Hadron spin structure in lattice QCD

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Extended Twisted Mass Collaboration

STIMULATE European Joint Doctorates

CFNS Workshop: High Luminosity-EIC (EIC-Phase II)
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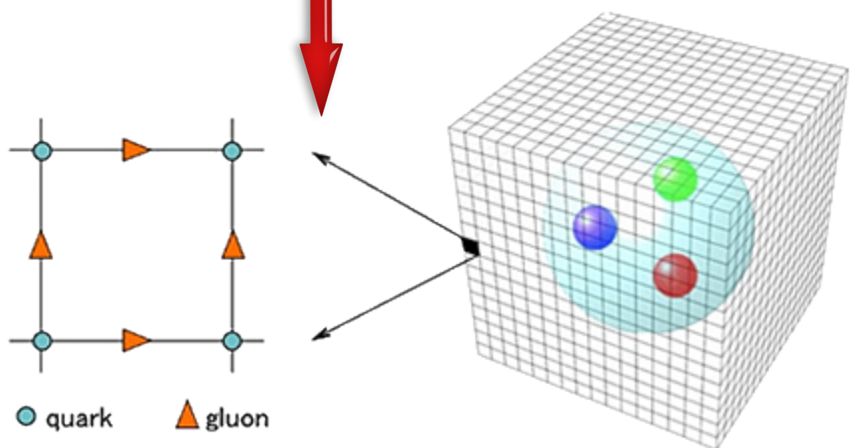
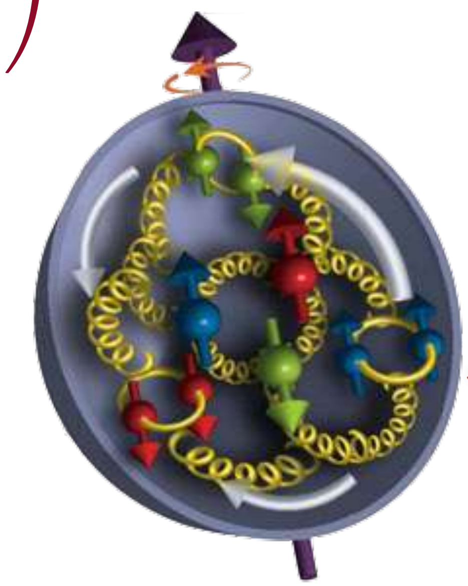
Outline

- Introduction
- Spin structure of the nucleon using one gauge ensemble - momentum and spin sums (also momentum sum of pion) and preliminary result in the continuum limit
- Mellin moments of pion and kaon
- Quasi-parton distributions in large momentum effective theory (LaMET) - next talks in this morning session
- Conclusions

Extraction of matrix elements in lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$

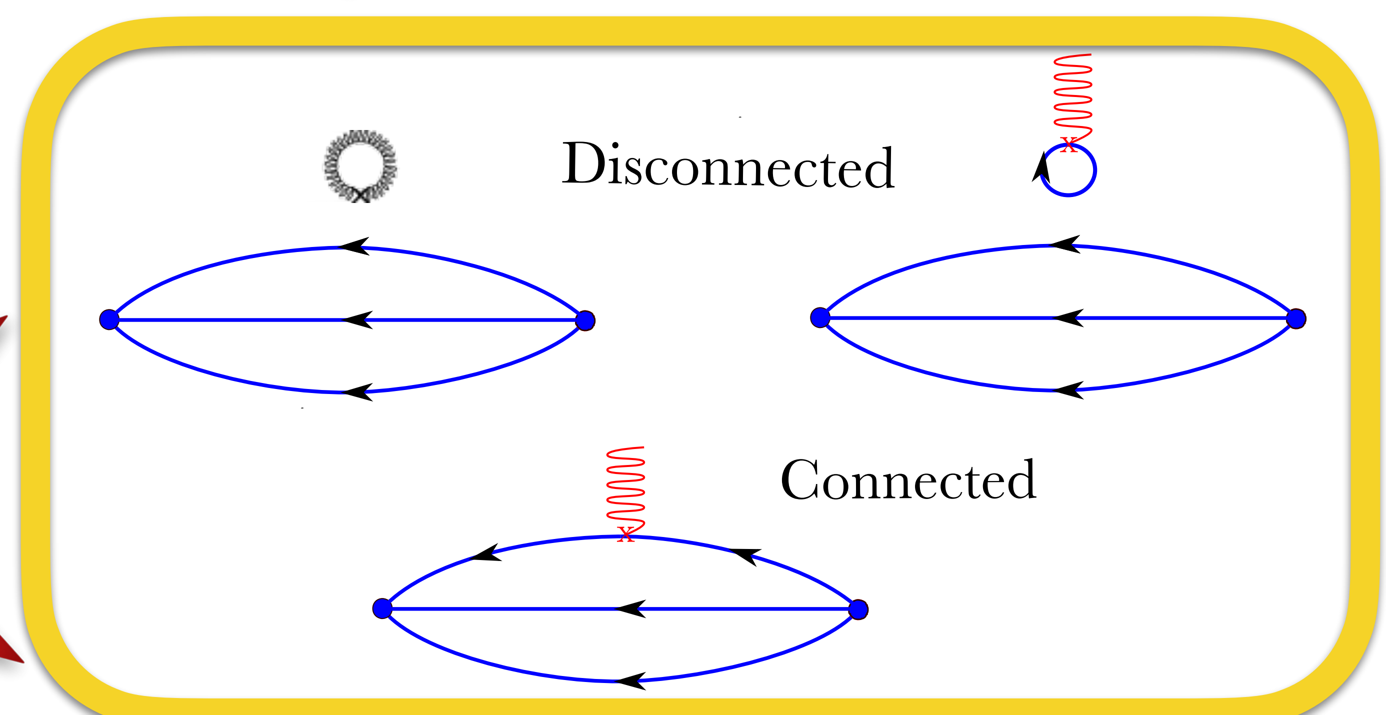




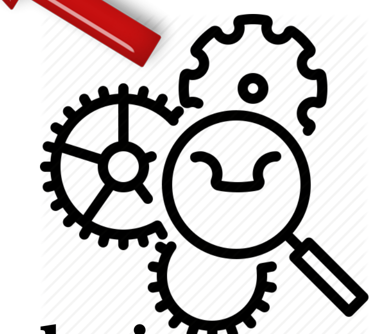
Simulation of gauge configurations U

Quark propagators

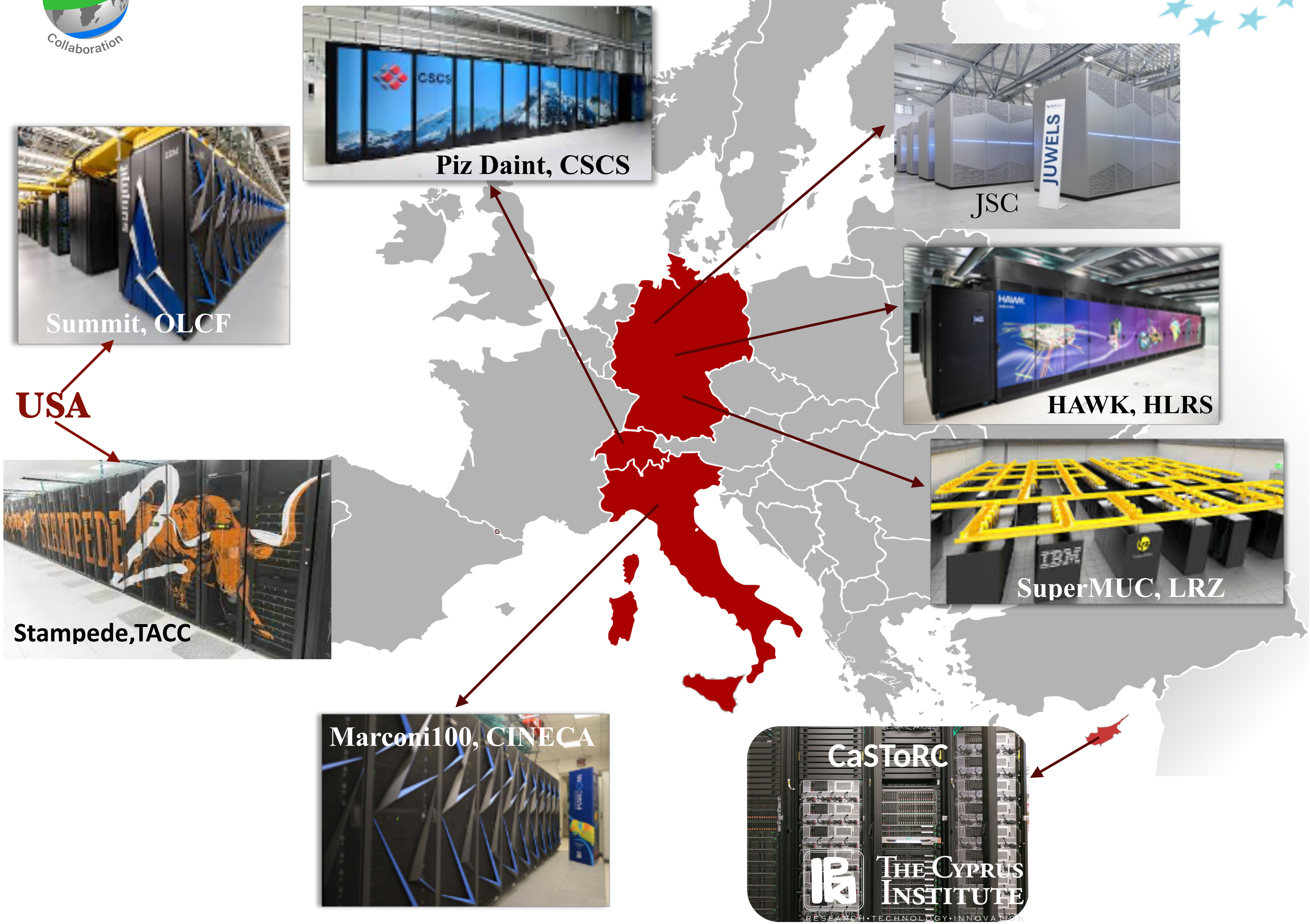
contractions



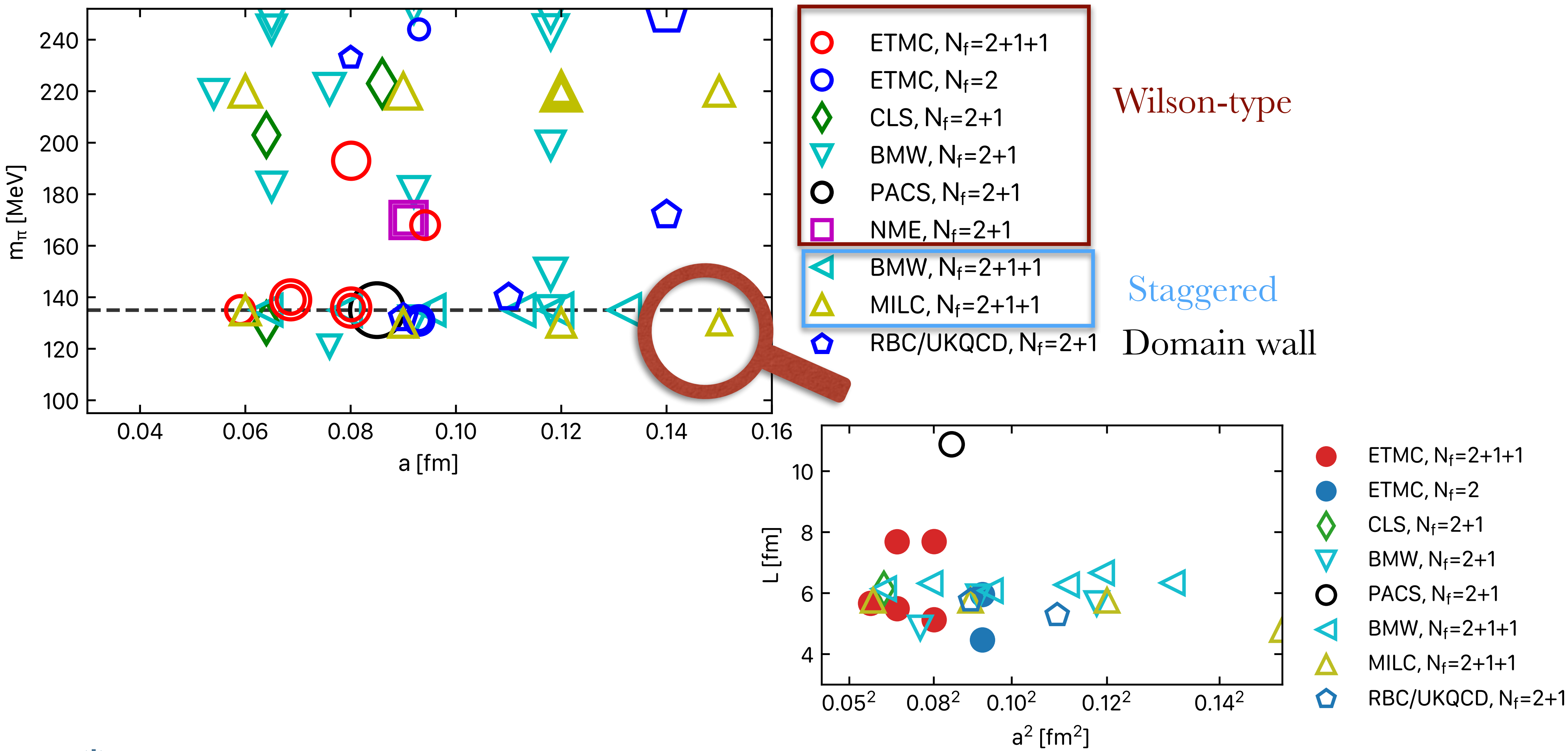
Data Analysis



Computational resources



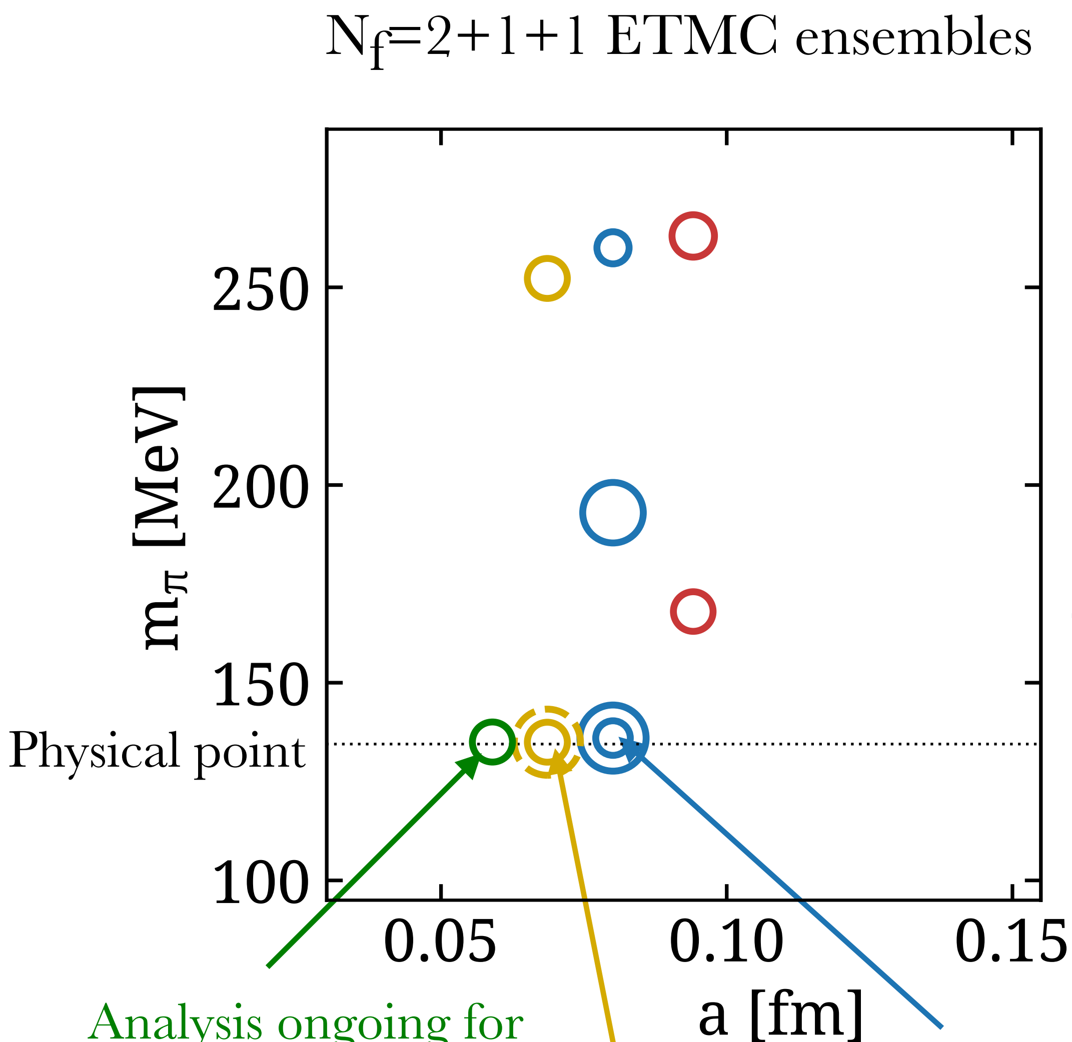
Status of current simulations



❋ A number of collaborations has physical point ensembles:

- Wilson-type: BMW, ETMC, CLS, PACS
    - BMW and ETMC have multiple lattice spacings 0.05<a<0.1 fm
    - PACS has a large volume ensemble at a single lattice spacing
- Staggered at physical point: MILC with 4 and BMW with 6 lattice spacings 0.05<a<0.15 fm
- Domain wall at physical point: RBC/UKQCD with 2 lattice spacings

Gauge ensembles generated by ETMC



Analysis ongoing for 96^3x192, a~0.06 fm

- Analysis ongoing for 80^3x160, a=0.07 fm
- Simulation ongoing for 112^3x224, a=0.07 fm

- Analysis completed for 64^3x128 a=0.08 fm
- Analysis ongoing for 96^3x192 a=0.08 fm

- 5 ensembles at physical pion mass
  - 3 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
  - Two volumes at a=0.08 fm and 0.07 fm of Lm_π~3.6 (5.1 fm) and Lm_π ~5.4 (7.7 fm)

- Algorithmic improvements needed to go to a<0.05 fm due to critical slow down in HMC (long autocorrelations) —> new approaches e.g. Machine learning approaches using equivariant flows

G. Kanwar, et al., Phys. Rev. Lett. 125 (2020) 121601, arXiv:2003.06413; D. Boyda, et al., Phys.Rev.D 103 (2021) 074504, arXiv: 2008.05456; M. S. Albergo *et al.*, *Phys.Rev.D* 104 (2021) 114507, arXiv:2106.05934

C. A. *et al.* (ETMC), "Simulating twisted mass fermions at physical light, strange and charm quark masses" Phys. Rev. D98 (2018) 054518

Systematics & Challenges

- **Simulations directly at the physical point**

  Systematic effects from chiral extrapolation are eliminated

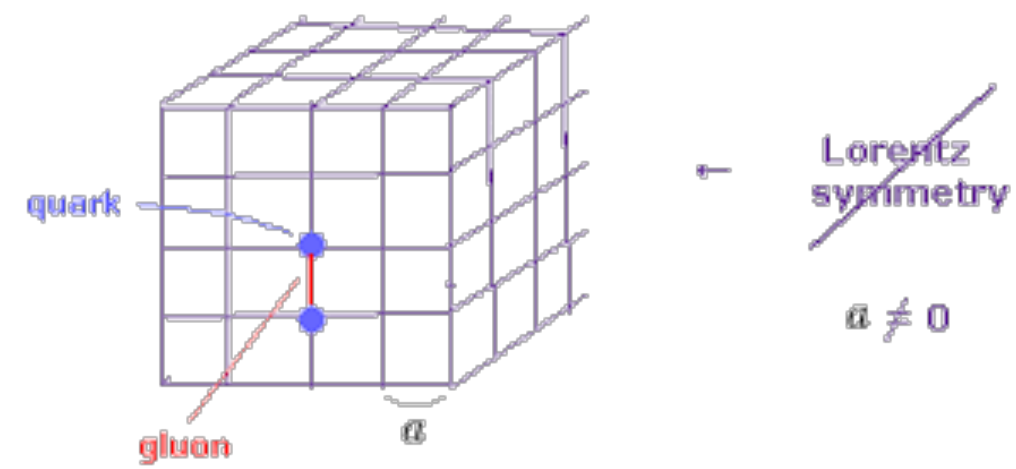
- **Discretisation effect:** Continuum limit

  —> need simulations for at least 3 lattice spacings

- **Finite volume effects:** Infinite volume limit

  —> need simulations for at least 3 volumes

Typically done using simulations for heavier than physical values of the pion mass



- **Ground-state identification**

  Cross-check (one-, two- and three-state fits, summation), but two or more particle states create difficulties

- **Renormalisation**

  Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts - more complicated for extended operators

- **Large momentum limit - important for direct evaluation of GPDs**

  Challenging since statistical errors grow exponentially

- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects**

Spin structure

Energy and momentum tensor (EMT)

Energy and momentum tensor taken to be symmetric

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

X. Ji, PRD 52, 271,1995, hep-ph/9502213

Traceless and trace parts

- **Physical matrix elements of the traceless part can be written in terms of two gauge invariant terms**

$$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g$$

$$\bar{T}^{\mu\nu}_q = \bar{\psi} i\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi\,, \qquad \bar{T}^{\mu\nu}_g = F^{\{\mu\rho}F^{\nu\}}{}_{\rho}$$

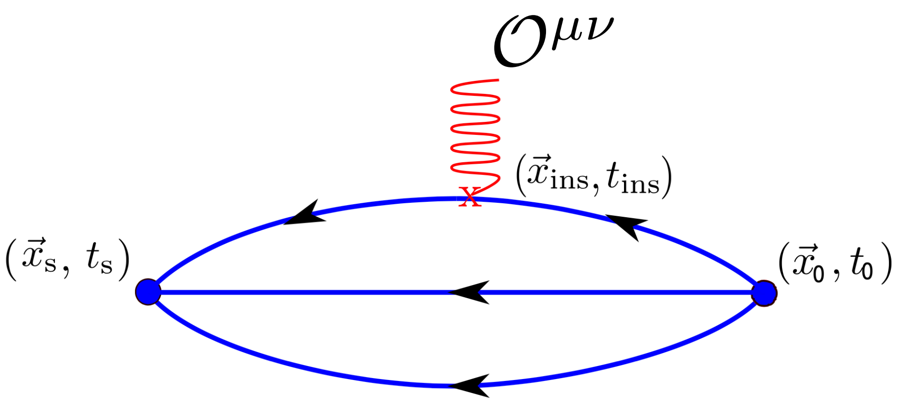
Symmetrisation over indices and subtraction of trace

Scheme and scale dependent

Related to quark and gluon total angular momentum

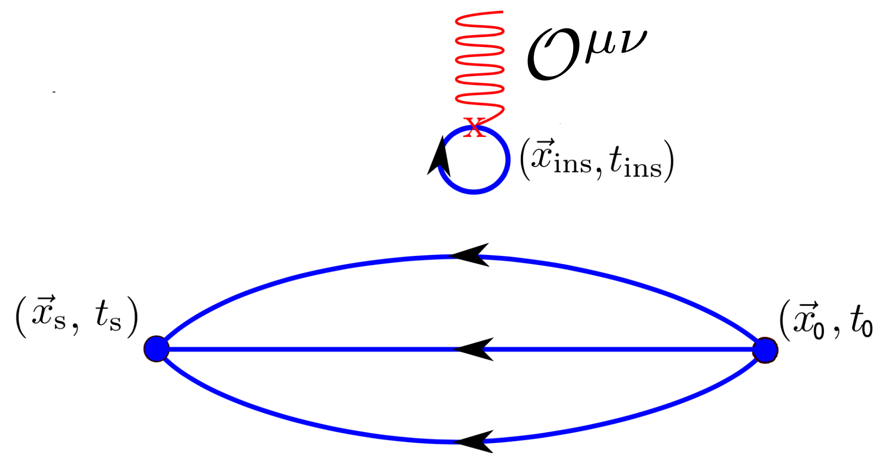
Nucleon matrix elements

$$C^{\mu\nu}_{3\mathrm{pt}}(\Gamma;\vec{q}=0,t_s,t_{\mathrm{ins}})=\sum_{\vec{x}_{\mathrm{ins}},\vec{x}_s}\mathrm{Tr}\left[\langle \Gamma J_N(t_s,\vec{x}_s)\mathcal{O}^{\mu\nu}(t_{\mathrm{ins}},\vec{x}_{\mathrm{ins}})\bar{J}_N(t_0,\vec{x}_0)\rangle\right]$$



connected

$+$



disconnected

❋ Identification of nucleon matrix element *M (t_0=0)*

Plateau and two-state fit:

$$R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\mathrm{ins}})=\frac{C^{\mu\nu}_{3\mathrm{pt}}(t_s,t_{\mathrm{ins}})}{C_{2\mathrm{pt}}(\Gamma_0,t_s)}\longrightarrow\boxed{\mathcal{M}}+\mathcal{O}(e^{-\Delta E(t_s-t_{\mathrm{ins}})})+\mathcal{O}(e^{-\Delta E t_{\mathrm{ins}}})$$

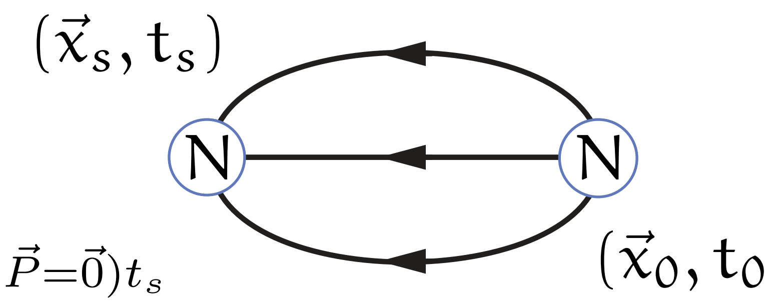
Included in the two-state fit

Summation:

$$\sum_{t_{\mathrm{ins}}=a}^{t_s-a}R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\mathrm{ins}})\longrightarrow c+\boxed{\mathcal{M}}t_s+\mathcal{O}(e^{-\Delta E t_s})$$

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys., B293: 420, 1987

Nucleon propagator

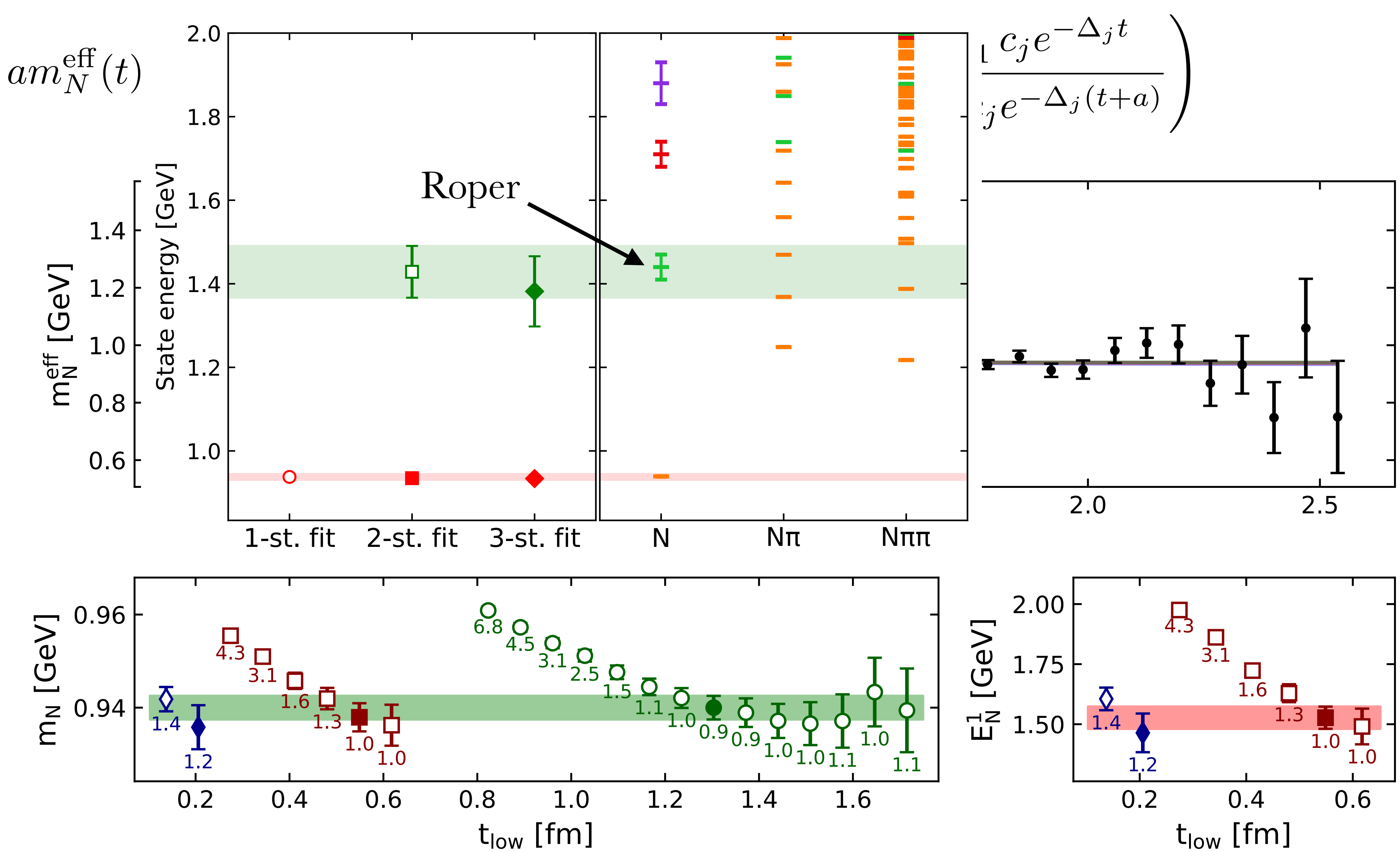


Analysis of two- and three-point functions C_{2pt} and C_{3pt}

$$C_{2\mathrm{pt}}(\Gamma_0; \vec{P}=\vec{0}, t_s) = \sum_{\vec{x}_s} \mathrm{Tr}\left[\langle \Gamma_0 J_N(t_s, \vec{x}_s) \bar{J}_N(t_0, \vec{x}_0)\rangle\right] = \sum_{n=0}^{\infty} a_n e^{-E_n(\vec{P}=\vec{0})t_s}$$

$$\xrightarrow{t_s \to \infty} a_0 e^{-m_N t_s} + \mathcal{O}\left(e^{-E_1(\vec{P}=\vec{0})t_s}\right)$$

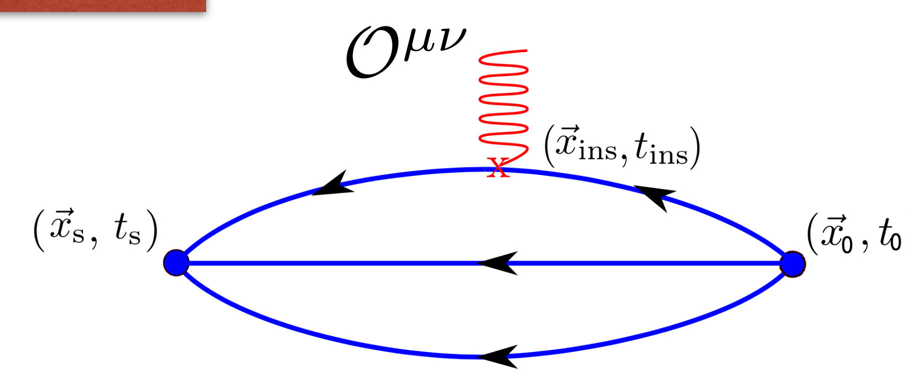
Fit the nucleon two-point function or effective mass keeping up to two excited states



Isovector momentum fraction

N_f=2+1+1 twisted mass fermions with a clover term: B-ensemble

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6



Statistics for connected contribution

t_s/a	$N_{\rm cnfs}$	$N_{\rm srcs}$	$N_{\rm meas}$
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

0.64 fm

Needed for studying excited states

1.6 fm

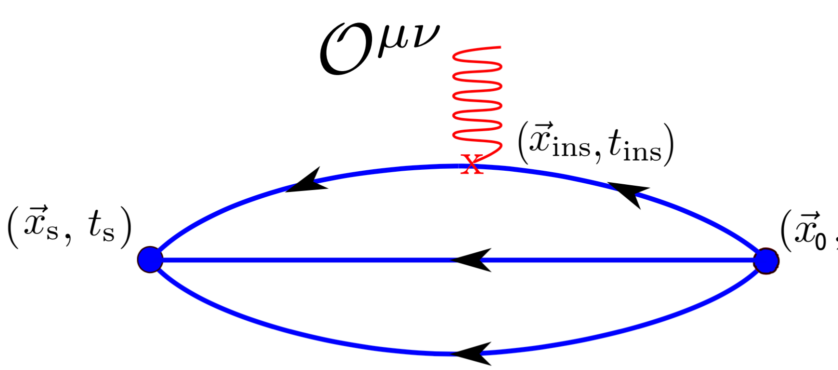
Increase statistics to keep approx. constant error

Nucleon matrix elements of EMT

$$\langle N(p',s')|\bar{T}^{\mu\nu}_{\rm q}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A^{\rm q}_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B^{\rm q}_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C^{\rm q}_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

$$\langle x\rangle_q = A^q_{20}(0) \quad \text{and} \quad J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

D-term



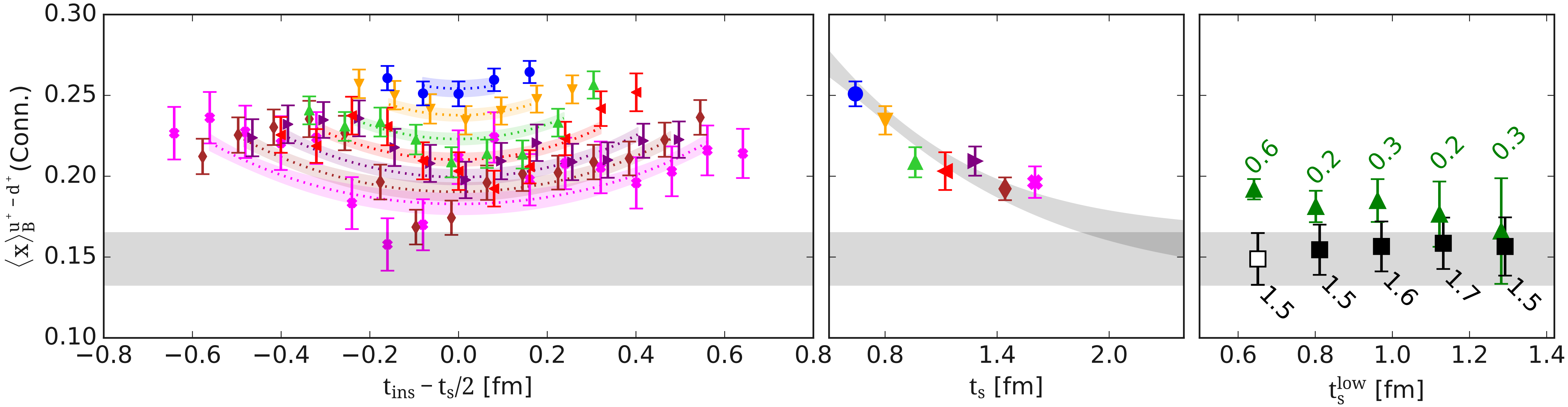
Ground-state dominance

- Plateau - one state
- Two- state fits
- Summation

$$\frac{C_{3pt}}{C_{2pt}}$$

Isovector

Plateau values



C. Alexandrou et al., PRD 101, 094513 (2020), arXiv: 2003.08486

Second Mellin moments

$\langle x\rangle_q,\ \langle x\rangle_{\Delta q},\ \langle x\rangle_{\delta q}$ Spin sum: $\frac{1}{2} = S_q + L_q + J_q$

$$S_q = \frac{1}{2}\sum_q \int_0^1 \left[\Delta q(x) + \Delta\bar{q}(x)\right]\, dx = \frac{1}{2}\sum_q g_A^q$$

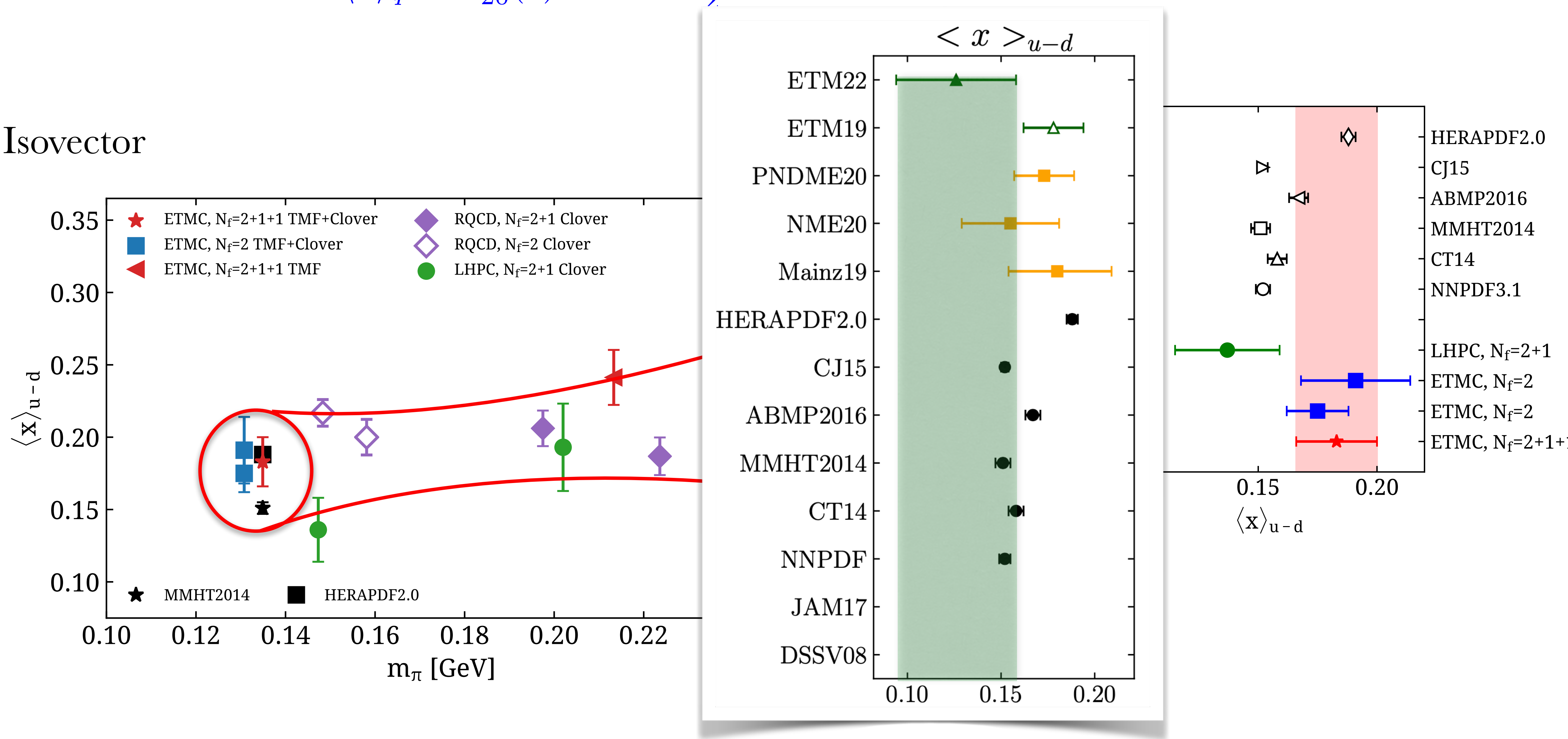
Momentum fraction - best measured

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^{q}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^{q}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

$$\mathcal{O}_V^{\mu\nu} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$$

$\langle x\rangle_q = A_{20}^q(0)$ $J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$ D-term

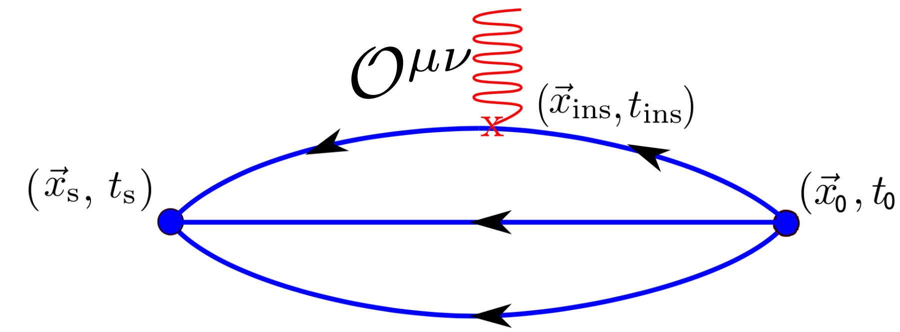
Isovector

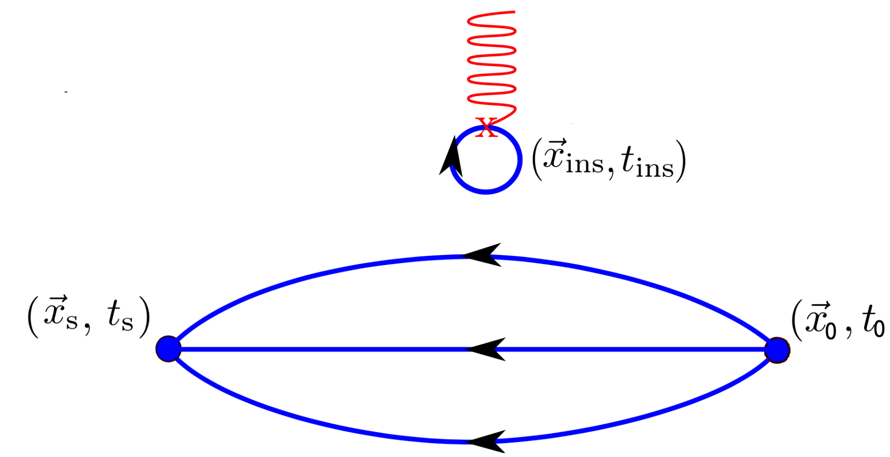


Momentum fraction for each quark

N_f=2+1+1 twisted mass fermions with a clover term

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6





Statistics for connected contribution

t_s/a	$N_{\rm cnfs}$	$N_{\rm srcs}$	$N_{\rm meas}$
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

0.64 fm

Needed for studying excited states

1.6 fm

Increase statistics to keep approx. constant error

Use hierarchical probing

no. of Hadamard vectors

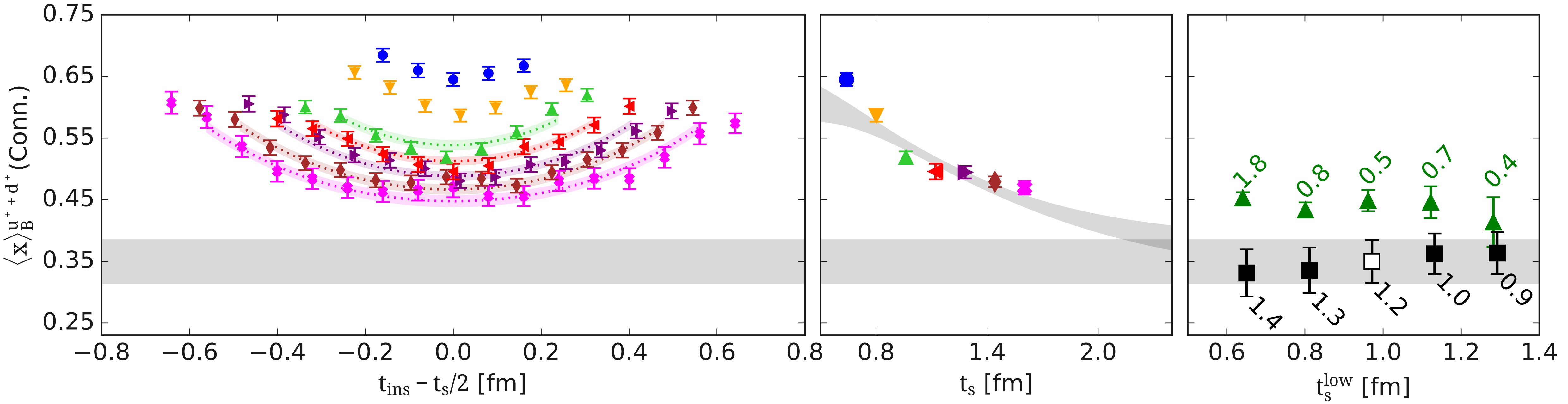
Statistics for disconnected contribution

2pt	(u+d)-quark loop	s-quark loop	c-quark loop
600000	750×512 + deflation of 200 modes	750×512	9000×32

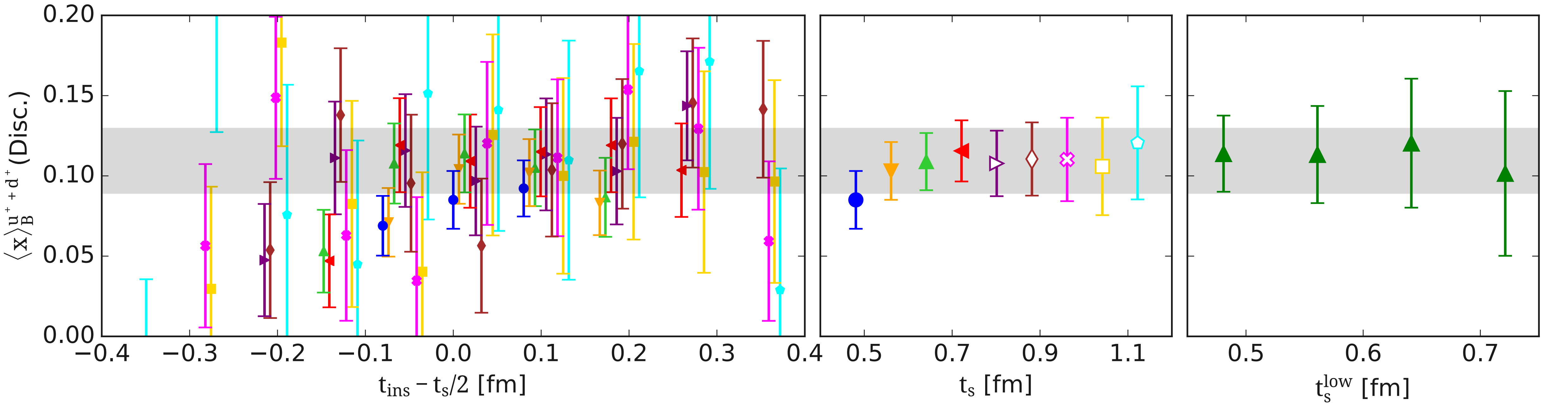
no. of stochastic vectors

Nucleon isoscalar momentum fraction

Connected



Disconnected

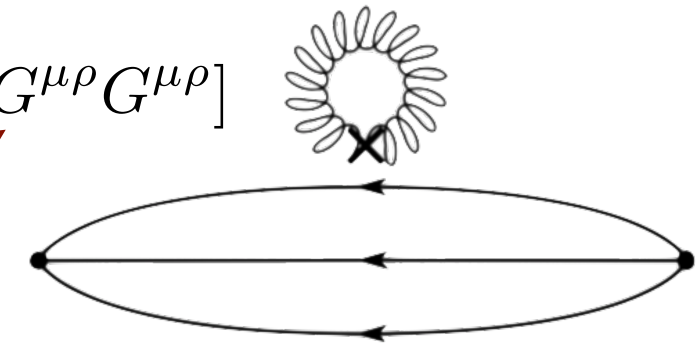


Gluon momentum fraction

$\langle x \rangle_g = A_{20}^g(0)$

$\mathcal{O}_g^{\mu\nu} = 2\mathrm{Tr}\left[G^{\mu\rho}G^{\mu\rho}\right]$

Field strength tensor



Use stout smearing to reduce UV noise: n_s=10

- In the rest frame of the nucleon: $\dfrac{\langle N|\mathcal{O}_g^{44} - \frac{1}{3}\mathcal{O}_g^{jj}|N\rangle}{\langle N|N\rangle} = -2m_N < x >_g$
- In a moving frame: $\dfrac{\langle N|\mathcal{O}_g^{4i}|N\rangle}{\langle N|N\rangle} = p_i < x >_g$
- Quark isoscalar & gluon momentum fractions mix —> need 2x2 matrix:

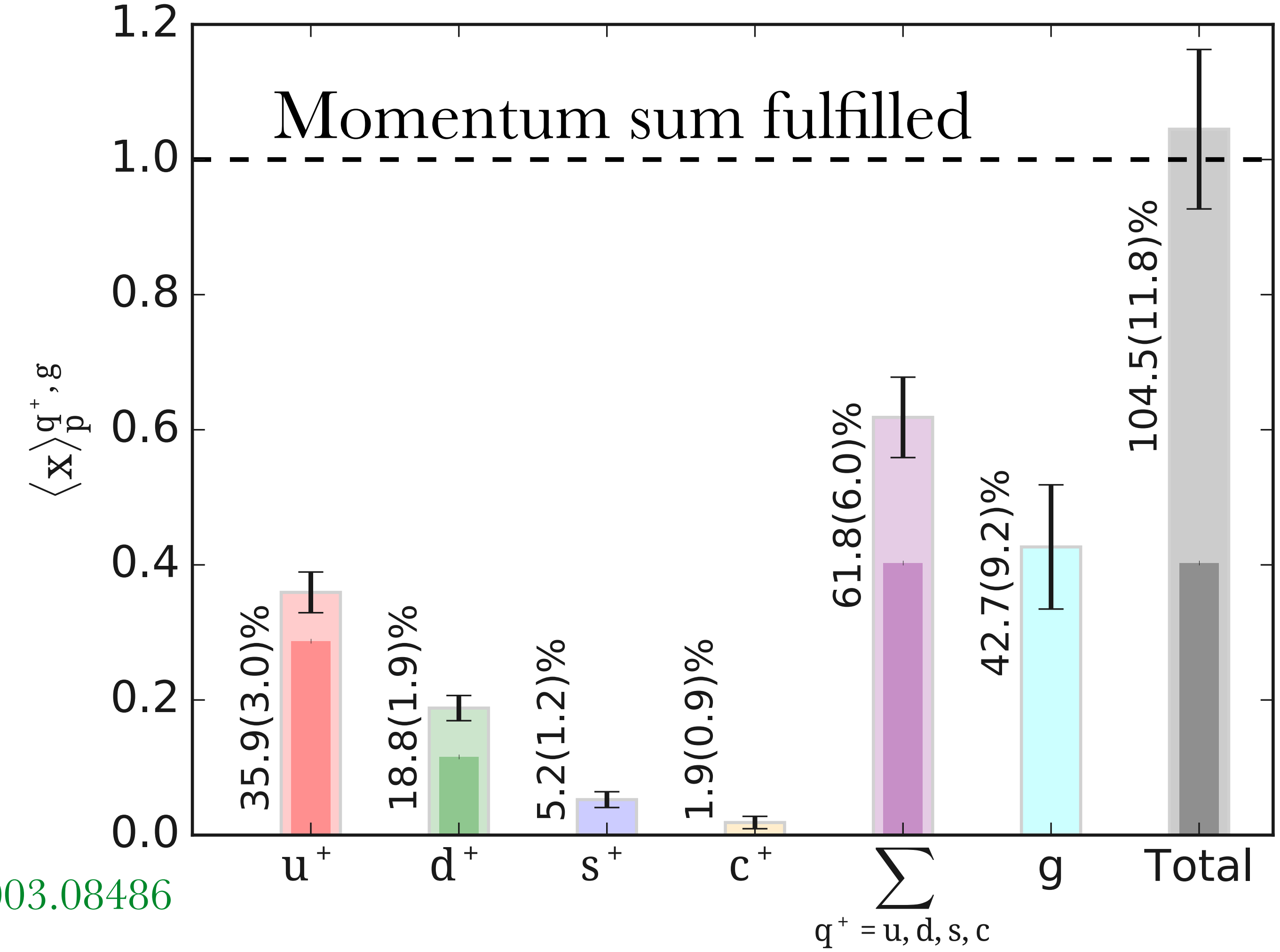
$$\langle x \rangle_g^R = Z_{gg}\langle x \rangle_g^B + Z_{gq}\sum_q \langle x \rangle_q^B$$

- Z_{qq} and Z_{gg} computed non-perturbatively

$$Z_{gq} = \frac{g^2 C_f}{16\pi^2}\left(0.8114 + 0.4434 c_{SW} - 0.2074 c_{SW}^2 + \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

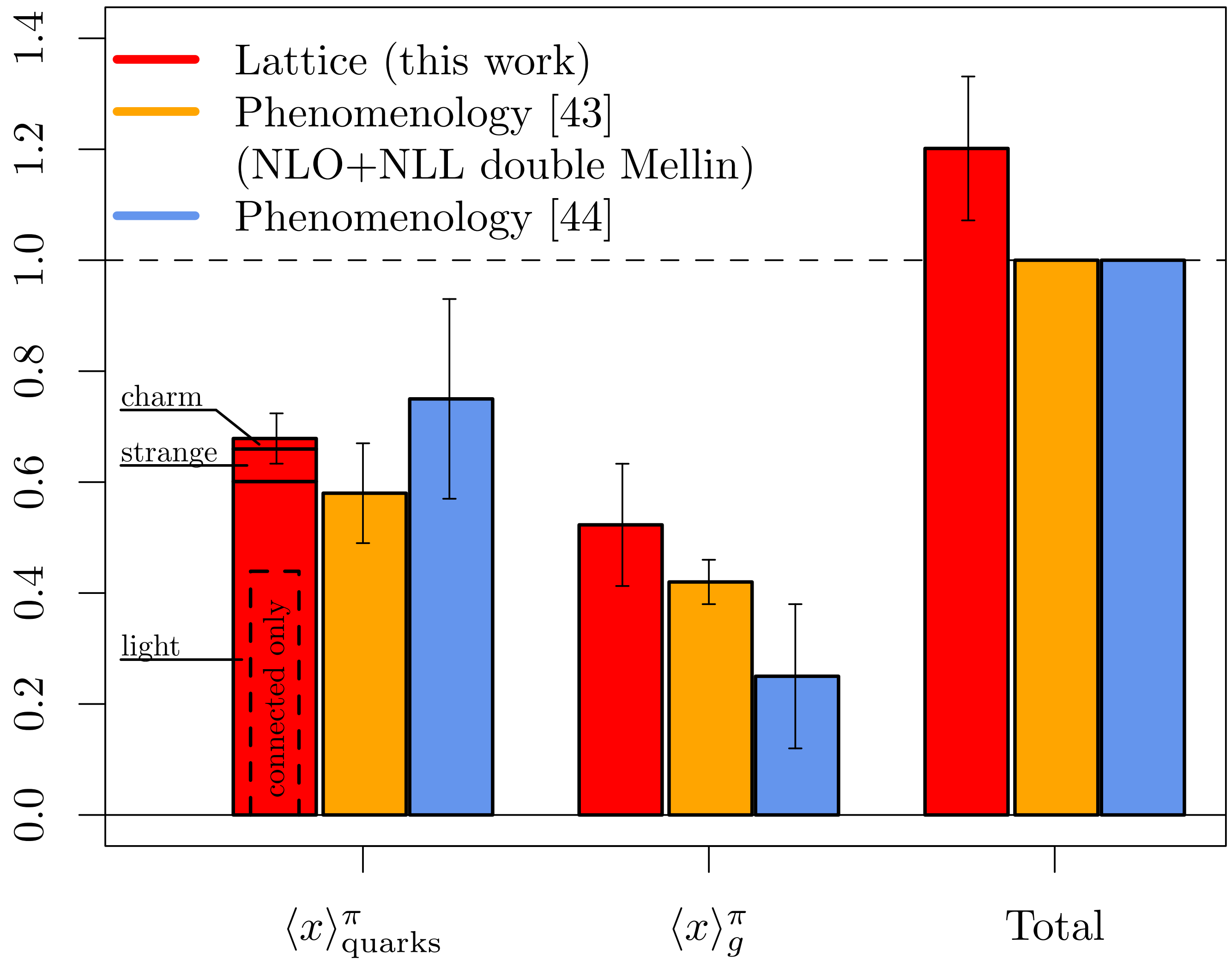
$$Z_{qg} = \frac{g^2 C_f}{16\pi^2}\left(0.2164 + 0.4511 c_{SW} + 1.4917 c_{SW}^2 - \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

computed perturbatively



C. Alexandrou et al., PRD 101, 094513 (2020), arXiv: 2003.08486

Momentum fraction of pion



N_f=2+1+1 twisted mass fermions

- Lattice size 64^3 x 128
- a=0.08 fm determined from the nucleon mass
- m_π=139 MeV
- Lm_π=3.6

$\overline{\text{MS}}$ scheme at 2 GeV

$$x_{u+d} = 0.601(28),\ x_s = 0.059(13),\ x_c = 0.019(05),\ \text{and}\ x_g = 0.52(11)$$

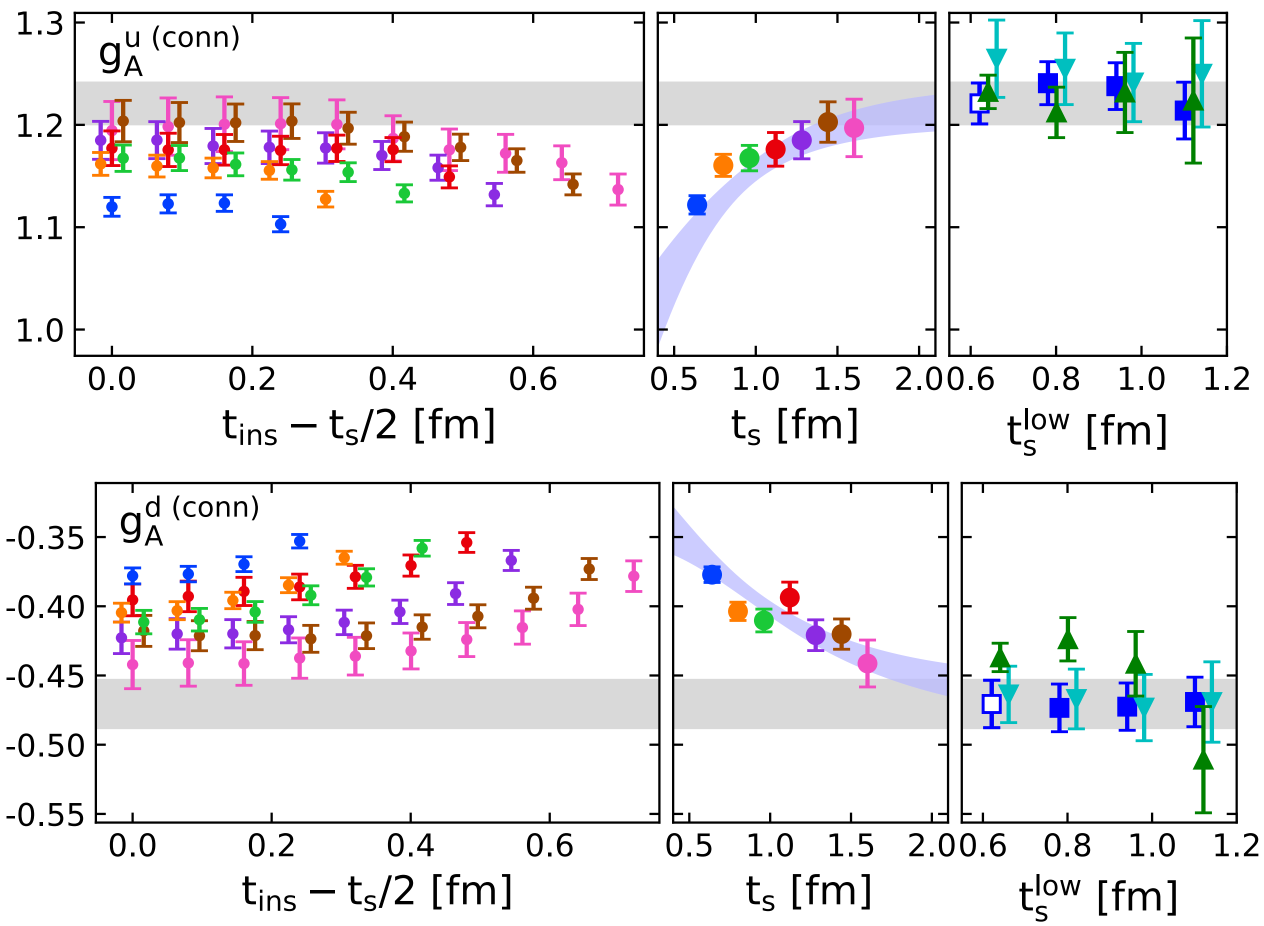
C. A. *et al.* (ETMC), Phys. Rev. Lett. 127 (2021) 25, 252001, arXiv: 2109.10692

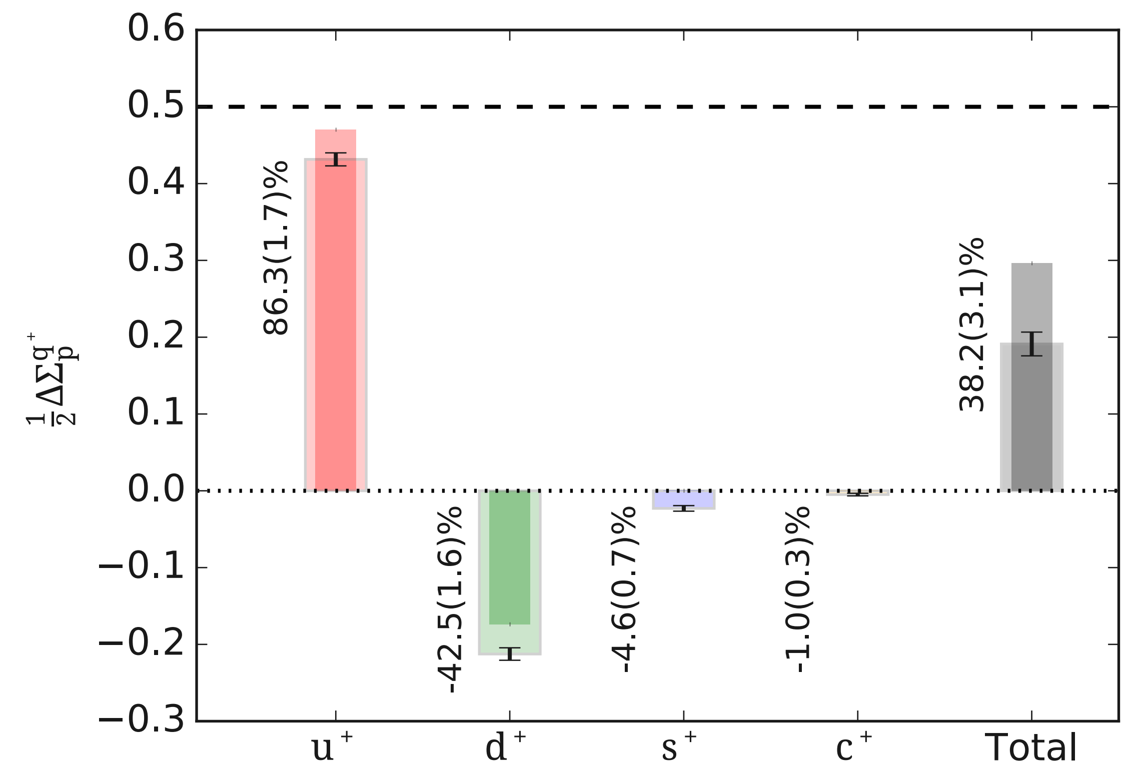
Flavor decomposition of axial charge

- Determines intrinsic spin carried by each quark

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$

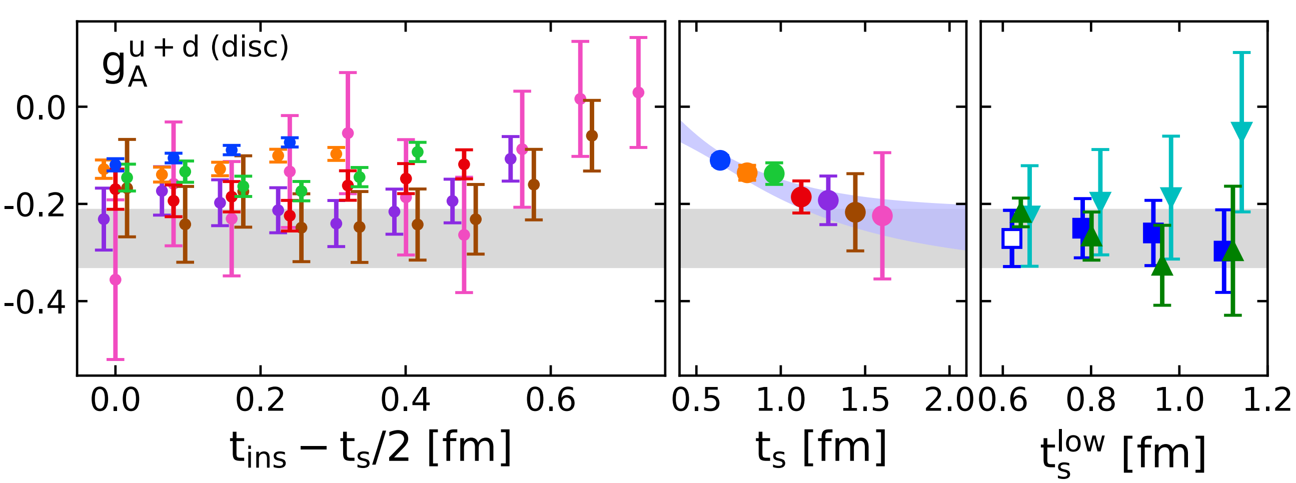
Connected up & down

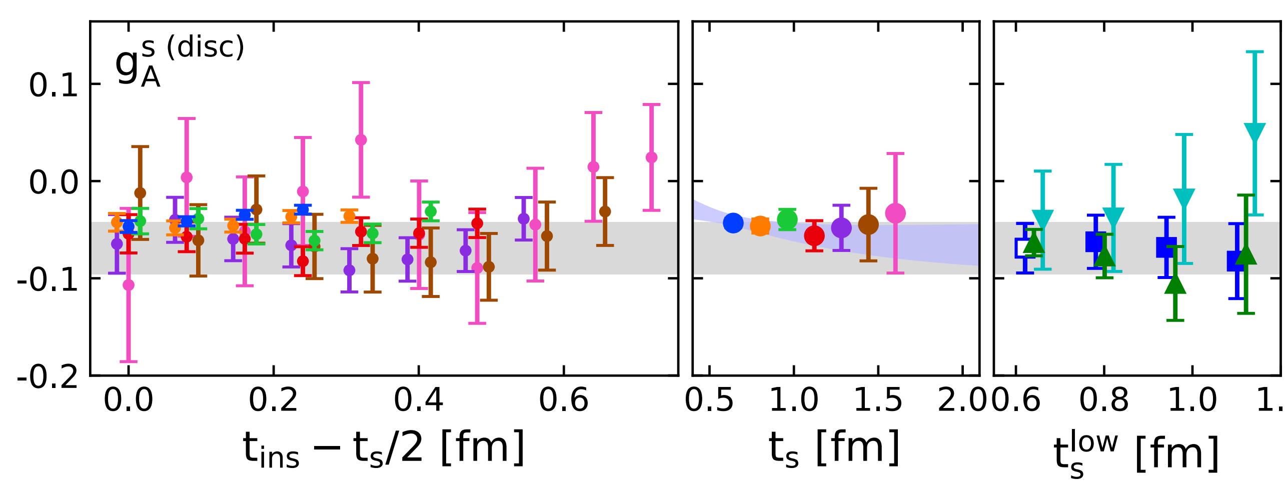




C. A. *et al.* (ETMC) Phys.Rev.D 102 (2020) 5, 054517, 1909.00485
C. A. *et al.* (ETMC) Phys.Rev.D 101 (2020) 9, 094513, 2003.08486

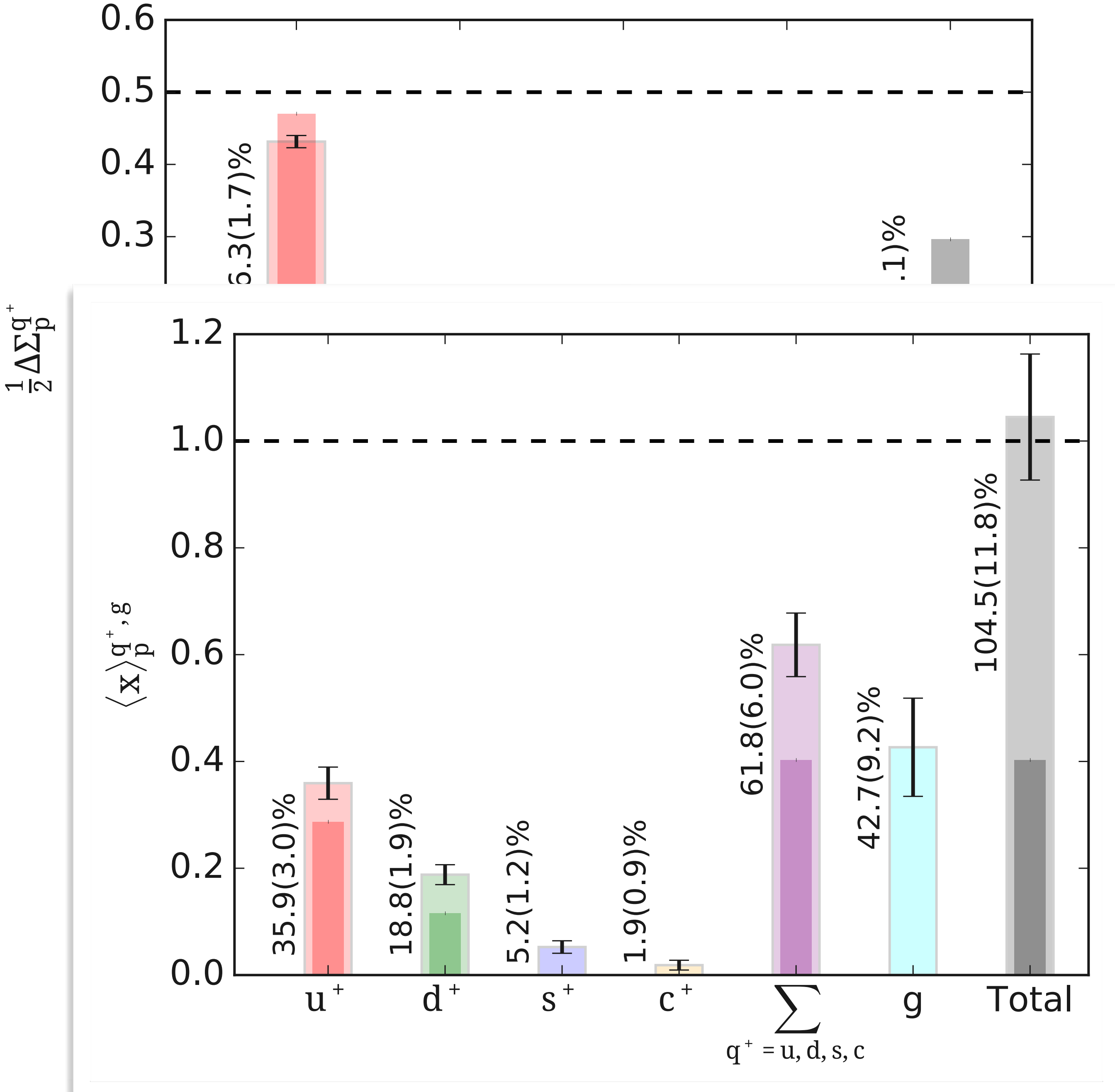
Disconnected



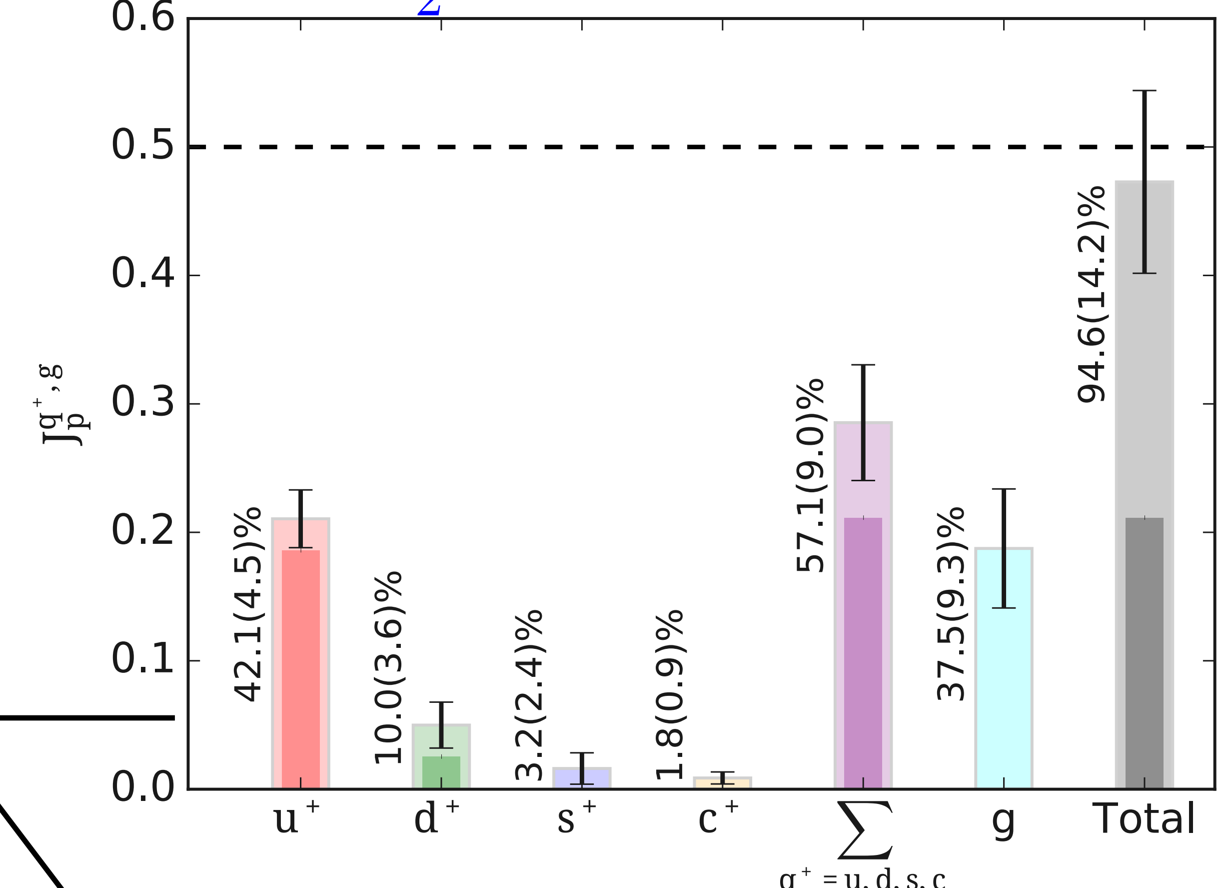


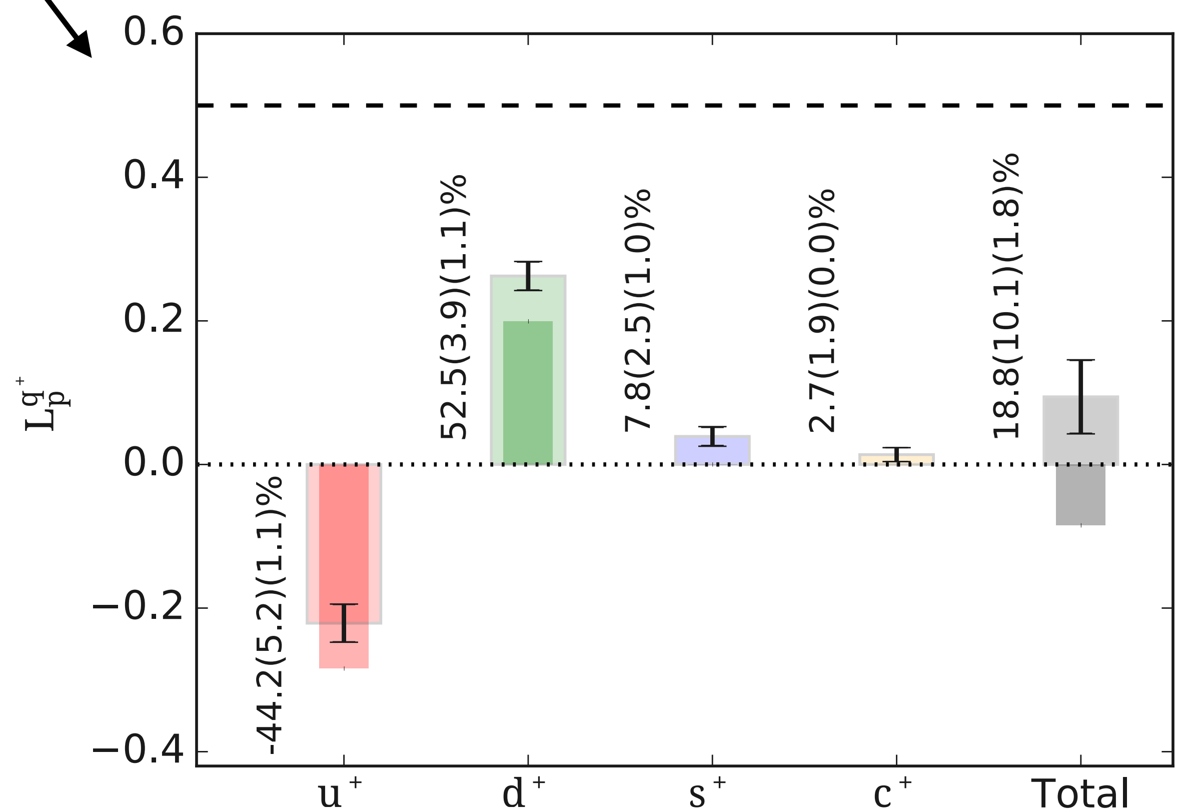
Spin sum and orbital angular momentum

$$\Delta\Sigma_{q+}(\mu^2) = \int_0^1 dx\,[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] = g_A^q$$



$$J_q = \frac{1}{2}[A_{20}^q(0) + B_{20}^q(0)]$$





Nucleon momentum and spin sums verified from lattice QCD

C. A. *et al.* (ETMC) Phys.Rev.D 102 (2020) 5, 054517, arXiv:1909.00485
C. A. *et al.* (ETMC) Phys.Rev.D 101 (2020) 9, 094513, arXiv:2003.08486

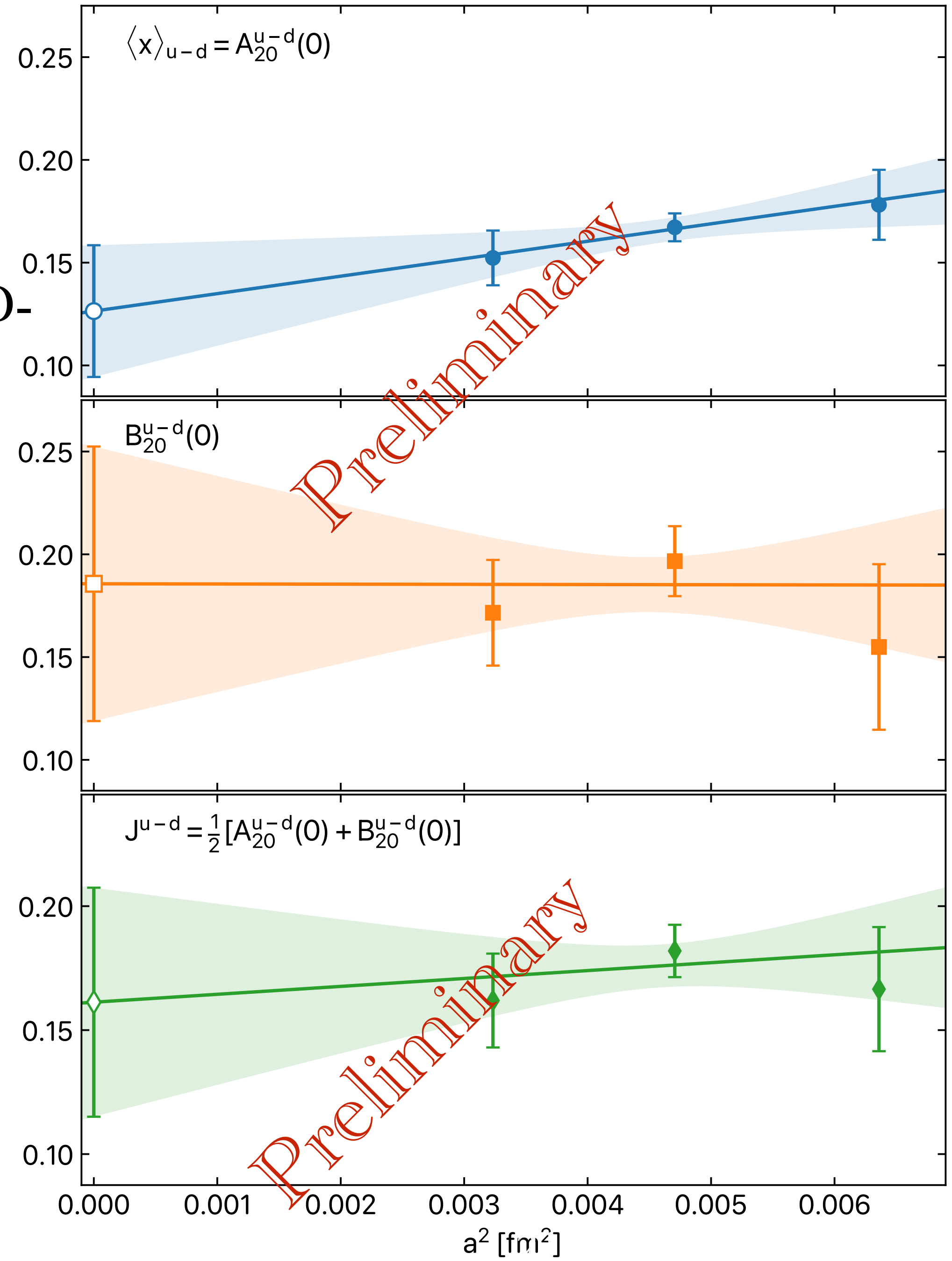
Continuum extrapolation

- Use three gauge ensembles simulated directly at physical pion mass - avoids uncontrolled chiral extrapolations

ensemble	$(L/a)^3.T/a$	a (fm)	m_π (MeV)	L (fm)	$m_\pi L$
cB211.072.64	$64^3 \cdot 128$	0.07961 (13)	140.2 (2)	5.09	3.62
cC211.060.80	$80^3 \cdot 160$	0.06821 (12)	136.7 (2)	5.46	3.78
cD211.054.96	$96^3 \cdot 192$	0.05692 (10)	140.8 (2)	5.46	3.90

- Isovector spin

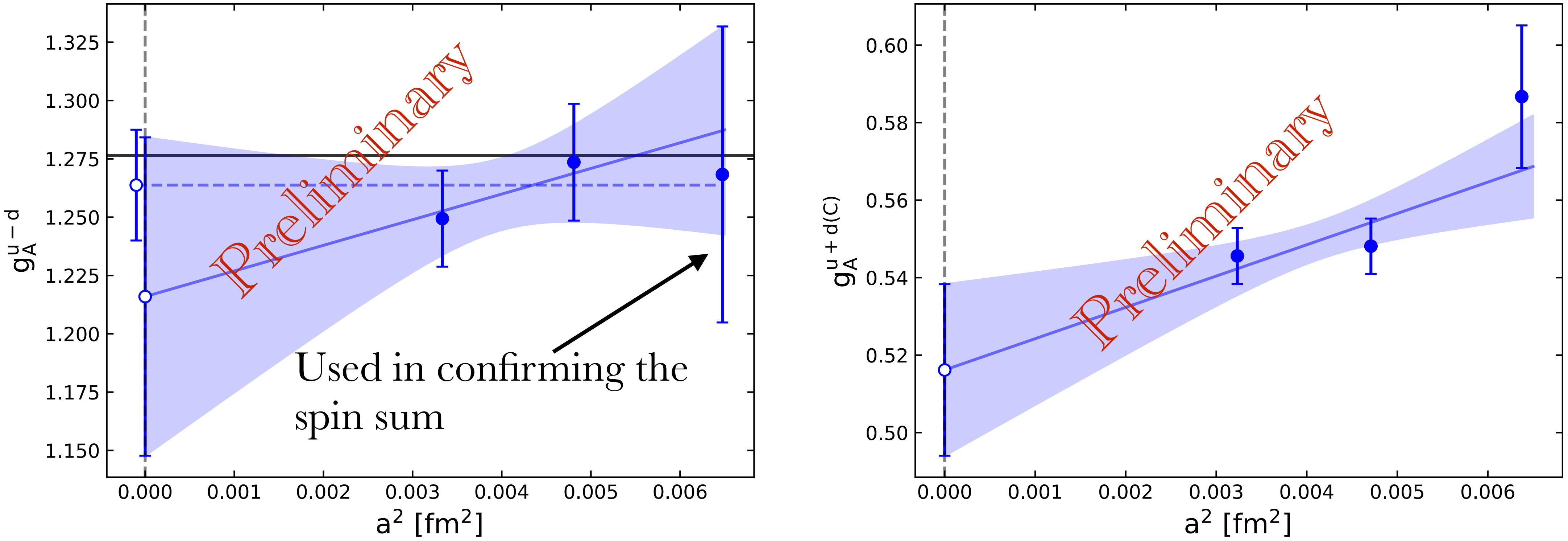
- Mild a²-dependence
- Increase statistics of B- and D-ensembles to reduce error of continuum result



Continuum limit of intrinsic spin ΔΣ

$$\Delta\Sigma_{q^+} = g_A^q$$

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$



Next step to perform the continuum extrapolation of the disconnected contributions

Pion and kaon moments

- Limited studies of pion and kaon PDFs despite important role in understanding e.g. dynamics of chiral symmetry breaking, isospin breaking, etc.

- First lattice QCD study of pion moments more than 30 years ago

C. Best *et al.* (QCDSF) Phys. Rev. D 56, 2743 (1997), hep-lat/9703014
D. Broemmel *et al.* (QCDSF-UKQCD) PoS LATTICE2007, 140 (2007)

- Compute <x>, <x²> and <x³> using N_f=2+1+1 twisted mass ensemble with m_π=260 MeV, only connected

$$\langle M(p)|\mathcal{O}^{44}|M(p)\rangle = \frac{1}{2E_M(p)}\left(\frac{m_M^2}{2} - 2(E_M(p))^2\right)\langle x\rangle_M$$

$$\langle M(p)|\mathcal{O}^{jk4}|M(p)\rangle = -p_j\, p_k \langle x^2\rangle_M \,,\; j \neq k$$

Indices chosen to avoid mixing —> Require boosted frame

$$\langle M(p)|\mathcal{O}^{\{1234\}}|M(p)\rangle = -i\, p^1\, p^2\, p^3\, \langle x^3\rangle_M$$

$$\frac{\langle x\rangle_\pi^{u^+}}{\langle x\rangle_K^{s^+}} = 0.823(8)(10)\,,\; \frac{\langle x^2\rangle_\pi^{u^+}}{\langle x^2\rangle_K^{s^+}} = 0.795(45)(80)\,,\; \frac{\langle x^3\rangle_\pi^{u^+}}{\langle x^3\rangle_K^{s^+}} = 0.325(244)(23)$$

s-quark distribution has support at larger x

C. A. *et al.,* Phys. Rev. D 103 (2021) 1, 014508, 2010.03495 & 2104.02247

Reconstruction of PDFs using moments

- Proof of concept using pion and kaon and simulations with pion mass 260 MeV
- Use moments up to <x^3>

q^f_M	$\langle x\rangle$	$\langle x^2\rangle$	$\langle x^3\rangle$
q^u_π	0.261(3)(6)	0.110(7)(12)	0.024(18)(2)
q^u_K	0.246(2)(2)	0.096(2)(2)	0.033(6)(1)
q^s_K	0.317(2)(1)	0.139(2)(1)	0.073(5)(2)

General form:

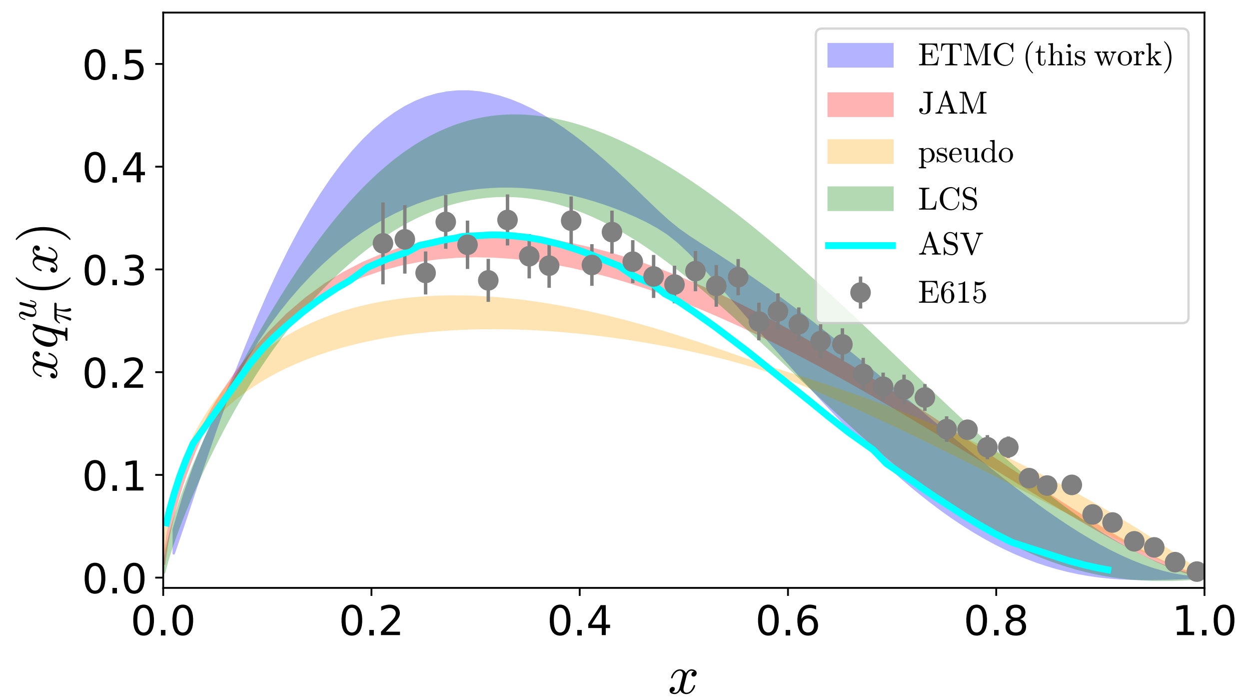
$$q^f_{\pi,K}(x) = N x^{\alpha}(1-x)^{\beta}(1+\rho\sqrt{x}+\gamma x)$$

Charge conservation determines N:

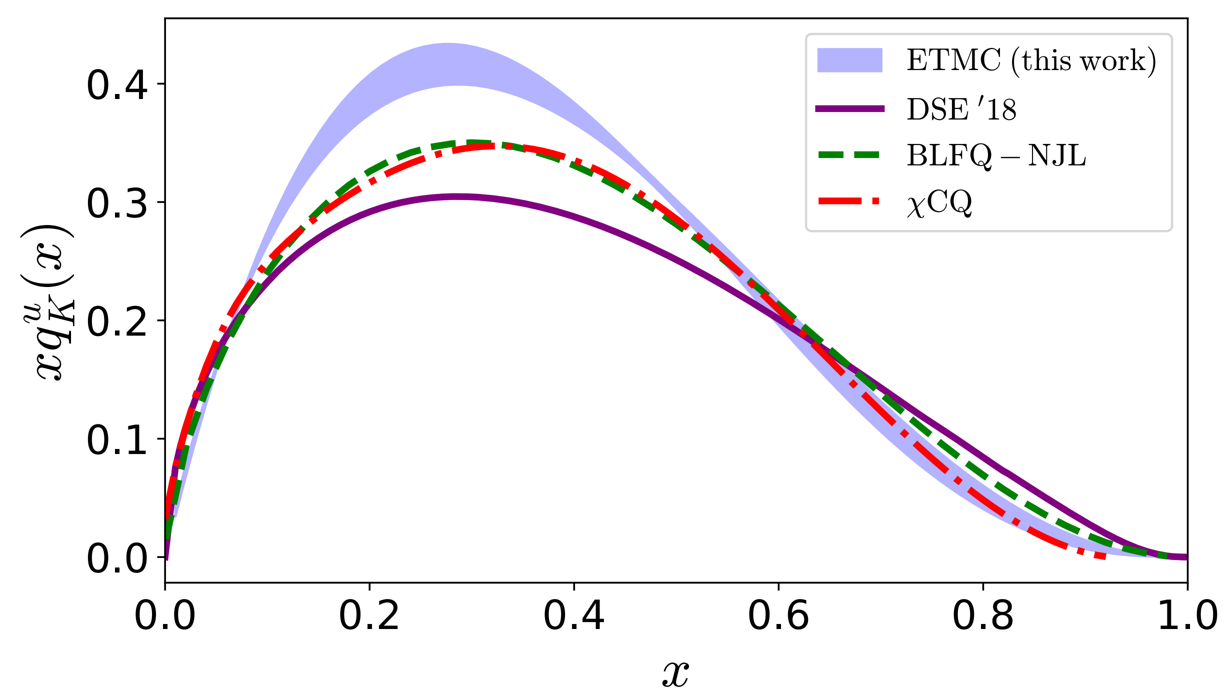
$$\langle 1\rangle_M = \int_0^1 q_M(x) = 1$$

- ρ is generally small so we take ρ=0
- we do a 3- and 2- (setting γ=0) parameter fit

We find the 3-parameter fit yields γ with large errors and gives compatible results with the 2-parameter fit
—> we show results for the 2-parameter fit
—> we find β~2



$\overline{\mathrm{MS}}$ at 27 GeV



C. A. *et al.*, (ETMC) Phys. Rev. D 103 (2021) 014508; 2010.03495
C. A. *et al.*, (ETMC) Phys. Rev. D 104 (2021) 054504; 2104.02247

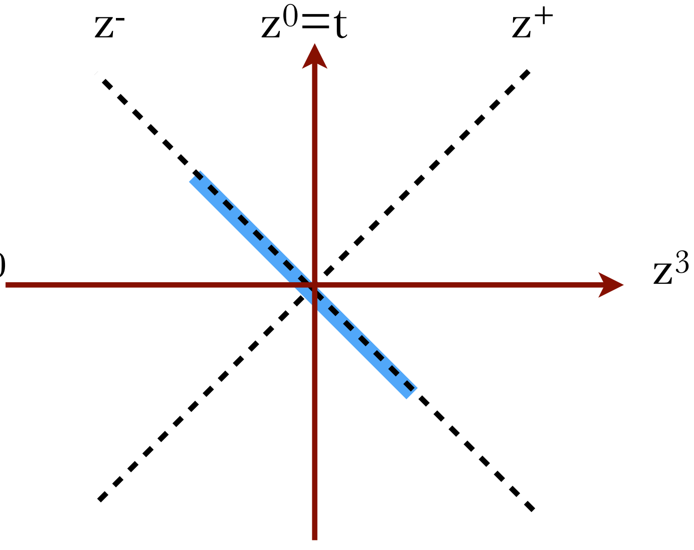
New era of direct computation of x-dependencne of parton distributions

Reviews

- M. Constantinou *et al*. (2020) 2006.08636
- H.-W. Lin *et al.* Prog. Part. Nucl. Phys. 100, 107, 1711.07916
- X. Ji, Y. Liu, J.-H. Zhang, 2004.03543
- K. Cichy and M. Constantinou, *Adv.High Energy Phys.* (2019) 3036904,1811.07248

Quasi-parton distributions

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]



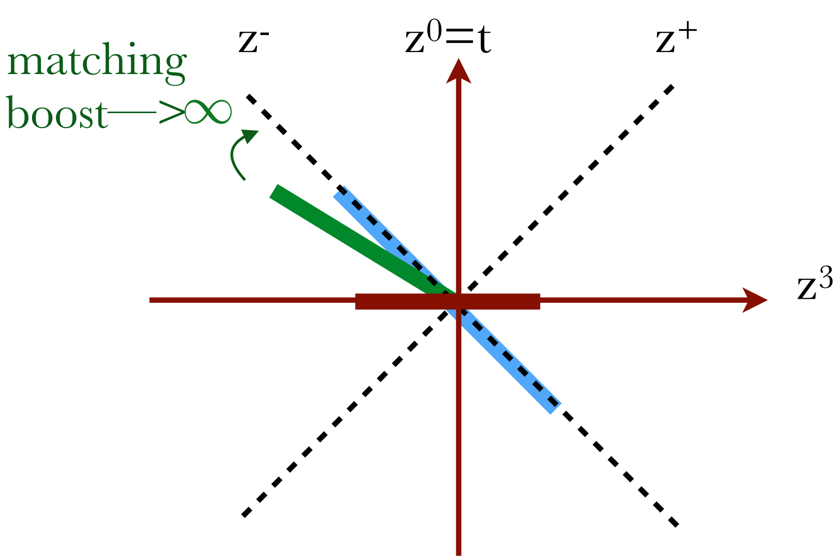
- PDFs light-cone correlation matrix elements

$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p)|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$

- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum

- Renormalise quasi-PDFs and increase nucleon boost

- Use LaMET to match to the infinite momentum frame using a matching kernel computed in perturbation theory



Computation of quasi-PDFs

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle|_\mu$$

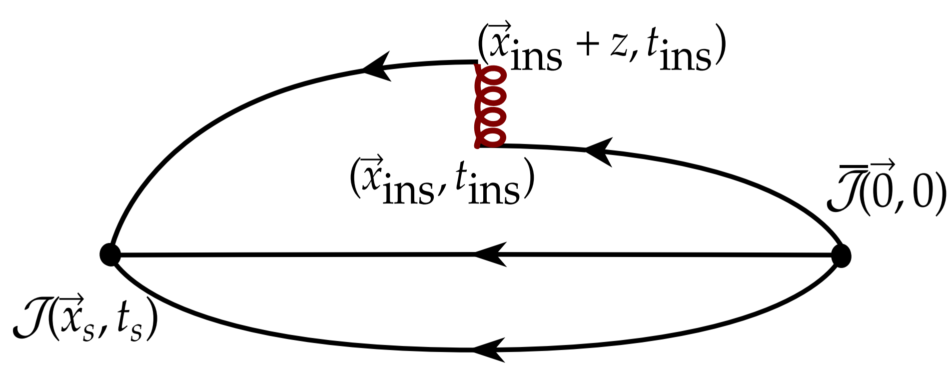
Renormalise non-perturbatively, $Z(z,\mu)$
Eliminates both UV and linear divergences

- Match using LaMET

Perturbative kernel

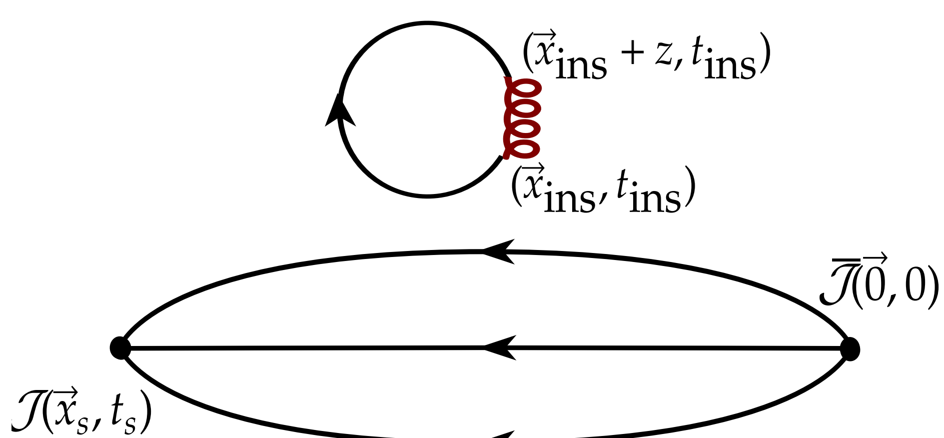
$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

Isovector (u-d) and isoscalar (u+d) connected

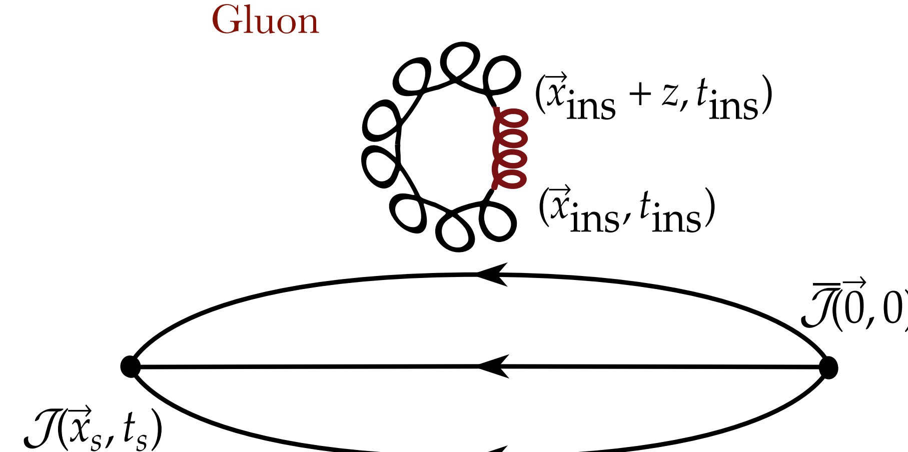


$$\Gamma = \begin{array}{ll} \gamma_0 & \text{unpolarised} \\ \gamma_5\gamma_3 & \text{helicity} \\ \sigma_{3i}, i=1,2 & \text{transversity} \end{array}$$

Isoscalar (u+d) disconnected, s and c

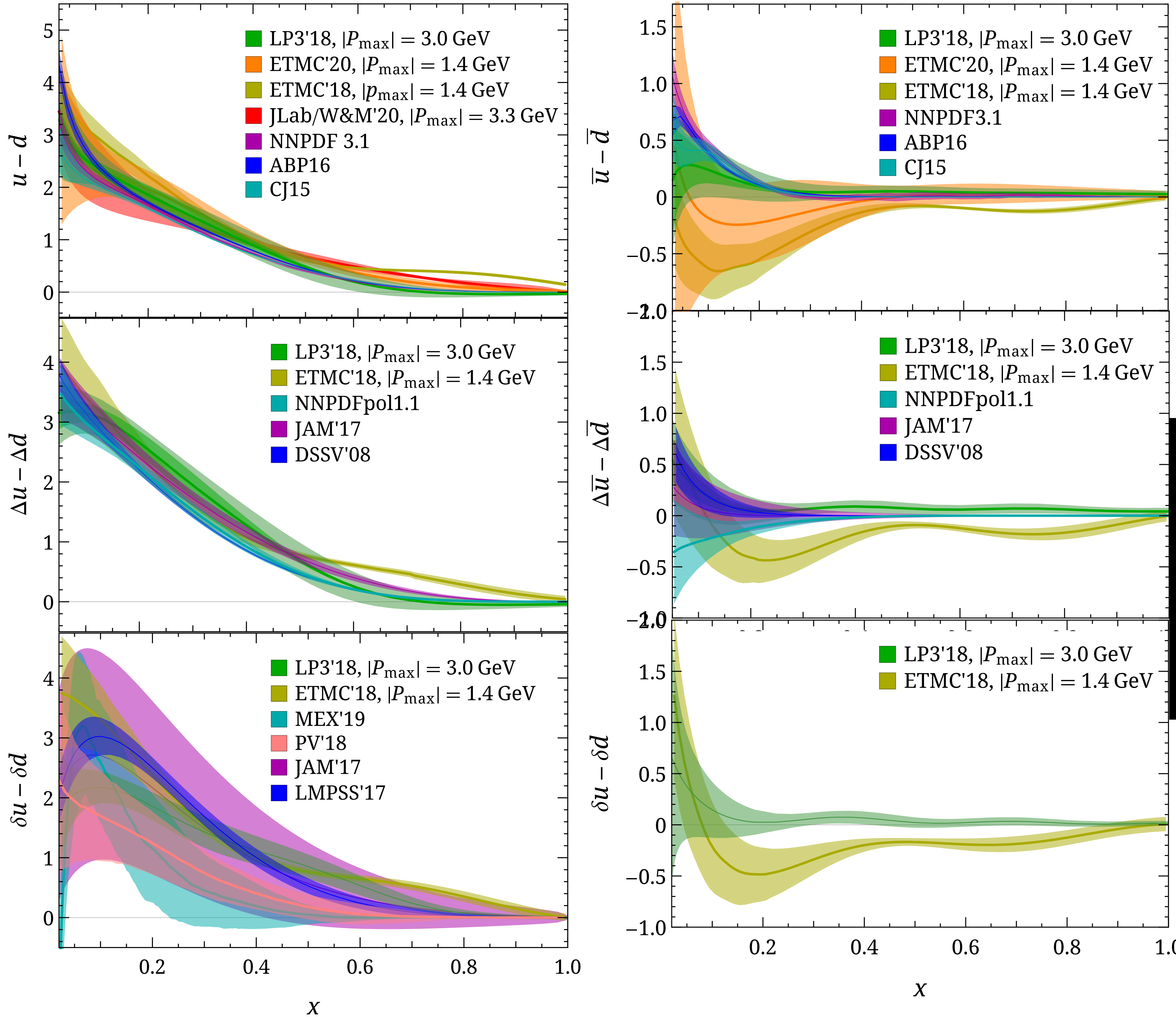


Gluon



Nucleon isovector PDFs

State-of-the-art results



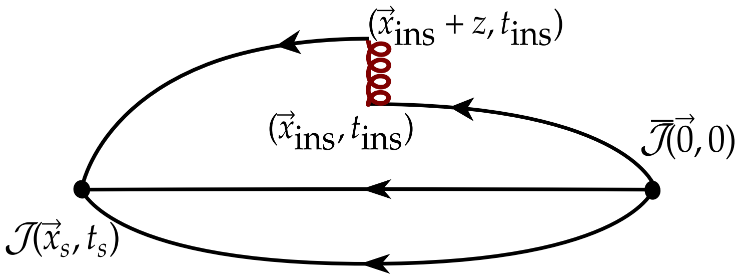
No continuum extrapolation yet

Also pion PDFs

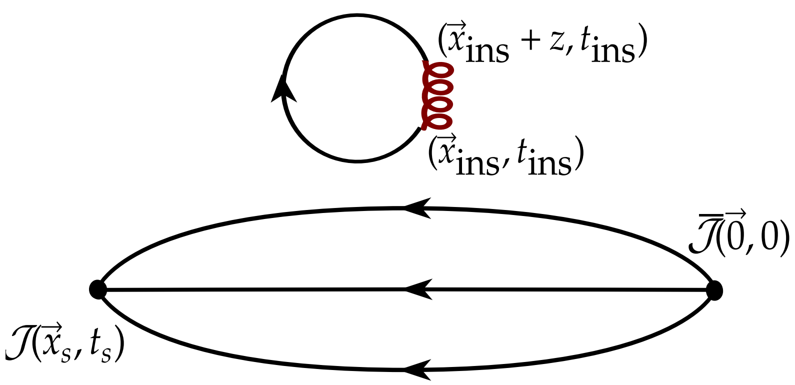
- X. Gao *et al*, Phys.Rev.D 102 (2020) 9, 094513, 2007.06590; 2112.02208
- T. Izubuchi, Phys.Rev.D 100 (2019) 3, 034516, 1905.06349
- R. Zhang, Phys.Rev.D 102 (2020) 9, 094519, 2005.13955

M. Constantinou *et al*. (2020) 2007.08636

Isoscalar and strange PDFs



Connected isoscalar: compute like isovector



$$\mathcal{L}(t_{\rm ins}, z) = \sum_{\vec{x}_{\rm ins}} \mathrm{Tr}\left[D_q^{-1}(x_{\rm ins}; x_{\rm ins}+z)\gamma^3\gamma^5 W(x_{\rm ins}, x_{\rm ins}+z)\right]$$

Two studies on disconnected with heavier than physical pion mass:

- Mixed action - clover valence on staggered sea, m_π=310 and m_π=690 MeV, only strange

R. Zhang, H.W. Lin, B. Yoon (2020), 2005.011

- Twisted mass fermions

$32^3 \times 64$	a=0.0938(3)(2) fm $\quad m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV $\quad m_\pi L \approx 4.0$

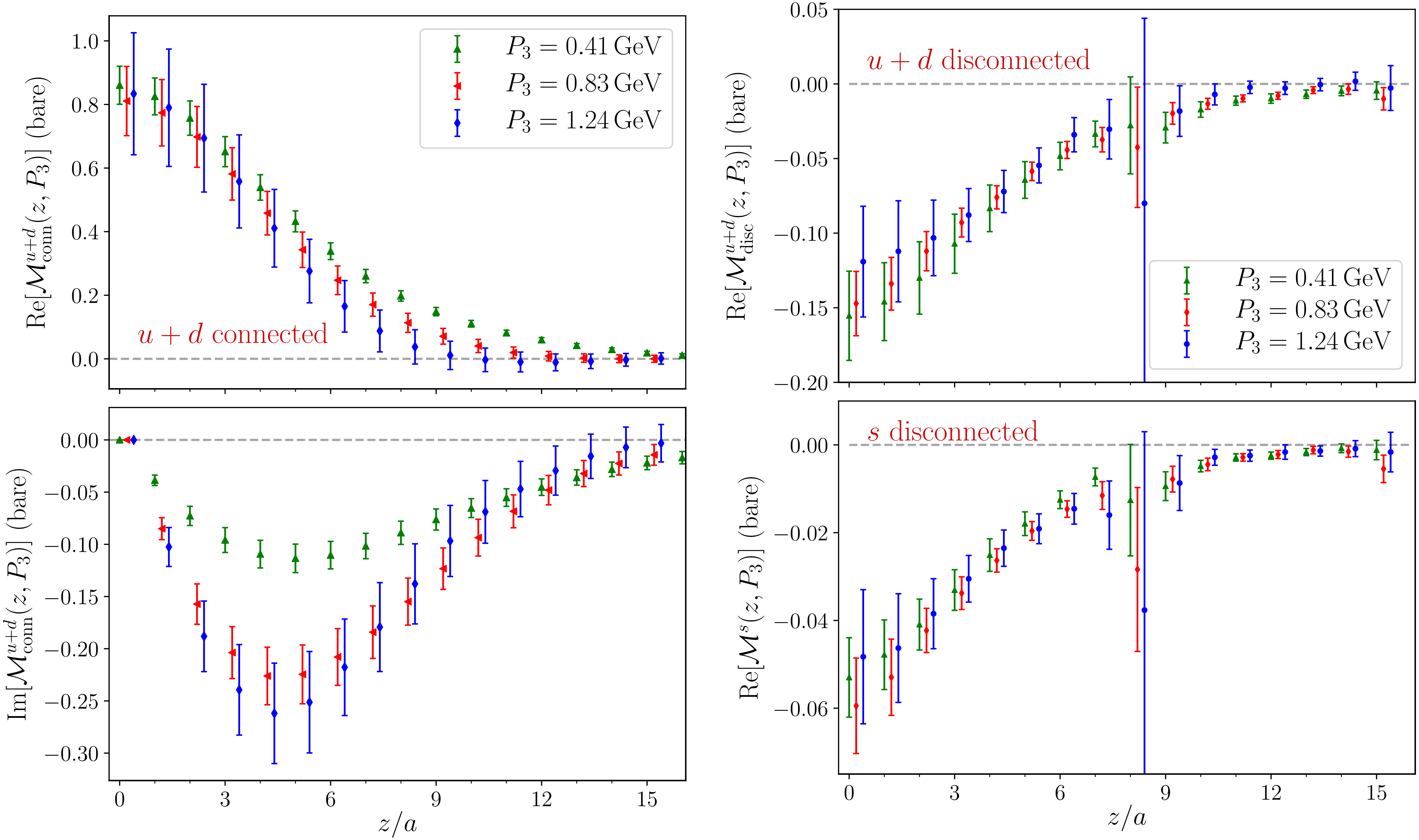
C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Manigrasso (2020), 2009.13061

Quark helicity PDFs

$$\mathcal{O}(z) = \overline{\psi}(z)\,\gamma^3\gamma^5 W(0,z)\psi(0)$$

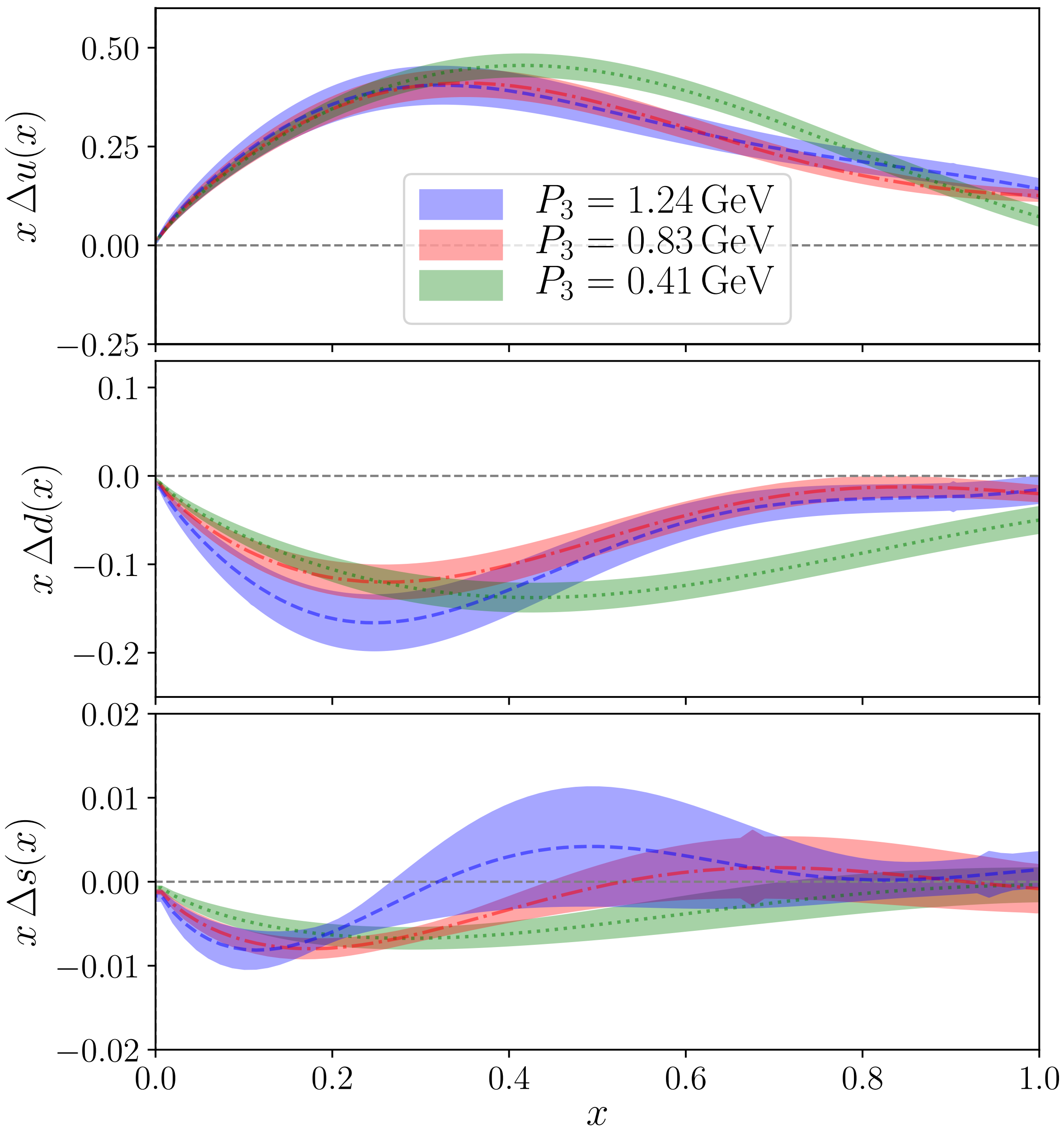
$$\frac{C_{3pt}(t_s, t_{\text{ins}}, P_3)}{C_{2pt}(t_s, P_3)} \xrightarrow{t_{\text{ins}}\gg 1; t_s \gg 1} \mathcal{M}(z, P_3)$$

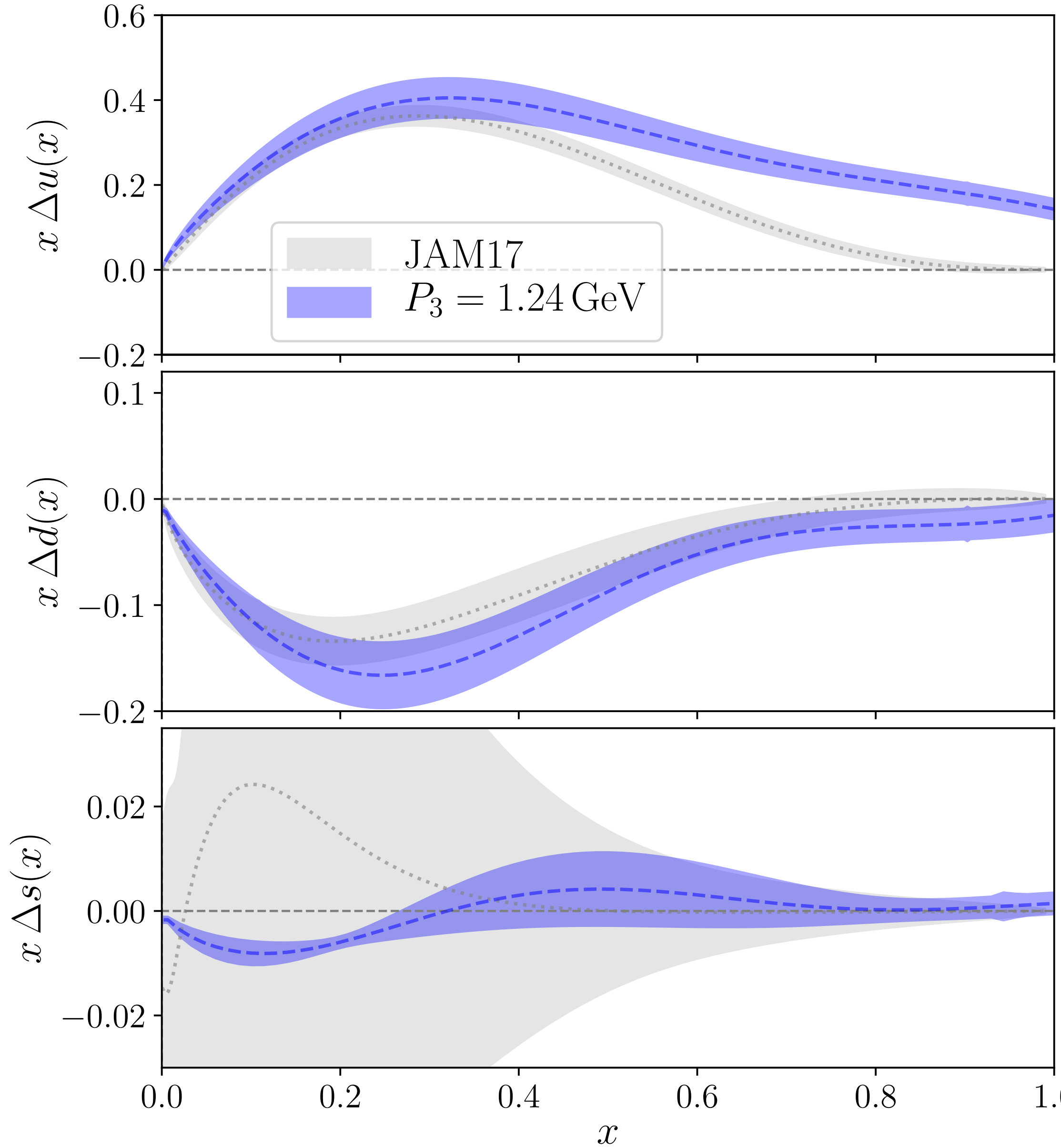
Bare matrix elements



small but clearly non-zero

Helicity distributions





C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.13061

- Convergence of results seen as we increase the momentum from P_3=0.83 to P_3=1.24 GeV
- Computation at the physical point is currently on-going

Generalised parton distributions

- Compute space-like matrix element with different initial and final nucleon boosts

$$h_\Gamma(z,\tilde{\xi},Q^2,P_3) = \langle N(P_3\hat{e}_z+\vec{Q}/2)|\,\overline{\psi}(z)\,\Gamma W(z,0)\,\psi(0)|\,N(P_3\hat{e}_z-\vec{Q}/2)\rangle$$

$$\tilde{\xi} = -\frac{Q_3}{2P_3}\ :\ \text{quasi}-\text{skewness} \quad \tilde{\xi} = \xi + \mathcal{O}(\frac{1}{P_3^2})$$

- Rest of the steps are the same as for quasi-PDFs: i.e. renormalise, take the Fourier transform and match

$$\tilde{F}_\Gamma(z,\tilde{\xi},Q^2,P_3,\mu^0,\mu_3^0) = \int_{-1}^{1} \frac{dy}{y} C_\Gamma\left(\frac{x}{y},\frac{\mu}{yP_3},\frac{\mu_3^0}{yP_3},\frac{(\mu^0)^2}{(\mu_3^0)^2}\right) F_\Gamma(y,Q^2,\xi,\mu)+\mathcal{O}\left(\frac{m^2}{P_3^2},\frac{Q^2}{P_3^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_3^2}\right)$$

RI-scale

$\overline{\rm MS}-\text{scale}$

Reduces to the matching kernel for ξ=0
Does not depend on Q^2

X.Ji *et al.*, Phys.Rev. D92 (2015) 014039
X.Xiong, J-H. Zhang, Phys.Rev. D92 (2015) 054037
Y-S. Liu *et al.*, Phys.Rev. D100 (2019), 034006

- Two first studies for pion and nucleon GPDs

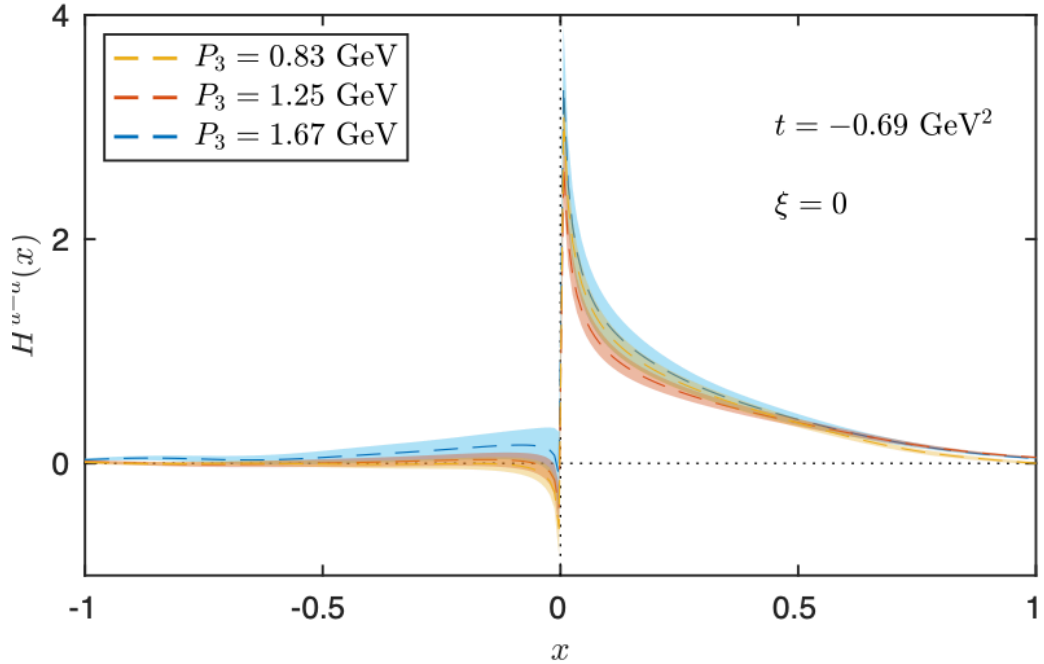
J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376
C. A. *et al.*, Phys.Rev.Lett. 125 (2020) 26, 262001, 2008.10573

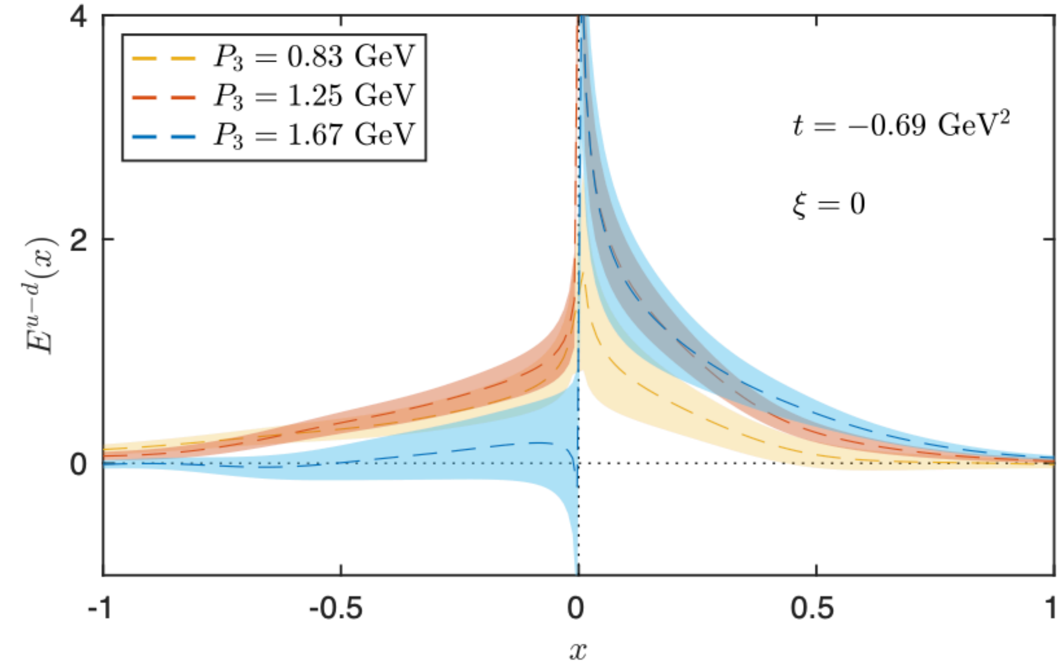
Results on unpolarised and helicity GPDs

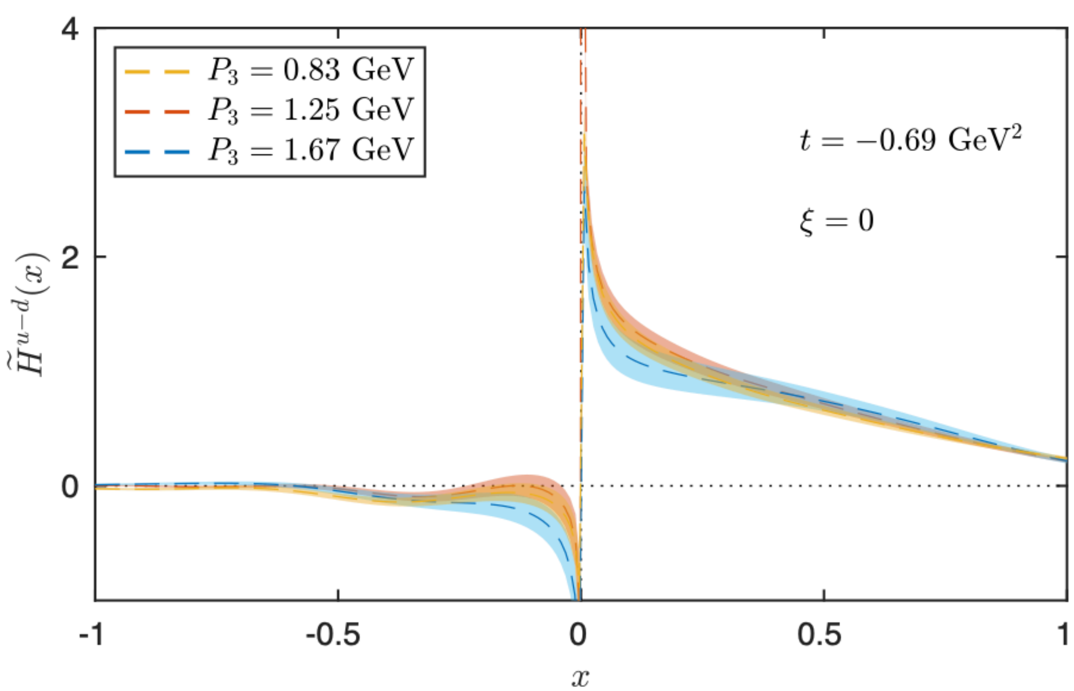
Convergence with momentum

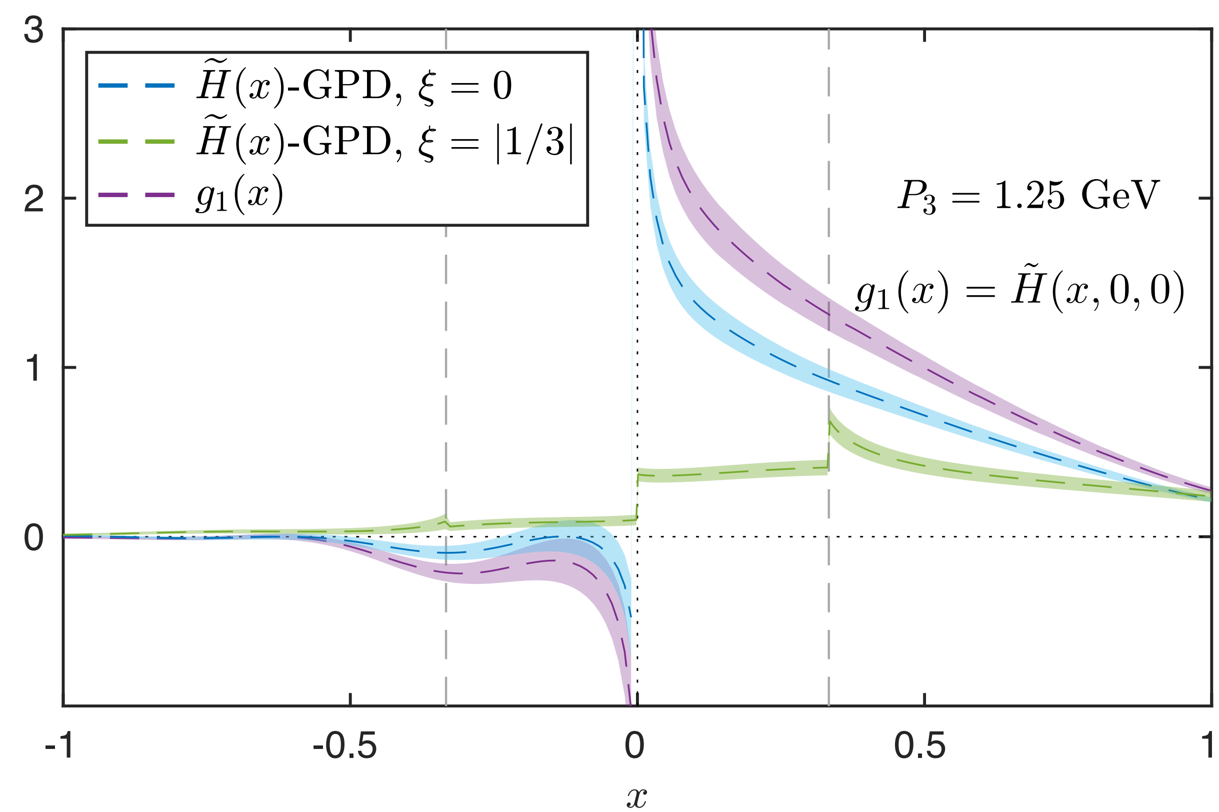
t=-Q[2]

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$









Conclusions

❋ **Precision era of lattice QCD**:

- Mellin moments of PFDs can be extracted precisely including sea quark effects

- Moments yield a lot of interesting physics and also reconstruct the PDFs - demonstrated for the pion and kaon

❋Continuum extrapolations **directly at the physical mass point** is now reality —> yield results in the continuum limit for the spin eliminating cut-off artefacts. Additional ensembles will allow taking the infinite volume limit

❋ Direct computation of PDFs, GPDs, TMDs, other hadrons is possible - see next talks during the day

Extended Twisted Mass Collaboration