Polarized and unpolarized gluon distributions in the nucleon from Lattice QCD

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CFNS Workshop: High Luminosity-EIC (EIC-Phase II)

WILLIAM & MARY
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- Origin of proton spin : “Proton spin crisis”

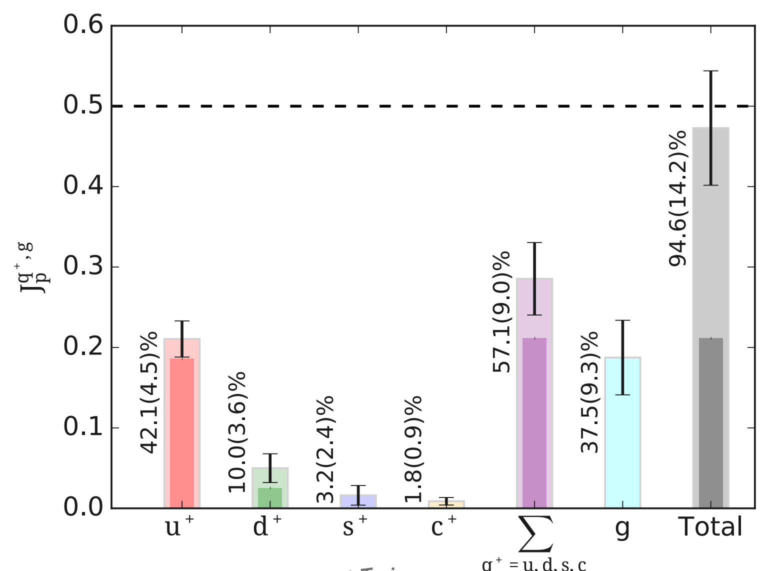
$$J = \frac{1}{2}\Delta\Sigma + L_q^{JM} + \Delta G + L_G$$

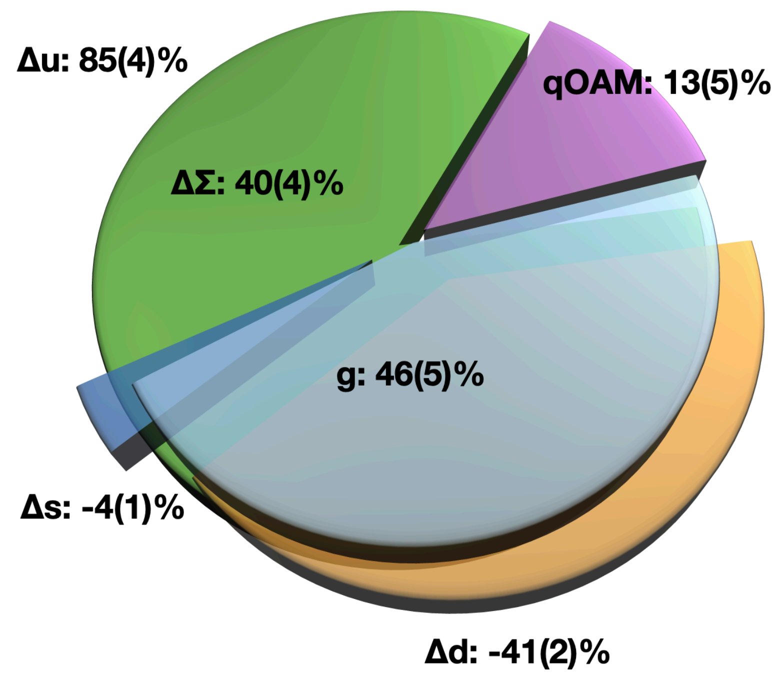
Jaffe & Manohar [NPB 1990]

- Proton spin decomposition & Lattice QCD

$$J = J_q + J_G = \frac{1}{2}\Delta\Sigma + L_q^{Ji} + J_G$$

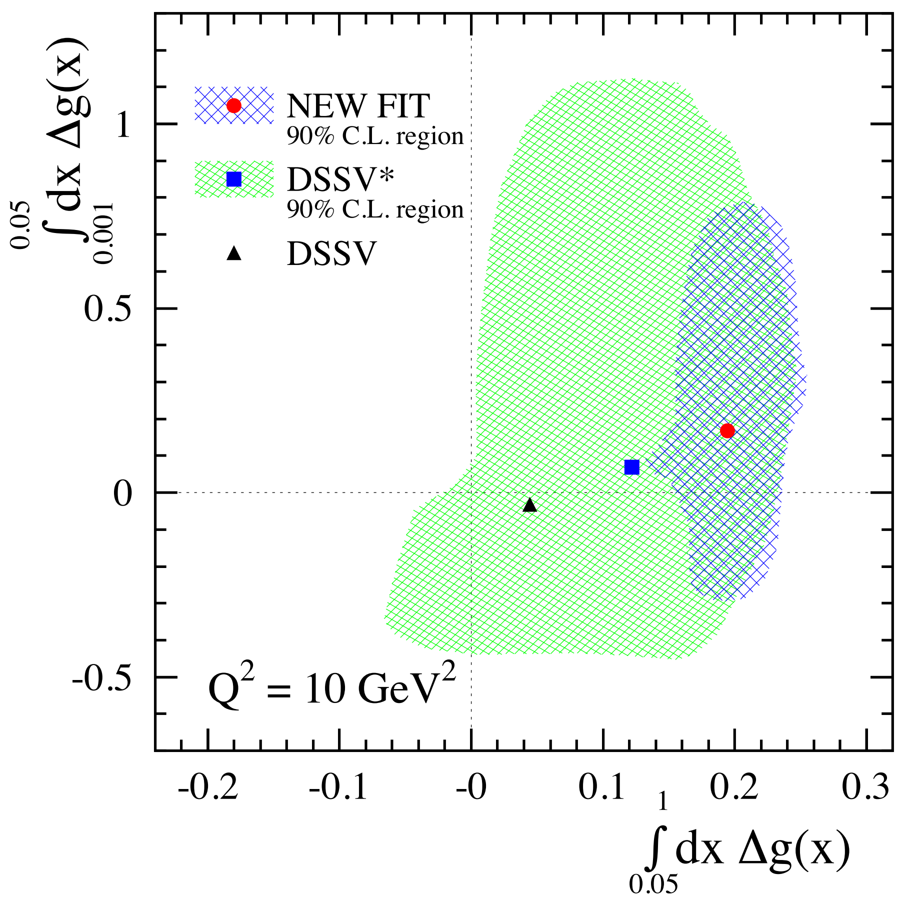
Ji [PRL 1997]



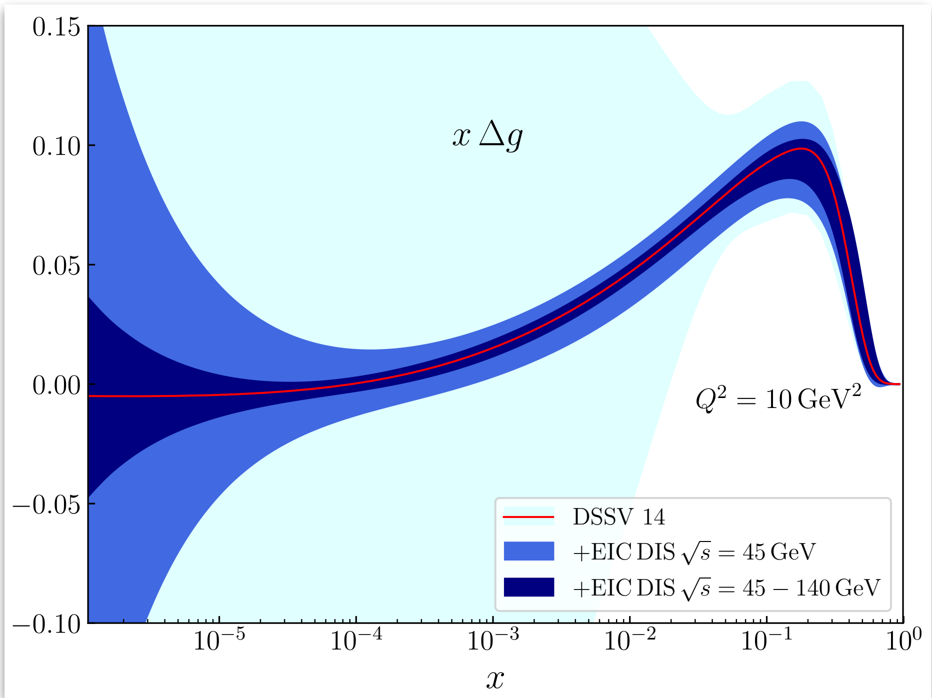


Status of gluon helicity distribution

- Gluon contribution to proton spin is not well-constrained from experimental data



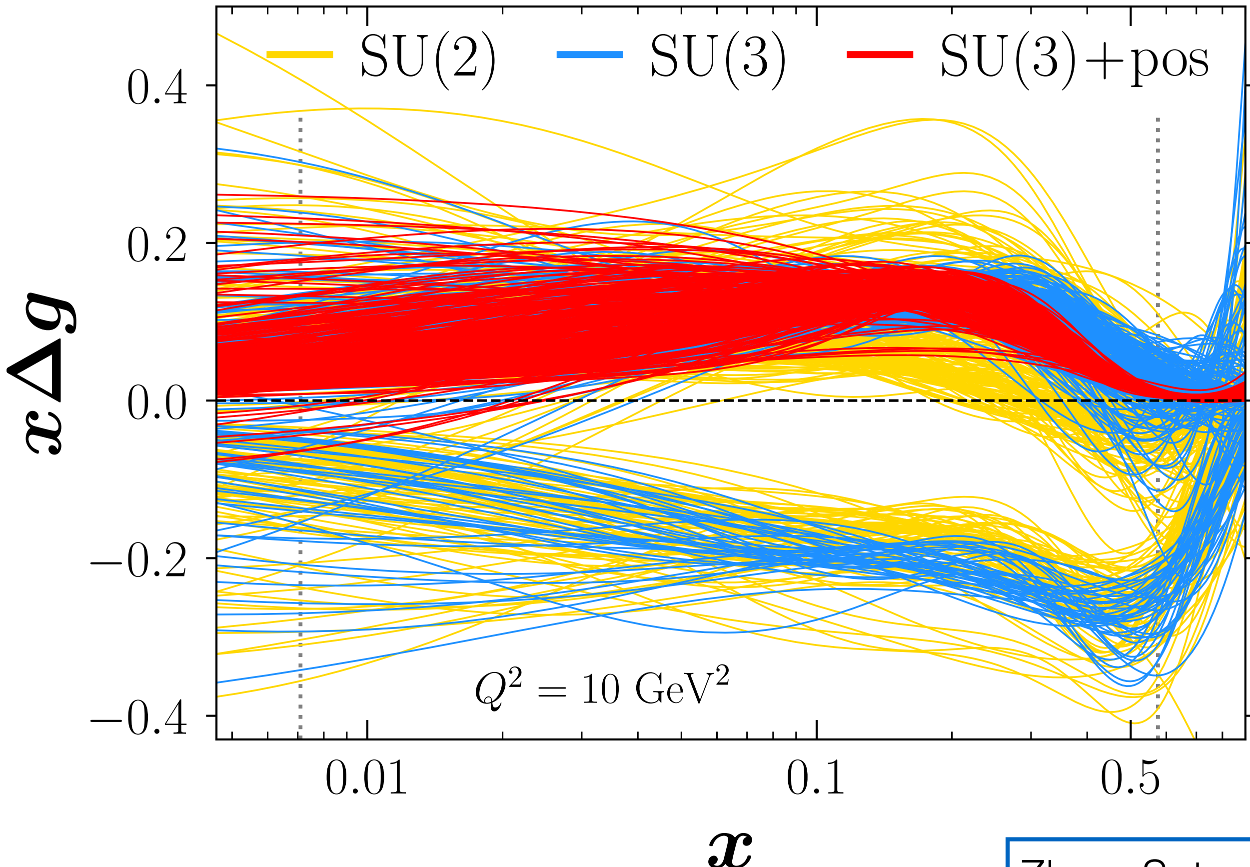
Florian et al [PRL 2014]



Impact of projected EIC data (EIC Yellow Report)

Status of gluon helicity distribution

- Recent JAM Collaboration analysis w/o positivity constraints on PDFs



Zhao, Sato, Melnitchouk [PRD 2022]

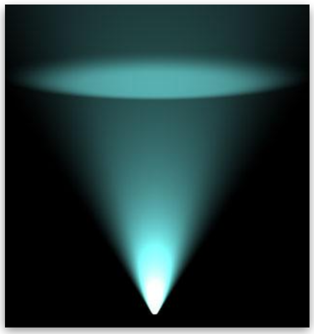
- LQCD can provide stringent constraint

Gluon contribution to proton spin from lattice QCD

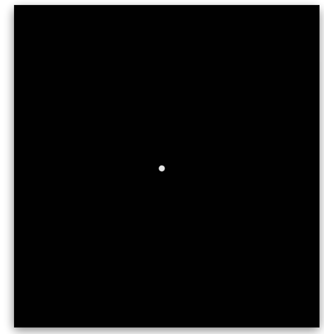
- ΔG as the matrix element of nonlocal operator of light cone correlation (not directly calculable on the lattice):

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}_{\alpha,b}^{\ \ +}(0)|PS\rangle$$

$r^2 - c^2t^2 = 0$



$t \to it$



- LQCD determination of gluon spin from local matrix element:

$$\vec{S}_g \to \vec{E} \times \vec{A}_{\text{phys}}$$

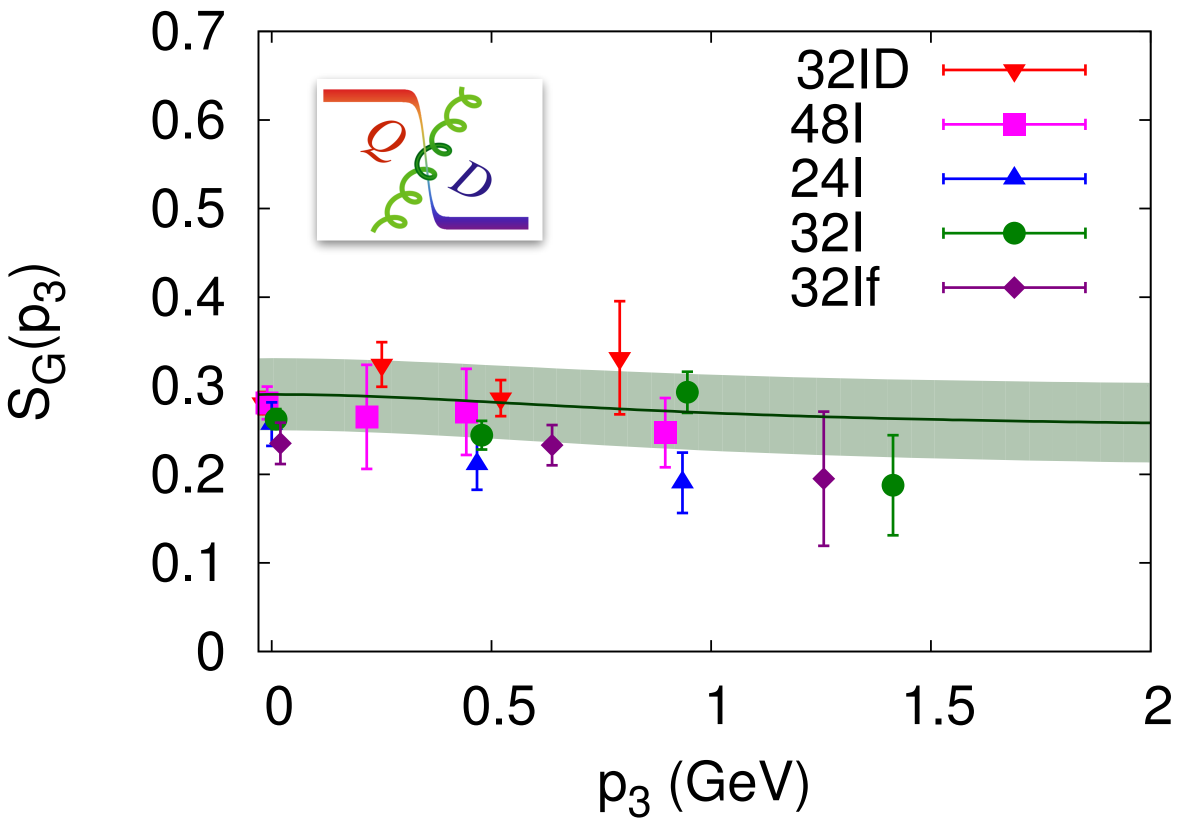
Ji, Zhang, Zhao [PRL 2013]

Hatta, Ji, Zhao [PRD 2014]

Matching using Large Momentum Effective Theory (LaMET) [Ji, Sci. China Phys 2014]

$$\Delta G$$

Lattice QCD determination of gluon spin

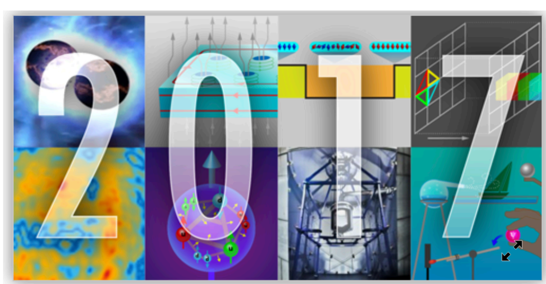


- After 1-loop matching

$$\Delta G(\mu^2 = 10\,\mathrm{GeV}^2) = 0.251(47)(16)$$

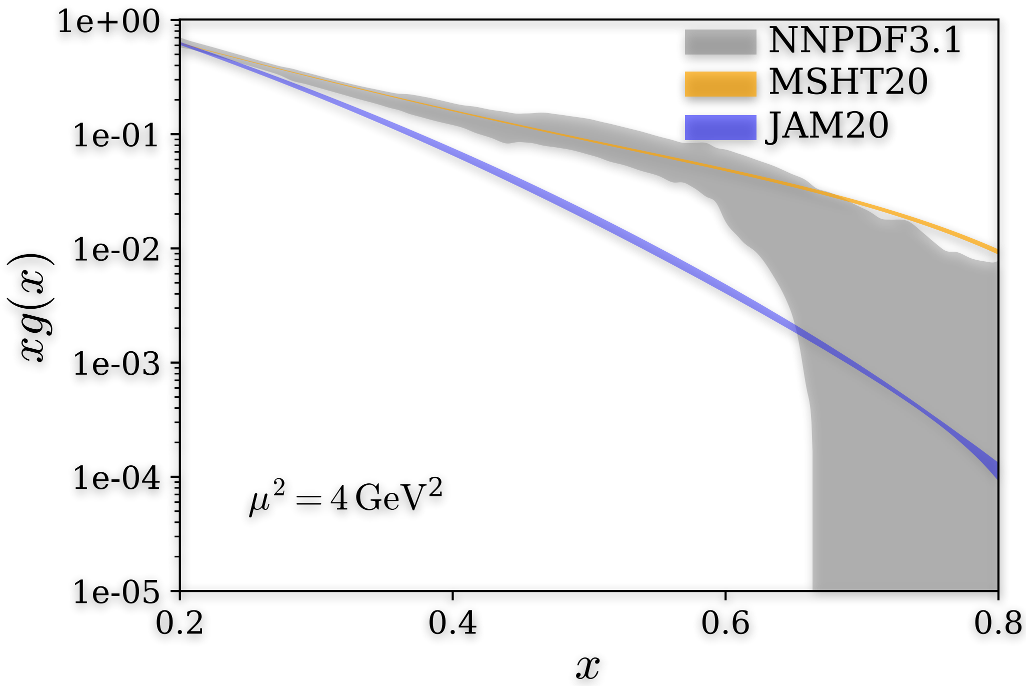
RSS, Glatzmaier, et al *PoS* LATTICE(2014)
Yang, RSS, et al (PRL 2017)

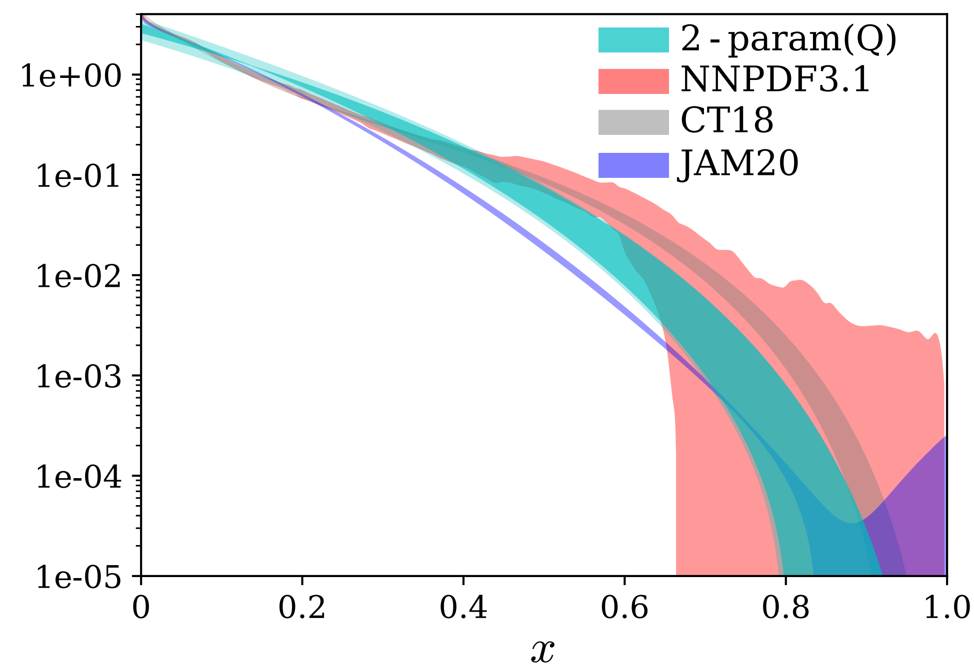
Highlights of the Year



Status of unpolarized gluon distribution & lattice QCD

- Gluon PDF is less explored in LQCD calculations and there is difference between PDF fits





Khan, **RSS**, et al [Hadstruc Collaboration] (PRD 2021)

- More on LQCD determination of unpolarized gluon PDF …

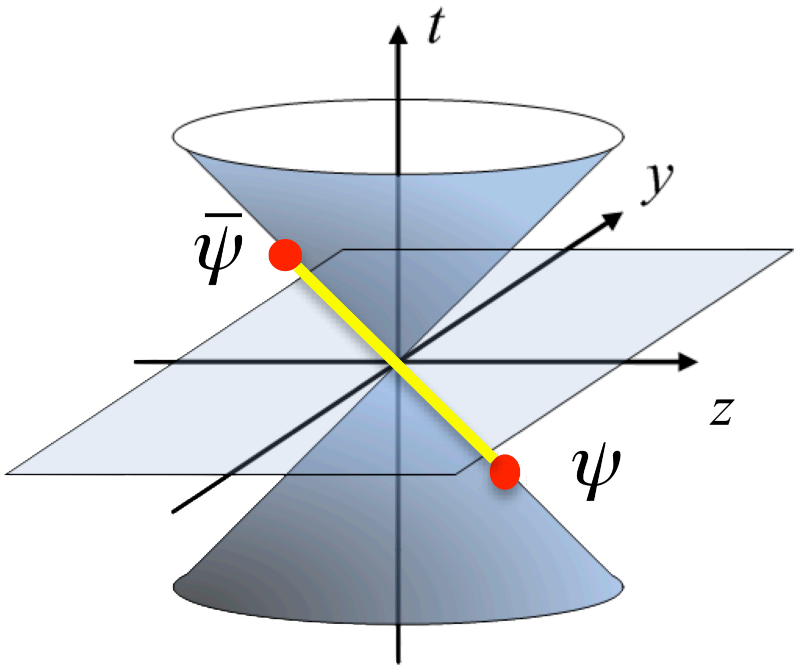
Lattice QCD formalism for calculating gluon PDFs

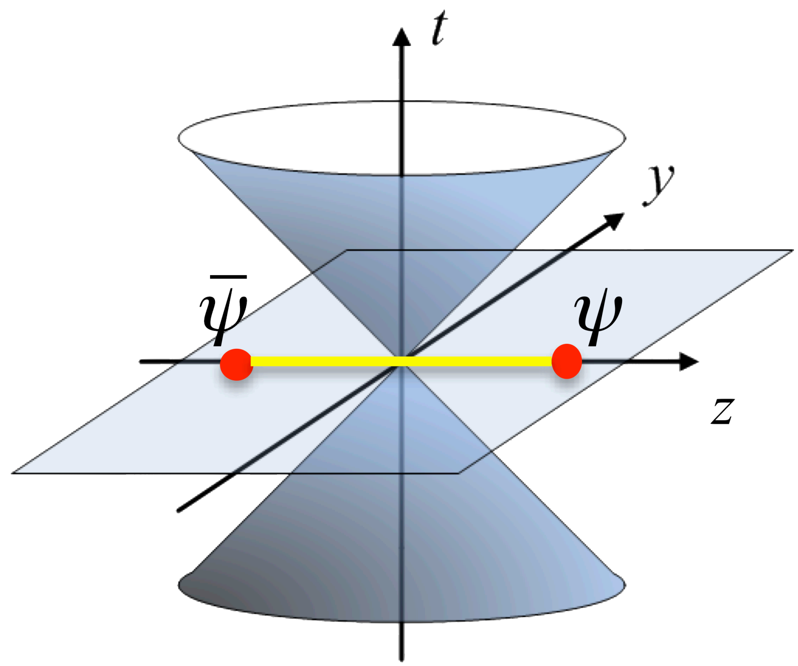
- Direct evaluation of light-cone correlations precluded by Euclidean metric of lattice QCD

- Lattice QCD proposals for x-dependent hadron structure:

  1. Hadronic tensor (Liu [PRL1994])
  2. Position-space correlators (Braun & Mueller [EPJ 2008])
  3. Quasi-PDFs (Ji [PRL 2013])
  4. Pseudo-PDFs (Radyushkin [PRD 2017])
  5. Lattice cross sections (Ma & Qiu [PRD 2018, PRL 2018]

Lattice QCD formalism from calculating gluon PDFs





- On the lattice, calculate spatial correlation in coordinate space

X. Ji [PRL 2013]

- For unpolarized gluon PDF: $M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$

- For polarized gluon PDF: $\widetilde{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s|\, G_{\mu\alpha}(z)\, W[z,0]\, \widetilde{G}_{\lambda\beta}(0)\, |p,s\rangle$

Lattice QCD formalism for calculating gluon PDFs

- LQCD calculated spatial correlation functions for gluon distributions are shown to be multiplicatively renormalizable

Zhang, Ji, et al [PRL 2019]
Li, Ma, Qiu [PRL 2019]

- LQCD matrix elements

X. Ji [PRL 2013]

Matching to LC distributions

Matching using Large Momentum Effective Theory (LaMET) [Ji, 2014]

Matching using short distance factorization (pseudo-PDF) [Radyushkin, 2017]

Today's talk

Lattice QCD formalism for calculating gluon PDFs

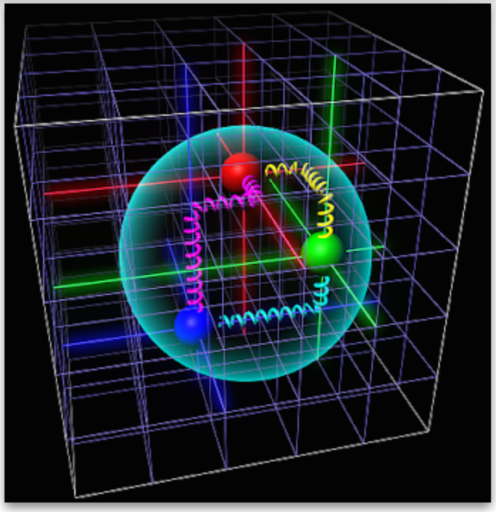
- Write **renormalized** LQCD matrix elements in terms of Lorentz invariant variables
  - z^2 and
  - Ioffe time, $\nu = p_z z$ (convention from Braun, et al [PRD 1995])

$$\mathfrak{M}(\nu, z^2) \xrightarrow[\text{matching}]{\text{perturbative}} \mathcal{I}_g(\nu, \mu^2) = \frac{1}{2}\int_{-1}^{1} dx\, e^{ix\nu}\, x\, g(x, \mu^2)$$

$$\widetilde{\mathfrak{M}}(\nu, z^2) \xrightarrow[\text{matching}]{\text{perturbative}} \widetilde{\mathcal{I}}_p(\nu, \mu^2) = \frac{i}{2}\int_{-1}^{1} \mathrm{d}x\, e^{-ix\nu}\, x\, \Delta g(x, \mu^2)$$

- Gluon helicity: $\Delta G(\mu^2) = \int_0^\infty \mathrm{d}\nu\, \widetilde{\mathcal{I}}_p(\nu, \mu^2) = \int_0^1 \mathrm{d}x\; \Delta g(x, \mu^2)$

Lattice QCD calculation



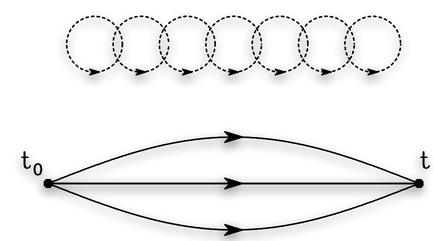
- Lattice size, $L \times T = 32^3 \times 64$
- Lattice spacing, $a \approx 0.094$ fm
- Pion mass, $m_\pi = 358$ MeV
- 351 configurations for unpolarized gluon distribution
- **1901** configurations for polarized gluon distribution
- Hadron boosted along z-direction, $p = \frac{2\pi n}{La} = [0, 2.46]$ GeV

Special features: optimized operators and nucleon correlator

- Nucleon correlation function using "Distillation" Peardon, et al [PRD 2009]
- Extended basis of operators (positive/negative parity, hybrid, ...
- Gluonic operator using "Wilson flow" M. Luscher, JHEP 2010
- Use summed generalized eigenvalue problem (sGEVP)

J. Bulava, et al, JHEP 2012

  - $C\exp(-\Delta E t/2)$ (GEVP)
  - $D\,t\exp(-\Delta E t)$ (sGEVP)



Lattice QCD calculation of unpolarized gluon distribution

- Matrix element for unpolarized gluon PDF

$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

- A proper combination on the lattice:

$$M_{0i;i0} = \langle p|\, G_{0i}(z)\, [z,0]\, G_{i0}(0)\, |p\rangle = 2\, p_0^2 \mathcal{M}_{pp} + 2\mathcal{M}_{gg}$$

$$M_{ji;ij} = -2\mathcal{M}_{gg}$$

$i, j \to x, y$

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2\, \mathcal{M}_{pp}$$

- Combination is multiplicatively renormalizable

Balitsky, et al [PLB 2020]

Lattice QCD calculation of unpolarized gluon distribution

- Renormalization: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}}\right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}}\right)$

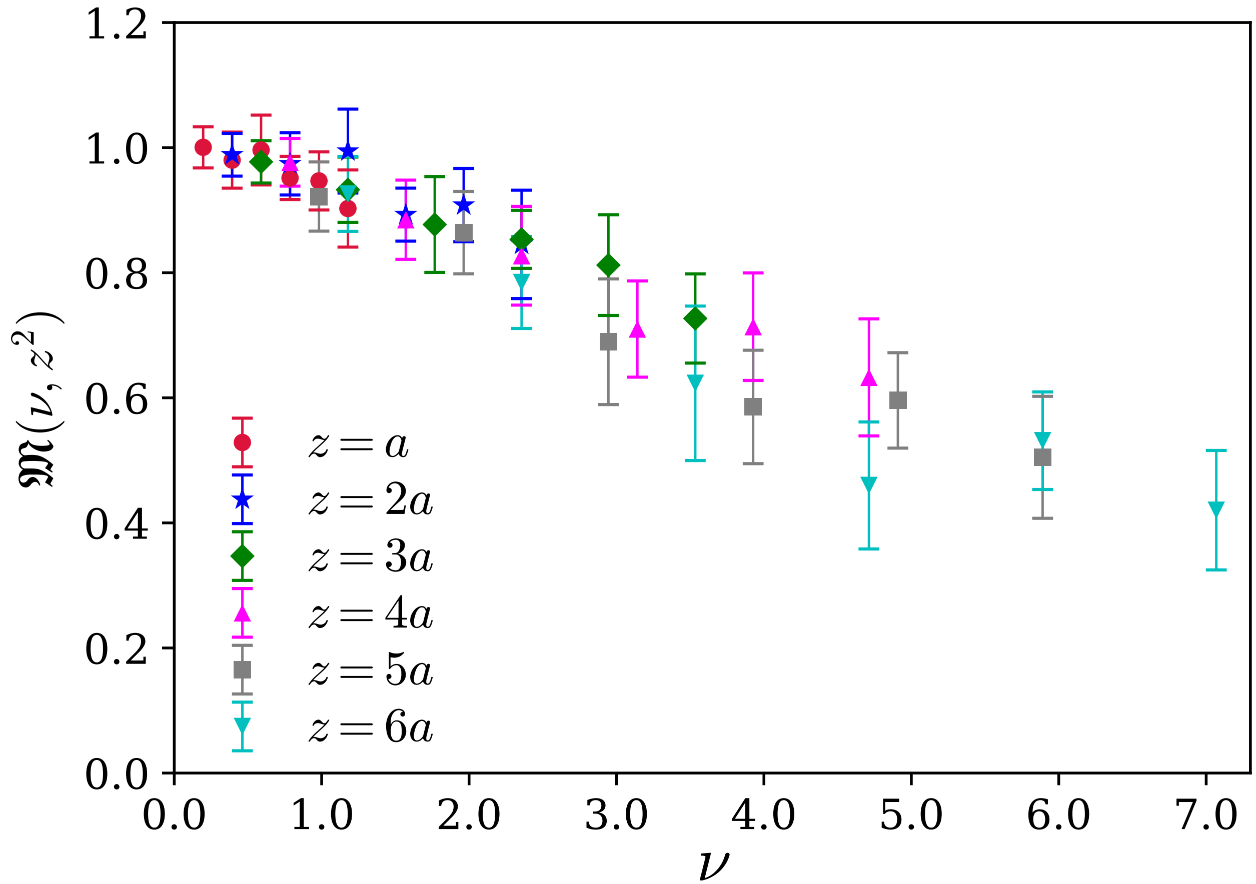
Reduced Ioffe-time distribution

Radyushkin [PLB 2017]
Orginos, et al [PRD 2017]

- After renormalization and perturbative matching

$$\mathcal{M}_{pp}(\nu, z^2) \to \mathcal{I}_g(\nu, \mu^2) = \int_0^1 dx \, \cos(x\nu) \, x \, g(x, \mu^2)$$

- Unpolarized gluon pseudo-ITD



- Recall the inverse problem:

$$\mathfrak{M}(\nu, z^2) \xrightarrow[\text{matching}]{\text{perturbative}} \mathcal{I}_g(\nu, \mu^2) = \frac{1}{2}\int_{-1}^{1} dx\, e^{ix\nu}\, x\, g(x, \mu^2)$$

Some phenomenology

- Fit NNPDF3.1 gluon PDF using ansatz

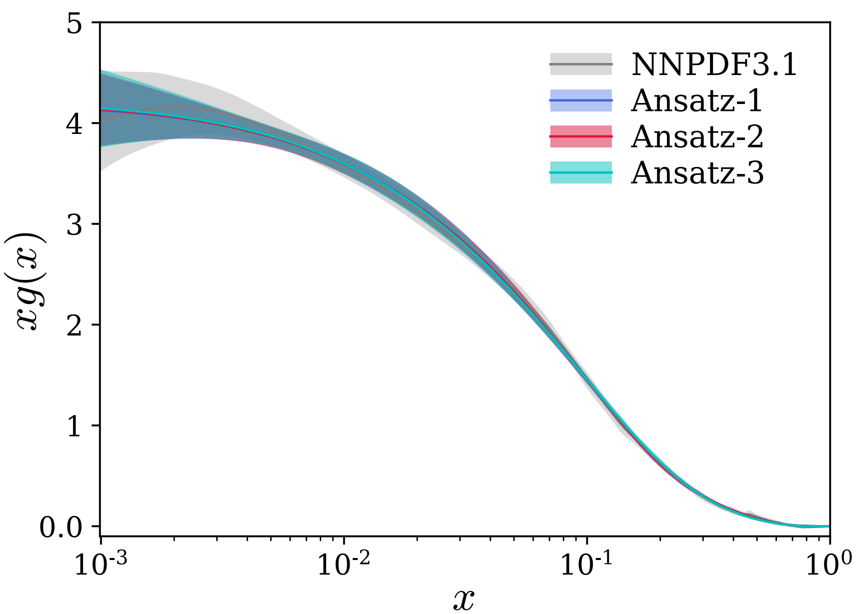
$$xg^{+}(x) = x^{\alpha}\left[A(1-x)^{4+\beta} + B(1-x)^{5+\beta}\right] \times (1+\gamma\sqrt{x}+\delta x)$$
$$xg^{-}(x) = x^{\alpha}\left[A(1-x)^{6+\beta} + B(1-x)^{7+\beta}\right] \times (1+\gamma\sqrt{x}+\delta x)$$

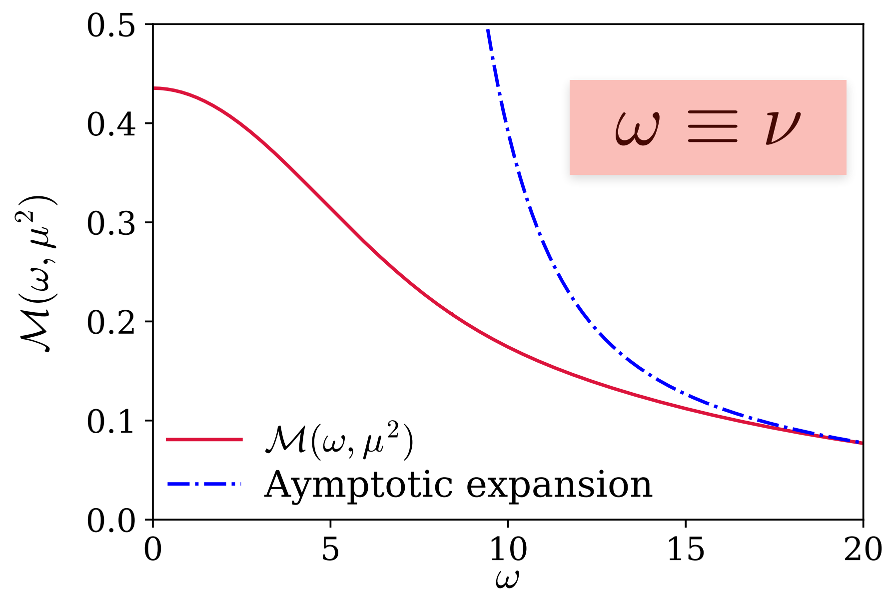
Brodsky, Burkardt, Schmidt [NPB 95]

$$xg(x) \equiv xg^{+}(x) + xg^{-}(x)$$
$$x\Delta g(x) \equiv xg^{+}(x) - xg^{-}(x)$$

RSS, **Liu, Paul**
PRD 2021

- Asymptotic form: $\mathcal{M}(\omega,\mu^2) = A\,\Big[\,\big(\,C_R(\alpha, 4+\beta;\,\omega)$
$+\gamma\, C_R(\alpha+1/2, 4+\beta;\,\omega) + \delta\, C_R(\alpha+1, 4+\beta;\,\omega)\,\big)$
$+(\,\beta\to\beta+2\,)\,\Big] + B\,\big[\,\beta\to\beta+1\,\big] + \mathcal{O}\big(1/\omega^{a+R+1}\big)$





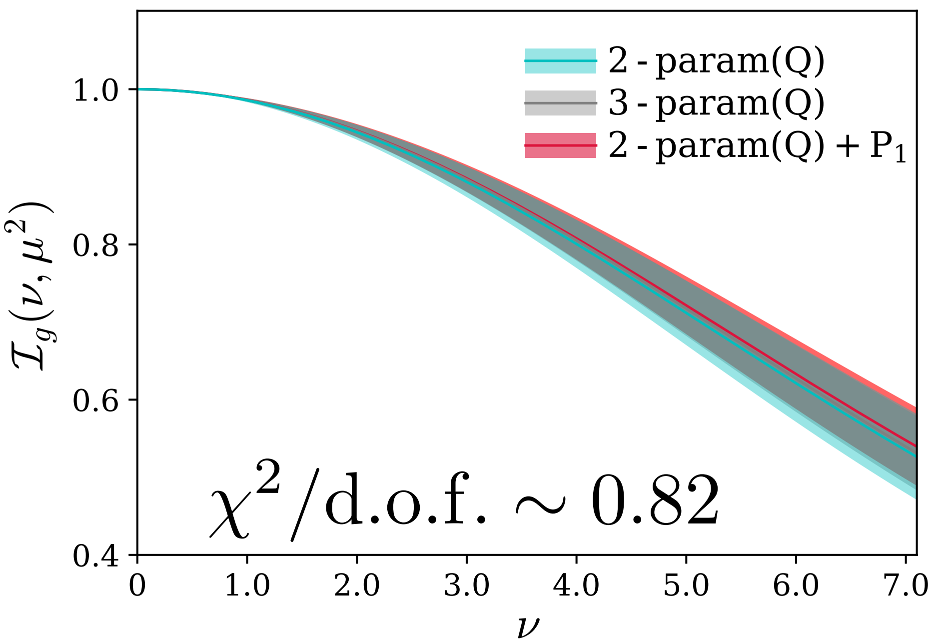
- PDF fits:

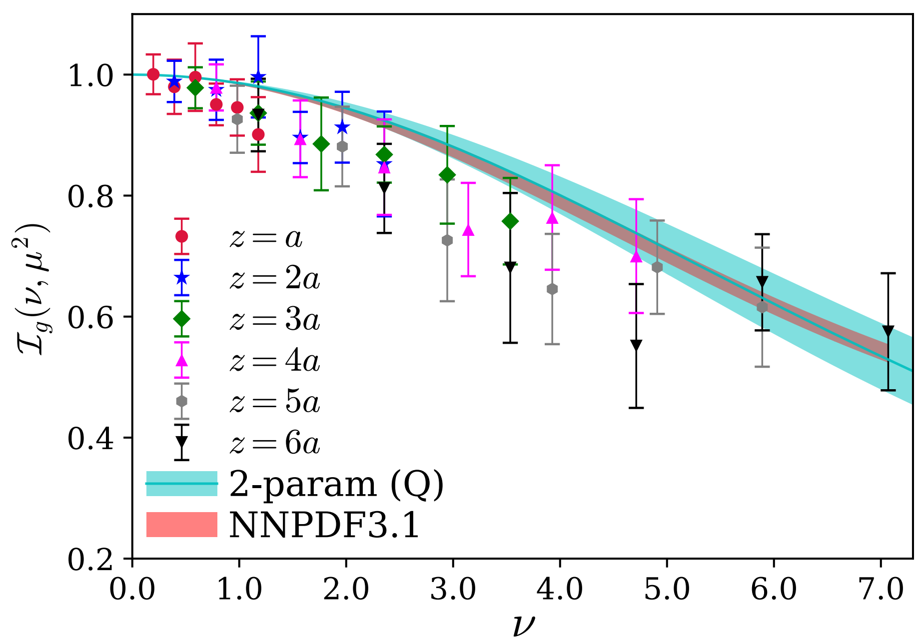
2-parameter form : $x^{\alpha}\,(1-x)^{\beta}$

+ correction to 2-parameter form

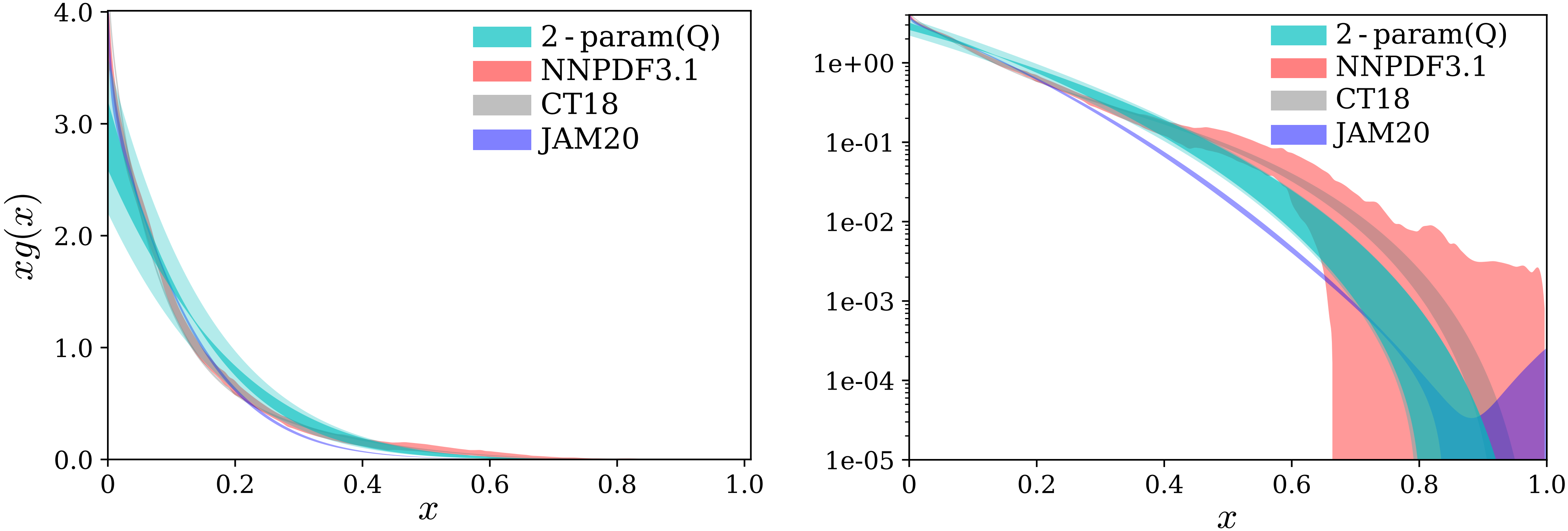
+ discretization error

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\; \mathcal{K}(x\nu, \mu^2 z^2)\, \frac{x^{\alpha}\,(1-x)^{\beta}}{B(\alpha+1, \beta+1)}$$





Determination of unpolarized gluon distribution



Khan, **RSS**, et al [Hadstruc Collaboration]
(PRD 2021)

Lattice QCD calculation of polarized gluon distribution

- Matrix element for polarized gluon distribution:

$$\widetilde{M}_{00}(z,p_z) \equiv [\widetilde{M}_{ti;ti}(z,p_z) + \widetilde{M}_{ij;ij}(z,p_z)]$$

- Renormalization:

$$\widetilde{\mathfrak{M}}(z,p_z) \equiv i\frac{[\widetilde{M}_{00}(z,p_z)/p_z p_0]/Z_{\mathrm{L}}(z/a_L)}{M_{00}(z,p_z=0)/m_p^2}$$

Balitsky, Morris, Radyushkin [JHEP 2022]

- The PROBLEM:

$$\widetilde{\mathfrak{M}}(\nu,z^2) = [\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu,z^2) - (1+m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)]$$

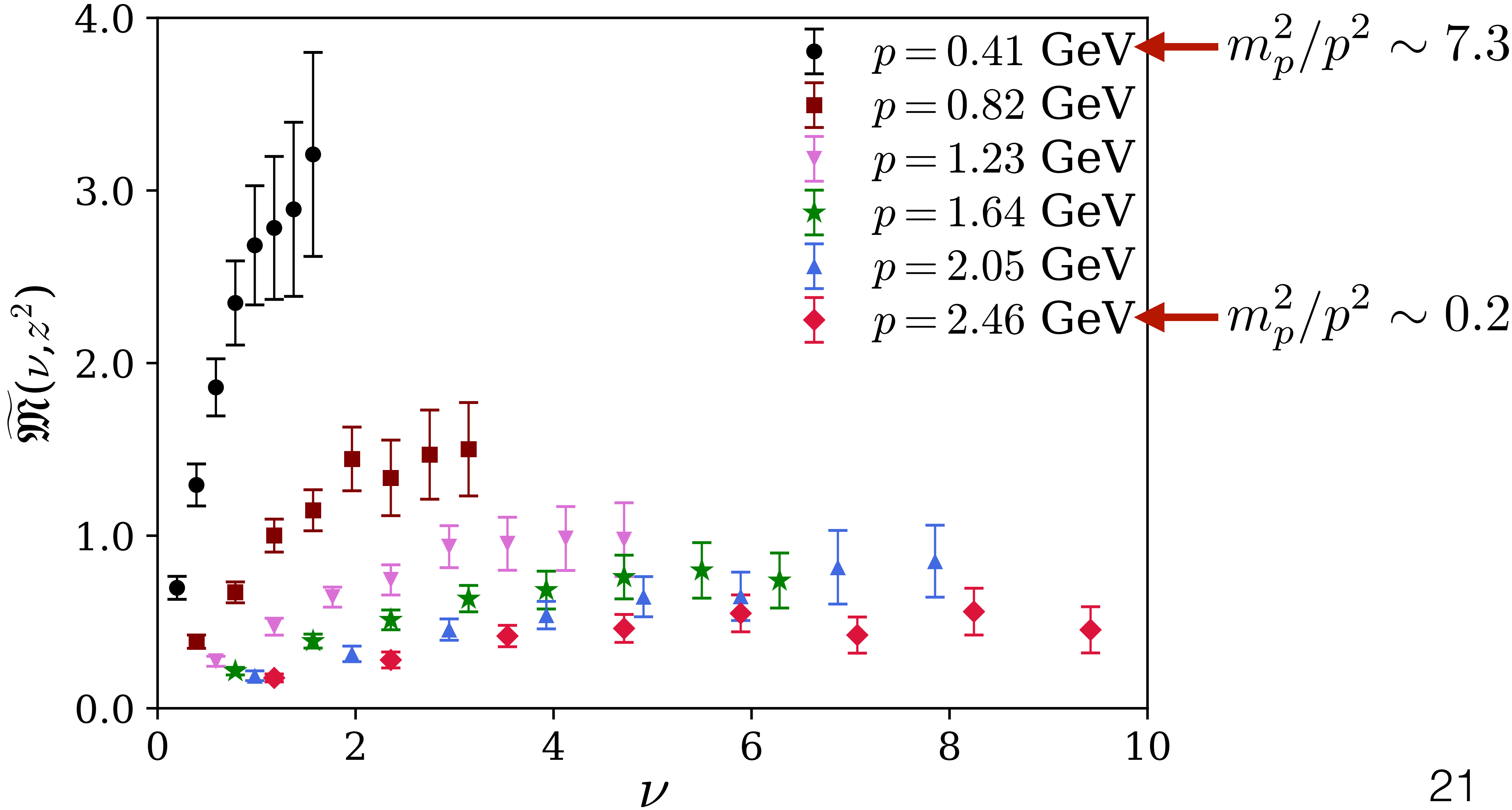
- What we want:

$$\widetilde{\mathcal{I}}_p(\nu) \equiv i\,[\widetilde{\mathcal{M}}_{ps}^{(+)}(\nu) - \nu\widetilde{\mathcal{M}}_{pp}(\nu)]$$

Ioffe time pseudo-distribution in the zero flow time limit

- Contamination term present in LQCD matrix element dominates

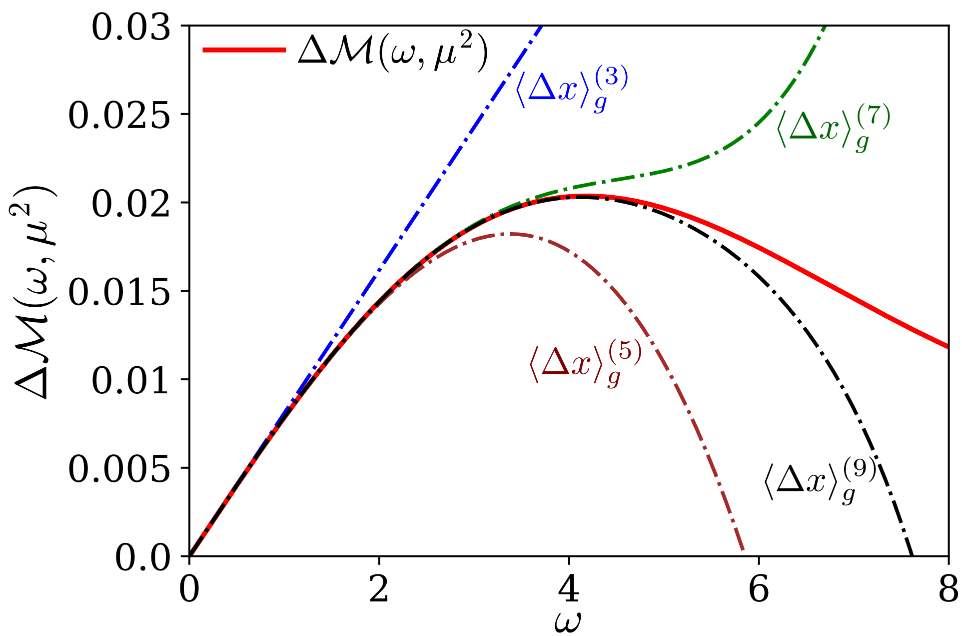
$$\widetilde{\mathfrak{M}}(\nu, z^2) = [\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu, z^2) - (1 + m_p^2/p_z^2)\nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2)]$$

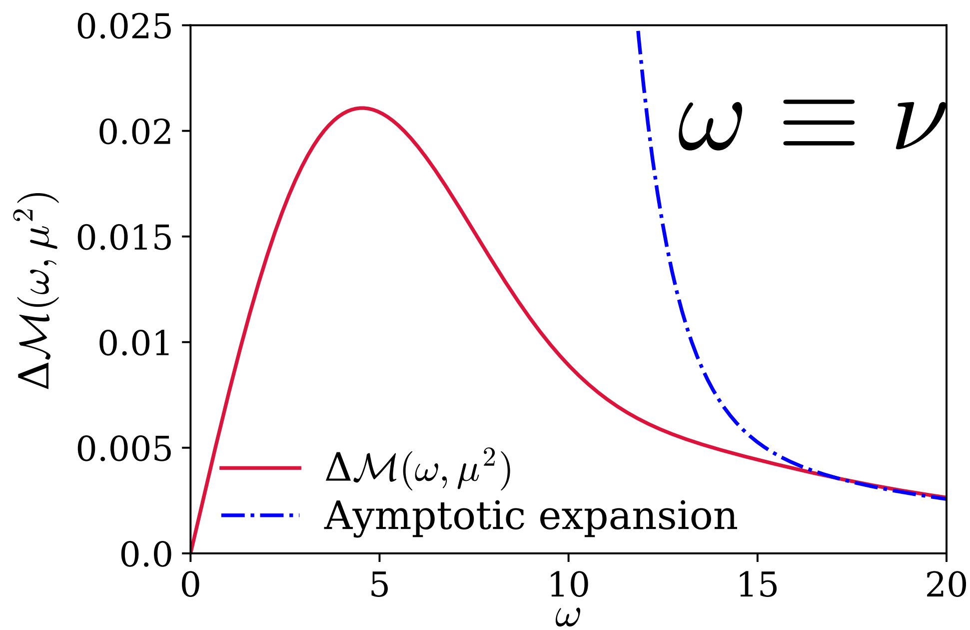


Another challenge for Lattice QCD calculation of x-dependent gluon helicity distribution

- Lattice data in a limited range of ν
- Available lattice data is sensitive up to first few moments

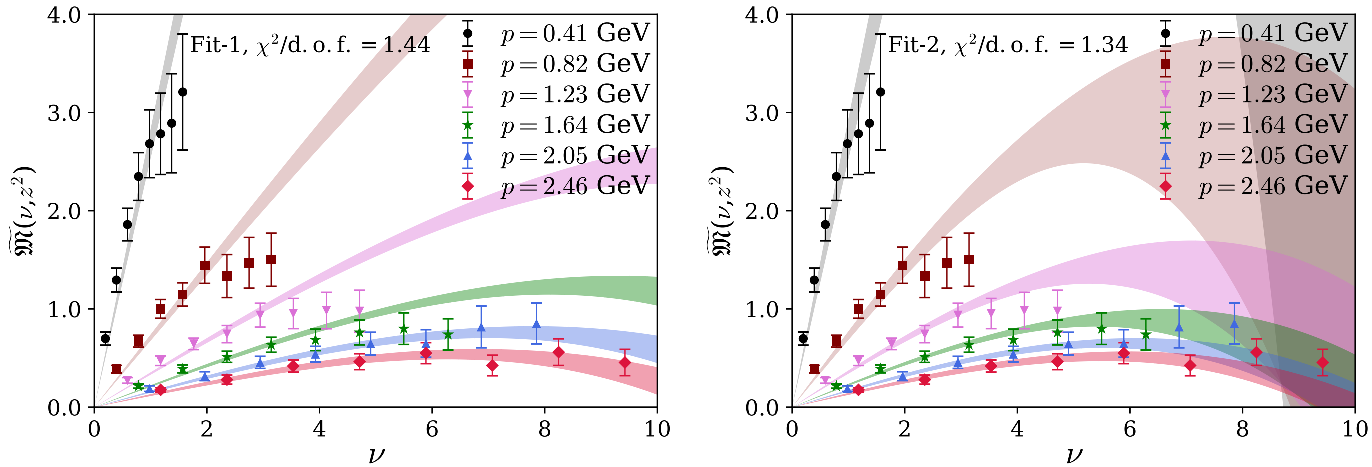
RSS, Liu, Paul [PRD 2021]



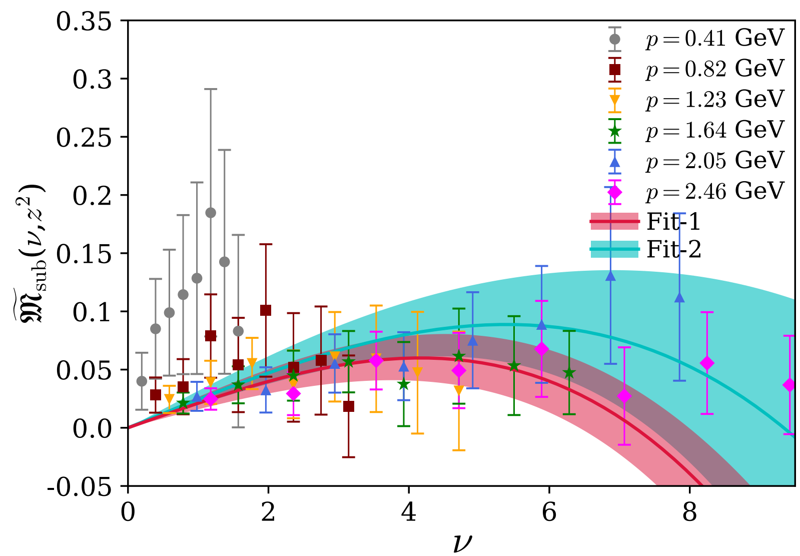


- Correction through fits using moments

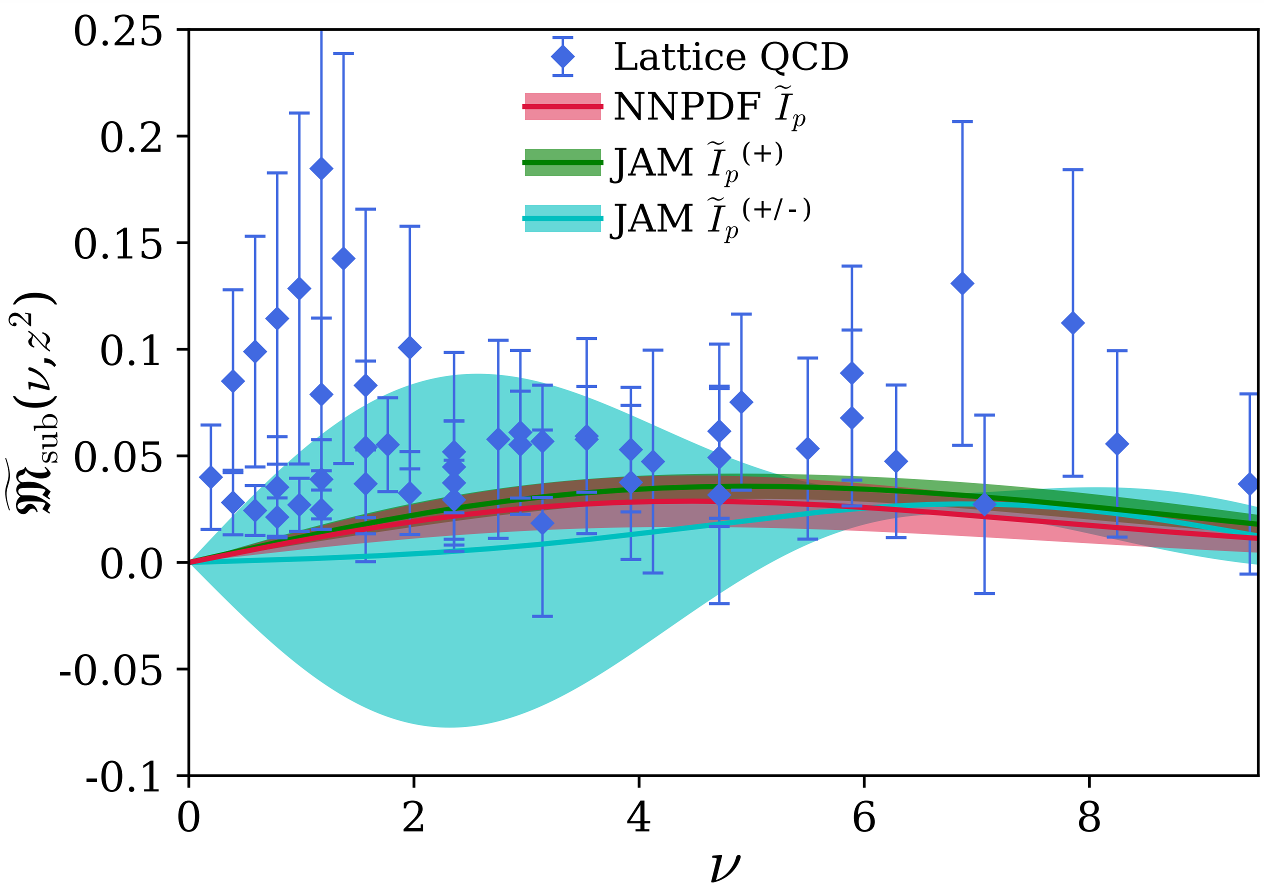
$$\widetilde{\mathfrak{M}}(\nu) = \sum_{i=0} \frac{(-1)^i}{(2i+1)!} a_i \nu^{2i+1} + \nu \frac{m_p^2}{p^2} \sum_{j=0} \frac{(-1)^j}{(2j)!} b_j \nu^{2j}$$



- Correction by subtracting zero momentum matrix elements



Comparison with phenomenology



In preparation

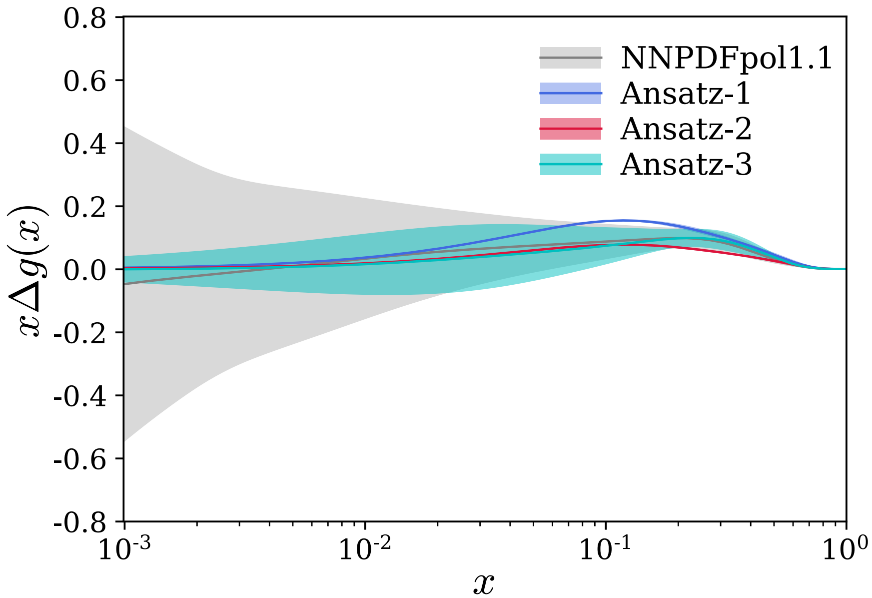
- With controlled statistics, next step is factorization & matching

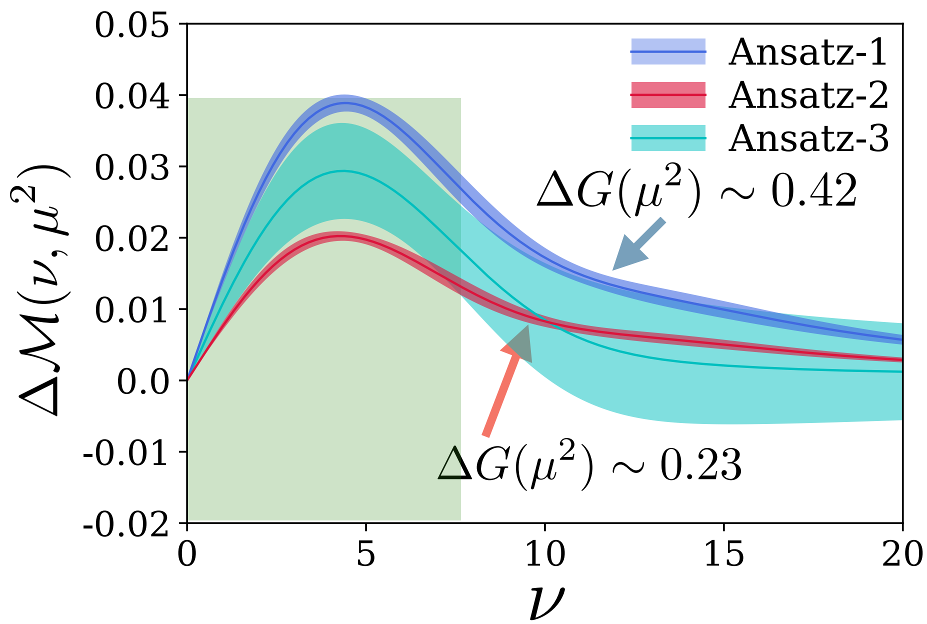
LQCD data = **Short-distance physics (pQCD)** $\otimes$ **Long-distance PDFs** + **Power correction**

Prospect of Lattice QCD on gluon helicity distribution

- Gluon helicity from light cone Ioff-time distribution

$$\Delta G(\mu^2) = \int_0^\infty d\nu \; \Delta\mathcal{M}_{\rm light-cone}(\nu, \mu^2)$$





RSS, Liu, Paul [PRD 2021]

- LQCD determination of polarized gluon ITD, even at small Ioffe-time window can have important impact

Summary & Outlook

- LQCD determination of unpolarized gluon PDF looks promising!

- We have presented the first LQCD determination of polarized gluon Ioffe-time distribution

- Future calculation: With precise LQCD matrix elements,, perform factorization and obtain light-cone Ioffe-time distribution

- Challenge: many systematics to understand

- Goal: determination of gluon contribution to proton spin & x-dependent helicity distribution

Thank you!