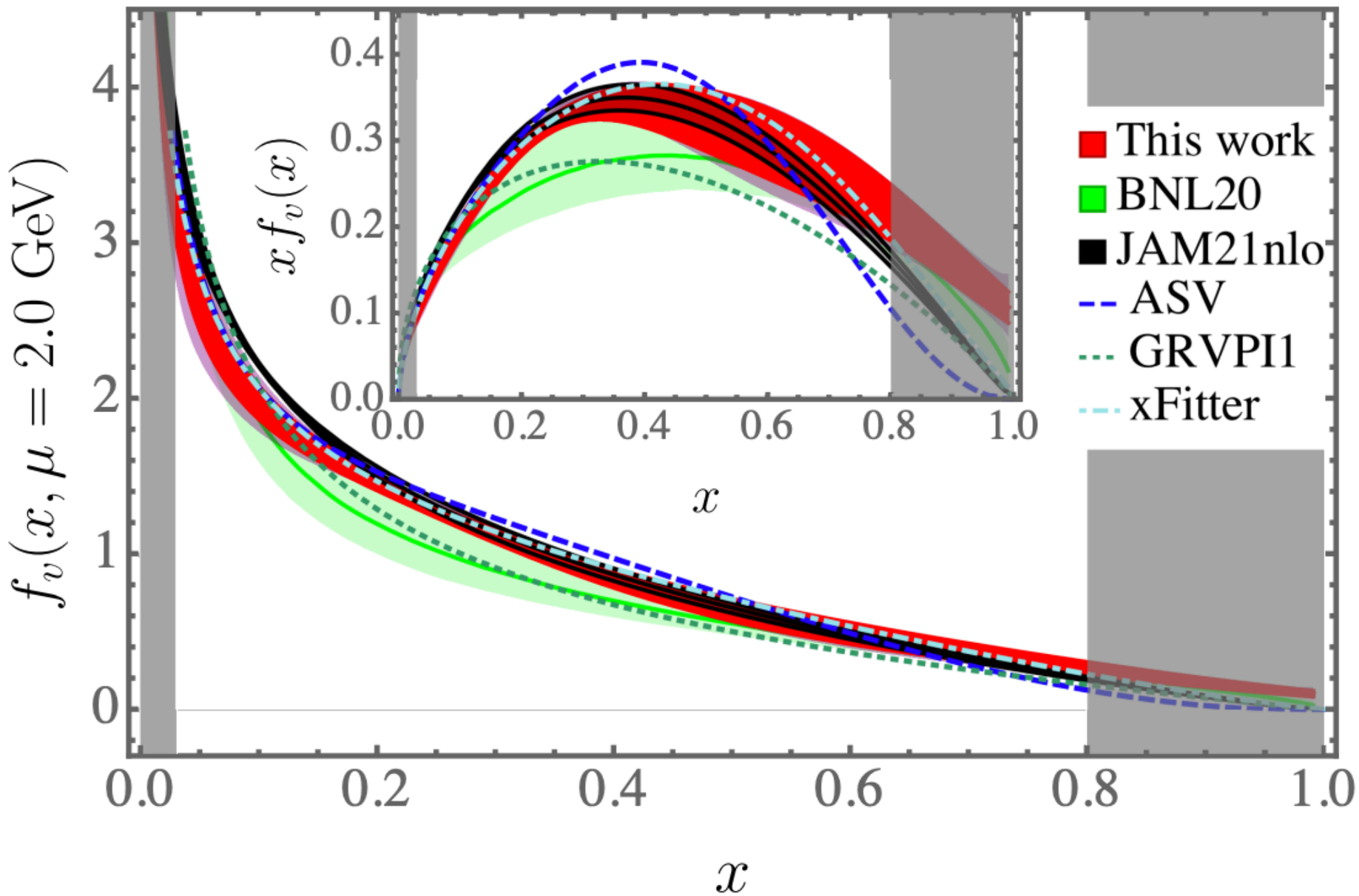


- Two fine lattice spacing with a^2 corrections
- Systematic errors: range of quark-antiquark distance, ren. scheme, PDF ansatz..

Roadmap for precision lattice studies



Hybrid LaMET x-space matching



Recent work using exactly the same lattice data I showed in the last few slides, but analysis in x-space

Y. Zhao et al, PRL128 (2022) 14, 142003

Lattice meets experiments: universality

Universality → PDF in different processes

Universality → Wilson coefficients in different hadronic M.E.'s

$\mathcal{M}_{\text{hadron}-1}$

$\mathcal{M}_{\text{hadron}-2}$

Twist-2 OPE $\sum_n \langle x^n \rangle c_n(\mu^2 z^2) \frac{(-iP_z z)^n}{n!}$ → $c_n(\mu^2 z^2)$ → **Twist-2 OPE**

$f_{\text{hadron}-1}(x)$

An experimental determination

$f_{\text{hadron}-2}(x, t, \xi = 0)$

NK and R. S. Sufian, PRD104 (2021) 7, 074506

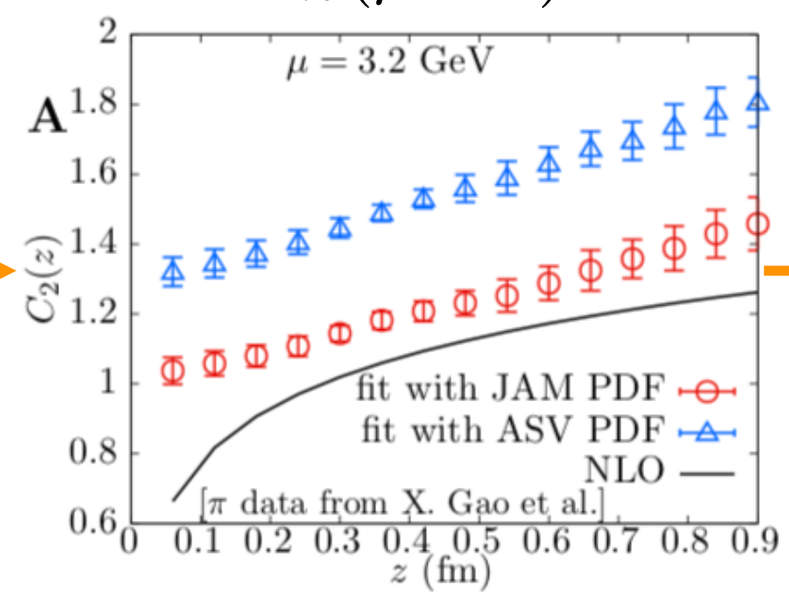
Lattice meets experiments: universality

Lattice data for pion matrix element (from BNL group)

Lattice data for proton matrix element (from ETMC)

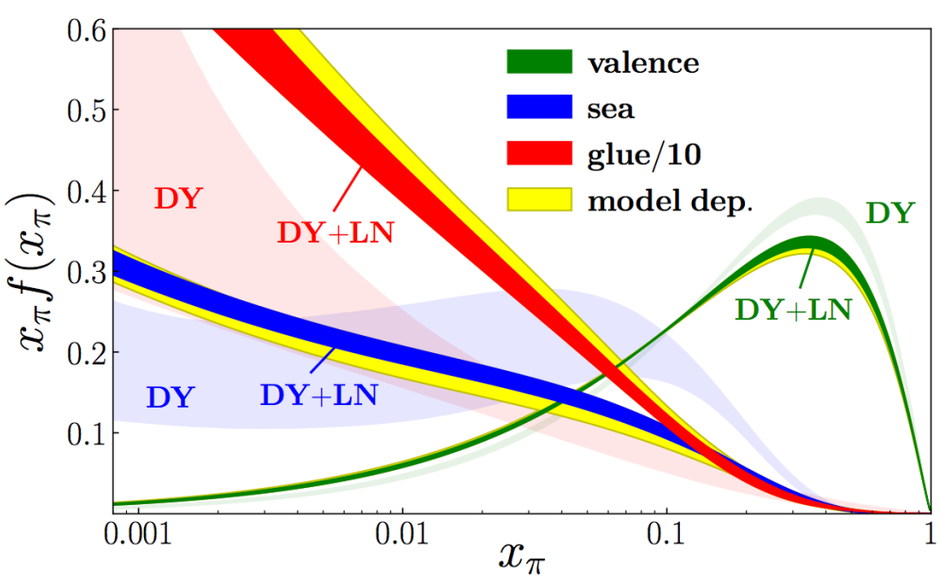
Twist-2 OPE

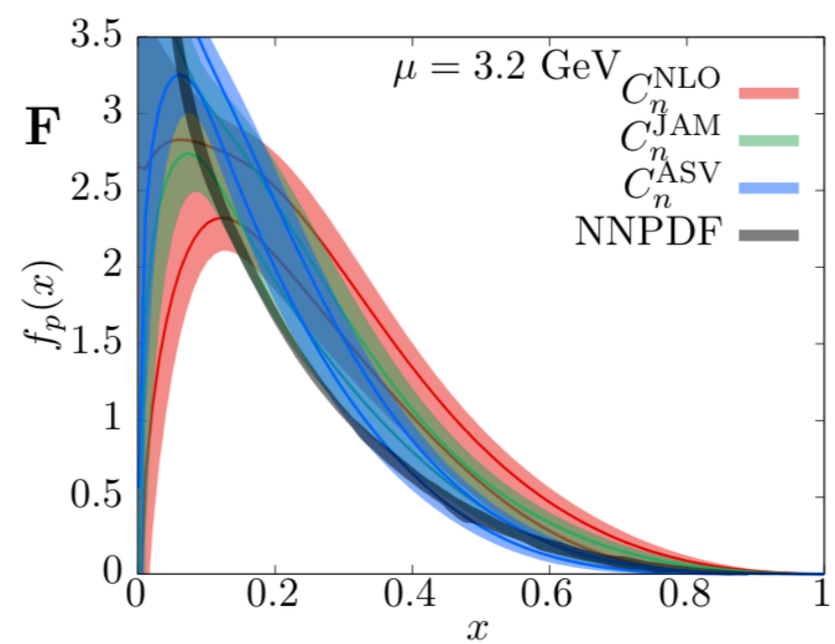
$$c_n(\mu^2 z^2)$$



Twist-2 OPE

JAM and ASV pion PDF data





Towards exclusive processes at large-Q^2 :Distribution amplitude of pion

Pion distribution amplitude from leading-twist OPE

Theoretical framework:

V.Braun, D.Müller '07
X.Ji '13
A.Radyushkin '17
Y.S.Liu et al, '18

$$\phi(x,\mu)$$

pert. matching in x-space

$$\langle 0|\bar{u}(-\tfrac{z_3}{2})\,\gamma_z\gamma_5\,d(\tfrac{z_3}{2})|\pi^+;P_3\rangle = f_\pi P_3 \mathcal{M}(\lambda = z_3 P_3, z_3^2)$$

(equal-time quasi-DA matrix element)

Mellin OPE at NLO

Conformal OPE at LL

$$\sum_{n=0} \frac{(-i\lambda)^n}{n!} \sum_{m=0}^{n} C_{n,m}(\mu^2 z^2)\langle x^m\rangle$$

Same up to
Finite NLO terms + truncation

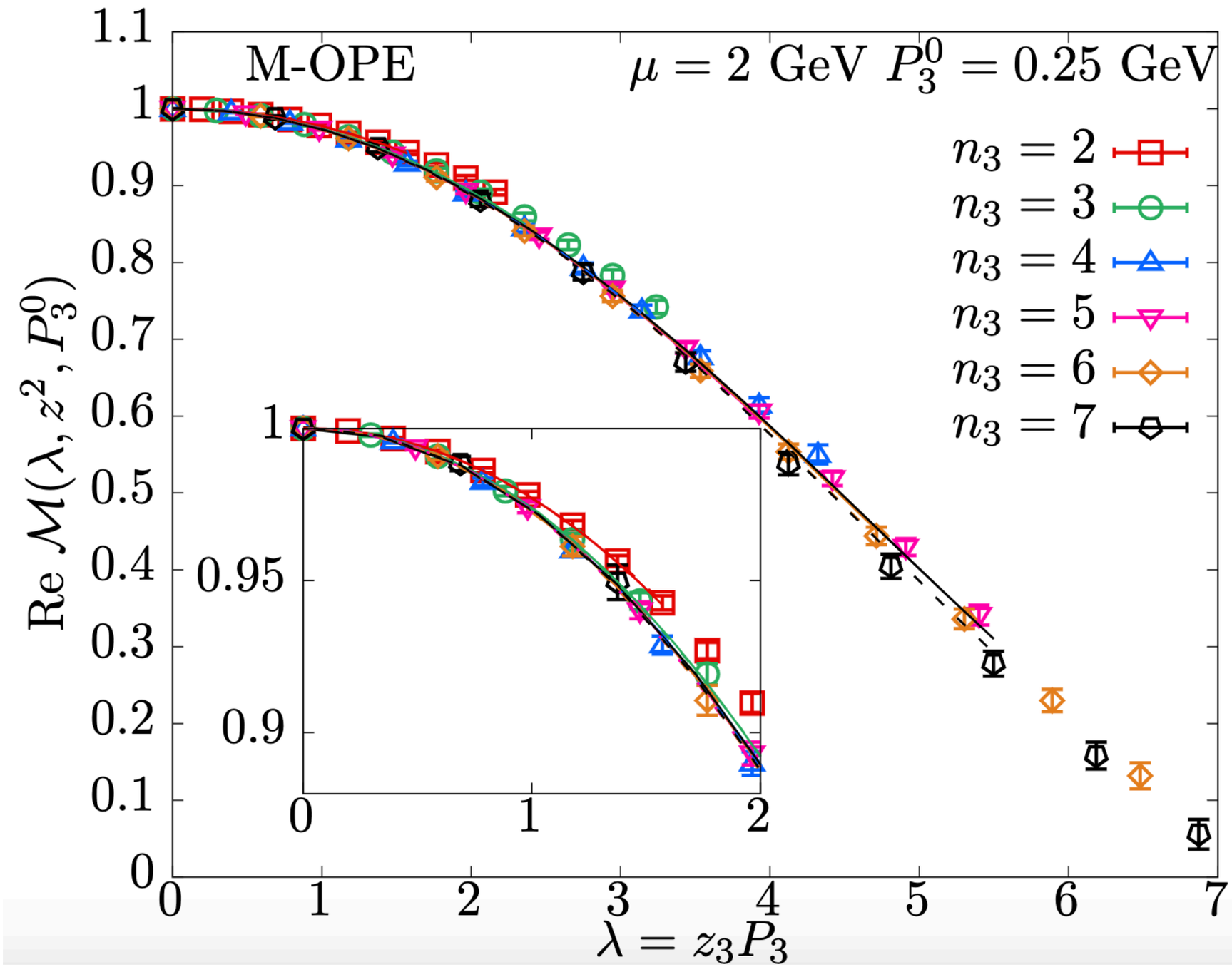
$$\sum_{n=0} a_n \mathcal{F}_n\left(\frac{\lambda}{2}, \mu^2 z^2\right)$$

Ratio of pion quasi-DA matrix elements and OPE fits

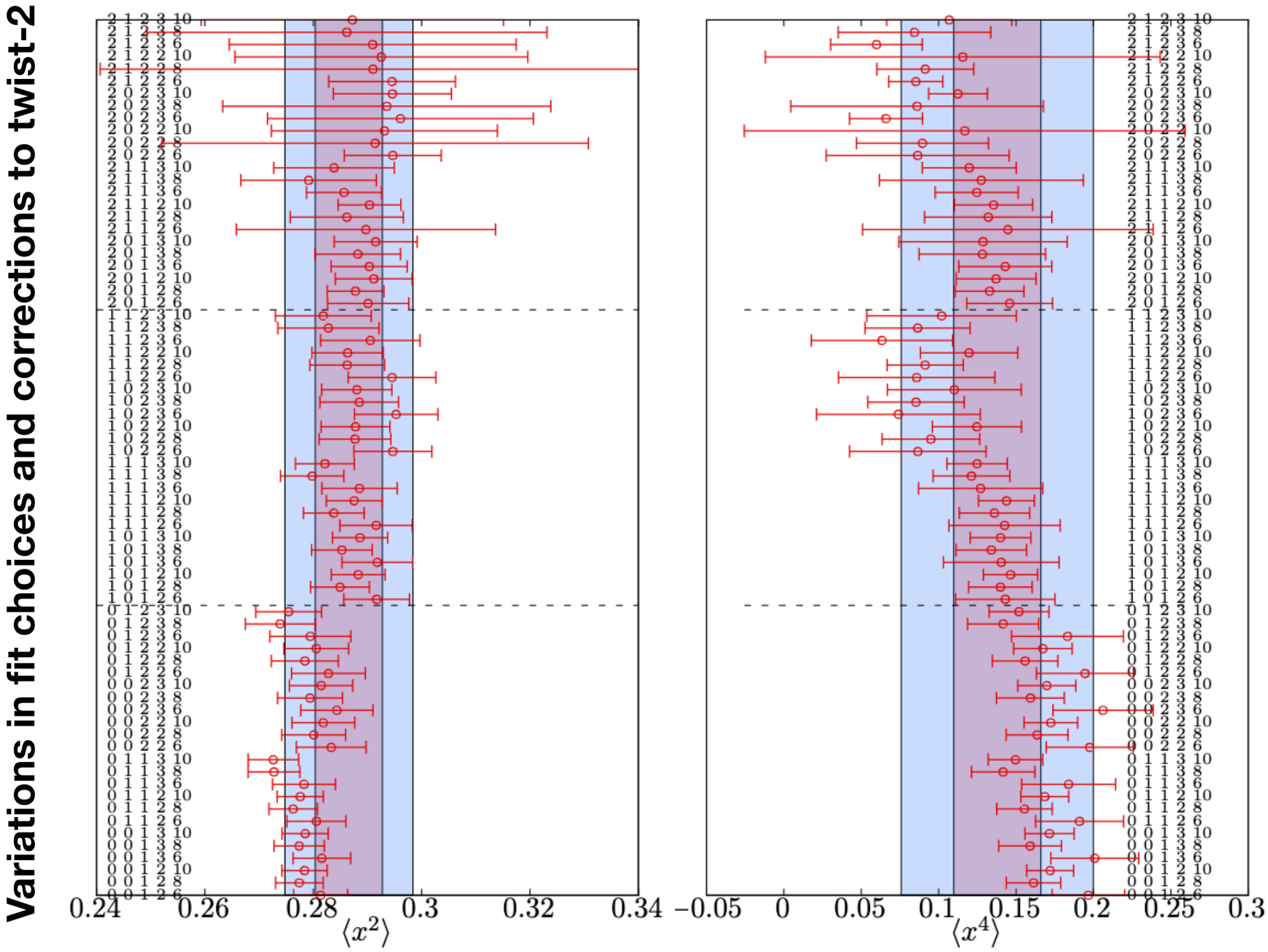
Renormalize: $\mathcal{M}\left(z_3 P_3, z_3^2, P_3^0\right) = \dfrac{h^B(z_3, P_3)}{h^B(z_3, P_3^0)}$

Fit OPE: $z \in [z_{\min}, z_{\max}]$

Lattice artifact? Higher twist?



Estimating first two Mellin moments of pion DA



0.2866(62)(56)

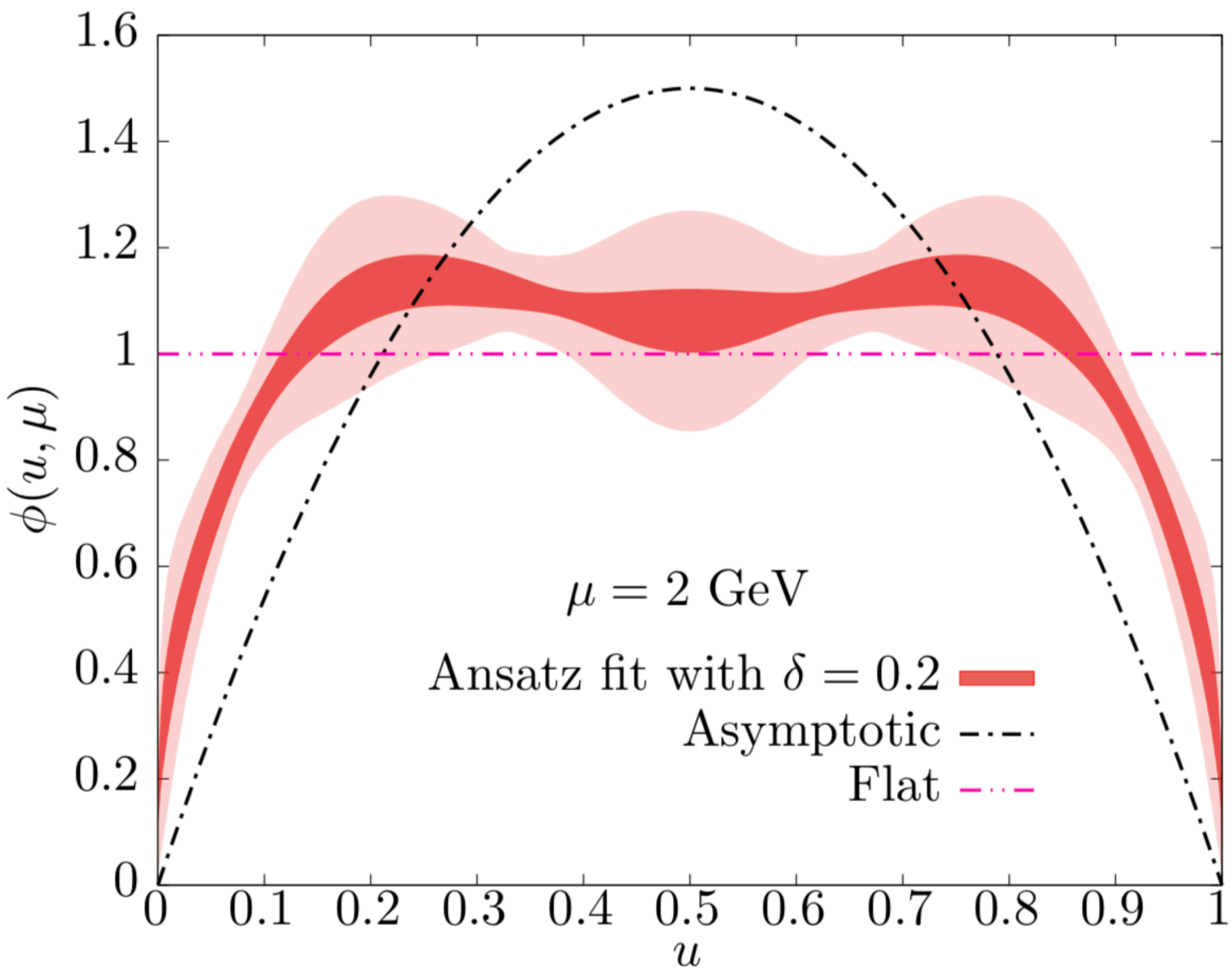
0.138(28)(34)

X.Gao, NK et al, 2206.04084

Reconstruction of x-dependent pion DA

$$\phi(x,\mu) = \mathcal{N} x^{\alpha}(1-x)^{\alpha} \sum_n s_n C_n^{\frac{1}{2}+\alpha}(1-2x) \qquad s_n \lessapprox \delta$$

[See works by HadStruc collaboration on using Jacobi polynomial basis]

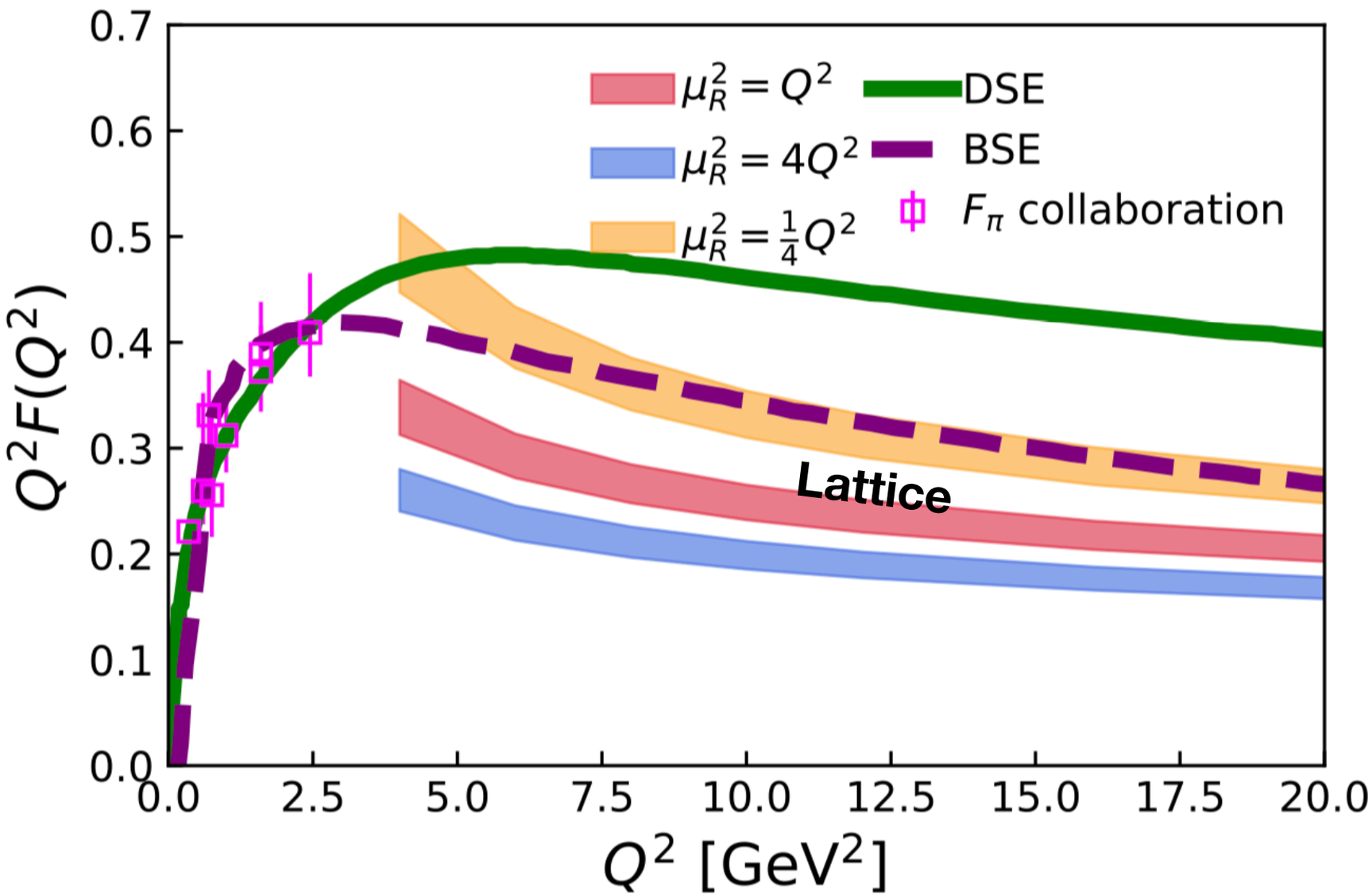


X.Gao, NK et al, 2206.04084

Factorization of form-factors: lattice can play a part

$$F_\pi(Q^2) = \int_0^1 \int_0^1 dx dy \, \phi(x,Q) E(x,y,Q^2) \phi(y,Q)$$

Input fitted DA from lattice



X.Gao, NK et al, 2206.04084

Outlook

- There is both theoretical and experimental Interest in pion physics, especially from the perspective of EIC — PDF of pion, form factors at large Q^2

- The leading-twist perturbative framework applied to lattice matrix elements is a promising direction to do parton physics on the lattice. Comes with its own drawbacks — fit systematics, perturbative convergence, large P_z, small z. Progress underway in all these directions.

- Presented lattice results on valence PDF and DA for pion using quasi-operator using fine lattices.

- Is it essential to extract x-dependent PDF from twist-2 saturated lattice matrix elements? How to extract relevant experimental quantities in model independent manner (e.g., large-x behavior, convolutions that enter FF) ?

- What is the best way to make use of experimental results in lattice QCD Parton computations?

- The question on emergence of mass-gap and hadron-spin is general to Yang-Mills theories. How to develop relevant theoretical models that lattice can probe?