Photoproduction of $\eta\pi$

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Joint Physics Analysis Center

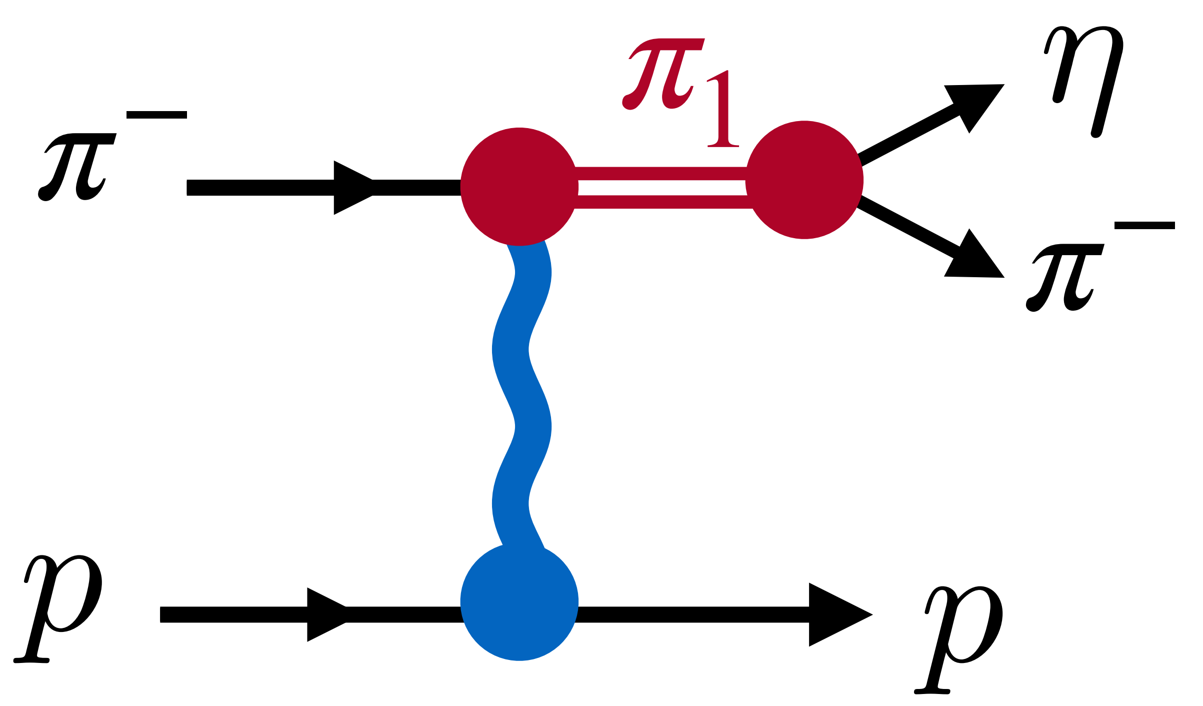
Exotic heavy meson spectroscopy
and structure with EIC

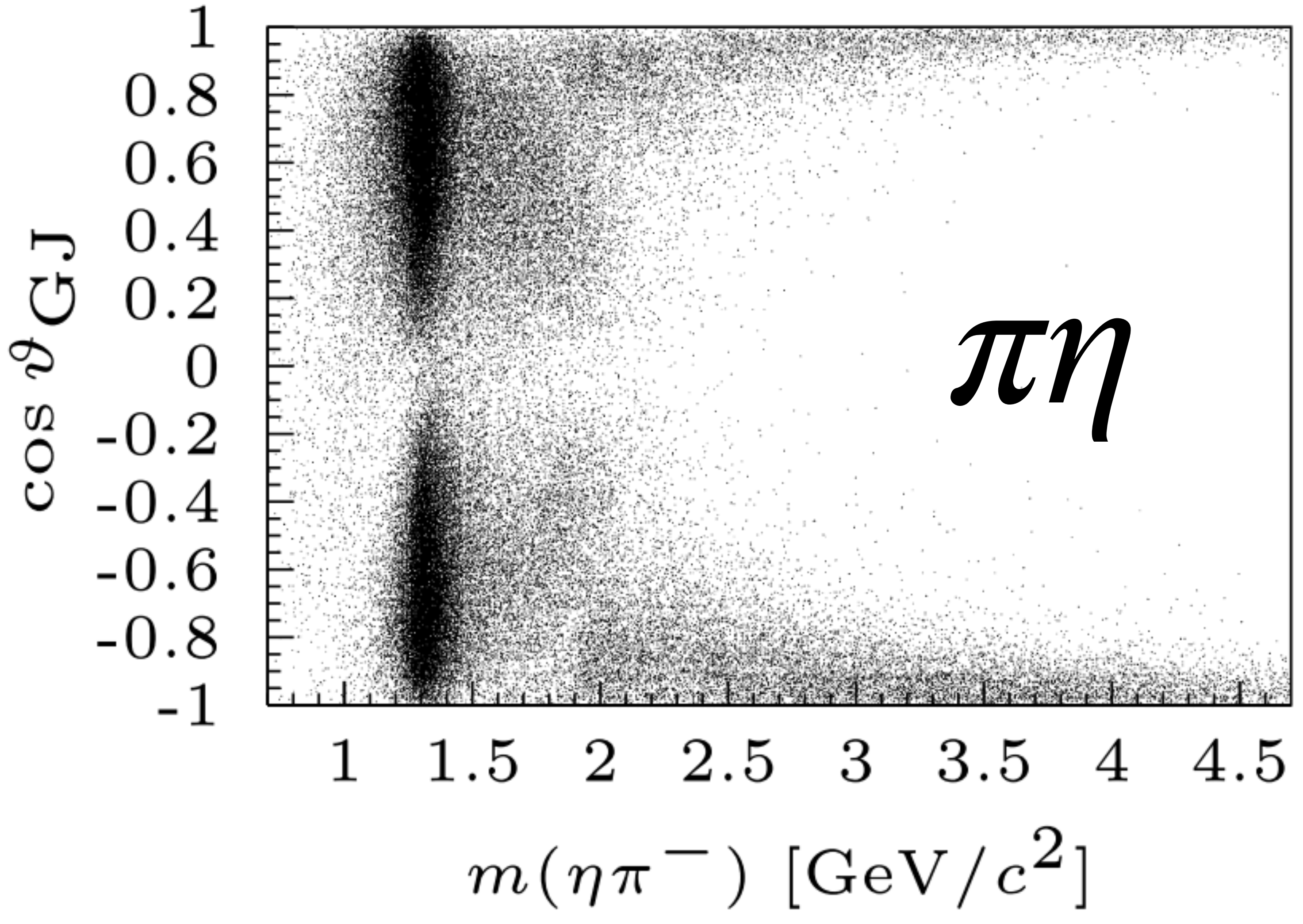
August 2022

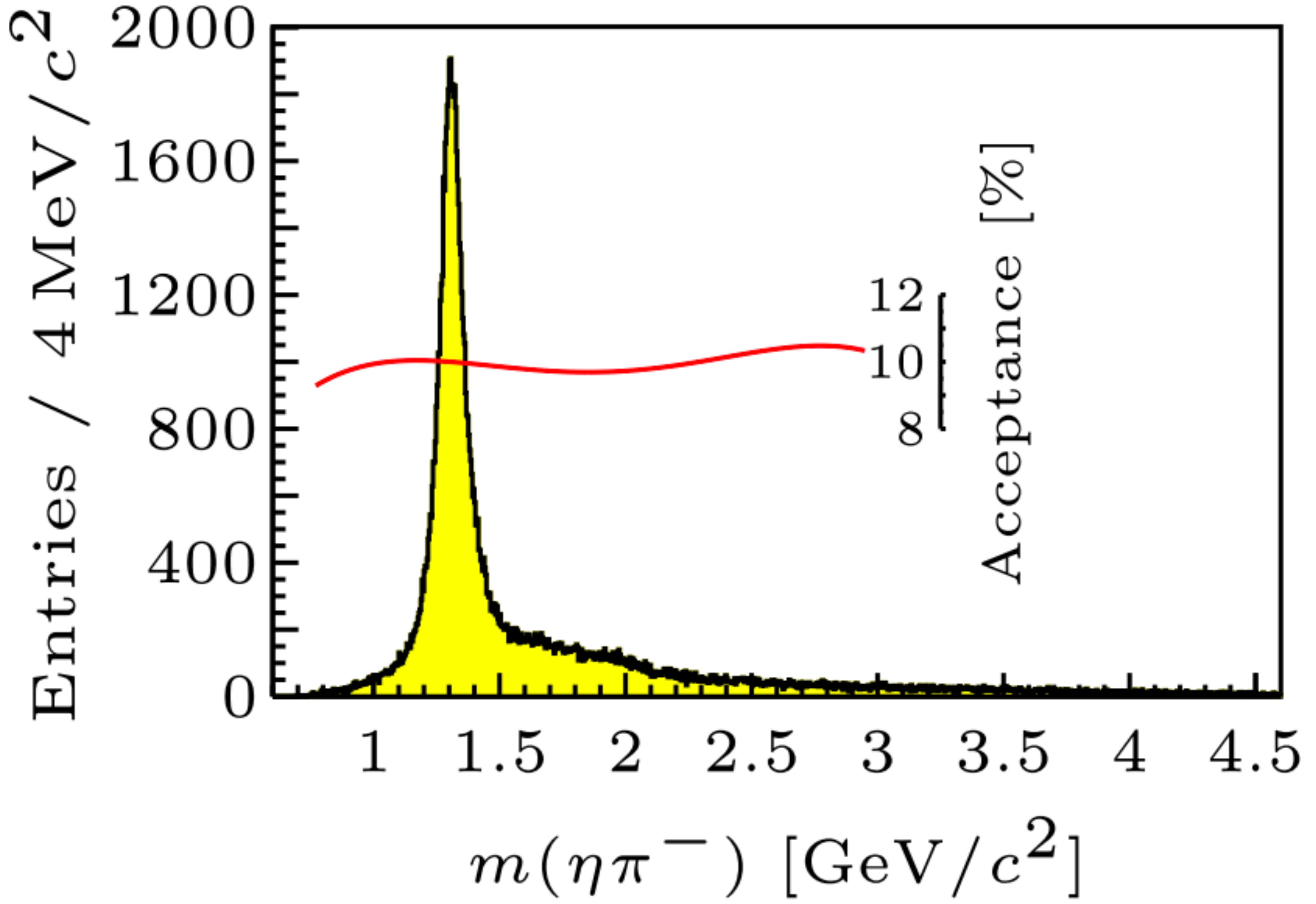
UNIVERSITAT DE BARCELONA

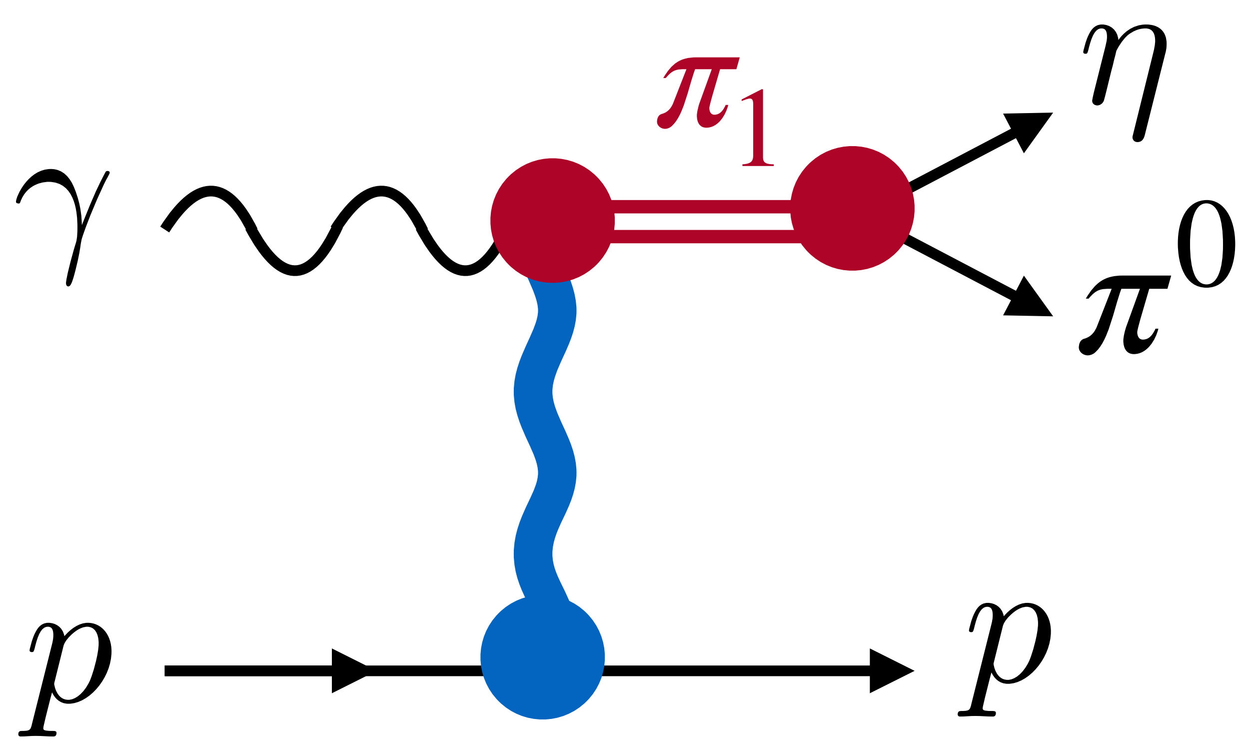
Joint Physics Analysis Center

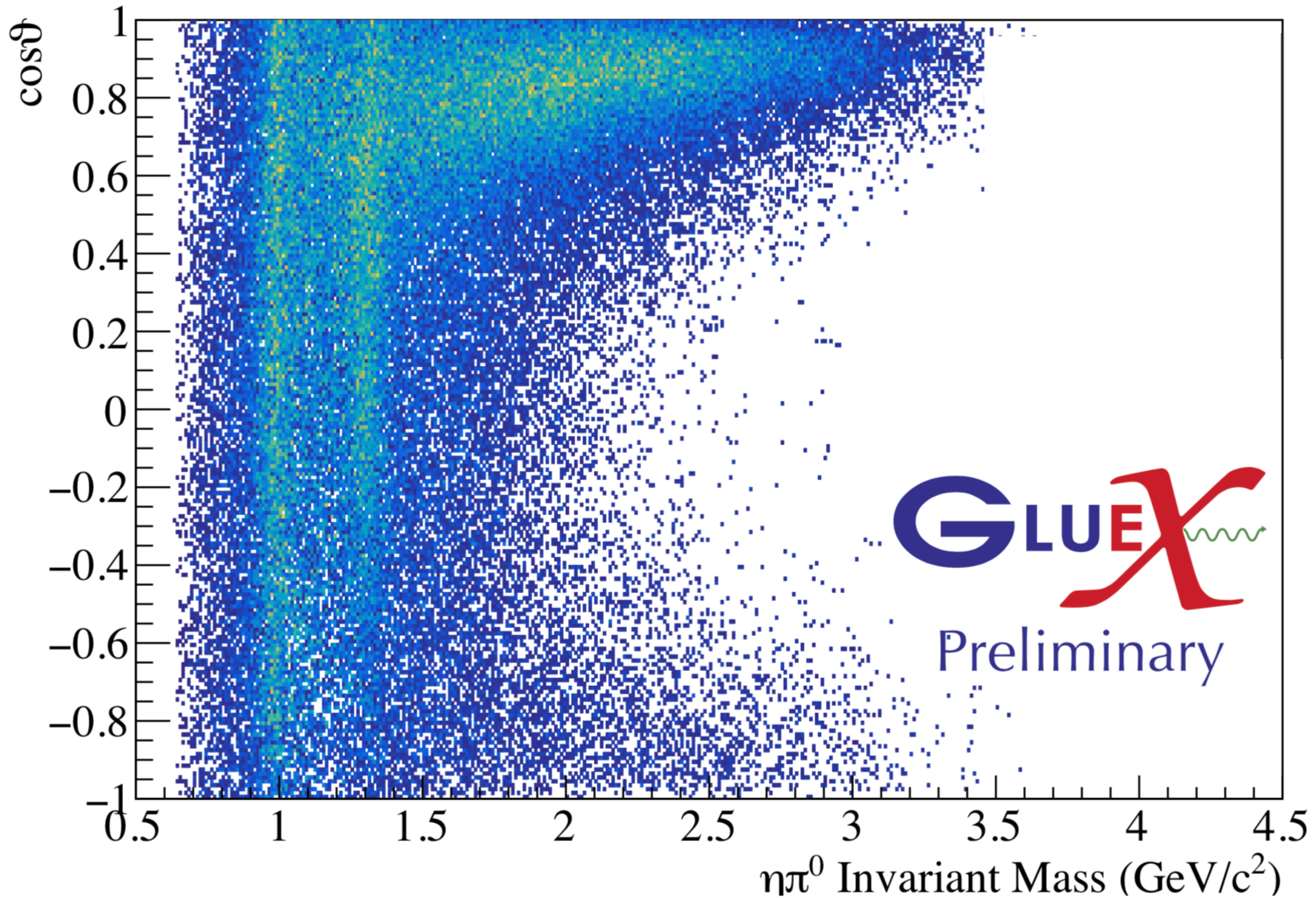
Eta-Pi @COMPASS and @GlueX

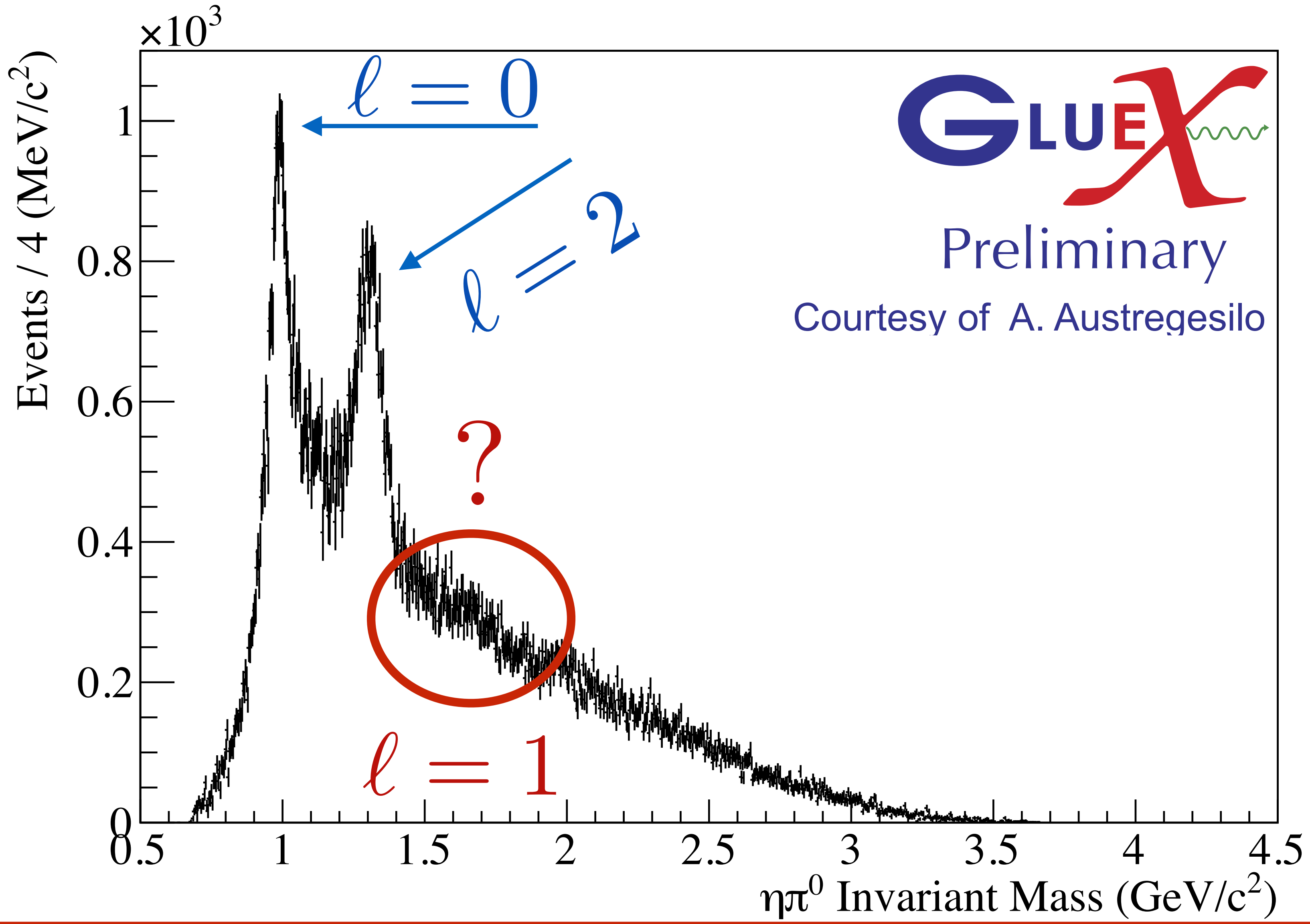




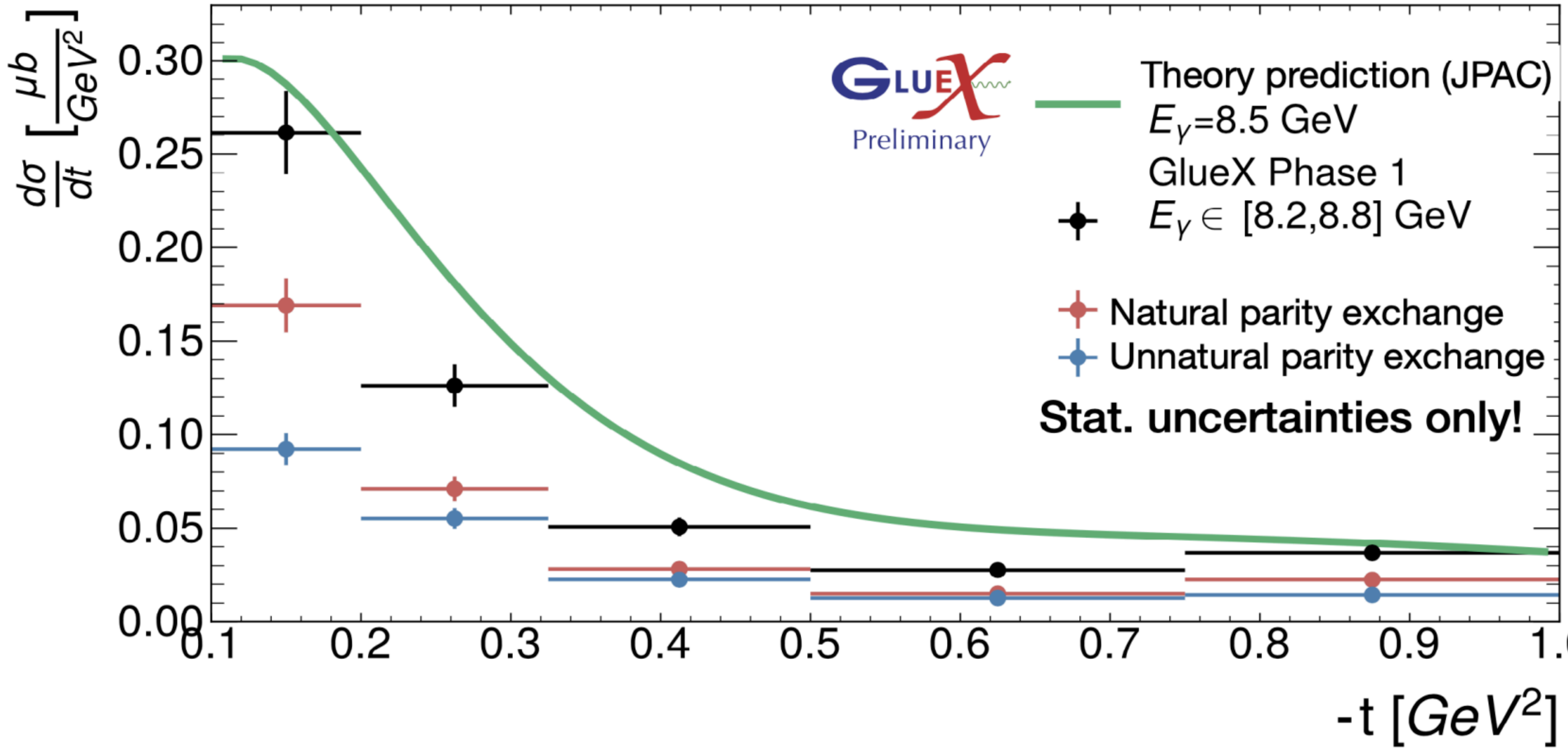


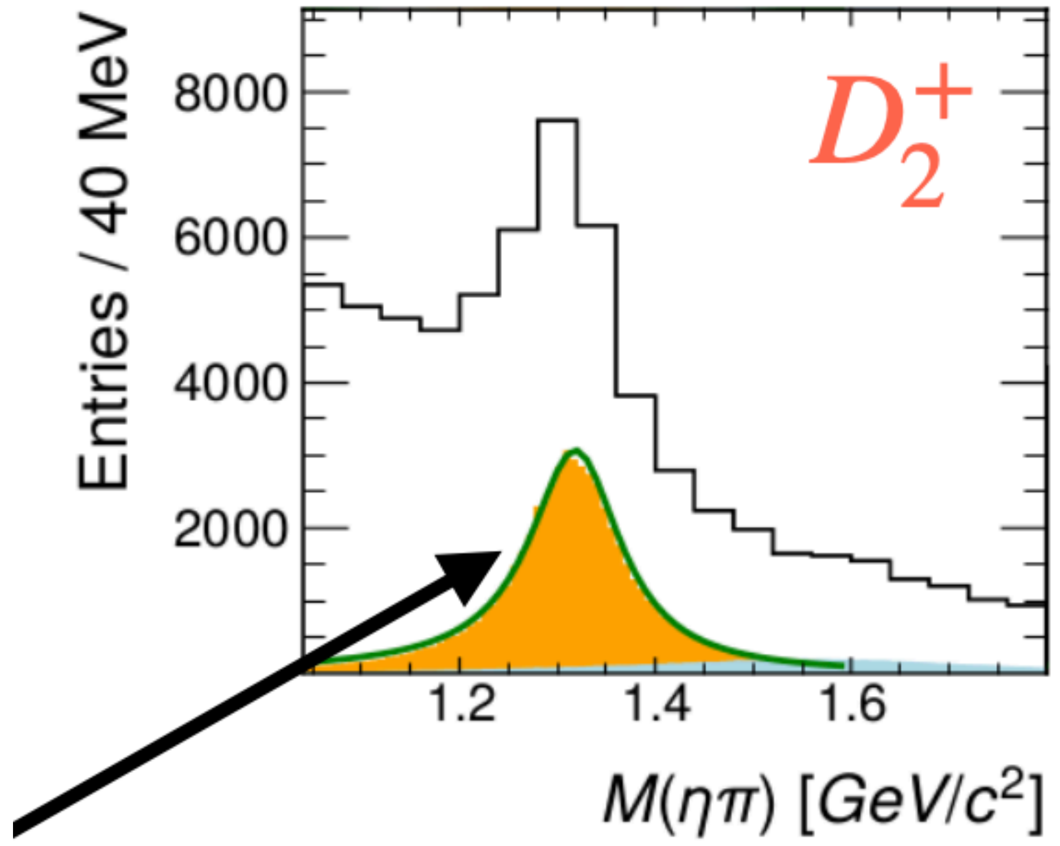


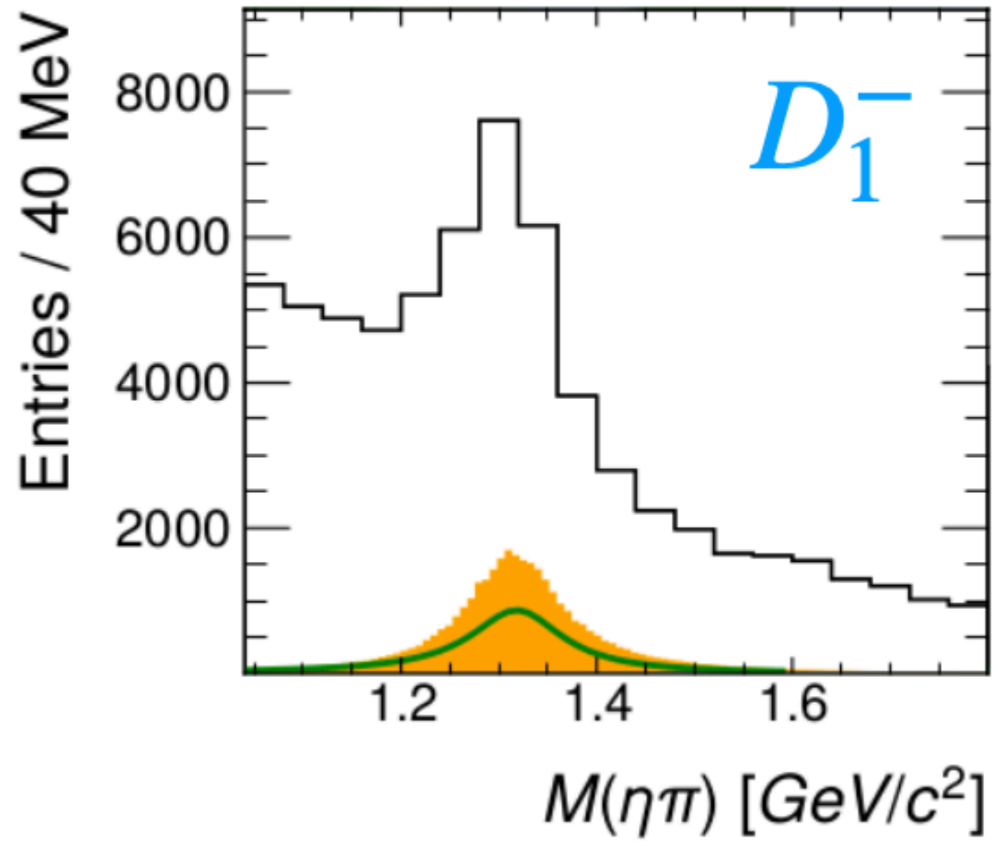




$a_2(1320)$ Photoproduction @GlueX

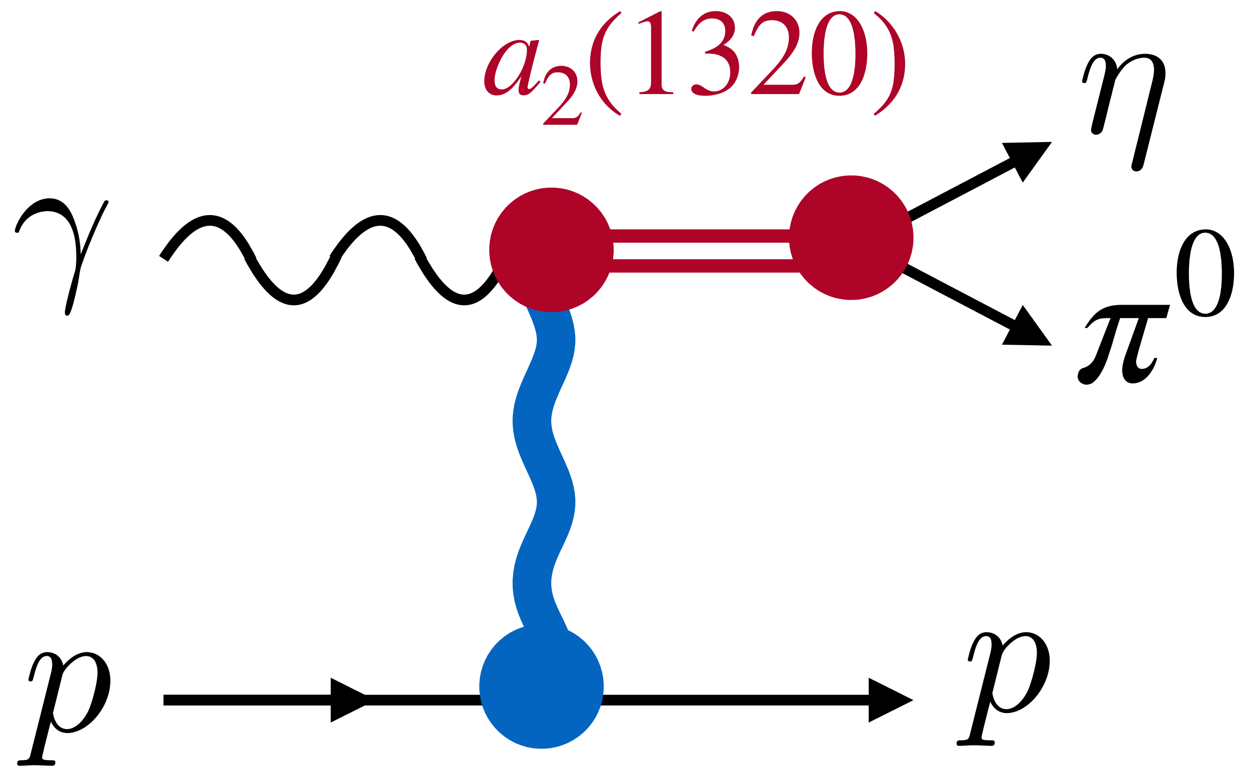






From S. Dobbs' talk in this meeting

Strong $a_2(1320)$ signal in $\pi\eta$



Extraction of the cross-section

Extraction of (all?) D-waves

"A step further" than SDME

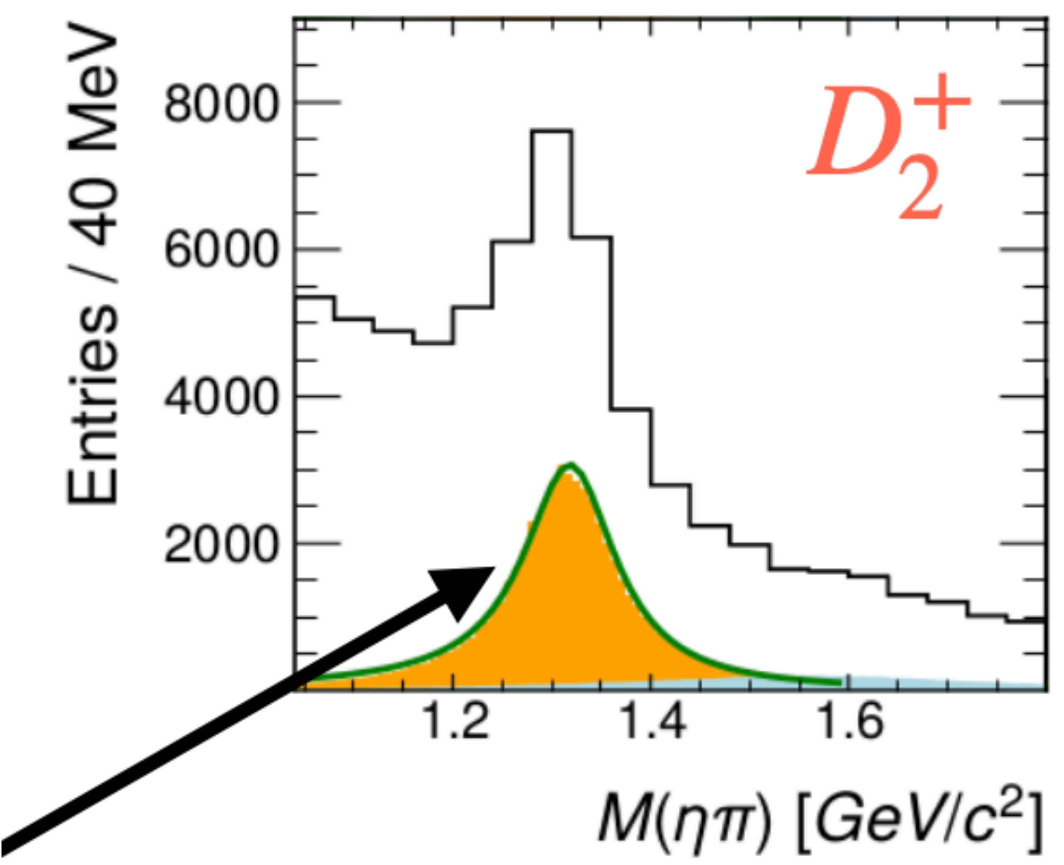
(spin density matrix elements)

Should compare with direct extraction of SDME

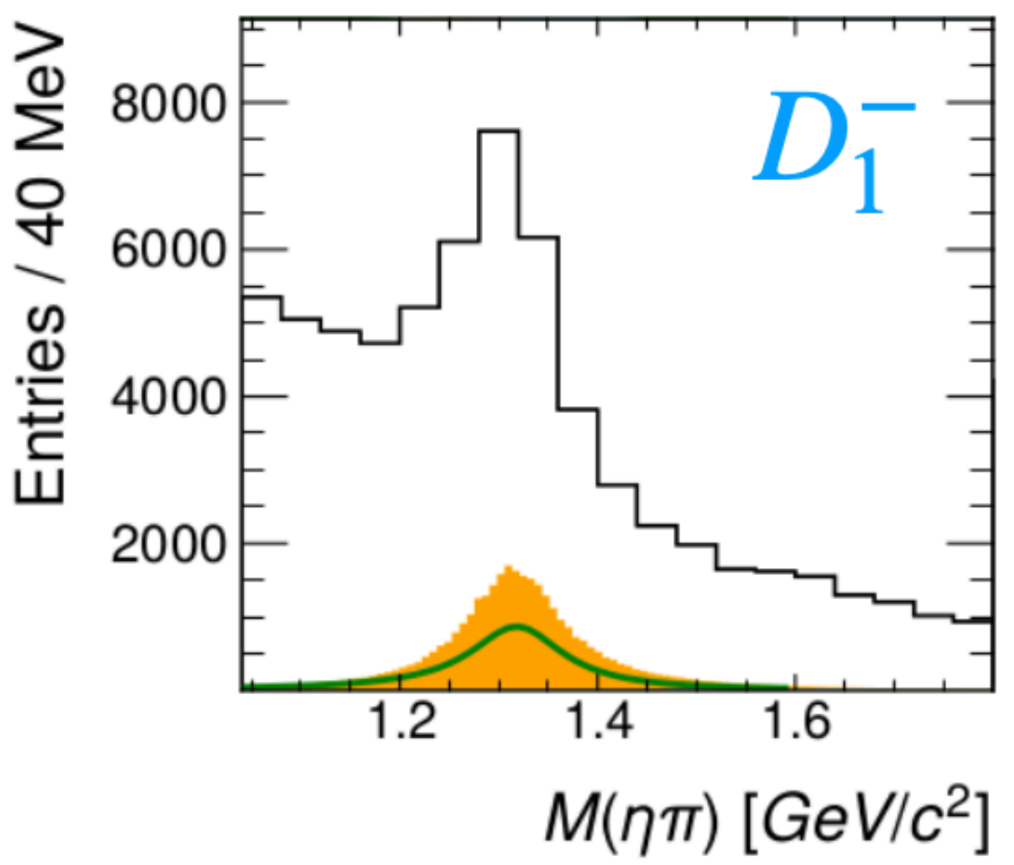
a2(1320)@GlueX

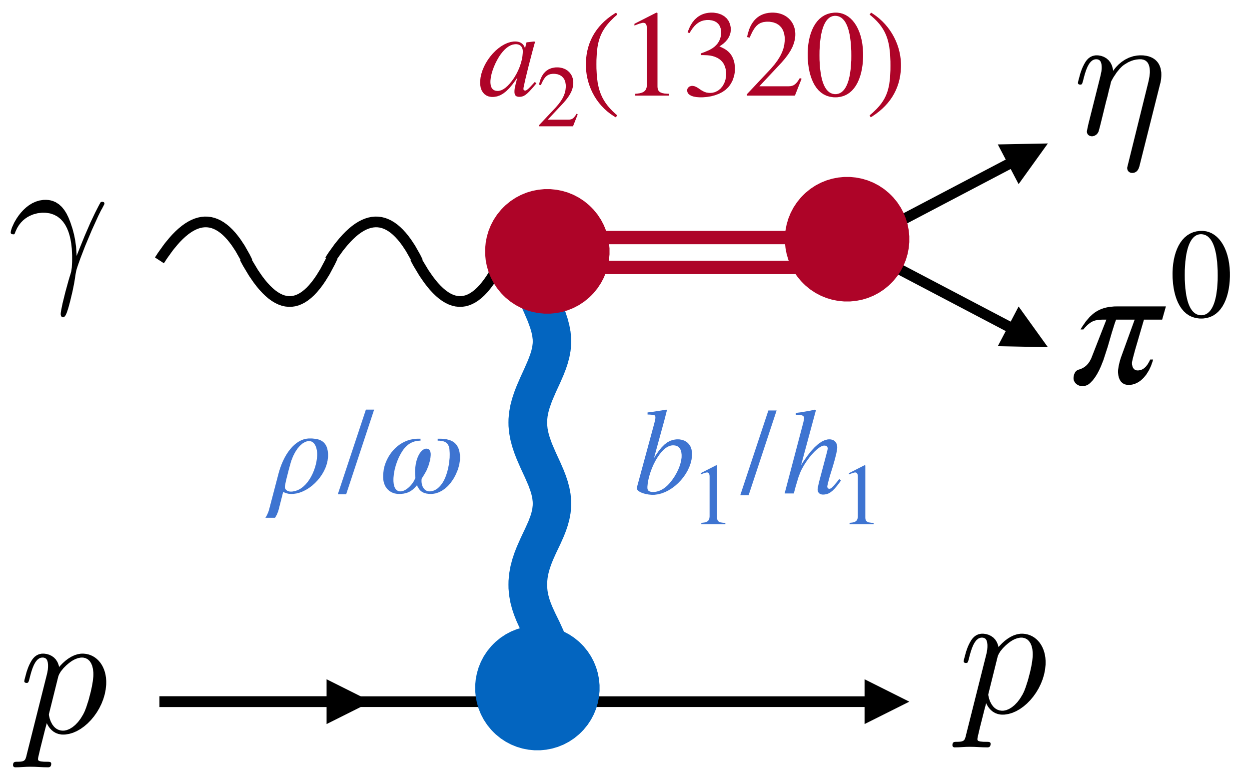
VM et al (JPAC) PRD102 (2020) 014003

Vector exchange



Axial-Vector exchange





Notation D_m^ϵ:

Reflectivity ϵ matches naturality of exchange

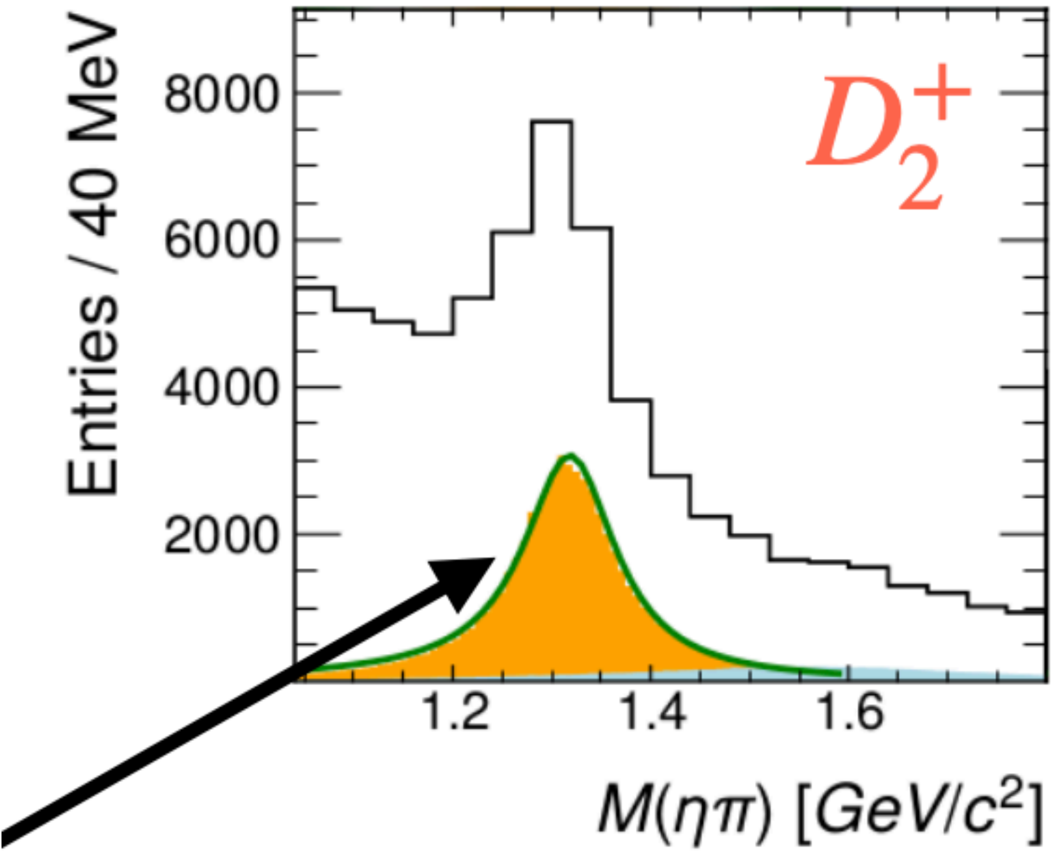
(At leading order in energy squared)

D-wave have 2*5 = 10 complex functions of t

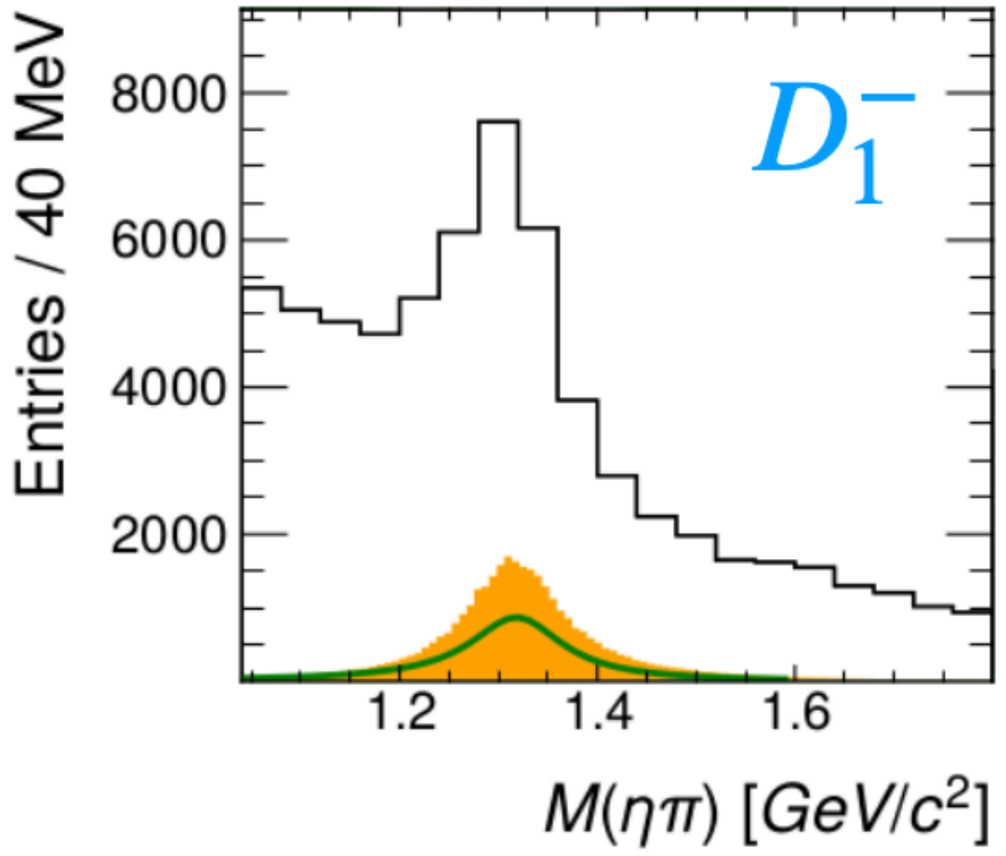
a2(1320)@GlueX

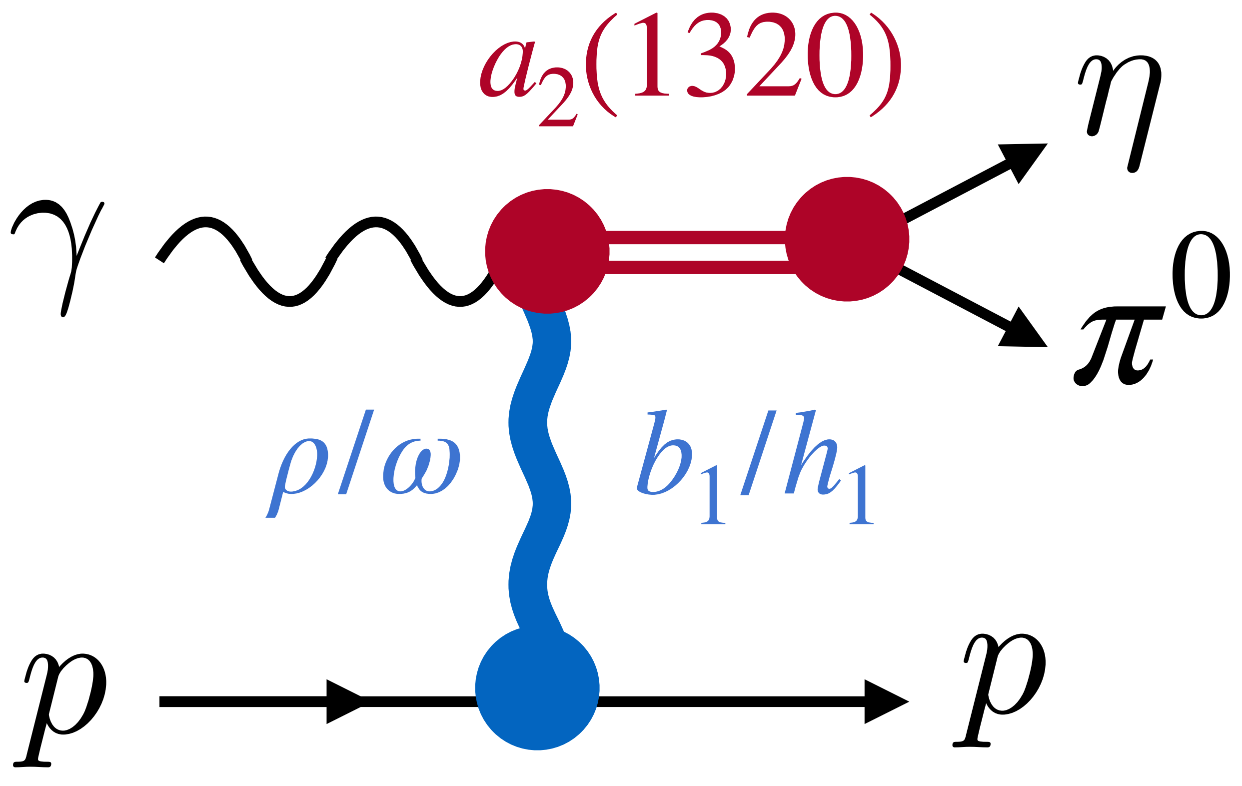
VM et al (JPAC) PRD102 (2020) 014003

Vector exchange



Axial-Vector exchange





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Assumptions of TMD to reduce nb. of couplings:

$$\mathscr{L}_{TVV} = \beta_N \, T^{\mu\nu} F_{\mu\rho} F_\nu{}^\rho \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Leads to

$$D_2^+ = -\frac{\beta_N}{2}\sqrt{\frac{-t}{m_{a_2}^2}} \qquad D_0^+ = \frac{\beta_N}{2}\frac{t}{\sqrt{6}m_{a_2}^2}\sqrt{\frac{-t}{m_{a_2}^2}}$$

$$D_1^+ = \frac{\beta_N}{2}\frac{-t}{m_{a_2}^2} \qquad D_{-1}^+ = D_{-2}^+ = 0$$

a2(1320)@GlueX

VM et al (JPAC) PRD102 (2020) 014003

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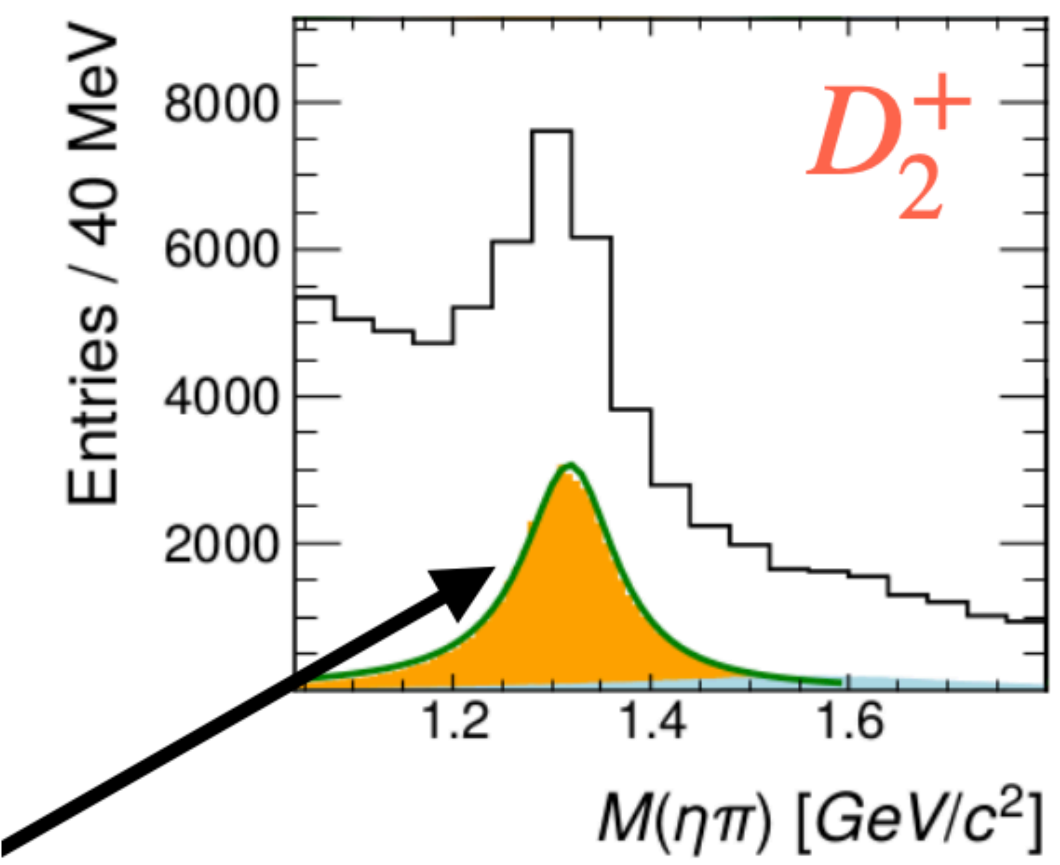
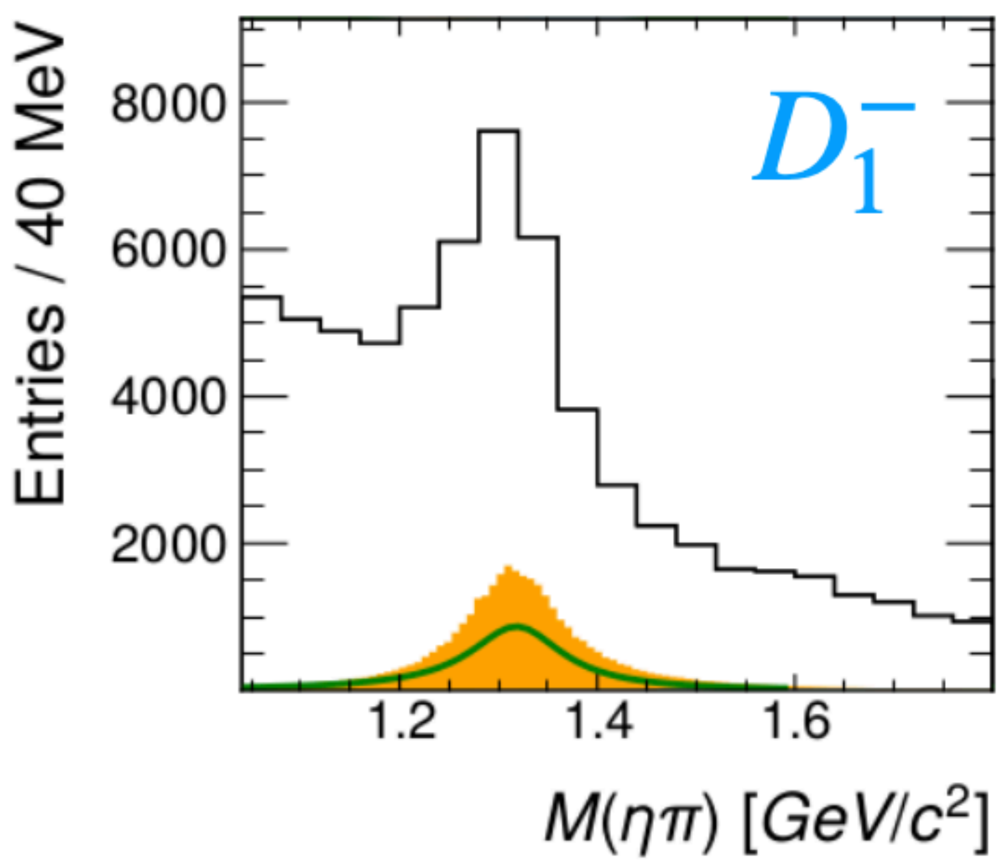
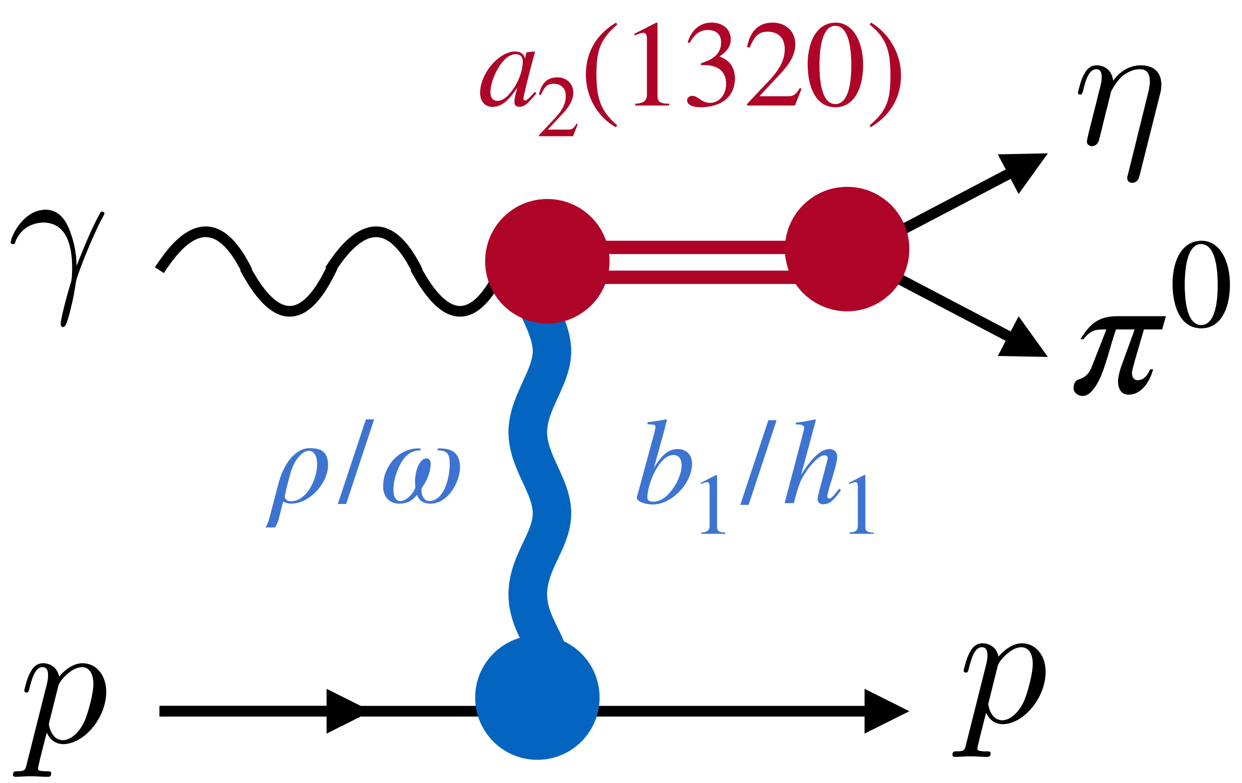


Axial-Vector exchange





For axial, M1 transition: $D_2^- = D_{-2}^- = 0 \quad D_{\pm 1,0}^- \neq 0$

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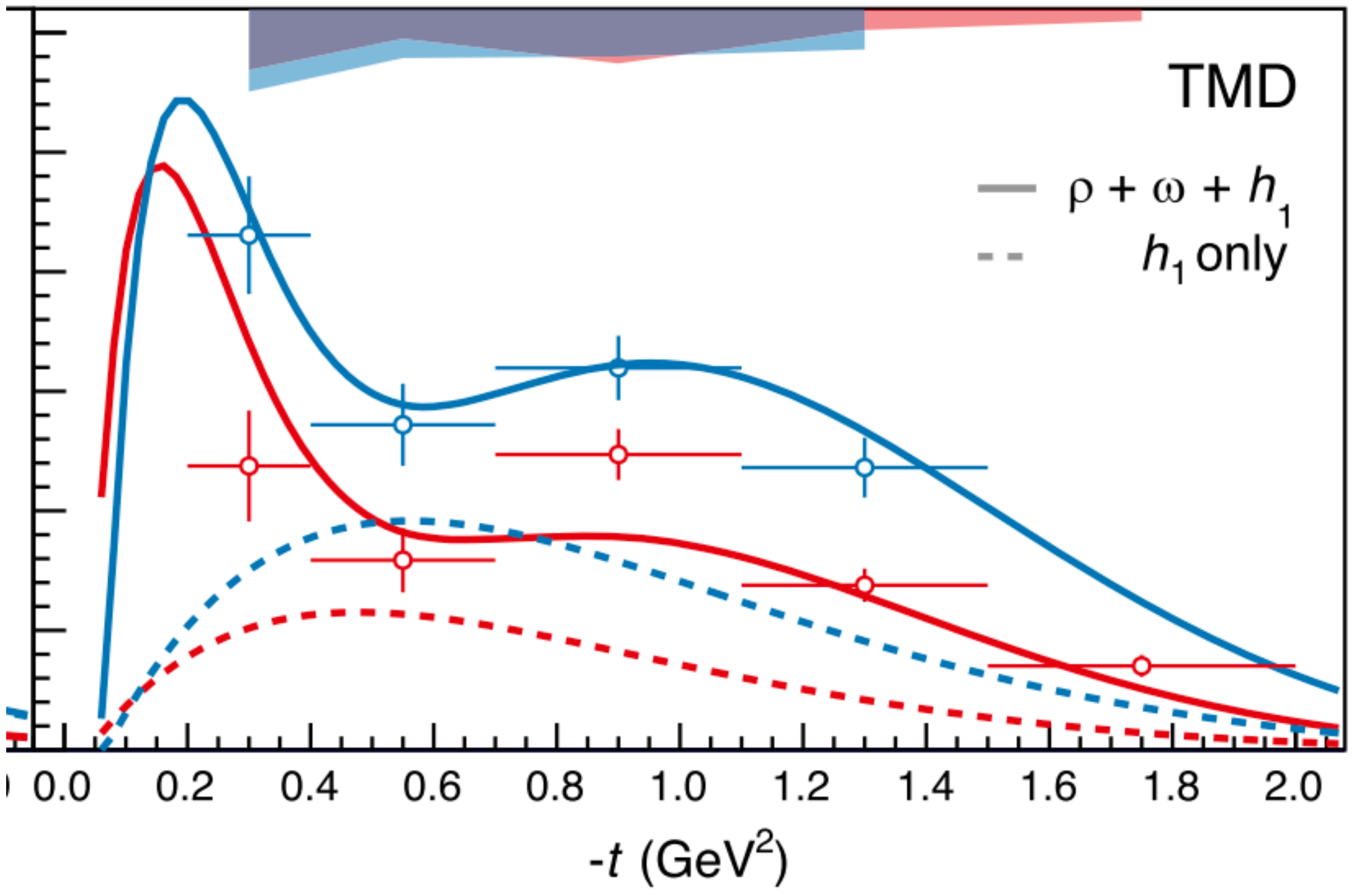
Tensor Meson Photoproduction @CLAS

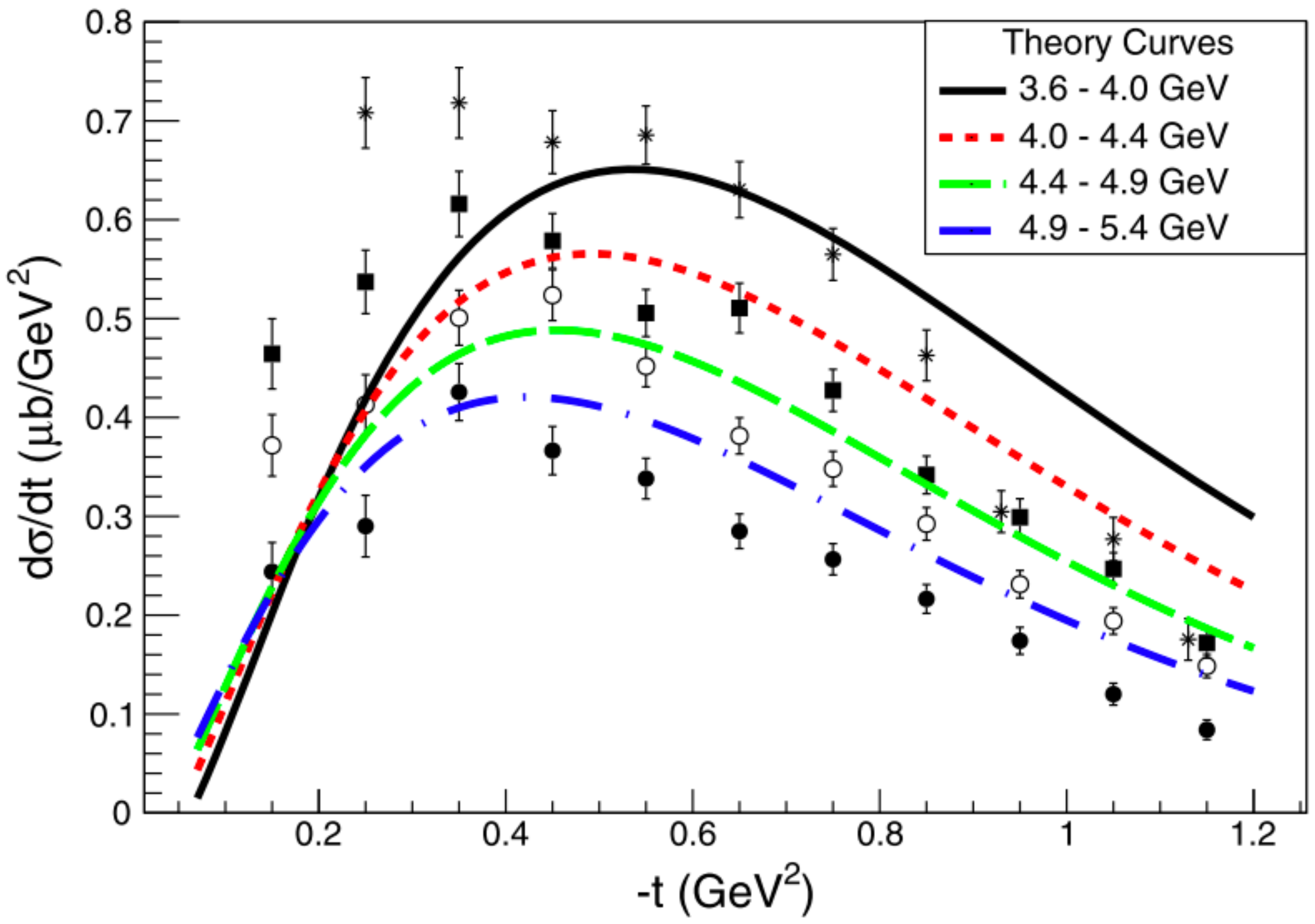
VM et al (JPAC) PRD102 (2020) 014003

CLAS PRC 102 (2020)

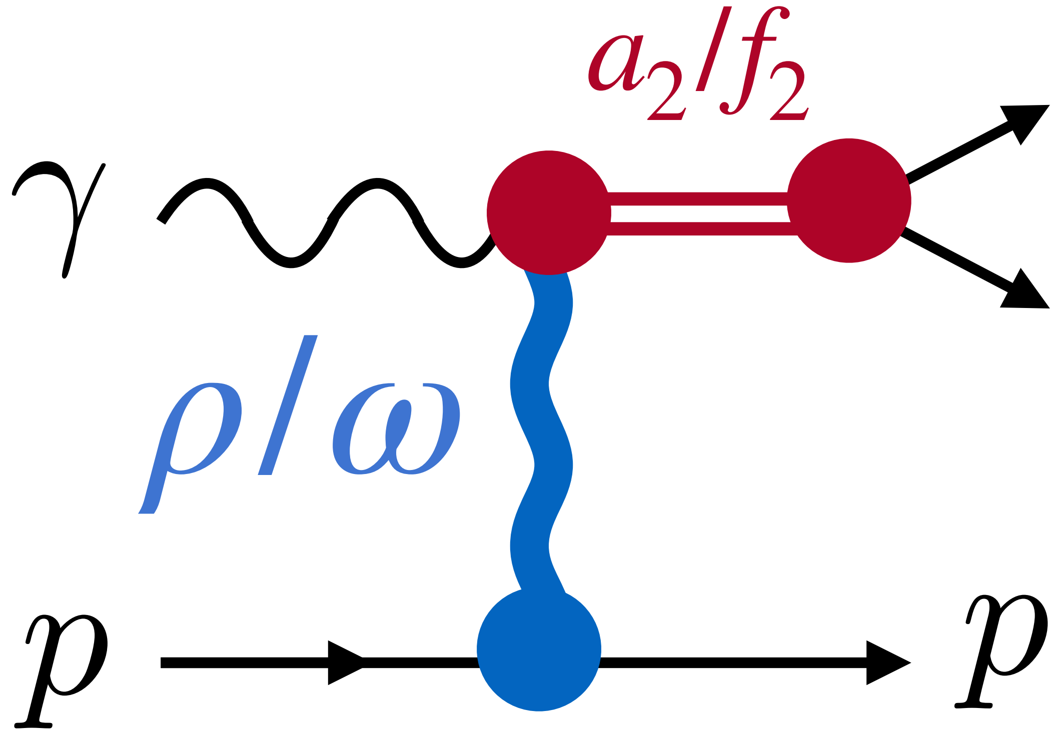


$$\gamma p \to a_2(1320)p :$$

CLAS PRL126 (2021)



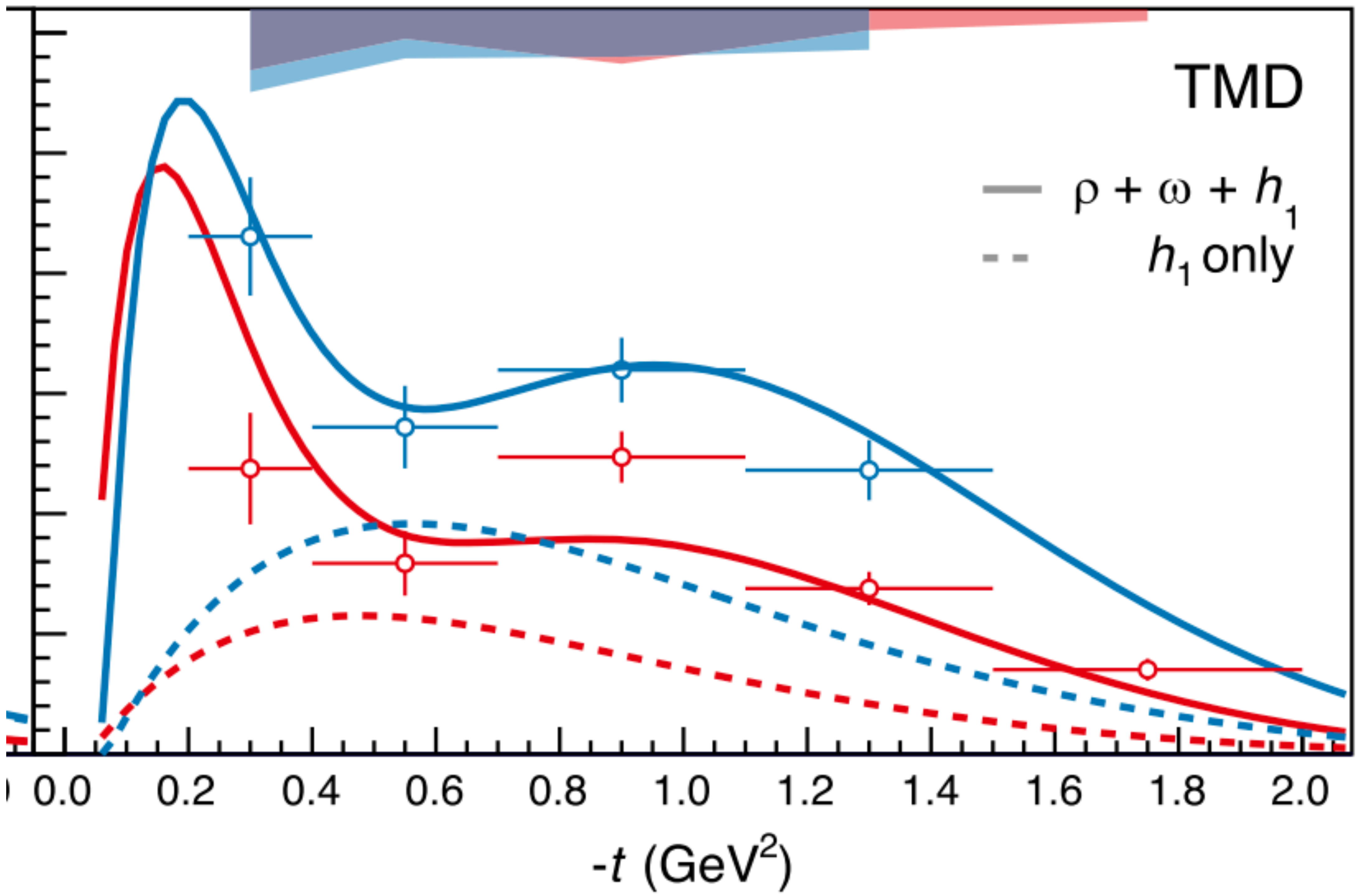
$$\gamma p \to f_2(1270)p :$$



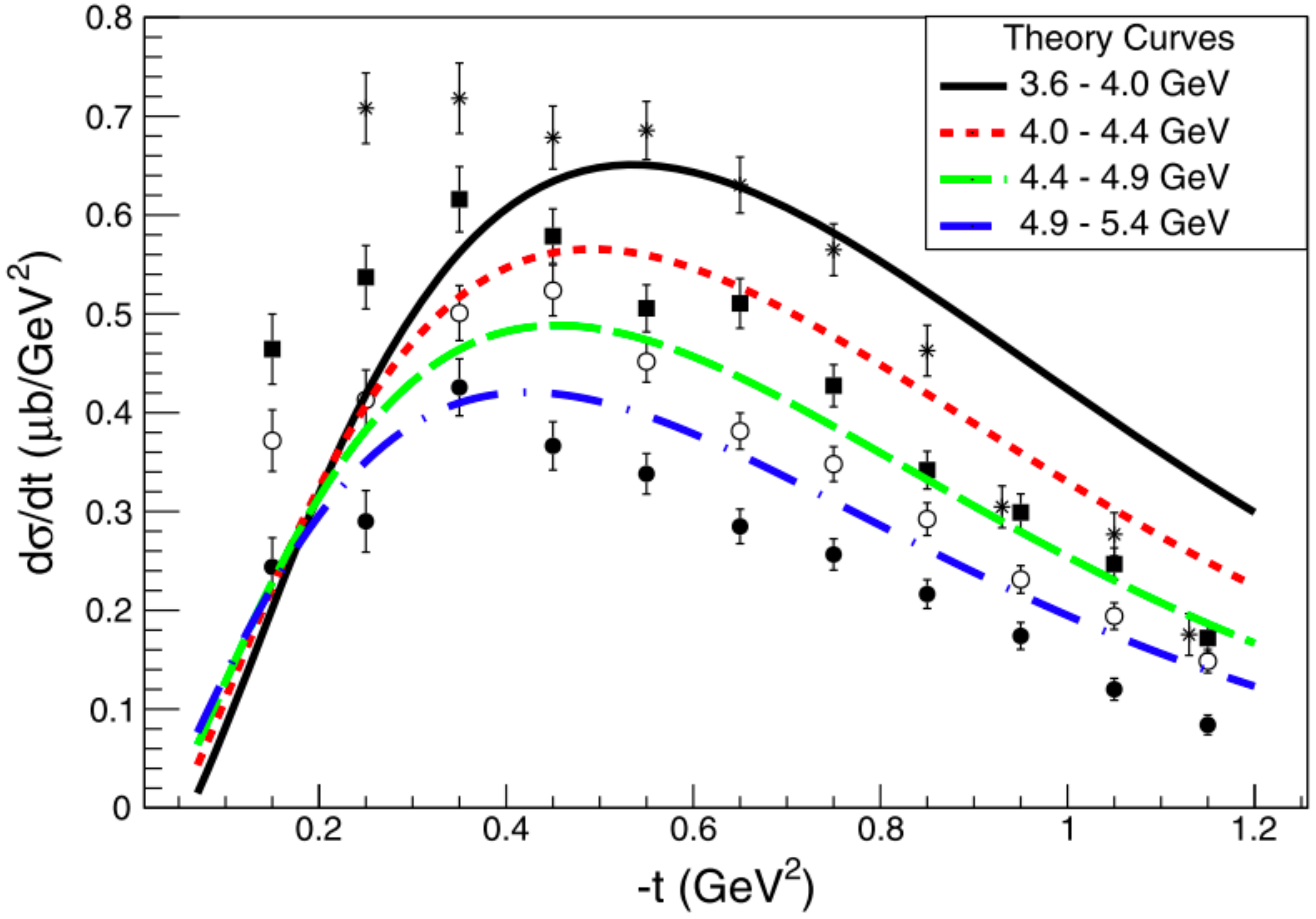
Tensor Meson Photoproduction @CLAS

VM et al (JPAC) PRD102 (2020) 014003

CLAS PRC 102 (2020)

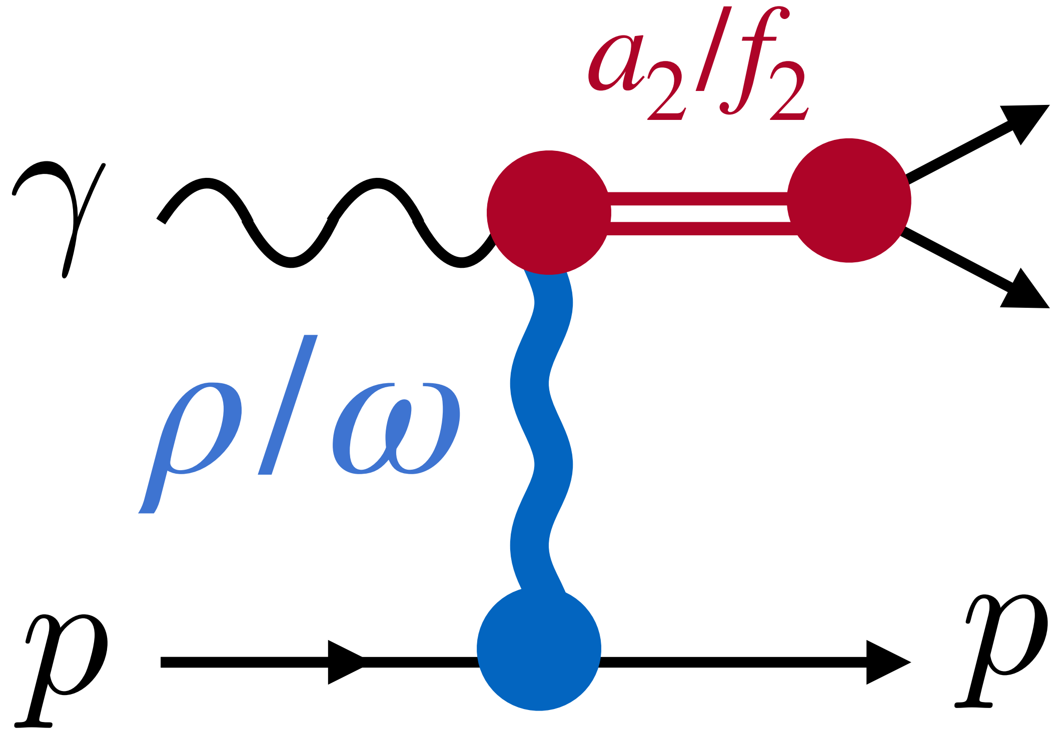


CLAS PRL126 (2021)



$$\gamma p \to a_2(1320)p :$$

$$\omega + \frac{1}{3}\rho$$



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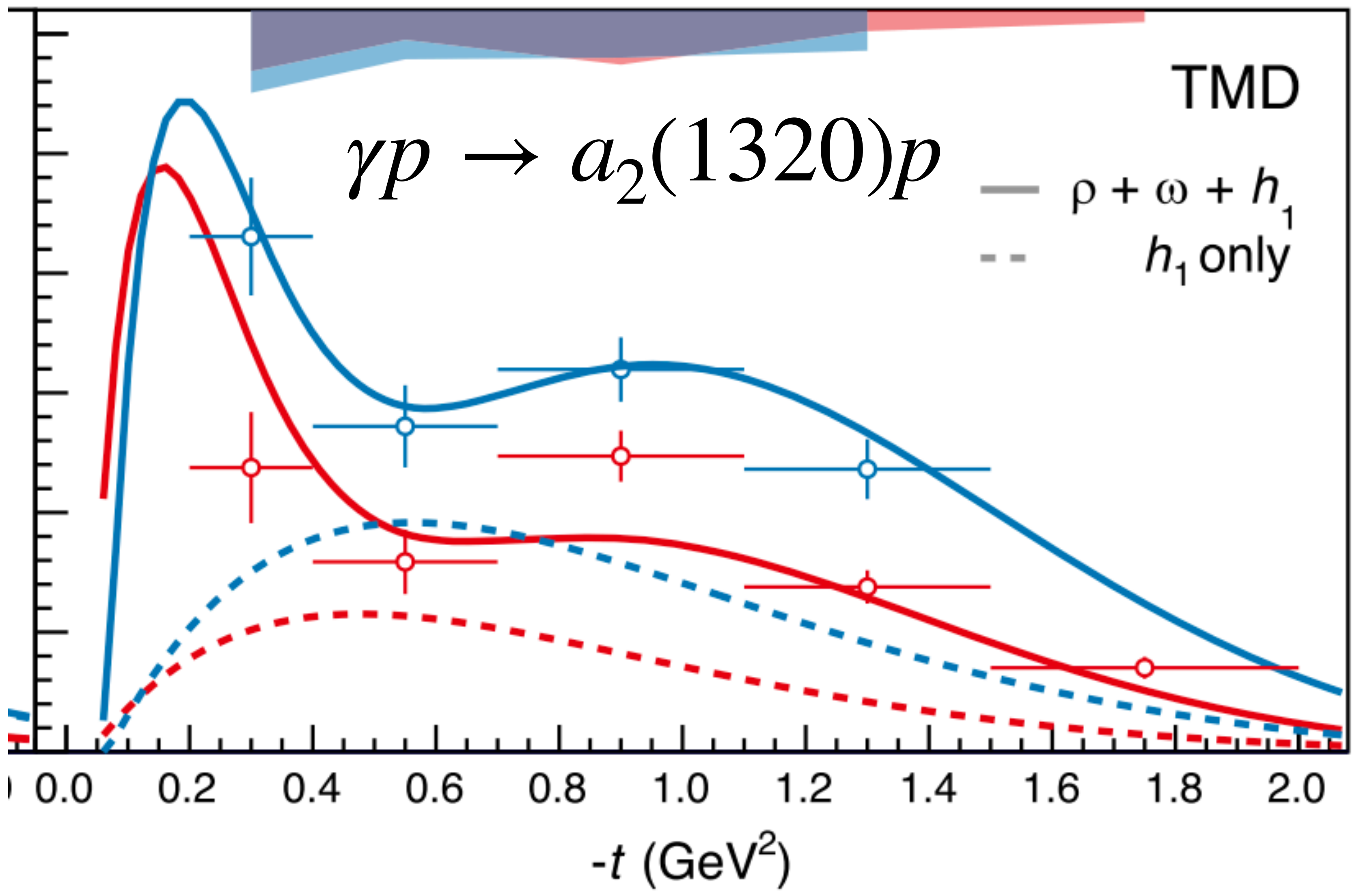
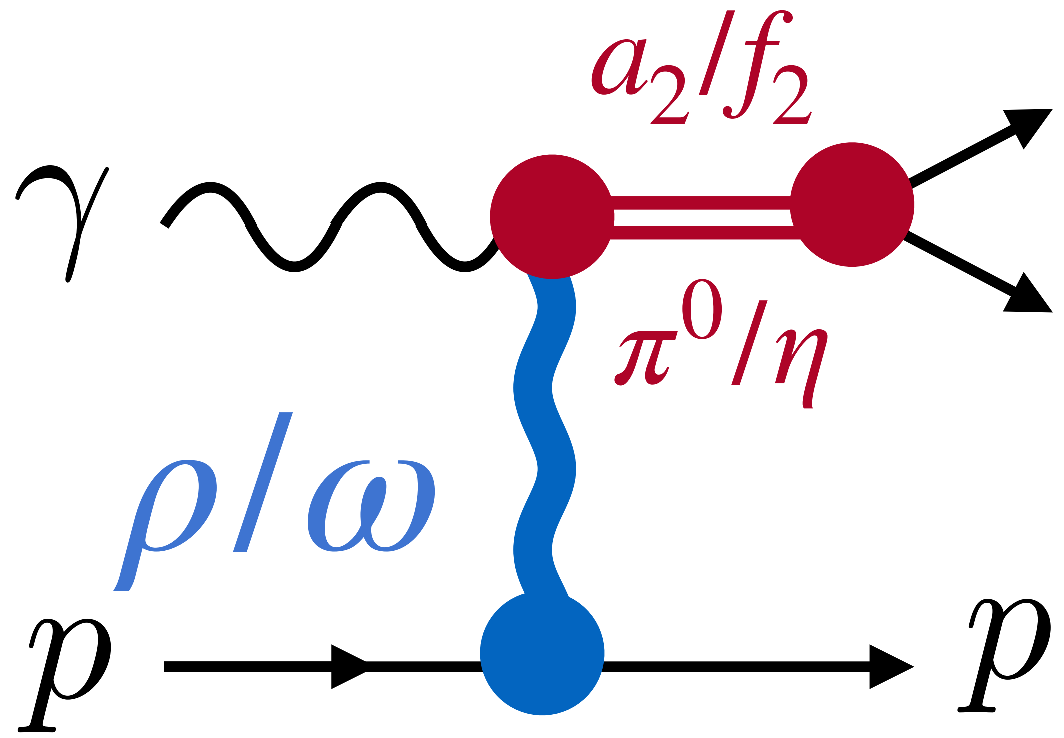
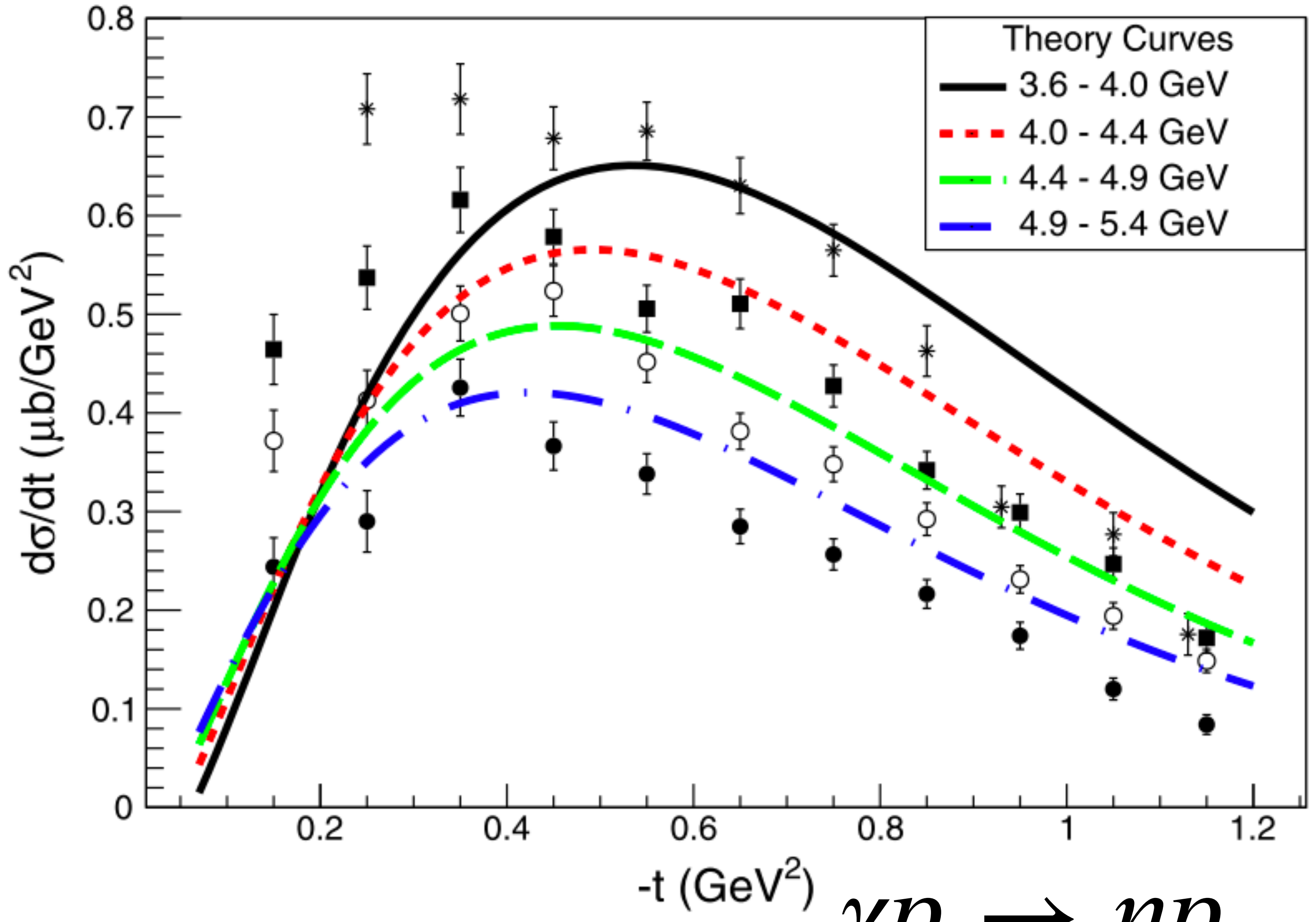
Tensor Meson Photoproduction @CLAS

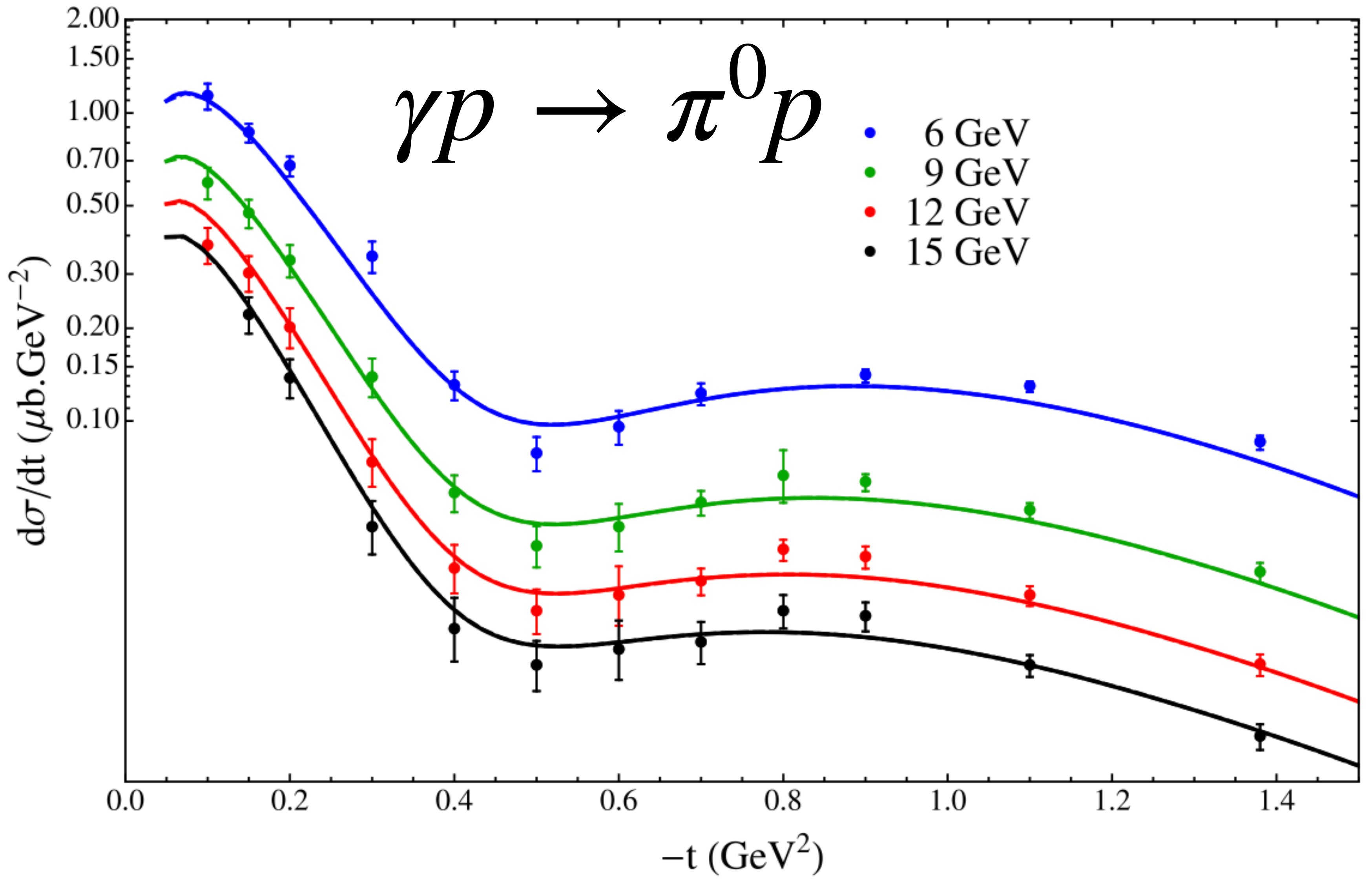
VM et al (JPAC) PRD102 (2020) 014003





$\gamma p \to f_2(1270)p$





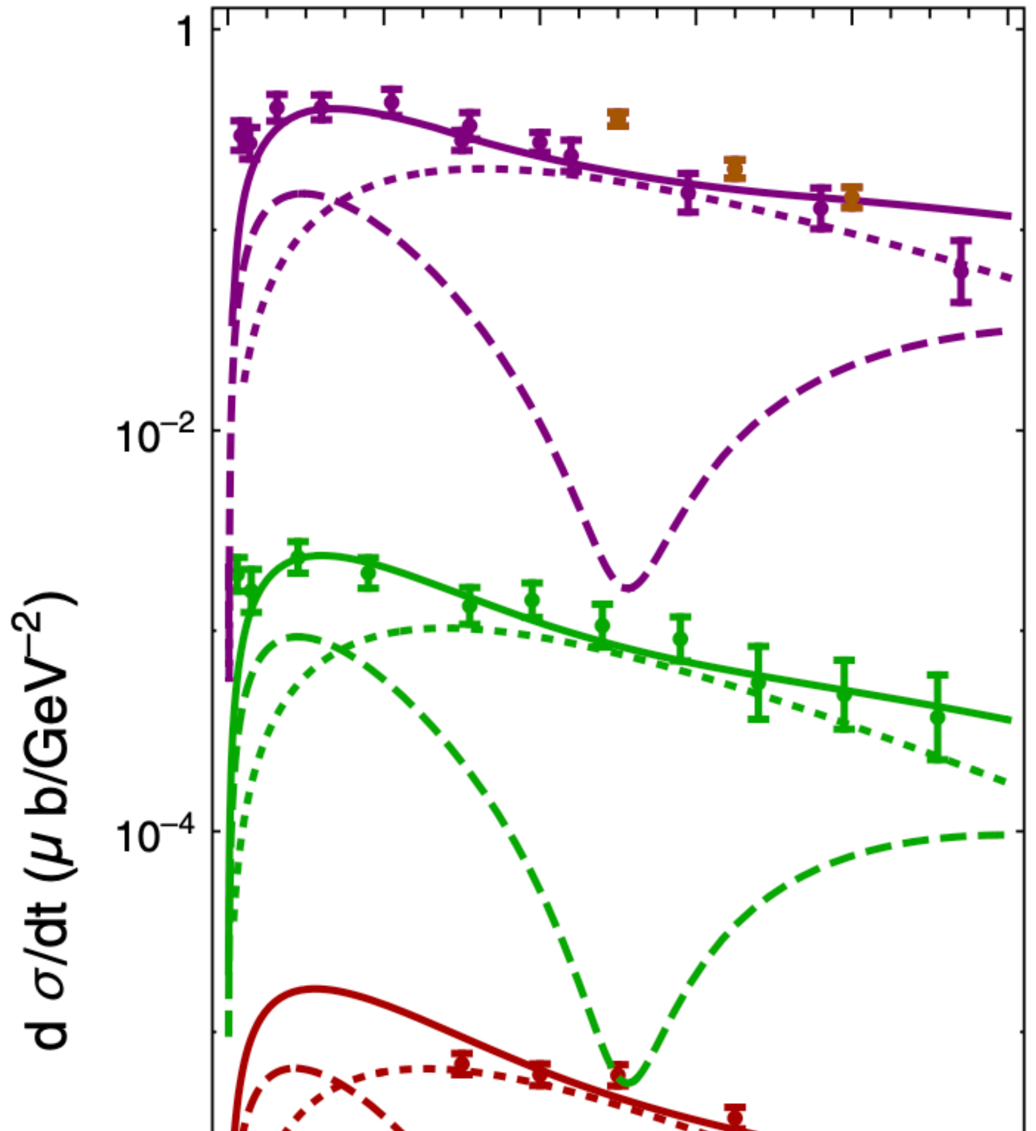
isovector:
dipping

$$\omega + \frac{1}{3}\rho$$

isoscalar:
non-dipping

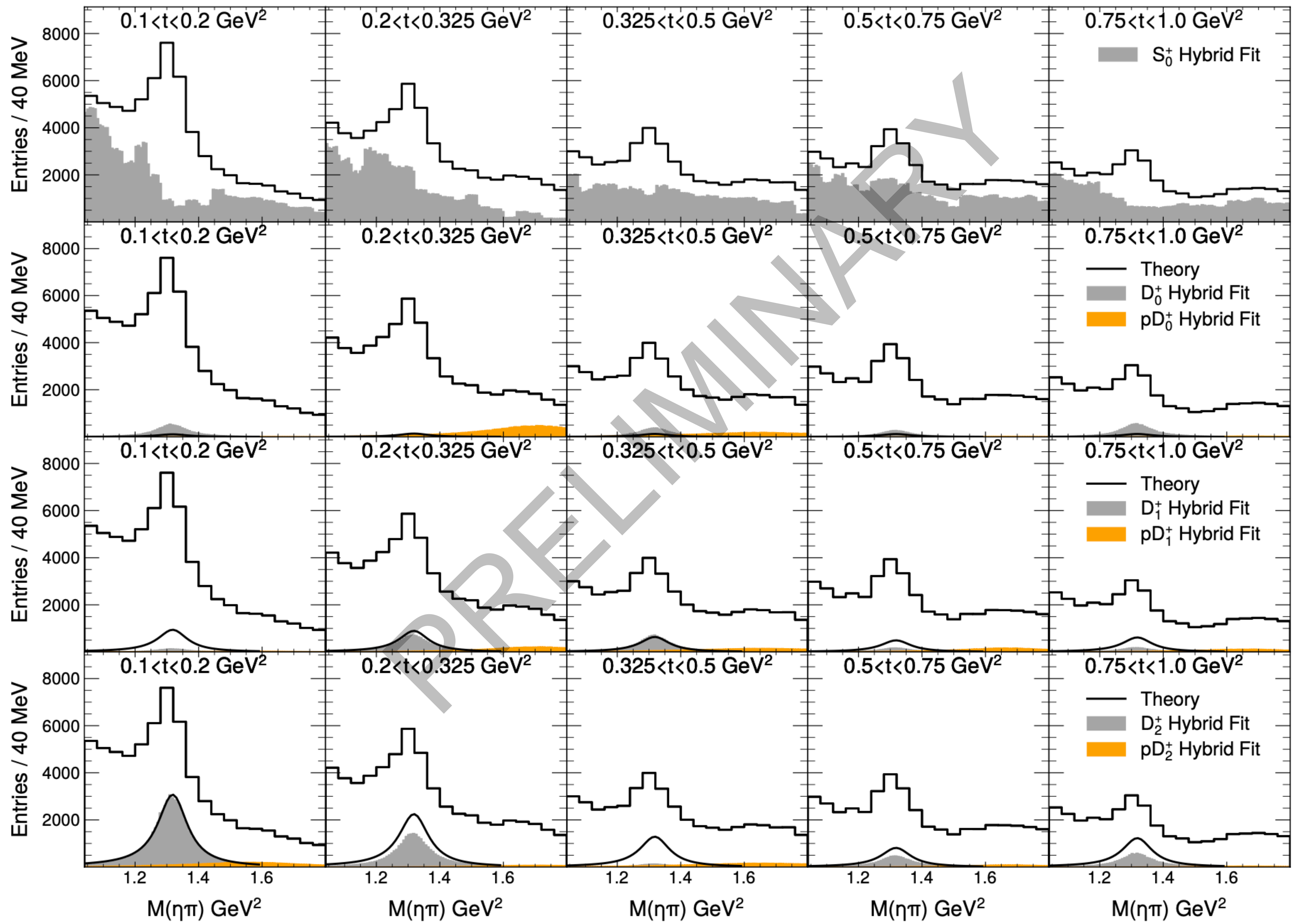
$$\rho + \frac{1}{3}\omega$$

$\gamma p \to \eta p$



$a_2(1320)$ Photoproduction @GlueX

In collaboration with L. Ng and M. Albrecht



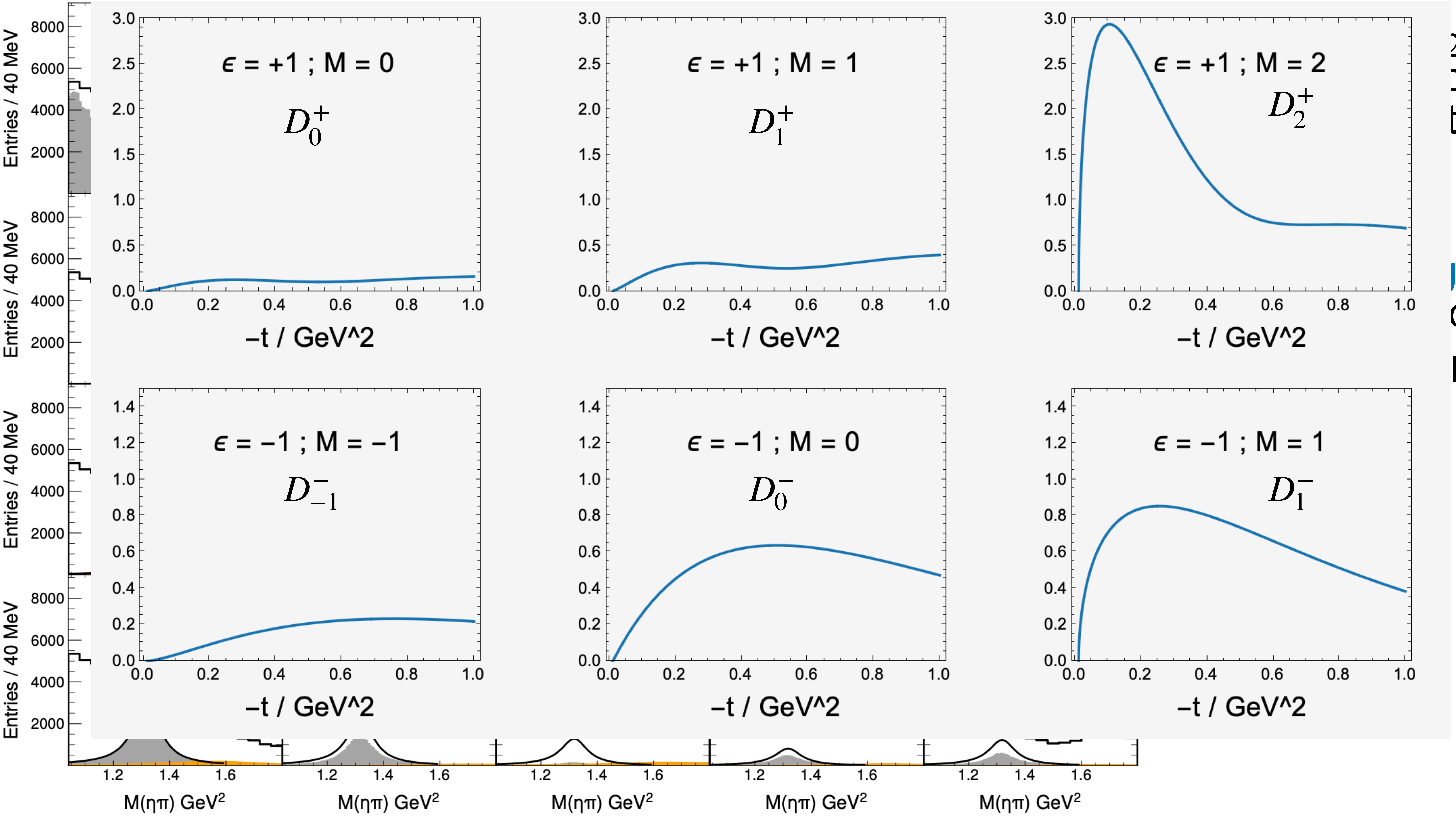
Extraction of $a_2(1320)$ production amplitudes from GlueX data

Reasonable agreement with model predictions from

VM et al (JPAC) PRD102 (2020)

$a_2(1320)$ Photoproduction @GlueX

In collaboration with L. Ng and M. Albrecht

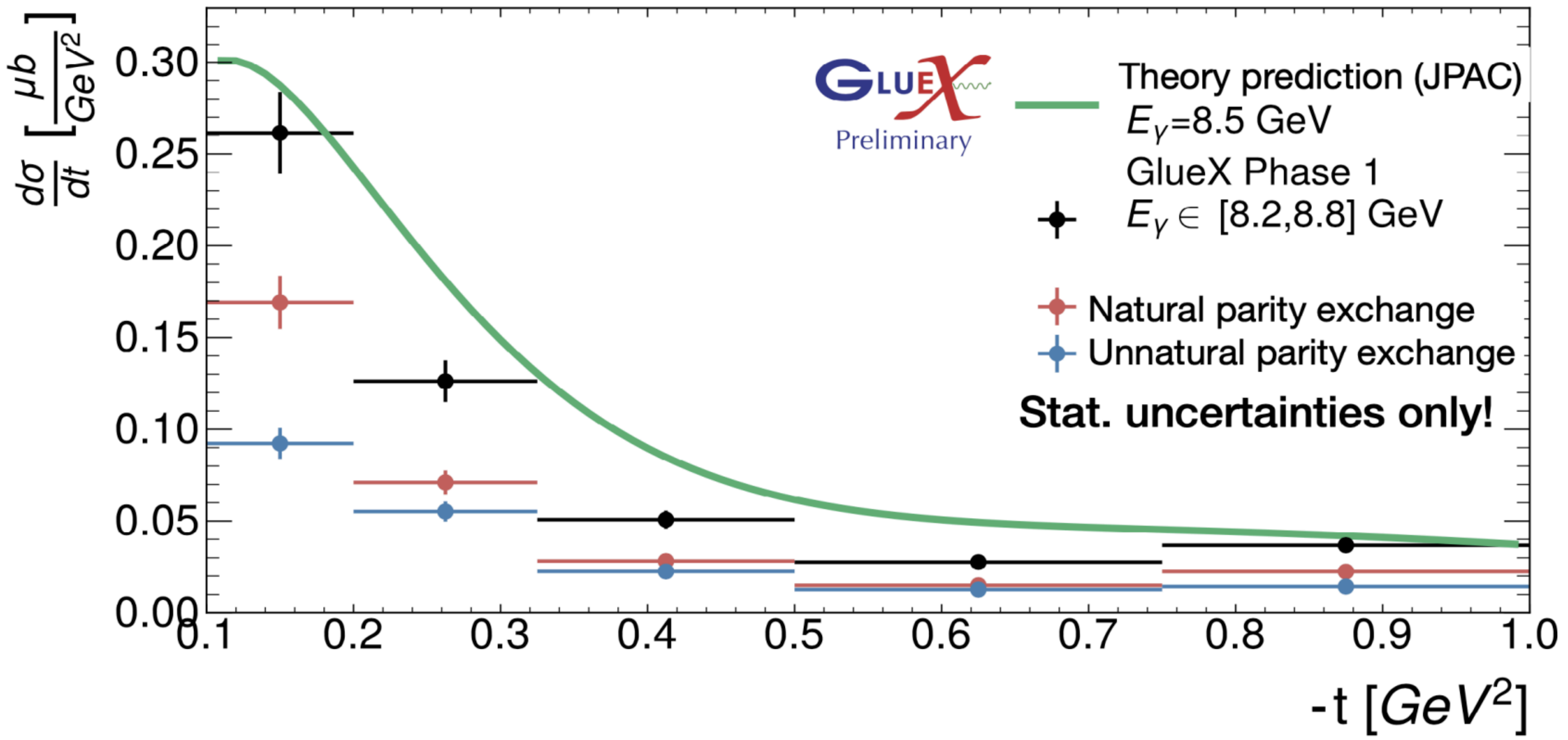


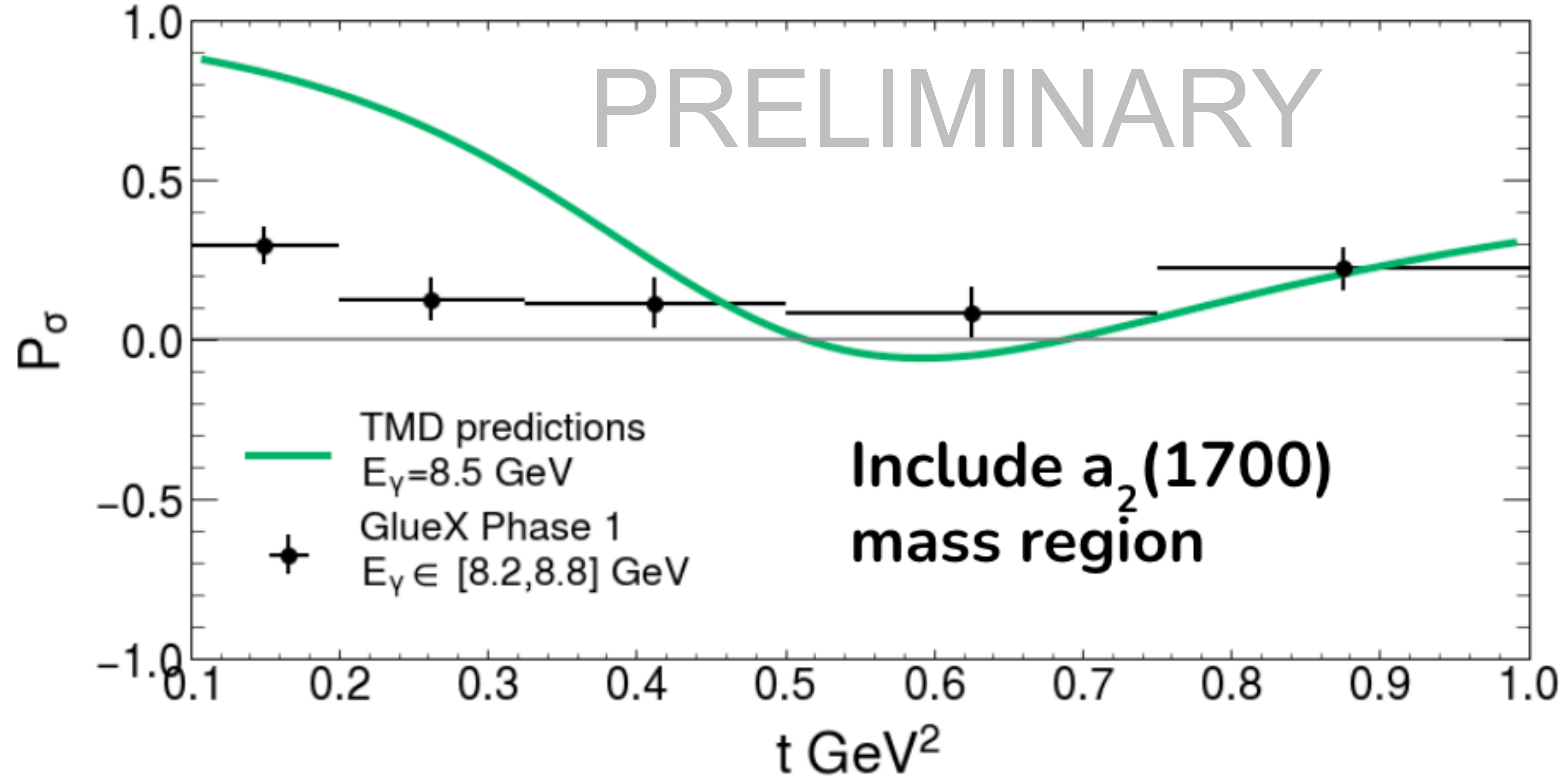
₂(1320)
plitudes
ta

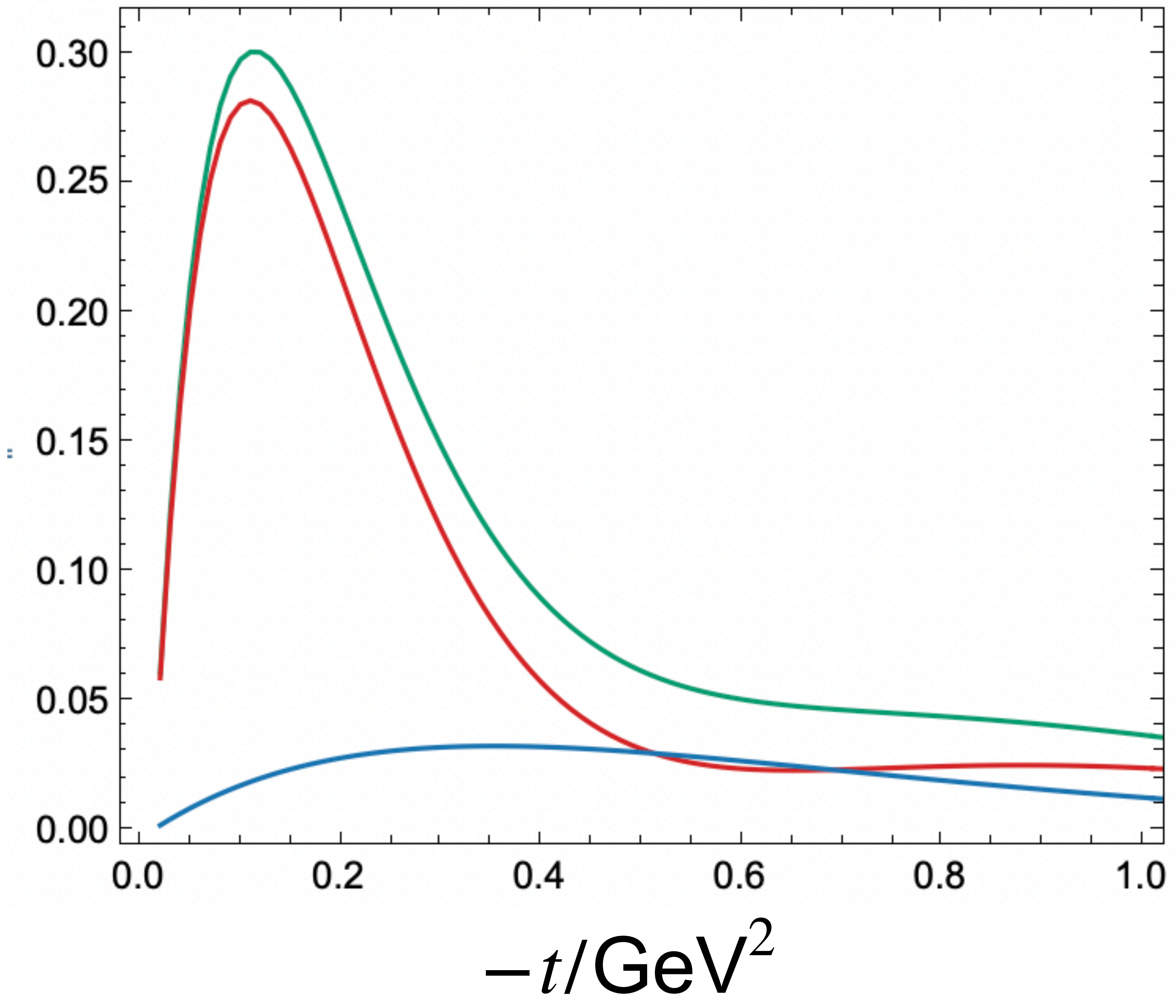
greement
dictions from
PRD102 (2020)

a2(1320)@GlueX

VM et al (JPAC) PRD102 (2020) 014003



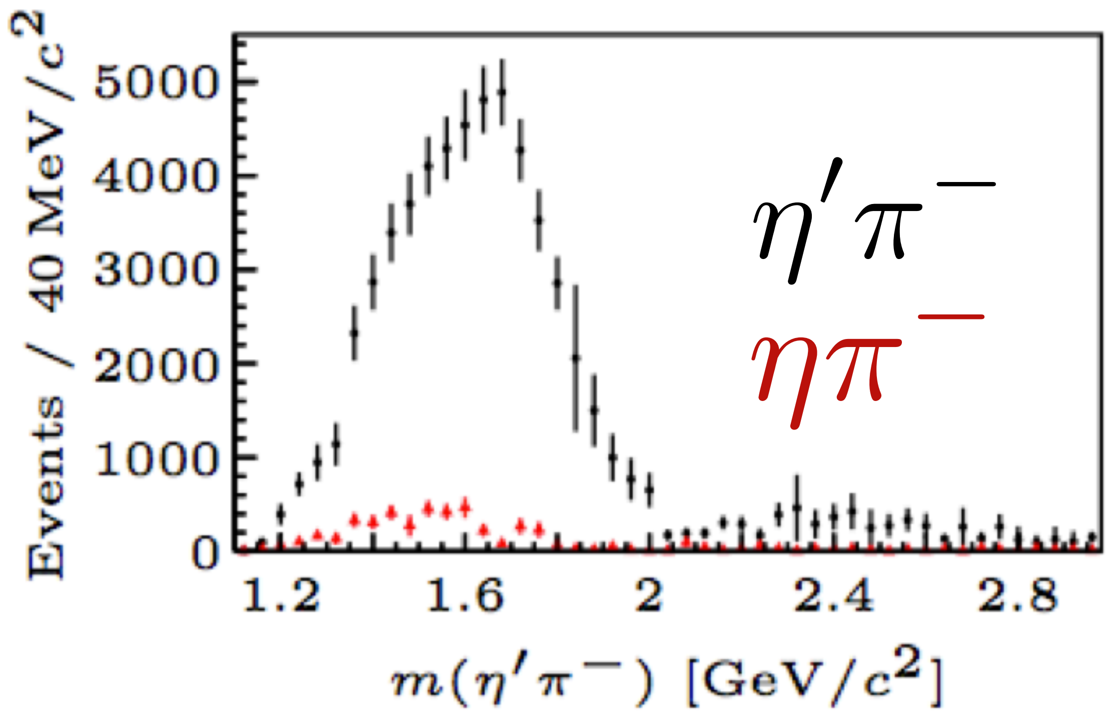


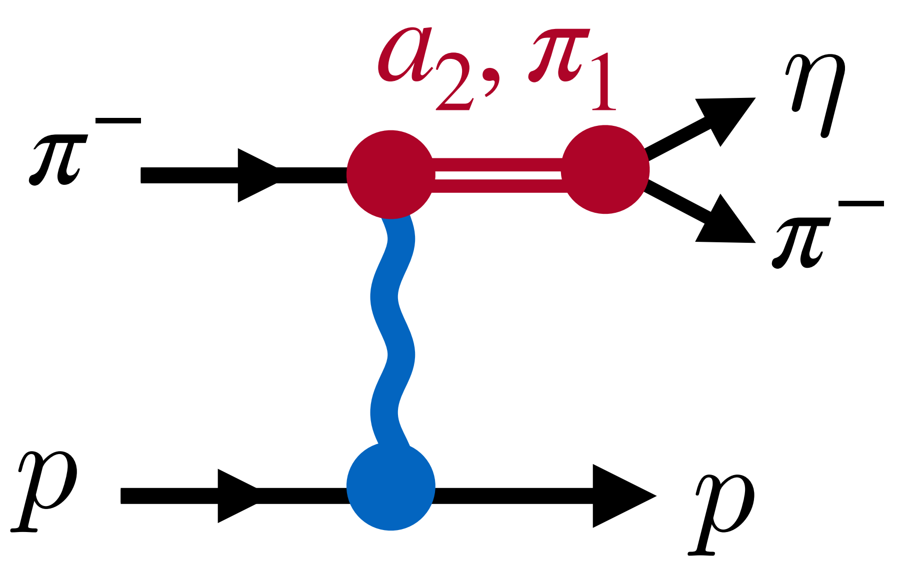


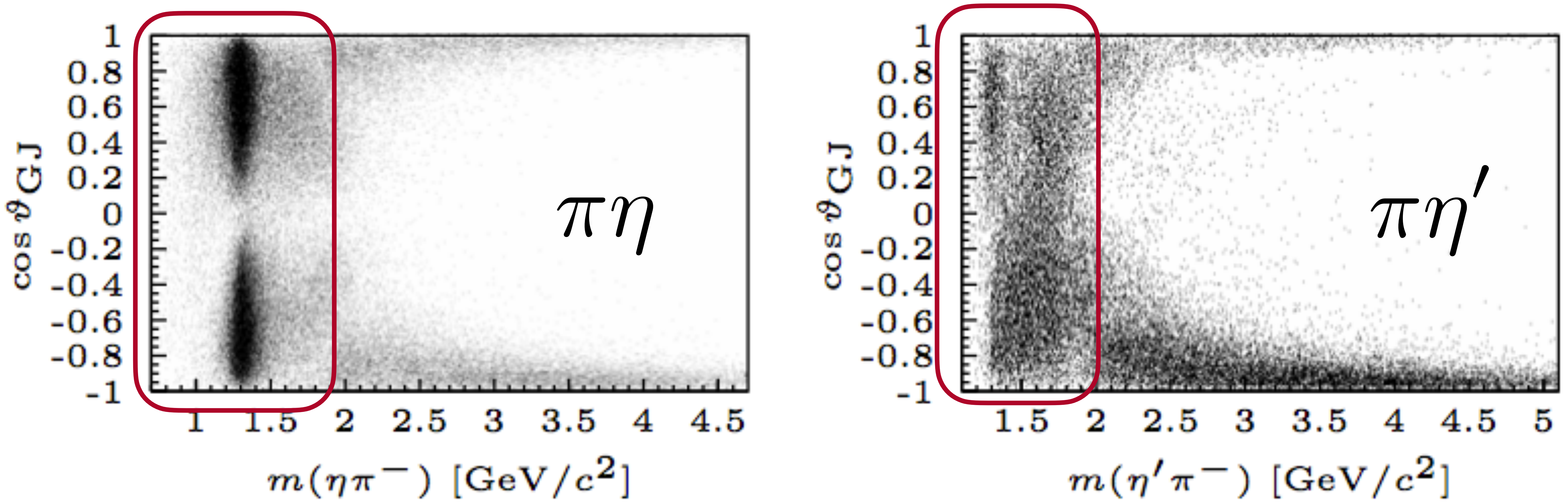
Dominance of the vector exchange at low t in the model

Origin of the Exotic Meson

COMPASS PLB740 (2015)







Exotic meson related to
Forward-backward asymmetry

Asymmetry related to
even-odd waves interferences

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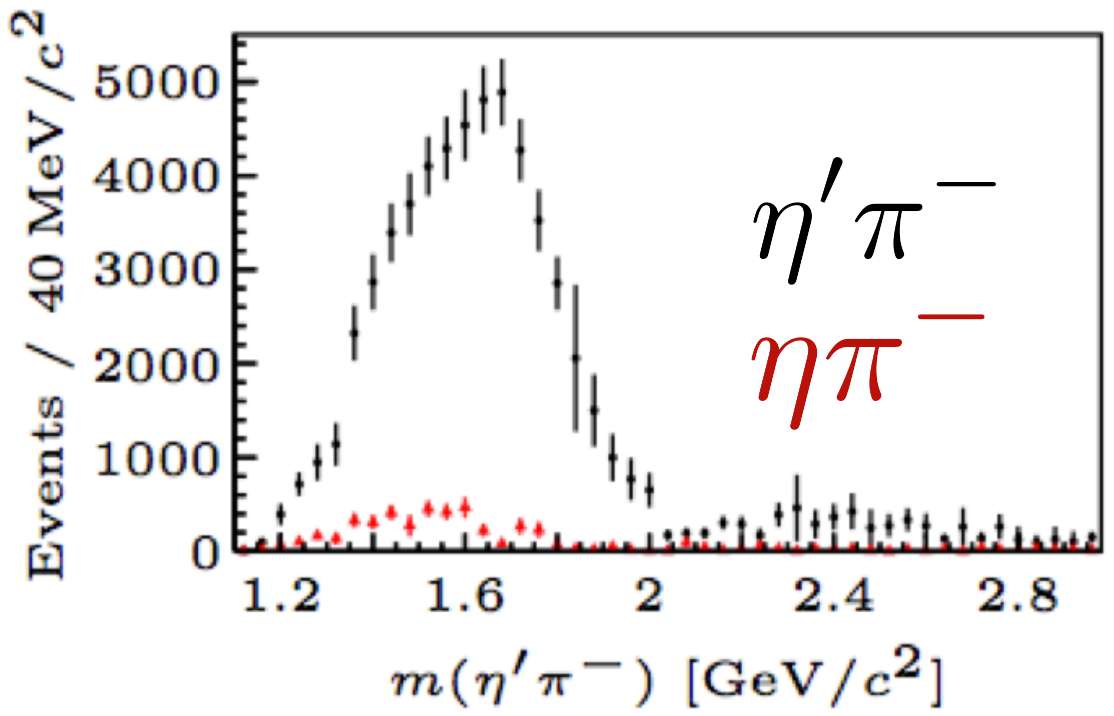
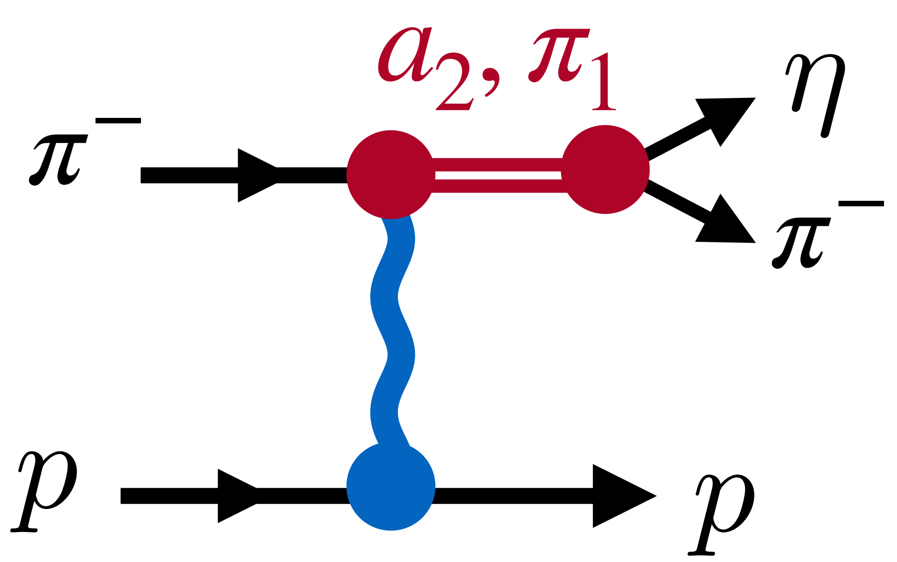
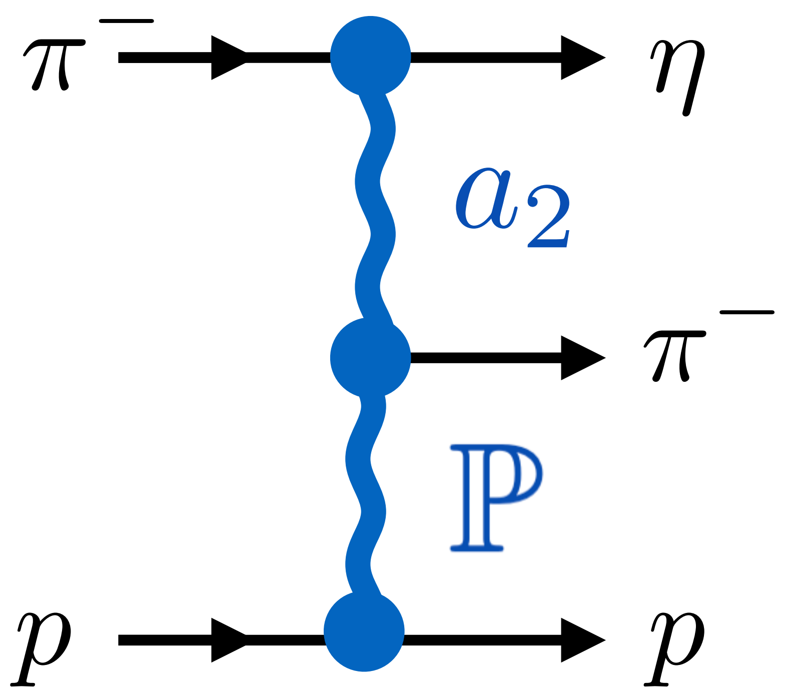
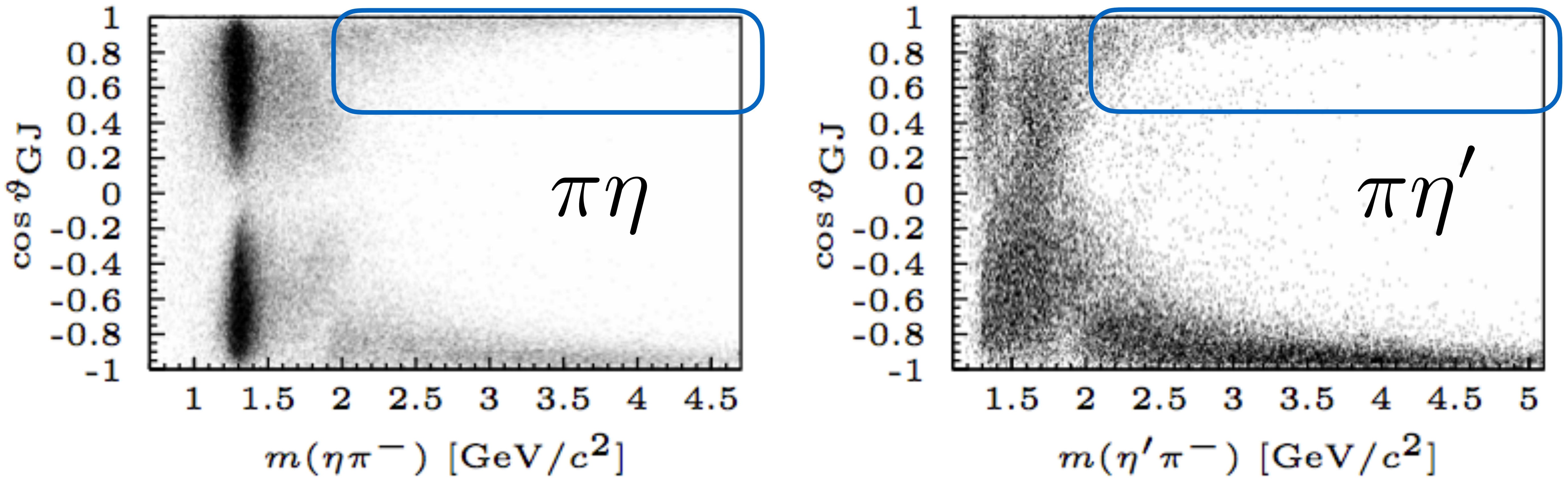






$\cos\theta_{GF} \sim 1 \rightarrow \eta$ forward



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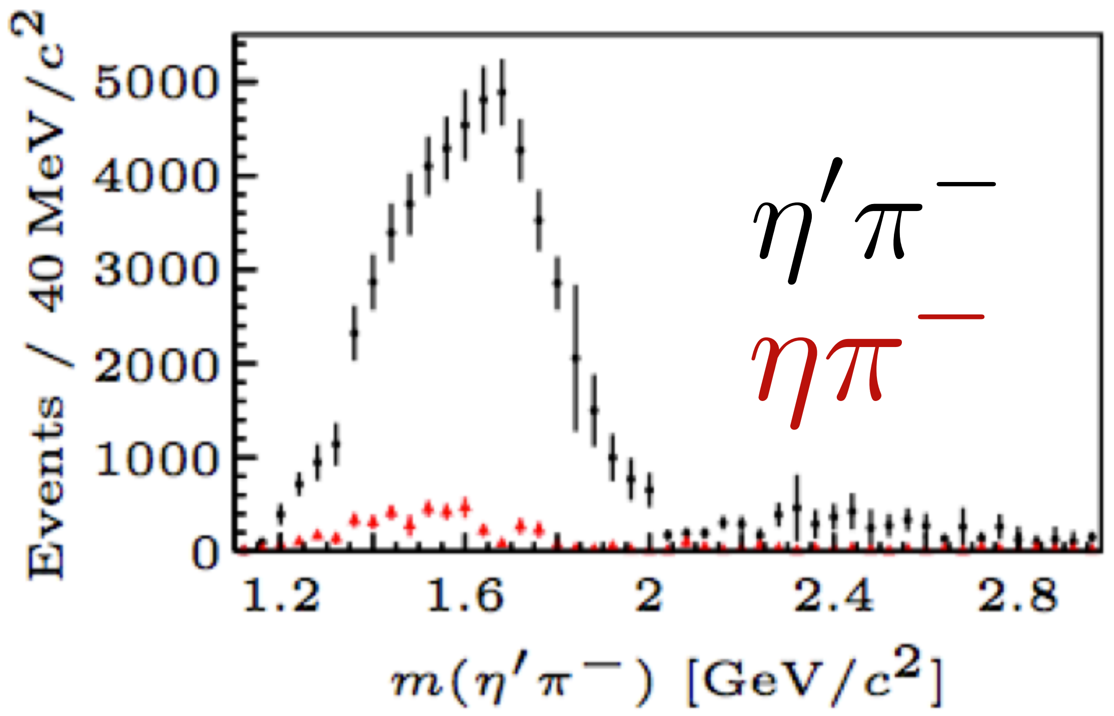

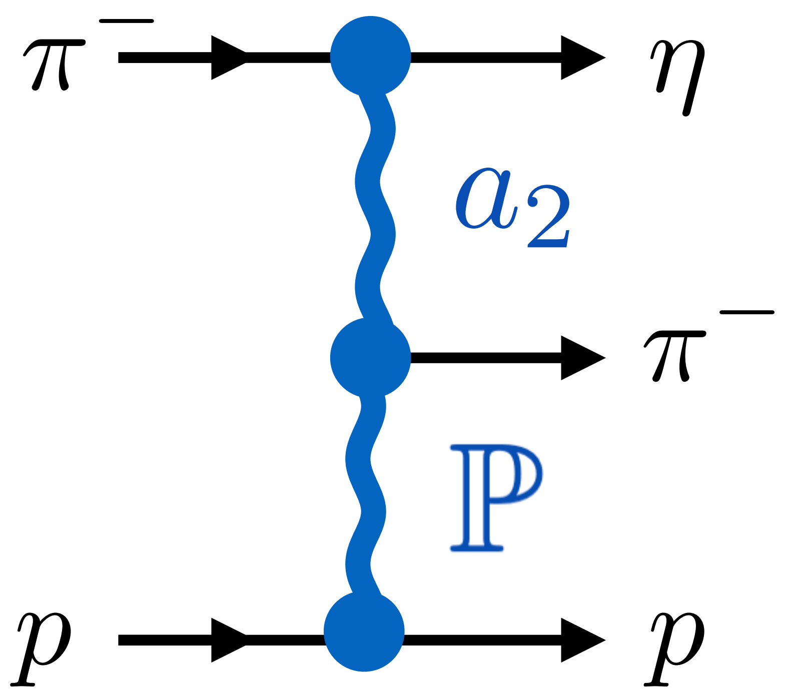

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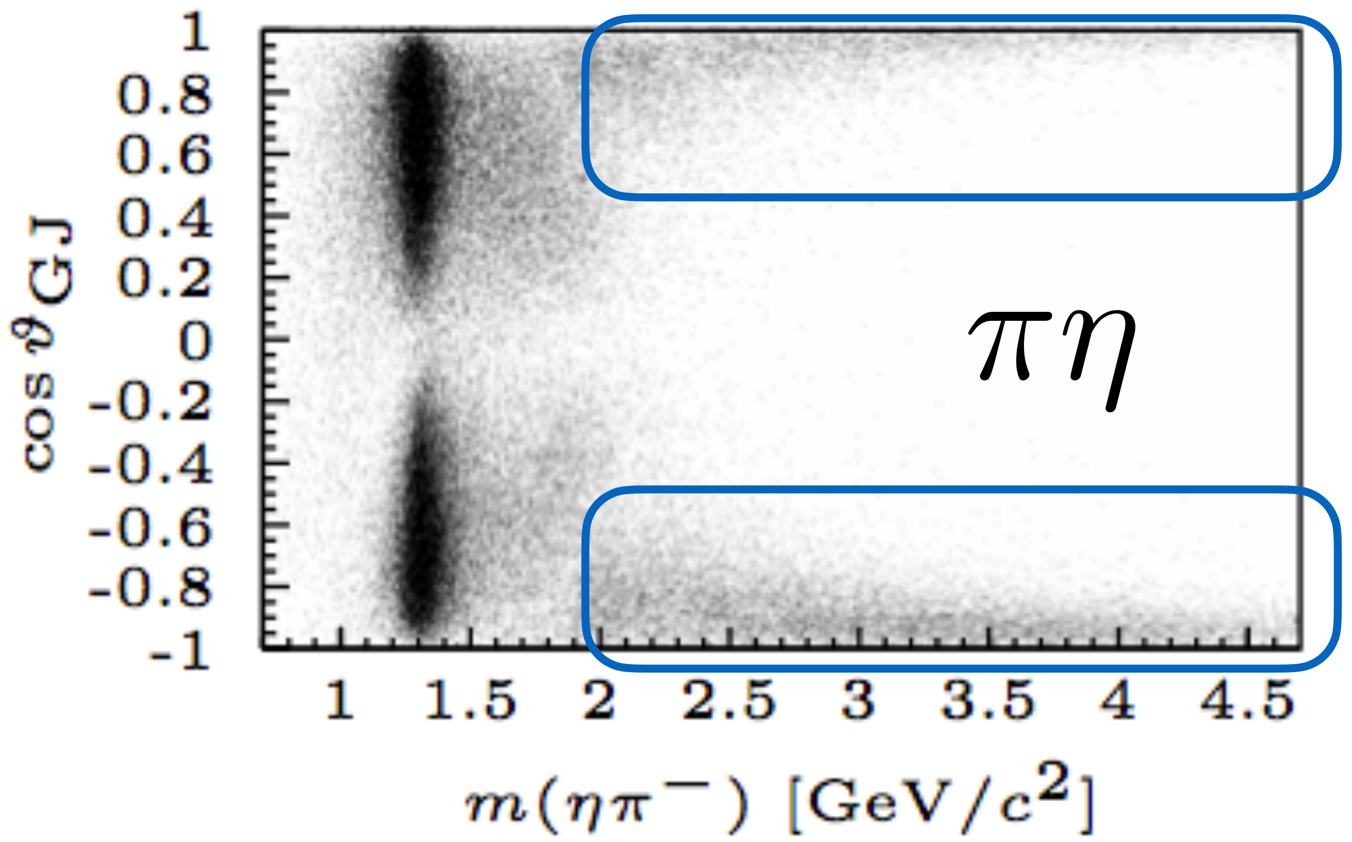
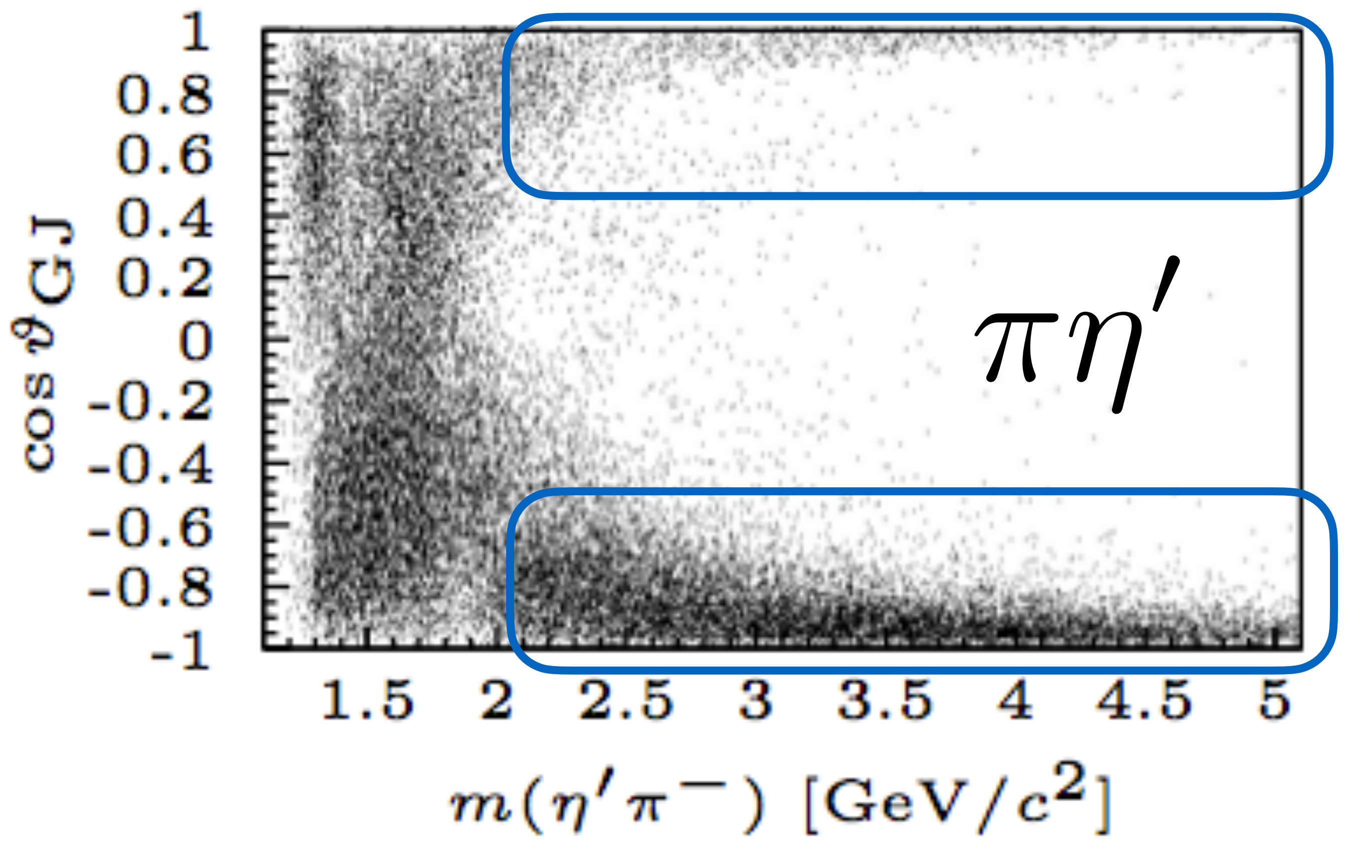
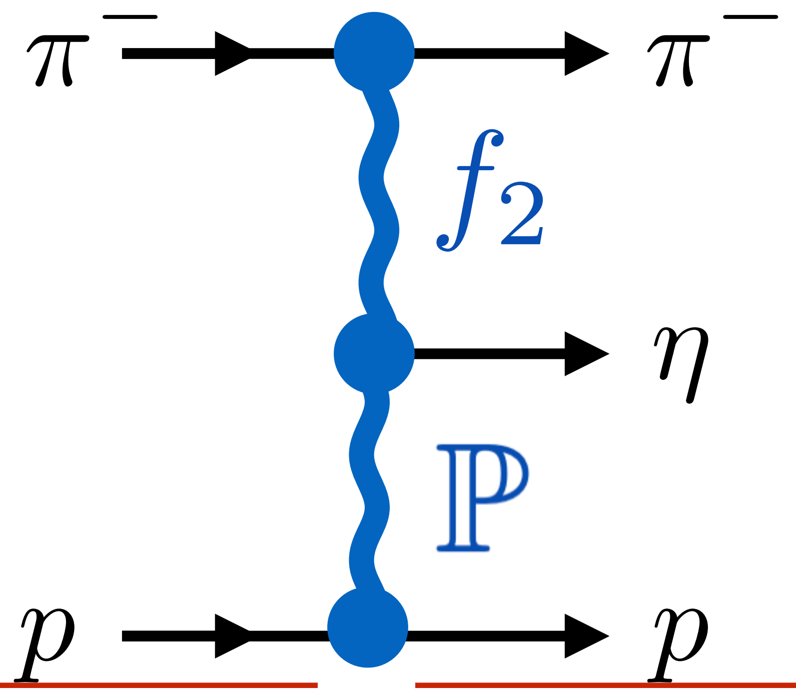


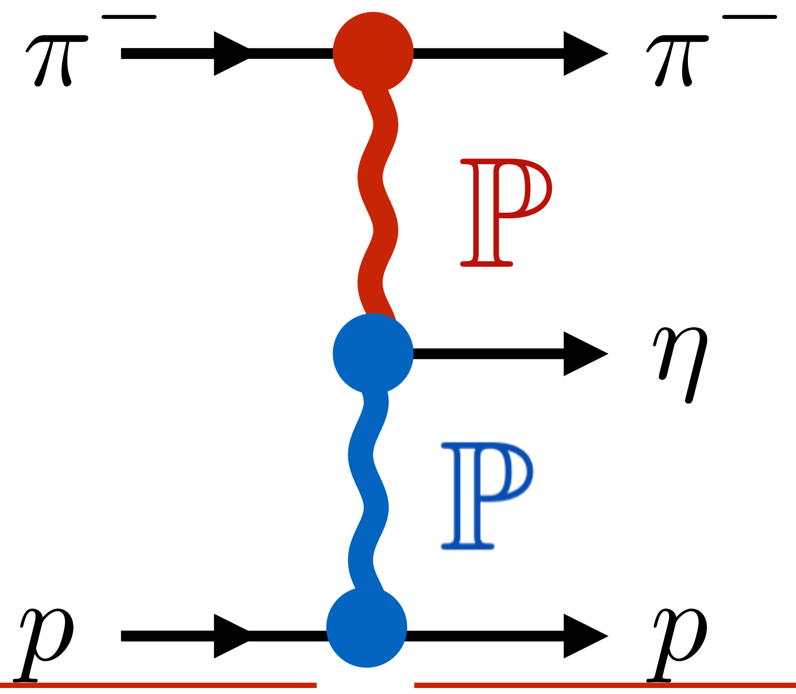


$\cos\theta_{GF} \sim -1 \to \eta$ backward

Exotic meson related to
Forward-backward asymmetry

Asymmetry related to
even-odd waves interferences





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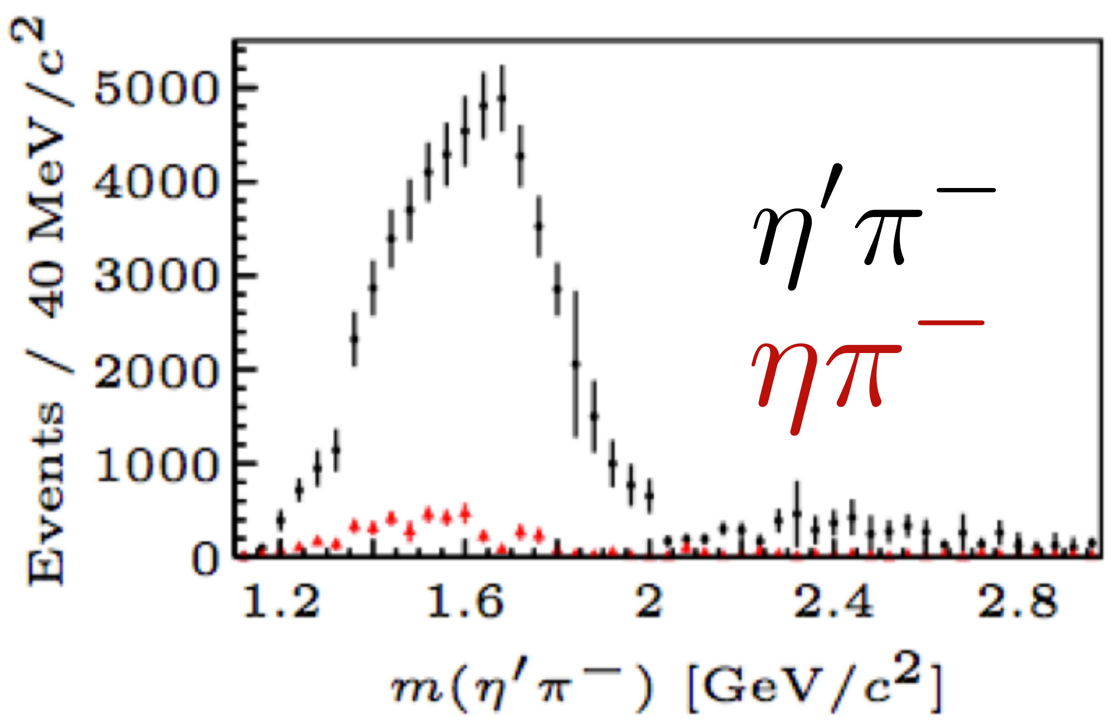
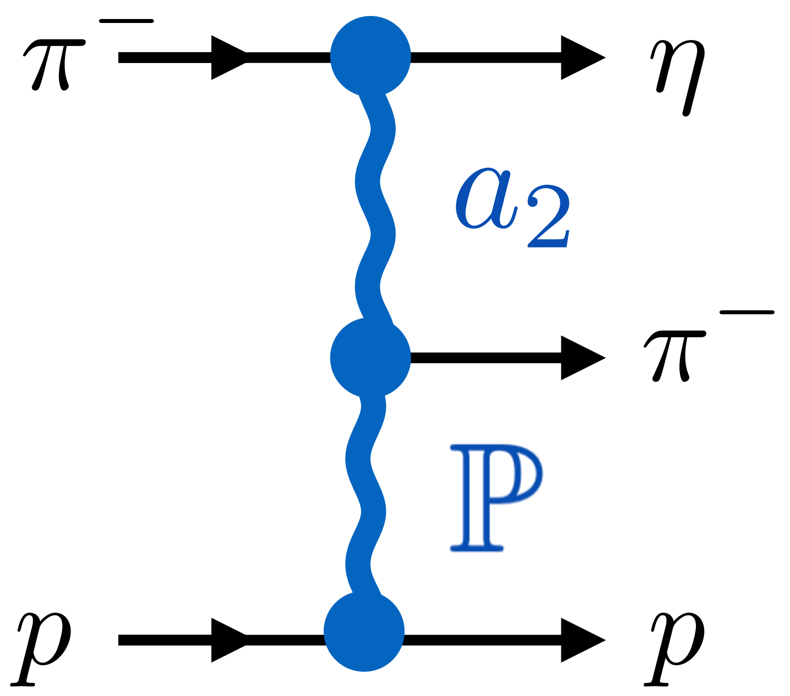


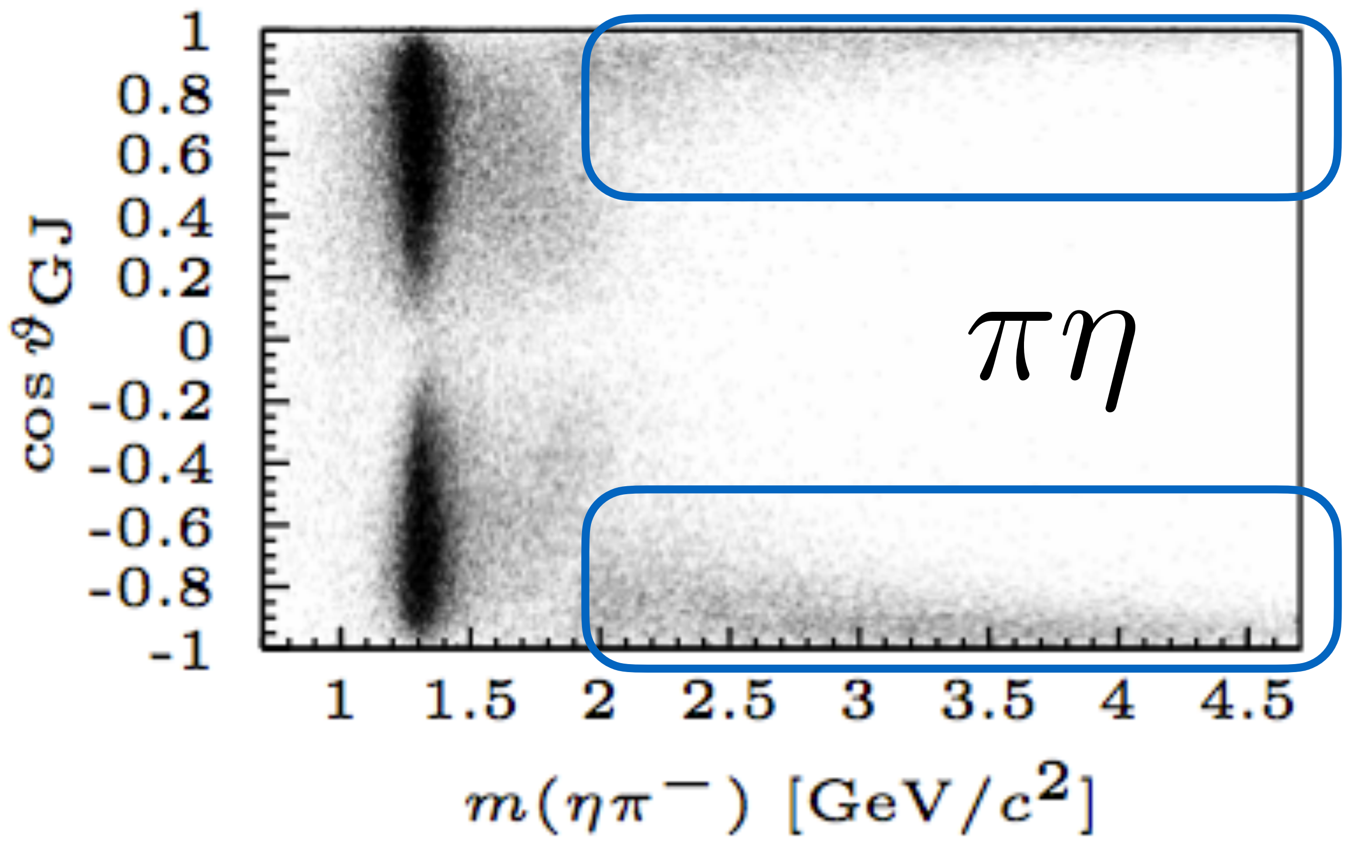


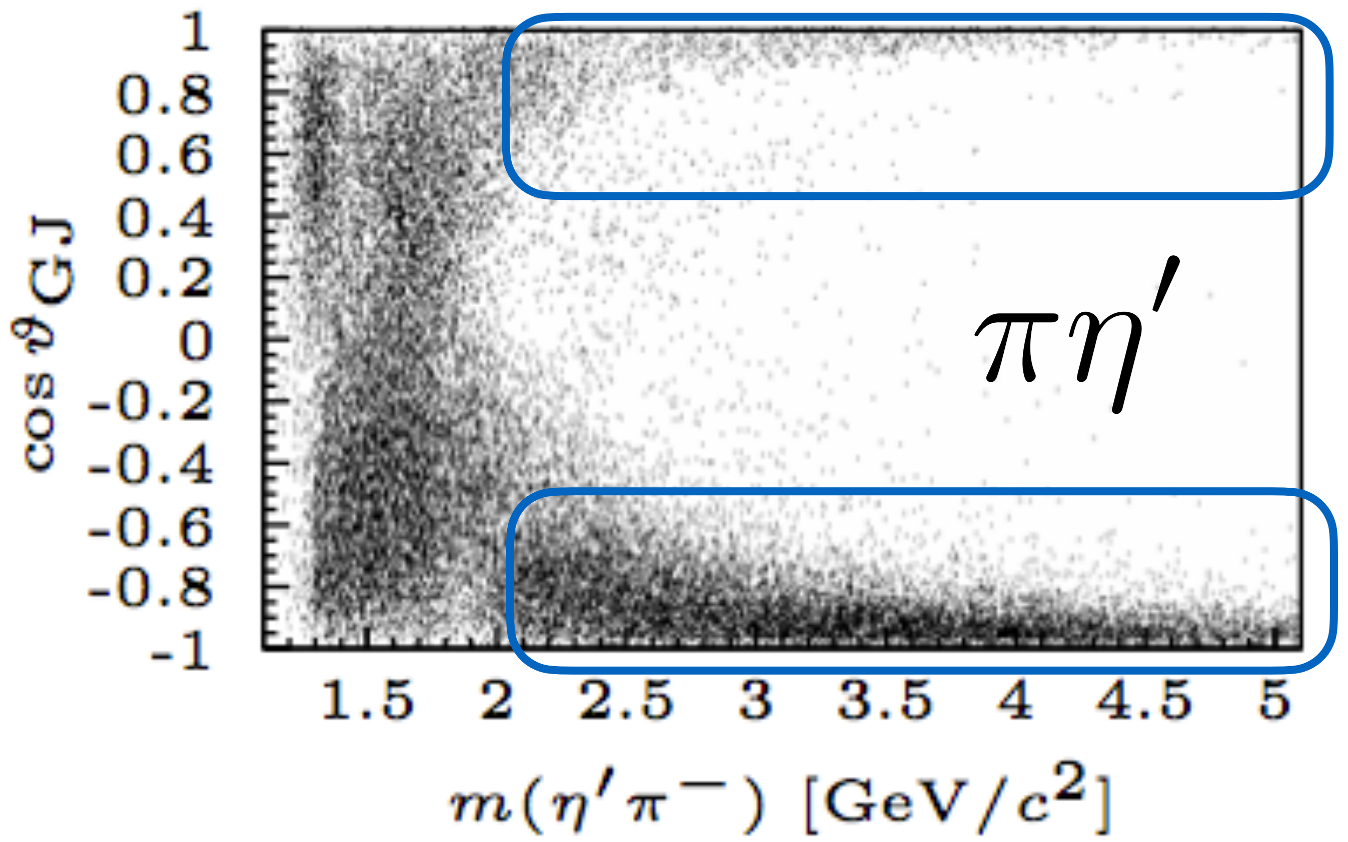
If a2 and f2 diagrams equal and no Pomeron

NO ASYMMETRY and NO π_1

$\cos\theta_{GF} \sim 1 \to \eta$ forward

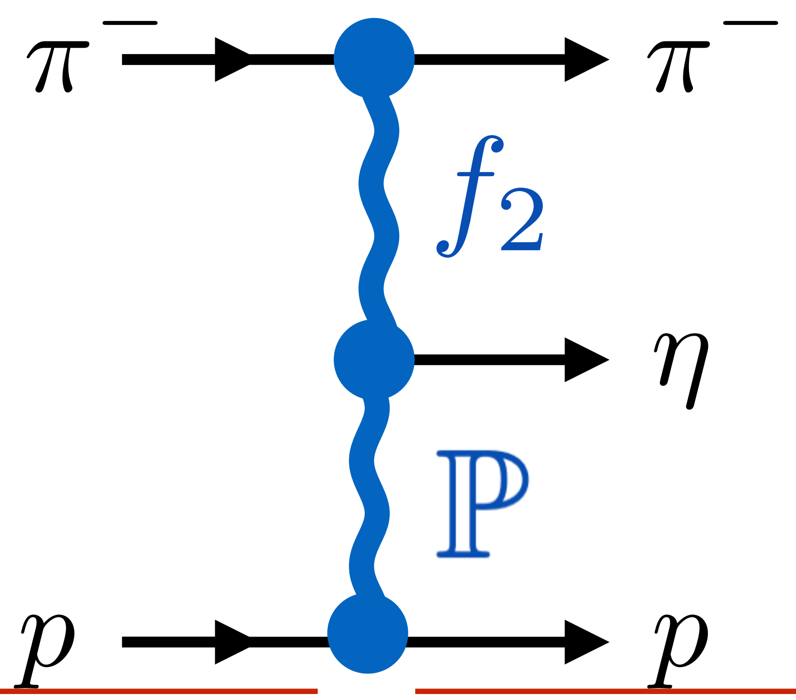
$\cos\theta_{GF} \sim -1 \to \eta$ backward

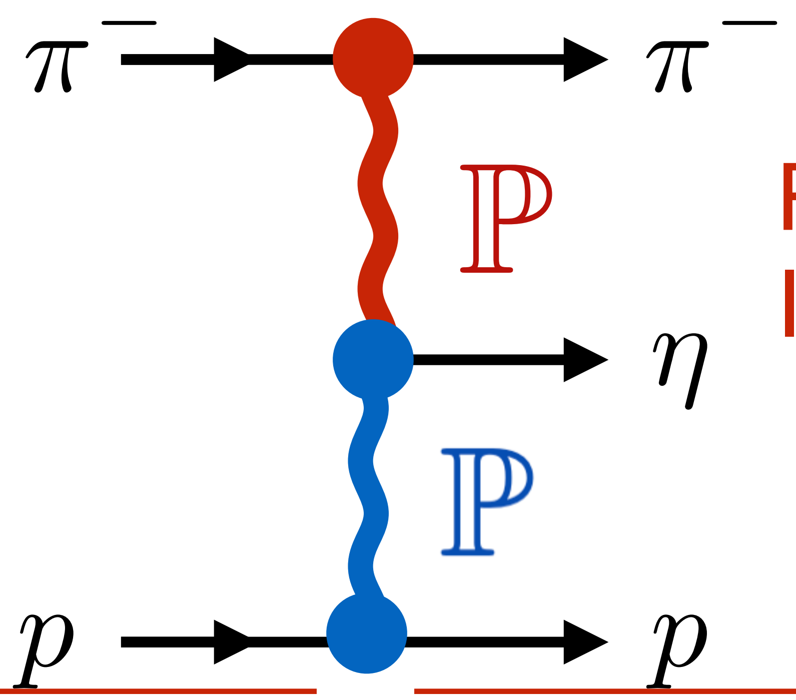




Exotic meson related to Forward-backward asymmetry

Asymmetry related to even-odd waves interferences

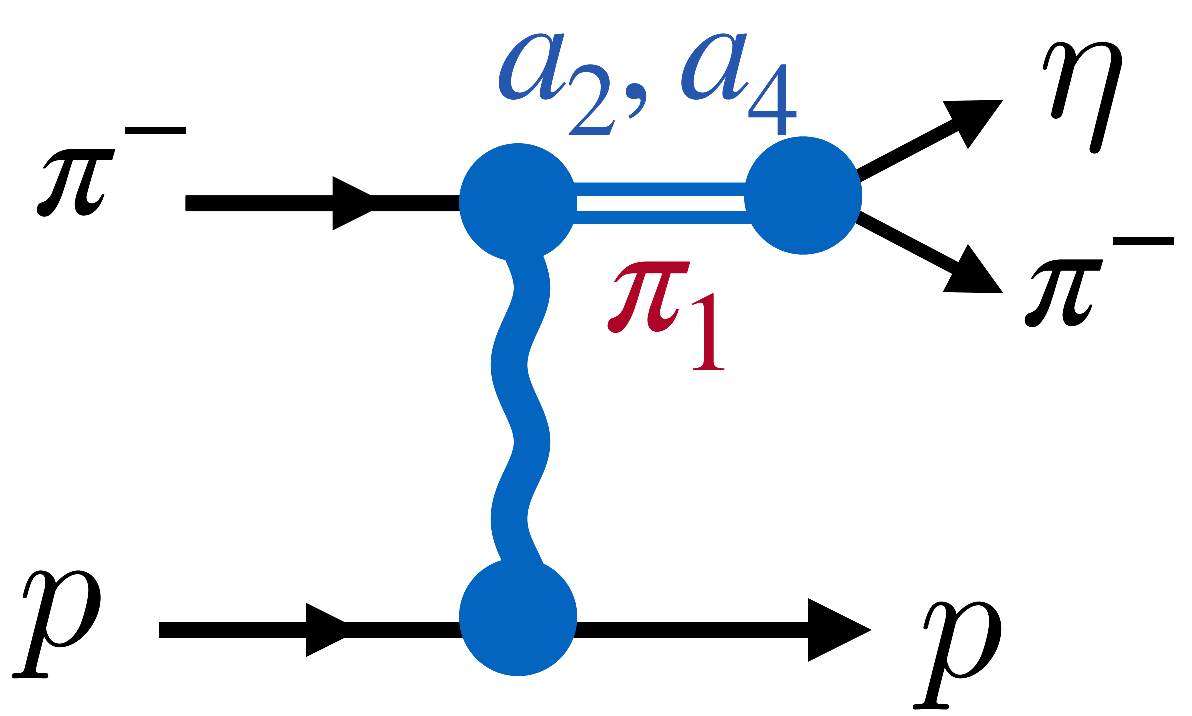


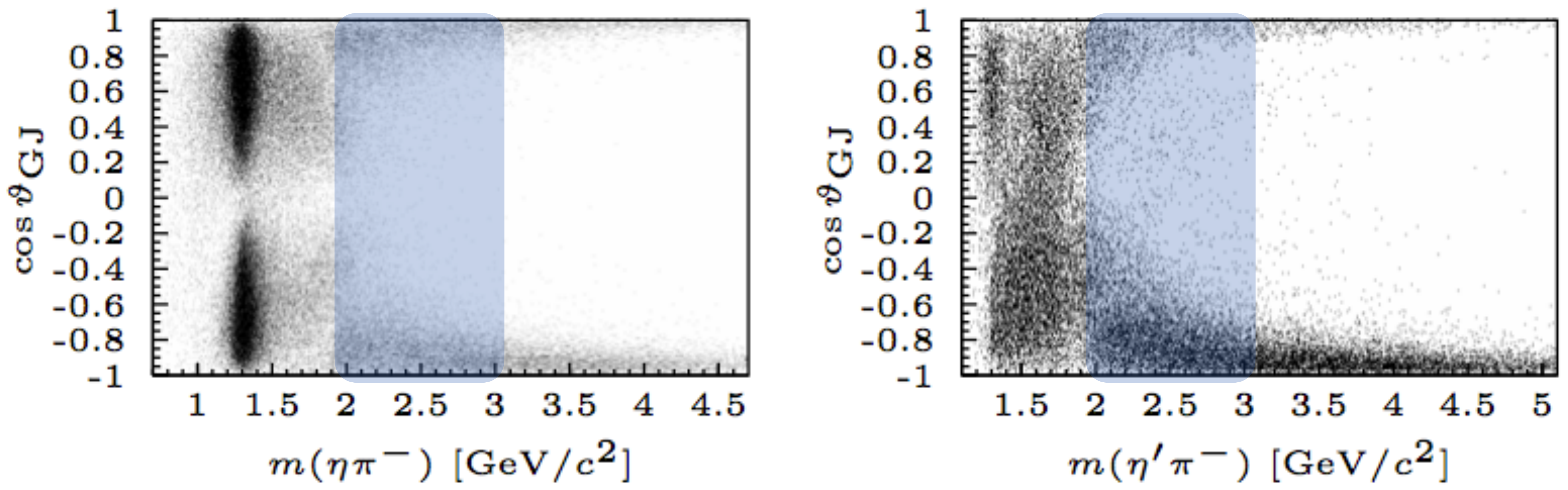


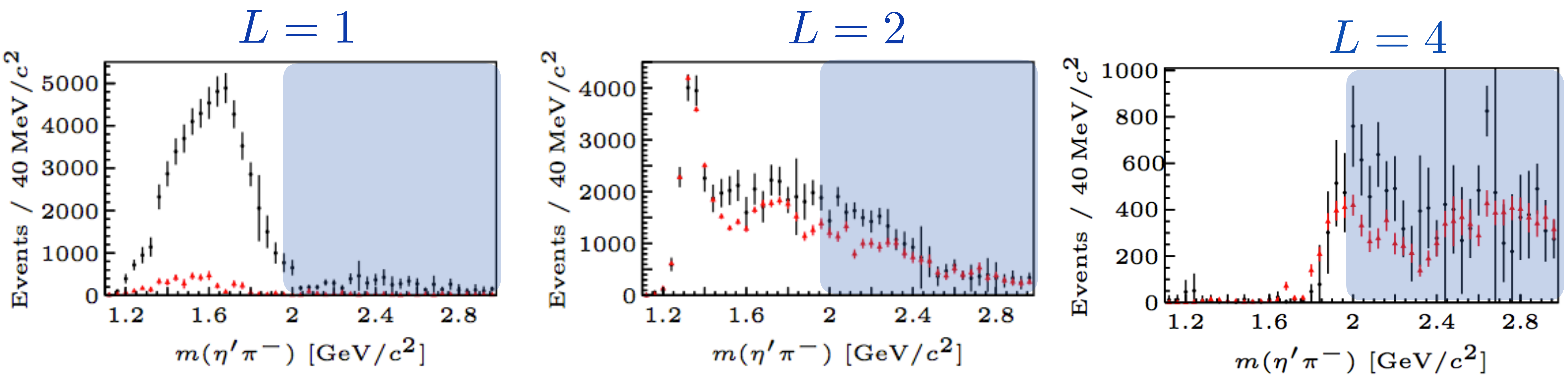
Responsible for asymmetry In eta-Prime - pion?

Partial Waves Expansion

COMPASS PLB740 (2015)







Resonance in angular mom. L = 1 ?

black: $\pi\eta'$ red: $\pi\eta$ (scaled)

Partial Waves of Double Regge Diagrams

Bibrzycki et al (JPAC) EPJC81 (2021) 915

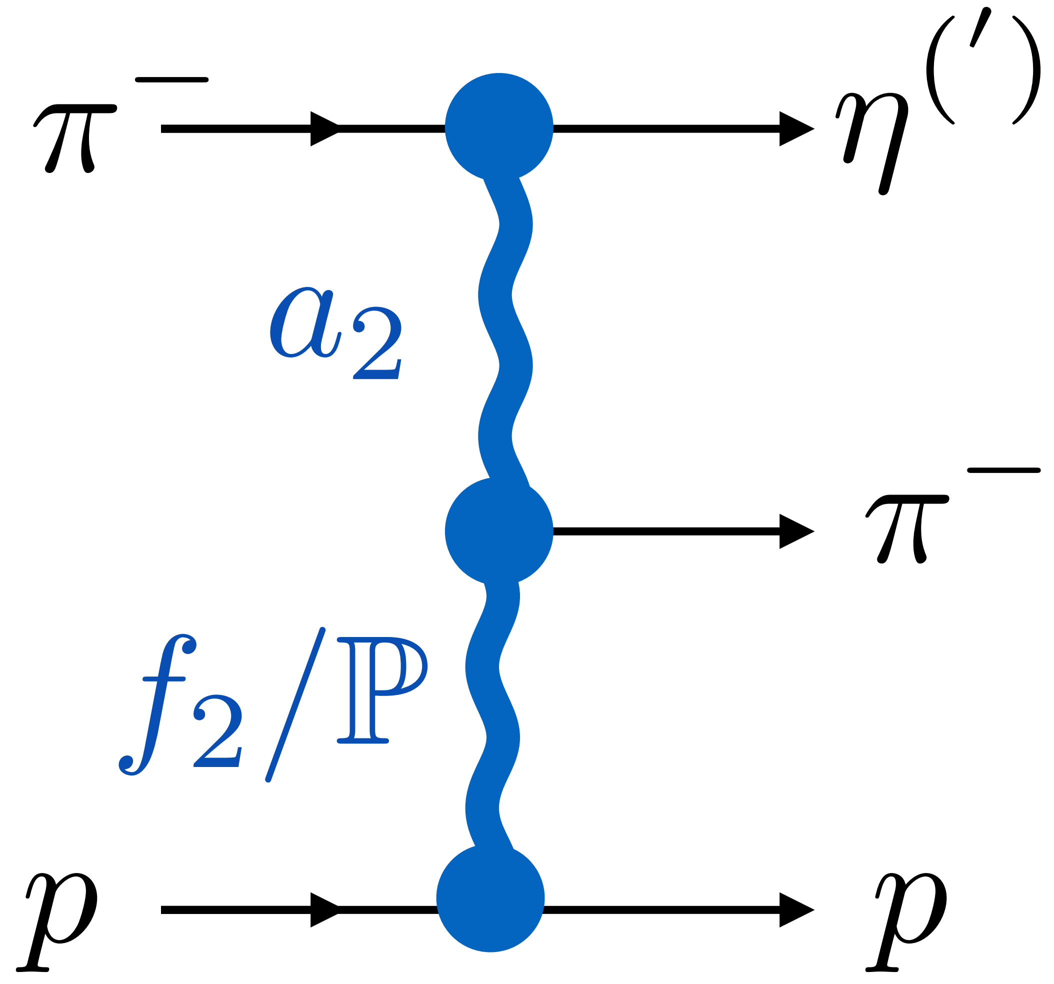
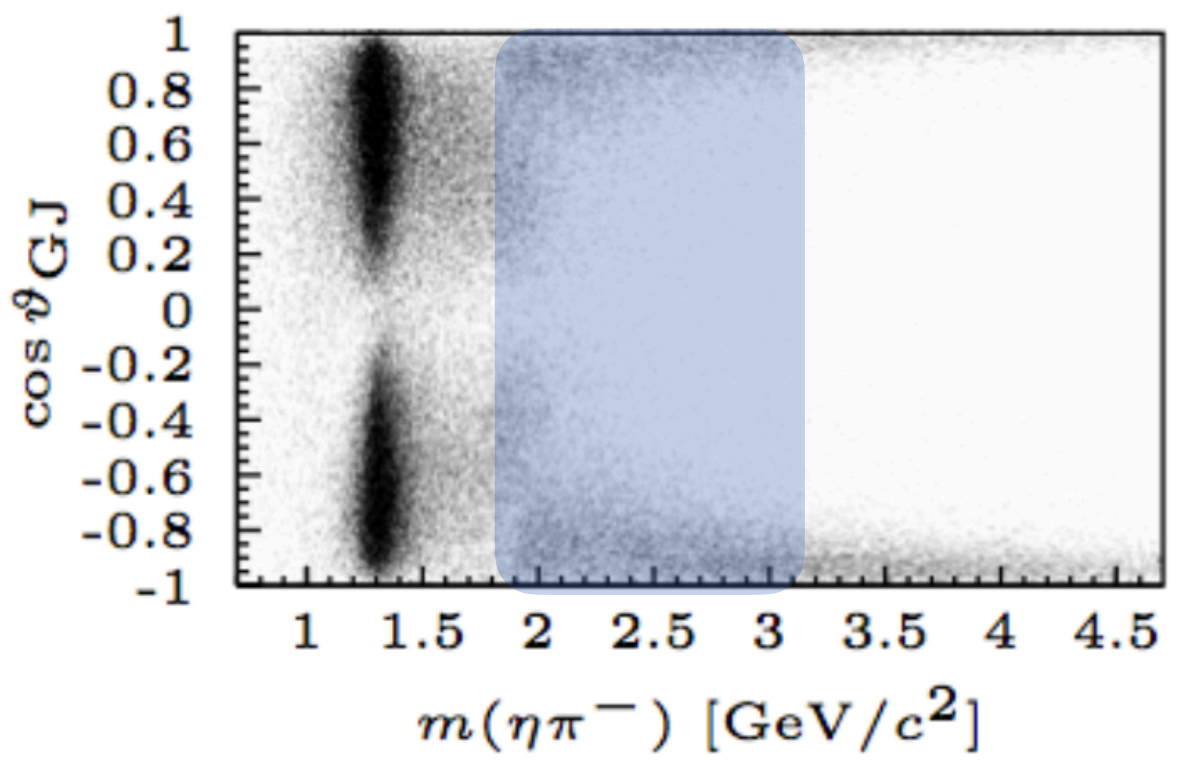
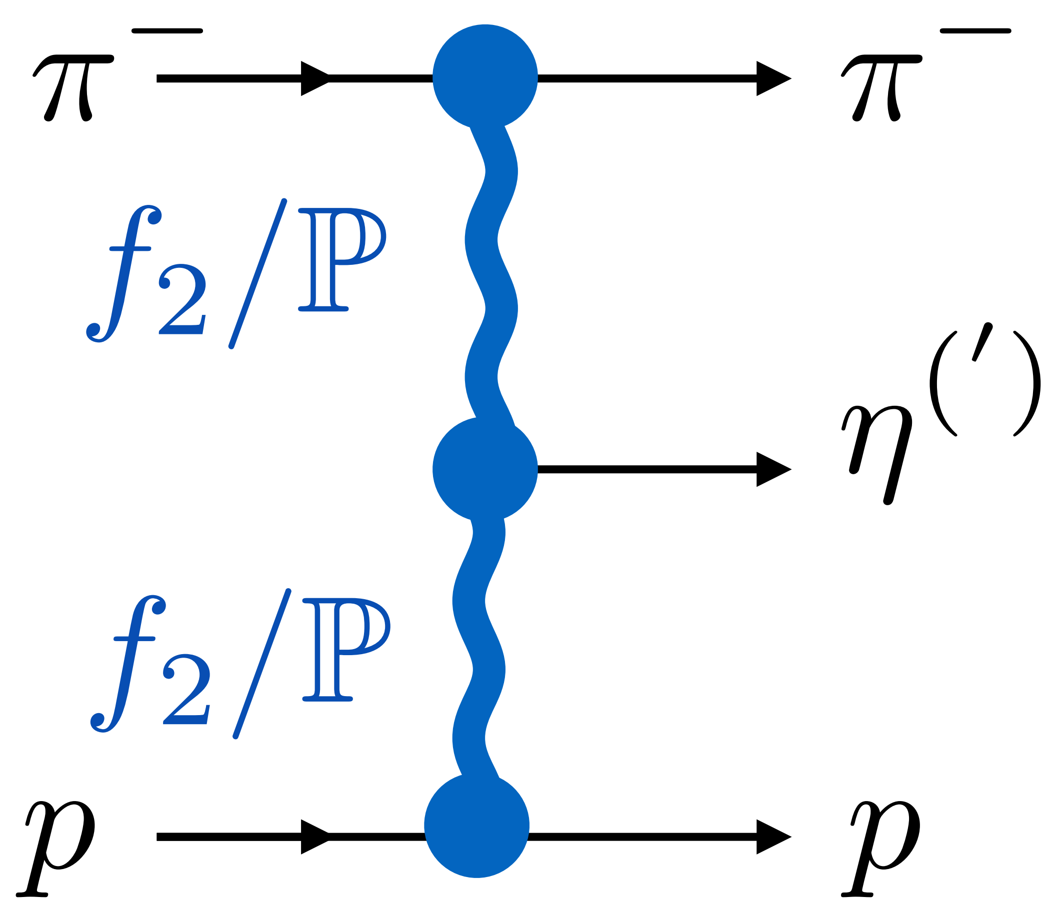


Diagram includes all partial waves

COMPASS describes intensity with $L \leqslant 6$ waves





Partial Waves of Double Regge Diagrams

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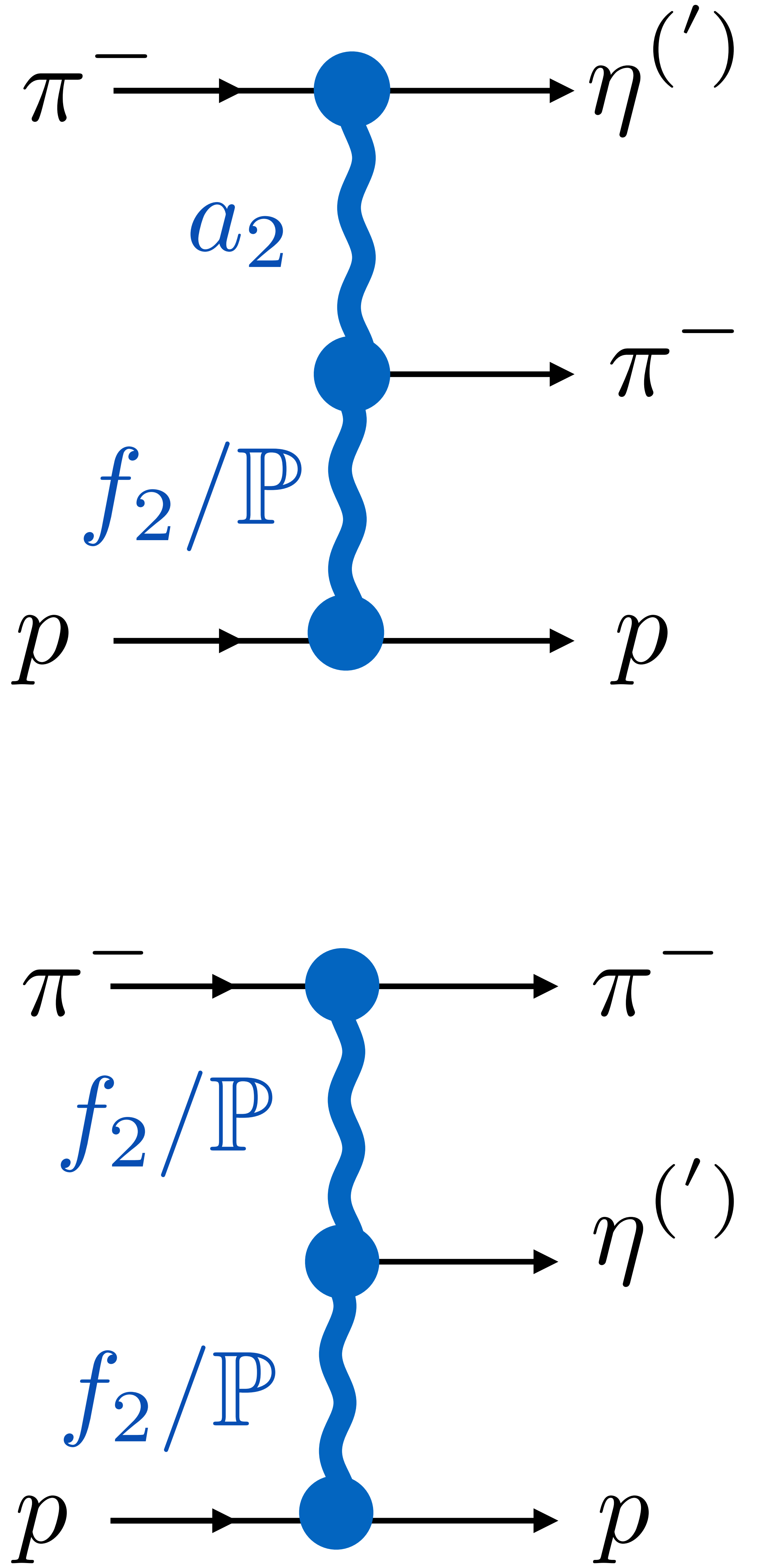
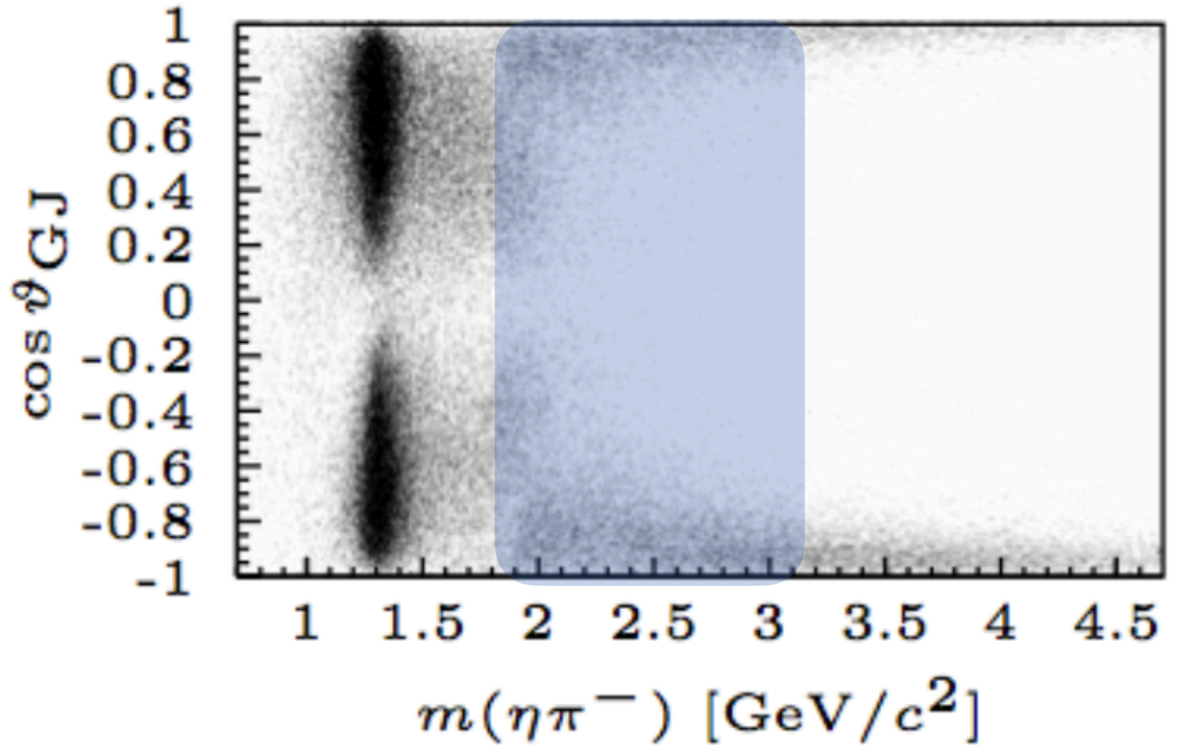


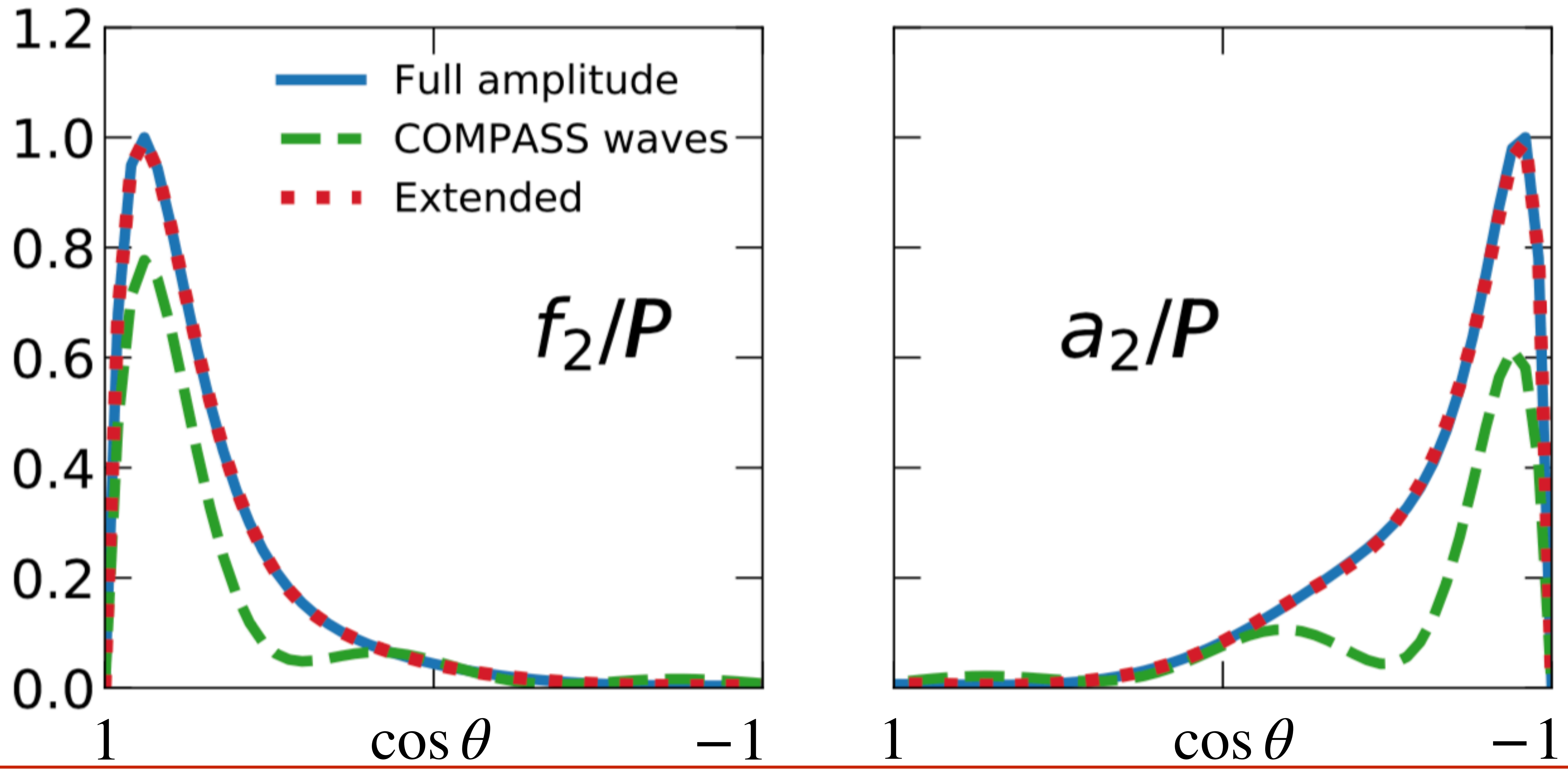
Diagram includes all partial waves

COMPASS describes intensity with $L \leqslant 6$ waves



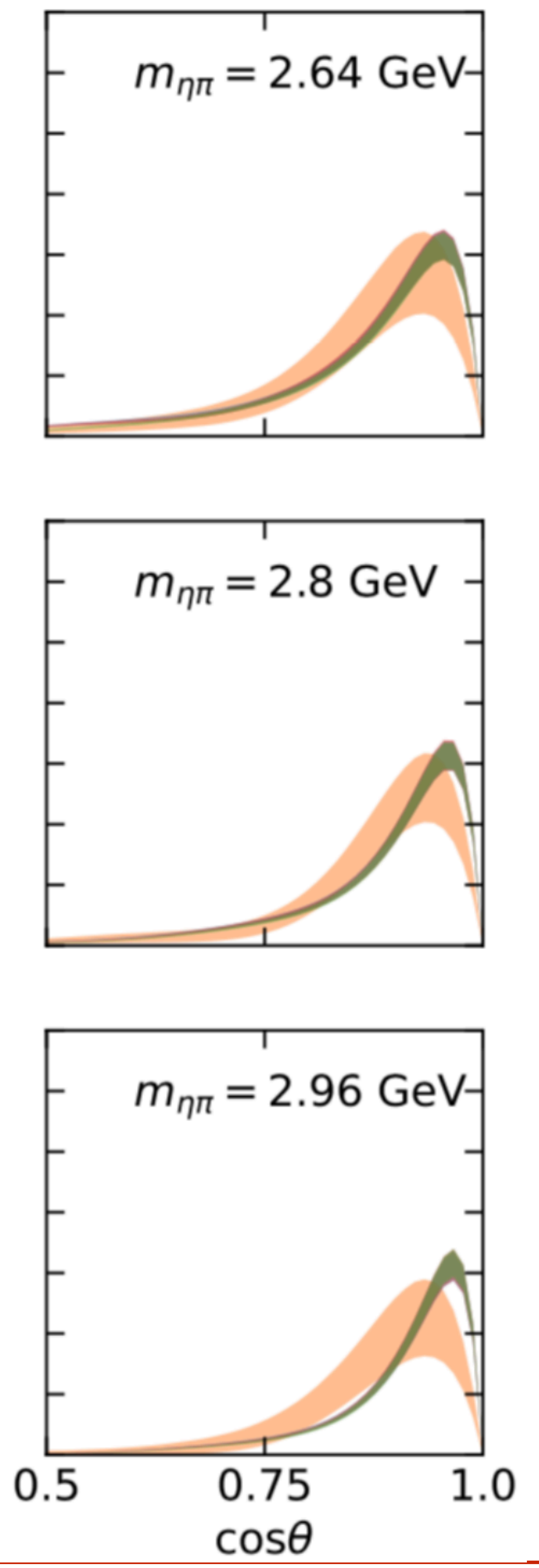
$L \leqslant 6$ waves only accounts for 60-80% of the diagram

→ Reconstruct intensity from partial waves



Eta-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



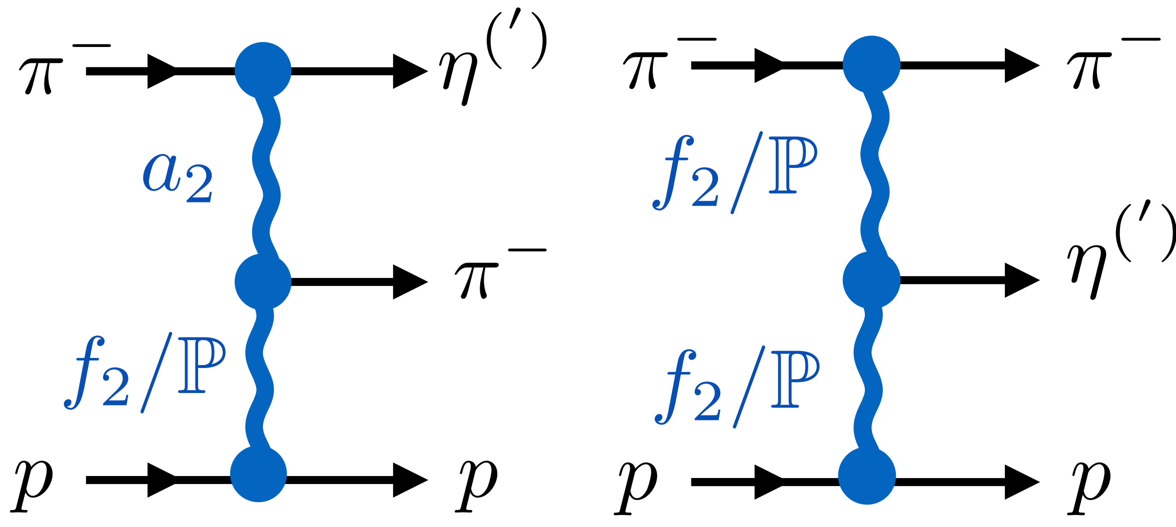
$(a_2, f_2 + \mathbb{P}) + (f_2, f_2)$

$(a_2, f_2 + \mathbb{P}) + (f_2, f_2 + \mathbb{P})$

$(a_2, f_2 + \mathbb{P}) + (f_2, f_2) + (\mathbb{P}, \mathbb{P})$

Data (reconstructed)

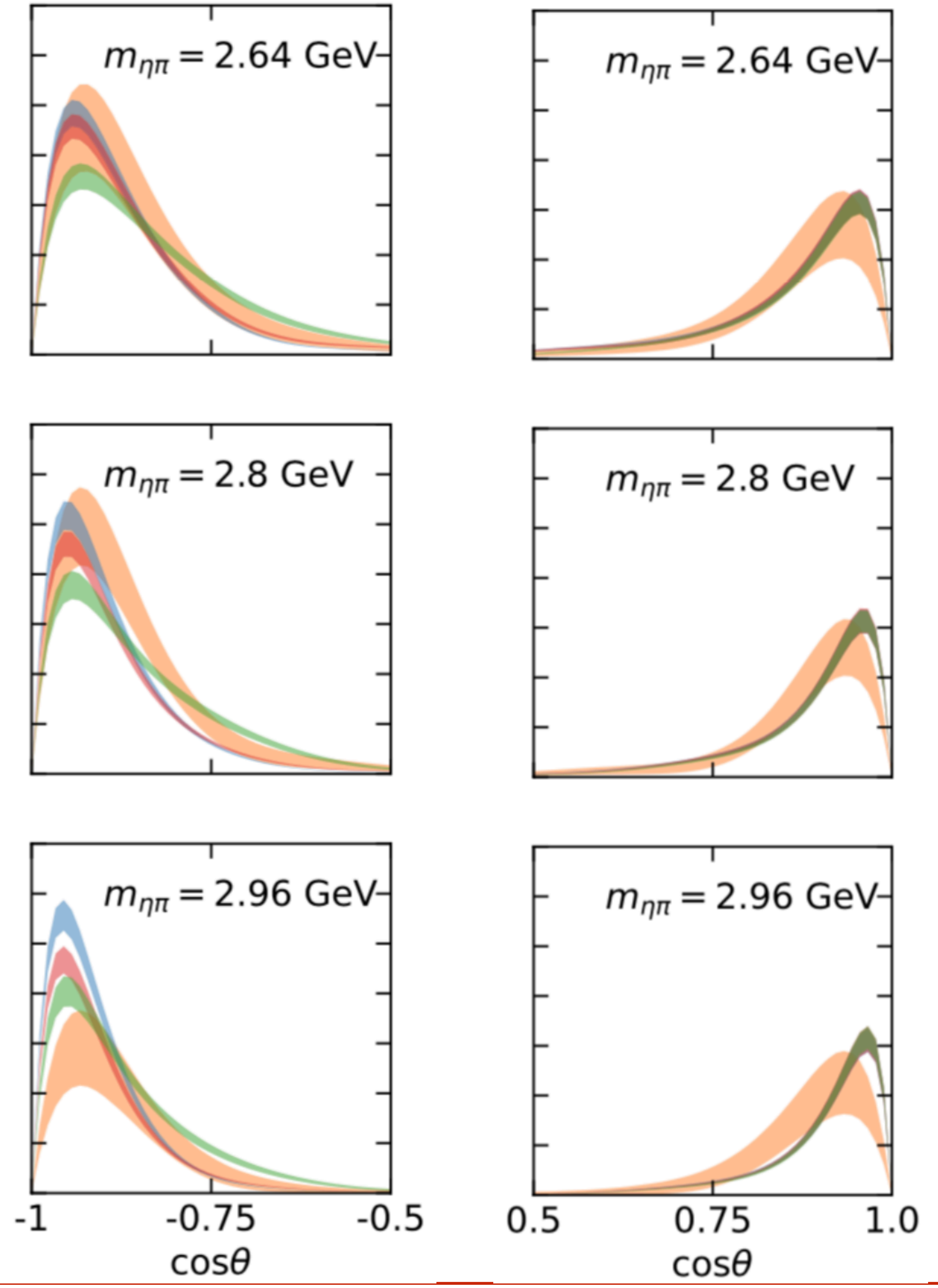
6 diagrams total



Forward intensity requires both (a_2, f_2) and $(a_2, \mathbb{P})$

Eta-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



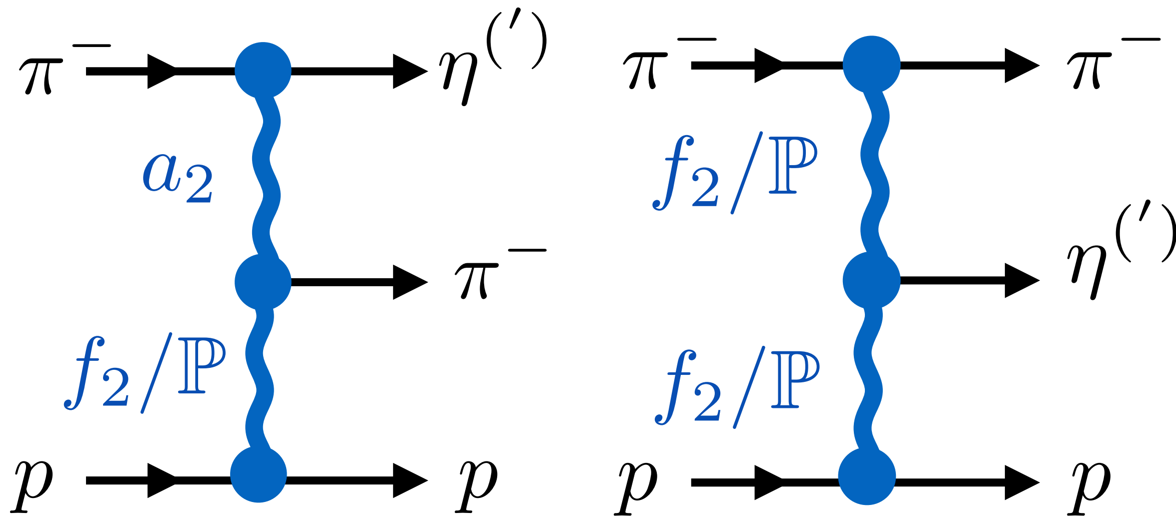
$(a_2,f_2 + \mathbb{P}) + (f_2,f_2)$

$(a_2,f_2 + \mathbb{P}) + (f_2,f_2 + \mathbb{P})$

$(a_2,f_2 + \mathbb{P}) + (f_2,f_2) + (\mathbb{P},\mathbb{P})$

Data (reconstructed)

6 diagrams total

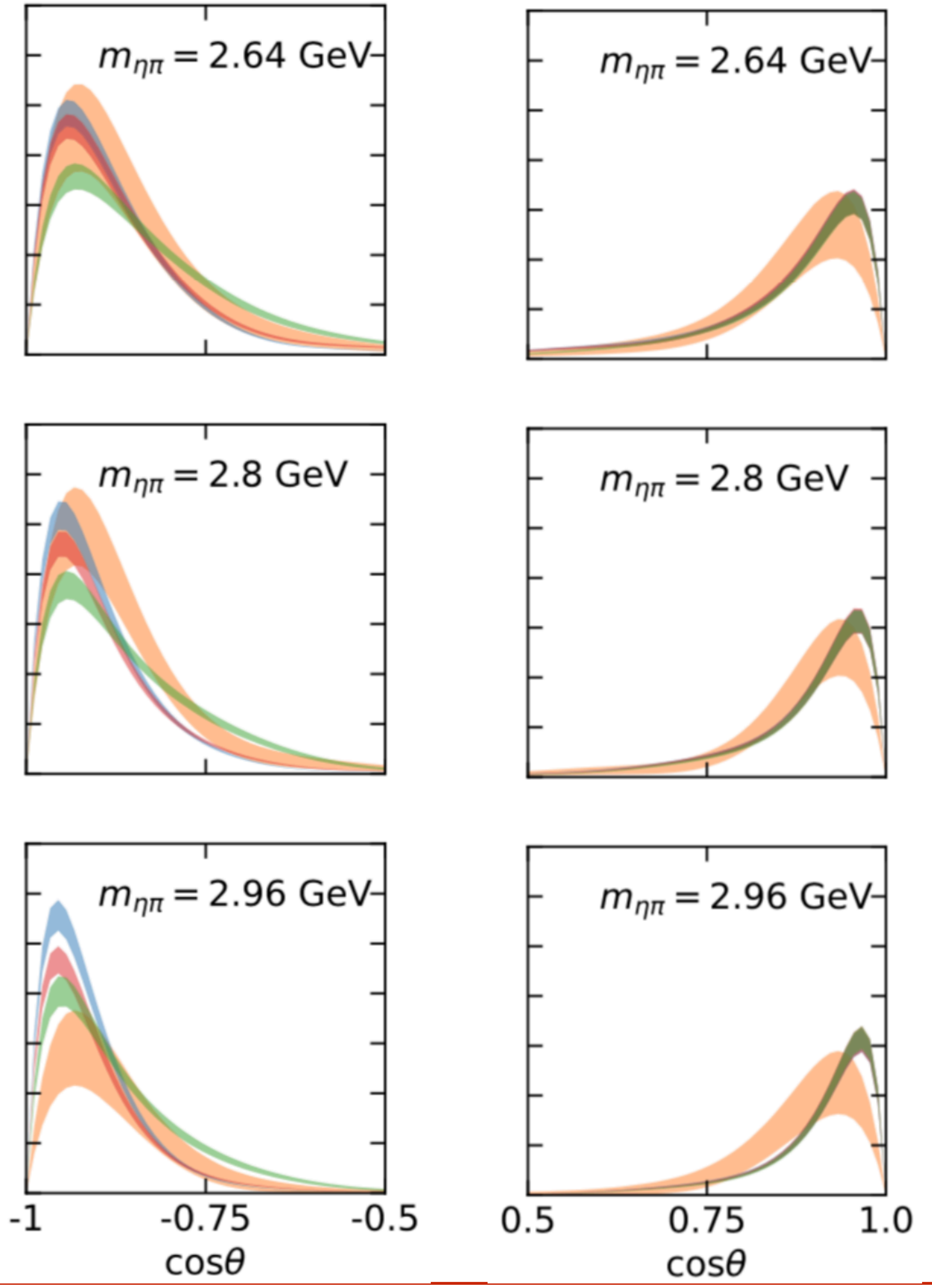


Forward intensity requires both (a_2,f_2) and $(a_2,\mathbb{P})$

Backward intensity requires (f_2,f_2) and either $(f_2,\mathbb{P})$ or $(\mathbb{P},\mathbb{P})$

Eta-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



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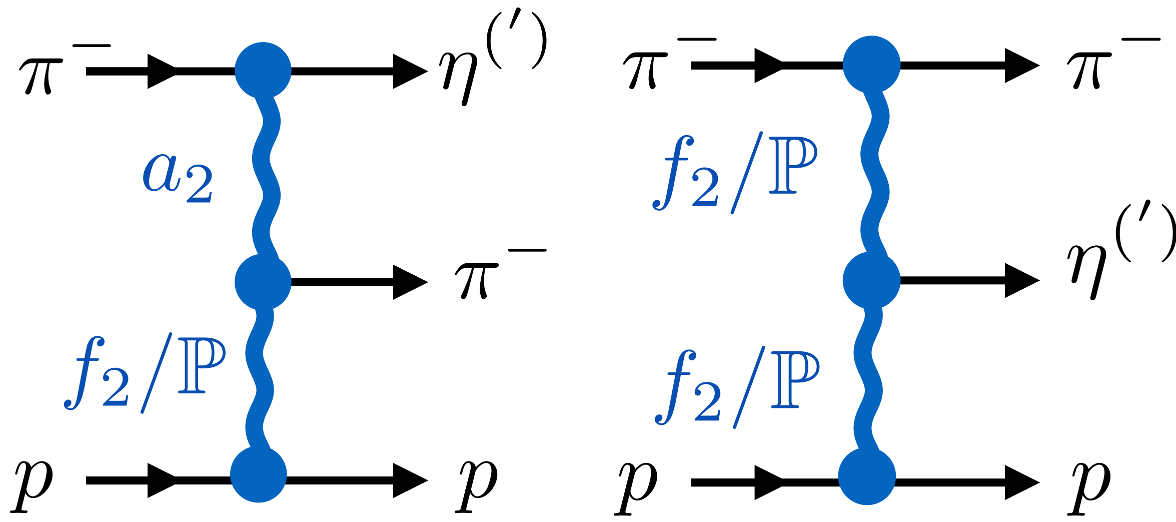
6 diagrams total



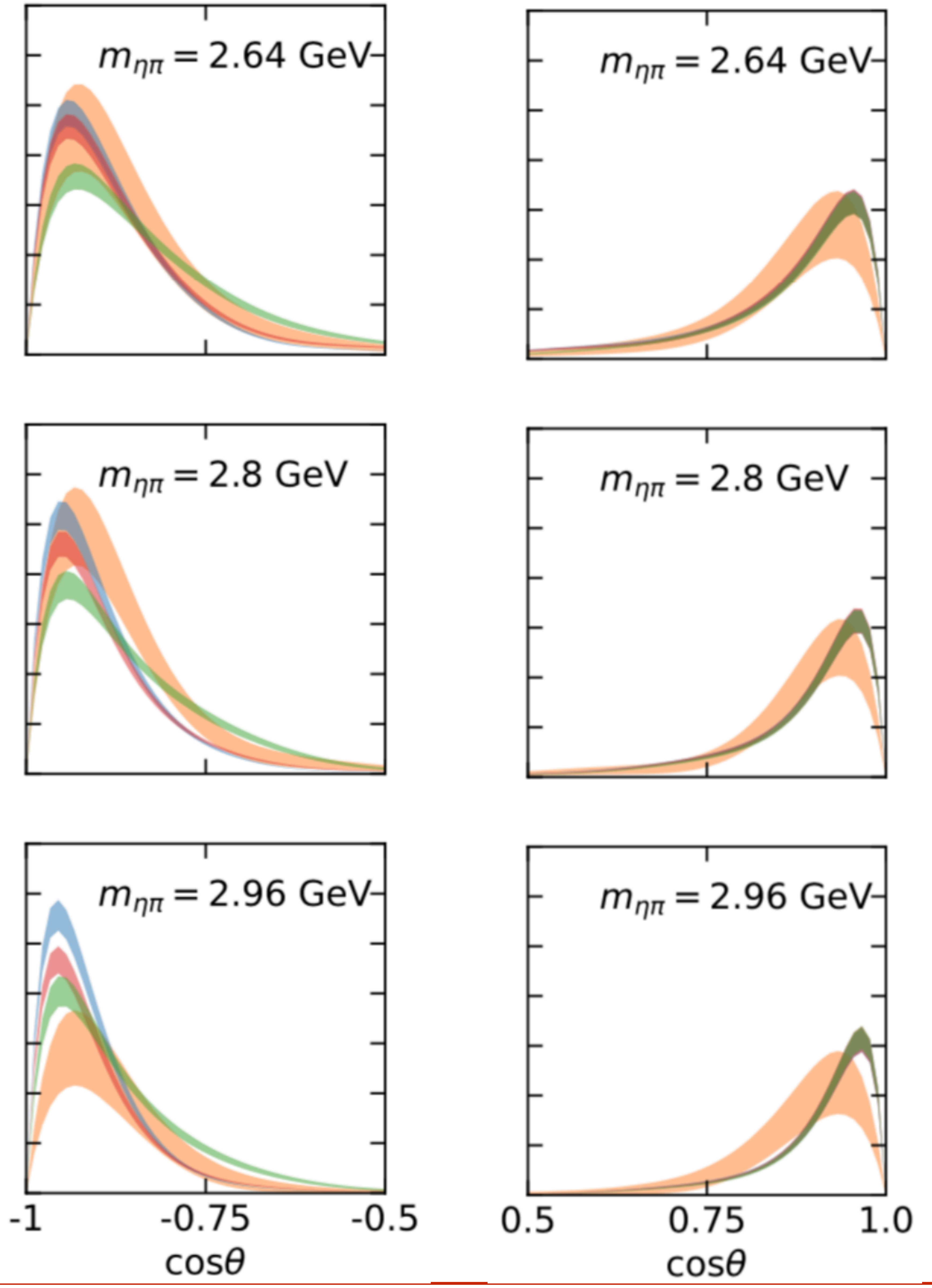
Forward intensity requires both (a_2,f_2) and $(a_2,\mathbb{P})$

Backward intensity requires (f_2,f_2) and either $(f_2,\mathbb{P})$ or $(\mathbb{P},\mathbb{P})$

$(a_2,\mathbb{P})$	0.40 ± 0.04
(a_2,f_2)	3.4 ± 0.4
$(f_2,\mathbb{P})$	-0.30 ± 0.05
(f_2,f_2)	-6.6 ± 0.7

Eta-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



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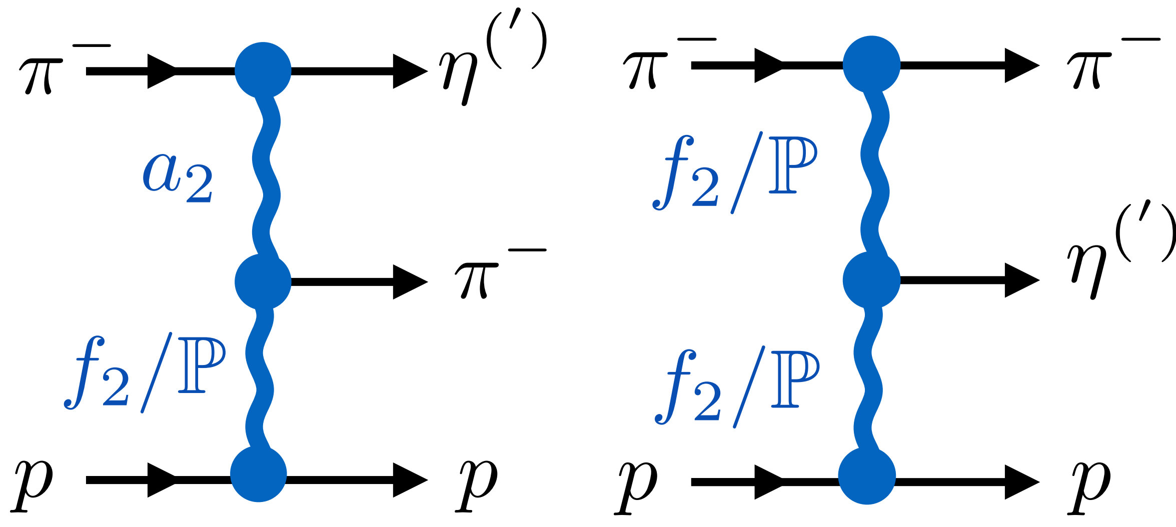

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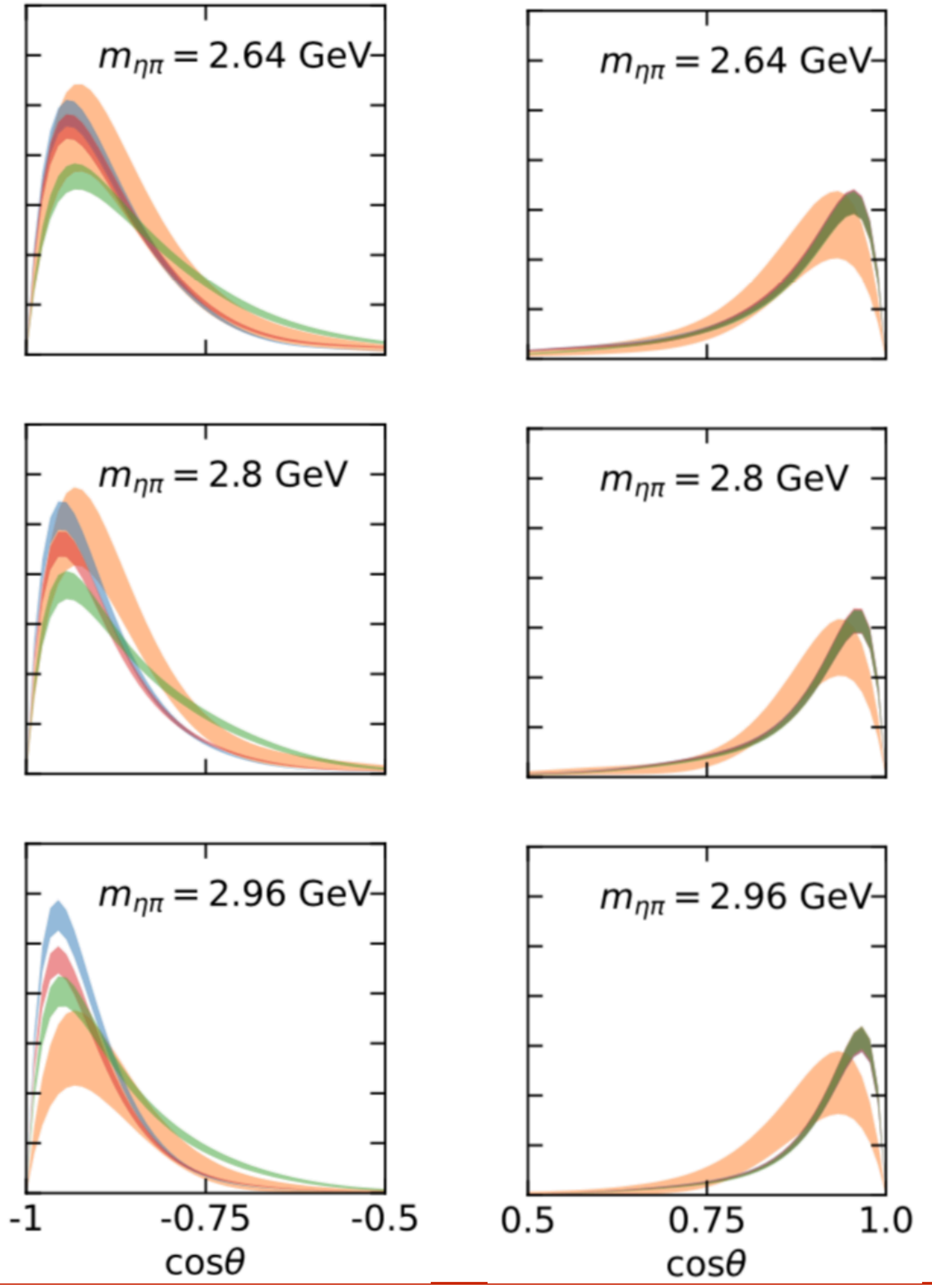
Eta-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



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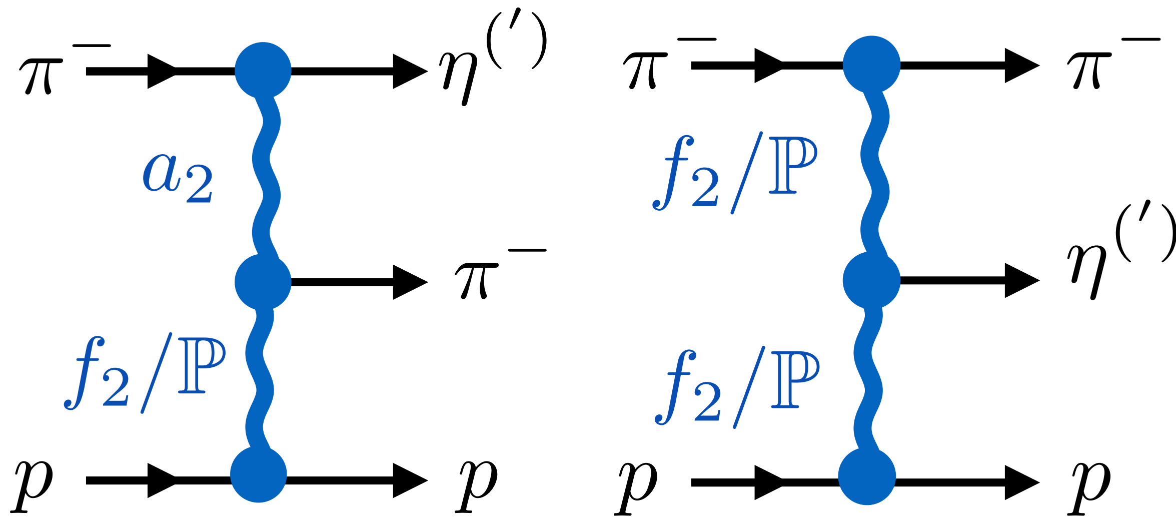
$(a_2, f_2 + \mathbb{P}) + (f_2, f_2) + (\mathbb{P}, \mathbb{P})$

Data (reconstructed)

6 diagrams total



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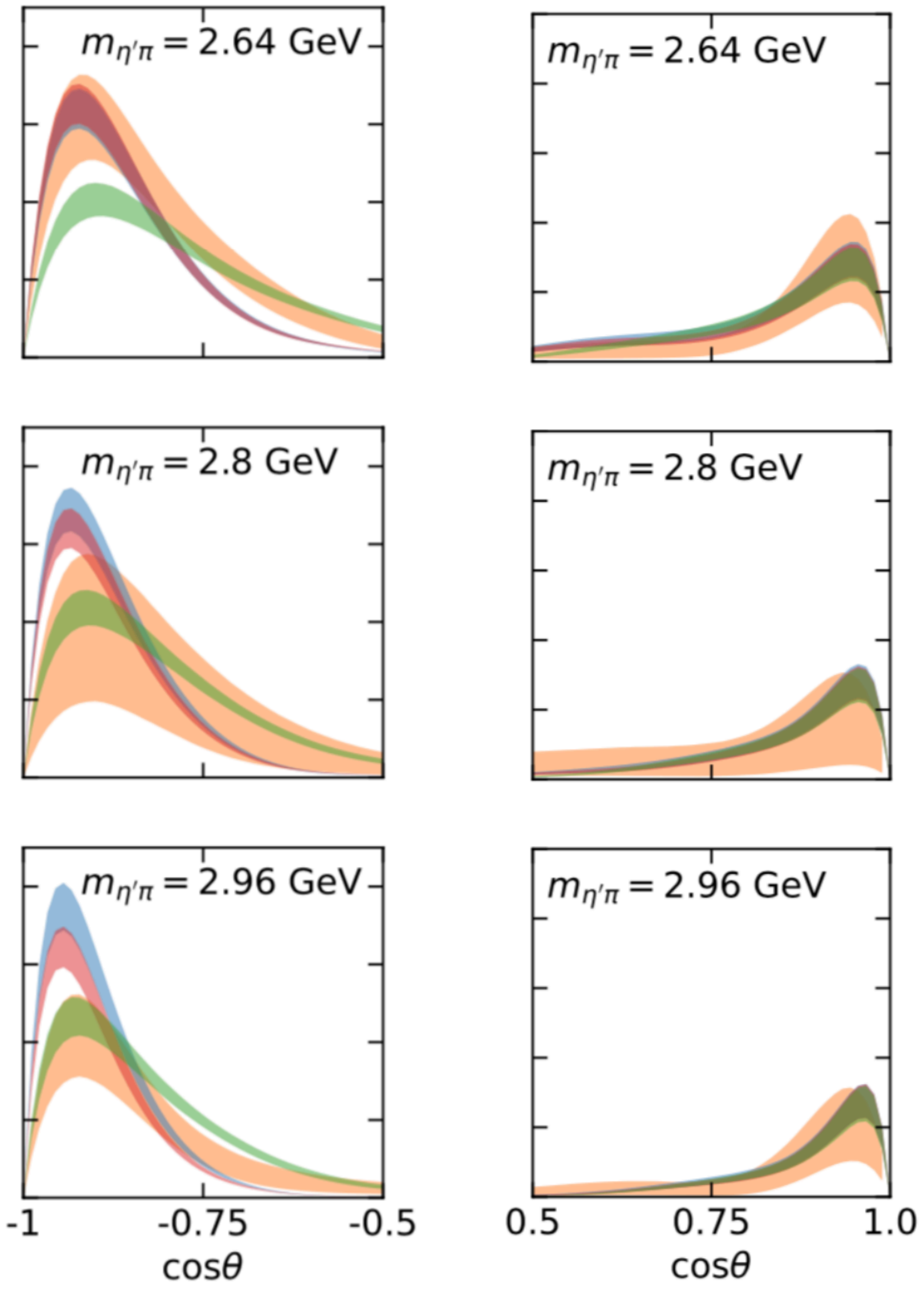
Asymmetry coming from difference between

(a_2, f_2) and (f_2, f_2)

Eta'-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



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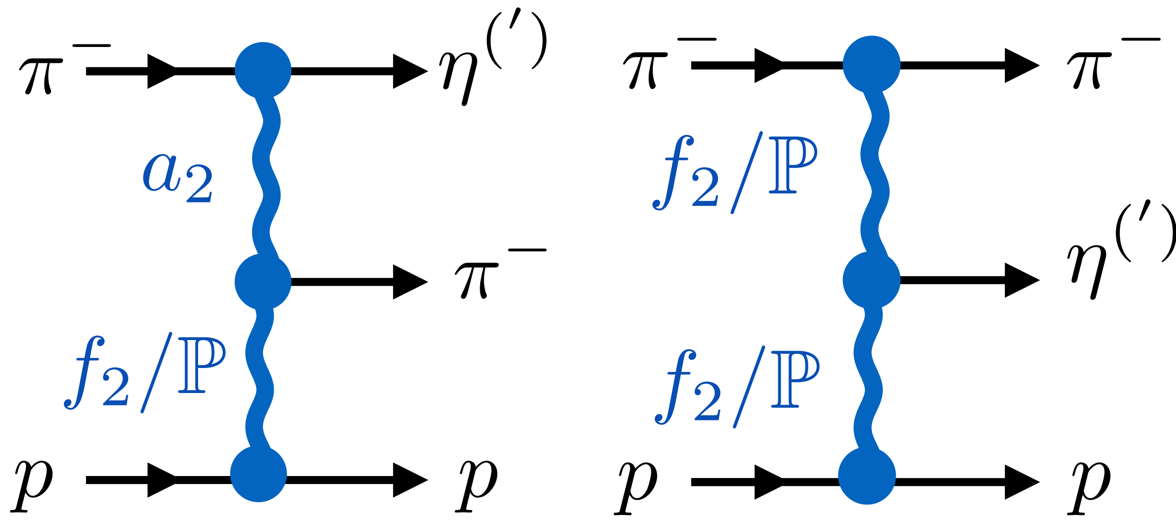
$(a_2, f_2 + \mathbb{P}) + (f_2, f_2 + \mathbb{P})$

$(a_2, f_2 + \mathbb{P}) + (f_2, f_2) + (\mathbb{P}, \mathbb{P})$

Data (reconstructed)

6 diagrams total



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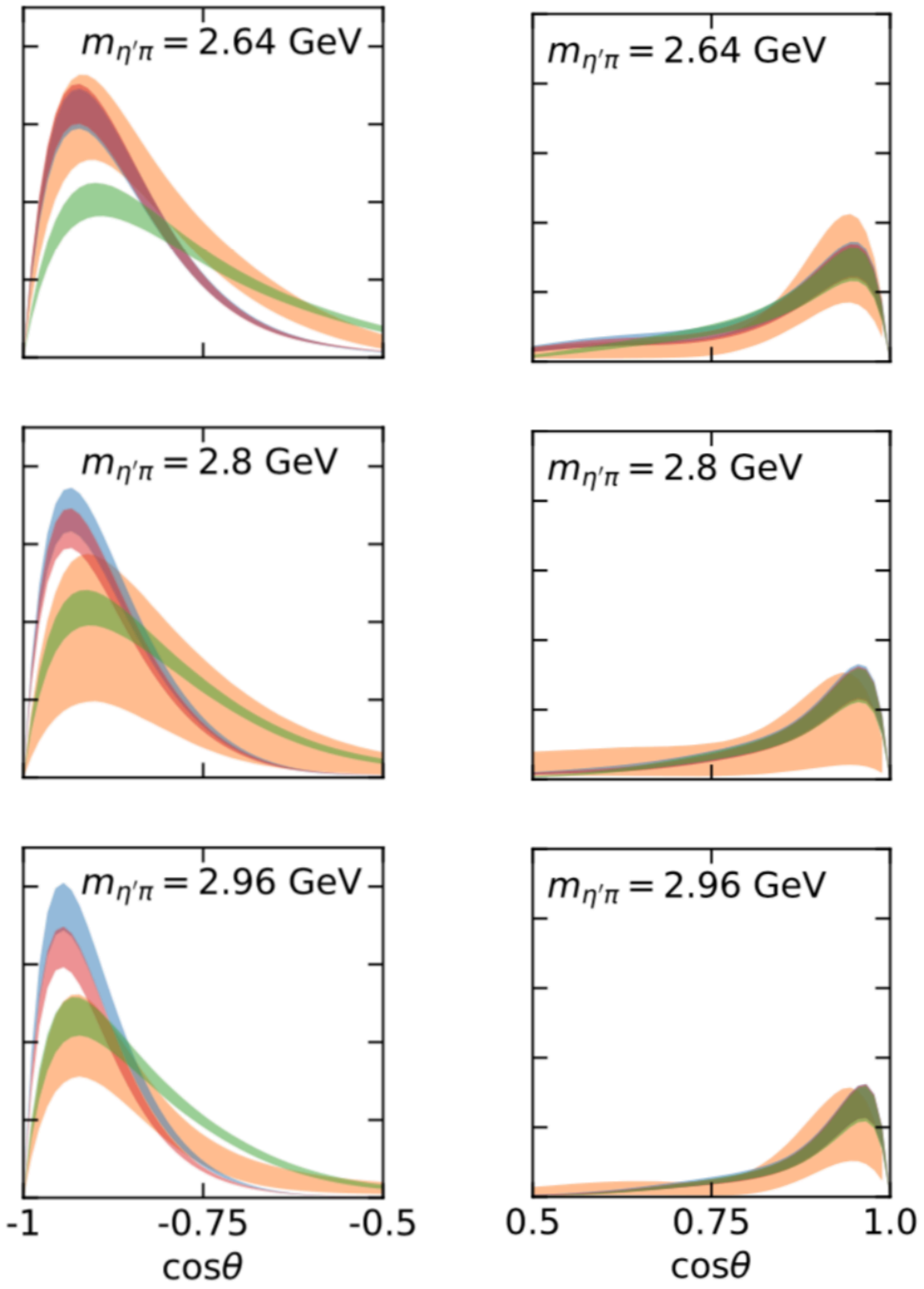
Backward intensity requires (f_2, f_2) and $(\mathbb{P}, \mathbb{P})$

Eta'-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



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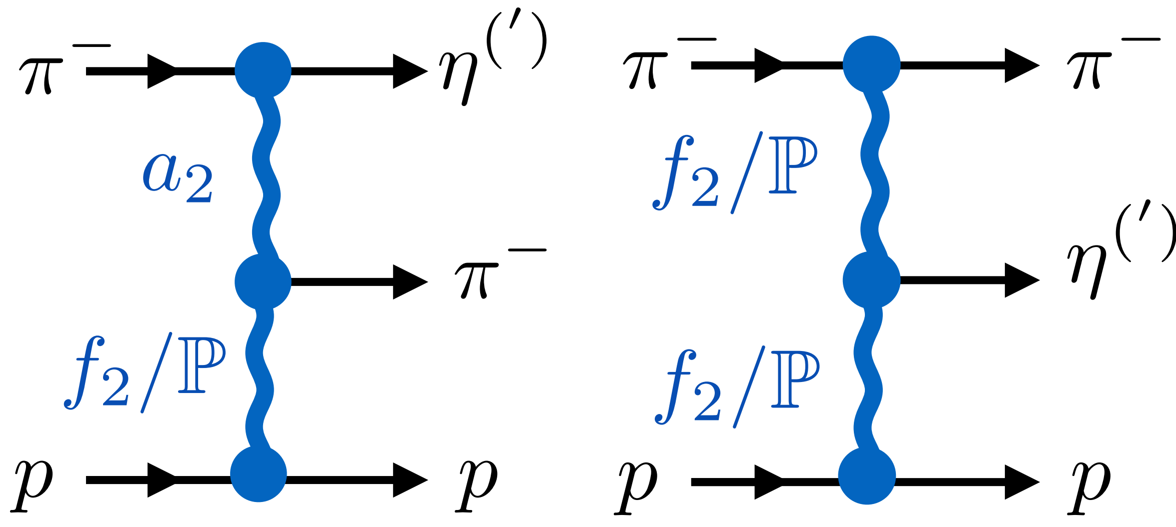
$(a_2,f_2 + \mathbb{P}) + (f_2,f_2) + (\mathbb{P},\mathbb{P})$

Data (reconstructed)

6 diagrams total



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Backward intensity requires (f_2,f_2) and $(\mathbb{P},\mathbb{P})$

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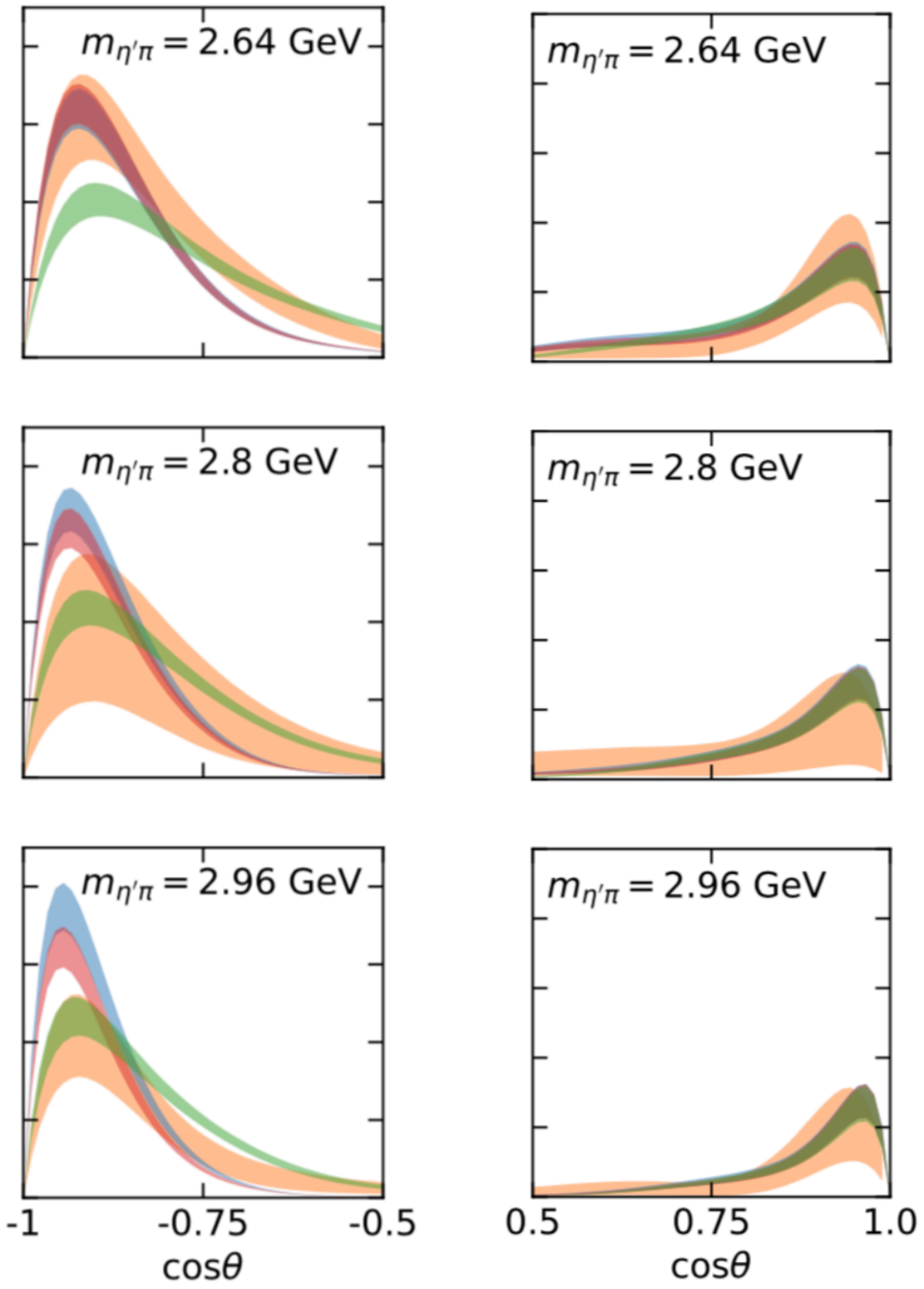
(a_2,f_2)	0.6 ± 0.5
(f_2,f_2)	-7.1 ± 0.6
$(\mathbb{P},\mathbb{P})$	0.018 ± 0.002

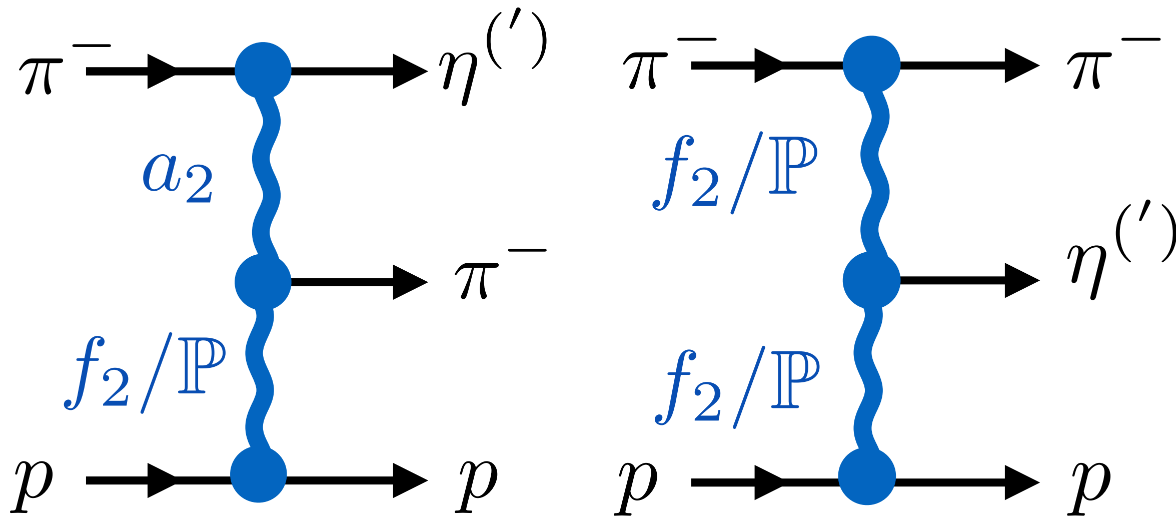
Eta'-Pi results

Bibrzycki et al (JPAC) EPJC81 (2021) 915



- (blue) $(a_2,f_2+\mathbb{P})+(f_2,f_2)$
- (red) $(a_2,f_2+\mathbb{P})+(f_2,f_2+\mathbb{P})$
- (green) $(a_2,f_2+\mathbb{P})+(f_2,f_2)+(\mathbb{P},\mathbb{P})$
- (orange band) Data (reconstructed)

6 diagrams total



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Asymmetry coming from difference between

(a_2,f_2) and (f_2,f_2)

+ contribution from $(\mathbb{P},\mathbb{P})$

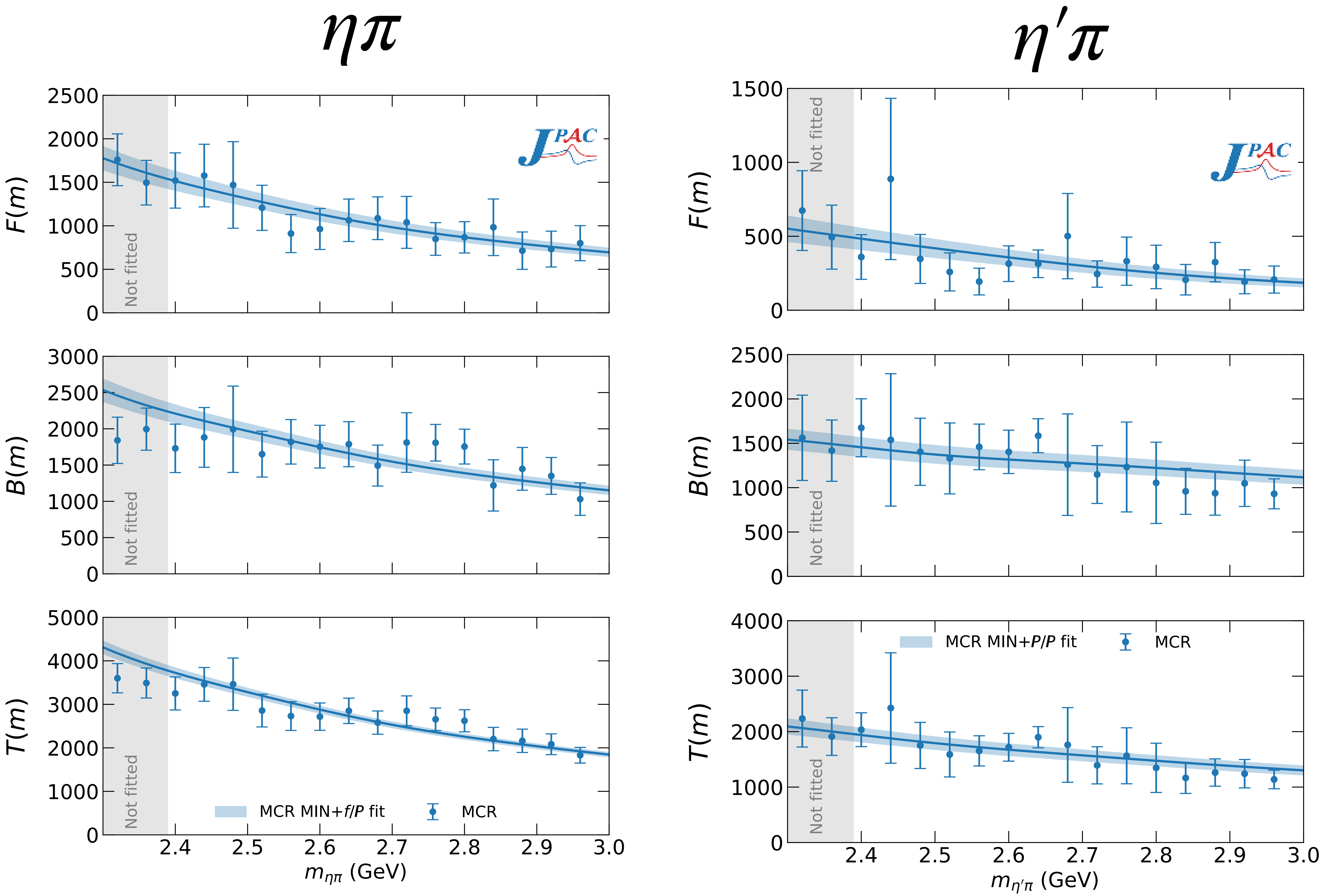
Forward-Backward Asymmetry

Bibrzycki et al (JPAC) EPJC81 (2021) 915

Forward intensity

Backward intensity

Total intensity

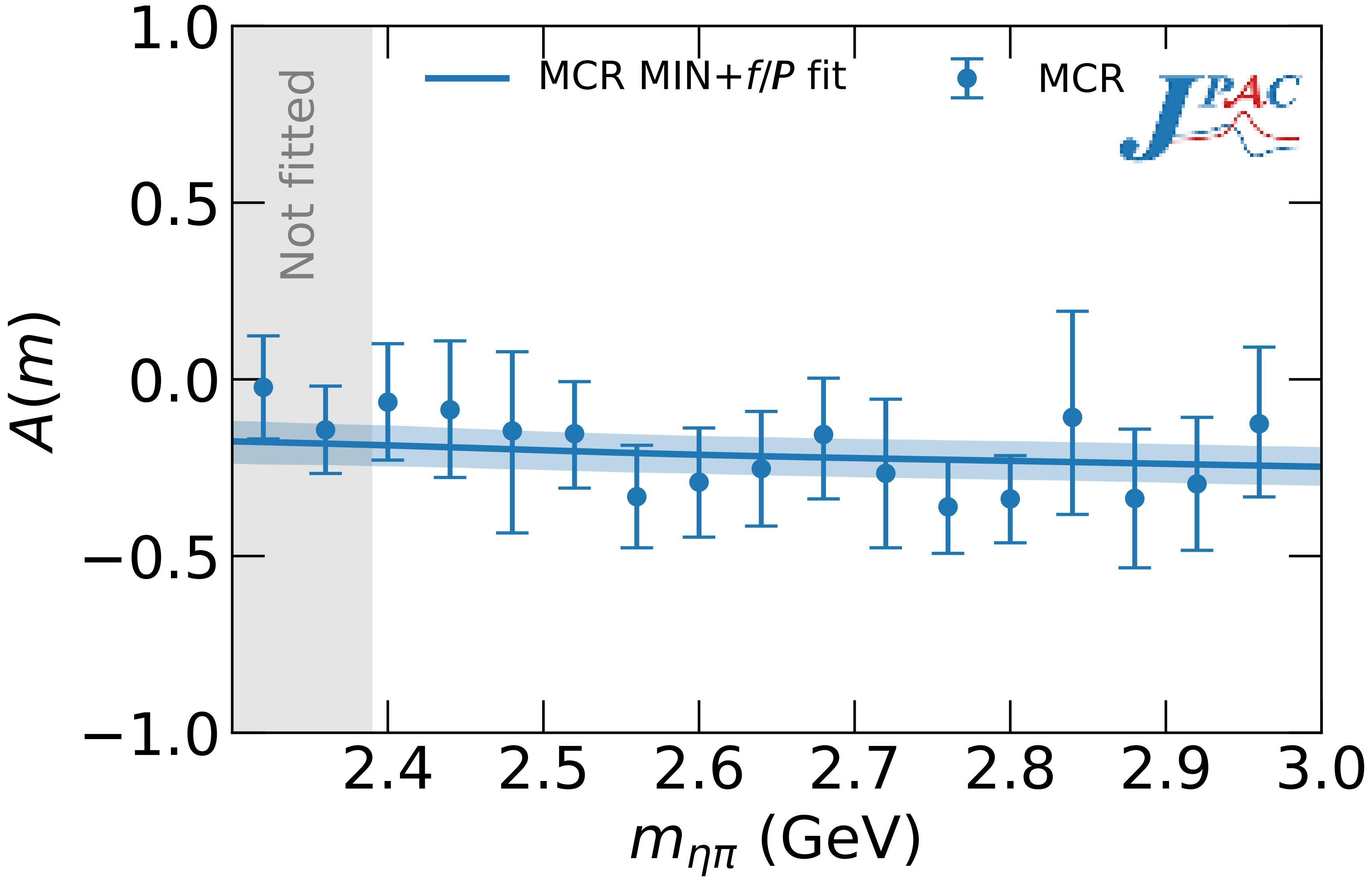


Forward-Backward Asymmetry

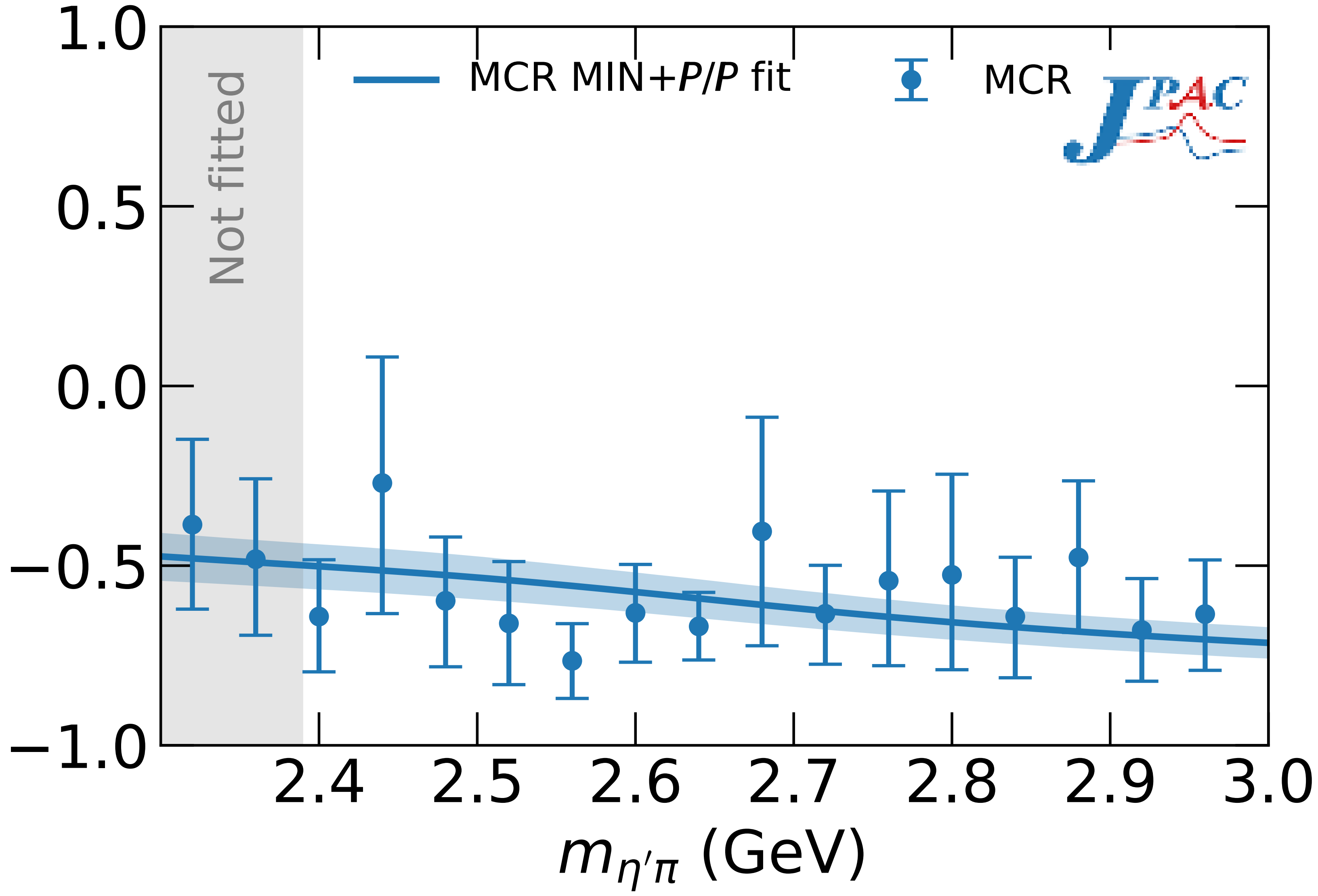
Fernández-Ramírez et al arXiv:2104.10646

$$A = \frac{\text{Forward} - \text{Backward}}{\text{Forward} + \text{Backward}}$$

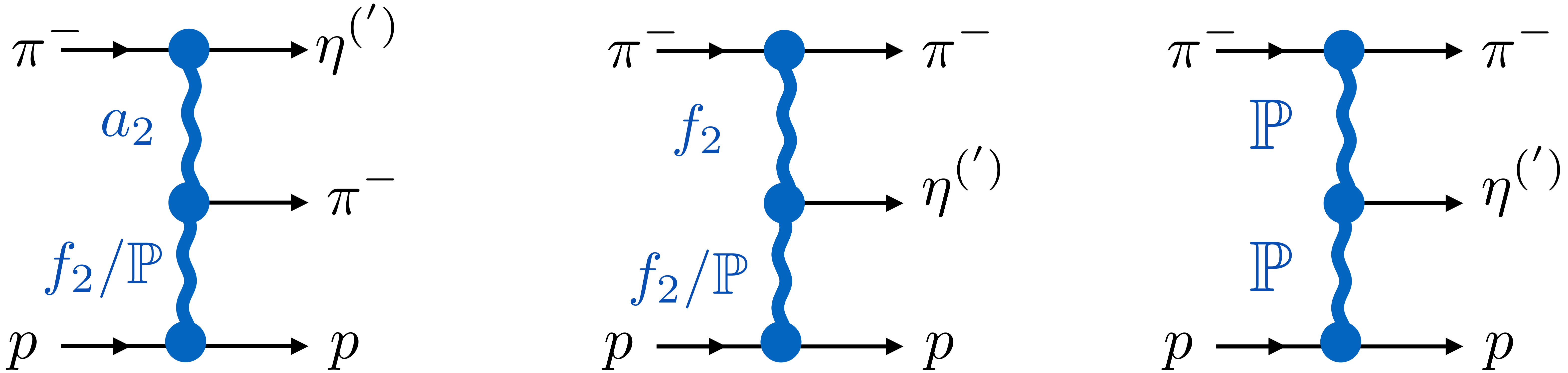
$\eta\pi$



$\eta'\pi$



Asymmetry originating mainly from $(a_2, f_2/\mathbb{P}) \neq (f_2, f_2/\mathbb{P})$ and from $(\mathbb{P}, \mathbb{P})$ in $\eta'\pi$

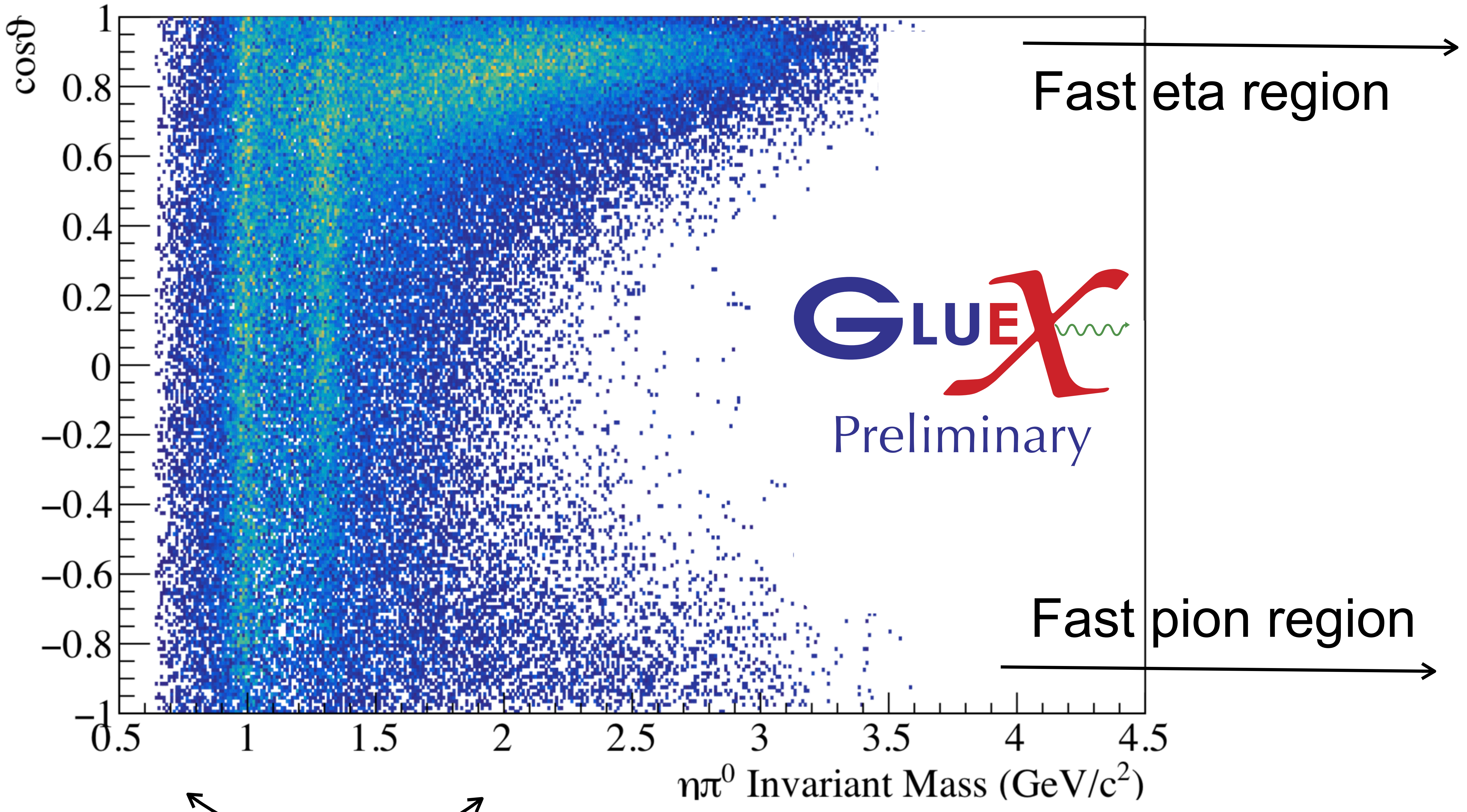


Eta-Pi @GlueX

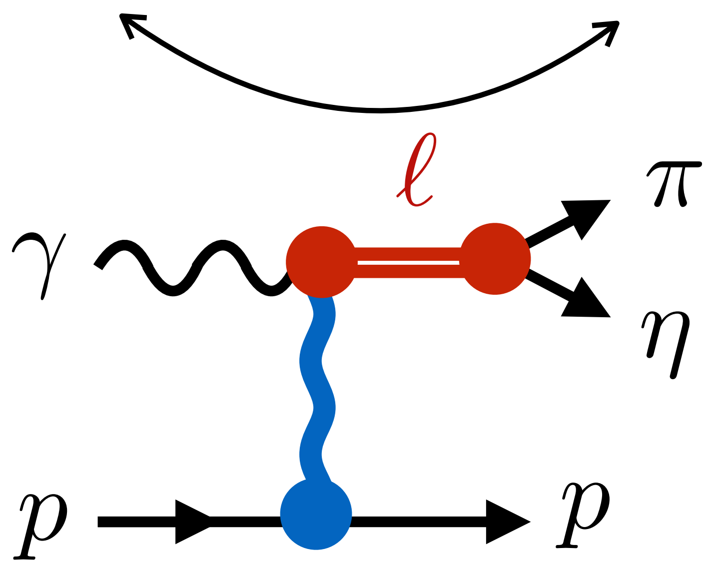
Ongoing study with GlueX

Fit data directly with double Regge amplitudes (with Barsotti and Shepherd)

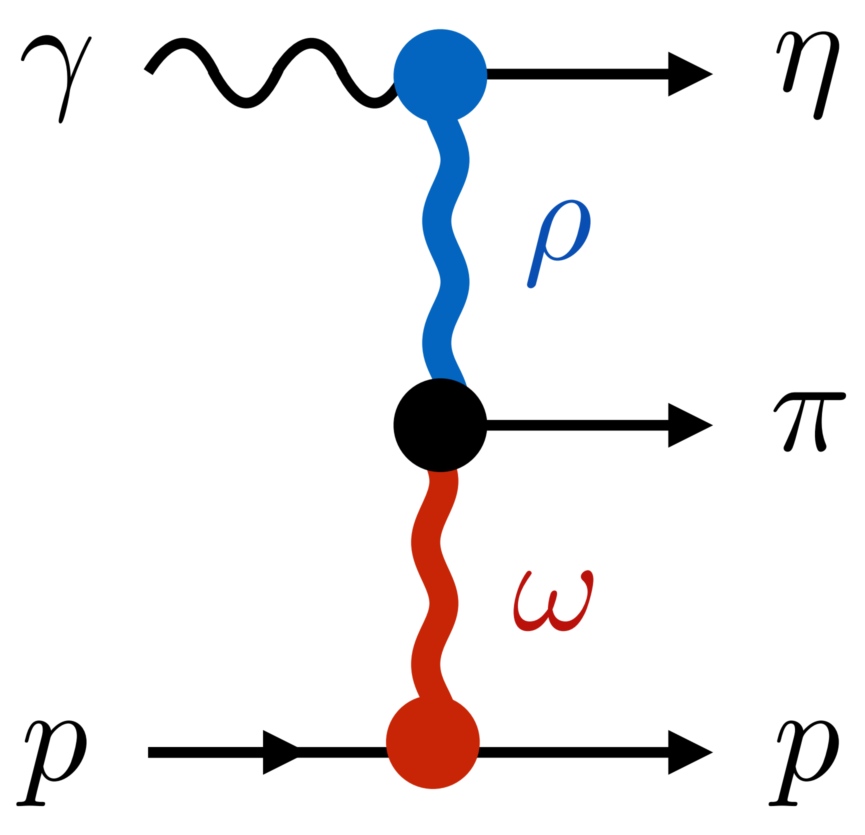
Exotic mesons originate from asymmetry in the double Regge region

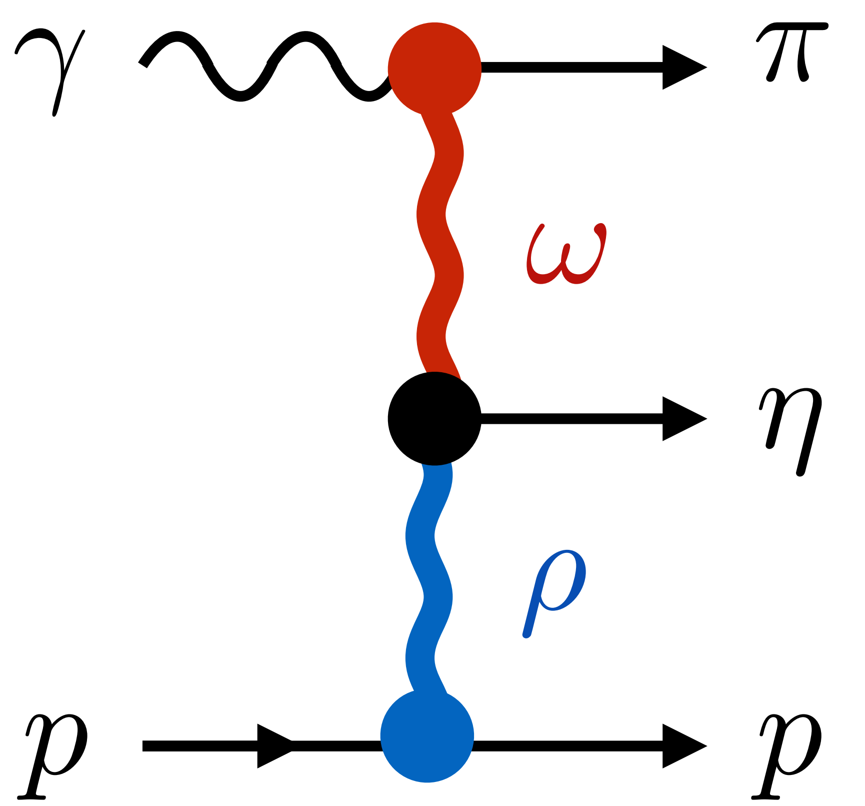


(not correct for detector acceptance)



dominant diagram





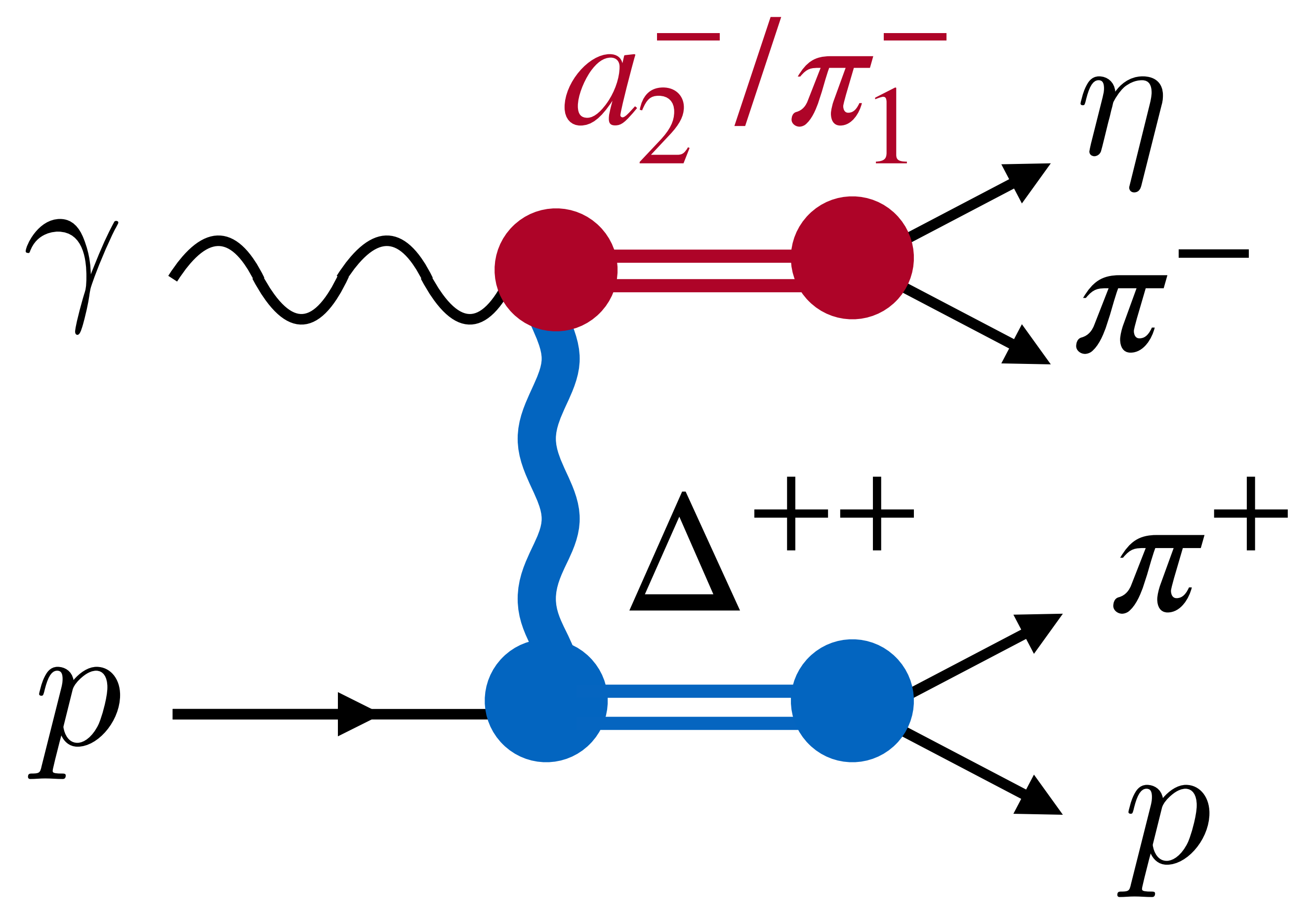
Conclusions & Future directions

Theoretical model to constraints extraction of D-waves

(Should compare with direct extraction of SDME)

Extraction of D-wave cross section

Ongoing: similar procedure for charged reaction:



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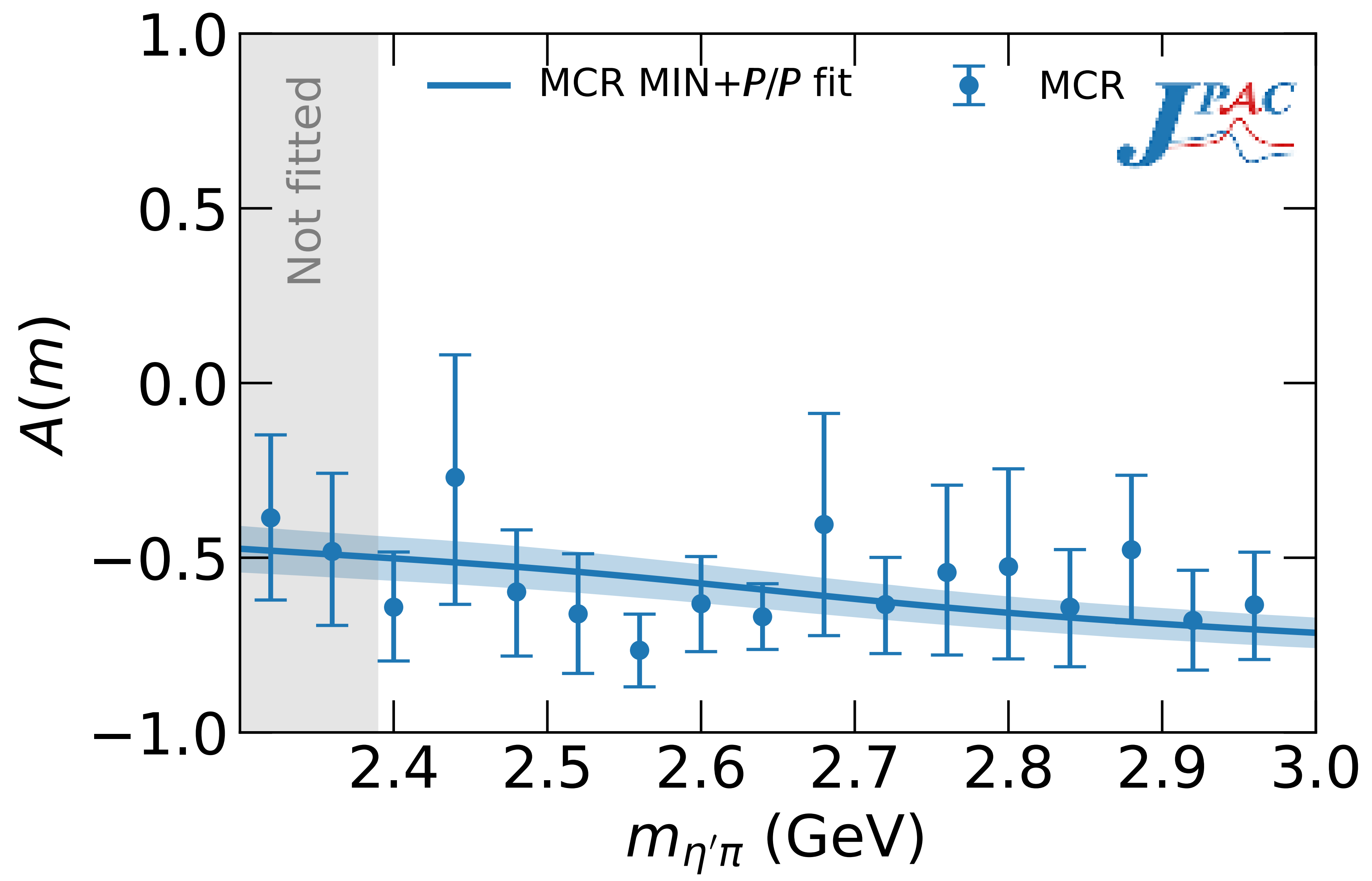


Ongoing: similar procedure for neutral reaction @GlueX

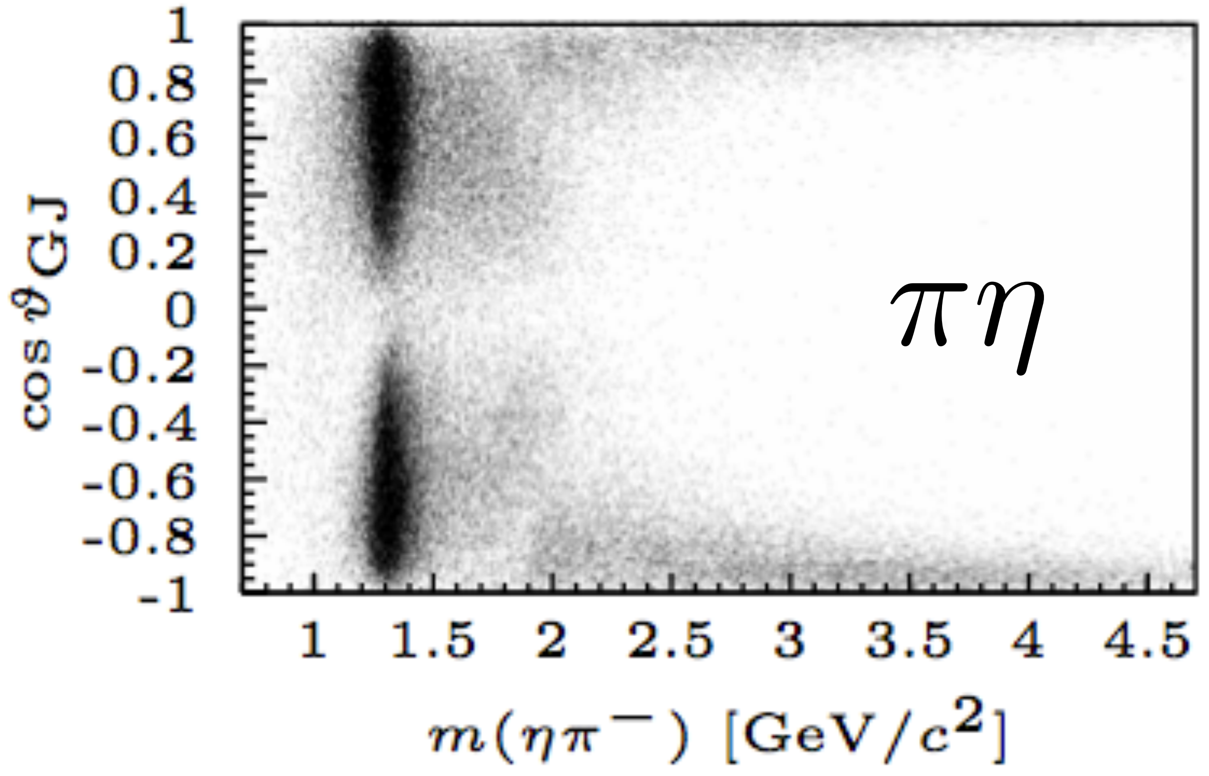
Backup Slides

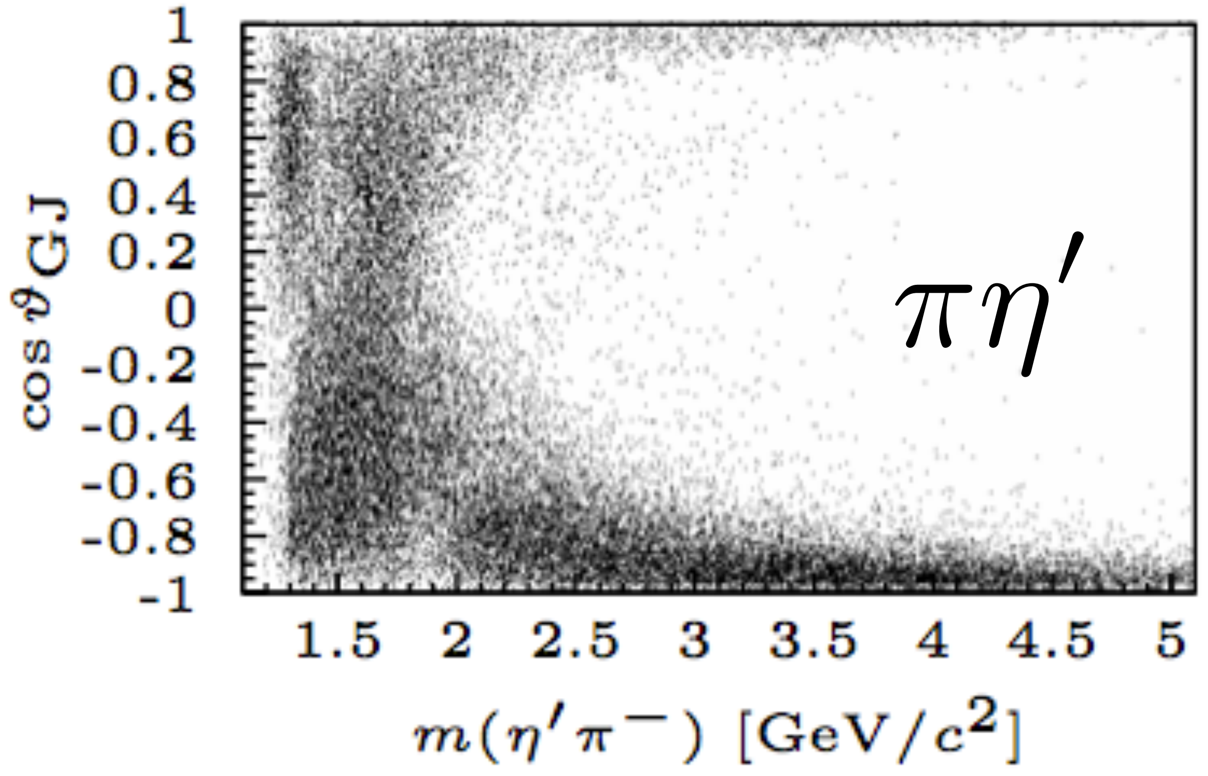
Summary

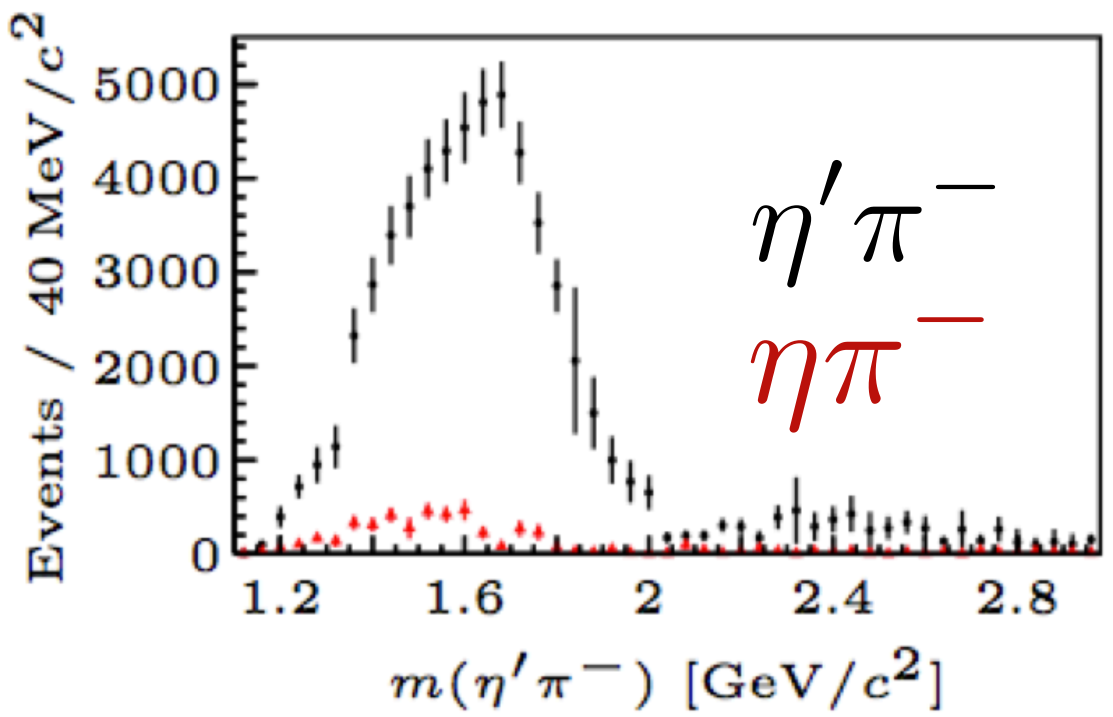
Fernández-Ramírez et al arXiv:2104.10646

Exotic mesons originate from asymmetry in double Regge region

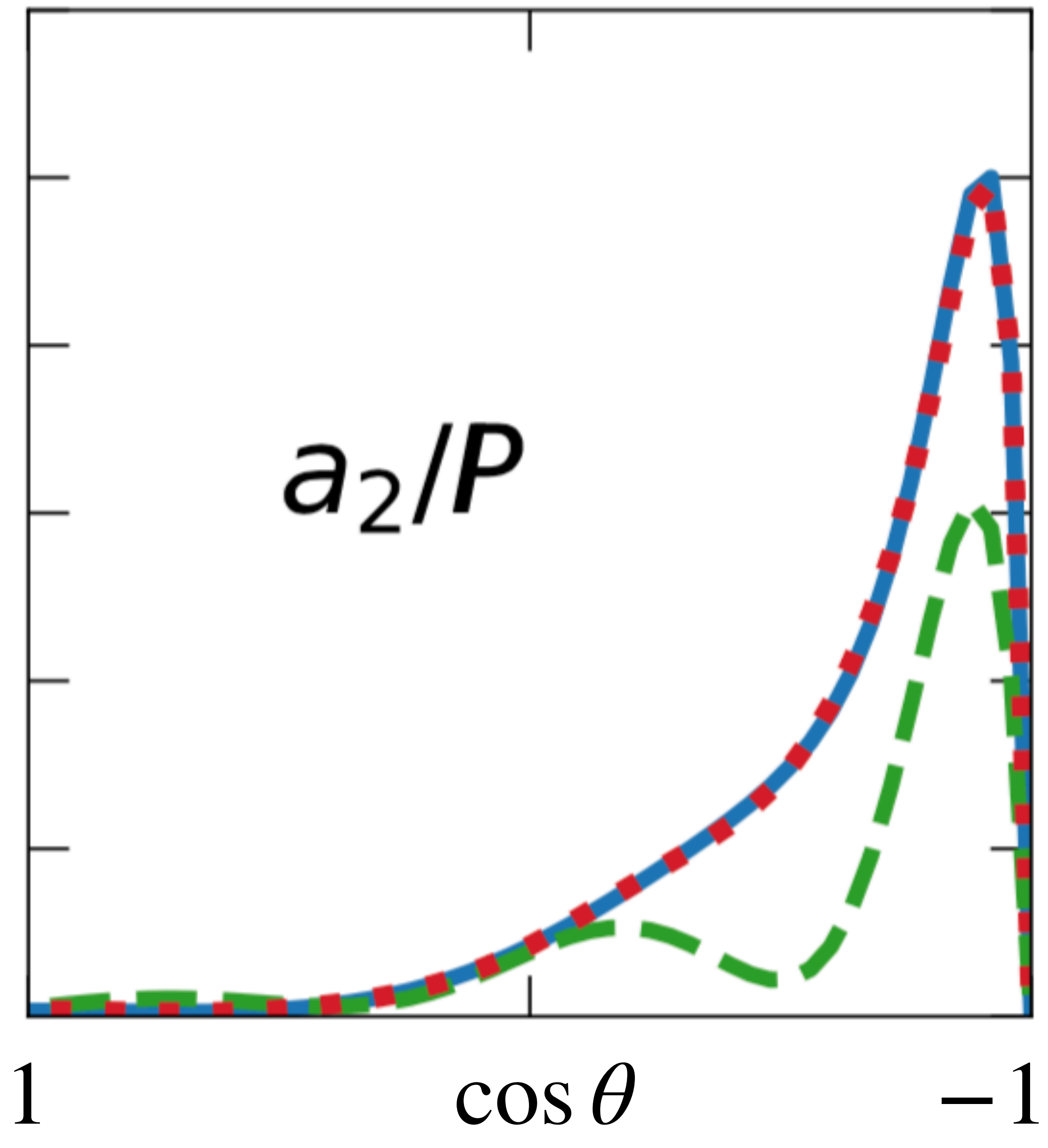
Can be formalised mathematically (in progress)

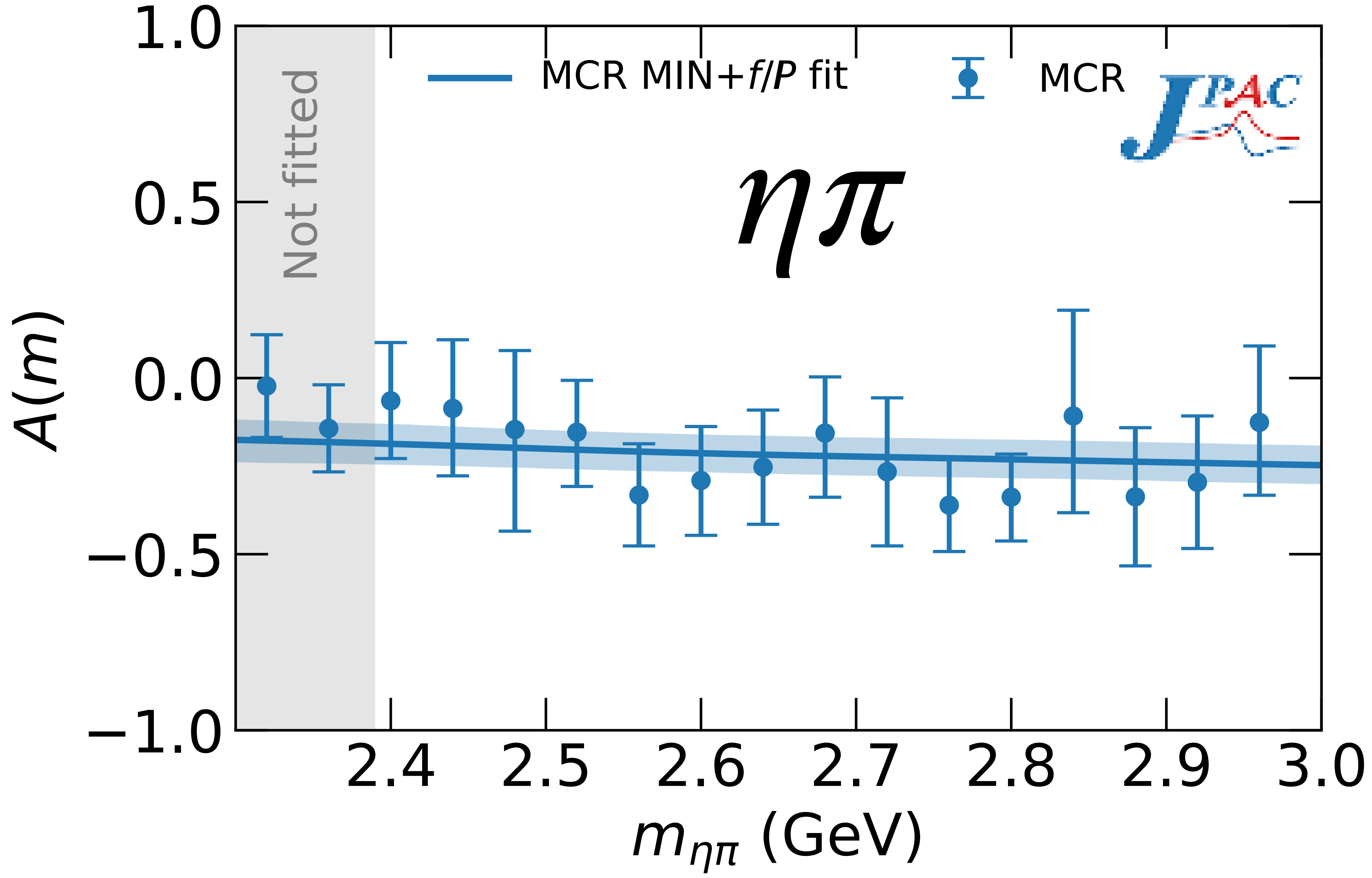


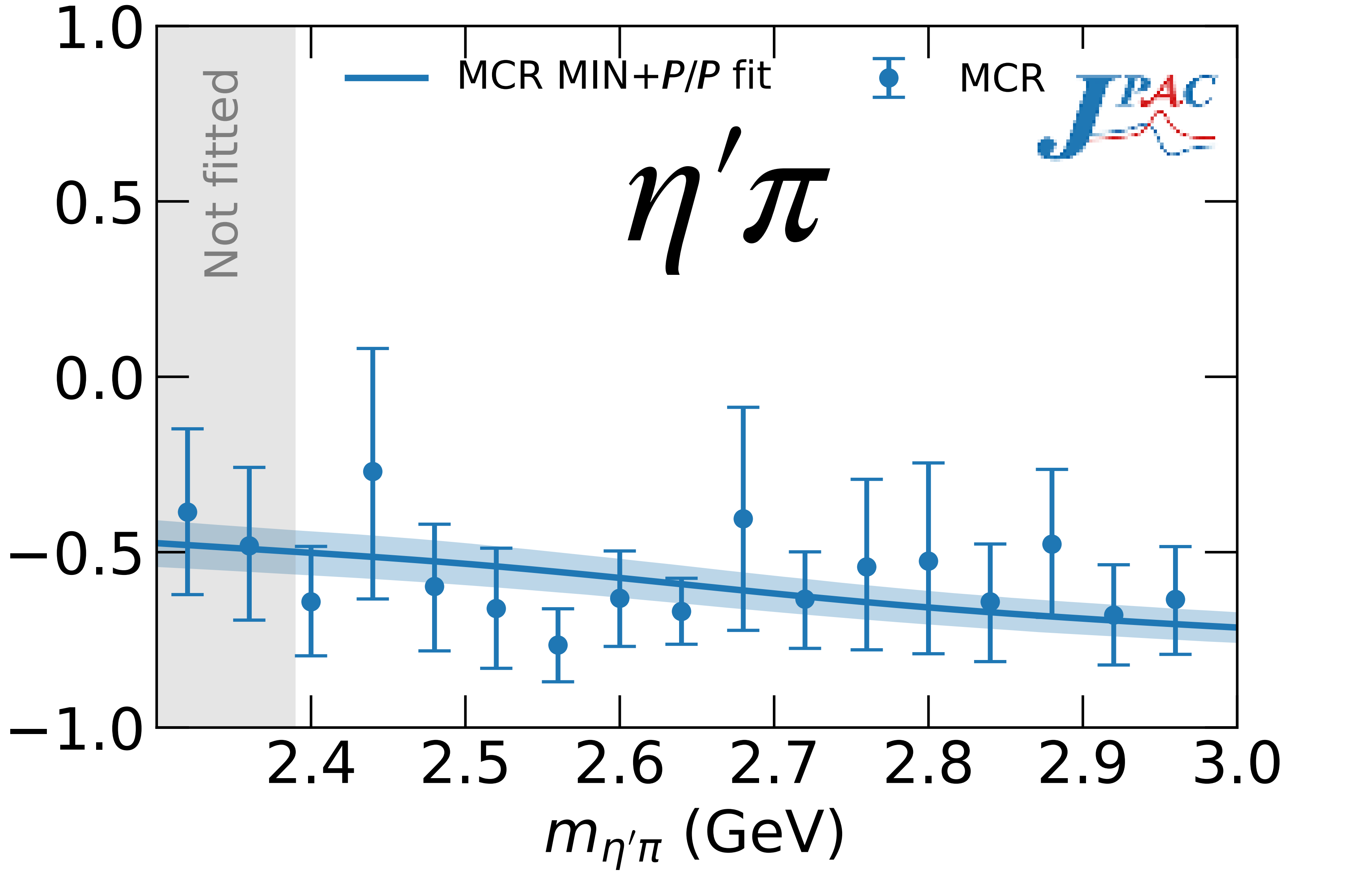




Partial waves not relevant in the double Regge region





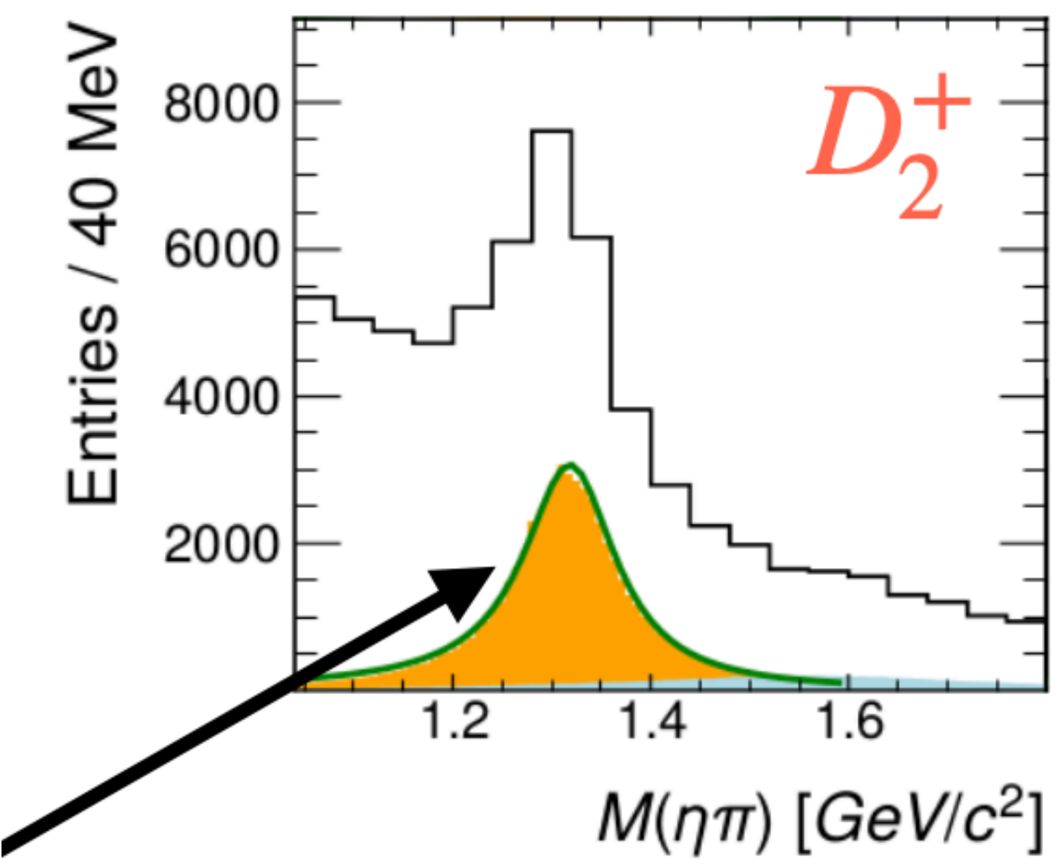


Asymmetry originating from $(a_2, f_2/\mathbb{P}) \neq (f_2, f_2/\mathbb{P})$ and from $(\mathbb{P}, \mathbb{P})$ in $\eta'\pi$

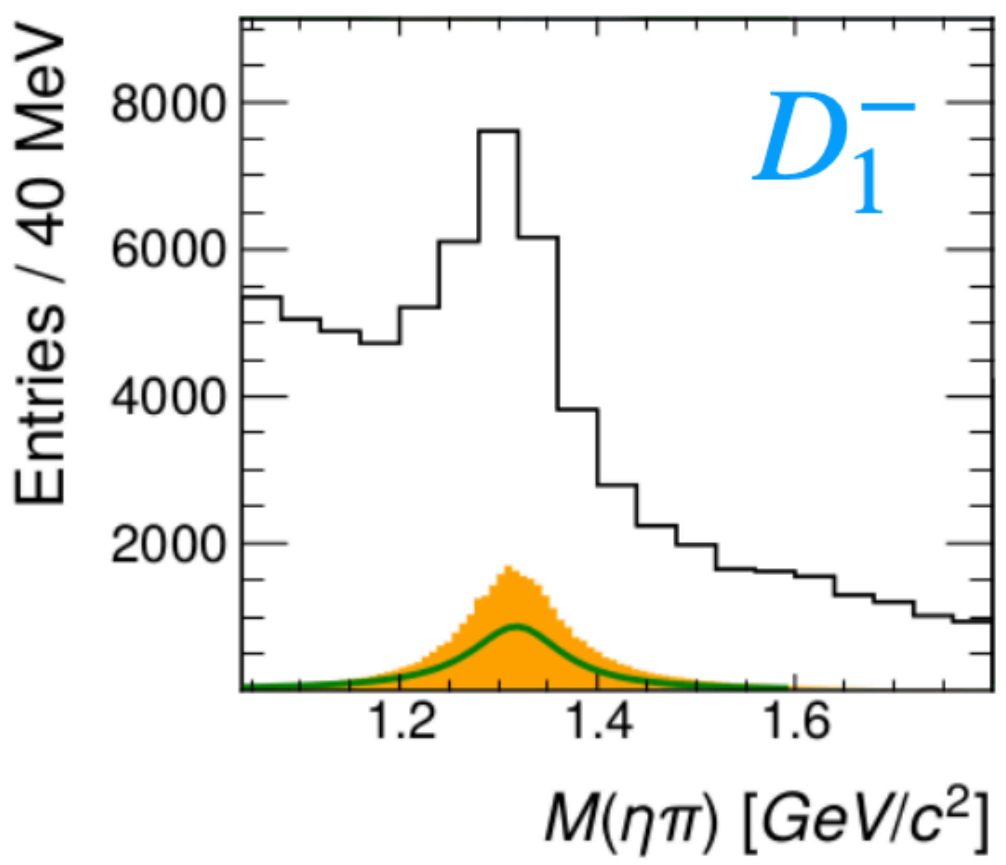
a2(1320)@GlueX

VM et al (JPAC) PRD102 (2020) 014003

Vector exchange



Axial-Vector exchange

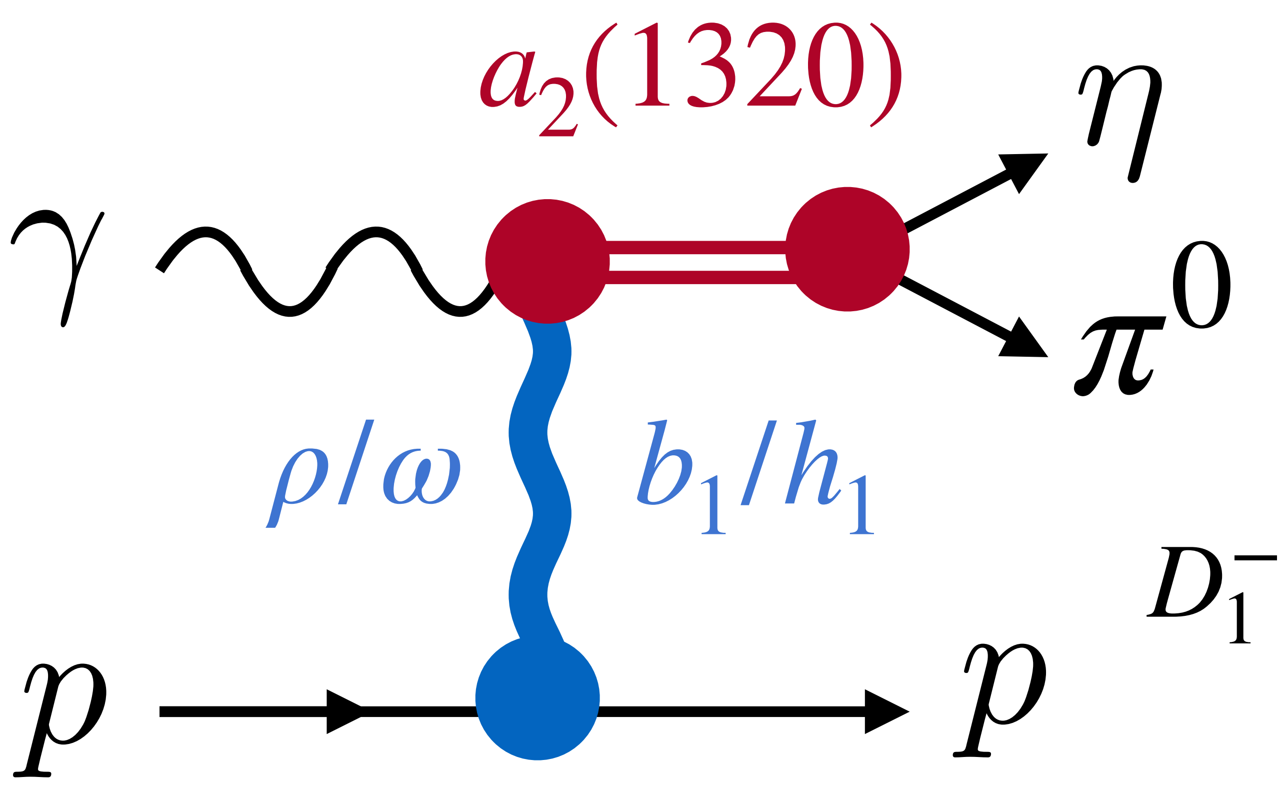


Notation D_m^ϵ:

Reflectivity ϵ matches naturality of exchange

(At leading order in energy squared)

D-wave have 2*5 = 10 complex functions of t



$$D_1^- = -\frac{\beta^U}{\sqrt{6}}\sqrt{\frac{-t}{m_{a_2}^2}}$$

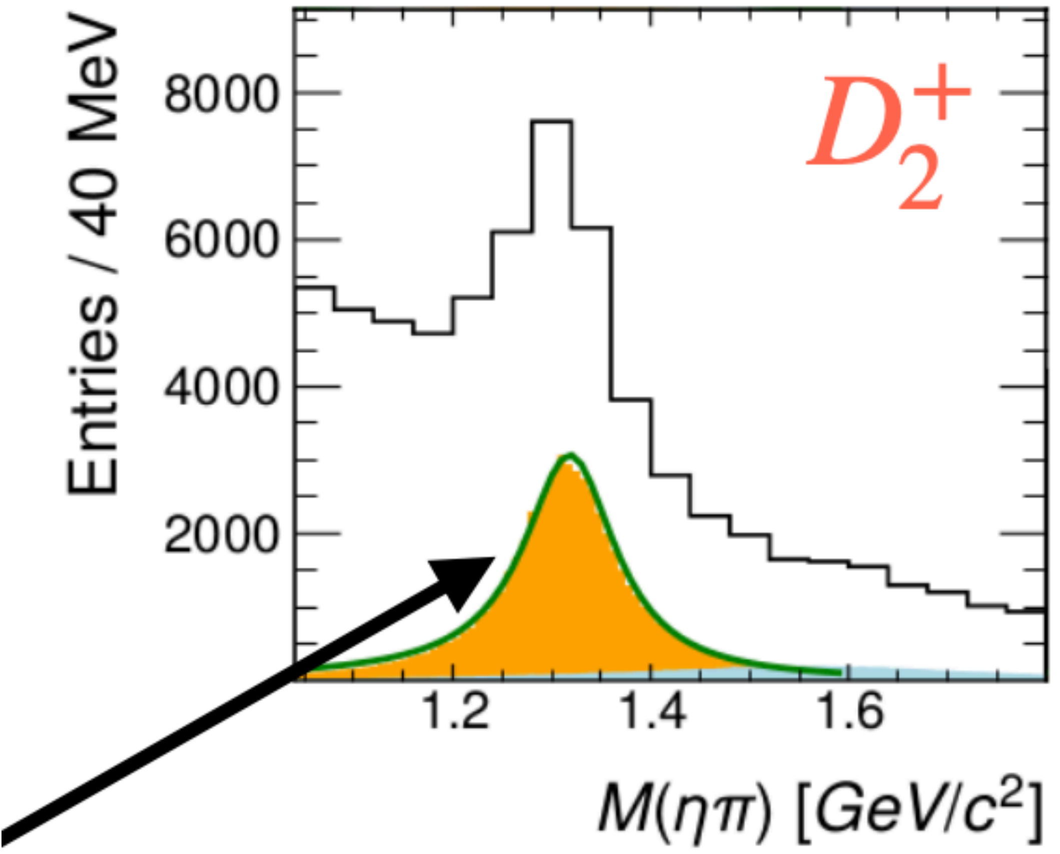
For axial, M1 transition:

$$D_1^- = \frac{\beta^U}{4} \quad D_{-1}^- = \frac{\beta^U}{4}\frac{-t}{m_{a_2}^2}$$

$$D_2^- = D_{-2}^- = 0$$

a2(1320)@GlueX

Vector exchange



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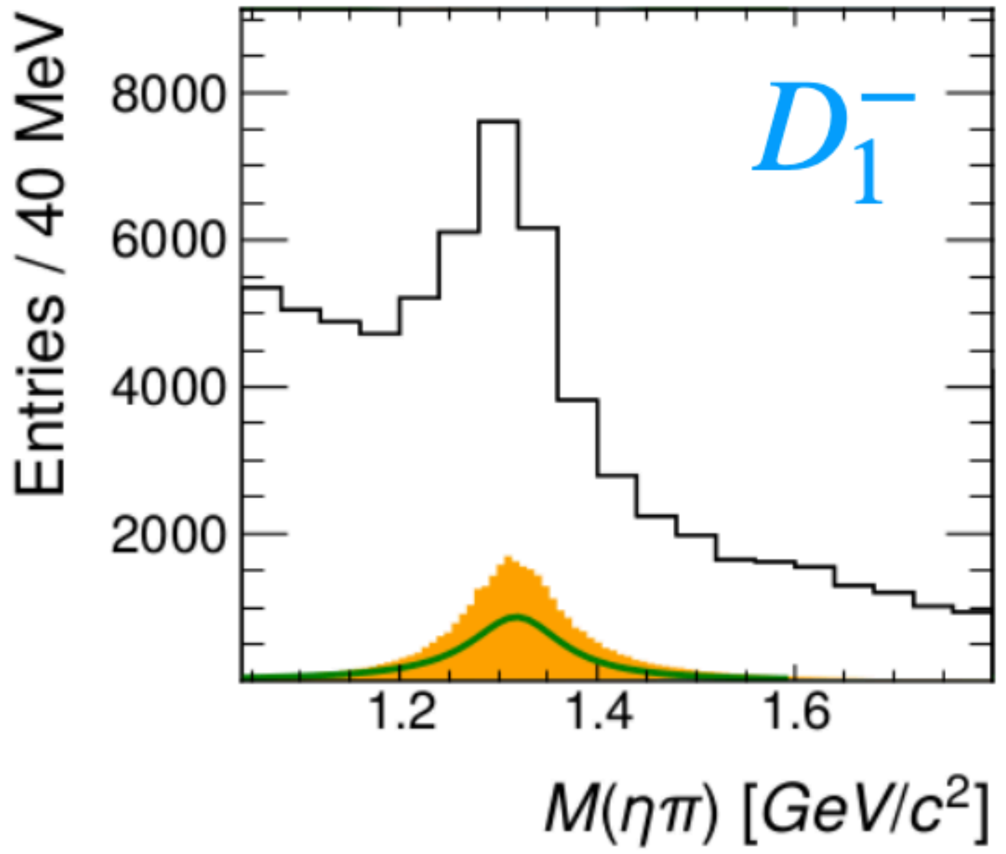

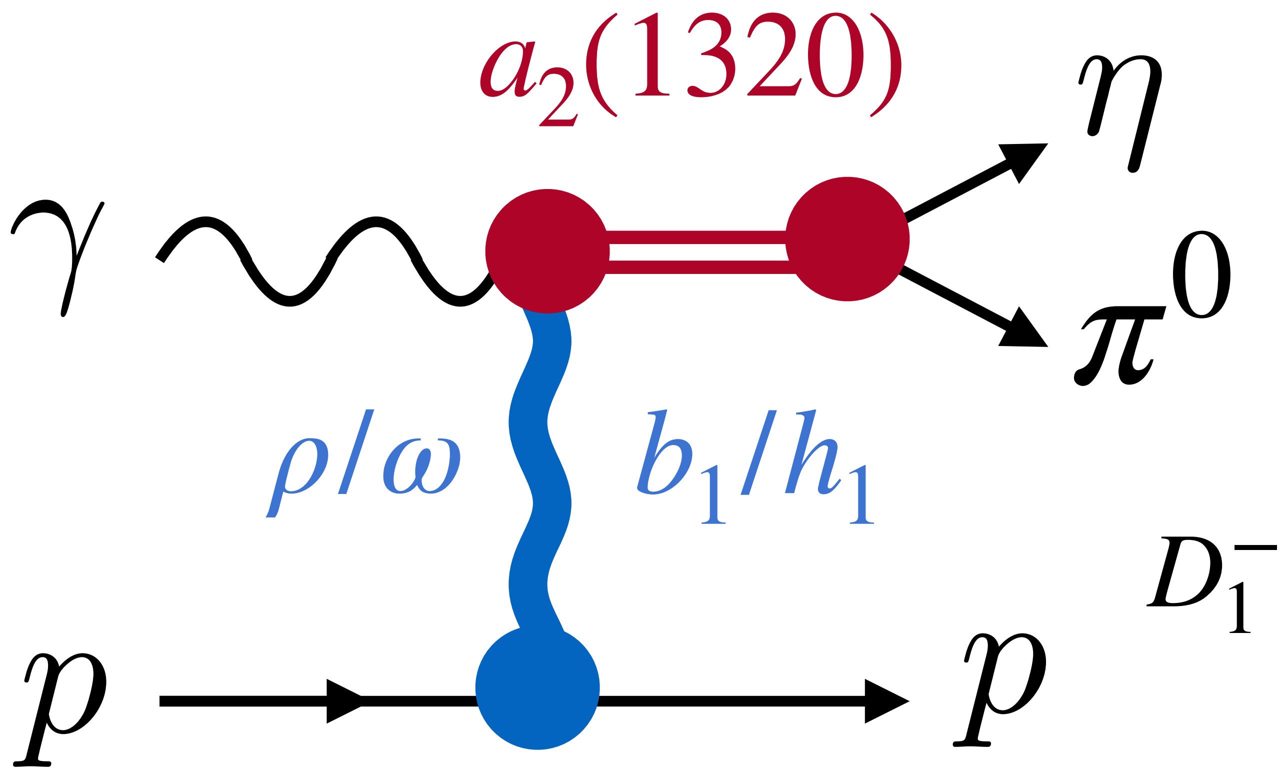

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D-wave have 2*5 = 10 complex functions of t

Assumptions of TMD to reduce nb. of couplings:

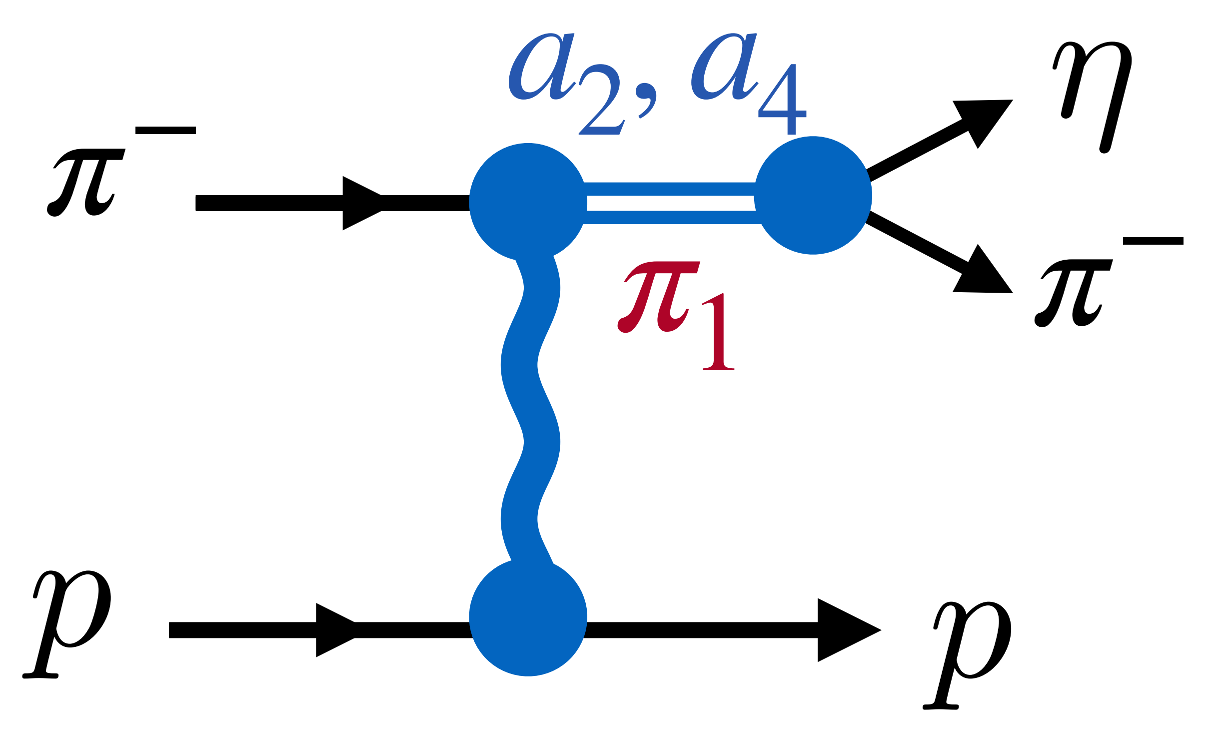
$$\mathscr{L}_{TVV} = \beta_N\, T^{\mu\nu} F_{\mu\rho} F_\nu{}^\rho \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Leads to

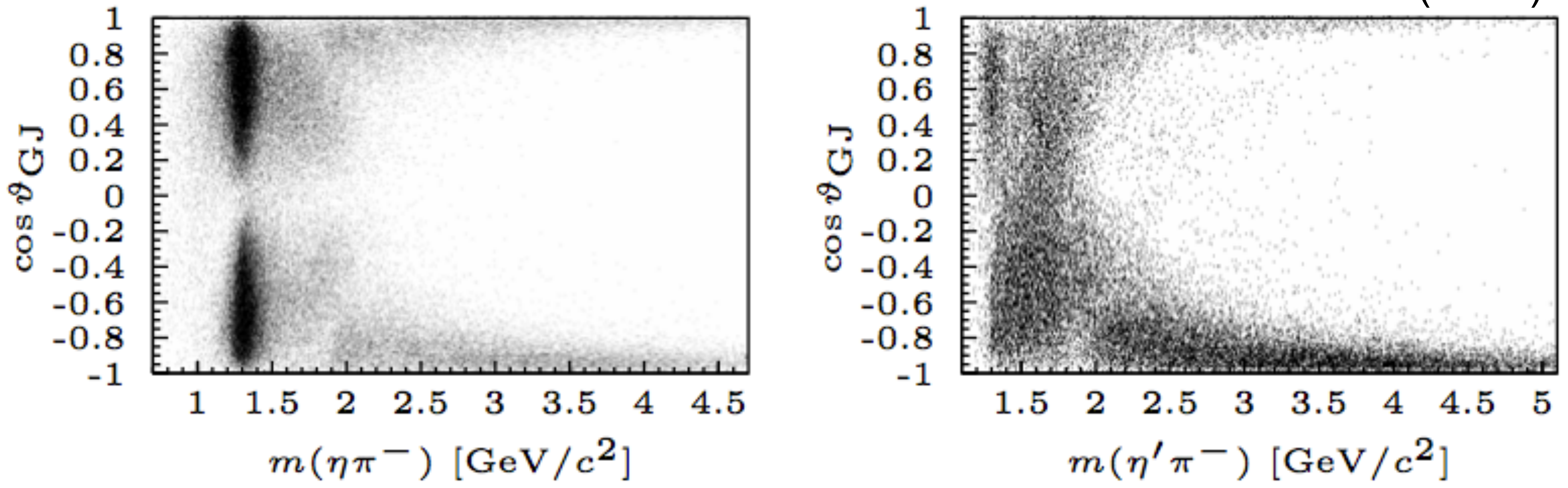
$$D_2^+ = -\frac{\beta_N}{2}\sqrt{\frac{-t}{m_{a_2}^2}} \qquad D_0^+ = \frac{\beta_N}{2}\frac{t}{\sqrt{6}m_{a_2}^2}\sqrt{\frac{-t}{m_{a_2}^2}}$$

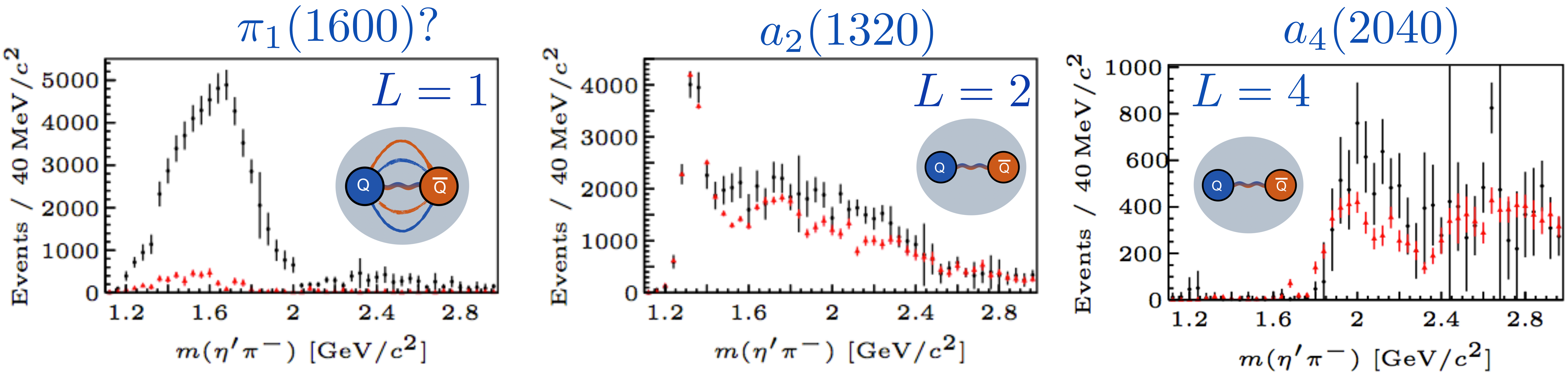
$$D_1^+ = \frac{\beta_N}{2}\frac{-t}{m_{a_2}^2} \qquad D_{-1}^+ = D_{-2}^+ = 0$$

Partial Waves Expansion



COMPASS PLB740 (2015)



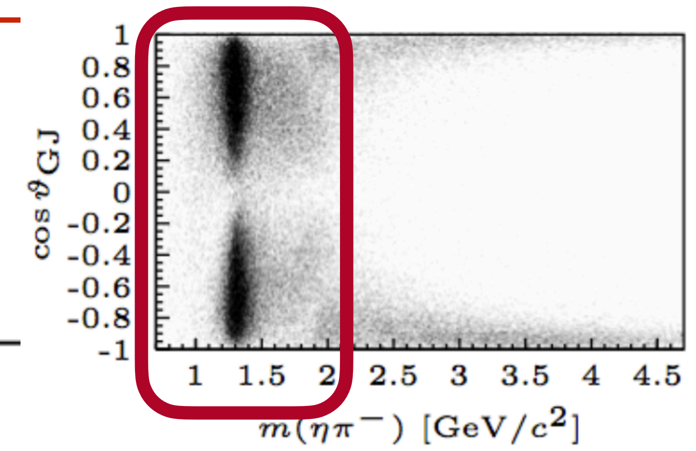


Resonance in angular mom. L = 1 ?

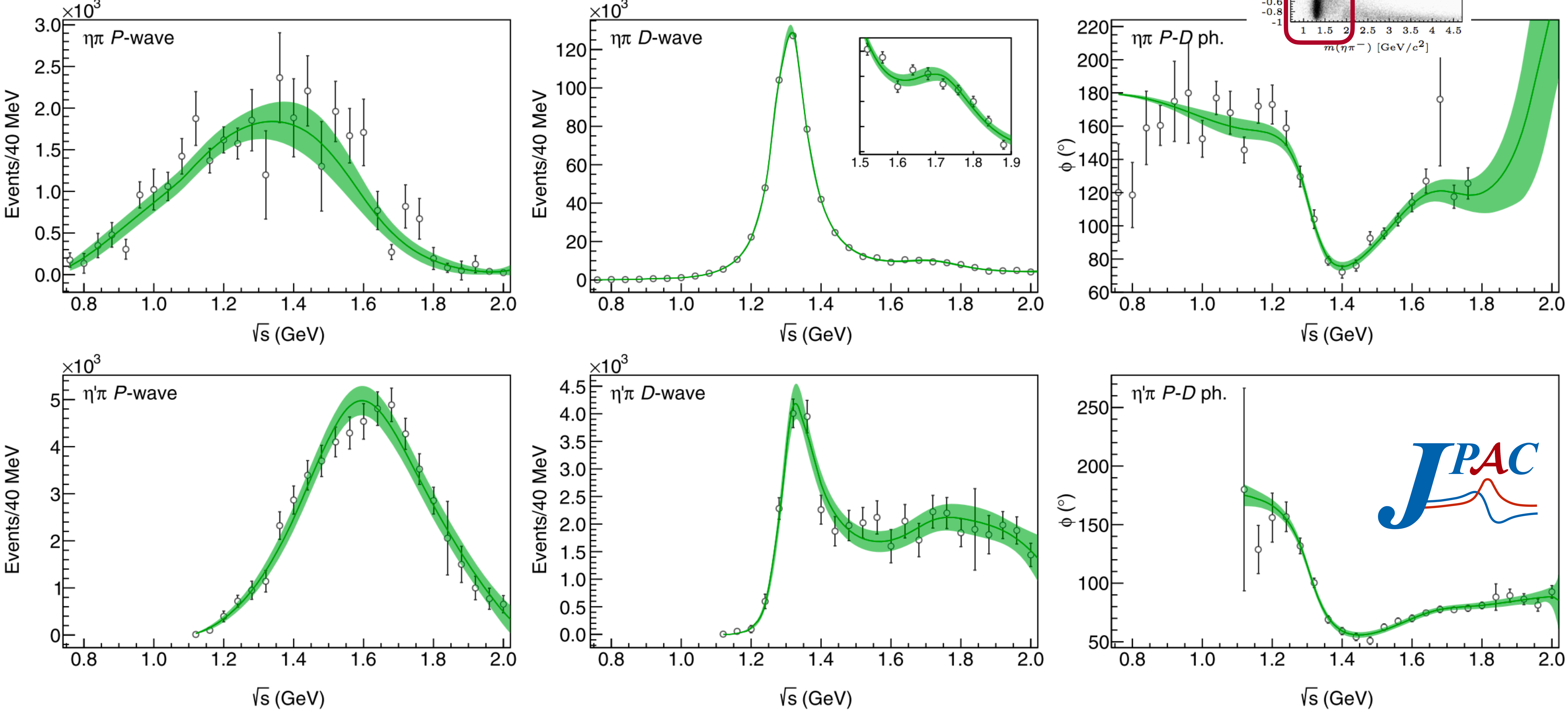
black: $\pi\eta'$ red: $\pi\eta$ (scaled)

Low Energy Fit of $L = 1,2$

Rodas et al PRL122 (2019)



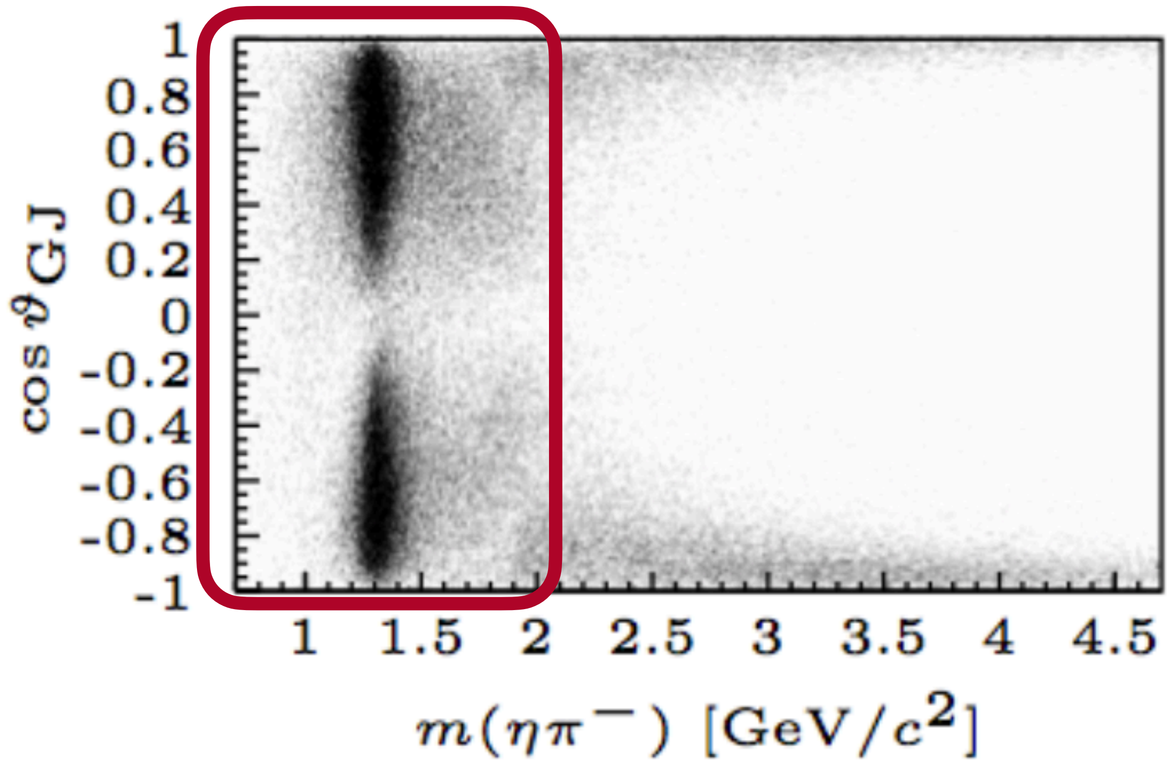
$\pi_1(1400)$ vs $\pi_1(1600)$

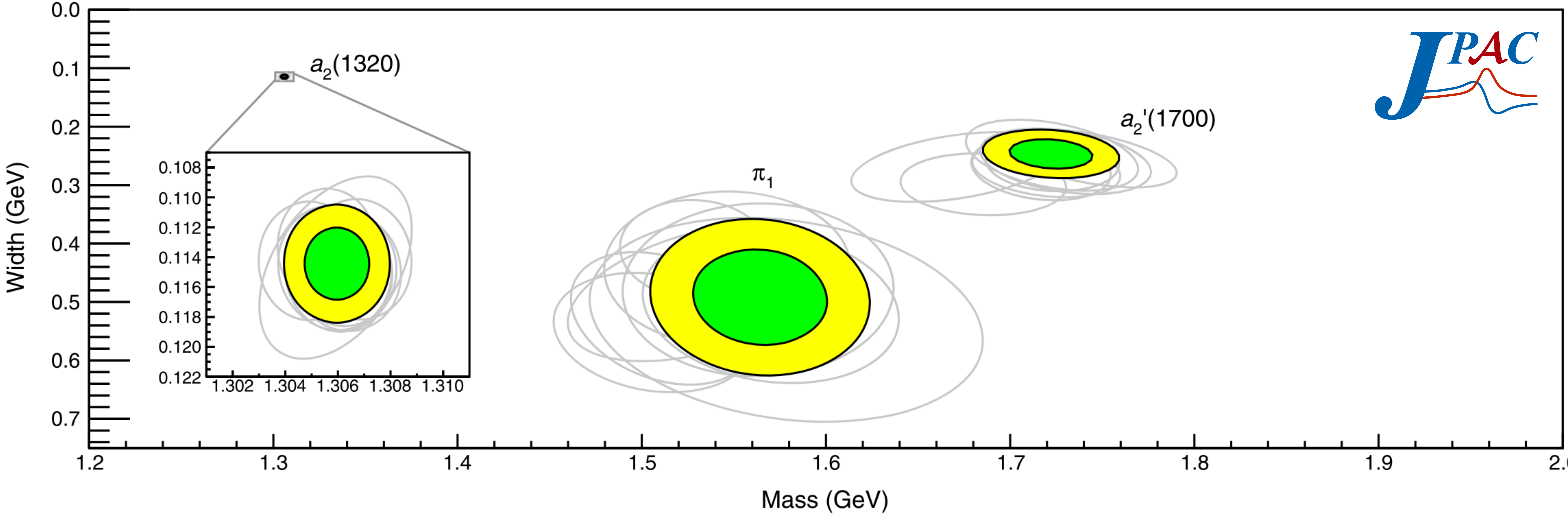


Low Energy Fit of $L = 1,2$

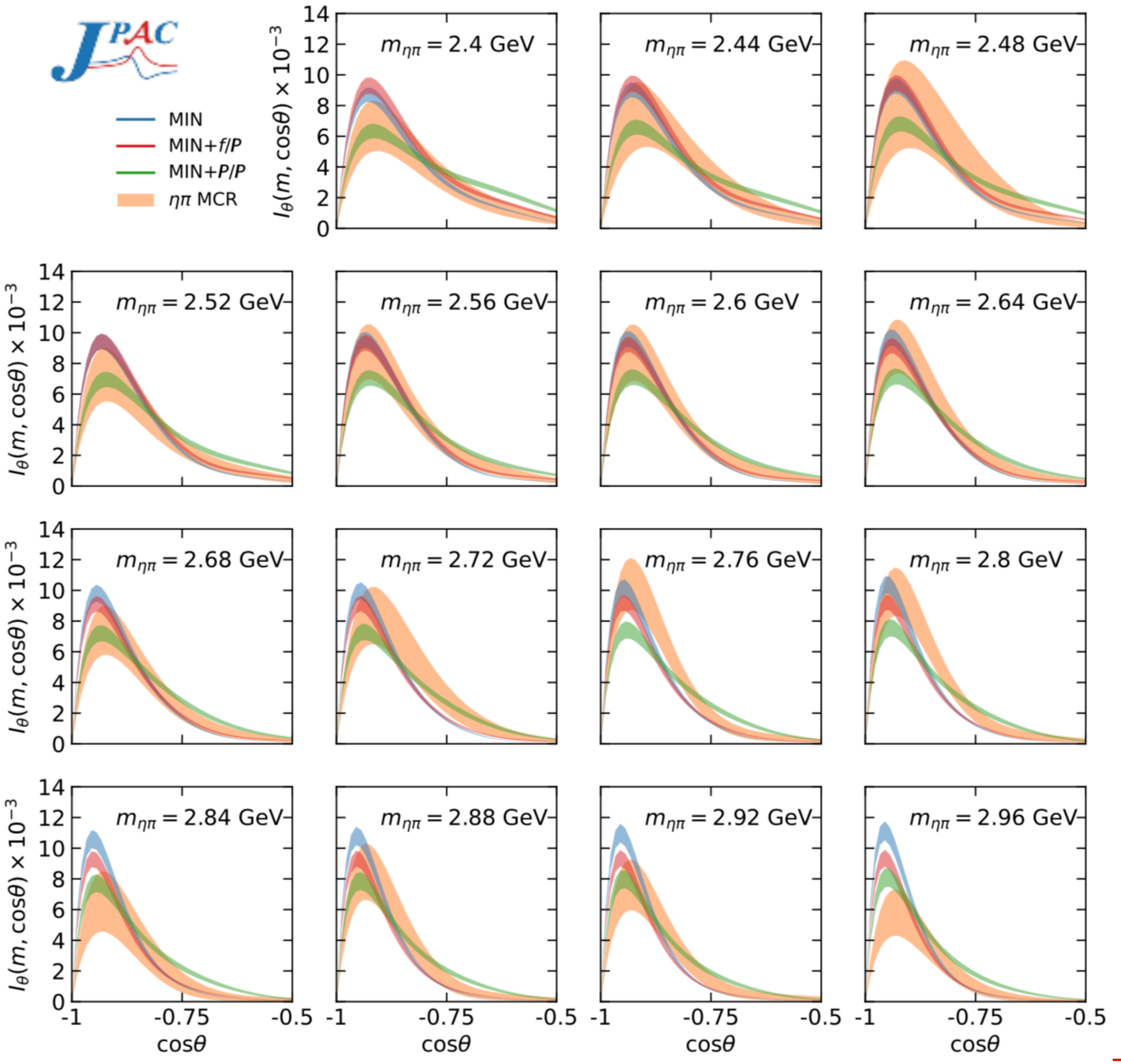
Rodas et al PRL122 (2019)

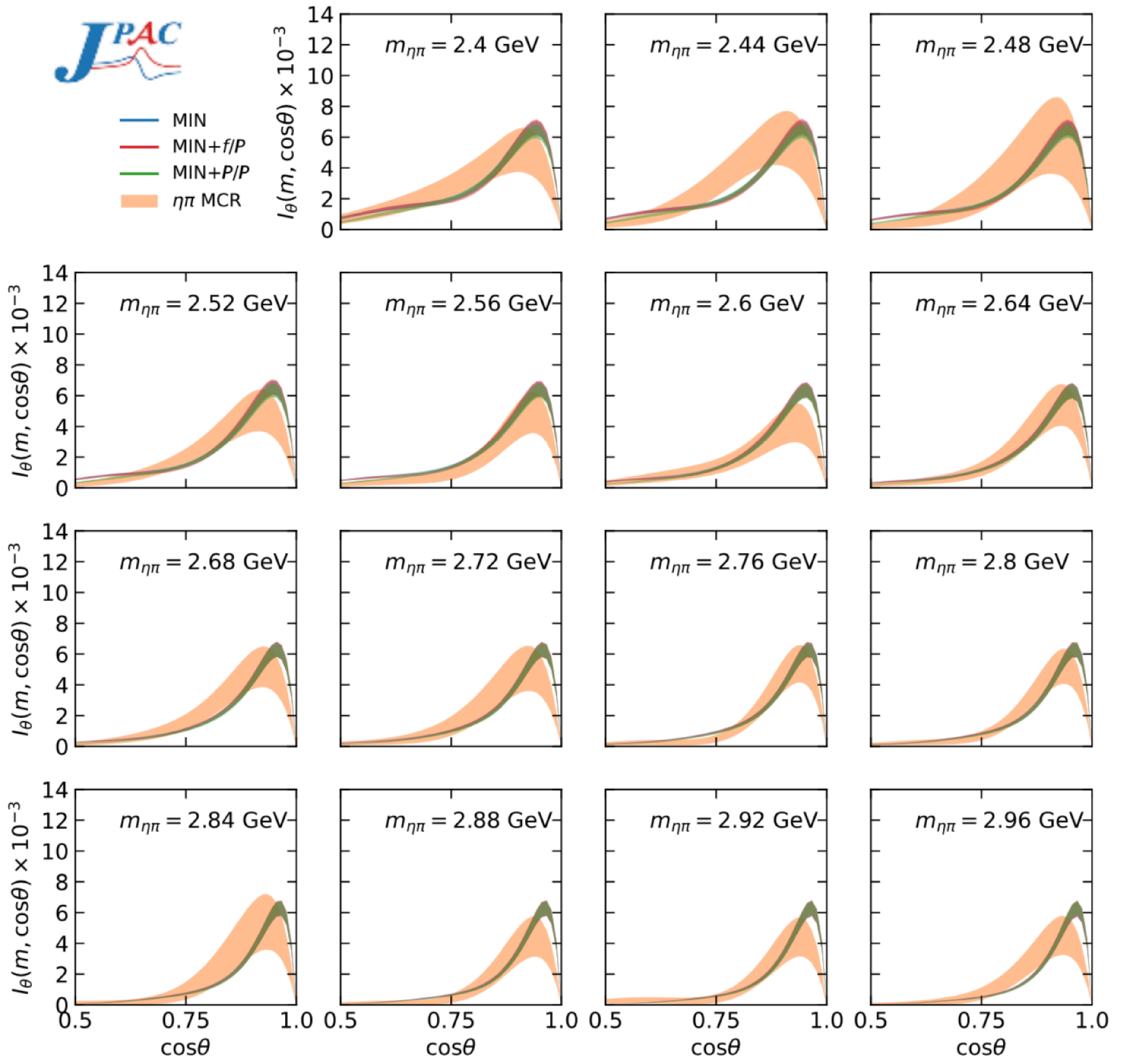
Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$





$\eta\pi$


JPAC
MIN
MIN+f/P
MIN+P/P
ηπ MCR
m_ηπ = 2.4 GeV
m_ηπ = 2.44 GeV
m_ηπ = 2.48 GeV
m_ηπ = 2.52 GeV
m_ηπ = 2.56 GeV
m_ηπ = 2.6 GeV
m_ηπ = 2.64 GeV
m_ηπ = 2.68 GeV
m_ηπ = 2.72 GeV
m_ηπ = 2.76 GeV
m_ηπ = 2.8 GeV
m_ηπ = 2.84 GeV
m_ηπ = 2.88 GeV
m_ηπ = 2.92 GeV
m_ηπ = 2.96 GeV
I_θ(m, cosθ) × 10^−3
cosθ


JPAC
MIN
MIN+f/P
MIN+P/P
ηπ MCR
m_ηπ = 2.4 GeV
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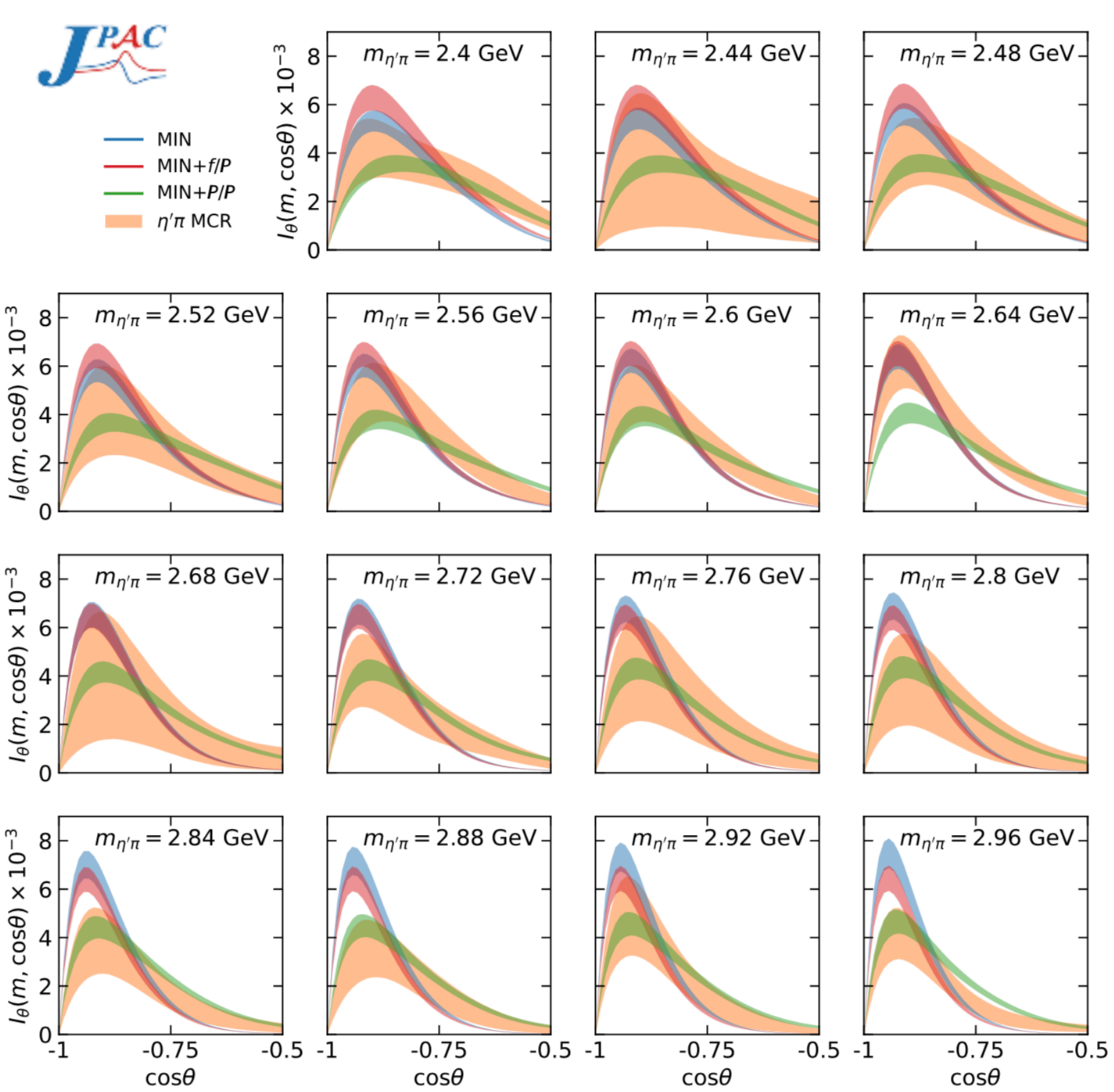
m_ηπ = 2.72 GeV
m_ηπ = 2.76 GeV
m_ηπ = 2.8 GeV
m_ηπ = 2.84 GeV
m_ηπ = 2.88 GeV
m_ηπ = 2.92 GeV
m_ηπ = 2.96 GeV
I_θ(m, cosθ) × 10^−3
cosθ

$\eta'\pi$


JPAC
MIN
MIN+f/P
MIN+P/P
η′π MCR
$m_{\eta'\pi} = 2.4$ GeV
$m_{\eta'\pi} = 2.44$ GeV
$m_{\eta'\pi} = 2.48$ GeV
$m_{\eta'\pi} = 2.52$ GeV
$m_{\eta'\pi} = 2.56$ GeV
$m_{\eta'\pi} = 2.6$ GeV
$m_{\eta'\pi} = 2.64$ GeV
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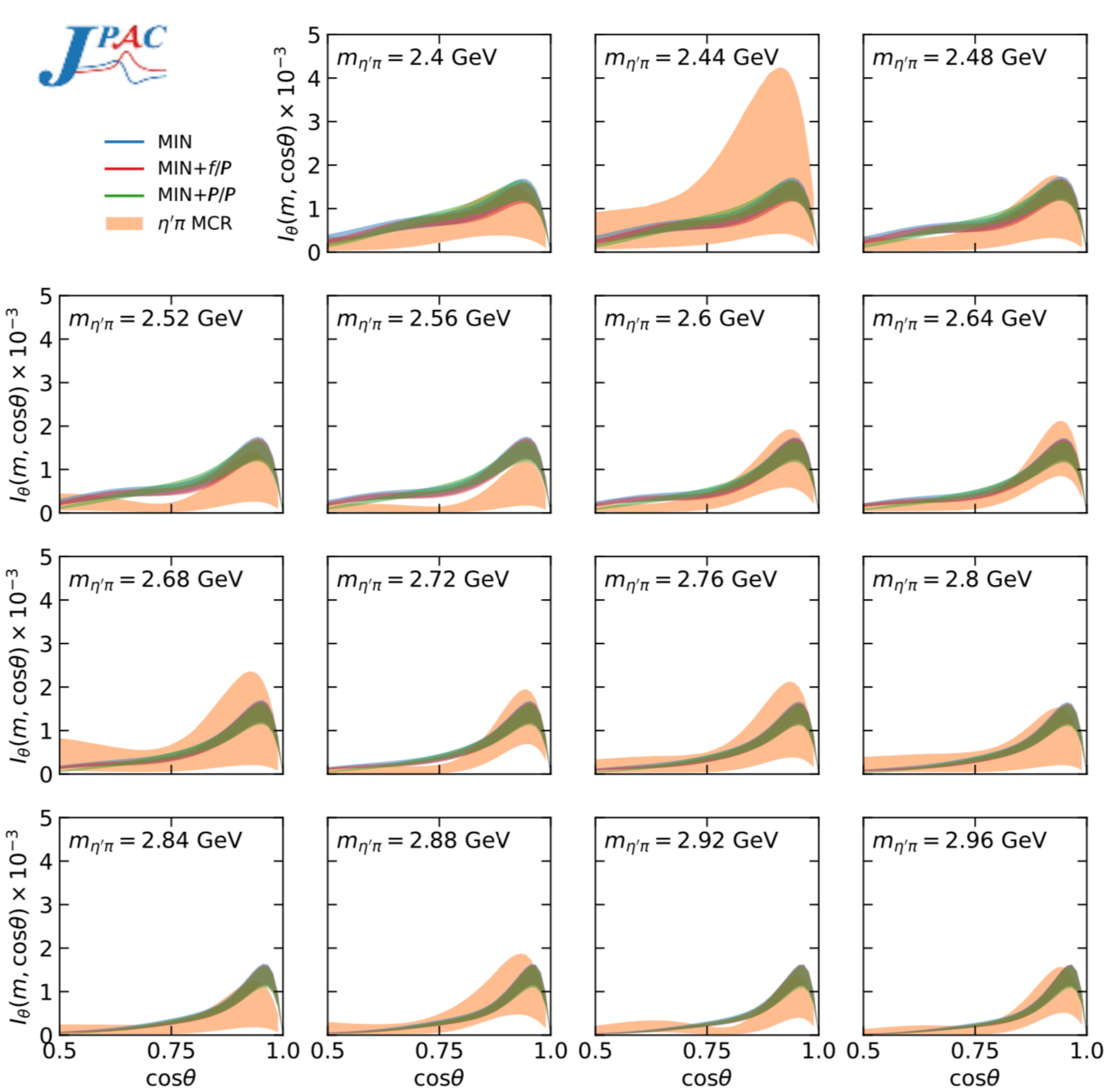
$m_{\eta'\pi} = 2.72$ GeV
$m_{\eta'\pi} = 2.76$ GeV
$m_{\eta'\pi} = 2.8$ GeV
$m_{\eta'\pi} = 2.84$ GeV
$m_{\eta'\pi} = 2.88$ GeV
$m_{\eta'\pi} = 2.92$ GeV
$m_{\eta'\pi} = 2.96$ GeV
$I_\theta(m, \cos\theta) \times 10^{-3}$
$\cos\theta$


JPAC
MIN
MIN+f/P
MIN+P/P
η′π MCR
$m_{\eta'\pi} = 2.4$ GeV
$m_{\eta'\pi} = 2.44$ GeV
$m_{\eta'\pi} = 2.48$ GeV
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$m_{\eta'\pi} = 2.96$ GeV
$I_\theta(m, \cos\theta) \times 10^{-3}$
$\cos\theta$