Exotic hadrons at LHCb

Misha Mikhasenko
on behalf of LHCb

August 15th, 2022
EIC workshop in Stony Brook
LHCb beauty

Muon Stations
Muon ID ∼ 97%
pi/mu mis ID ∼ 1-3%

Ring Imaging CHerenkov
Koan ID ∼ 95%
pi/K mis ID ∼ 5%

Dipole Magnet

Tracking stations

pp collider (7+7 TeV)
1 Pentaquarks
- $\Lambda_b^0 \rightarrow J/\psi pK^-$
- $\Xi_b^- \rightarrow J/\psi \Lambda K^-$
- $B^- \rightarrow J/\psi \Lambda \bar{p}$

2 Tetraquarks
- prompt $D^0 D^0 \pi^+$
- $B^+ \rightarrow D^+ D^- K^+$
- $B^+ \rightarrow D_s^+ D_s^- K^+$
- $B^+ \rightarrow D^- D_s^+ \pi^+$ and $B^0 \rightarrow \bar{D}^0 D_s^+ \pi^-$

3 Plans
Summary of Pentaquarks studies

(*) will be discussed today

\[ X_b \rightarrow (J/\psi p) \ldots \]

\[ \Lambda_b^0 \rightarrow (J/\psi p)K^- \quad (*) \]
\[ \Lambda_b^0 \rightarrow (J/\psi p){\pi}^- \]
\[ B_s^0 \rightarrow (J/\psi p){\bar{\rho}} \]

Thresholds:
\[ \Sigma_c^{(*)} + D^{(*)0} / \Sigma_c^{(*)} + D^{(*)-} \]

\[ P_{\psi N}^\Lambda : \]

\[ P_{\psi S}^\Lambda : \]

\[ \Xi_b^- \rightarrow (J/\psi \Lambda)K^- \quad (*) \]
\[ B^- \rightarrow (J/\psi \Lambda){\bar{\rho}} \quad (*) \]

Thresholds (?):
\[ \Xi_c^{(*)0} D^{(*)0} / \Xi_c^{(*)} + D^{(*)-} \]

LHCb proposal for the new name convention of exotic hadrons [arxiv: 2206.15233]
$\Lambda_b^0 \rightarrow J/\psi p K^-$
The first pentaquarks

- Close to the $\Sigma_c \bar{D}(\star)$ threshold,
- have “right” multiplicity: $1/2 \otimes 1 = 1/2 \oplus 3/2$
- Narrow(!): 10, 20, and 5 MeV for $\Gamma_{BW}$

![Graph showing LHCb data and fitted curves for exotic hadrons](image-url)
The first pentaquarks

- Close to the $\Sigma_c \bar{D}^{(*)}$ threshold,
- have “right” multiplicity: $1/2 \otimes 1 = 1/2 \oplus 3/2$
- Narrow(!): 10, 20, and 5 MeV for $\Gamma_{\text{BW}}$

$\Sigma_c^+ \bar{D}^0$
$\Sigma_c^{**} \bar{D}^{*0}$
$\Sigma_c^{**} \bar{D}^{*0}$

LHCb

@9 fb$^{-1}$

$P_c(4312)^+$
$P_c(4440)^+$
$P_c(4457)^+$

$\psi(4440)$

$P_c(4312)$

$\psi J/\psi m$
$\Xi_b^− \rightarrow J/ψΛK^−$

$P^Λ_{ψs}$
Hint for the strange partners

$\Xi_b^- \to J/\psi(\to \mu^+\mu^-)\Lambda(\to p\pi^-)K^- \text{ data sample} \ [\text{Sci.Bull. 66 (2021) 1278-1287}]

- Full data sample 1750 signals with purity 80%.
- The amplitude model includes: $\Xi$
- $P_{\psi}^{\Lambda}(4459): m = 4458.8 \pm 2.9^{+4.7}_{-1.1} \text{ MeV}, \Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7} \text{ MeV}$

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**LHCb**

9 fb$^{-1}$

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$\Xi_{c}^{0} \bar{D}^{*0}$

---

$\Xi^{*-}(1690)$

---

$\Xi^{*-}(1820)$

---

$\Lambda K^-$

---

$28$

---

$26$

---

$24$

---

$22$

---

$20$

---

$18$

---

$4.5$

---

$5$

---

$m_{J/\psi\Lambda} [\text{GeV}]$

---

$m_{J/\psi\Lambda} [\text{GeV}]$

---

$m_{\Lambda K^-} [\text{GeV}]$

---

$m_{\Lambda K^-} [\text{GeV}]$
\[ B^- \rightarrow J/\psi \Lambda \bar{p} \]

\[ P^\Lambda_{\psi s} \]
$B^- \rightarrow J/\psi(\rightarrow \mu^+\mu^-)\Lambda(\rightarrow p\pi)\bar{p}$

- **Amplitudes:**
  - NR($J/\psi p$), $84.0 \pm 2.2\%$
  - NR($\Lambda\bar{p}$), $11.3 \pm 1.3\%$
  - New $P_{\psi s}^\Lambda$, $12.5 \pm 0.7\%$, with parameters
    - $m(P_{\psi s}^\Lambda) = 4338.2 \pm 0.7$ MeV
    - $\Gamma(P_{\psi s}^\Lambda) = 7.0 \pm 1.2$ MeV
- $J^P = 1/2^-$ is preferred
- BW mass is close to $\Xi_c \bar{D}$ thresholds:
  - $0.8$ MeV above $\Xi_c^+D^-$
  - $2.9$ MeV above $\Xi_c^0\bar{D}^0$
Tetraquarks candidates

(*) will be discussed today

\[ T^{b}_{\psi} \quad (Z_c) \]
3900, 4430, . . .

\[ T^{\theta}_{\psi_s} \quad (Z_{cs}) \]
4000, 4220

\[ T^{J/\psi K^+} \quad (\star) \]

\[ T^{J/\psi} \quad (\star) \]

\[ T^{J/\psi J/\psi} \quad (\star) \]

\[ X^{(T_{\psi \phi})} \]
4140, 4274, 4500, . . .

\[ T^{D^0 D^0 \pi^+} \quad (\star) \]

\[ T^{D^+ K^-} \quad (\star) \]

\[ T^{D_s^\pm \pi^+} \quad (\star) \]

\[ T^{T_{cc}} \]
3874

\[ T^{T_{cs}} \quad (X) \]
2900

\[ T^{T_{c\bar{s}}} \quad (X) \]
2900
prompt $D^0 D^0 \pi^+$
The landmark of 2021: a signal in $D^0 D^0 \pi^+$

Event selection

- Select $D^0 D^0 \pi^+$ candidates from primary vertex with detached $D^0 \to K^- \pi^+$
- Require detached $K^- \pi^+$ with high $p_T$
- Require good quality of tracks, vertexes, and particle IDs.
- Ensure no $K/\pi$ candidates belong to one track
- Ensure no reflections via mis-ID
- Subtraction / sWeight for fake-D background

A narrow peak near $D^{*+} D^0$ threshold!
Fit to the spectrum

Unitarized model

Two models: Native & Th.-motivated

- The peak position is well constrained.
- The width is not, limit to the coupling: $|g| > 7.7(6.2) \text{GeV}$ at 90(95)% CL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$117 \pm 16$</td>
<td>$N$</td>
<td>$186 \pm 24$</td>
</tr>
<tr>
<td>$\delta m_{BW}$</td>
<td>$-273 \pm 61 \text{ keV/c}^2$</td>
<td>$\delta m_{U}$</td>
<td>$-359 \pm 40 \text{ keV/c}^2$</td>
</tr>
<tr>
<td>$\Gamma_{BW}$</td>
<td>$410 \pm 165 \text{ keV}$</td>
<td>$</td>
<td>g</td>
</tr>
</tbody>
</table>

Naive BW

Advanced Model

Excellent agreement with the data. Reaction amplitude is fully fixed.
Predicted mass spectrum

The resolution removed

- Peak position: $-359 \pm 40$ keV
  (The most precise ever wrt to the threshold)
- FWHM: $47.8 \pm 1.9$ keV,
- Lifetime: $\tau \approx 10^{-20}$ s.
  (Unprecedentedly large for exotic hadrons)
- The pole parameters:
  \[
  \delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}, \quad
  \Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}.
  \]

- Nearly-isolated resonance below the $D^{*+}D^0$ threshold
- Long tail with cusps at the $D^{*+}D^0$ and $D^{*0}D^+$ thresholds
\[ B^+ \rightarrow D^+ D^- K^+ \]

\[ T_{CS} \]
Dalitz plot for $B^+ \to D^+ D^- K^+$

- Horizontal bands are resonances in $D^+ D^-$
- Hint for a vertical band around 8.5 GeV$^2$ in $m^2(D^-K^+)$
- Exotic candidate $T_{cs}(2900)$: $[\bar{c}\bar{s}ud]$
- Both quantum numbers $J^P = 0^+$ and $1^-$ are wanted by the fit
Horisontal bands are resonances in $D^+D^-$

- Hint for a vertical band around 8, 5 GeV$^2$ in $m^2(D^-K^+)$

- Exotic candidate $T_{cs}(2900)$: $[\bar{c}\bar{s}ud]$

- Both quantum numbers $J^P = 0^+$ and $1^-$ are wanted by the fit
\[ B^+ \rightarrow D_s^+ D_s^- K^+ \]

\[ \chi_{c0}/T_{\psi\phi} \]
Threshold enhancement at $D_s^+D_s^-$ in $B^+ \rightarrow D_s^+D_s^-K^+$ decays

Main features of the data:
- significant structure at the threshold
- a prominent dip at $m(D_s^+D_s^-) = 4.15$ GeV.

Baseline model: $D_s^+D_s^-$ resonances
- $1^{--}$: $\psi(4260) \sim 4\%$, $\psi(4660) \sim 2\%$
- $0^{++}$: $X(3960) \sim 24\%$, $X(4140) \sim 18\%$, NR $\sim 50\%$

LHCb data: $9 \text{ fb}^{-1}$

Exotic hadrons at LHCb
August 15th
Is $X(3960)$ the same as $\chi_{c0}(3930)$ from $D^+D^-$?

Assuming to be the same, $\mathcal{B}(\chi_{c0} \to D^+D^-)/\mathcal{B}(\chi_{c0} \to D_s^+D_s^-P) \sim 0.3$

large molecular component, or large tetraquark component, $T_{\psi\phi}$

[JHEP 06 (2021) 035] finds a state coupled to $D_s^+D_s^-$ on the lattice
Is $X(3960)$ the same as $\chi_{c0}(3915)$?

$B^+ \to (D_s^+ D_s^-) K^+$ by LHCb:

$\gamma \gamma \to J/\psi \omega$ by Belle:

- Belle sees a clean state in $J/\psi \omega$ with $J^P = 0^+$
- The $D_s^+ D_s^-$ signal might be a tail of the $\chi_{c0}(3915)$ state
\[ B^+ \rightarrow D^- \quad D_S^+ \pi^+ \]

\[ T_{c\bar{s}}^{a_+}(\ldots)^{++} \]

\[ B^0 \rightarrow \bar{D}^0 \quad D_S^+ \pi^- \]

\[ T_{c\bar{s}}^{a_-}(\ldots)^0 \]
$T_{cs}^a(2900)$ in the $D_s^{\pm}\pi^+$ system
[LHCb-PAPER-2022-026 (in preparation)]

- 4420 $B^0$ decays and 3940 $B^-$ decays, including charge-conjugated reactions
- Simultaneous fit using the isospin symmetry
- $T_{cs}^a \sim 2\%$ needed ($>5\sigma$)
- $T_{cs}^a$: $J^P = 0^+$ is favored ($7.5\sigma$)
- Mass and width are close to those of $T_{cs}^a(2900)$
  - $T_{cs}^{a0}$: $m = 2892 \pm 14 \pm 15$ MeV, $\Gamma = 119 \pm 26 \pm 12$ MeV;
  - $T_{cs}^{a++}$: $m = 2921 \pm 17 \pm 19$ MeV, $\Gamma = 137 \pm 32 \pm 14$ MeV
$T^{a}_{cs}(2900)$ in the $D^+_s\pi^+$ system

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Summary of the new hadronic states

Patrick Koppenburg (ORIGINS Cluster)
Exotic hadrons at LHCb
August 15th

T_{sa}(6900)
T_{sa}(6600)
T_{ac}(3875)
T_{ac}(3875)'
P_{N}(4457)'
T_{ac}(4220)'
T_{ac}(4220)''
P_{N}(4338)'
P_{N}(4338)''
P_{N}(3842)
X(4140)
P_{N}(4450)'
P_{N}(4380)'
X(4700)
X(4500)
X(4274)

Mass [MeV/c^2]

Date of arXiv submission

X(3960)
P_{N}(4312)'
P_{N}(4440)'
P_{N}(4440)''
X(4685)
X(4630)
P_{N}(4140)

Patrick.koppenburg@cern.ch 2022-07-20
Summary of the new hadronic states

59 new hadrons at LHCb

Date of arXiv submission

Mass [MeV/c²]

Patrick.Koppenburg@cern.ch 2022-07-20

Misha Mikhasenko (ORIGINS Cluster)
Exotic hadrons at LHCb
August 15th
17/22
**Updated timeline for LHC**

[W. Altmannshofer, F. Archilli, arxiv::2206.11331]

<table>
<thead>
<tr>
<th>Large Hadron Collider (LHC)</th>
<th>High Luminosity LHC (HL-LHC)</th>
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<tbody>
<tr>
<td><strong>Run 1</strong></td>
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<tr>
<td>7 TeV → 8 TeV</td>
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<tr>
<td>LHCb</td>
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<tr>
<td>ATLAS/CMS</td>
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<tr>
<td>2011 13 TeV</td>
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<tr>
<td>9 fb⁻¹</td>
<td>2011 13 TeV</td>
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<td>190 fb⁻¹</td>
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<td><strong>LS1</strong></td>
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<td>2012 13 TeV</td>
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<td><strong>Run2</strong></td>
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<td>Upgrade I</td>
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<td>35 fb⁻¹</td>
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<td>450 fb⁻¹</td>
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<td><strong>LS2</strong></td>
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<td>2014 13.6 TeV</td>
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<td><strong>Run3</strong></td>
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<tr>
<td>2017 13.6 TeV</td>
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<td>2026 2027 2028 2029 2030</td>
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<td>2031 2032 2033 2034 2035</td>
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<tr>
<td>2036 2037 2038 2039 2040</td>
<td></td>
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<tr>
<td>2051...</td>
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</table>

**LHCb:**
- **ramping up after major Upgrade I**
- **×5 statistics in Run 3(2023-2025) @13.6 TeV + Run 4(2029-2032) @14 TeV**

Misha Mikhasenko (ORIGINS Cluster)  Exotic hadrons at LHCb  August 15th
SMOG(2): Fixed Targer LHCb

- System for Measuring Overlap with Gas (SMOG)
- LHC beam collides with the injected gas ($10^{-7}$ mbar)
- Used to reconstruct the beams transverse profile
- LHCb + gas target $\Rightarrow$ Fixed-target LHCb!

- Unique energy scale: $68 < \sqrt{s} < 110$ GeV
- Access to unexplored kinematic region: high-$x$ and intermediate $Q^2$

Some physics results

- Detached antiprotons in $p$He, [LHCb, arXiv:2205.09009]
- Charmonium production in $p$Ne, [LHCb-PAPER-2022-014]
- $J/\psi$ production in PbNe, [LHCb-PAPER-2022-011]
Thank you for your attentions
CMS confirms $T_{\psi\psi}$ structures

CMS Preliminary

135 fb$^{-1}$ (13 TeV)

Data  Fit
BW1  BW2[X(6900)]
BW3  Background

4.2$\sigma$

[CMS-PAS-BPH-21-003]

- Clear dips is present that makes the incoherent fit struggles
- Third state is significant

CMS confirms $T_{\psi\psi}$ structures

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Data  Fit
BW1  BW2 [X(6900)]
BW3  Background

Clear dips is present that makes the incoherent fit struggles
Third state is significant

[CMS-PAS-BPH-21-003]

ATLAS also finds structures in $J/\psi J/\psi$

![Graph showing data, Sig.+Bkg., Background, Sig. w/o Int., Sig. Int. for $J/\psi$ di-J/$\psi$ with $\sqrt{s} = 13$ TeV, 139 fb$^{-1}$]
ATLAS also finds structures in $J/\psi J/\psi$ and $\psi' J/\psi$

- Hints for the near-threshold structure
- Resonances in $\psi' J/\psi$ might produce structures in $J/\psi J/\psi$ as partial-reconstructed decays, $\psi' \rightarrow J/\psi + $ neutrals

[ATLAS-CONF-2022-040]
Proposal for $J/\psi J/\psi$ angular analysis

$\Delta \phi = \phi_1 + \phi_2$

Two main angles:

- $\theta_1, \theta_2$ decay angle of $J/\psi$ (helicity angle)
- $\Delta \phi$ the azimuthal angle between the decay plans

Symmetry constraints: permutation of identical $J/\psi$, parity
Four categories of possible helicity matrices

<table>
<thead>
<tr>
<th>group</th>
<th>$\eta \chi(-1)^J, (-1)^J$</th>
<th>$J^P$</th>
<th>symmetry</th>
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<tbody>
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<td>I</td>
<td>+, +</td>
<td>$0^+, 2^+, 4^+, 6^+$</td>
<td>symmetric, $S$</td>
</tr>
<tr>
<td>II</td>
<td>−, +</td>
<td>$0^-, 2^-, 4^-, 6^-$</td>
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</tr>
<tr>
<td>III</td>
<td>+, −</td>
<td>$1^-, 3^-, 5^-, 7^-$</td>
<td>antisymmetric, $A$</td>
</tr>
<tr>
<td>IV</td>
<td>−, −</td>
<td>$1^+, 3^+, 5^+, 7^+$</td>
<td>antisymmetric, $A$</td>
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\[
H^{(I)} = \begin{pmatrix} b & a & c \\ a & d & a \\ c & a & b \end{pmatrix}_S \\
H^{(II)} = \begin{pmatrix} b & a \\ a & -a \\ -a & -b \end{pmatrix}_S \\
H^{(III)} = \begin{pmatrix} a \\ -a \\ -a \end{pmatrix}_A \\
H^{(IV)} = \begin{pmatrix} a & c \\ -a & a \\ -c & -a \end{pmatrix}_A
\]

$a, b, c, d$ are still unknown coefficients, complex in general.
Four categories of possible helicity matrices

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\[
\begin{pmatrix} b & d \\ d & b \end{pmatrix}_S, \quad \begin{pmatrix} b \\ -b \end{pmatrix}_S, \quad \begin{pmatrix} a \\ -a \\ a \\ -a \end{pmatrix}_A
\]

$a, b, c, d$ are still unknown coefficients, complex in general.
$T^+_c$ decay amplitude

$|T^+_c\rangle = \frac{1}{\sqrt{2}} \left( |D^0 D^+\rangle - |D^* D^0\rangle \right)$

Model assumptions:

- $J^P = 1^+$: $S$-wave decay to $D^*D$
- $T^+_c$ is an isoscalar: $|T^+_c\rangle_{I=0} = \left\{ |D^0 D^+\rangle - |D^* D^0\rangle \right\} / \sqrt{2}$
- No isospin violation in couplings to $D^{*+}D^0$ and $D^{*0}D^+$
Natural parity chanmonia above $D^+D^-$ threshold:

- $\psi(3770)$, $\chi_{c0}(3930)$, $\chi_{c2}(3930)$,
- $\psi(4040)$, $\psi(4160)$, $\psi(4415)$

Compare to inclusive $DD$ spectra:

$X(3872) \rightarrow D^0\bar{D}^0\pi(\gamma)$ [LHCb, JHEP 1907 (2019) 035]
Dalitz Plot Decomposition (DPD)

Update to the angular analysis formalism

Spin in 3-body decays

- [MM et al. PRD (2019)]
- [M. Wang et al., CPC (2021)]

\[ A_{\lambda_0, \lambda_1, \lambda_2} = A^{(12)}_{\lambda_0, \lambda_1, \lambda_2} + A^{(23)}_{\lambda_0, \lambda_1, \lambda_2} + A^{(31)}_{\lambda_0, \lambda_1, \lambda_2} \]

Used in the past

- unphysical inhomogeneity
- spin 1/2: \( A(\pi) \neq A(-\pi) \)
- range of \( \phi \) matters \([-\pi, \pi]\) vs \([0, 2\pi]\)

Proposed in DPD

\[ A_{\lambda_0, \lambda_1, \lambda_2} = \sum_{\nu} D^{1/2*}_{\lambda_0, \nu}(\alpha, \beta, \gamma) O^{(23)}_{\nu, \lambda_1, \lambda_2}(m^2_{12}, m^2_{23}) \]

- correct \( \phi \) dependence by construction