Twist-3 PDFs from Lattice QCD with a Phenomenological component

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BNL

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CFNS Workshop:
High Luminosity-EIC (EIC-Phase II)

Center for Frontiers in Nuclear Science

Stony Brook University

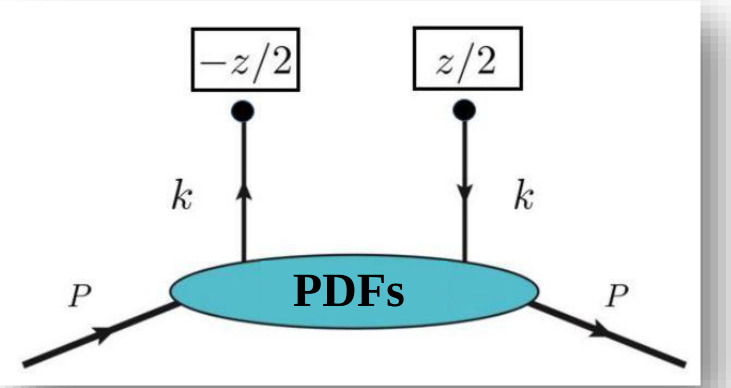
21-24 June 2022
Online

Outline

- **Brief overview of twist-3 PDFs**
- **Essence of the quasi-PDF approach**
- **Complications in the "Matching" for the twist-3 PDFs**
- **Lattice QCD results for twist-3 PDFs**
- **Summary**

Brief overview of twist-3 PDFs

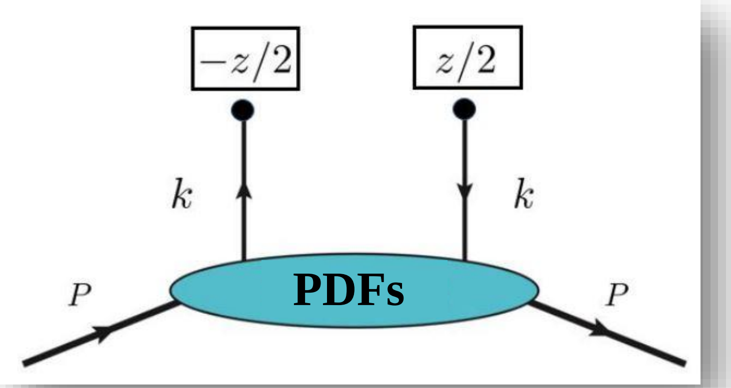
Brief overview of twist-3 PDFs



Twist-classification of PDFs

$$\boxed{d\sigma = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} + \ldots}$$

Brief overview of twist-3 PDFs



Twist-classification of PDFs

$$d\sigma = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} + \ldots$$

Twist-2 **Twist-3**

Q **: Hard scale for a process**

Brief overview of twist-3 PDFs

Twist-2 PDFs	Twist-3 PDFs
Order of contribution: $\mathcal{O}(1)$	**Order of contribution:** $\mathcal{O}(1/\mathrm{Q})$

Twist-2 PDFs

PDFs	Dirac structure
$f_1(x)$	$\Gamma = \gamma^+$
$g_1(x)$	$\Gamma = \gamma^+\gamma_5$
$h_1(x)$	$\Gamma = i\sigma^{i+}\gamma_5$

Twist-3 PDFs

Jaffe, Ji (PRL 67, 552)/ Jaffe, Ji (Nucl. Phys. B 375, 527)

PDFs	Dirac structure
$e(x)$	$\Gamma = 1$
$g_T(x)$	$\Gamma = \gamma_\perp^i\gamma_5$
$h_L(x)$	$\Gamma = i\sigma^{+-}\gamma_5$

Density interpretation:

$f_1(x)$

$g_1(x)$

$h_1(x)$

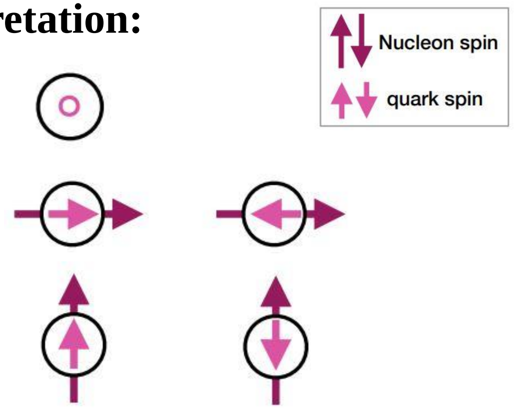


Fig. courtesy:
M. Constantinou

No density interpretation:

Burkardt (arXiv: 0810.3589)

$$\int dx\, x^2 g_T(x) \;\to \perp$$ **force**

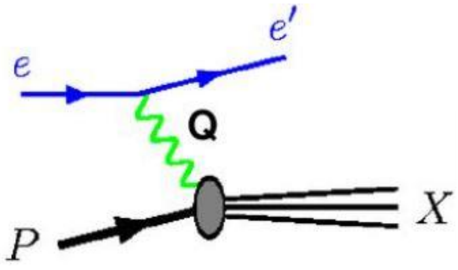
$$\int dx\, x^2 e(x) \;\to \perp$$ **force**

Brief overview of twist-3 PDFs

Processes sensitive to twist-3 PDFs (list not exhaustive)

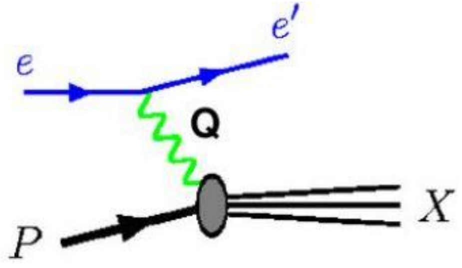
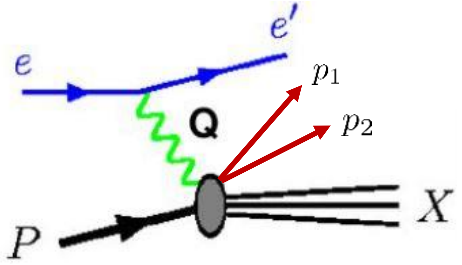
Brief overview of twist-3 PDFs

Processes sensitive to twist-3 PDFs (list not exhaustive)

PDFs	Processes	Data
$g_T(x)$		**For instance:** **Hall A, 2016/ Hall C, 2018**

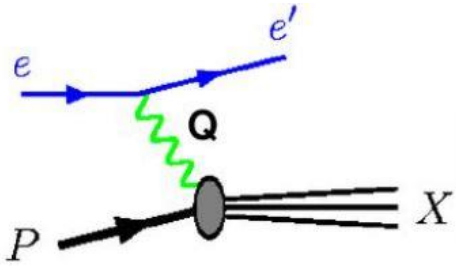
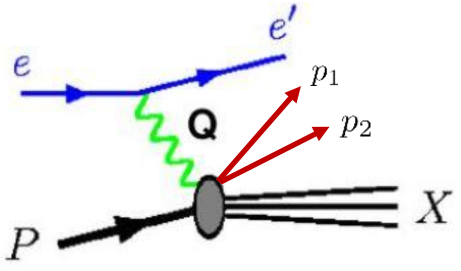
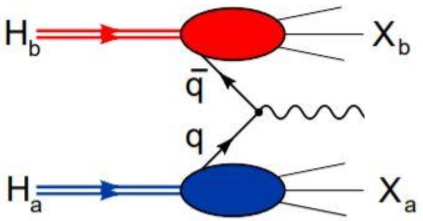
Brief overview of twist-3 PDFs

Processes sensitive to twist-3 PDFs (list not exhaustive)

PDFs	Processes	Data
$g_T(x)$		**For instance:** **Hall A, 2016/ Hall C, 2018**
$e(x)$		**For instance:** **CLAS12 (2021)**

Brief overview of twist-3 PDFs

Processes sensitive to twist-3 PDFs (list not exhaustive)

PDFs	Processes	Data
$g_T(x)$		**For instance:** **Hall A, 2016/** **Hall C, 2018**
$e(x)$		**For instance:** **CLAS12 (2021)**
$h_L(x)$		**None**

Brief overview of twist-3 PDFs

Some model studies of twist-3 PDFs (list not exhaustive)

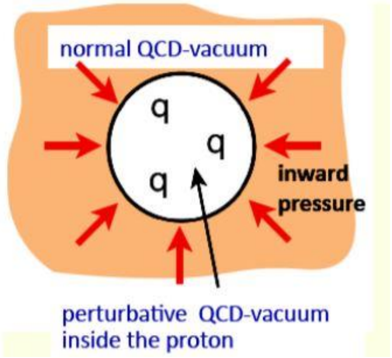
Brief overview of twist-3 PDFs

Some model studies of twist-3 PDFs (list not exhaustive)

PDFs	Model & $\delta(x)$?

Brief overview of twist-3 PDFs

Some model studies of twist-3 PDFs (list not exhaustive)

PDFs	**Model & $\delta(x)$?**
$e(x)$ $h_L(x)$	 $\cancel{\delta(x)}$ **Jaffe, Ji, 1991**

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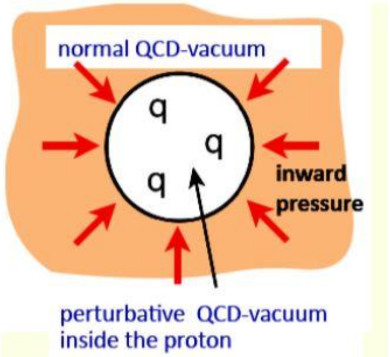
Some model studies of twist-3 PDFs (list not exhaustive)

PDFs	**Model &** $\delta(x)$?		
$e(x)\ \ h_L(x)$	 $\cancel{\delta(x)}$ **Jaffe, Ji, 1991**	$\chi\mathbf{QSM}$ $e(x) \sim \delta(x)$ **Ohnishi, Wakamatsu, 2003**	

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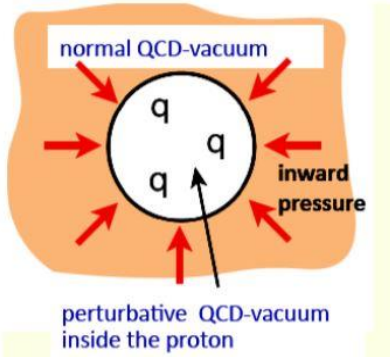
$e(x)\ \ h_L(x)$	 $\cancel{\delta(x)}$ **Jaffe, Ji, 1991**	χ**QSM** $e(x) \sim \delta(x)$ **Ohnishi, Wakamatsu, 2003**	 $\cancel{\delta(x)}$ **Jakob, et. al, 1997** $\delta(x)$ **Aslan, Burkardt, 2018**
$g_T(x)$			$\cancel{\delta(x)}$ **Jakob, et. al, 1997** $\delta(x)$ **Aslan, Burkardt, 2018**

Brief overview of twist-3 PDFs

Some model studies of twist-3 PDFs (list not exhaustive)

PDFs	Model & $\delta(x)$?	
$e(x)$ h_L	$\cancel{\delta(x)}$ Jaffe, Ji, 1991 · 2003	2018
$g_T(x)$		$\cancel{\delta(x)}$ Jakob, et. al, 1997 $\delta(x)$ Aslan, Burkardt, 2018

Sounds interesting!
Can we extract these quantities from lattice QCD?

Quasi-PDF approach

Light-cone (standard) correlator

$$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **Cannot be computed on Euclidean lattice**

Quasi-PDF approach

Light-cone (standard) correlator	Correlator for quasi-PDFs (Ji, 2013)
$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p \vert \bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2}) \vert p\rangle \Big\vert_{z^+=\vec{z}_\perp=0}$	$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p \vert \bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2}) \vert p\rangle \Big\vert_{z^0=\vec{z}_\perp=0}$
• **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$ • **<u>Cannot</u> be computed on Euclidean lattice**	• **Non-local correlator depending on position** z^3 • **<u>Can</u> be computed on Euclidean lattice**

- **Quasi-PDF approach made it possible to directly extract light-cone PDFs from lattice QCD**

Quasi-PDF approach

Light-cone (standard) correlator	Correlator for quasi-PDFs (Ji, 2013)
$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p \vert \bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2}) \vert p\rangle \Big\vert_{z^+=\vec{z}_\perp=0}$	$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p \vert \bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2}) \vert p\rangle \Big\vert_{z^0=\vec{z}_\perp=0}$
• **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$ • **Cannot be computed on Euclidean lattice**	• **Non-local correlator depending on position** z^3 • **Can be computed on Euclidean lattice**

- **Quasi-PDF approach made it possible to directly extract light-cone PDFs from lattice QCD**
- **By now, enormous progress has taken place: See previous talks by Constantia, Nikhil, Raza & Krzysztof**

Quasi-PDF approach

Light-cone (standard) correlator

$$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**

Correlator for quasi-PDFs (Ji, 2013)

$$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

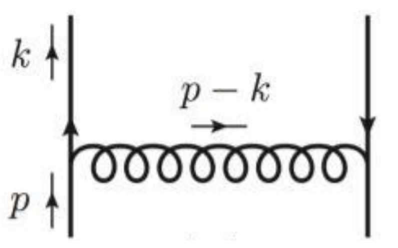
- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**

- **Quasi-PDF approach made it possible to directly extract light-cone PDFs from lattice QCD**
- **By now, enormous progress has taken place: See previous talks by Constantia, Nikhil, Raza & Krzysztof**
- **Other Euclidean approaches: Pseudo-PDF (Radyushkin, 2017), current-current correlators (Ma, Qiu, 2014) …**

Quasi-PDF approach

Quasi-PDF approach

Sample calculations for twist-2 PDF f_1 in Quark Target Model (QTM)



Quasi-PDF approach

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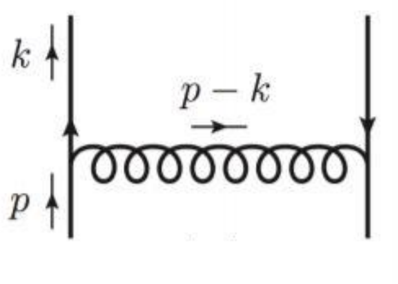

$$f_1(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k} + m_q)\,\gamma^+\,(\not{k} + m_q)\,\gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\left(x - \frac{k^+}{p^+}\right)\frac{1}{p^+}$$

Quasi-PDF approach

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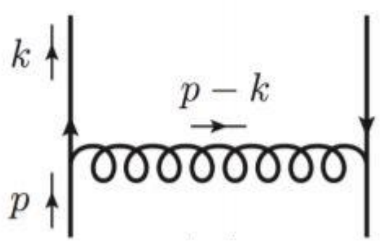

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$$k = \left(k^+ = \frac{k^0+k^3}{\sqrt{2}},\ k^- = \frac{k^0-k^3}{\sqrt{2}},\ k_\perp\right)$$

Quasi-PDF approach

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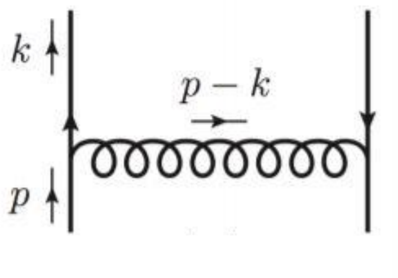

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- $\int dk^- \rightarrow$ Residue theorem

Quasi-PDF approach

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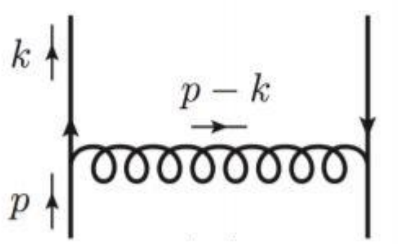

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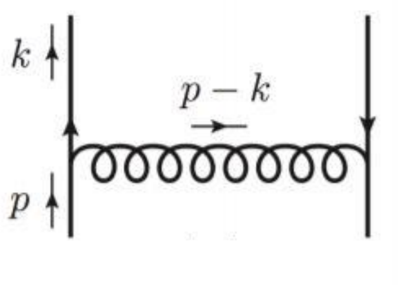

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- $\int dk^- \rightarrow$ Residue theorem
- Perform $\int d^2 k_\perp$

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

$$\mathcal{P}_{\mathrm{UV}} = \frac{1}{\epsilon_{\mathrm{UV}}} + \ln 4\pi - \gamma_E$$

$0 < x < 1$

$\int_0^\infty dk_\perp$

Quasi-PDF approach

Sample calculations for twist-2 PDF f_1 in Quark Target Model (QTM)

What if I calculate f_1 , with, a completely spatial correlator & with γ^3 in the Infinite Momentum Frame (IMF)?

Quasi-PDF approach

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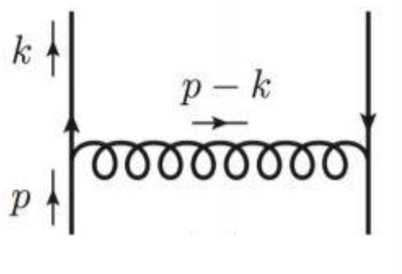

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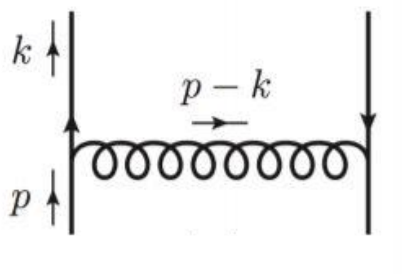

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- $k \to (k^0,\, k_\perp,\, k^3)$

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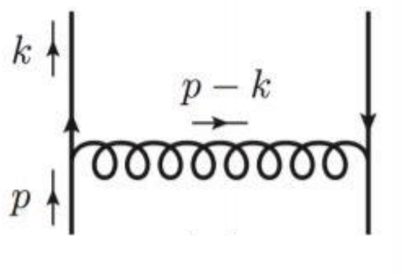

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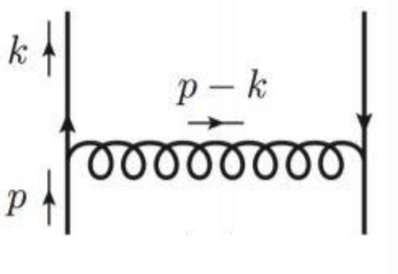

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- $k \to (k^0,\, k_\perp,\, k^3)$
- $\int dk^0 \to$ Residue theorem
- Take $p^3 \to \infty$
- Perform $\int d^2 k_\perp$

Quasi-PDF approach

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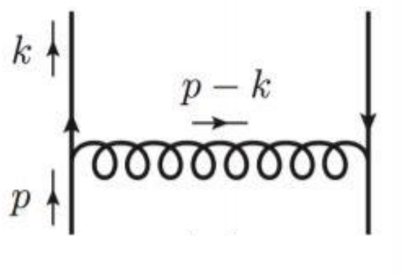

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$$\int_{\mathbf{0}}^{\infty} dk_\perp$$

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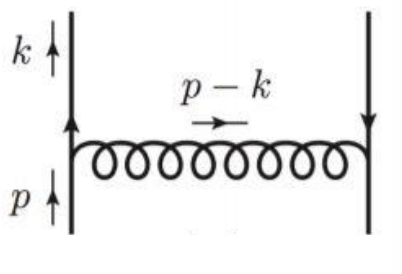
Sample calculations for twist-2 PDF f_1 in Quark Target Model (QTM)

What if I calculate f_1, with, a completely spatial correlator & with γ^3 in the Infinite Momentum Frame (IMF)?



$$f_1(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+\cancel{m_q})\,\gamma^3\,(\not{k}+\cancel{m_q})\,\gamma^\mu\right]}{(k^2-\cancel{m_q^2}+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^3}{p^3}\right)\frac{1}{p^3}$$

- $k \to (k^0,\, k_\perp,\, k^3)$
- $\int dk^0 \to$ Residue theorem
- Take $p^3 \to \infty$
- Perform $\int d^2k_\perp$

$\int_0^\infty dk_\perp$

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

Unfortunately, this cannot be calculated on lattice

Quasi-PDF approach

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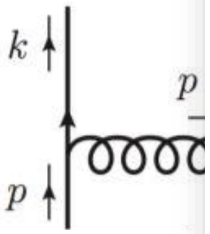

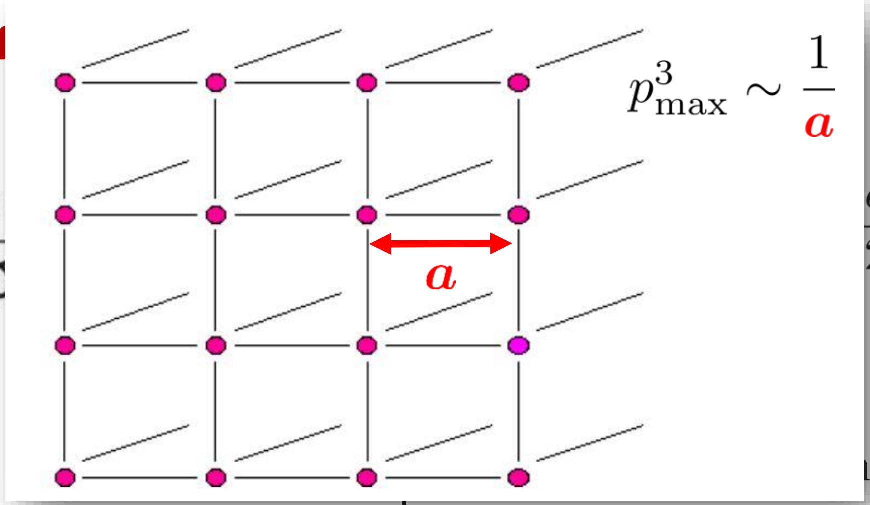

$$\frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k} + \cancel{m_q})\,\gamma^3\,(\not{k} + \cancel{m_q})\,\gamma^\mu\right]}{(k^2 - \cancel{m_q^2} + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\left(x - \frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right)\frac{1}{\boldsymbol{p^3}}$$

...eorem

- Take $p^3 \to \infty$
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$\int_{\mathbf{0}}^{\boldsymbol{\infty}} dk_\perp$

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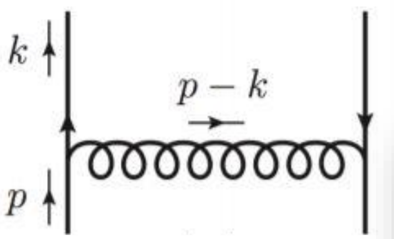

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What if I keep p^3 finite & repeat this calculation?

- $\int dk^0 \rightarrow$ Residue theorem
- Take $p^3 \rightarrow \infty$
- Perform $\int d^2 k_\perp$

$$\int_0^\infty dk_\perp$$

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

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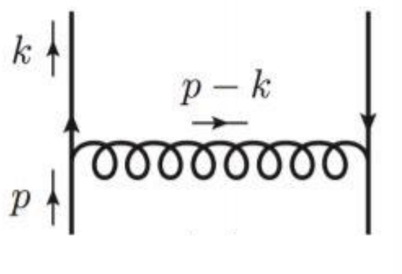

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$$\boxed{f_1(x, p^3) \quad = \quad \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x)\,\ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\,\ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\,\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}}$$

Quasi-PDF approach

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IR singularities of quasi-PDFs & light-cone PDFs are same

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UV-finite!

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Divergences will manifest after we integrate over x

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Matching for twist-3 PDFs

SB, Cichy, Constantinou, Metz, Scapellato, Steffens

Matching formula:

$$q_{\mathrm{Q}}(x;P_3) = \int_{-1}^{+1} \frac{d\,y}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

(Scale dependence omitted)

**(Xiong, Ji, Zhang, Zhao, 2013/
Stewart, Zhao, 2017/
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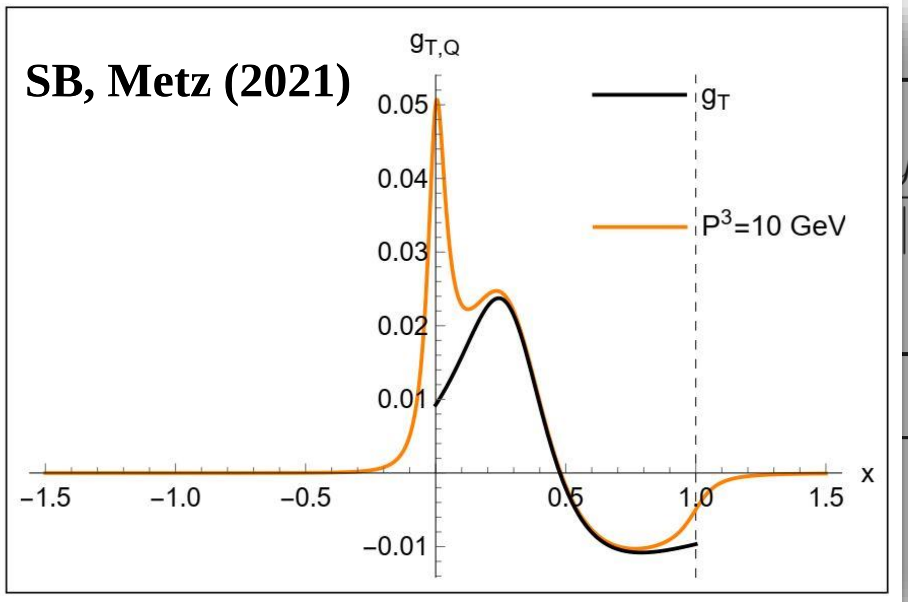
Contributions in a nutshell:

Not covered in this talk

- **Derived the one-loop matching coefficient for the twist-3 PDFs** $(g_T(x),\ e(x),\ h_L(x))$

Matching for twist-3 PDFs

SB, Cichy, Constantinou, Metz, Scapellato, Steffens



Ma

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Co

- **Derived the one-loop matching coefficient for the twist-3 PDFs** $(g_T(x),\ e(x),\ h_L(x))$
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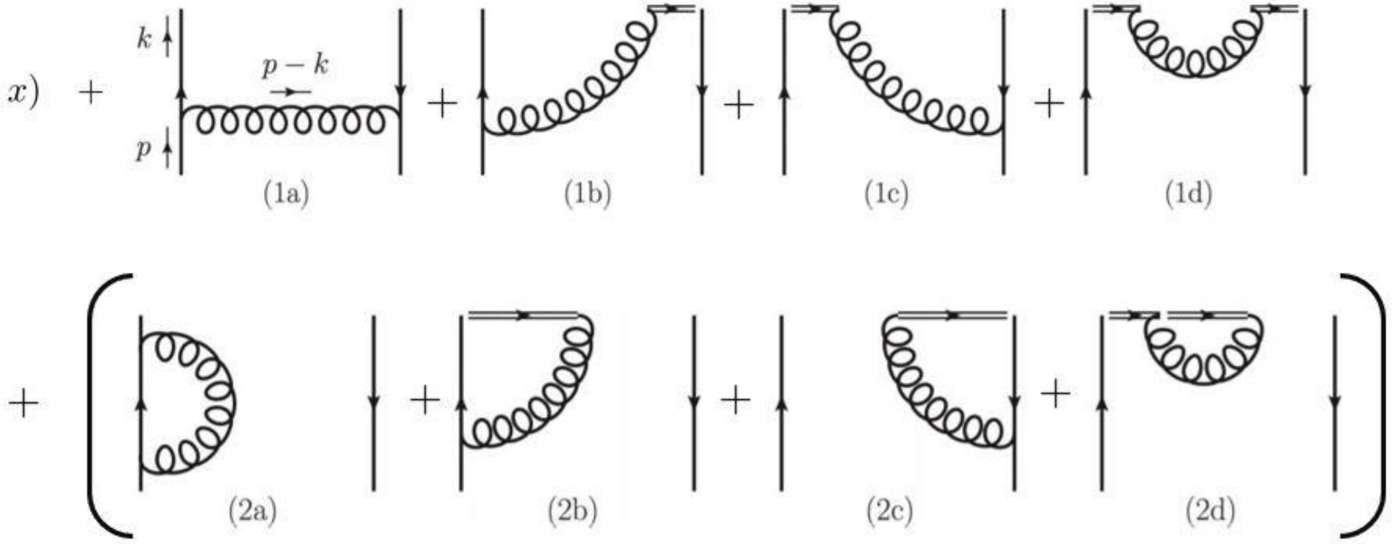
Matching for twist-3 PDFs

SB, Cichy, Constantinou, Metz, Scapellato, Steffens

Set-up for our calculation

1-loop corrections (Feynman Gauge) $= \delta(1-x) +$



Ultra-violet: $\int^{\infty} d^2 k_{\perp} \longrightarrow \epsilon_{\mathrm{UV}}$

IR: $\int_0 d^2 k_{\perp} \longrightarrow m_{q,g} \neq 0, \ \epsilon_{\mathrm{IR}}$

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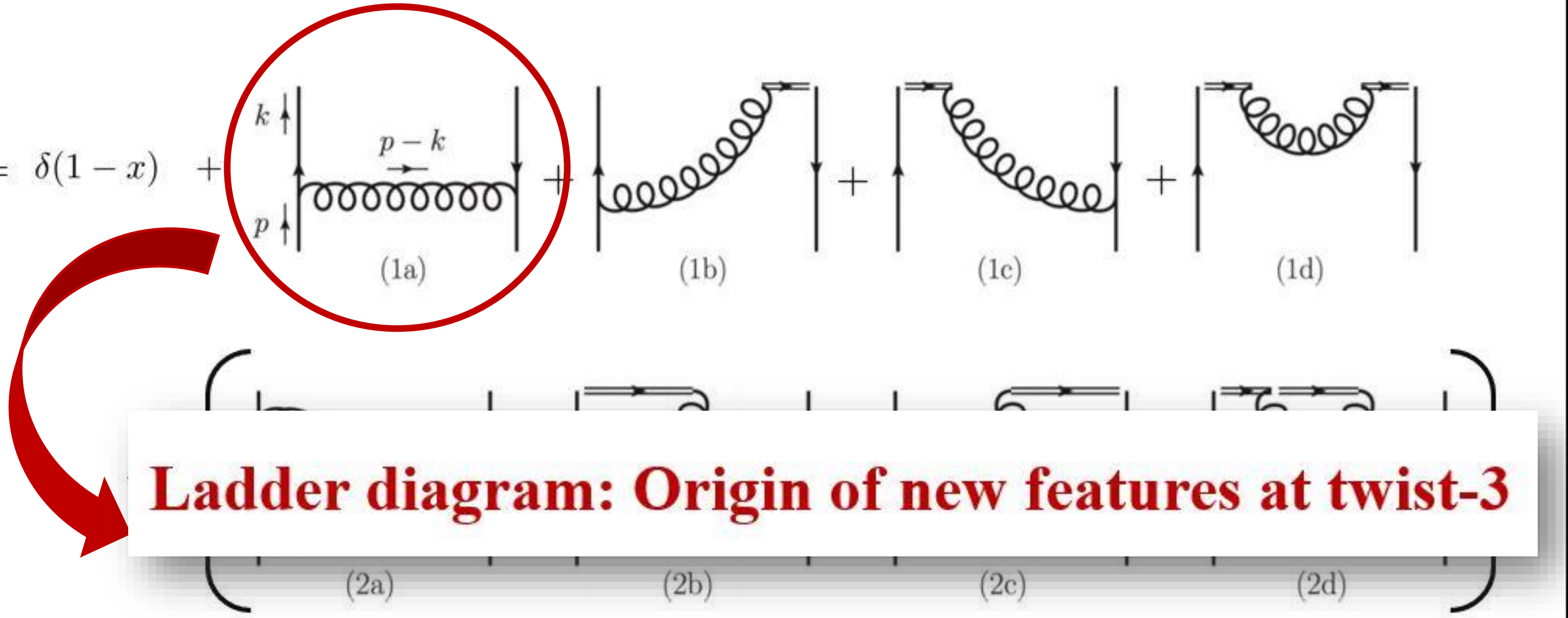

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Non-trivial role of zero-modes in matching for $e(x)$ & $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Light-cone PDF	Features
Example: $$h^{(1a)}_{L(\mathrm{s})}(x)\Big\vert_{m_q} = -\frac{\alpha_s C_F}{2\pi}\,\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2_{\mathrm{UV}}}{m_q^2} - 1\right)$$	• **Zero modes are unavoidable**

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Quasi-PDF	**Features**
Example: $$h^{(1a)}_{L,\mathrm{Q(s)}}(x)\Big\vert_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{p^3}{\sqrt{x^2p_3^2+m_q^2}}$$	• Seemingly different looking IR pole structure

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Example: $$h^{(1a)}_{L,\mathrm{Q(s)}}(x)\big\|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{p^3}{\sqrt{x^2 p_3^2 + m_q^2}}$$	• **Seemingly different looking IR pole structure** • **Do quasi-PDFs and LC PDFs share same IR physics?**

Non-trivial role of zero-modes in matching for $e(x)$ & $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Treatment of IR singularity for quasi-PDFs (non-zero quark mass)
$$h^{(1a)}_{L,\mathrm{Q(s)}}(x)\Big\vert_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{1}{\sqrt{x^2+\eta^2}} \qquad \text{where,} \quad \eta^2 = \frac{m_q^2}{p_3^2} \quad -1<x<1$$

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- **Doing a twist-expansion before we calculate the matching coefficient gives rise to an <u>incorrect conclusion</u> of mismatch in the IR between quasi & LC PDFs!**

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(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

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Incorrect approach	**Correct approach**
$h^{(1a)}_{L,\mathrm{Q(s)}}(x) \approx -\frac{\alpha_s C_F}{2\pi}\frac{1}{x}$	$\int_{-1}^{1} dx\, \frac{f(x)}{\sqrt{x^2+\eta^2}} = \int_{-1}^{1} dx\, f(x)\,\delta(x)\left(\ln\frac{4}{\eta^2}\right) + \int_{-1}^{1} dx\, f(x)\left[\frac{1}{\lvert x\rvert}\right]_{+[0]} + \mathcal{O}(\eta^2)$
• **Doing a twist-expansion before we calculate the matching coefficient gives rise to an <u>incorrect conclusion</u> of mismatch in the IR between quasi & LC PDFs!**	• **By convoluting with a well-behaved test-function, it is possible to isolate singularity at** $x=0$ • **Agreement in the IR poles between quasi & LC PDFs: Matching possible for** $e(x)$, $h_L(x)$

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Treatment of IR singularity for quasi-PDFs (non-zero quark mass)

$$h^{(1a)}_{L,\mathrm{Q(s)}}(x)\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{1}{\sqrt{x^2+\eta^2}} \qquad \text{where,} \quad \eta^2 = \frac{m_q^2}{p_3^2} \quad -1 < x < 1$$

Point $x = 0$ is extremely delicate for quasi-PDFs!

$$+ \int_{-1} dx\, f(x) \left[\frac{1}{|x|}\right]_{+[0]} + \mathcal{O}(\eta^2)$$

- **Doing a twist-expansion before we calculate the matching coefficient gives rise to an <u>incorrect conclusion</u> of mismatch in the IR between quasi & LC PDFs!**

- **By convoluting with a well-behaved test-function, it is possible to isolate singularity at $x = 0$**

- **Agreement in the IR poles between quasi & LC PDFs: Matching possible for $e(x)$, $h_L(x)$**

Matching for twist-3 PDFs

SB, Cichy, Constantinou, Metz, Scapellato, Steffens

Matching formula:

$$q_{\mathrm{Q}}(x;P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

Contributions in a nutshell:

- **Derived the one-loop matching coefficient for the twist-3 PDFs** $(g_T(x),\ e(x),\ h_L(x))$
- **Provided the necessary theoretical tools to deal with complications due to singular zero-mode contributions**
- **These contributions led to the first-ever extraction of** $(g_T(x), h_L(x))$ **from lattice QCD**

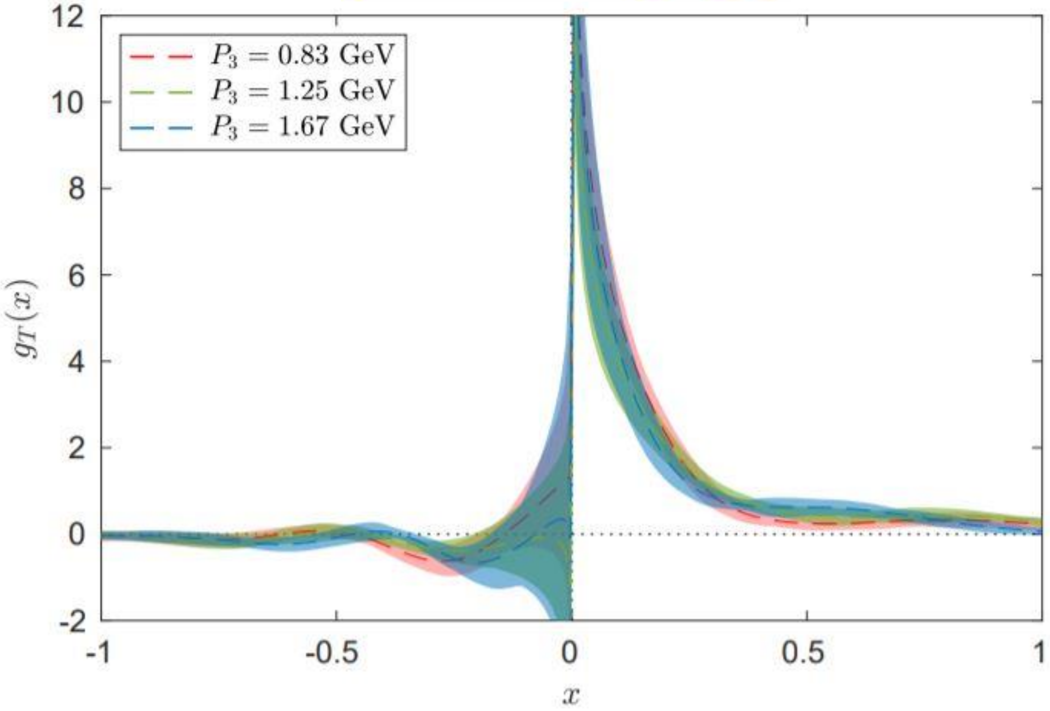
Lattice QCD results for $g_T^{\mathrm{u-d}}(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD Rapid 102 (2020)

Ensemble:

$a = 0.093$ fm, $L \approx 3$ fm, $m_\pi \approx 260$ MeV

(PRD Editor's highlight)



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Extended Twisted Mass Collaboration

Ensemble:

$$a = 0.093 \text{ fm}, \; L \approx 3 \text{ fm}, \; m_\pi \approx 260 \text{ MeV}$$

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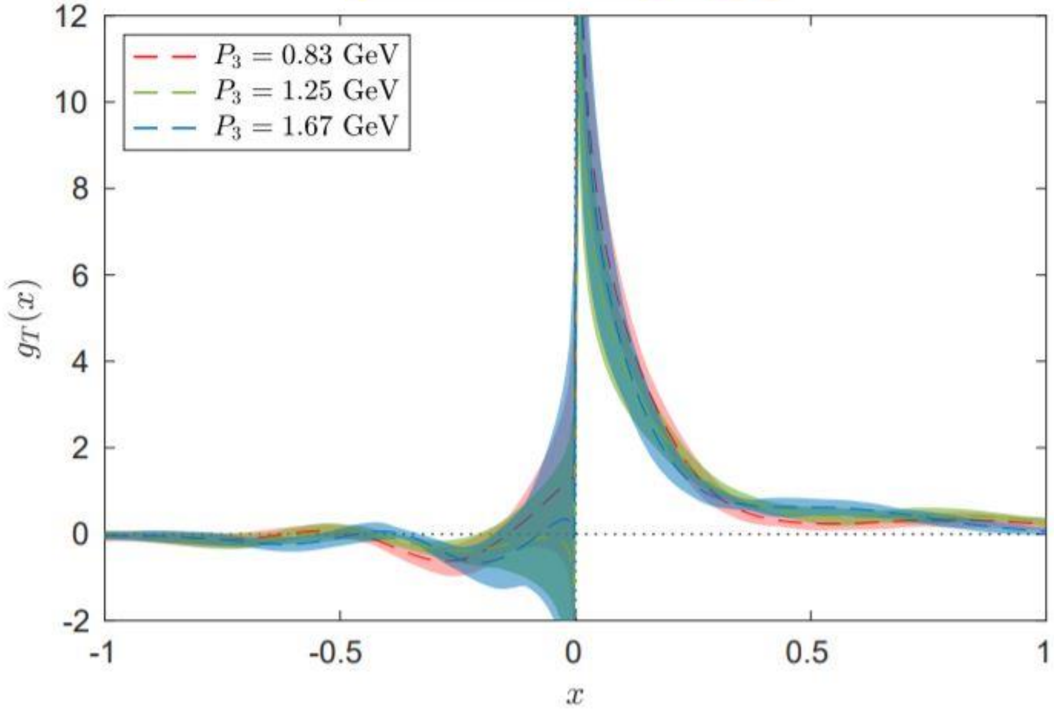

Check of WW approximation

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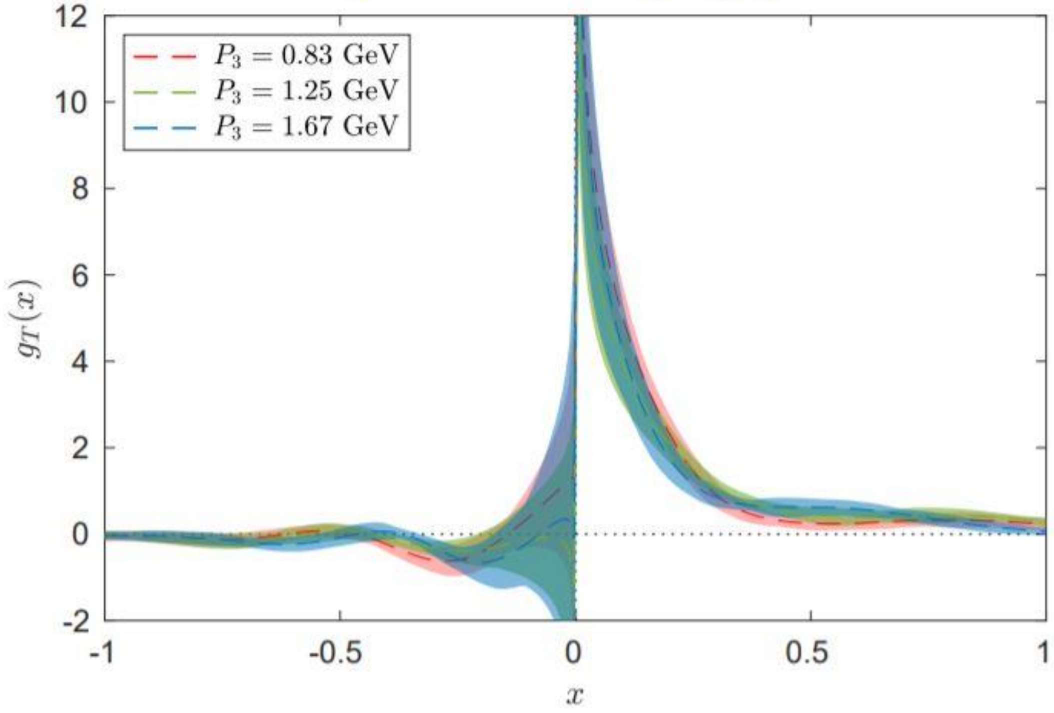

Check of WW approximation

$$g_T(x) = \int_x^1 \frac{dy}{y} g_1(y) + \tilde{g}_T(x)$$

Lattice QCD results for $g_T^{\mathrm{u-d}}(x)$

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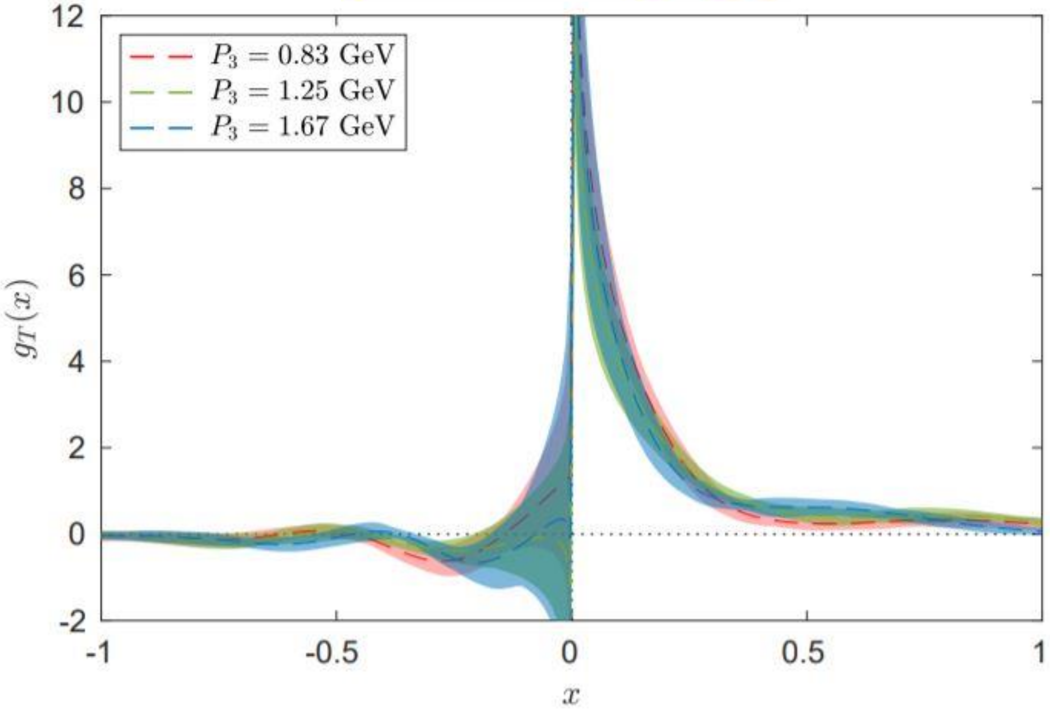

Check of WW approximation

$$g_T(x) = \int_x^1 \frac{dy}{y} g_1(y) + \cancel{\tilde{g}_T(x)}$$

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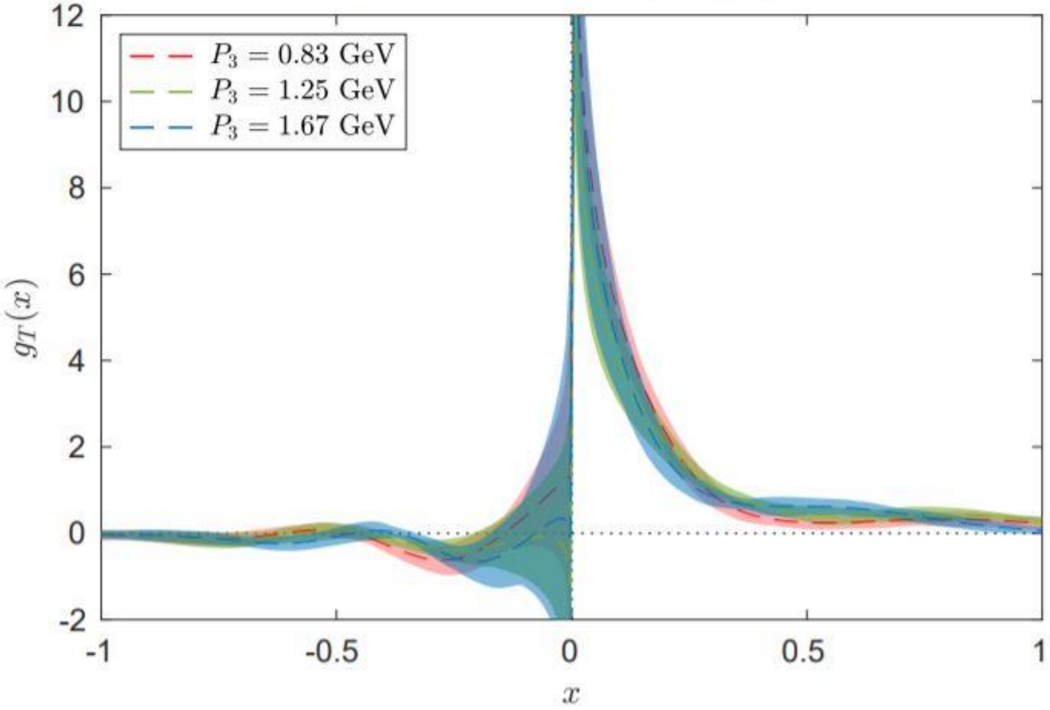

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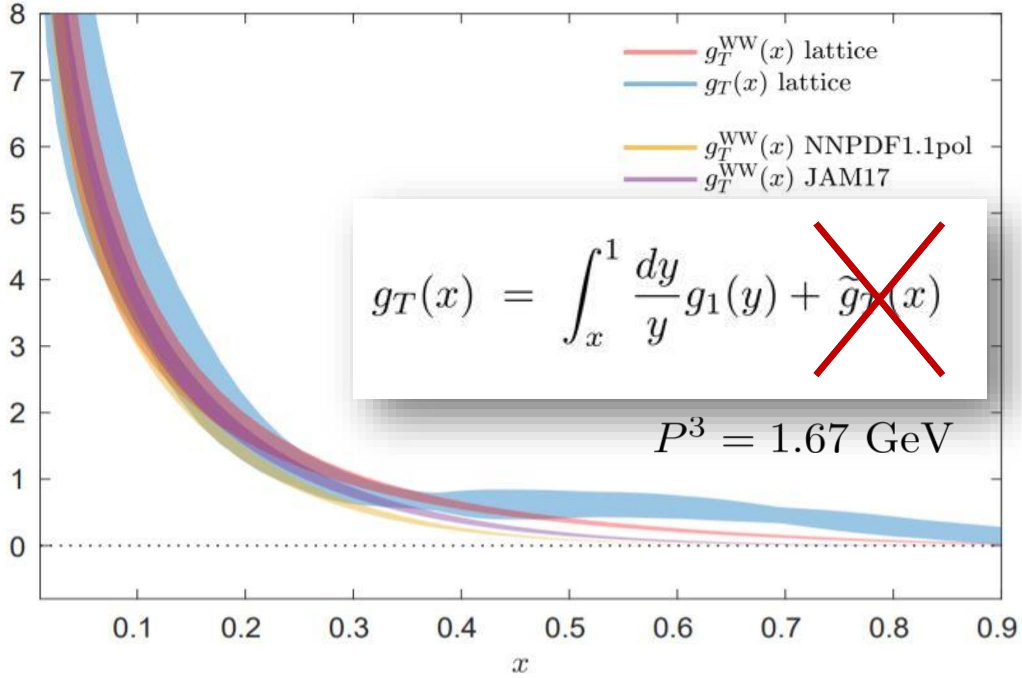

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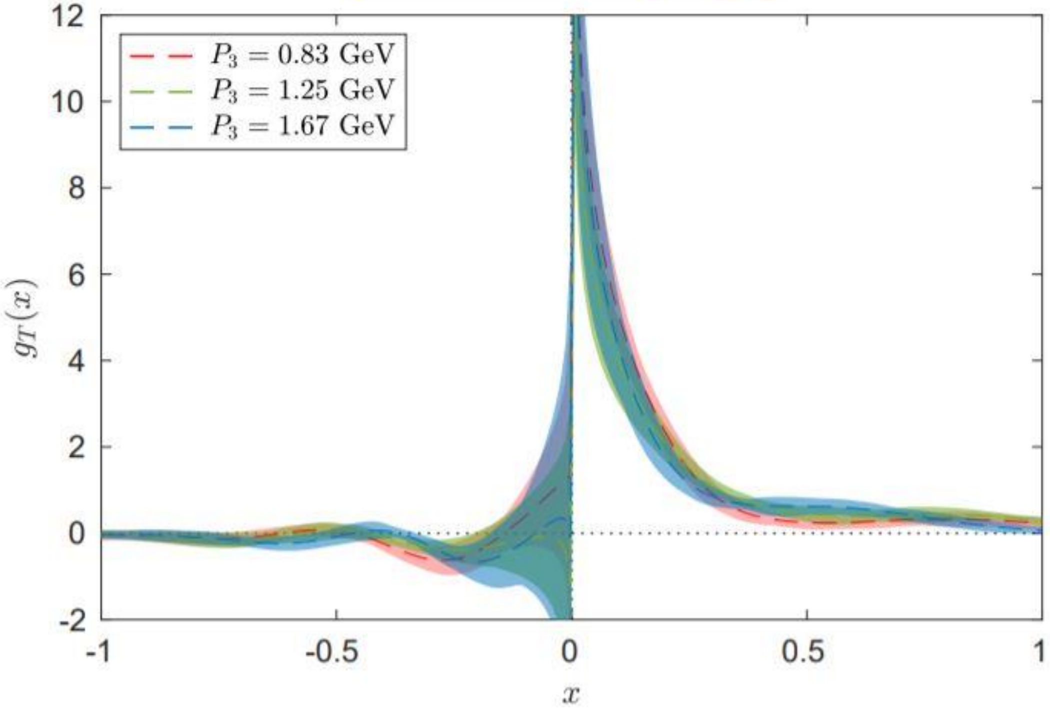

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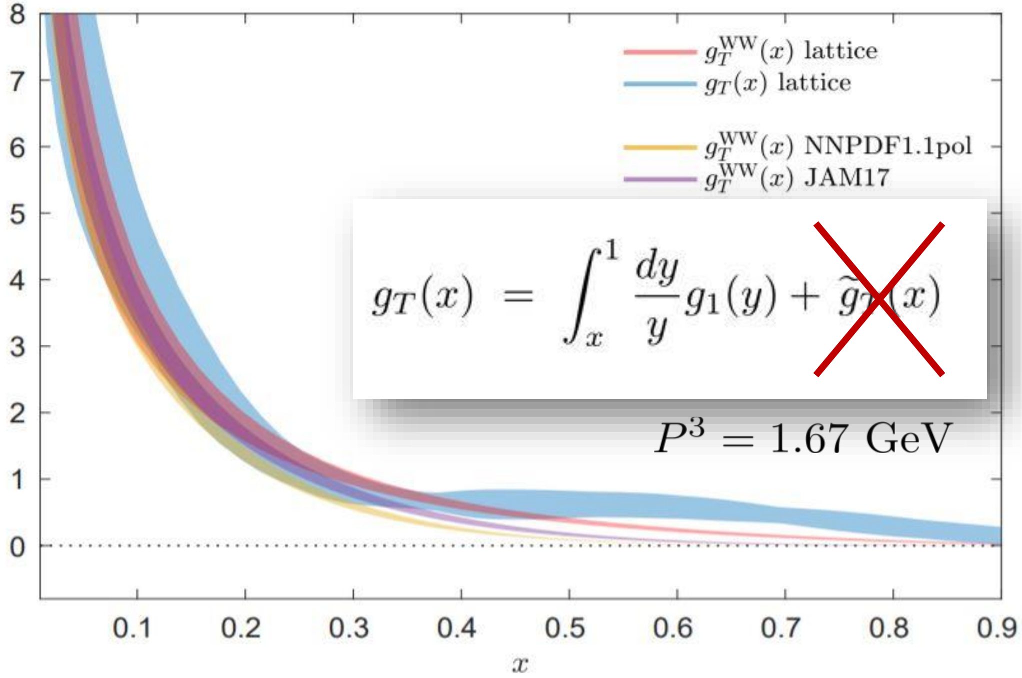

$$g_T(x) = \int_x^1 \frac{dy}{y} g_1(y) + \cancel{\tilde{g}_T(x)}$$

Good agreement between $g_T(x)$ **&** $g_T^{\mathrm{WW}}(x)$

Lattice QCD results for $g_T^{\mathrm{u-d}}(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD Rapid 102 (2020)

Ensemble:

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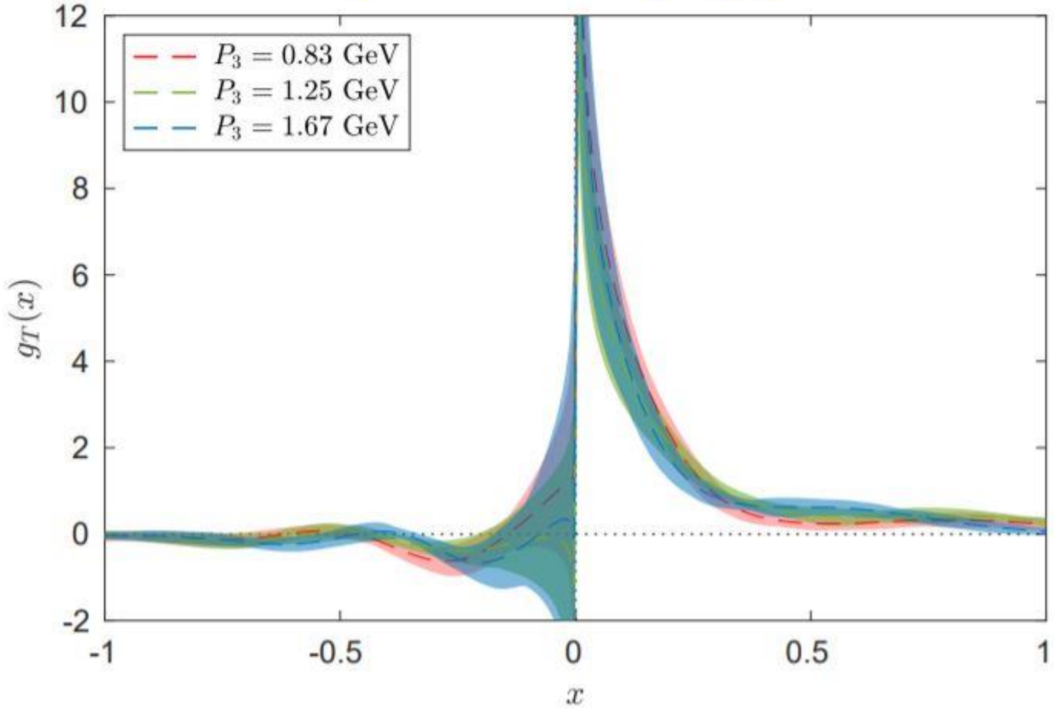

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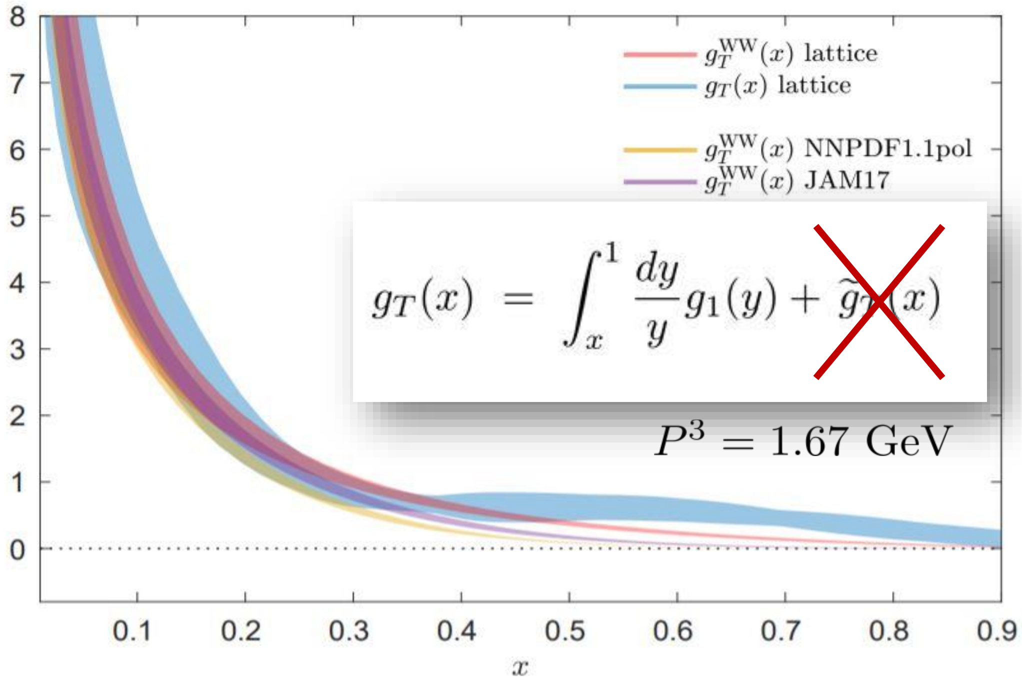


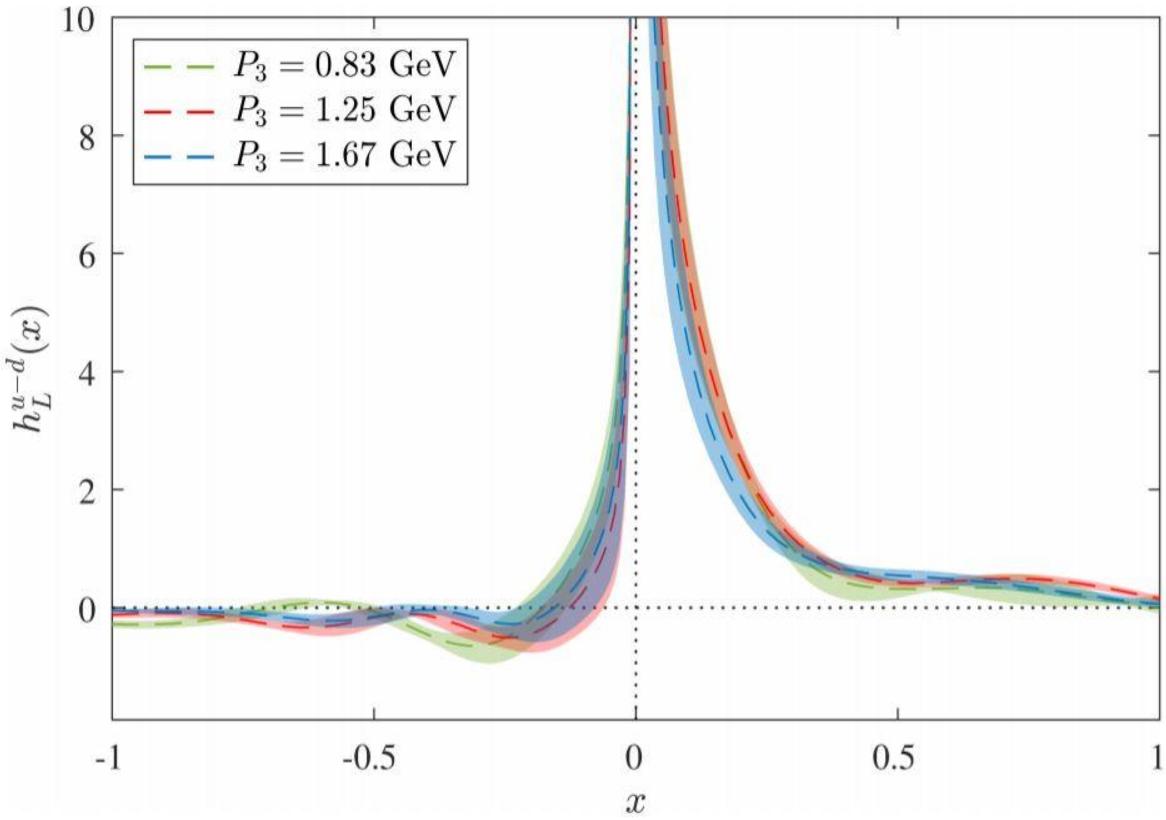
Good agreement between $g_T(x)$ **&** $g_T^{\mathrm{WW}}(x)$

Still, possible violation of up to $30\% - 40\%$ **perceivable**

Matching for twist-3 PDF $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 104 (2021)

Ensemble: $a = 0.093$ fm, $L \approx 3$ fm, $m_\pi \approx 260$ MeV



Matching for twist-3 PDF $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 104 (2021)

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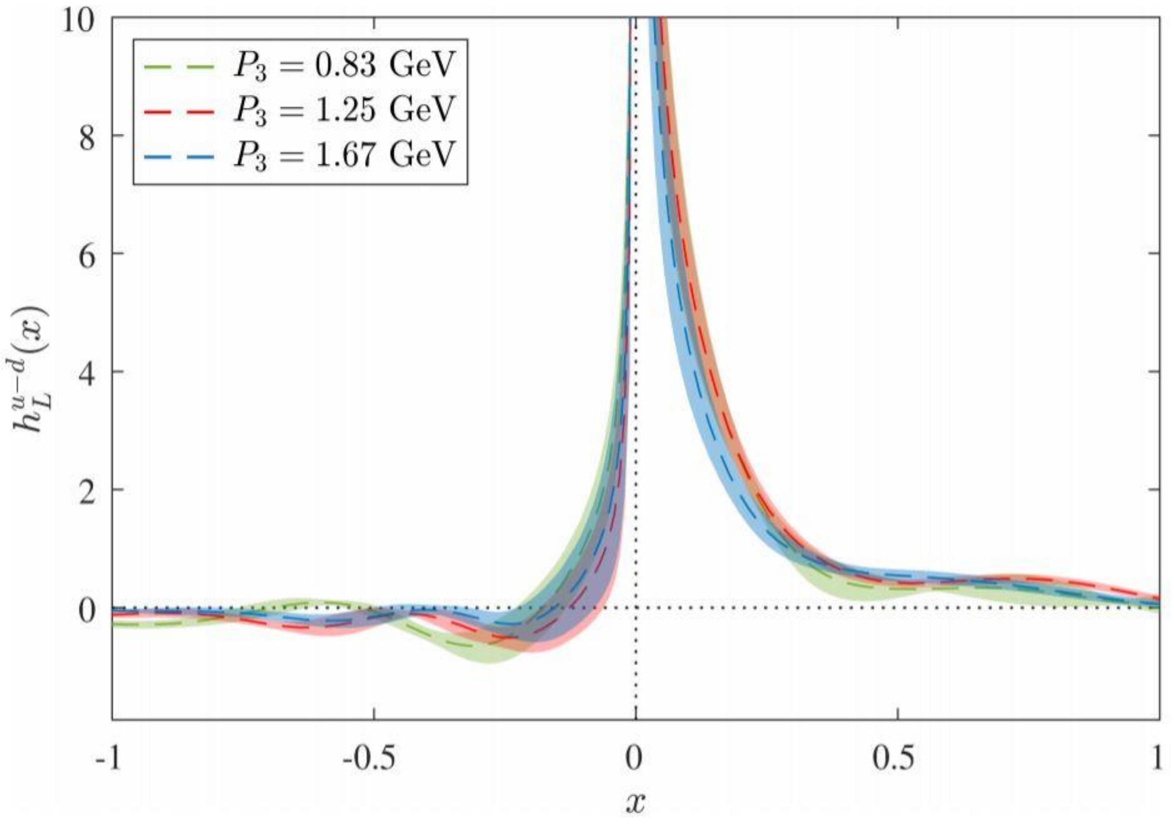
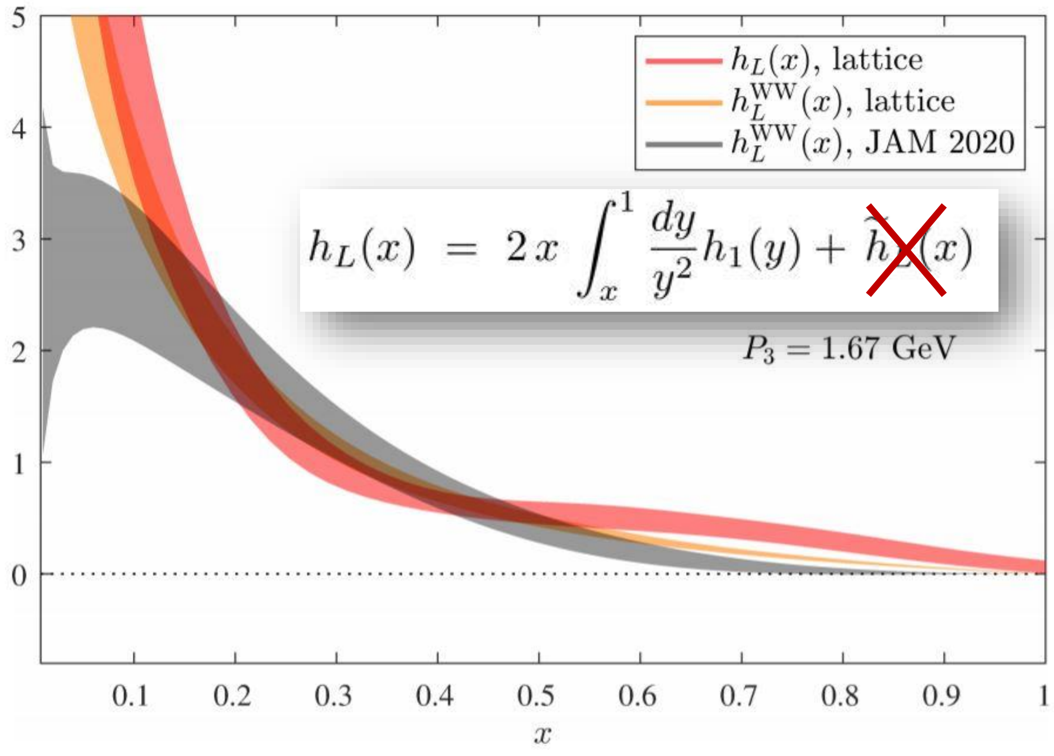


Check of WW approximation



Qualitatively, same findings as $g_T(x)$

Summary

- **Euclidean-correlator approaches have made it possible to directly access PDFs from lattice QCD**
- **Extracted matching coefficient for the twist-3 PDFs for the first time**
- **Presence of singular zero-modes in perturbative results makes the extraction of matching coefficient non-trivial**
- **We laid the necessary theoretical foundation to deal with zero-modes in matching**
- **We provided the first lattice results of $g_T(x)$ and $h_L(x)$**