Precision Physics & Proton Structure at the LHC

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High Energy Theory Seminar
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Precision Physics & Proton Structure at the LHC

based on

[PRL 127 (2021) 7, 072001; 2102.08039]

[2201.07237]

[work in preparation]

in collaboration with

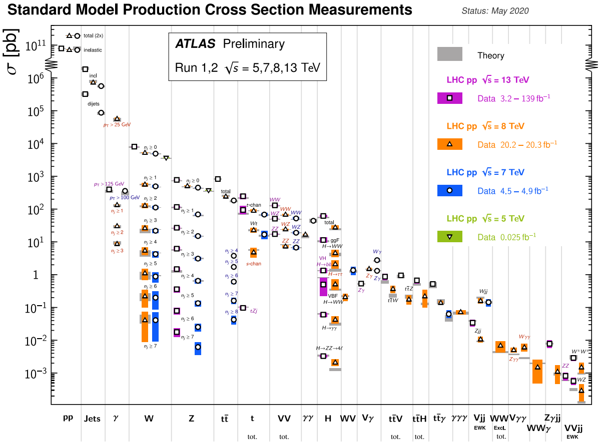
G. Billis, B. Dehnadi, M. Ebert, I. Stewart, Z. Sun, F. Tackmann






The Standard Model – a victim of its own success?

- No hints of new degrees of freedom at the LHC
- No indication from the theory itself at what scales it breaks down
- But neutrino oscillations & dark matter remind us we're not yet done …



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A path forward

- Precision measurements probe BSM beyond direct energy reach $Q \lesssim 0.1\, E_{\rm cm}$:

$$\frac{\Delta\mathcal{O}}{\mathcal{O}} \sim \frac{Q^2}{\Lambda^2_{\rm BSM}} \quad \Leftrightarrow \quad \Lambda_{\rm BSM} \sim Q\sqrt{\frac{\mathcal{O}}{\Delta\mathcal{O}}}$$

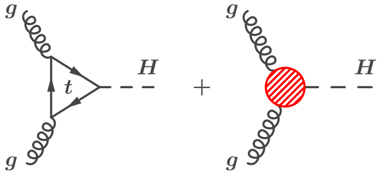
- To control theory uncertainties in $\Delta\mathcal{O}$, need to understand, at the percent level, the QCD radiation patterns that accompany the process of interest at the LHC

- Measure fiducial Higgs cross sections at the LHC, e.g.:

$$gg \to H \to \gamma\gamma \text{ with } p_T^\gamma > p_T^{\text{cut}} \sim 25\,\text{GeV},\, |\eta_\gamma| < \eta_{\text{cut}} \sim 2.4$$

  - Avoids (model dependent) extrapolation to full phase space
  - Most model-independent way we have to search for BSM in the Higgs sector

- Total fiducial cross section measures deviations from SM gluon-fusion rate:



$$= \left(\frac{\alpha_s}{12\pi v} C_t + \frac{v}{\Lambda_{\text{BSM}}^2} C_{HG}\right) H\, G_{\mu\nu}^a G^{a,\mu\nu}$$

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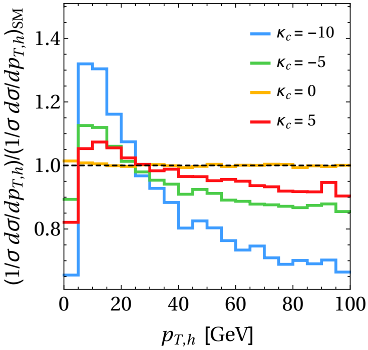
- Total fiducial cross section measures deviations from SM gluon-fusion rate:

	Individual			Marginalised		
SMEFT Coeff.	Best fit [$\Lambda = 1$ TeV]	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]	Best fit [$\Lambda = 1$ TeV]	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]
C_{Hq}	0.00	[-0.017, +0.012]	8.3	-0.05	[-0.11, +0.012]	4.1
$C_{Hq}^{(1)}$	0.02	[-0.1, +0.14]	2.9	-0.04	[-0.27, +0.18]	2.1
C_{Hd}	-0.03	[-0.13, +0.071]	3.1	-0.39	[-0.91, +0.13]	1.4
C_{Hu}	0.00	[-0.075, +0.073]	3.7	-0.19	[-0.63, +0.25]	1.5
$C_{H\Box}$	-0.27	[-1, +0.47]	1.2	-0.9	[-3, +1.2]	0.69
C_{HG}	0.00	[-0.0034, +0.0032]	17.0	0.00	[-0.014, +0.0086]	9.4
C_{HW}	0.00	[-0.012, +0.006]	11.0	0.12	[-0.38, +0.62]	1.4
C_{HB}	0.00	[-0.0034, +0.002]	19.0	0.07	[-0.09, +0.22]	2.5

[Ellis, Madigan, Mimasu, Sanz, You, 2012.02779; Tab. 6]

- Next-to-most basic thing: measure the Higgs transverse momentum
- High $p_T^H \sim \sqrt{\hat{s}} \gg m_H$ increases sensitivity to new operators, but low statistics
- Focus of this talk: $p_T^H \lesssim m_H \sim \sqrt{\hat{s}} \ll 2m_t$ (or p_T^H integrated over)
  - Measure/put bounds on anomalous b, c, and light quark Yukawa couplings
    [Bishara, Haisch, Monni, Re, 1606.09253; Soreq, Zhu, Zupan, 1606.09621]



Motivation: Precision Higgs Physics at the LHC

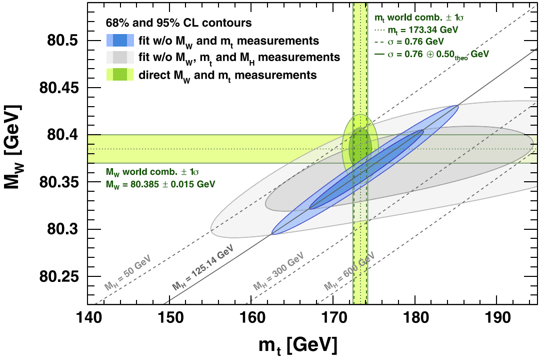
- Recall: Uncertainty $\Delta\sigma$ on SM prediction translates into LHC discovery reach

$$\frac{\Delta\sigma}{\sigma} \sim \frac{m_H^2}{\Lambda_{\mathrm{BSM}}^2} \quad \Leftrightarrow \quad \Lambda_{\mathrm{BSM}} \sim m_H \sqrt{\frac{\sigma}{\Delta\sigma}}$$

Challenges for theory

- QCD corrections to $gg \to H$ are large: $\sigma/\sigma_{\mathrm{LO}} \approx 3$
  - Calculation of inclusive cross section has been pushed to N^{3}LO
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '15-'18]
- But LHC experiments apply kinematic selection cuts on Higgs decay products
  - Need complete interplay of QCD corrections and $\mathcal{O}(1)$ fiducial acceptance
  - Leads to an interesting connection to the physics of **T**ransverse **M**omentum **D**ependent **P**arton **D**istribution **F**unctions!

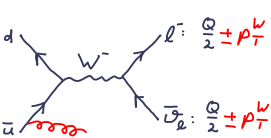
Measuring the W mass as a precision test of the SM:

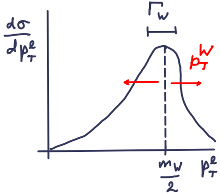


[Baak et al., gfitter collaboration, '14]

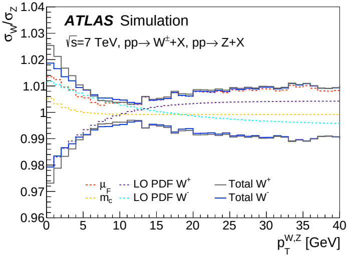
- Blue: Global fit to LEP EW precision observables, use $\frac{m_W}{m_Z} = \cos\theta_W$
  (+ higher-order corrections ...)
- Green: Measure m_W directly
- ▶ Compare!

To measure m_W at the LHC, need theory input because the neutrino is lost:





$\Rightarrow$ Experiments need precise theory predictions for $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$ to model the p_T^W spectrum using precisely measured p_T^Z as input



$$
\begin{aligned}
m_W &= 80370 \pm 7_{\text{stat.}} \\
&\quad \pm 11_{\text{exp. syst.}} \\
&\quad \pm 14_{\text{modelling syst.}}\,\mathrm{MeV} \\
&= 80370 \pm 19\,\mathrm{MeV}
\end{aligned}
$$

[ATLAS, 1701.07240]

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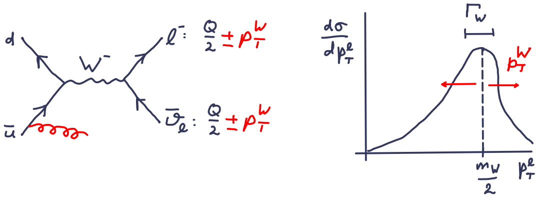

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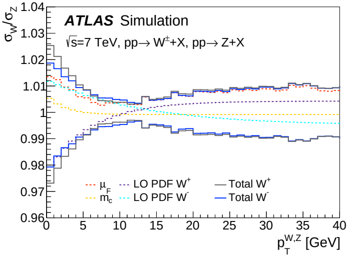
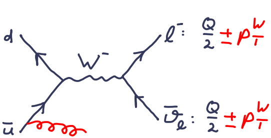


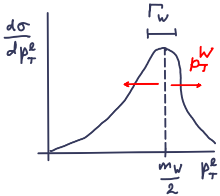
$$
\begin{aligned}
m_W^{\mathrm{LHCb}} &= 80354 \pm 23_{\text{stat.}} \\
&\quad \pm 10_{\text{exp. syst.}} \\
&\quad \pm 17_{\text{theory}} \\
&\quad \pm 9_{\text{PDF}}\ \mathrm{MeV} \\
&= 80354 \pm 32\ \mathrm{MeV}
\end{aligned}
$$

[LHCb, 2109.01113]

Motivation: Measuring m_W at the LHC

To measure m_W at the LHC, need theory input because the neutrino is lost:





$\Rightarrow$ Experiments need precise theory predictions for $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$ to model the p_T^W spectrum using precisely measured p_T^Z as input

~~Challenges~~ Opportunities for theory

- Need sub-percent precision on $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$
  - Leave no stone unturned: QCD three-loop corrections, QED radiative corrections, quark mass effects, nonperturbative transverse structure of the proton
- … and of course, this effort involves:
  Transverse **M**omentum **D**ependent **P**arton **D**istribution **F**unctions!

Outline

1. TMD Factorization and Resummation
2. Third-order Predictions for Fiducial Higgs Production
3. The Drell-Yan q_T Spectrum and Its Uncertainty at $\mathrm{N}^3\mathrm{LL}'$

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1. TMD Factorization and Resummation
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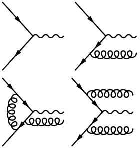
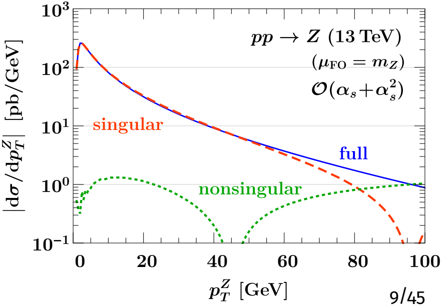
Structure of the fixed-order spectrum for $p_T \ll Q \equiv \sqrt{q^2} = m_{\ell\ell}$:

$$\frac{d\sigma}{dp_T} = \delta(p_T) + \alpha_s \Big[\frac{L}{p_T} + \frac{1}{p_T} + \delta(p_T) + f_1^{\rm nons}(p_T)\Big] + \alpha_s^2 \Big[\frac{L^3}{p_T} + \frac{L^2}{p_T} + \frac{L}{p_T} + \frac{1}{p_T} + \delta(p_T) + f_2^{\rm nons}(p_T)\Big] + \vdots \quad \vdots \quad \vdots \quad \vdots \quad \ddots + \ldots \Big]$$

$$\sim (1/p_T)\Big[\qquad \mathcal{O}(1) \qquad + \mathcal{O}(p_T^2/Q^2)\Big]$$



- "Singular" or "leading power" terms
  - Large logarithms $L \equiv \ln p_T/Q$ left after real/virtual IR poles cancel
  - To be resummed to all orders
- "Nonsingular" or "subleading power"
  - Suppressed by relative p_T^2/Q^2
  - Supplied by matching to full FO

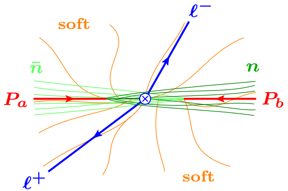


Leading-power terms factorize into hard, collinear, and soft contributions:

[Collins, Soper, Sterman ’85; many different formulations] $x_{a,b} \equiv (Q/E_{\rm cm})e^{\pm Y}$

$$\frac{\mathrm{d}\sigma_{\rm sing}}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}p_T^2} = \sum_{a,b} H_{ab}(Q^2,\mu) \times [B_a B_b S](Q^2, x_a, x_b, \vec{p}_T, \mu)$$

$$[B_a B_b S] \equiv \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta^{(2)}(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s) \times B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$



- Hard function $H_{ab} = \sigma^{\rm LO}_{ab\to Z} \times$ $\overline{\rm MS}$-renormalized quark (form factor)^2
- Beam and soft functions individually feature so-called rapidity divergences
- Regularize and renormalize $\Rightarrow$ 2D renormalization group in (μ, ν)

Resummation basics: Factorization at leading power

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$$\times\; B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$
$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu)$$
$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2}\, e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{f}_a^{\rm TMD}(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_b^{\rm TMD}(x_b, b_T, \mu, \zeta_b)$$

- Fourier transform to turn convolution into a product in $\vec{b}_T$ space
- Often: Combine beam and $\sqrt{\text{soft}}$ function into a TMD PDF that runs as a function of (μ, ζ)
- Collins-Soper scales $\zeta_{a,b}$ = energies² of the scattering partons, e.g. $= Q^2$ in Z rest frame
  - Evolution with respect to ζ governed by Collins-Soper kernel $\gamma_\zeta^q(b_T, \mu)$

Leading-power terms factorize into hard, collinear, and soft contributions:

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$$\times\; B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$

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- For $b_T \ll 1/\Lambda_{\rm QCD}$, TMD PDFs can be calculated in terms of collinear PDFs:

$$\tilde{f}_i^{\rm TMD}(x, b_T, \mu, \zeta) = \sum_j \int \frac{\mathrm{d}z}{z}\, C_{ij}(z, b_T, \mu, \zeta)\, f_j\Big(\frac{x}{z}, \mu\Big)\, F_i^{\rm NP}(x, b_T, \zeta)$$

$$F_i^{\rm NP}(x, b_T, \zeta) = 1 + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2)$$

Resummation basics: Breaking up large logs and solving evolution equations

$$\frac{d\sigma_{\rm sing}}{p_T} = H(Q,\mu) \times B(p_T,\mu,\nu/Q)^2 \otimes S(p_T,\mu,\nu/p_T)$$

$$\ln^2\frac{p_T}{Q} = 2\ln^2\frac{Q}{\mu} + 2\ln\frac{p_T}{\mu}\ln\frac{\nu}{Q} + \ln\frac{p_T}{\mu}\ln\frac{\mu\, p_T}{\nu^2}$$

- For generic μ, ν, each function contains (potentially large) logs
- Resummation follows from solving RGEs, and evolving each function from starting scales μ_i, ν_i to common arbitrary μ, ν

$$H(\mu) = H(\mu_H) \times U_H(\mu_H, \mu)$$
$$B(\mu,\nu) = B(\mu_B,\nu_B) \otimes U_B(\mu_B,\nu_B;\mu,\nu)$$
$$S(\mu,\nu) = S(\mu_S,\nu_S) \otimes U_S(\mu_S,\nu_S;\mu,\nu)$$

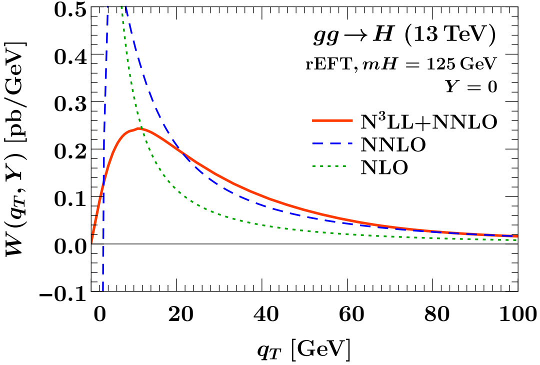
- In b_T space: $\mu_B, \mu_S \sim 1/b_T$, evolution is multiplicative

$$\Rightarrow \quad \frac{d\sigma^{\rm res}_{\rm sing}}{p_T} = \frac{1}{p_T}\exp\left(-\alpha_s \ln\frac{\mu_H}{\mu_S}\right)\left[1+\alpha_s\ln\frac{p_T}{\mu_S}\right]\left[1+\alpha_s\ln\frac{\mu_H}{Q}\right]$$

[Very schematic!]

$$\Rightarrow \quad \frac{d\sigma_{\rm sing}^{\rm res}}{p_T} = \frac{1}{p_T} \exp\left(-\alpha_s \ln \frac{\mu_H}{\mu_S}\right) \left[1 + \alpha_s \ln \frac{p_T}{\mu_S}\right] \left[1 + \alpha_s \ln \frac{\mu_H}{Q}\right]$$

[Very schematic!]



Order	Boundary cond. (FO singular)	Anomalous dimensions γ_i (noncusp)	Γ_{cusp}, β	FO matching (nonsingular)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL′ (+NLO$_0$)	α_s	1-loop	2-loop	α_s
NNLL (+NLO$_0$)	α_s	2-loop	3-loop	α_s
NNLL′ (+NNLO$_0$)	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL (+NNLO$_0$)	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL′ (+N^{3}LO$_0$)	α_s^3	3-loop	4-loop	α_s^3
N^{4}LL (+N^{3}LO$_0$)	α_s^3	4-loop	5-loop	α_s^3

- Resummation order is uniquely specified by perturbative order of boundary coefficients and anomalous dimensions (each is convergent on its own)
- Can show that with these ingredients, all (next-to)^{n}leading logarithmic terms $\alpha_s^k L^{k+1+n}$ in $\ln(\mathrm{d}\sigma/\mathrm{d}q_T)$ are captured for all $k \geq 1$
- At "primed" orders, boundary conditions are included to α_s^n higher in addition
  - Improves residual dependence on boundary scales
  - Ensures integral of (reexpanded) matched spectrum is N^{n}LO cross section

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Fiducial predictions for $gg \to H \to \gamma\gamma$ at three loops

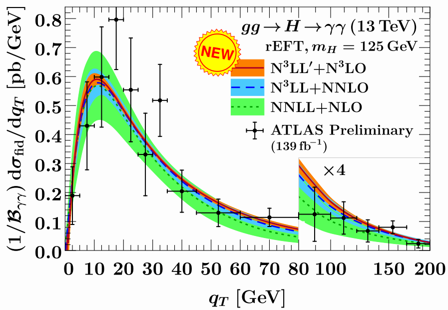
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

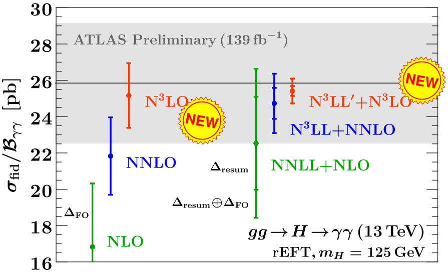
$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

Goal

[Billis, Dehnadi, Ebert, JM, Tackmann, PRL 127 (2021) 7, 072001, 2102.08039]

- Compute fiducial spectrum for $q_T \equiv p_T^H = p_T^{\gamma\gamma}$ at N^{3}LL′+N^{3}LO
- Compute total fiducial cross section at N^{3}LO, and improved by resummation





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- Compute total fiducial cross section at N^{3}LO, and improved by resummation

- Previous state of the art was N^{3}LL(+NNLO$_1$) and NNLO, respectively
  [Chen et al. ’18; Bizoń et al. ’18; Gutierrez-Reyes et al. ’19; Becher, Neumann ’20]

Kicked off a recent push for fiducial color singlet at complete three-loop accuracy:

- Complementary N^{3}LO results for fiducial $Y_{\gamma\gamma}, \eta_{\gamma 1}, \Delta\eta_{\gamma\gamma}$ (with different method)
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]
- Fiducial N^{3}LL′ results for Higgs q_T spectrum
  [Re, Rottoli, Torrielli, 2104.07509]
  [For Drell-Yan, $\gamma\gamma$, see also 2103.04974, 2106.11260, 2107.12478, 2111.14509]

Fiducial predictions for $gg \to H \to \gamma\gamma$ at three loops

Conside… $p_T^{\gamma 1}$ … ci… , … 52]

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- Com… p_T^H
- Com… t N… on
- Prev… NL…
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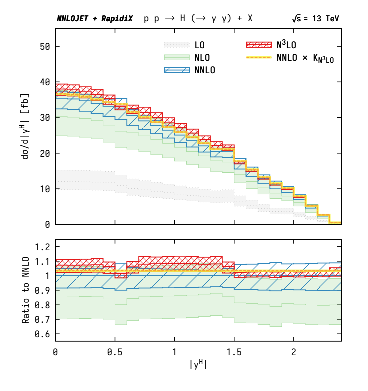
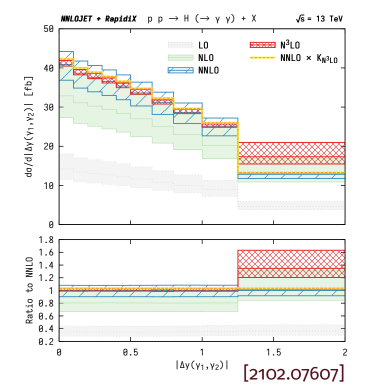




Kicked off a recent push for fiducial color singlet at complete three-loop accuracy:

- Complementary N^{3}LO results for fiducial $Y_{\gamma\gamma}, \eta_{\gamma 1}, \Delta\eta_{\gamma\gamma}$ (with different method)
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]
- Fiducial N^{3}LL$'$ results for Higgs q_T spectrum
  [Re, Rottoli, Torrielli, 2104.07509]
  [For Drell-Yan, $\gamma\gamma$, see also 2103.04974, 2106.11260, 2107.12478, 2111.14509]

Fiducial predictions for $gg \to H \to \gamma\gamma$ at three loops

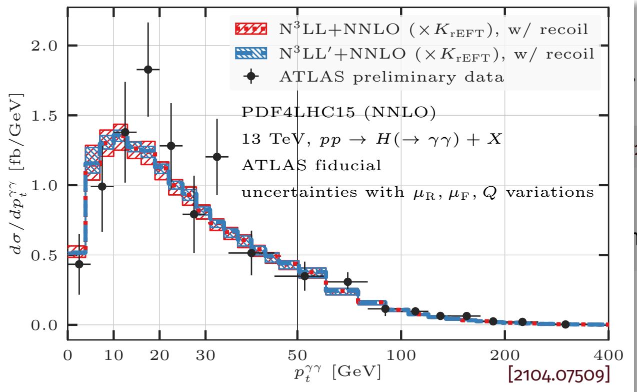
Consider gg

$p_T^{\gamma 1} \geq$ (… $.37, 1.52]$

Goal … 2001, 2102.08039]

- Compute …
- Compute … mmation

- Previous …
  [Chen et al. …



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Set up some notation, use that production and decay (acceptance) factorize:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W(q_T, Y)\,, \quad A_{\mathrm{incl}} = 1\,, \quad W(q_T, Y) = \frac{\mathrm{d}\sigma_{\mathrm{incl}}}{\mathrm{d}q_T\, \mathrm{d}Y}$$

Fiducial power corrections are those power corrections to TMD factorization coming from the q_T-dependent acceptance:

$$\frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T} \equiv \int \mathrm{d}Y \Big[A(q_T, Y; \Theta) - A^{(0)}(Y; \Theta)\Big] W^{(0)}(q_T, Y)$$

- These uniquely predict all linear power corrections $\mathrm{d}\sigma^{(1)}$ because

$$A(q_T, Y; \Theta) = A^{(0)}(Y; \Theta) \quad \Big[1 + \mathcal{O}\Big(\frac{q_T}{m_H}\Big)\Big]$$

$$W(q_T, Y) = W^{(0)}(q_T, Y) \Big[1 + \mathcal{O}\Big(\frac{q_T^2}{m_H^2}\Big)\Big]$$

[Presence of linear terms pointed out in Ebert, Tackmann, 1911.08486]

[Factorization/resummation & use in subtractions: Ebert, JM, Stewart, Tackmann, 2006.11382]

[Analytic results in double-logarithmic approximation: Salam, Slade, 2106.08329]

[See also Alekhin et al., 2104.02400; Buonocore et al., 2111.13661; Camarda et al., 2111.14509]

Challenge

Fiducial power corrections upset fixed-order perturbative convergence of *total* σ_{fid}

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T}$:

$$\sigma^{\mathrm{FO}}_{\mathrm{incl}} = 13.80\,[1 + 1.291 + 0.783 + 0.299]\ \mathrm{pb}$$

$$\sigma^{\mathrm{FO}}_{\mathrm{fid}} = 6.928\,[1 + 1.429 + 0.723 + 0.481]\ \mathrm{pb}$$

$$= 6.928\,[1 + (1.300 + 0.129_{\mathrm{fpc}}) + (0.784 - 0.061_{\mathrm{fpc}}) + (0.331 + 0.150_{\mathrm{fpc}})]\ \mathrm{pb}$$

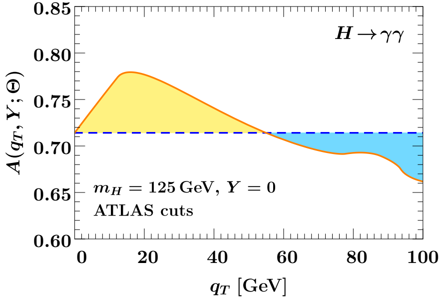
- Fiducial power corrections show no convergence, remainder is similar to inclusive

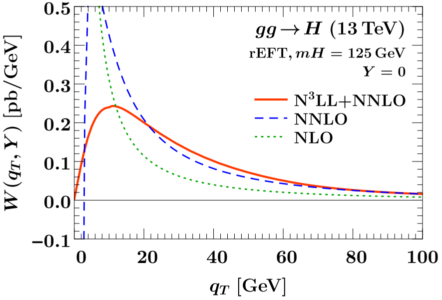
Challenge

Fiducial power corrections upset fixed-order perturbative convergence of *total* σ_{fid}

Two ways to understand the effect of small q_T on total cross section:

1. Acceptance acts as a weight under the q_T integral





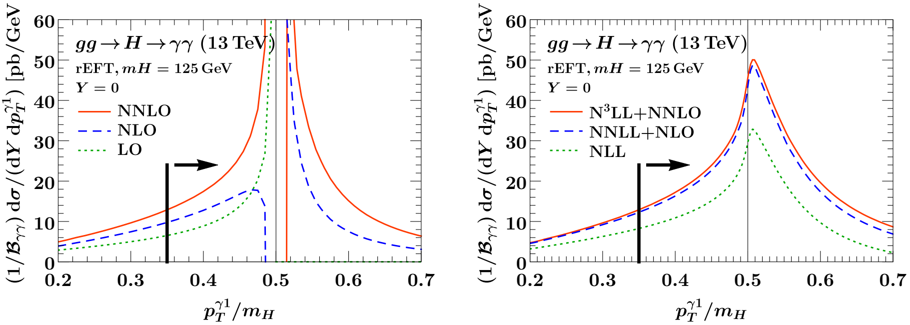
$$\sigma_{\text{incl}} = \int \mathrm{d}q_T\, W(q_T) \qquad \sigma_{\text{fid}} = \int \mathrm{d}q_T\, A(q_T)\, W(q_T)$$

Challenge

Fiducial power corrections upset fixed-order perturbative convergence of *total* σ_{fid}

Two ways to understand the effect of small q_T on total cross section:

1. Acceptance acts as a weight under the q_T integral
2. We're cutting on the resummation-sensitive photon p_T



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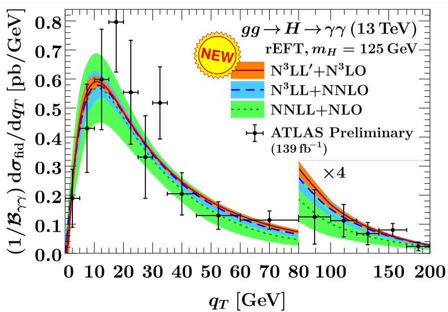
Solution

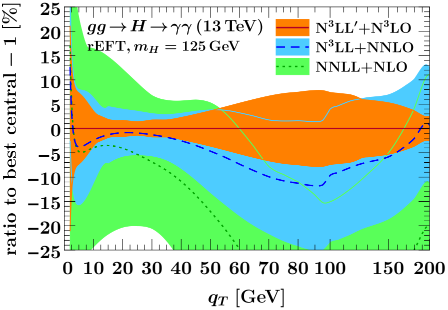
Resum fiducial power corrections to the same N^{3}LL′ accuracy as leading power by resumming $W^{(0)} = H\, B \otimes B \otimes S$ and keeping $A(q_T, Y; \Theta)$ exact:

$$\frac{\mathrm{d}\sigma_{\mathrm{res}}^{\mathrm{fpc}}}{\mathrm{d}q_T} = \int \mathrm{d}Y \Big[A(q_T, Y; \Theta) - A^{(0)}(Y; \Theta)\Big] W_{\mathrm{res}}^{(0)}(q_T, Y)$$

- Effect is fully predicted by TMD factorization & resummed perturbation theory

Results: The fiducial q_T spectrum at N^{3}LL′+N^{3}LO



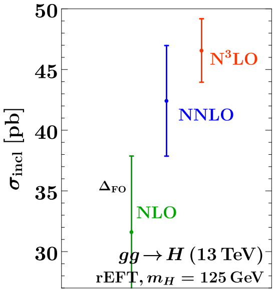


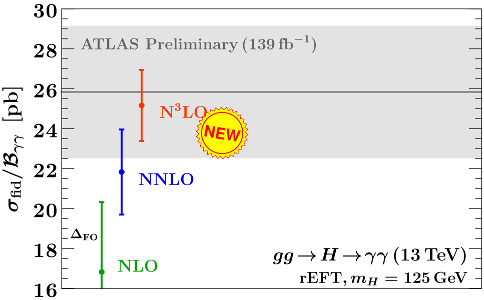
- Good agreement with (at the time) preliminary ATLAS Run 2 data
  - Divide $H \to \gamma\gamma$ branching ratio $\mathcal{B}_{\gamma\gamma}$ out of data [LHC Higgs Cross Section WG, 1610.07922]
  - Data are corrected for other production channels, photon isolation efficiency [ATLAS, 1802.04146]

- Observe excellent perturbative convergence & uncertainty coverage

▶ Perturbative uncertainties estimate by varying RG boundary scales

Results: The total fiducial cross section at N^{3}LO and N^{3}LL′+N^{3}LO

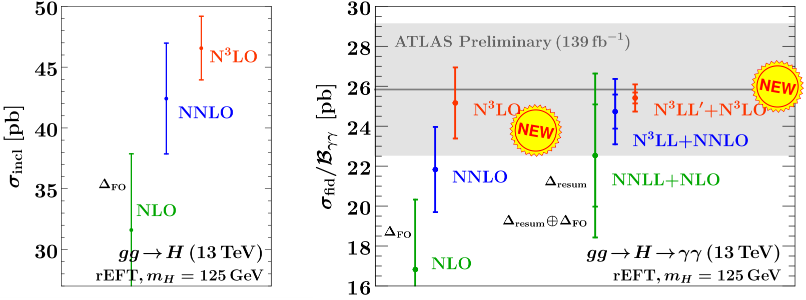




- Large N^{3}LO correction to fiducial cross section (larger than inclusive)
  - Caused by fiducial power corrections, *not* captured by rescaling inclusive N^{3}LO result
- TMD resummation restores convergence, gives detailed handle on uncertainty:

N^{3}LO: $$\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.16 \pm 1.78_{\rm FO} \pm 0.12_{\rm nons})\,\text{pb}$$

N^{3}LL′+N^{3}LO: $$\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.41 \pm 0.59_{\rm FO} \pm 0.21_{q_T} \pm 0.17_{\varphi} \pm 0.06_{\rm match} \pm 0.20_{\rm nons})\,\text{pb}$$



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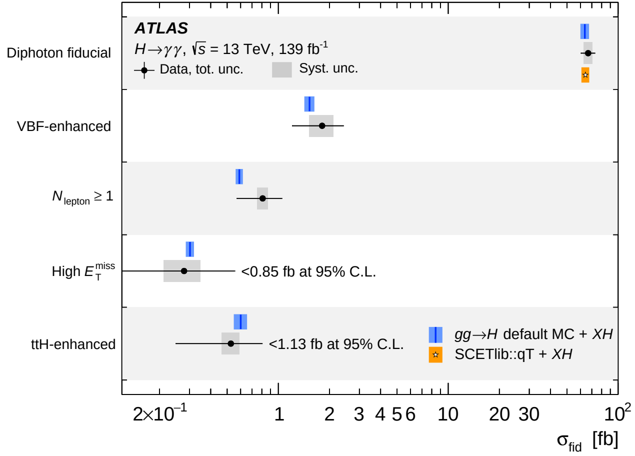
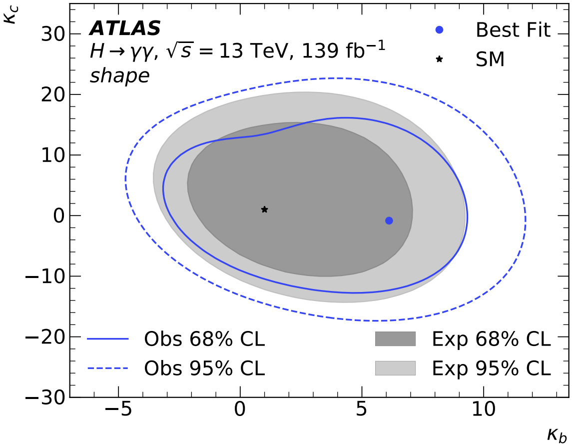


[ATLAS, 2202.00487]

- Default MC = resummed fiducial two-loop prediction reweighted to N^3LO inclusive
  - A priori unclear whether this is three-loop accurate

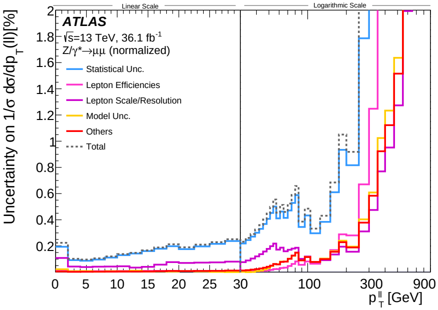


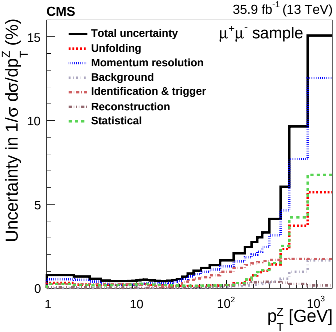
[ATLAS, 2202.00487]

- Limits on anomalous y_b, y_c couplings use our precise N^{3}LL$'$ fiducial y_t^2 baseline
- Also provided consistent two-loop $y_{b,c}y_t$ and $y_{b,c}^2$ signal templates to ATLAS

1. TMD Factorization and Resummation
2. Third-order Predictions for Fiducial Higgs Production
3. The Drell-Yan q_T Spectrum and Its Uncertainty at $\mathrm{N^3LL'}$

LHC data for fiducial Drell-Yan production are *exquisite*:





Goal

Sub-percent-level perturbative description of $pp \to Z/\gamma^* \to \ell^+\ell^-$ at the LHC.

Perturbative ingredients: Factorized TMD cross section at N^3LL'

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\rm res}_{\rm fact}}{dq_T} + \left[\frac{d\sigma^{\rm FO}_{\rm full}}{dq_T} - \frac{d\sigma^{\rm FO}_{\rm fact}}{dq_T}\right] \equiv \frac{d\sigma^{\rm res}_{\rm fact}}{dq_T} + \frac{d\sigma^{\rm nons}_{\rm fact}}{dq_T}$$

$$\frac{d\sigma_{\rm fact}}{dQ\,dY\,dq_T} = \sum_q H_{q\bar{q}}(Q,\mu)\;\; q_T \int_0^\infty db_T\, b_T\, J_0(q_T b_T) \times f_q^{\rm TMD}(x_a, b_T, \mu, \zeta)\, f_{\bar{q}}^{\rm TMD}(x_b, b_T, \mu, \zeta) + (q \leftrightarrow \bar{q})$$

Implemented in SCETlib C++ numerical library [Ebert, JM, Tackmann]:

- Three-loop hard function [Baikov et al. ’09; Lee et al. ’10; Gehrmann et al. ’10, ’20; Czakon et al. ’21]
- Three-loop matching of TMD PDFs onto collinear PDFs [Li, Zhu, ’16; Luo, Yang, Zhu, Zhu ’19; Ebert, Mistlberger, Vita ’20]
  - Prediction includes complete three-loop RG boundary conditions (N^3LL')
  - Integral of spectrum is N^3LO-accurate
- Four-loop cusp, three-loop noncusp anomalous dimensions [Brüser, Grozin, Henn, Stahlhofen ’19; Henn, Korchemsky, Mistlberger ’20; v. Manteuffel, Panzer, Schabinger ’20] [Moch, Vermaseren, Vogt ’05; Idilbi, Ma, Yuan ’06]
- Three-loop Collins-Soper kernel [Li, Zhu, ’16; Vladimirov ’16]

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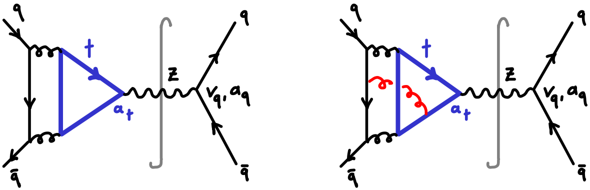

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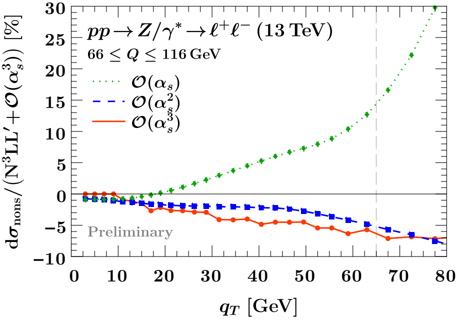
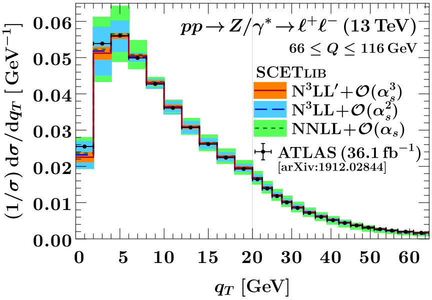
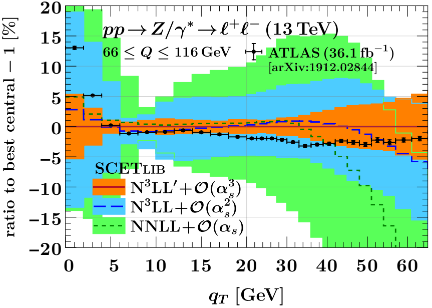
$$\frac{\mathrm{d}\sigma_{\mathrm{nons}}}{\mathrm{d}q_T} = \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{sing}}}{\mathrm{d}q_T} = \frac{1}{q_T}\mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big)$$



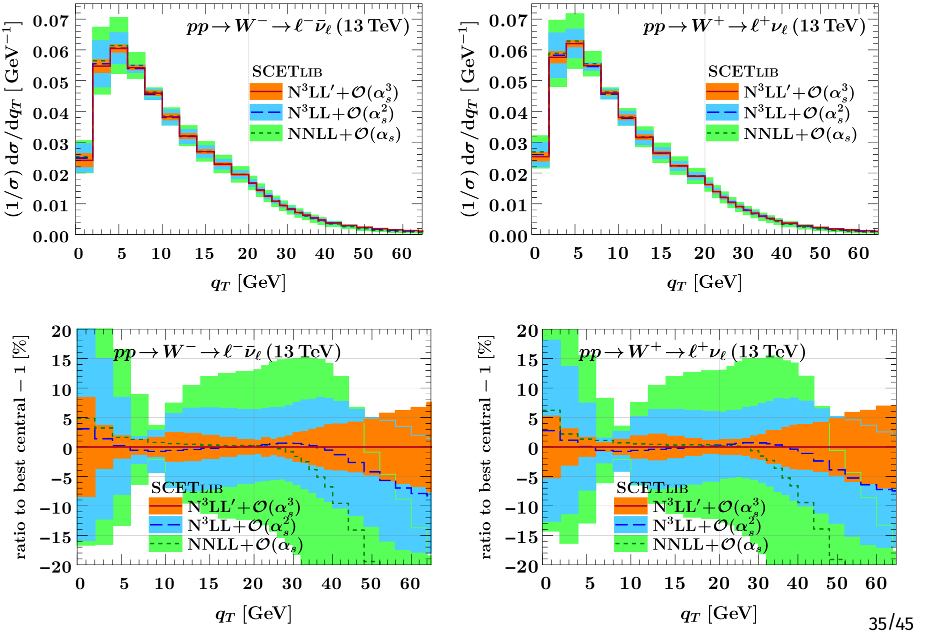
- In-house analytic implementation of all helicity structure functions at $\mathcal{O}(\alpha_s)$
- Fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^2)$ from MCFM
  [Campbell, Ellis, et al. '99, '15]
- Very recently: Precise fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^3)$ from NNLOjet
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]
- Use that $\left|\int_0^{8\,\mathrm{GeV}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{(3)}_{\mathrm{nons}}}{\mathrm{d}q_T}\right| \leq 1\mathrm{pb}$ to drop $\mathrm{d}\sigma^{(3)}_{\mathrm{nons}}$ below 8 GeV

- Central results use MSHT20nnlo with $\alpha_s(m_Z) = 0.118$, $n_f = 5$
- NNLO (= three-loop!) PDF evolution formally sufficient at N^{3}LL′:
  - DGLAP kernels are a noncusp anomalous dimension
  - Scale dependence cancels within three-loop beam function
  - Separate question whether PDFs should have been extracted using three-loop $\hat{\sigma}_{ij}$

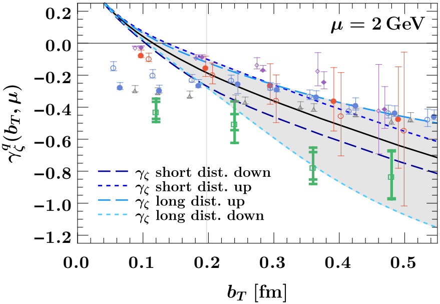




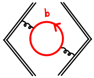
- Excellent perturbative convergence towards three-loop result
- Higher orders are covered by uncertainty estimate at lower orders
  …see backup slides for how they are estimated


$pp \to W^- \to \ell^- \bar{\nu}_\ell$ (13 TeV)
$pp \to W^+ \to \ell^+ \nu_\ell$ (13 TeV)
SCETLIB
N^{3}LL$'$+$\mathcal{O}(\alpha_s^3)$
N^{3}LL+$\mathcal{O}(\alpha_s^2)$
NNLL+$\mathcal{O}(\alpha_s)$
$(1/\sigma)\, d\sigma/dq_T$ [GeV^{-1}]
ratio to best central − 1 [%]
q_T [GeV]

$$\frac{1}{2}\gamma^q_{\nu,\mathrm{NP}}(b_T) = \gamma^q_{\zeta\,\mathrm{NP}}(b_T) = c^i_\zeta \tanh\left(\frac{\omega^2_{\zeta,i}}{|c_\zeta|} b_T^2\right) = \mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i} b_T^2 + \mathcal{O}(b_T^4)$$



- Vary either ω_ζ (“short distance”) or c_ζ (“long distance”) to cover lattice results
  [Collection of lattice data reproduced from Shanahan, Wagman, Zhao, 2107.11930]
- Pick central value of $\mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i}(1 \pm 2)$ to serve as bias correction for known leading (NNLL) bottom quark mass effect in γ^q_ζ:



$$\Delta\gamma^q_\zeta(b_T, m_b, \mu) = \frac{\alpha_s^2}{\pi^2} C_F T_F (m_b b_T)^2 \left(\ln \frac{b_T^2 m_b^2}{4e^{-2\gamma_E}} - 1\right) \approx -(0.25\,\mathrm{GeV})^2 b_T^2$$

NOTE Compatible with [Scimemi, Vladimirov ’19; Bacchetta et al. ’19], but aim for a-priori prediction

- Most general structure of leading NP correction $b_T^2 \Lambda_i^{(2)}(x)$ is complicated
- However, can show that for a given process and fiducial volume, only *a single average coefficient* $\overline{\Lambda}$ remains after the integral over hard phase space Φ_B:
  [Ebert, JKLM, Stewart, Sun '22]

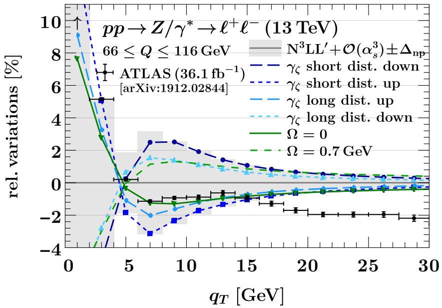
$$\tilde{\sigma}(b_T) = \tilde{\sigma}^{(0)}(b_T)\Big\{1 + b_T^2\Big(2\overline{\Lambda}^{(2)} + \gamma_{\zeta,q}^{(2)} L_{Q^2}\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\rm QCD} b_T)^4\big]$$

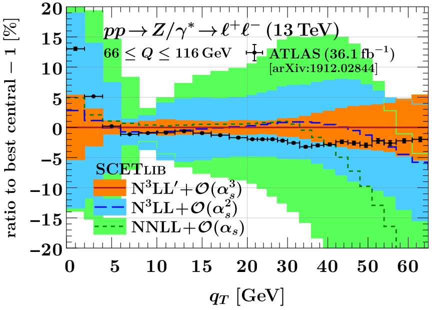
$$\overline{\Lambda}^{(2)} = \frac{\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)\big[\Lambda_i^{(2)}(x_a) + \Lambda_j^{(2)}(x_b)\big]}{2\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)}$$

- ▶ Idea: Promote $\overline{\Lambda}^{(2)}$ to a single-parameter Gaussian model

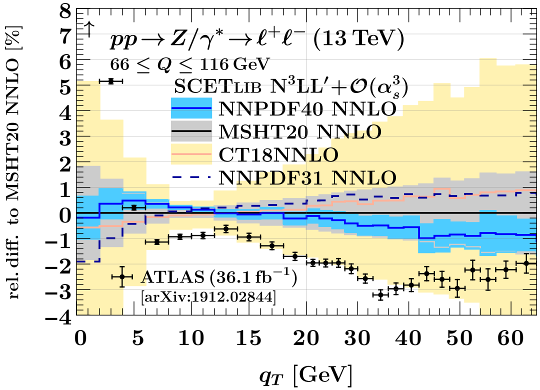
$$f_i^{\rm NP}(x, b_T) = \exp(-\Omega^2 b_T^2) \quad \text{with} \quad \overline{\Lambda}^{(2)} = -\Omega^2$$

- Take central $\Omega = 0.5\,\mathrm{GeV}$ and vary it as $\Omega = \{0, 0.7\}\,\mathrm{GeV}$
- ▶ For $q_T \gg \Lambda_{\rm QCD}$, this captures the most general form of the leading NP correction to the rapidity-integrated q_T spectrum

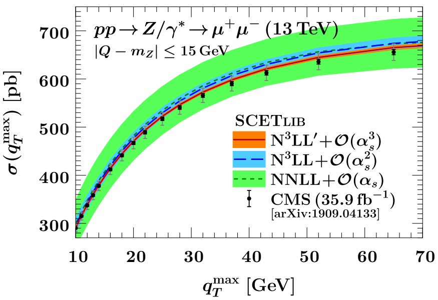


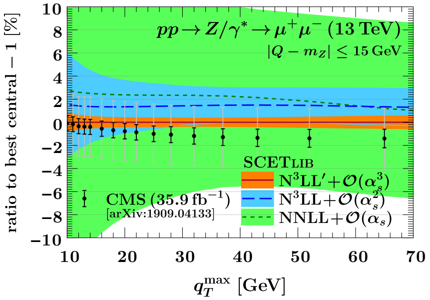


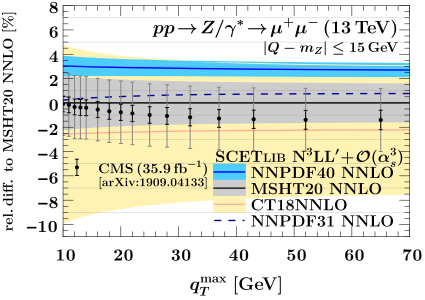
- Taken at face value, the lowest bins seem to prefer *weaker* NP effects
- N^{3}LL closer to data for $q_T \le 15\,\mathrm{GeV}$ with our default NP parameters, suggesting that three-loop and NP corrections can be traded off for one another
- Overshoot data at $q_T = 20 - 30\,\mathrm{GeV}$, way outside NP effect strength



- PDF uncertainty largely cancels in normalized spectrum
- Cannot explain overshoot at $q_T = 20 - 30\,\mathrm{GeV}$

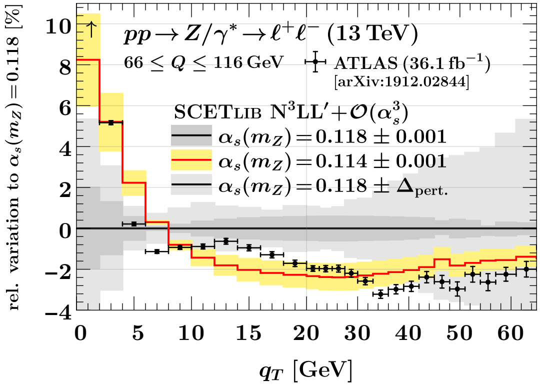






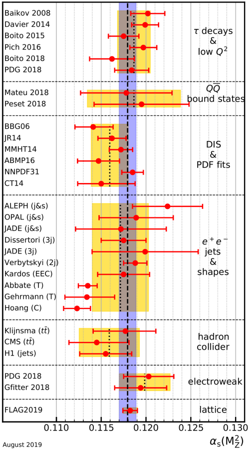
- Cumulative cross section distinguishes recent PDF sets
- $\mathcal{O}(b_T^2 \Lambda_{\mathrm{QCD}}^2)$ effects $\leq 0.1\%$ past $q_T^{\max} \sim 20\,\mathrm{GeV}$
- Hold small $\alpha_s^{2,3}$ nonsingular fixed
  - Whole figure at few 100 CPUh
- Promising target for N^{3}LO PDF fits

Results: Impact of α_s on normalized Z spectrum



- Parametric uncertainty due to $\alpha_s(m_Z)$ on par with perturbative uncertainty
- Overshoot at $q_T = 20 - 30\,\text{GeV}$ is naturally explained by lower $\alpha_s(m_Z)$

This is not unprecedented …



- Lower values of $\alpha_s(m_Z)$ have previously been reported in fits to e^+e^- event shapes (thrust and C parameter)

  DISCLAIMER: This was *not* an actual fit to $\{\alpha_s(m_Z), \Omega, \omega_\zeta^{(2)}\}$.

- Like $p_T^{Z/W}$, these are driven by all-order resummation …

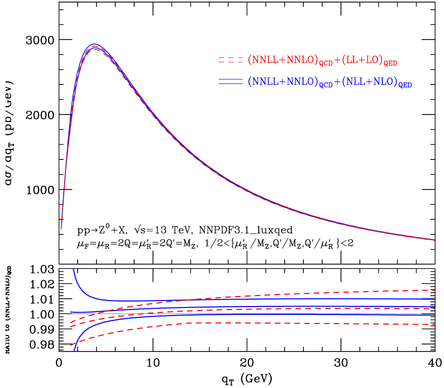
T. Rex Might Have Had Close Cousins

New York Times, March 1, 2022

"That's not the kind of thing you should be doing based on femur robusticity and the presence or absence of a tooth," Dr. Hone added. "If you're going to shoot for the king, don't miss."

Systematics for $Z/W^{\pm}$ production at the theory frontier:

- QED effects for on-shell Z well understood
  [Bacchetta, Echevarria '18; Cieri, Ferrera, Sborlini '18; Billis, Tackmann, Talbert '19]
  - Expected to be $\sim 1\%$, but would bring the tail up *more*



[Cieri, Ferrera, Sborlini 1805.11948]

Systematics for $Z/W^{\pm}$ production at the theory frontier:

- QED effects for on-shell Z well understood
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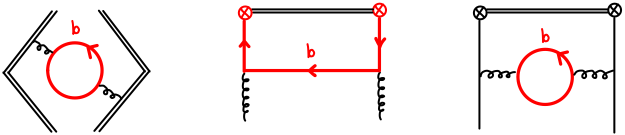
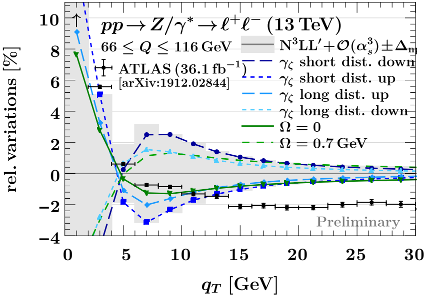
  - Expected to be $\sim 1\%$, but would bring the tail up *more*

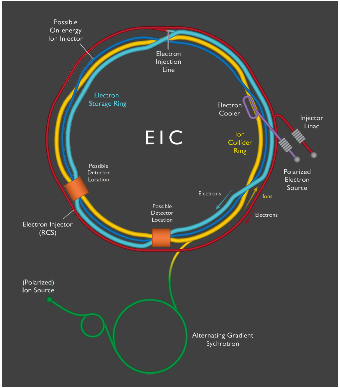
- QED radiative corrections to full process and interplay with TMDs are *challenging*
- Subleading power TMD factorization of *nonsingular* cross section
  [Progress towards doing this at least for $\mathcal{O}(q_T/Q)$ azimuthal correlations!]
  [Moos, Scimemi, Vladimirov '21-'22; Ebert, Gao, Stewart '21]
- Probably most important: Full treatment of mass effects/flavor thresholds
  - Expect impact on spectrum (and cumulative cross section) to be suppressed by $\# \, m_b^2/q_T^2$?



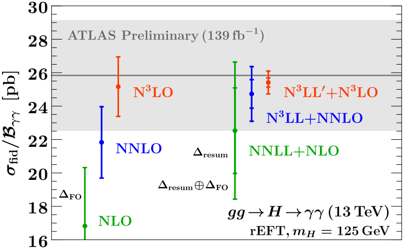
Important challenge: How to connect TMD NP parameter Ω between Z and $W^{\pm}$?

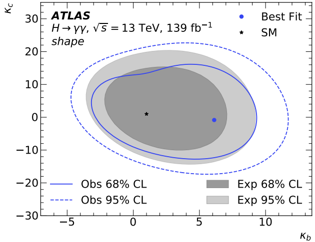
- ▶ How similar are the transverse distributions of u vs. d, $\bar{u}$ vs. $\bar{d}$ *really*?
  - Essentially unconstrained in current global TMD PDF fits
  - Flavor structure can have $\pm 10\,\mathrm{MeV}$ impact on p_T^ℓ-based m_W measurement [Bacchetta, Bozzi, Radici, Ritzmann, Signori '18]
- ▶ EIC to the rescue!

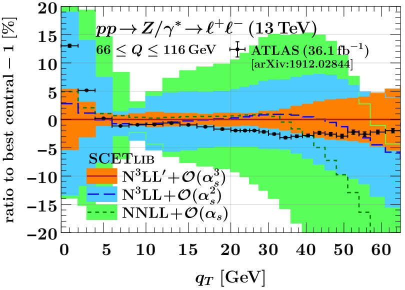


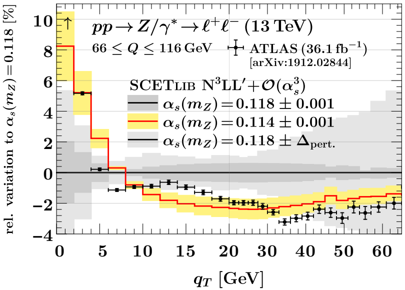


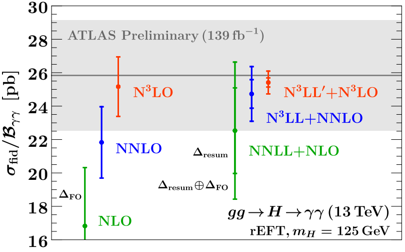
Summary (in pictures)

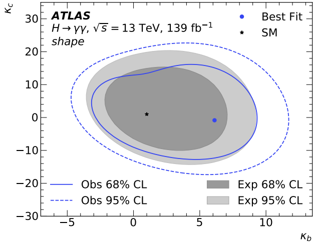


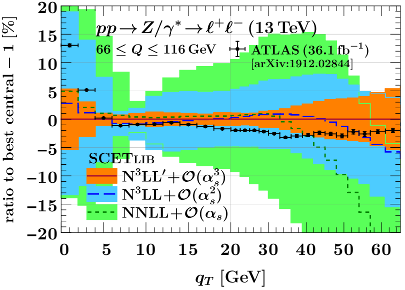


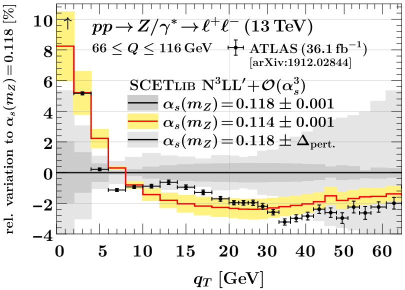








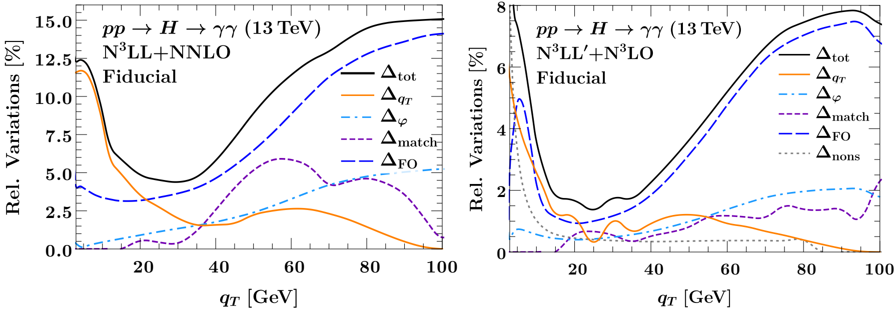




Thank you for your attention!

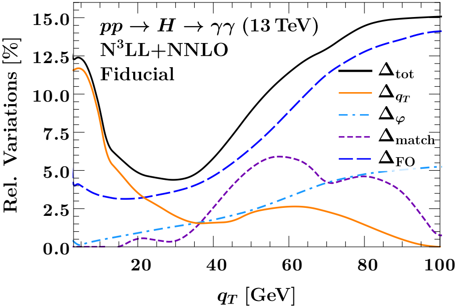
Backup (Higgs)

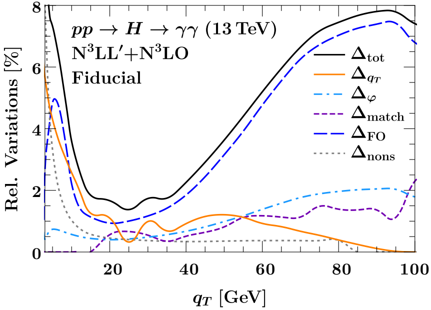
Uncertainty breakdown



$$\Delta_{\rm tot} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{\rm match} \oplus \Delta_{\rm FO} \oplus \Delta_{\rm nons}$$

- Probes higher-order resummed terms $\sim \ln q_T / m_H$
- Estimated by envelope of 36 different combinations of independently varying $\{\mu_B, \mu_S, \dots\}$ in $W^{(0)} = H\, B \otimes B \otimes S$

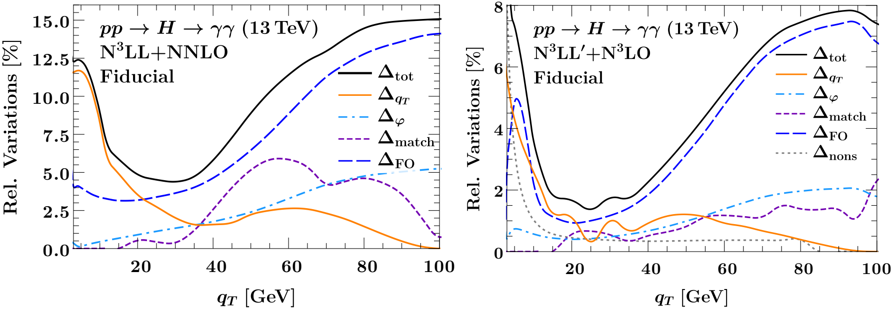




$$\Delta_{\rm tot} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{\rm match} \oplus \Delta_{\rm FO} \oplus \Delta_{\rm nons}$$

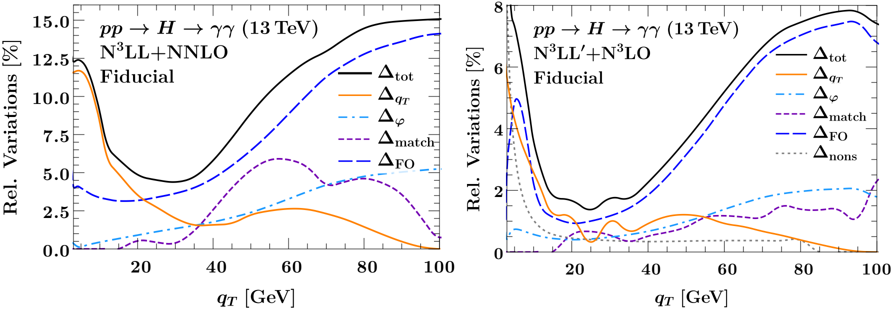
- Probes higher-order terms $\sim \ln \frac{-m_H^2 - i0}{\mu_H^2} = -i\pi$ in timelike gluon form factor
- Estimated by varying phase of complex hard scale over $\arg \mu_H \in \{\pi/4, 3\pi/4\}$

Uncertainty breakdown



$$\Delta_{\text{tot}} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{\text{match}} \oplus \Delta_{\text{FO}} \oplus \Delta_{\text{nons}}$$

- Uncertainty from matching scheme between resummed peak and fixed-order tail
- Estimated by varying the transition points governing resummation turn-off
- Turn-off implemented by *profile scales* $\mu_{B,S} \to \mu_{\text{FO}}$
  - Ambiguity manifestly reduces at each order



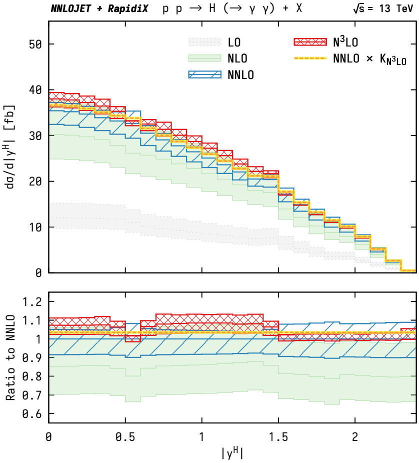
$$\Delta_{\mathrm{tot}} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{nons}}$$

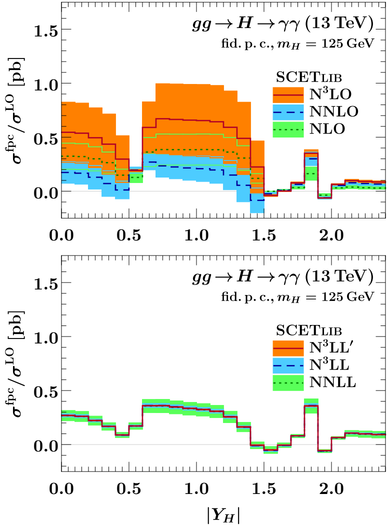
- Fixed-order uncertainty
- Estimated by standard variations of overall $\mu_{\mathrm{FO}} = \mu_R$ (dominates over μ_F)
- Fit/MC uncertainty on extraction of nonsingular terms $\sim q_T^2/m_H^2$

  …see separate backup slide

- "Infrared sensitivity" observed also in other Higgs observables at N^{3}LO
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

⇔ Precisely the fiducial power corrections we can analytically deal with and resum





Note: Plots on the right show *only* σ^{fpc}.

Resummation effects in other $H \to \gamma\gamma$ observables

- "Infrared sensitivity" observed also in other Higgs observables at $\mathrm{N^3LO}$
  [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

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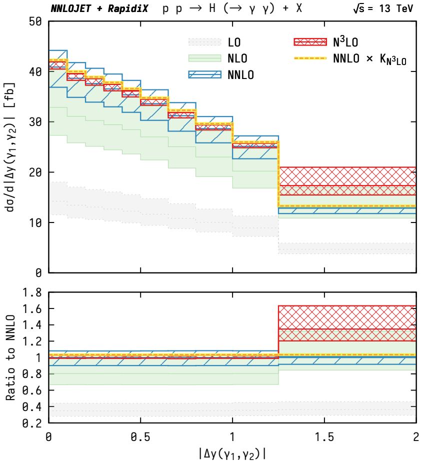

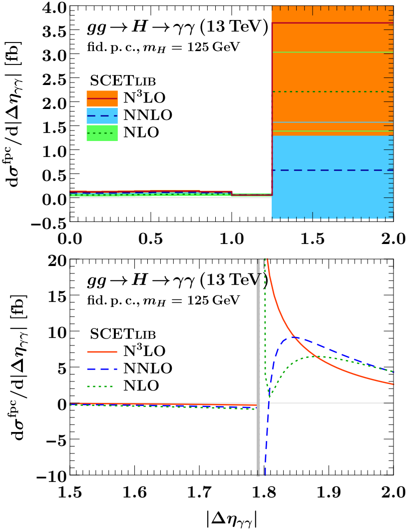

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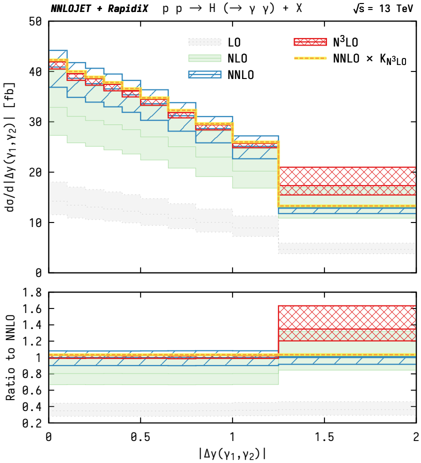
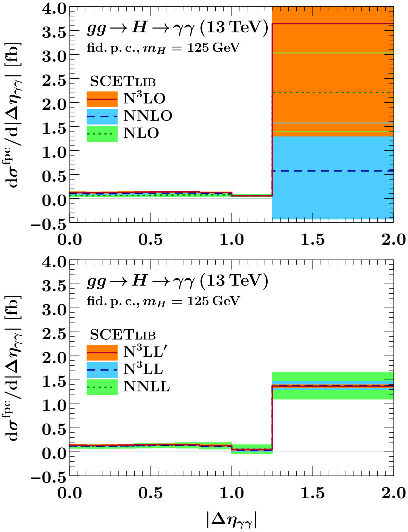
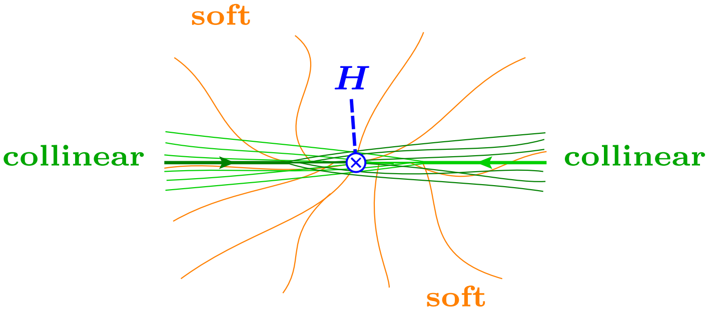




Note: Plots on the right show *only* σ^{fpc}.

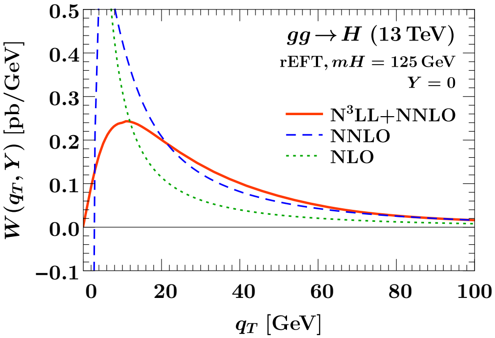
At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a \, \mathrm{d}^2\vec{k}_b \, \mathrm{d}^2\vec{k}_s \, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big) \times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu) \, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu) \, S(\vec{k}_s, \mu, \nu)$$



Leading-power factorization & resummation to N^{3}LL$'$

- Renormalization group evolution between (e.g.) $\mu_S \sim q_T$ and $\mu_H \sim m_H$ resums large $\frac{\alpha_s^n}{q_T} \ln^{2n} \frac{q_T}{m_H}$ to all orders $\Rightarrow$ Sudakov peak $\sim \frac{1}{q_T} e^{-\alpha_s \ln^2 q_T/m_H}$



- N^{3}LL$'$ $\Leftrightarrow$ complete three-loop anomalous dimensions and boundary conditions
  [See backup for a complete list of ingredients and references]

- Apart from high order, this is completely standard for inclusive q_T spectrum

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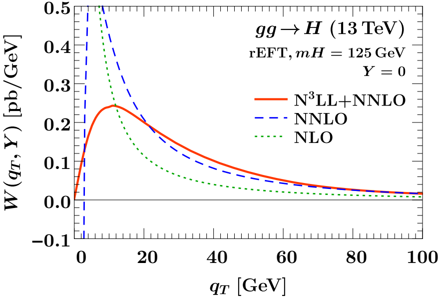
Ingredients satisfy 2D renormalization group equations, e.g. soft function:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_S^g(\mu, \nu) \qquad \nu \frac{\mathrm{d}}{\mathrm{d}\nu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_\nu^g(b_T, \mu)$$

- Solve recursively at fixed order
  - Complete log structure of $\mathrm{d}\sigma^{(0)}$
- Closed-form all-order solution
  - Resummed Sudakov peak
- Resummation order specified by perturbative order of anom. dims. and boundary conditions



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To reach N^3LL' for $W^{(0)}$, implemented in `SCETlib`:

- Three-loop soft and hard function ...includes in particular the three-loop virtual form factor
  [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions
  [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
  [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions
  [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- N^3LL solutions to virtuality/rapidity RGEs in b_T space
- Hybrid profile scales for fixed-order matching
  [Lustermans, JM, Tackmann, Waalewijn '19]

Differential q_T subtractions

$$\sigma = \int_0^{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} + \int_0^{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T} + \int_{q_T^{\rm off}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T}$$

Include $\mathrm{d}\sigma^{\rm fpc}$ in differential subtraction:

$$\frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y) = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\rm fpc}}{\mathrm{d}q_T}$$

Remaining (nonsingular) terms:

$$\frac{\mathrm{d}\sigma_{\rm FO}^{\rm nons}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta) \Big[W_{\rm FO}^{(2)}(q_T, Y) + \cdots\Big] = \Big[\frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma_{\rm FO}^{\rm sing}}{\mathrm{d}q_T}\Big]_{q_T > 0}$$

Challenges:

- Obtaining stable $H + 1j$ results for $q_T \to 0$ is *hard* …in particular at NNLO_1
- Dropping the nonsingular below $q_T \leq q_T^{\rm cut}$ is not viable, either …as we'll see shortly
  - Crucial to use differential subtraction, not slicing

Differential q_T subtractions

$$\sigma = \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}q_T} + \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\text{FO}}^{\text{nons}}}{\mathrm{d}q_T} + \int_{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\text{FO}_1}}{\mathrm{d}q_T}$$

Include $\mathrm{d}\sigma^{\text{fpc}}$ in differential subtraction:

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Remaining (nonsingular) terms:

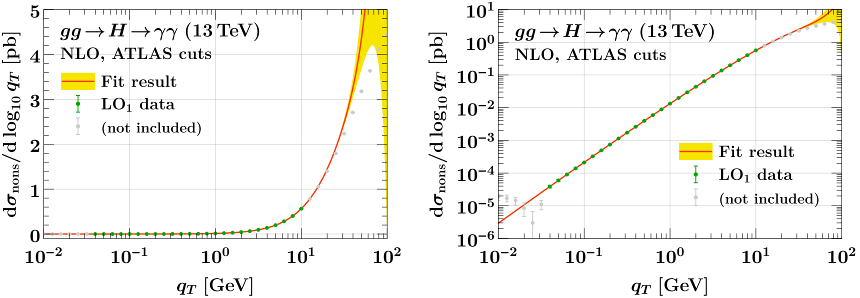
$$\frac{\mathrm{d}\sigma_{\text{FO}}^{\text{nons}}}{\mathrm{d}q_T} = \int \mathrm{d}Y \, A(q_T, Y; \Theta) \left[W_{\text{FO}}^{(2)}(q_T, Y) + \cdots \right] = \left[\frac{\mathrm{d}\sigma_{\text{FO}_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma_{\text{FO}}^{\text{sing}}}{\mathrm{d}q_T} \right]_{q_T > 0}$$

Key idea

Fit nonsingular data to known form at subleading power and integrate *analytically*:

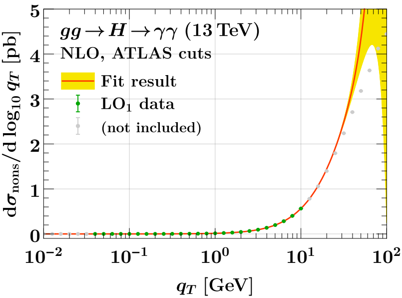
$$q_T \frac{\mathrm{d}\sigma_{\text{FO}}^{\text{nons}}}{\mathrm{d}q_T} \bigg|_{\alpha_s^n} = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \left(a_k + b_k \frac{q_T}{m_H} + c_k \frac{q_T^2}{m_H^2} + \cdots \right) \ln^k \frac{q_T^2}{m_H^2}$$

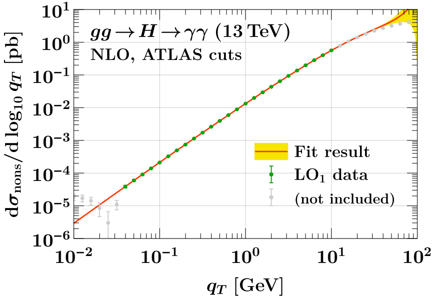
- Include higher-power b_k, c_k to get unbiased a_k
  - Allows us to use more precise data at higher q_T as lever arm in the fit



Fixed-order inputs:

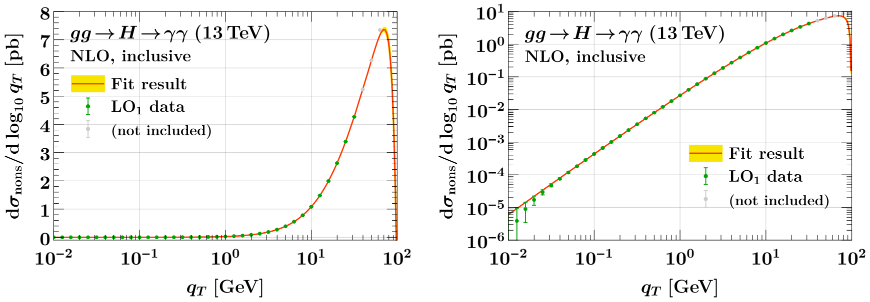
- NLO contribution to $W(q_T, Y)$ at $q_T > 0$ (LO$_1$) is easy
- At NNLO (NLO$_1$), renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia ’17]





Fit procedure:

- Perform separate χ^2 fits of $\{a_k^{\mathrm{incl,fid}}\}$ to inclusive and fiducial nonsingular data
  [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combinations to ensure fit is stable
  [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu '15-'16]



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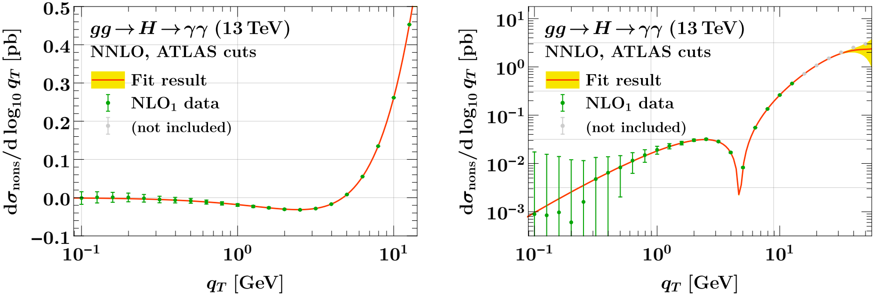

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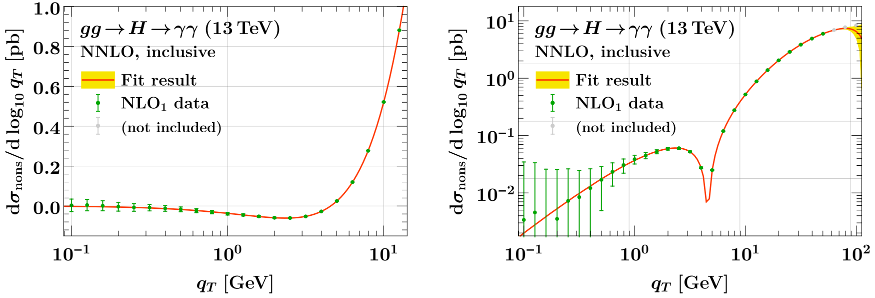
Fit results at (N)NLO



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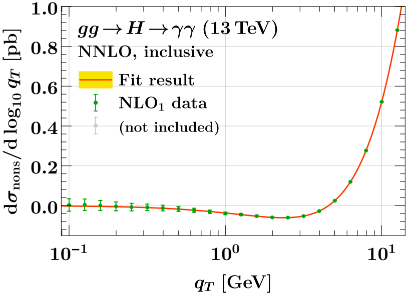
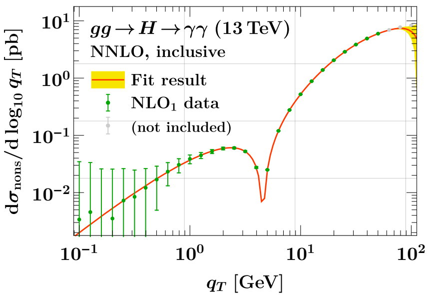
  [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combinations to ensure fit is stable
  [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu '15-'16]

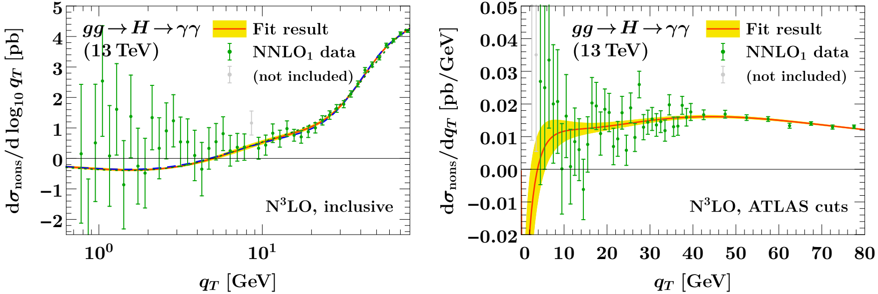




- Check the purely hadronic a_k^{fid} by directly fitting them to

$$q_T \int \mathrm{d}Y\, A^{(0)}(Y;\Theta)\left[W - W^{(0)}\right] = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \left(a_k^{\mathrm{fid}} + c_k' \frac{q_T^2}{m_H^2} + \cdots \right) \ln^k \frac{q_T^2}{m_H^2} \quad ✓$$

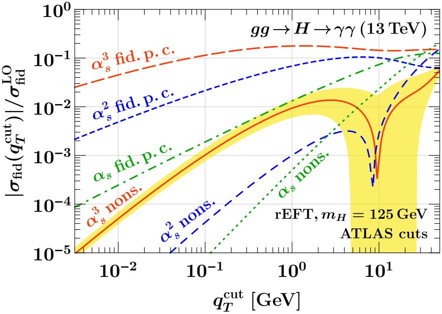
- Recover analytic (N)NLO coefficient of σ_{incl} at 10^{-5} (10^{-4}) ✓
- Analytic implementation gives us awesome precision on *all* NLP coefficients
  (all logs at NLO *and* NNLO, also differential in Y, broken down by color structure, ...)
  - Can serve as benchmark for q_T resummation at subleading power



Setup:

- Combined fit to existing binned inclusive and fiducial NNLO$_1$ data from NNLOjet
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
- Empirically find $0.4 \leq a_k^{\text{fid}}/a_k^{\text{incl}} \leq 0.55$ at (N)NLO $\Rightarrow$ use as weak 1σ constraint
- Add $\sigma_{\text{incl}}(q_T \leq q_T^{\text{cut}}) = \sigma_{\text{incl}}^{\text{N}^3\text{LO}} - \sigma_{\text{incl}}(q_T > q_T^{\text{cut}})$ as additional incl. data point
  [Mistlberger '18]

Comparison to other methods: q_T slicing

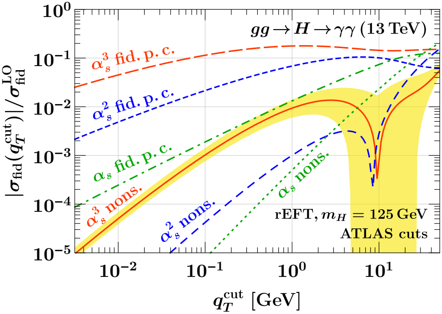


Slicing approach to q_T subtractions:

[used e.g. in Cieri, Chen, Gehrmann, Glover, Huss, 1807.11501; Camarda, Cieri, Ferrera, 2103.04974]

$$\sigma = \sigma^{(0)}(q_T^{\rm cut}) + \sigma^{\rm fpc}(q_T^{\rm cut}) + \sigma^{\rm nons}(q_T^{\rm cut}) + \int_{q_T^{\rm cut}} dq_T \, \frac{d\sigma_{\rm FO_1}}{dq_T}$$

- Slicing uses finite $q_T^{\rm cut} \sim 2$ GeV and neglects *both* $\sigma^{\rm fpc}(q_T^{\rm cut}), \sigma^{\rm nons}(q_T^{\rm cut}) \approx 0$
- This is a catastrophic approximation even at α_s^2, and definitely at α_s^3
- Even without $\sigma^{\rm fpc}$ (e.g., without cuts), this is a bad approximation at α_s^3
  - $q_T^{\rm cut}$ variations only scan local maximum around 2 GeV …



Projection-to-Born method:

[used e.g. in Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, 2102.07607]

$$\frac{d\sigma}{dY} = A(0, Y)\,\frac{d\sigma_{\rm incl}}{dY} + \int_{\approx q_T^{\rm cut}} dq_T \left[A(q_T, Y) - A(0, Y)\right] W(q_T, Y)$$

- First term from analytic (threshold expansion of) inclusive rapidity spectrum
- Second term numerically from $H + 1j$ MC, dominated by $\sigma^{\rm fpc}$ at small q_T
- Need to integrate down to $q_T^{\rm cut} \ll 0.1\,{\rm GeV}$ to get error below 10% of $\sigma_{\rm LO}^{\rm fid}$!
  [See also Salam, Slade, 2106.08329 for an explicit/analytic estimate at double-logarithmic level]

Backup (Drell-Yan)

- Use exact analytic solutions of virtuality and rapidity RG equation, combined with fast numerically exact solution of β function [Ebert '21]

  ...will come back to this

- Choose RG boundary scales as *hybrid profile scales* $\mu_X(b_T, q_T, Q)$: [Lustermans, JKLM, Tackmann, Waalewijn '19]

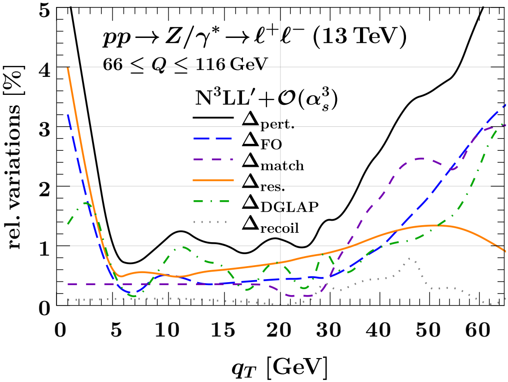
$$\mu_X(b_T, q_T \ll Q) = \frac{b_0}{b_T} \quad \text{but} \quad \mu_X(b_T, q_T \to Q) \to \mu_{\rm FO} = Q$$

- Apply "local" b^* prescription starting at $\mathcal{O}(b_T^4)$ to virtuality scales *only*:

$$\mu_X \to \mu_X^* = \Big[\big(\mu_X^{\min}\big)^4 + \Big(\frac{b_0}{b_T}\Big)^4\Big]^{1/4} = \frac{b_0}{b_T}\Big\{1 + \mathcal{O}\Big[\big(\mu_i^{\min} b_T\big)^4\Big]\Big\}$$

  - Avoids contaminating nonperturbative corrections at quadratic order [Conflict with b_T-space renormalon structure: Scimemi, Vladimirov '18] [Translation back to momentum space: Ebert, JKLM, Stewart, Sun '22]

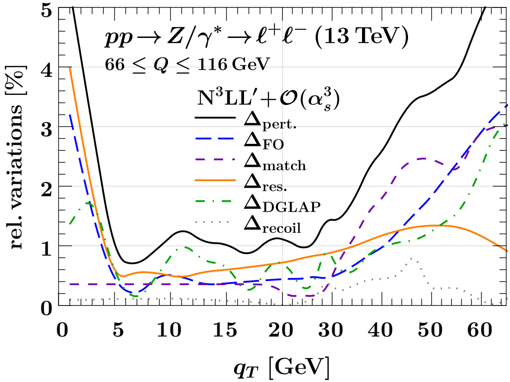
- For PDFs inside beam functions, use $\mu_f^{\min} = \min\{Q_0, m_c\}$



$$\Delta_{\text{pert.}} = \Delta_{\text{FO}} \oplus \Delta_{\text{match}} \oplus \Delta_{\text{res}} \oplus \Delta_{\text{DGLAP}} \oplus \Delta_{\text{recoil}}$$

- Fixed-order uncertainty, keeps resummed logarithms unchanged
- Estimated by standard variations of overall $\mu_R = \mu_{\text{FO}}$
- All scales (except μ_f) are chosen $\propto \mu_{\text{FO}}$, so e.g. μ_H/μ_S unchanged
- Frozen out at $b_T \lesssim 1/\Lambda_{\text{QCD}}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP

Breakdown of perturbative uncertainties



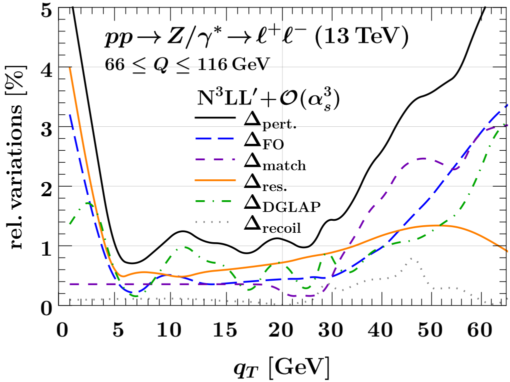
$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Uncertainty from matching scheme between resummed peak and fixed-order tail
- Estimated by varying the $x = q_T/Q$ transition points in hybrid profile as

$$\{x_1, x_2, x_3\} = \{0.3, 0.6, 0.9\} \pm \{0.1, 0.15, 0.2\}$$

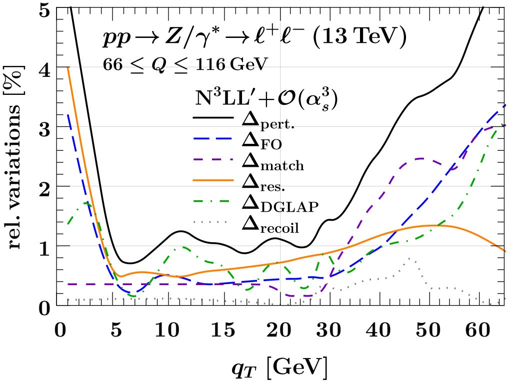
- Checked that *inclusive* integrated cross section is recovered within Δ_{match}

Breakdown of perturbative uncertainties



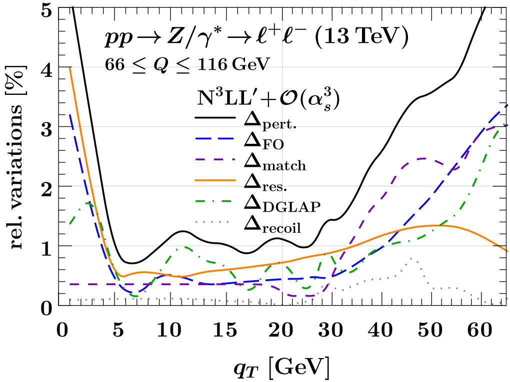
$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Probes higher-order resummed logarithms
- Estimated by envelope of 36 different combinations of independently varying $\{\mu_B, \mu_S, \nu_B, \ldots\}$ in $\sigma^{(0)} = H\, B \otimes B \otimes S$
- Also frozen out at $b_T \lesssim 1/\Lambda_{\mathrm{QCD}}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP



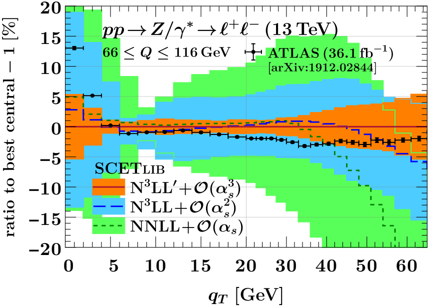
$$\Delta_{\rm pert.} = \Delta_{\rm FO} \oplus \Delta_{\rm match} \oplus \Delta_{\rm res} \oplus \Delta_{\rm DGLAP} \oplus \Delta_{\rm recoil}$$

- Estimate of missing higher orders (four loops) in DGLAP running
- Estimated both in peak and tail by joint variations of $\mu_f(b_T, q_T, Q)$ and $\mu_F(Q)$
- Oscillatory due to b_T-space features at uncancelled m_b threshold

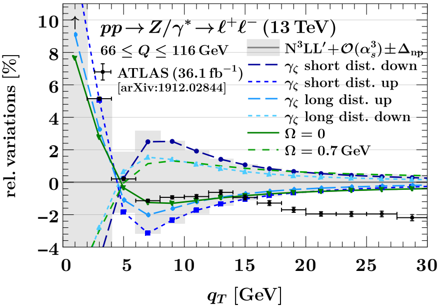


$$\Delta_{\text{pert.}} = \Delta_{\text{FO}} \oplus \Delta_{\text{match}} \oplus \Delta_{\text{res}} \oplus \Delta_{\text{DGLAP}} \oplus \Delta_{\text{recoil}}$$

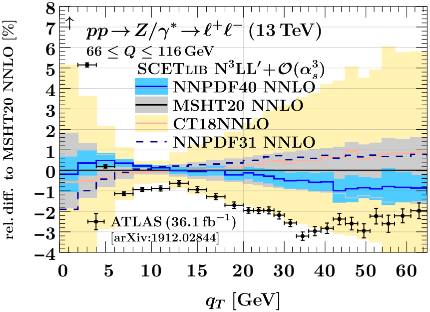
- RPI-I transformation of n_a^μ, n_b^μ in $W^{\mu\nu}_{\text{LP}} \sim g_\perp^{\mu\nu}(n_a, n_b)$
- Induces $O(q_T^2/Q^2)$ change in spectrum due to fiducial cuts on $L_{\mu\nu}$
  [Ebert, JKLM, Stewart, Tackmann '20]
- Equivalent to changing "recoil prescription"/choice of Z rest frame by $\mathcal{O}(q_T/Q)$



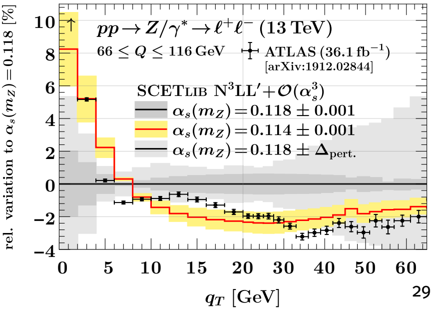
$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$66 \le Q \le 116$ GeV
ATLAS (36.1 fb^{-1})
[arXiv:1912.02844]
SCETLIB
$\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$
$\mathrm{N^3LL} + \mathcal{O}(\alpha_s^2)$
$\mathrm{NNLL} + \mathcal{O}(\alpha_s)$
ratio to best central − 1 [%]
q_T [GeV]



$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$66 \le Q \le 116$ GeV
ATLAS (36.1 fb^{-1})
[arXiv:1912.02844]
$\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3) \pm \Delta_{\mathrm{np}}$
γ_ζ short dist. down
γ_ζ short dist. up
γ_ζ long dist. up
γ_ζ long dist. down
$\Omega = 0$
$\Omega = 0.7$ GeV
rel. variations [%]
q_T [GeV]

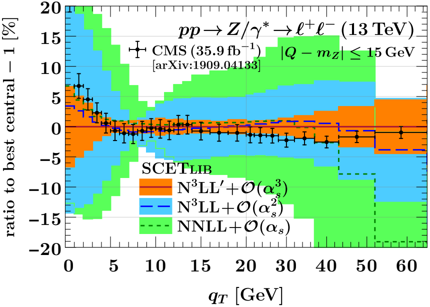


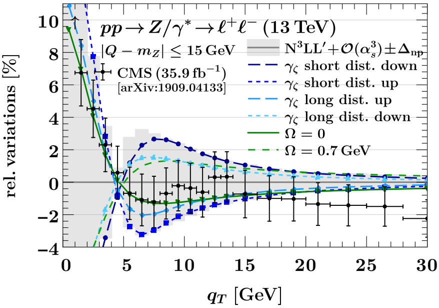
$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$66 \le Q \le 116$ GeV
SCETLIB $\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$
NNPDF40 NNLO
MSHT20 NNLO
CT18NNLO
NNPDF31 NNLO
ATLAS (36.1 fb^{-1})
[arXiv:1912.02844]
rel. diff. to MSHT20 NNLO [%]
q_T [GeV]

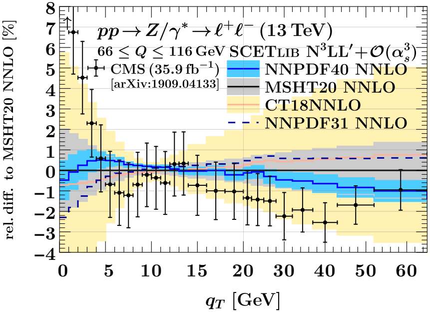


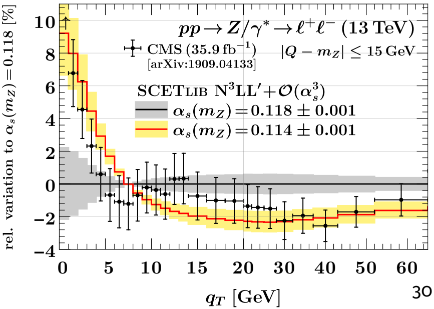
$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$66 \le Q \le 116$ GeV
ATLAS (36.1 fb^{-1})
[arXiv:1912.02844]
SCETLIB $\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$
$\alpha_s(m_Z) = 0.118 \pm 0.001$
$\alpha_s(m_Z) = 0.114 \pm 0.001$
$\alpha_s(m_Z) = 0.118 \pm \Delta_{\mathrm{pert.}}$
rel. variation to $\alpha_s(m_Z) = 0.118$ [%]
q_T [GeV]

CMS normalized spectrum (dressed leptons)

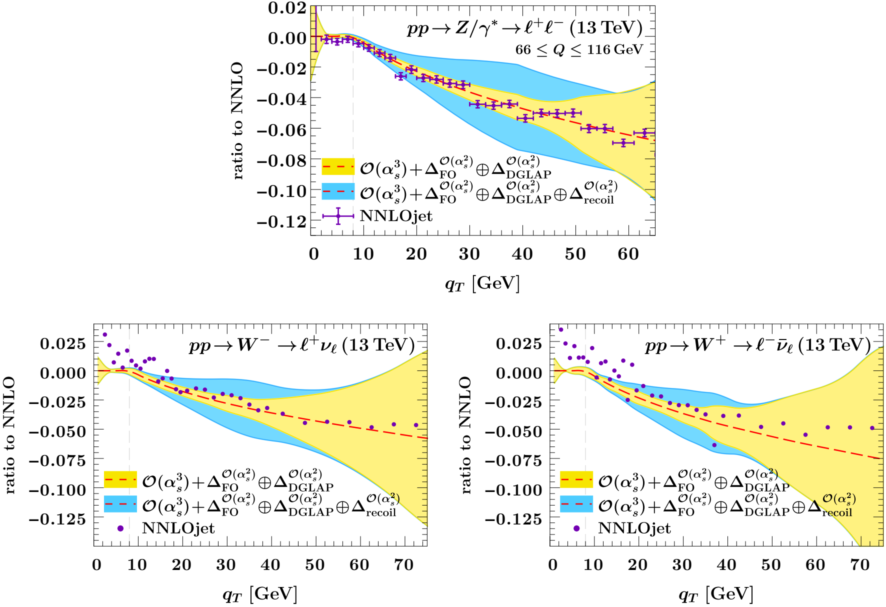




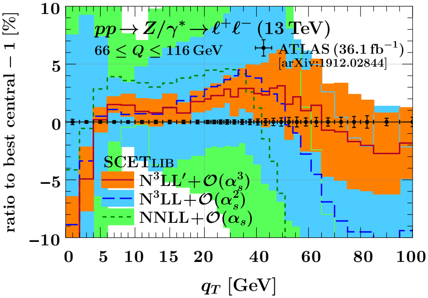


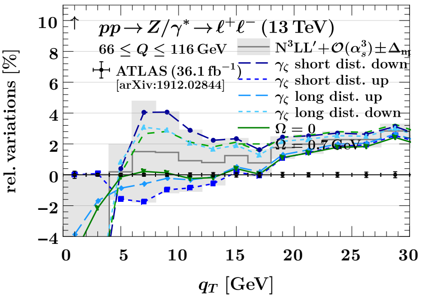


$\mathcal{O}(\alpha_s^3)$ nonsingular interpolations

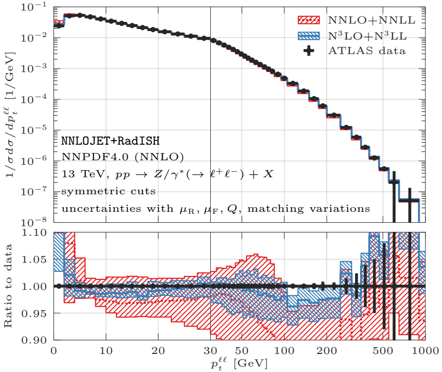


Comparison with RadISH (using identical NNLOjet fixed-order matching)





[Chen et al., 2203.01565]



- Can recover the data for $q_T \leq 4\,\mathrm{GeV}$ with NP model $\approx$ *off*
- To recover the RadISH result at $\leq 4\,\mathrm{GeV}$, would need large positive $\gamma_\zeta^{(2)}$ or $\bar{\Lambda}^{(2)}$
- In either case, cannot recover $\geq 20\,\mathrm{GeV}$ due to $\Lambda^2_{\mathrm{QCD}}/q_T^2$ scaling imposed by TMD factorization & OPE

Comparison with RadISH (using identical NNLOjet fixed-order matching)

- Common ingredient: Sudakov evolution kernels from $\mu_0 \sim Q$ to $\mu \sim 1/b_T, q_T$

  e.g.: $$K_\Gamma(\mu_0, \mu) = \int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \Gamma[\alpha_s(\mu')] \ln \frac{\mu'}{\mu_0}$$

- Implementation of Sudakov kernels in SCETlib is exactly equal to numerical solution of β function + numerical μ' integral
  - $\beta(\alpha_s)$ and $\Gamma(\alpha_s)$ truncated after α_s^4, no additional approximations or assumptions
  - Exact RGE closure $U(\mu_0, \mu)\, U(\mu, \mu_0) = 1$
  - Exact path independence in (μ, ν) or (μ, ζ) plane

- …but much faster, thanks to closed-form results in [Ebert, 2110.11360] in terms of a single polynomial root-finding problem

Comparison with RadISH (using identical NNLOjet fixed-order matching)

- Common ingredient: Sudakov evolution kernels from $\mu_0 \sim Q$ to $\mu \sim 1/b_T, q_T$

  e.g.: $$K_\Gamma(\mu_0, \mu) = \int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\, \Gamma[\alpha_s(\mu')] \ln \frac{\mu'}{\mu_0}$$

- Common to expand $K_\Gamma(\mu_0, \mu)$ in terms of $\alpha_s(\mu_0)$ throughout instead

  $\Rightarrow$ simpler analytic solution with $g^{(1)}$ a function of an $\mathcal{O}(1)$ argument:

  $$K_\Gamma^{\text{exp.}}(\mu_0, \mu) = L g^{(1)}\big(\alpha_s(\mu_0) L\big) + \text{NLL}\,, \qquad L = \ln \frac{\mu_0}{\mu}$$

- However, reexpanding in terms of $\alpha_s(\mu_R)$, $\mu_R \neq \mu_0$ (read: μ_0 = resummation scale) leads to large truncation errors [Billis, Tackmann, Talbert, 1907.02971]