x-dependent GPDs from Lattice QCD

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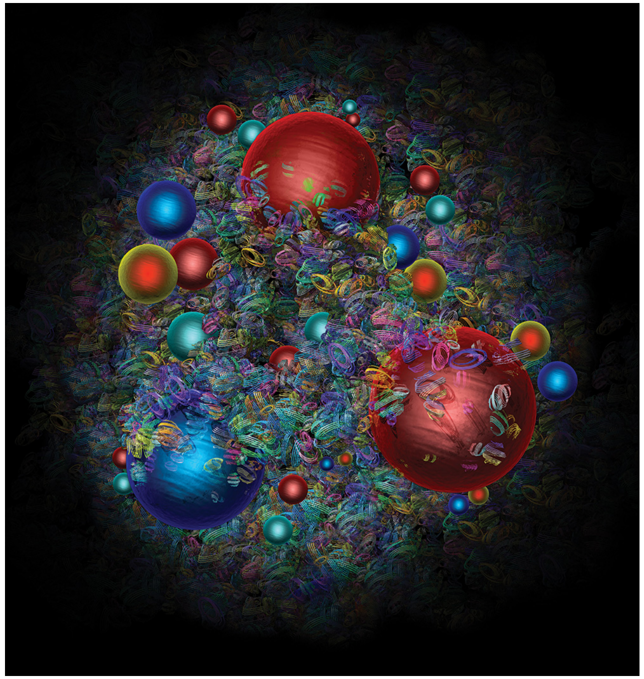
Outline:

Introduction
Quasi-GPDs:
– how it works
– twist-2 GPDs
– twist-3 GPDs
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson
K. Hadjiyiannakou, K. Jansen, A. Metz, A. Scapellato, F. Steffens

Nucleon structure

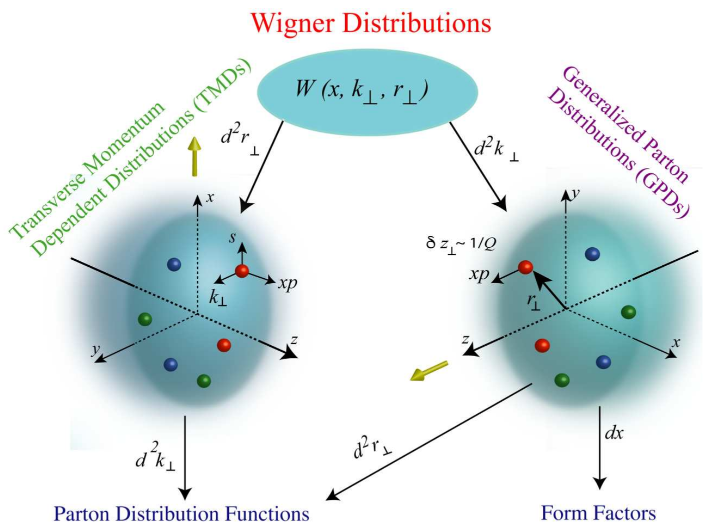


One of the central aims of hadron physics:
to understand better nucleon structure.

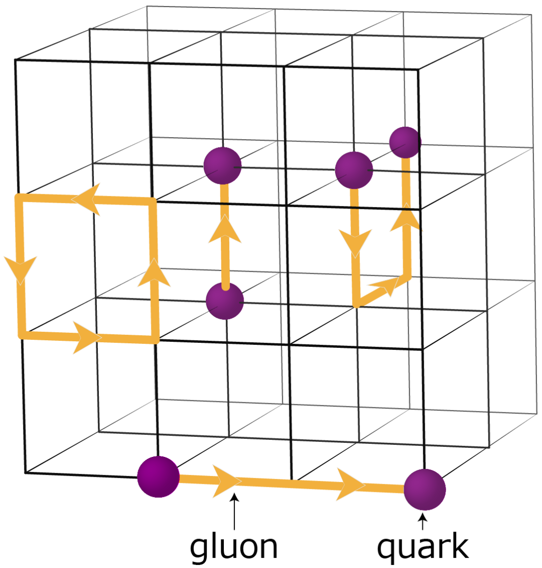
- This is one of the crucial expectations from the Electron-Ion Collider (EIC)!
- In particular, we want to probe the 3D structure.
- Thus, we need to access new kinds of functions: GPDs, TMDs.
- Also higher-twist is of growing importance for the full picture.
- Both theoretical and experimental input needed.

Lattice can provide *qualitative* and eventually *quantitative* knowledge of different functions and their moments:

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function



Lattice QCD – what one should keep in mind



- Lattice QCD offers a way for a careful *ab initio* study of non-perturbative aspects of QCD.
- Its huge strength: possibility to control all systematic effects: *cut-off effects, finite volume effects, quark mass effects, isospin breaking, excited states, . . .*
- For many aspects, already precision results with percent/per mille total uncertainty.
- However, many aspects (the difficult ones!) with only exploratory studies.
- Difficult problems need time to:
  - ⋆ find the proper way to address
  - ⋆ prove computational feasibility
  - ⋆ optimize the computational method
  - ⋆ acquire all data (long computations...)
  - ⋆ analyze all systematics
- Nucleon structure is mostly difficult... and very expensive computationally.
- Thus, do not expect miracles.
- Overall, **expect complementary role of lattice**.
- Robust quantitative statements: *low moments, form factors*.
- x-dependence: breakthrough in recent years, but a long way to go to solid quantitative statements.

Approaches to x-dependence

- Recent years (since ≈ 2013): breakthrough in accessing x-dependence.
- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y}\, C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$
  with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Lattice PDFs/GPDs: dynamical progress

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

- **Quasi-distributions**
  - Nucleon twist-2 PDF
    - unpolarized: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, arXiv:1803.04393; LPC, PRD101(2020)034020; BNL/MSU, PRD102(2020)074504; ETMC, PRD103(2021)094512; MSU/NTU, 2011.14971
    - helicity: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, PRL121(2018)242003; BNL/MSU, PRD102(2020)074504; ETMC, PRD103(2021)094512
    - transversity: ETMC, PRD98(2018)091503(R); ETMC, PRD99(2019)114504; LP3, arXiv:1810.05043
  - Nucleon twist-3 PDF: ETMC/Temple PRD102(2020)111501(R), PRD104(2021)114510
  - Nucleon GPD: ETMC, PRL125(2020)262001; ETMC, PRD105(2022)034501; MSU, PRL127(2021)182001
  - Nucleon singlet PDF: ETMC, PRL126(2021)102003; ETMC, PRD104(2021)054503; MSU/LANL, PRD104(2021)094511
  - Δ PDF: ETMC/PKU PRD102(2020)014508
  - Meson DA: LP3, PRD95(2017)094514; LP3, NPB939(2019)429; MSU/NTU, PRD102(2020)094519; LPC, PRL127(2021)062002
  - Pion/Kaon PDF: LP3, PRD100(2019)034505; BNL, PRD100(2019)034516; MSU/NTU/BNU, PRD103(2021)014516; CCNU/BNL/ANL, PRL128(2022)142003
  - Pion GPD: MSU/NTU/UR, NPB952(2020)114940
- **Pseudo-distributions**
  - Nucleon PDF: HadStruc, PRD96(2017)094503; HadStruc, JHEP12(2019)081; HadStruc, PRL125(2020)232003; ETMC, PRD103(2021)034510; HadStruc, JHEP11(2021)024; HadStruc, JHEP11(2021)148; adStruc, PRD105(2022)034507
  - Gluon PDF: MSU, IJMPA36(2021)13; MSU, PLB823(2021)136778; HadStruc, PRD104(2021)094516; MSU, arXiv:2112.03124
  - Pion PDF: HadStruc, PRD100(2019)114512; BNL, PRD102(2020)094513
- **Current-current**
  - Pion PDF: HadStruc, PRD99(2019)074507; HadStruc, PRD102(2020)054508
- **Auxiliary light quark**
  - Pion DA: UR, EPJC78(2018)217; UR, PRD98(2018)094507
- **Auxiliary heavy quark**
  - Pion DA: Taiwan/MIT, arXiv:1810.12194; Taiwan/MIT, PRD105(2022)034506
- **OPE without OPE**
  - Nucleon F_1: QCDSF, PRL118(2017)242001; QCDSF, PRD102(2020)114505
  - Nucleon GPD: QCDSF, PRD105(2022)014502
- **Hadronic tensor**
  - Nucleon W_{11}: χQCD, PRD101(2020)114503

Reviews:

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

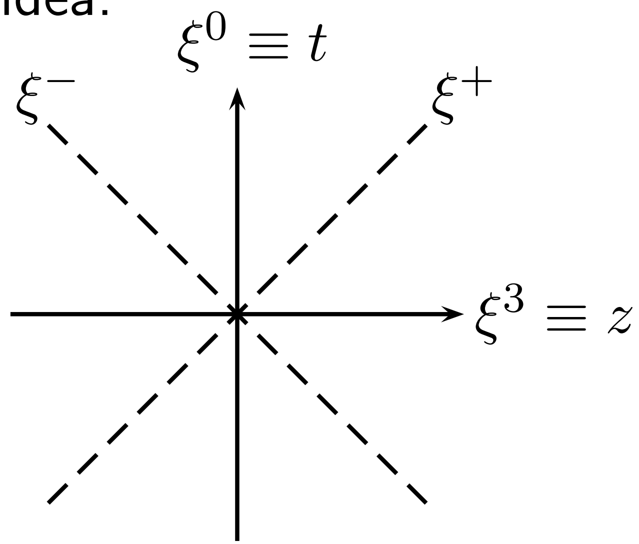
X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



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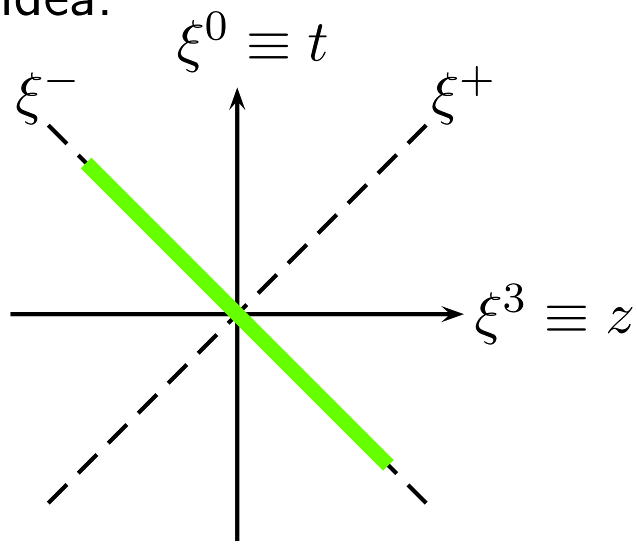

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

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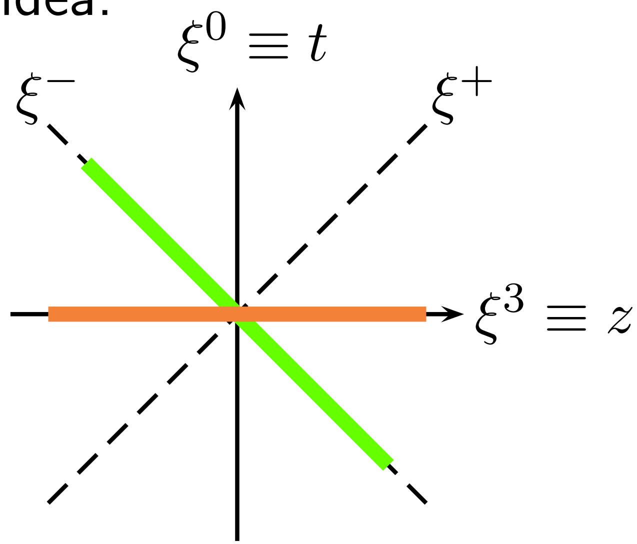

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$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

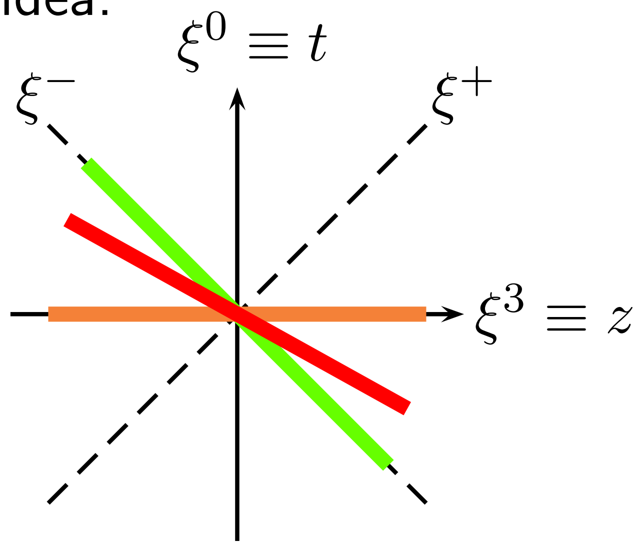
$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



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$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

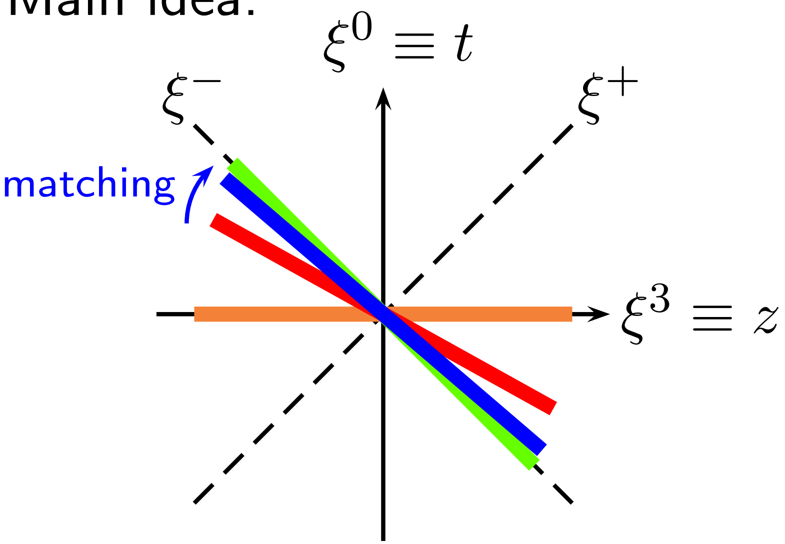
$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

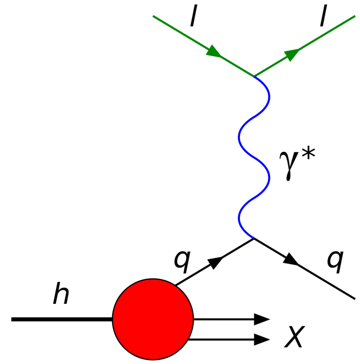
X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

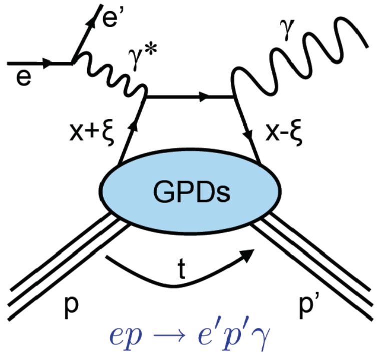
$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1}\frac{dy}{|y|}\,\underbrace{C\left(\frac{x}{y},\frac{\mu}{P_3}\right)}_{\text{pert.kernel}}\,\underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

Generalized parton distributions (GPDs)

- Parton distribution functions (PDFs) – formal definition:
  $f(x,\mu) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle P|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|P\rangle$
- Generalized parton distributions (GPDs):
  $F(x,\xi,t,\mu) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle P''|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|P'\rangle$
  The only difference: **momentum transfer**
  i.e. $P'' \neq P'$ ($P'' = P' + Q$, $t = -Q^2$).



- GPDs reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$
- Moments of GPDs are form factors, e.g. $\int dx H(x,\xi,t) = F_1(t)$
- Experimental access:
  - ⋆ PDFs – **Deep Inelastic Scattering (DIS)** – $ep \longrightarrow eX$
  - ⋆ GPDs – **Deeply Virtual Compton Scattering (DVCS)** – $ep \longrightarrow e'p'\gamma$



Quasi-GPDs: similar procedure to quasi-PDFs
Important new aspect: 2 or 4 GPDs need to be disentangled, e.g. H and E:

$$\mathcal{M}(z,t,\xi;\,\mu_R;\,\Gamma,\overline{\Gamma}) = \mathcal{K}_H(\Gamma,\overline{\Gamma})H(z,t,\xi;\mu_R) + \mathcal{K}_E(\Gamma,\overline{\Gamma})E(z,t,\xi;\mu_R).$$

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$

$\vec{P}' = \vec{P} + \vec{Q}$, $\quad \vec{Q}$ – momentum transfer

lattice computation of bare ME

most costly part of the procedure!

needs several $\vec{Q}$ vectors

↓

renormalization

intermediate RI scheme

conversion to $\mathrm{M\overline{MS}}$ scheme

(incl. evolution to $\mu = 2$ GeV)

logarithmic and power divergences in bare matrix elements

also: one needs to disentangle 2 GPDs types

H-GPDs and E-GPDs (different projectors)

↓

reconstruction of x-dependence

z-space$\to$$x$-space

Backus-Gilbert

non-trivial aspect: reconstruction of a continuous distribution from a finite set of ME ("inverse problem")

↓

matching to light cone

$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

needs a sufficiently large momentum

valid up to higher-twist effects

↓

light-cone GPD

the final desired object!

Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L = 3$ fm,
- $m_\pi \approx 260$ MeV.

P_3	P_3 [GeV]	N_{meas}
$4\pi/L$	0.83	4152
$6\pi/L$	1.25	42080
$8\pi/L$	1.67	112192

ETMC, Phys. Rev. Lett. 125 (2020) 262001
ETMC, Phys. Rev. D105 (2022) 034501
S. Bhattacharya et al., 2112.05538

Always: $u - d$ flavor combination

Kinematics:

- three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2,
- nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV,
- momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2.

Bare matrix elements

Lattice matrix elements need to be computed with 2 different projections (unpolarized/polarized).
Below for the unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M\overline{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space$\to$$x$-space
Backus-Gilbert

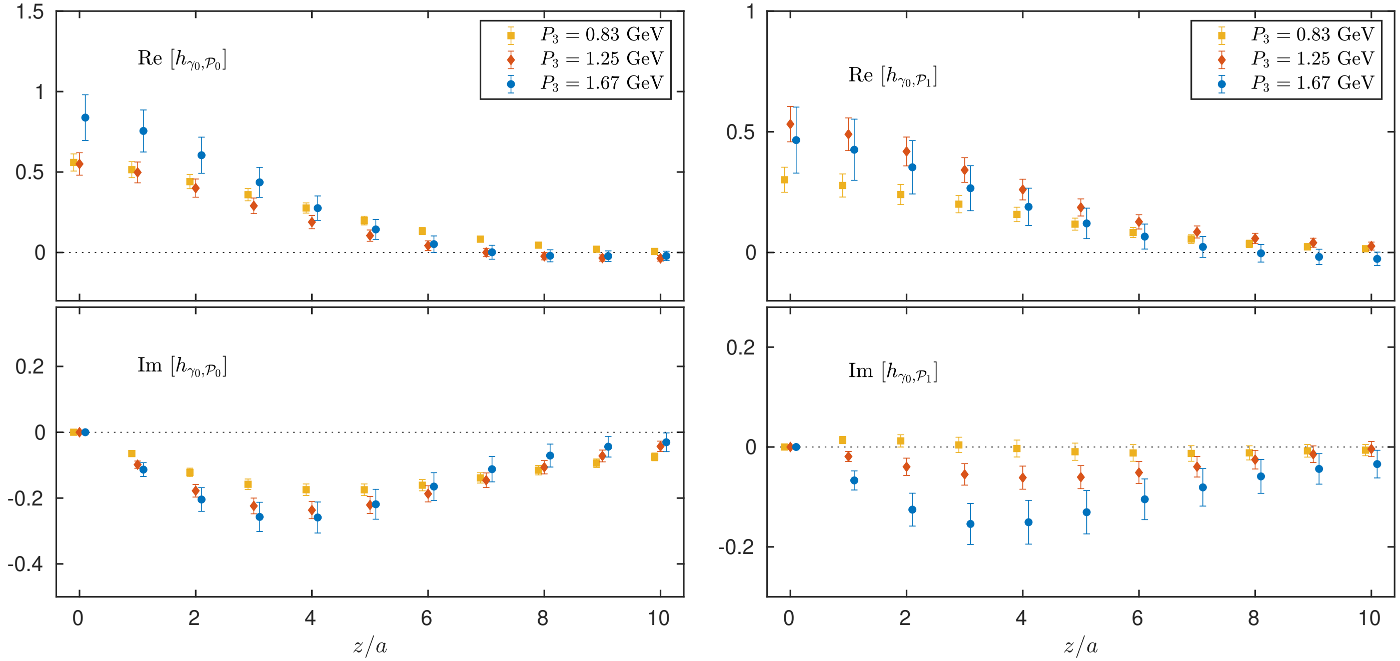
↓

matching to light cone
$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

↓

light-cone GPD

unpolarized projector | polarized projector



Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Disentangled renormalized matrix elements

Removal of divergences and disentangling of H- and E-GPDs.
Unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
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(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space$\to$$x$-space
Backus-Gilbert

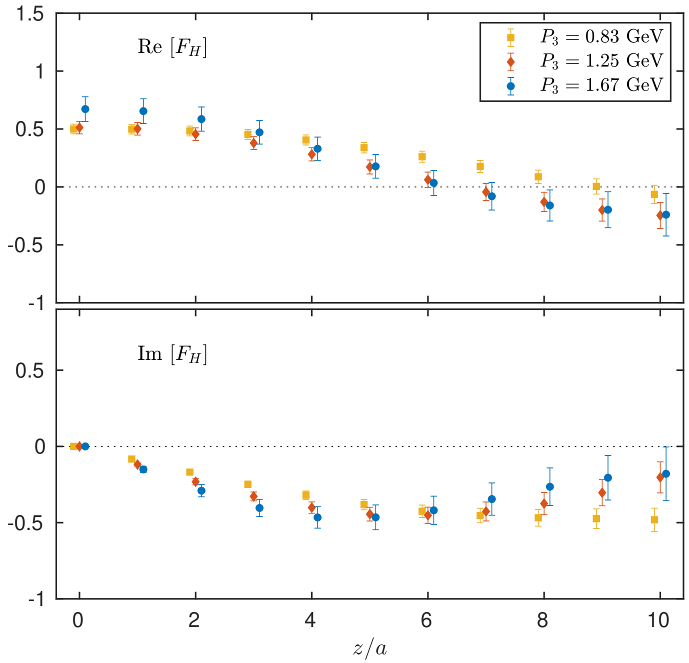
↓

matching to light cone
$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

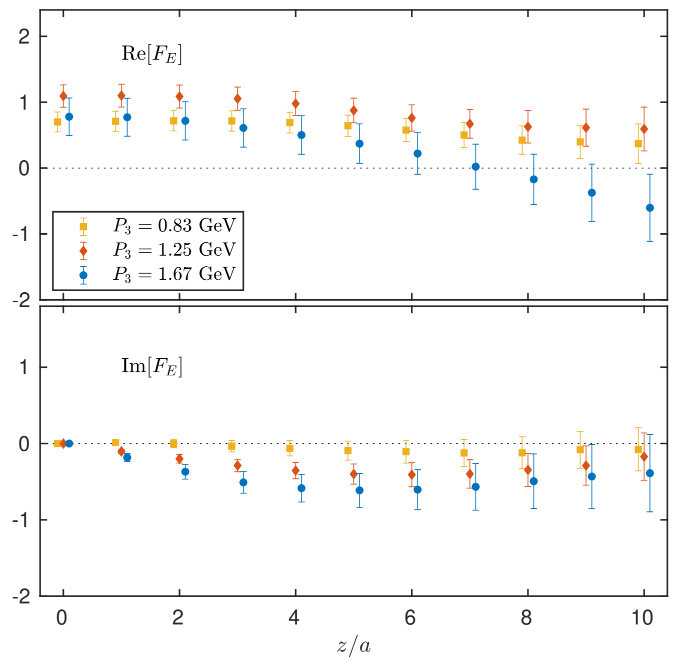
↓

light-cone GPD

ME of H-function



ME of E-function



Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Reconstruction of x-dependence and matching to light cone.
Unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M\overline{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

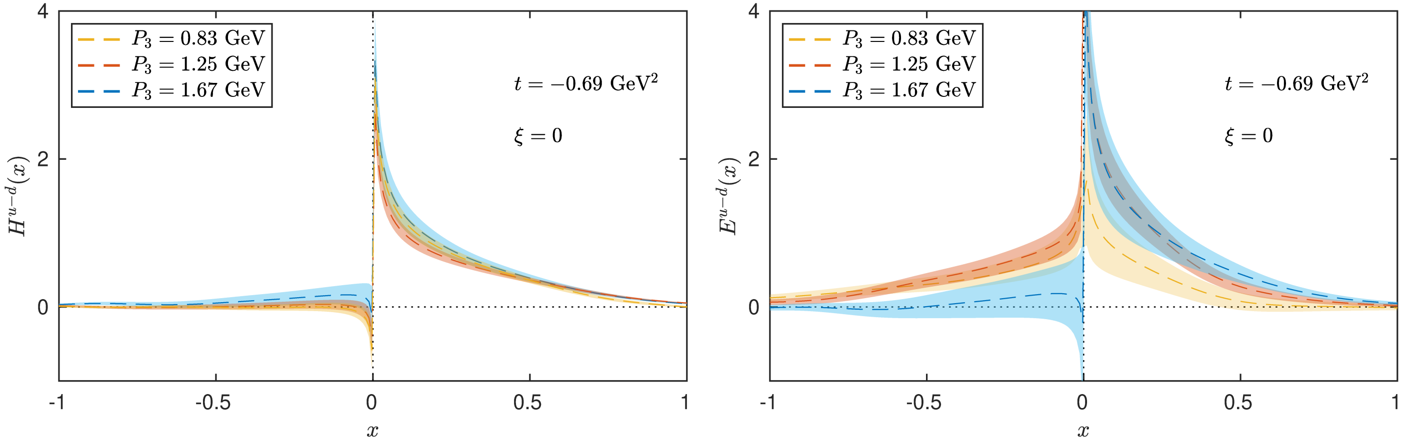
↓

matching to light cone
$\mathrm{M\overline{MS}} \rightarrow \overline{\mathrm{MS}}$

↓

light-cone GPD

H-GPD E-GPD



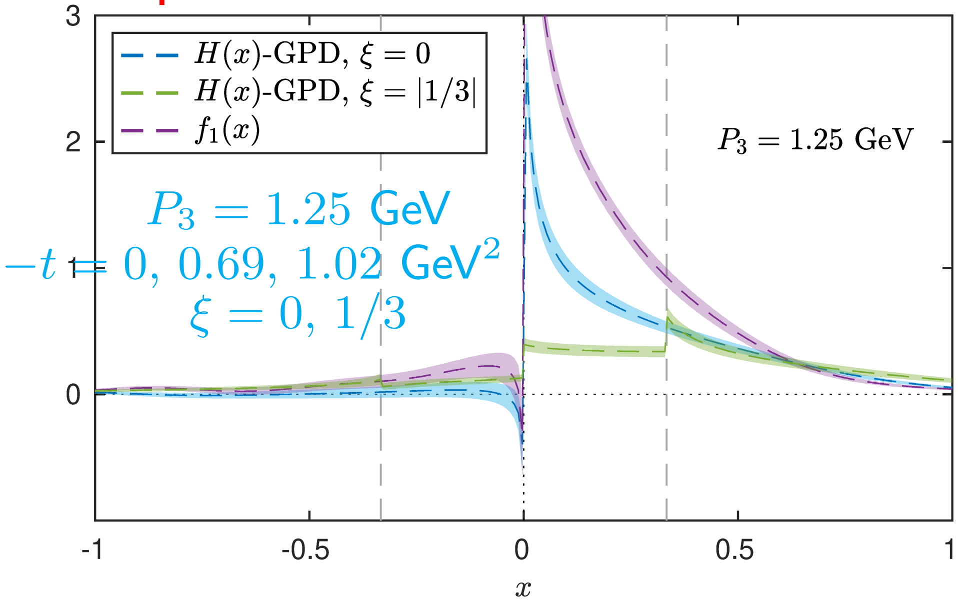
Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



Important insights from models:

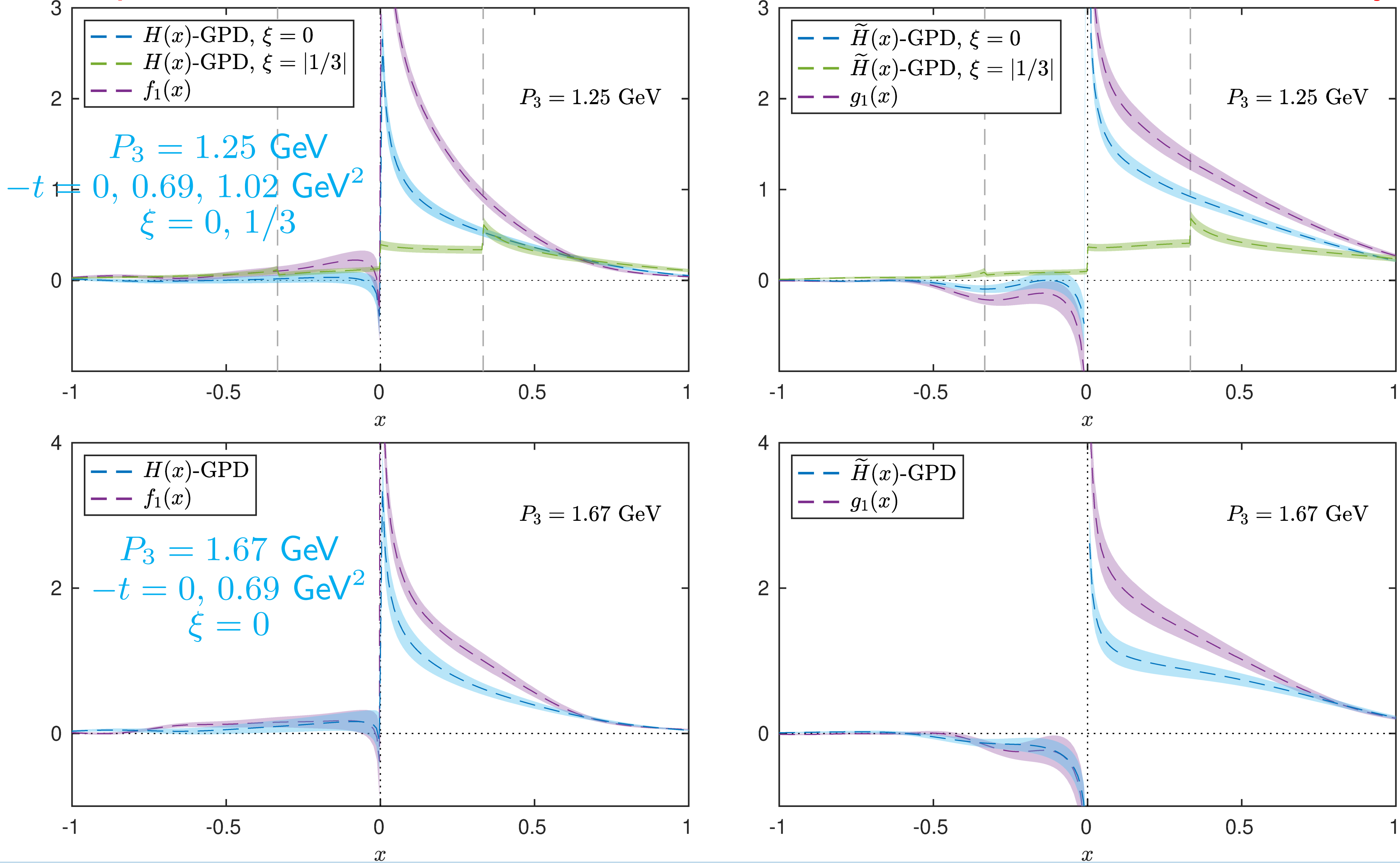
S. Bhattacharya, C. Cocuzza, A. Metz
Phys. Lett. B788 (2019) 453
Phys. Rev. D102 (2020) 054201

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity



Transversity GPDs

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
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(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

matching to light cone
$\overline{\rm MMS} \to \overline{\rm MS}$

↓

light-cone GPD

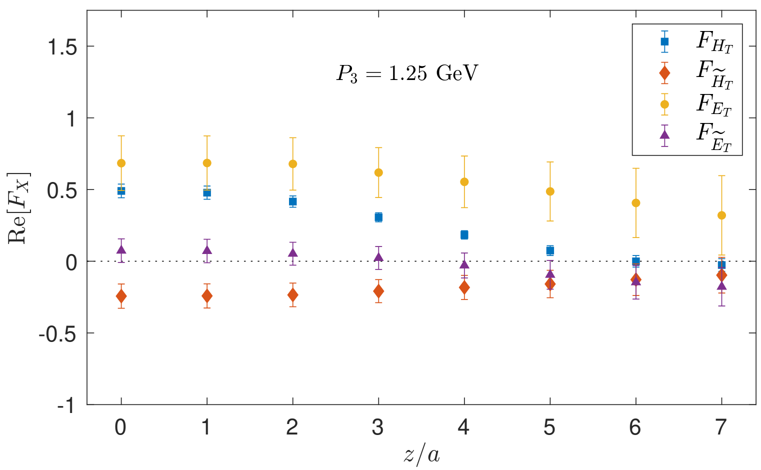
Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

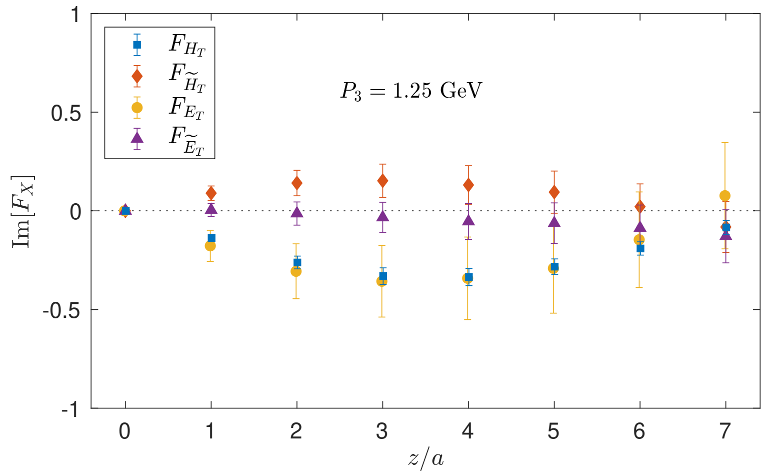
Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2

Renormalized ME

Real part | Imaginary part

$\xi = 1/3$





Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
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renormalization
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reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

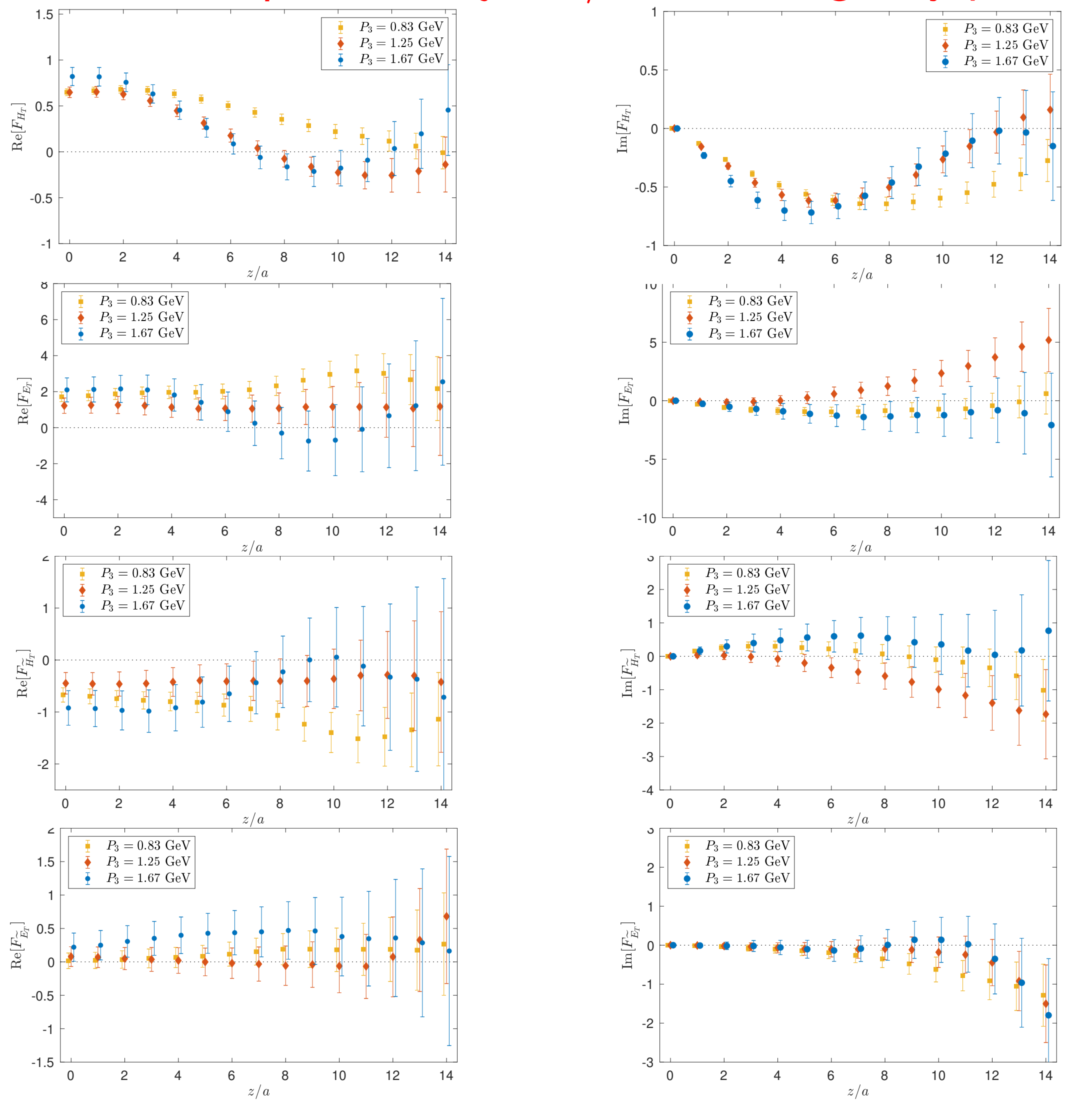
↓

matching to light cone
$\mathrm{M\overline{MS}} \rightarrow \overline{\mathrm{MS}}$

↓

light-cone GPD

Real part $\qquad \xi = 1/3 \qquad$ Imaginary part



ETMC, Phys. Rev. D105 (2022) 034501

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M\overline{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

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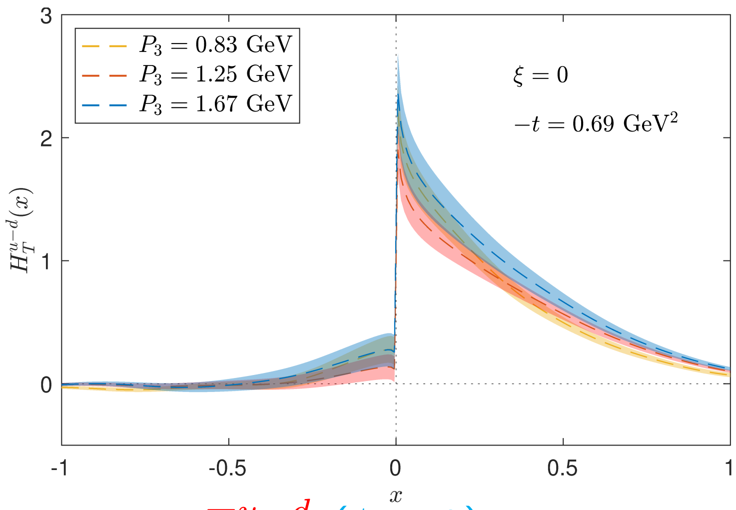
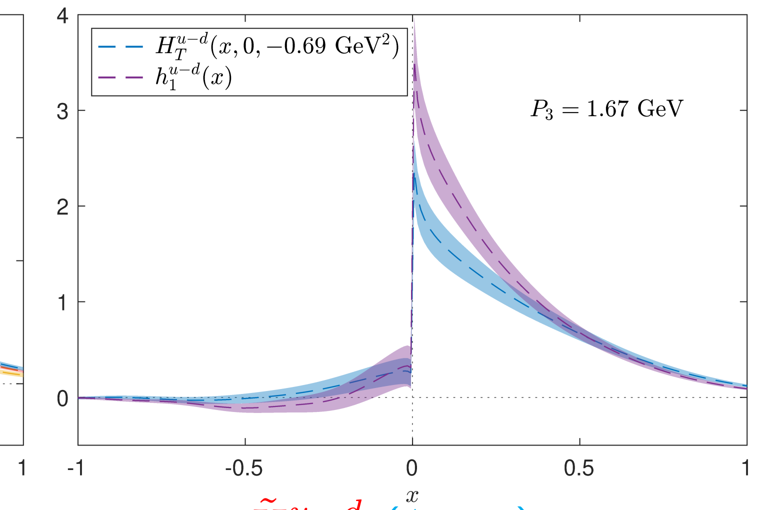
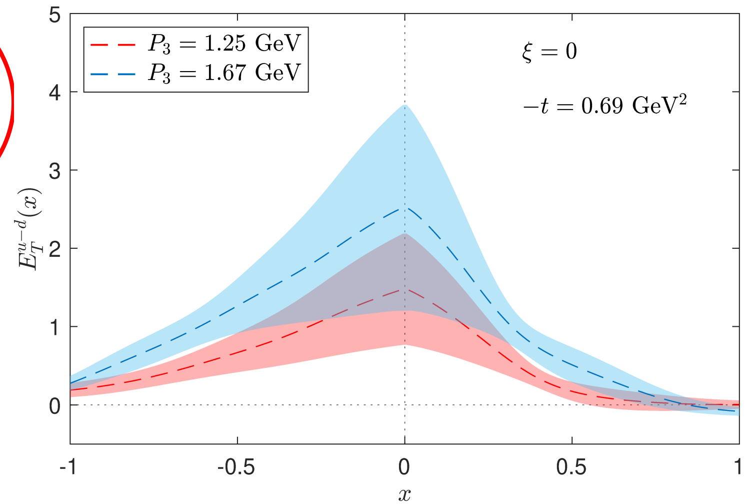
H_T^{u-d} $(\xi = 0)$

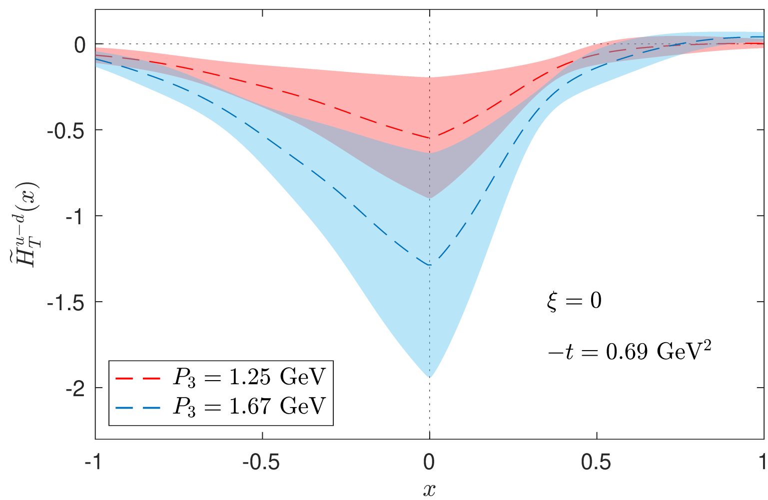




E_T^{u-d} $(\xi = 0)$



$\tilde{H}_T^{u-d}$ $(\xi = 0)$



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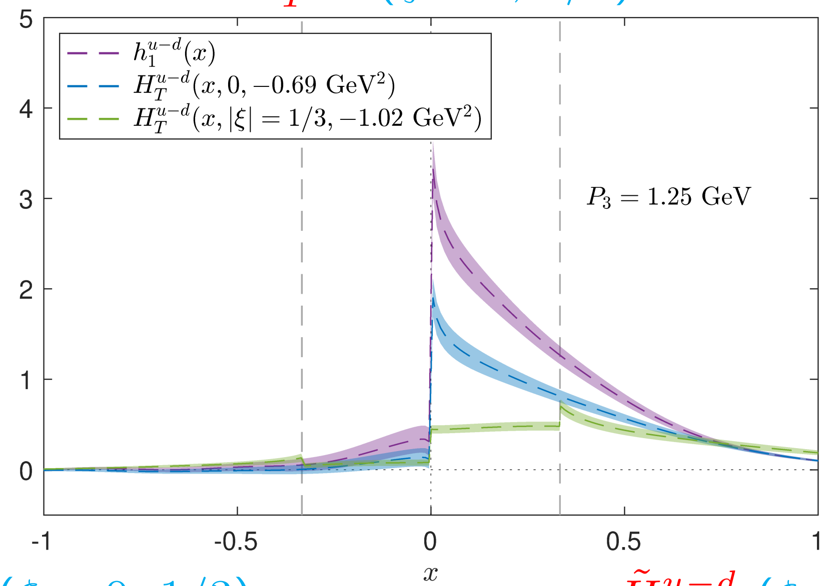
light-cone GPD

ETMC, Phys. Rev. D105 (2022) 034501

H_T^{u-d} $(\xi = 0,\ 1/3)$



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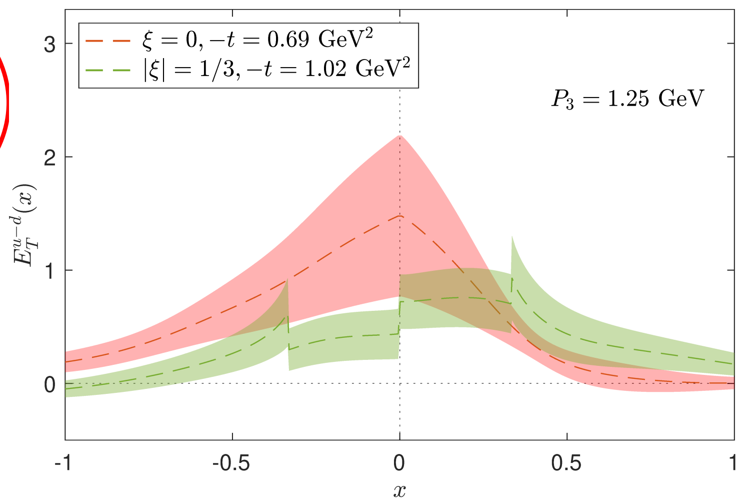
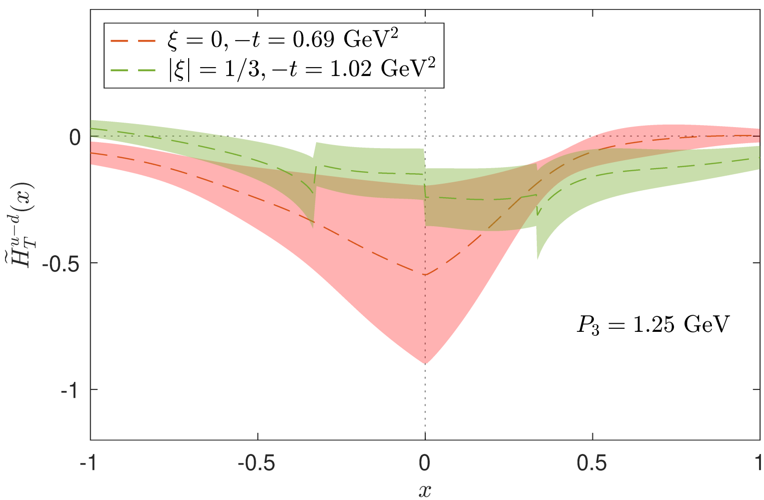


$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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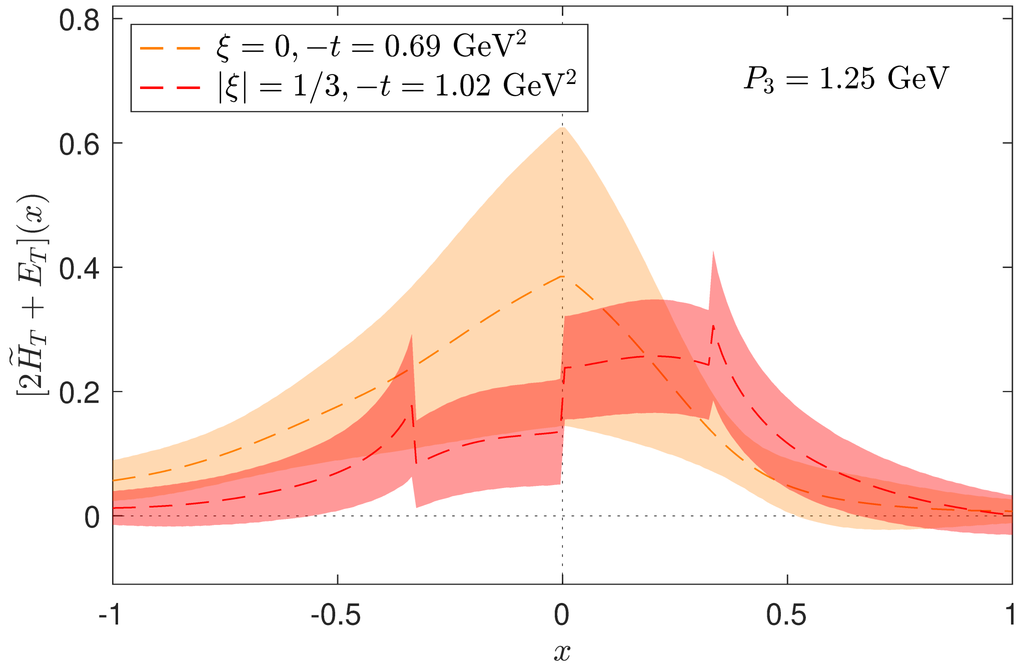
↓

light-cone GPD

More fundamental quantity: $E_T + 2\tilde{H}_T$

– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target (k_T)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton



Introduction
Results
Setup
Bare ME
Renorm ME
Matched GPDs
Transversity
Comparison
Twist-3
Summary

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,,\\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,,\\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,,\\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \quad (1)$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,,\\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,,\\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \quad (3)\\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \quad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only ξ-odd GPD).

Moments of transversity GPDs

Moments of	$H_T(x, \xi = 0, t = -0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
H_{Tq}	0.65(4)	0.64(6)	0.81(10)	0.49(5)
H_T	0.69(4)	0.67(6)	0.84(10)	0.45(4)
xH_T	0.20(2)	0.21(2)	0.24(3)	0.15(2)
A_{T10} $(z = 0)$	0.65(4)	0.65(6)	0.82(10)	0.49(5)

Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of	$E_T(x, \xi = 0, t = -0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
E_{Tq}		1.20(42)	2.05(65)	0.67(19)
E_T		1.15(43)	2.10(67)	0.73(19)
xE_T		0.06(4)	0.13(5)	0.11(11)
B_{T10} $(z = 0)$	1.71(28)	1.22(43)	2.10(67)	0.68(19)

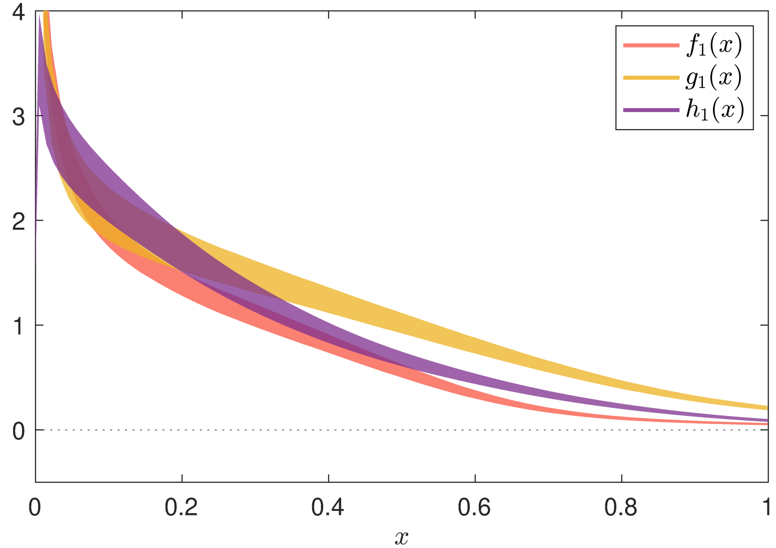
Moments of	$\widetilde{H}_T(x, \xi = 0, t = -0.69\,\mathrm{GeV}^2)$			$\widetilde{H}_T(x, \xi = 1/3, t = -1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
$\widetilde{H}_{Tq}$		-0.44(20)	-0.90(32)	-0.26(9)
$\widetilde{H}_T$		-0.42(21)	-0.92(33)	-0.27(9)
$x\widetilde{H}_T$		-0.17(8)	-0.30(10)	-0.05(5)
$\widetilde{A}_{T10}$ $(z = 0)$	-0.67(14)	-0.45(21)	-0.92(33)	-0.24(8)

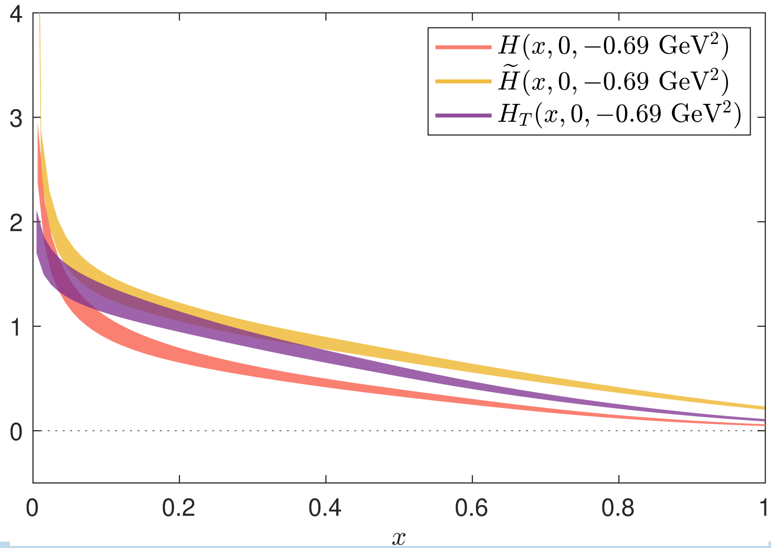
Similar conclusions (but very large errors).

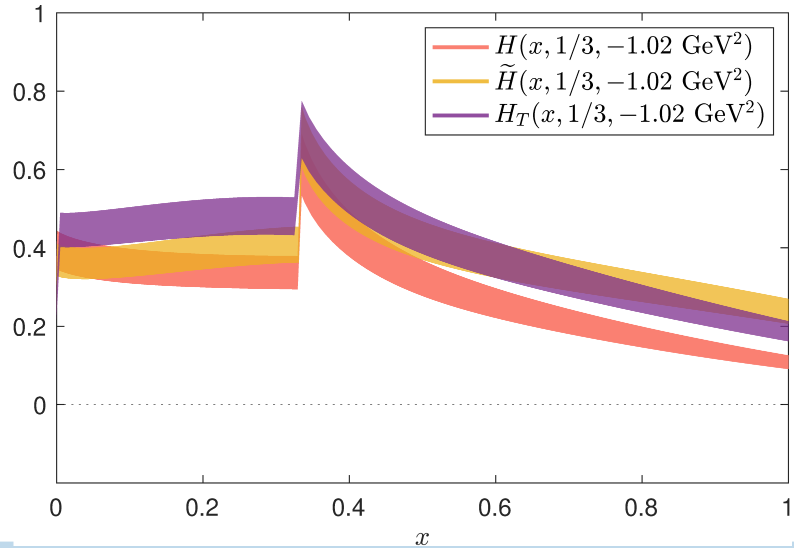
Comparison of PDFs and GPDs

ETMC, Phys. Rev. Lett. 125 (2020) 262001

ETMC, Phys. Rev. D105 (2022) 034501







Twist-3

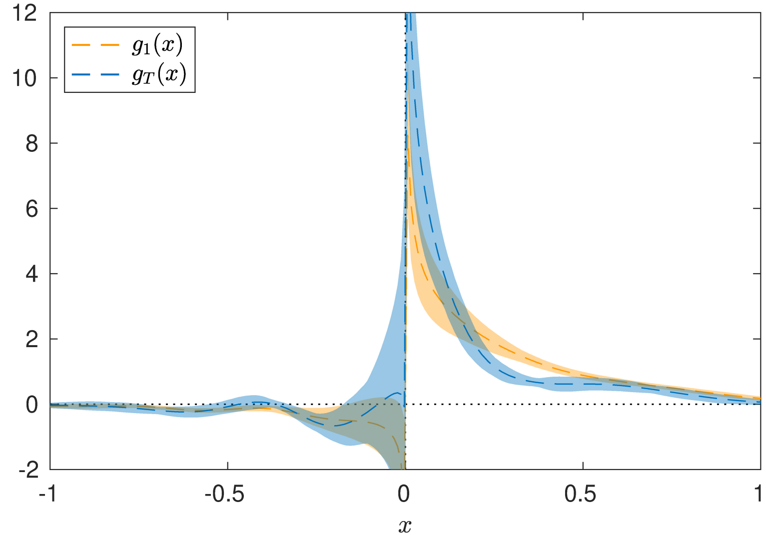
PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.



Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
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  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282
  Note: neglected qgq correlations
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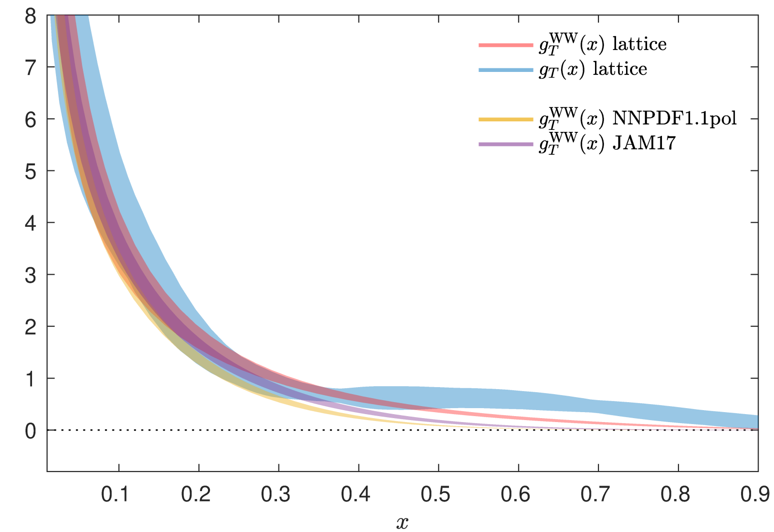

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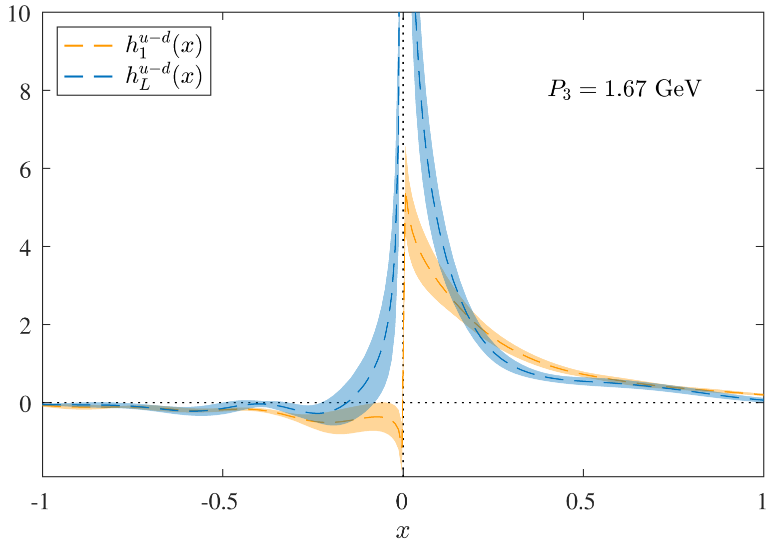

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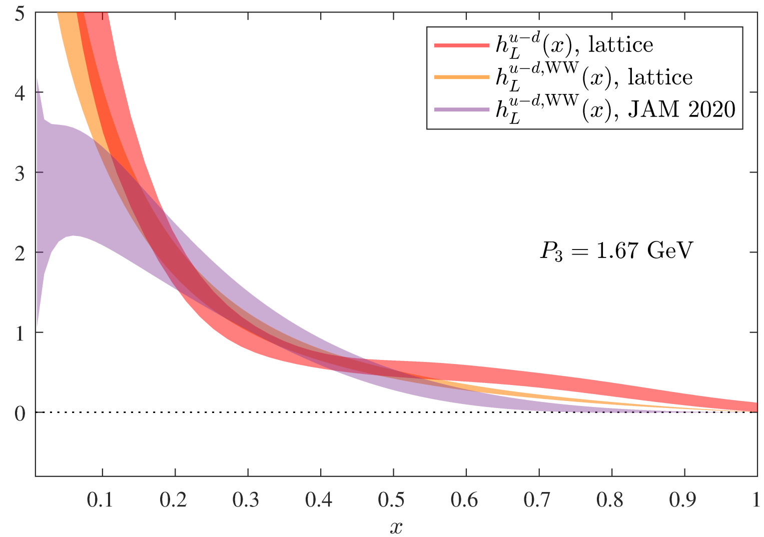
  S. Bhattacharya et al., 2107.02574 (PRD in press)
- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2112.05538



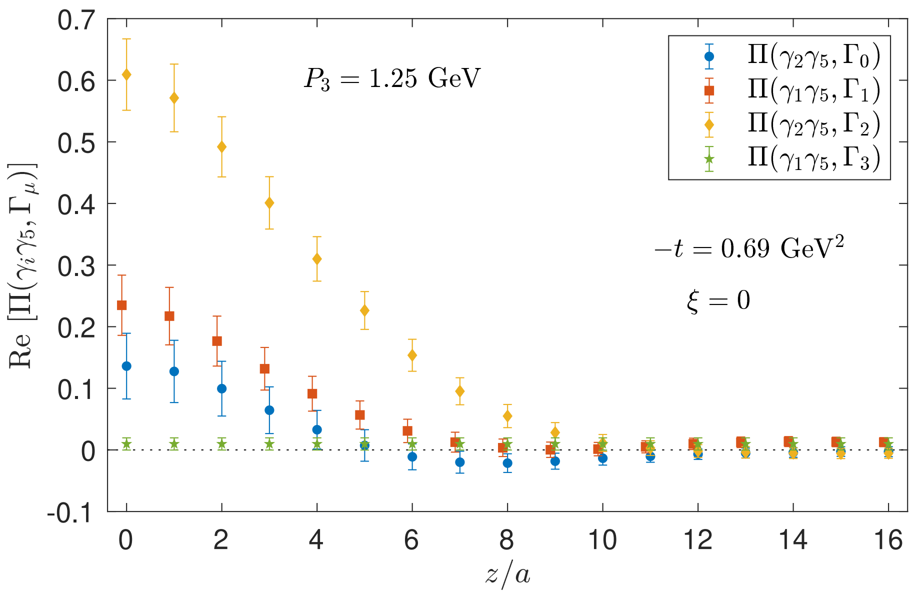
First exploration of twist-3 GPDs

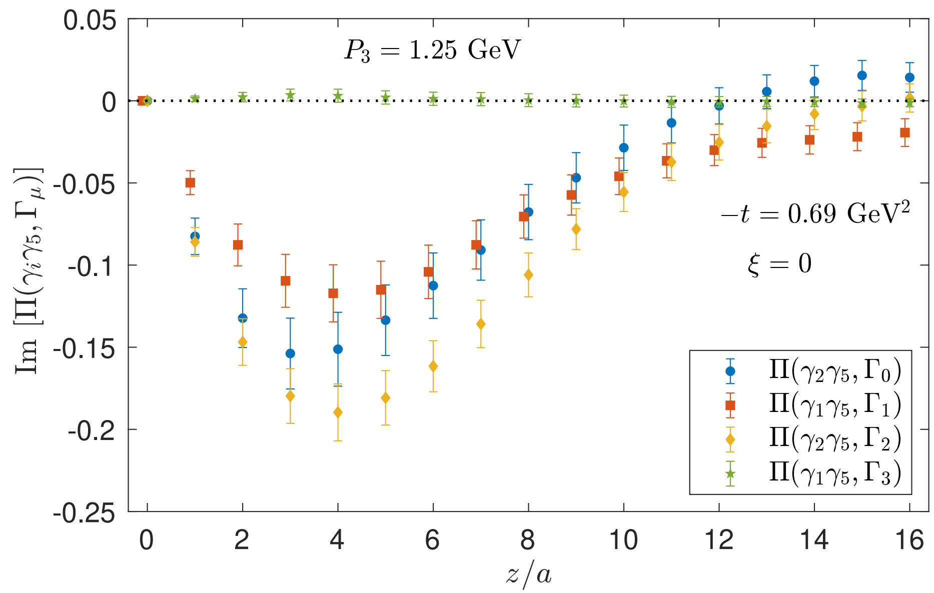
Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., 2112.05538

Twist-3 axial GPDs: $\widetilde{G}_1,\ \widetilde{G}_2,\ \widetilde{G}_3,\ \widetilde{G}_4$

$$h_{\gamma^j\gamma_5} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma_5}{2m}\rangle\rangle[F_{\widetilde{E}} + F_{\widetilde{G}_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho\gamma_5\rangle\rangle[F_{\widetilde{H}} + F_{\widetilde{G}_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+}\rangle\rangle F_{\widetilde{G}_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+}\rangle\rangle F_{\widetilde{G}_4}\,.$$

Bare ME: (same lattice setup)

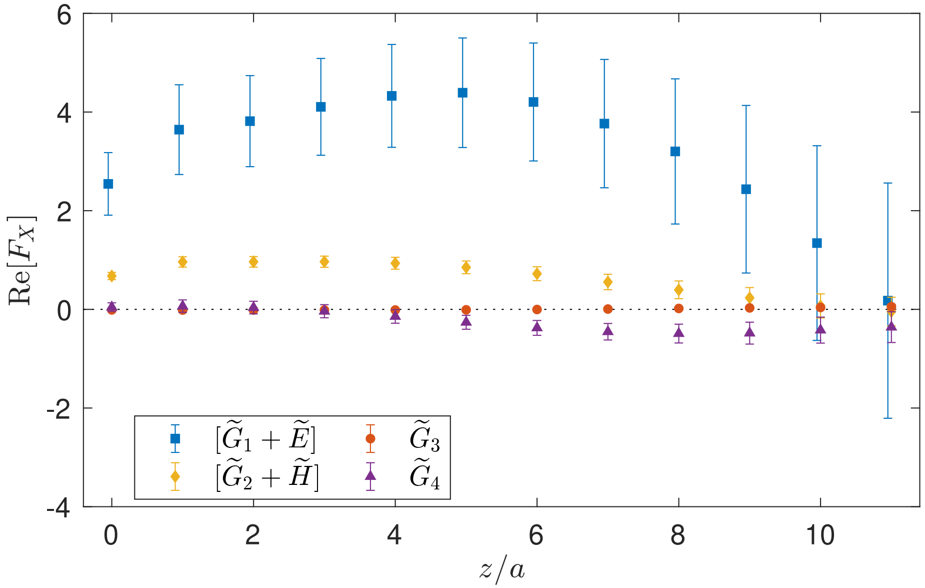


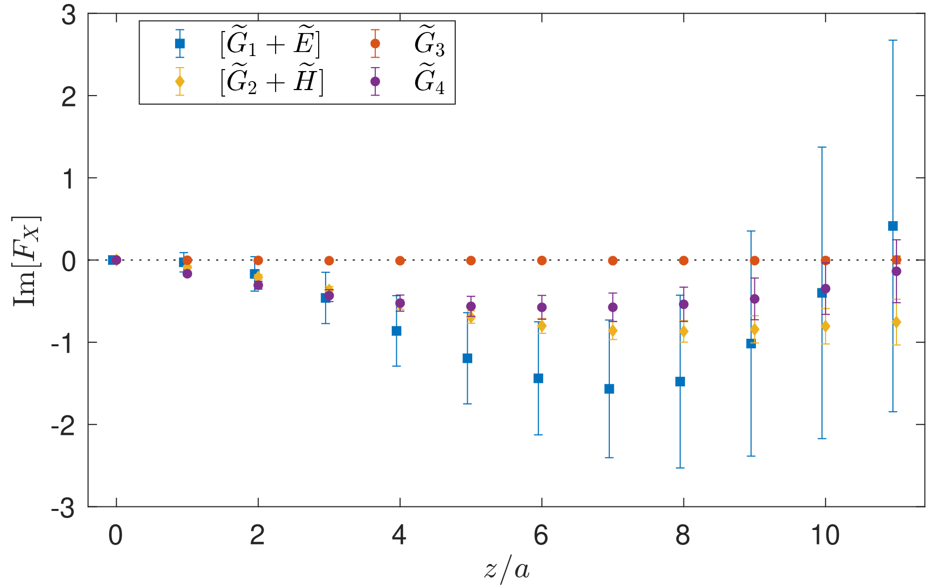


Contributions from different insertions and projectors ($\vec{Q} = (Q_x, 0, 0)$):

$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

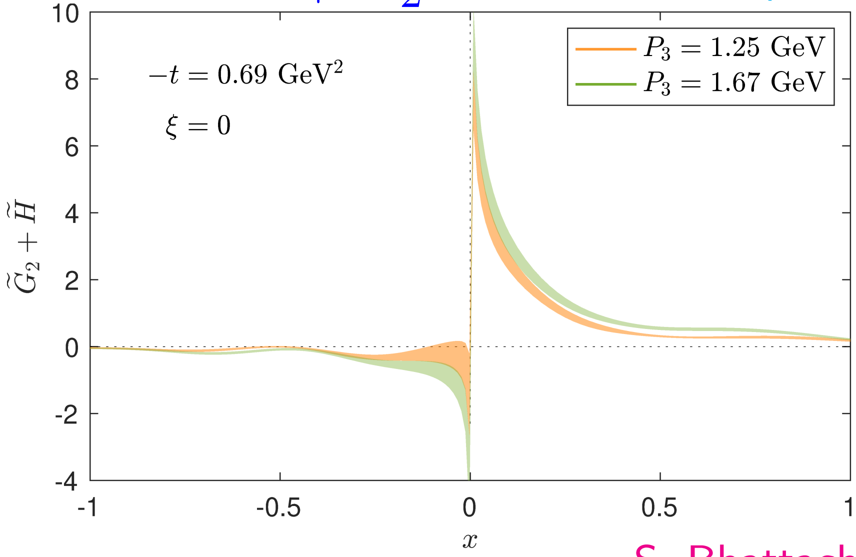
S. Bhattacharya et al., 2112.05538

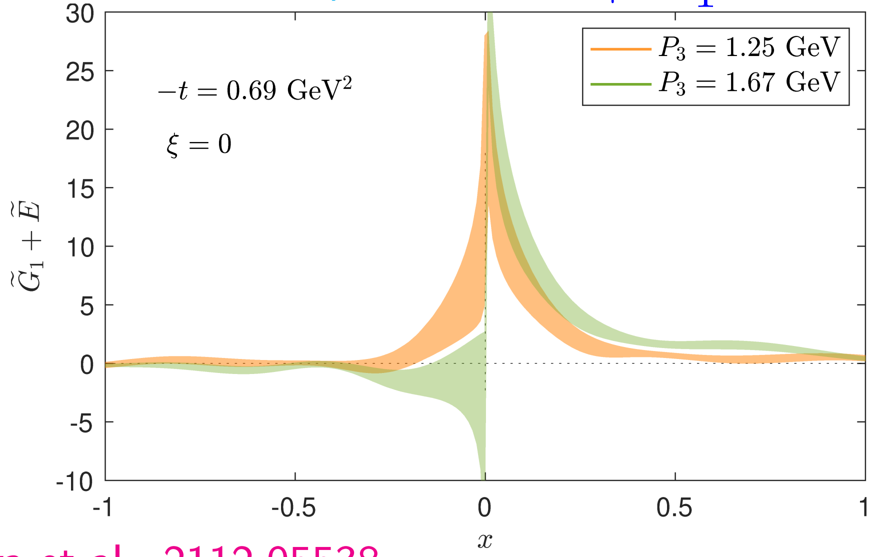




First exploration of twist-3 GPDs

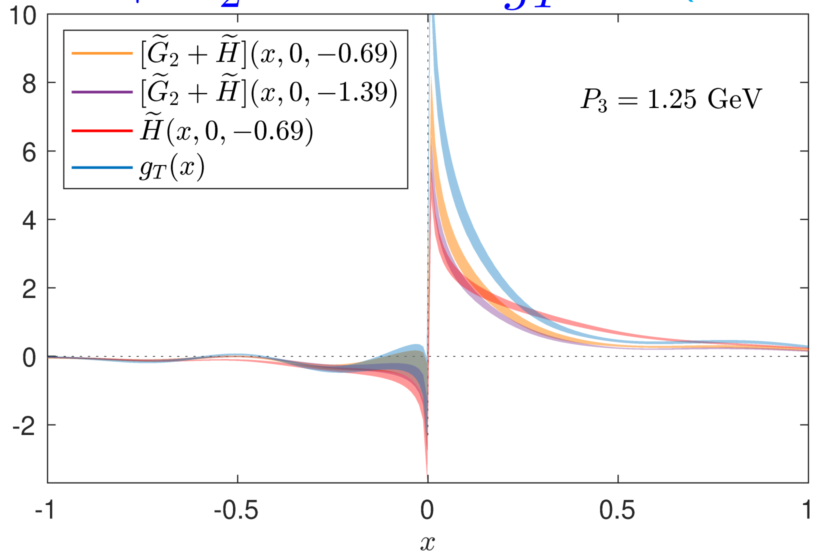
$\widetilde{H}+\widetilde{G}_2$ ($\xi = 0, -t = 0.69$ GeV^2) $\widetilde{E}+\widetilde{G}_1$

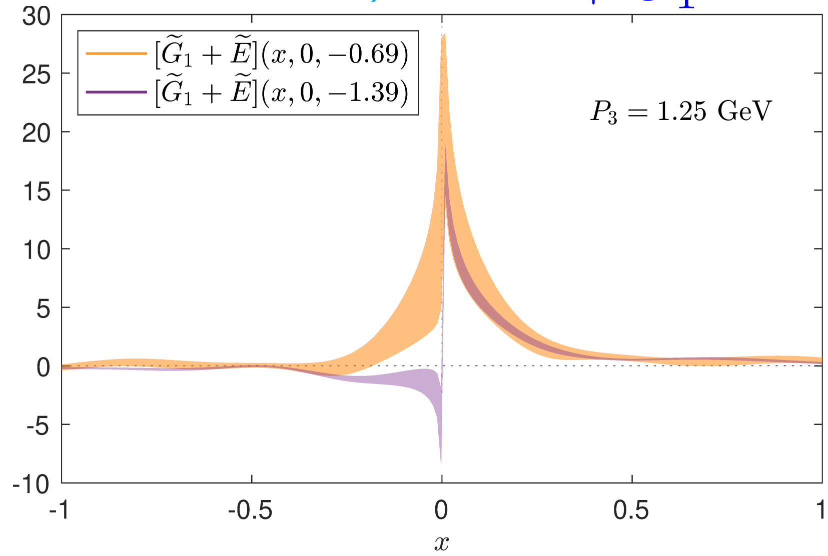




S. Bhattacharya et al., 2112.05538

$\widetilde{H}+\widetilde{G}_2$ vs. $\widetilde{H}$ vs. g_T ($\xi = 0, -t = 0.69, 1.39$ GeV^2) $\widetilde{E}+\widetilde{G}_1$





Conclusions and prospects

- **Enormous progress in lattice calculations of GPDs!**
- Very encouraging results, but there are still major challenges related to control of several sources of systematics.
- GPDs much more challenging than PDFs:
  - ⋆ signal decays with increasing $-t$,
  - ⋆ separate calculations for different $-t$,
  - ⋆ discreteness of $-t$,
  - ⋆ several projectors needed to disentangle GPDs,
  - ⋆ non-zero skewness – enhanced power corrections at the ERBL-DGLAP boundary.
- Expect:
  - ⋆ slow, but consistent progress,
  - ⋆ complementary role of LQCD and phenomenology.

Thank you for your attention!