# Production of $\chi_{c 1}(3872)$ and $T_{\psi \psi}(6900)$ from hadronic collisions to ultraperipheral collisions at EIC 

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## Outline

## Production of $\mathbf{X}(3872)$ in hadronic collisions - gluon gluon fusion

## Ultraperipheral collisions, $\gamma \gamma$ collisions at EIC

What is the mechanism of $T_{\psi \psi}(6900)$ in pp collisions?
$T_{\psi \psi}(6900)$ in UPCs
A. Cisek, W. Schäfer and A. Szczurek, "Structure and production mechanism of the enigmatic $X(3872)$ in high-energy hadronic reactions," [arXiv:2203.07827 [hep-ph]].
R. Maciuła, W. Schäfer and A. Szczurek, "On the mechanism of $T_{4 c}(6900)$ tetraquark production," Phys. Lett. B 812 (2021), 136010 [arXiv:2009.02100 [hep-ph]].
I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, "Light-front approach to axial-vector quarkonium $\gamma^{*} \gamma^{*}$ form factors," [arXiv:2208.05377 [hep-ph]].
Tin Baranov, A. Cisek, M. Kłusek-Gawenda, W. Schäfer and A. Szczurek, "The $\gamma \gamma \rightarrow J / \psi J / \psi$ reaction and the $J / \psi J / \psi$ pair production in exclusive ultraperipheral ultrarelativistic heavy ion collisions," Eur. Phys. J. C 73 (2013) no.2, 2335 [arXiv:1208.5917 [hep-ph]].

## Hadroproduction of $X(3872)$ (or $\chi_{c 1}(3872)$ )



- Structure of $\chi_{c 1}(3872)\left(J^{P C}=1^{++}\right)$still enigmatic. Its situation near the threshold of $D \bar{D}^{*}$ suggests its interpretation as the weakly bound "molecule".
- What about a $\chi_{c 1}(2 P)$ component?
- Production at large $p_{T}$ (hard process) is often suggested to serve as a probe of structure.
- $P_{T}$ distributions have been measured by ATLAS, CMS and LHCb in the $J / \psi \pi \pi$ channel.
- Do the sizeable production cross sections rule out the large size molecule?
- We use the $k_{T}$-factorization approach in which gluons carry transverse momentum and are off-shell. It efficiently includes some NLO corrections at small- $x$. Note that for on-shell gluons $g g \rightarrow 1^{++}$vanishes!


## $k_{T}$-factorization: fusion of off-shell gluons: $c \bar{c}$ state

The inclusive cross section for $X$ (3872)-production via the $2 \rightarrow 1$ gluon-gluon fusion mode is obtained from

$$
\begin{aligned}
\frac{d \sigma}{d y d^{2} \boldsymbol{p}} & =\int \frac{d^{2} \boldsymbol{q}_{1}}{\pi \boldsymbol{q}_{1}^{2}} \mathcal{F}\left(x_{1}, \boldsymbol{q}_{1}^{2}, \mu_{F}^{2}\right) \int \frac{d^{2} \boldsymbol{q}_{2}}{\pi \boldsymbol{q}_{2}^{2}} \mathcal{F}\left(x_{2}, \boldsymbol{q}_{2}^{2}, \mu_{F}^{2}\right) \delta^{(2)}\left(\boldsymbol{q}_{1}+\boldsymbol{q}_{2}-\boldsymbol{p}\right) \\
& \times \frac{\pi}{\left(x_{1} x_{2} s\right)^{2}} \overline{\left|\mathcal{M}_{\boldsymbol{g}^{*} g^{*} \rightarrow X(3872)}\right|^{2}}
\end{aligned}
$$

With

$$
\mathcal{M}_{g^{*} g^{*} \rightarrow X(3872)}=\frac{q_{1 \perp}^{\mu} \boldsymbol{q}_{2 \perp}^{\nu}}{\left|\boldsymbol{q}_{1}\right|\left|\boldsymbol{q}_{2}\right|} \mathcal{M}_{\mu \nu}=\frac{x_{1} x_{2} s}{\left|\boldsymbol{q}_{1}\right|\left|\boldsymbol{q}_{2}\right|} n_{+}^{\mu} n_{-}^{\nu} \mathcal{M}_{\mu \nu}
$$

Here the matrix element squared for the fusion of two off-shell gluons into the ${ }^{3} P_{1}$ color singlet $c \bar{c}$ charmonium is (Kniehl, Vasin, Saleev):

$$
\begin{aligned}
\overline{\left|n_{+}^{\mu} n_{-}^{\nu} \mathcal{M}_{\mu \nu}\right|^{2}} & =\frac{\left(4 \pi \alpha_{S}\right)^{2}}{N_{c}\left(N_{c}^{2}-1\right)} \frac{\left|R^{\prime}(0)\right|^{2}}{\pi M_{X}^{3}} \frac{\boldsymbol{q}_{1}^{2} \boldsymbol{q}_{2}^{2}}{\left(M_{X}^{2}+\boldsymbol{q}_{1}^{2}+\boldsymbol{q}_{2}^{2}\right)^{4}} \\
& \times\left(\left(\boldsymbol{q}_{1}^{2}+\boldsymbol{q}_{2}^{2}\right)^{2} \sin ^{2} \phi+M_{X}^{2}\left(\boldsymbol{q}_{1}^{2}+\boldsymbol{q}_{2}^{2}-2\left|\boldsymbol{q}_{1}\right|\left|\boldsymbol{q}_{2}\right| \cos \phi\right)\right)
\end{aligned}
$$

where $\phi$ is the azimuthal angle between $\boldsymbol{q}_{1}, \boldsymbol{q}_{2}$. The momentum fractions of gluons are fixed as $x_{1,2}=m_{T} \exp ( \pm y) / \sqrt{s}$, where $m_{T}^{2}=\boldsymbol{p}^{2}+M_{X}^{2}$. We use for the first radial $p$-wave excitation from $\left|R^{\prime}(0)\right|^{2}=0.1767 \mathrm{GeV}^{5}$ from Eichten \& Quigg.

## Production of molecule component

- Here we also start from the hard subprocess: production of the $c \bar{c}$-pair.

$$
\begin{aligned}
\frac{d \sigma(p p \rightarrow Q \bar{Q}+\text { anything })}{d y_{1} d y_{2} d^{2} \boldsymbol{p}_{1} d^{2} \boldsymbol{p}_{2}}= & \int \frac{d^{2} \boldsymbol{k}_{1}}{\pi \boldsymbol{k}_{1}^{2}} \mathcal{F}\left(x_{1}, \boldsymbol{k}_{1}^{2}, \mu_{F}^{2}\right) \int \frac{d^{2} \boldsymbol{k}_{2}}{\pi \boldsymbol{k}_{2}^{2}} \mathcal{F}\left(x_{2}, \boldsymbol{k}_{2}^{2}, \mu_{F}^{2}\right) \\
& \times \delta^{(2)}\left(\boldsymbol{k}_{1}+\boldsymbol{k}_{2}-\boldsymbol{p}_{1}-\boldsymbol{p}_{2}\right) \frac{1}{16 \pi^{2}\left(x_{1} x_{2} s\right)^{2}} \overline{\left|\mathcal{M}_{g^{*} g^{*} \rightarrow c \bar{c}}^{\text {off }- \text { hel }}\right|^{2}}
\end{aligned}
$$

where $\mathcal{M}_{g^{*} g^{*} \rightarrow Q \bar{Q}}^{\text {off-shell }}$ is the off-shell matrix element for the hard subprocess (Catani, Ciafaloni, Hautmann).

- we then hadronize charm quarks to $D, \bar{D}^{*}$ mesons, by assuming, in the pp-cm frame, that

$$
\vec{p}_{D}=\vec{p}_{c} .
$$

- We use fragmentation fractions:

$$
\begin{aligned}
& f\left(c \rightarrow D^{0}\right)=f\left(\bar{c} \rightarrow \bar{D}^{0}\right)=0.547, \\
& f\left(c \rightarrow D^{+}\right)=f\left(\bar{c} \rightarrow D^{-}\right)=0.227, \\
& f\left(c \rightarrow D^{* 0}\right)=f\left(\bar{c} \rightarrow \bar{D}^{* 0}\right)=0.237, \\
& f\left(c \rightarrow D^{*+}\right)=f\left(\bar{c} \rightarrow \bar{D}^{*-}\right)=0.237 .
\end{aligned}
$$

## Production of molecule component

- The molecule is the C-even combination

$$
\left|\Psi_{m o l}\right\rangle=\frac{1}{\sqrt{2}}\left(\left|D^{0} \bar{D}^{* 0}\right\rangle+\left|D^{* 0} \bar{D}^{0}\right\rangle\right)
$$

- To obtain $D, \bar{D}^{*}$, we multiply the $c \bar{c}$ cross section by

$$
\frac{1}{2}\left[f\left(c \rightarrow D^{0}\right) f\left(\bar{c} \rightarrow \bar{D}^{* 0}\right)+f\left(c \rightarrow D^{* 0}\right) f\left(\bar{c} \rightarrow \bar{D}^{0}\right)\right]=\left\{\begin{array}{l}
0.036 \text { direct } \\
0.13 \text { including feeddown } .
\end{array}\right.
$$

- In the spirit of effective range theory, we want to relate the continuum cross section to the bound state cross section.
- We are interested in low

$$
k_{r e l}=\frac{1}{2} \sqrt{M_{c \bar{c}}^{2}-4 m_{c}^{2}},
$$

We integrate the $D \bar{D}^{*}$ cross section over the relative momentum $k_{r e l}^{D \bar{D}^{*}}$ up to a cutoff $k_{\text {max }}^{D \bar{D}}$ (Artoisenet \& Braaten). Instead impose a cutoff $k_{\max }$ on the relative momentum $k_{r e l}$. Within our kinematics the latter will be similar to $k_{\text {rel }}^{D \bar{D}^{*}}$, but somewhat larger. We estimate, that $k_{\max }=0.2 \mathrm{GeV}$ corresponds roughly to $k_{\max }^{D D} \approx 0.13 \mathrm{GeV}$.

## Production of molecule component




Figure: Distribution in $k_{r e l}$ for different windows of $p_{t, c \bar{c}}=p_{t, X}$ (left panel) for the CMS kinematics. In the right panel we show the cross sections divided by $k_{\text {rel }}^{2}$. In these calculations the KMR UGDF with the MMHT NLO collinear gluon distribution was used. Branching fractions are included here.

- cross section $d \sigma \propto k_{r e l}^{2} d k_{r e l}$, cubic dependence on cutoff.
- debate in the literature on $k_{\max }^{D \bar{D}^{*}}$, some authors suggest $k_{X}=\sqrt{2 \mu \varepsilon_{X}} \sim 35 \mathrm{MeV}$, but more consistent with effective range theory is $k_{\max }^{D \bar{D}^{*}} \sim m_{\pi}$.
- better would be a calculation with a $D \bar{D}^{*}$ "wave function at the origin".


## Numerical results vs. data, CMS



Figure: Transverse momentum distribution of $X(3872)$ for the CMS experiment. Shown are results for 3 different gluon uPDFs. Here $\mathrm{BR}=0.038$. The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of $D^{0}, \bar{D}^{0}$. We also show corresponding distribution for the DPS mechanism (dotted line).

## Numerical results vs. data, ATLAS



Figure: Transverse momentum distribution of $X(3872)$ for the ATLAS experiment. Shown are results for 3 different gluon uPDFs. Here $\mathrm{BR}=0.038 \cdot 0.0596$. The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of $D^{0}, \bar{D}^{0}$.

## Numerical results vs. data, LHCb



Figure: Transverse momentum distribution of $X(3872)$ for the LHCb experiment. Shown are results for 3 different gluon uPDFs. Here $B R=0.038$. The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of $D^{0}, \bar{D}^{0}$.

## $c \bar{C}$-state, molecule, mixture of both...



Figure: Transverse momentum distribution of $X(3872)$ for the CMS, ATLAS and LHCb experiments. Shown are results for the KMR UGDF. Here $\mathrm{BR}=0.038$ for CMS and LHCb, and $\mathrm{BR}=0.038 \cdot 0.0596$ for ATLAS. We show results for different combinations of $\alpha$ and $\beta$ as specified in the figure legend. Here the feedown contributions are included.
-

$$
|X(3782)\rangle=\alpha|c \bar{c}\rangle+\frac{\beta}{\sqrt{2}}\left(\left|D \bar{D}^{*}\right\rangle+\left|\bar{D} D^{*}\right\rangle\right)
$$

## Ultraperipheral collisions

some examples of ultraperipheral processes:


- photoabsorption on a nucleus
- diffractive photoproduction with and without breakup/excitation of a nucleus
- $\gamma \gamma$-fusion.
- electromagnetic excitation/dissociation of nuclei. Excitation of Giant Dipole Resonances.
- the intact nuclei in the final state are not measured. Each of the photon exchanges is associated with a large rapidity gap.
- very small $p_{T}$ of the photoproduced system.


## Ultraperipheral collisions

Photon-photon process in the electron-proton or electron-ion collision:


- proton $(Z=1)$ or ion $(Z=82$ for Pb$)$ is a source of quasireal Weizsäcker-Williams photons)

$$
n(x)=\frac{Z^{2} \alpha_{\mathrm{em}}}{\pi} \int \frac{d \boldsymbol{q}^{2}}{\boldsymbol{q}^{2}}\left(\frac{\boldsymbol{q}^{2}}{\boldsymbol{q}^{2}+x^{2} m_{p}^{2}}\right)^{2} F_{\mathrm{ch}}^{2}\left(\boldsymbol{q}^{2}+x^{2} m_{p}^{2}\right)
$$

- "anti-tagged" electrons, say $Q^{2}<0.1 \mathrm{GeV}^{2}$, are also the source of quasireal photons $\longrightarrow$ $\gamma \gamma$-fusion.
- at finite $Q^{2}$ we have access to a whole polarization density matrix of virtual photons,
- the intact nuclei in the final state are not measured. Photon exchange is associated with a large rapidity gap.


## $\gamma^{*} \gamma^{*}$-transition form factors for $J^{P C}=1^{++}$axial mesons

$$
\begin{aligned}
\frac{1}{4 \pi \alpha_{\mathrm{em}}} \mathcal{M}_{\mu \nu \rho} & =i\left(q_{1}-q_{2}+\frac{Q_{1}^{2}-Q_{2}^{2}}{\left(q_{1}+q_{2}\right)^{2}}\left(q_{1}+q_{2}\right)\right)_{\rho} \tilde{G}_{\mu \nu} \frac{M}{2 X} F_{\mathrm{TT}}\left(Q_{1}^{2}, Q_{2}^{2}\right) \\
& +i e_{\mu}^{L}\left(q_{1}\right) \tilde{G}_{\nu \rho} \frac{1}{\sqrt{X}} F_{\mathrm{LT}}\left(Q_{1}^{2}, Q_{2}^{2}\right)+i e_{\nu}^{L}\left(q_{2}\right) \tilde{G}_{\mu \rho} \frac{1}{\sqrt{X}} F_{\mathrm{TL}}\left(Q_{1}^{2}, Q_{2}^{2}\right) .
\end{aligned}
$$

- Above we introduced

$$
\tilde{G}_{\mu \nu}=\varepsilon_{\mu \nu \alpha \beta} q_{1}^{\alpha} q_{2}^{\beta}
$$

and the polarization vectors of longitudinal photons

$$
e_{\mu}^{L}\left(q_{1}\right)=\sqrt{\frac{-q_{1}^{2}}{X}}\left(q_{2 \mu}-\frac{q_{1} \cdot q_{2}}{q_{1}^{2}} q_{1 \mu}\right), \quad e_{\nu}^{L}\left(q_{2}\right)=\sqrt{\frac{-q_{2}^{2}}{X}}\left(q_{1 \nu}-\frac{q_{1} \cdot q_{2}}{q_{2}^{2}} q_{2 \nu}\right) .
$$

- $F_{\mathrm{TT}}(0,0)=0$, there is no decay to two photons (Landau-Yang).
- $F_{\mathrm{LT}}\left(Q^{2}, 0\right) \propto Q$ (absence of kinematical singularities). $f_{\mathrm{LT}}=\lim _{Q^{2} \rightarrow 0} F_{\mathrm{LT}}\left(Q^{2}, 0\right) / Q$ gives rise to so-called "reduced width".
- we are interested in the situation with one virtual and one real photon.


## $\gamma^{*} \gamma^{*}$ cross sections

$$
\sigma_{i j}=\frac{32 \pi(2 J+1)}{N_{i} N_{j}} \frac{\hat{s}}{2 M \sqrt{X}} \frac{M \Gamma}{\left(\hat{s}-M^{2}\right)^{2}+M^{2} \Gamma^{2}} \Gamma_{\gamma^{*} \gamma^{*}}^{i j}\left(Q_{1}^{2}, Q_{2}^{2}, \hat{s}\right),
$$

where $\{i, j\} \in\{\mathrm{T}, \mathrm{L}\}$, and $N_{\mathrm{T}}=2, N_{\mathrm{L}}=1$ are the numbers of polarization states of photons. In terms of our helicity form factor, we obtain for the LT configuration, putting at the resonance pole $\hat{s} \rightarrow M^{2}$, and $J=1$ for the axial-vector meson:

$$
\begin{gathered}
\Gamma_{\gamma^{*} \gamma^{*}}^{\mathrm{LT}}\left(Q_{1}^{2}, Q_{2}^{2}, M^{2}\right)=\frac{\pi \alpha_{\mathrm{em}}^{2}}{3 M} F_{\mathrm{LT}}^{2}\left(Q_{1}^{2}, Q_{2}^{2}\right) \\
\tilde{\Gamma}(A)=\lim _{Q^{2} \rightarrow 0} \frac{M^{2}}{Q^{2}} \Gamma_{\gamma^{*} \gamma^{*}}^{\mathrm{LT}}\left(Q^{2}, 0, M^{2}\right)=\frac{\pi \alpha_{\mathrm{em}}^{2} M}{3} f_{\mathrm{LT}}^{2}, \text { with } f_{\mathrm{LT}}=\lim _{Q^{2} \rightarrow 0} \frac{F_{\mathrm{LT}}\left(Q^{2}, 0\right)}{Q},
\end{gathered}
$$

which provides a useful measure of size of the relevant $e^{+} e^{-}$cross section in the $\gamma \gamma$ mode. For a $c \bar{c}$ state:

$$
f_{\mathrm{LT}}=-e_{f}^{2} M^{2} \frac{\sqrt{3 N_{c}}}{8 \pi} \int_{0}^{\infty} \frac{d k k^{2} u(k)}{\left(k^{2}+m_{Q}^{2}\right)^{2}} \frac{1}{\sqrt{M_{Q \bar{Q}}}}\left\{\frac{2}{\beta^{2}}-\frac{1-\beta^{2}}{\beta^{3}} \log \left(\frac{1+\beta}{1-\beta}\right)\right\}
$$

with

$$
\beta=\frac{k}{\sqrt{k^{2}+m_{Q}^{2}}}, \quad M_{Q \bar{Q}}=2 \sqrt{k^{2}+m_{Q}^{2}}
$$

## Nonrelativistic limit

- Simple explicit expressions in terms of derivative of WF at the origin can be found in the nonrelativistic limit (Schuler, Berends, van Gulik '98):

$$
\begin{aligned}
F_{\mathrm{TT}}\left(Q_{1}^{2}, Q_{2}^{2}\right) & =2 e_{f}^{2} \sqrt{\frac{6 N_{c}}{\pi M^{3}}} R^{\prime}(0) \frac{Q_{1}^{2}-Q_{2}^{2}}{\nu} \\
F_{\mathrm{LT}}\left(Q_{1}^{2}, Q_{2}^{2}\right) & =-2 e_{f}^{2} \sqrt{\frac{6 N_{c}}{\pi M}} R^{\prime}(0) \frac{\left(\nu+Q_{2}^{2}\right) Q_{1}}{\nu^{2}} \\
F_{\mathrm{TL}}\left(Q_{1}^{2}, Q_{2}^{2}\right) & =2 e_{f}^{2} \sqrt{\frac{6 N_{c}}{\pi M}} R^{\prime}(0) \frac{\left(\nu+Q_{1}^{2}\right) Q_{2}}{\nu^{2}}
\end{aligned}
$$

with $\nu=\left(M^{2}+Q_{1}^{2}+Q_{2}^{2}\right) / 2$.

- reduced width:

$$
\tilde{\Gamma}(A)=\frac{2 \alpha_{\mathrm{em}}^{2} e_{f}^{4} N_{c}}{m_{Q}^{4}}\left|R^{\prime}(0)\right|^{2}
$$

- relativistic corrections in a light front approach in I.Babiarz, R. Pasechnik, WS, A. Szczurek, hep-ph/ 2208.05377.
- for one virtual \& one real photon one obtains

$$
F_{\mathrm{TT}}\left(Q^{2}, 0\right)=-\frac{Q}{M} F_{\mathrm{LT}}\left(Q^{2}, 0\right)
$$

## $\gamma^{*} \gamma^{*}$-transition form factors for $\chi_{c 1}(1 P)$ axial mesons



Figure: Form factors $F_{\mathrm{LT}}\left(Q^{2}, 0\right)$ for one virtual photon (left and middle panels) and $F_{\mathrm{LT}}\left(Q^{2}, 0\right) / Q$ (right panel). The top panels: our results in the LFWF approach and the bottom panels: nonrelativistic timit.

## $Q^{2}$-dependence of the $\gamma^{*} \gamma$ cross section




Figure: The square of the effective form factor as a function of photon virtuality within LFWF approach (on the I.h.s.) and in the nonrelativistic limit (on the r.h.s.).

$$
\begin{aligned}
\sigma_{\mathrm{tot}}^{\gamma^{*} \gamma}\left(Q^{2}, 0\right) & =16 \pi^{3} \alpha_{\mathrm{em}}^{2} \delta\left(\hat{s}-M^{2}\right) \frac{Q^{2}}{Q^{2}+M^{2}}\left(1+\frac{Q^{2}}{2 M^{2}}\right)\left(\frac{F_{\mathrm{LT}}\left(Q^{2}, 0\right)}{Q}\right)^{2} \\
& \equiv 16 \pi^{3} \alpha_{\mathrm{em}}^{2} \delta\left(\hat{s}-M^{2}\right) \mathrm{F}_{\mathrm{eff}}^{2}\left(Q^{2}\right)
\end{aligned}
$$

## (Rough) Estimates for EIC energies

- for $Q^{2} \ll 2 M^{2}$ longitudinal photons will dominate. Note, that the total cross section does not have the $d Q^{2} / Q^{2}$ logarithmic integral.

$$
Q^{2} \frac{d \sigma\left(e A \rightarrow e^{\prime} X(3872) A\right)}{d Q^{2}}=\frac{\alpha_{\mathrm{em}}}{\pi} \int_{y_{\min }}^{1} \frac{d y}{y} \frac{d x}{x} f_{L}(y) n_{\gamma / A}(x) \delta\left(x y s-M^{2}\right) 16 \pi^{3} \alpha_{\mathrm{em}}^{2} \mathrm{~F}_{\mathrm{eff}}^{2}\left(Q^{2}\right)
$$

- We assume the $Q^{2}$-dependence of a $c \bar{c}$-state, and a reduced width of $\tilde{\Gamma}=0.5 \mathrm{keV}$.
- limit on reduced width from Belle (updated, Achasov, 2022): $24 \mathrm{eV}<\tilde{\Gamma}\left(\chi_{c 1}(3872)\right)<615 \mathrm{eV}$.

Table: Cross sections on proton and ${ }^{208} \mathrm{~Pb}$

| $\sqrt{s_{e N}}[\mathrm{GeV}]$ | $\sigma(e p \rightarrow e p X)[\mathrm{pb}]$ | $\sigma(e A \rightarrow e A X)[\mathrm{pb}]$ |
| :--- | :---: | :---: |
| 50 | 0.06 | 60 |
| 140 | 0.16 | 340 |

- Hadronic contribution?


## Possible molecule contribution to $\tilde{\text { I }}$ ?



- apparently nothing (?) is known about the molecular contribution to the reduced width.
- What about the analogous contribution to the one we adopted in the hadronic case? Say $\gamma^{*} \gamma \rightarrow c \bar{c} \rightarrow \bar{D} D^{*}$, and FSI of $D \bar{D}^{*}$ generates the $X(3872)$.
- Spins of heavy quarks in $X$ (3872) are entangled to be in the spin-triplet state (M. Voloshin, 2004). But near threshold the $c \bar{c}$ state produced via $\gamma \gamma$-fusion is in the ${ }^{1} S_{0}$ state. (It's different for gluons, where color octet populates ${ }^{3} S_{1}$ !)
- $\rightarrow$ "handbag mechanism" suppressed in heavy quark limit.
- Purely hadronic models?


## Fully heavy tetraquark $T_{\psi \psi}(6900)$



Figure: Two dominant reaction mechanisms of production of $c \bar{c} c \bar{c}$ nonresonant continuum. The left diagram represent the SPS mechanism (box type) and the left diagram the DPS mechanism.

$$
d \sigma_{p p \rightarrow c \bar{c} c \bar{c} x}=\int d x_{1} \frac{d^{2} k_{1 t}}{\pi k_{1 t}^{2}} d x_{2} \frac{d^{2} k_{2 t}}{\pi k_{2 t}^{2}} \mathcal{F}_{g}\left(x_{1}, k_{1 t}^{2}, \mu^{2}\right) \mathcal{F}_{g}\left(x_{2}, k_{2 t}^{2}, \mu^{2}\right) d \hat{\sigma}_{g^{*} g^{*} \rightarrow c \bar{c} c \bar{c}}
$$

- Calculations performed using the code KaTie by A. van Hameren.


## $T_{4 c}(6900)$ single vs. double parton scattering



Figure: Distribution of $p_{t, 4 c}$ of four quark-antiquark system within invariant mass window $\left(M_{R}-0.1 \mathrm{GeV}, \mathrm{M}_{\mathrm{R}}+0.1 \mathrm{GeV}\right)$. Here $\sqrt{s}=13 \mathrm{TeV}$ and average rapidity of quarks and antiquarks in the interval $(2,4.5)$. The solid line is for SPS, the dashed line for $g g g g \rightarrow c \bar{c} c \bar{c}$ DPS contribution.

$$
\frac{d \sigma_{T_{4 c}}}{d^{3} \vec{P}_{T_{4 c}}}=F_{T_{4 c}} \int_{M_{T_{4 c}}-\Delta M}^{M_{T_{4 c}}+\Delta M} d^{3} \vec{P}_{4 c} d M_{4 c} \frac{d \sigma_{c \bar{c} c \bar{c}}}{d M_{4 c} d^{3} \vec{P}_{4 c}} \delta^{3}\left(\vec{P}_{T_{4 c}}-\frac{\left.M_{T_{4 c}} \vec{P}_{4 c}\right)}{M_{4 c}}\right)
$$

## Gluon-gluon fusion "color singlet model"



Figure: The mechanism of gluon-gluon fusion leading to the production of the $T_{4 c}(6900)$ tetraquark.

The off-shell gluon fusion cross sections is proportional to a form-factor, which depends on the virtualities of gluons, $Q_{i}^{2}=-k_{i}^{2}$ :

$$
\begin{aligned}
d \sigma_{g^{*} g^{*} \rightarrow 0^{-}} & \propto \frac{1}{k_{1 t}^{2} k_{2 t}^{2}}\left(\vec{k}_{1 t} \times \vec{k}_{2 t}\right)^{2} F^{2}\left(Q_{1}^{2}, Q_{2}^{2}\right) \\
d \sigma_{g^{*} g^{*} \rightarrow 0^{+}} & \propto \frac{1}{k_{1 t}^{2} k_{2 t}^{2}}\left(\left(\vec{k}_{1 t} \cdot \vec{k}_{2 t}\right)\left(M^{2}+Q_{1}^{2}+Q_{2}^{2}\right)+2 Q_{1}^{2} Q_{2}^{2}\right)^{2} \frac{F^{2}\left(Q_{1}^{2}, Q_{2}^{2}\right)}{4 X^{2}}
\end{aligned}
$$

with $X=\left(M^{4}+2\left(Q_{1}^{2}+Q_{2}^{2}\right) M^{2}+\left(Q_{1}^{2}-Q_{2}^{2}\right)^{2}\right) / 4$.

## Production of pseudoscalar or scalar



Figure: Transverse momentum distribution of the $T_{4 c}(6900)$ tetraquark for the $0^{-}$(left panel) and $0^{+}$(right panel) assignments. Here $\sqrt{s}=13 \mathrm{TeV}$. We show results for the KMR UGDF and $\Lambda=6 \mathrm{GeV}$ (solid line) and $\Lambda$ $=4 \mathrm{GeV}$ (dashed line).

$$
F\left(Q_{1}^{2}, Q_{2}^{2}\right)=\frac{\Lambda^{2}}{\Lambda^{2}+Q_{1}^{2}+Q_{2}^{2}},
$$

- Since the ratio of signal-to-background improves with transverse momentum of the tetraquark and knowing relatively well the behaviour of the SPS and DPS background we can conclude that the $0^{-}$assignment is disfavoured by the LHCb experimental results.


## Possibilities of photoproduction of $T_{4 c}(6900)$ in UPCs

- Production of $J / \psi J / \psi$ pairs in $\gamma \gamma$ collisions.

- Cross section in UPCs (lead-lead) at LHC (Baranov, Cisek, Kłusek-Gawenda, WS, Szczurek (2013)):


- Predictions for $T_{\psi \psi}$ in UPCs from the literature:

Goncalves \& Moreira (2021): $\sigma_{\text {tot }}=170 \mathrm{nb}\left(0^{+}\right), 206\left(2^{+}\right) \mathrm{nb}$, incl. branching to $\mathrm{J} / \psi \mathrm{J} / \psi$.
Esposito et al. (2021): $\sigma_{\text {tot }}=(282$ or 1165 $) \times B R(J / \psi J / \psi) \mathrm{nb}$ for $J^{P}=0^{+}, 2^{+}$.

## Possibilities of photoproduction of $T_{4 c}(6900)$ in UPCs

- Here the cross section is dominated by transverse photons.
- $Q^{2}$ dependence and correlation of electron/hadronic plane could be used to investigate different spin-parity assignments.
- We estimate photoproduction cross section with $Q_{\max }^{2}=0.1 \mathrm{GeV}$.
- No consensus on even the ballpark of the $\gamma \gamma$ width of $T_{4 c}(6900)$ in the literature.
- We use $\Gamma=2 \mathrm{keV}$

Table: Cross sections on proton and ${ }^{208} \mathrm{~Pb}$

| $\sqrt{s_{e N}}[\mathrm{GeV}]$ | $\sigma\left(e p \rightarrow e p T_{4 c}\right)[\mathrm{pb}]$ | $\sigma\left(e A \rightarrow e A T_{4 c}\right)[\mathrm{pb}]$ |
| :--- | :---: | :---: |
| 50 | 0.06 | 21.3 |
| 140 | 0.256 | 360 |

## Summary

- We considered prompt hadroproduction of $\chi_{c 1}(3872)$ at LHC energies for a $c \bar{c}$ state, a molecule, or a mixture of both.
- The molecule production has the hardest behaviour as a function of $p_{T}$. This is expected from to the color-octet contribution. Shape of molecule alone does not agree well with data.
- Production of $c \bar{c}$ state gives reasonable behaviour, as does a mixture of $c \bar{c}$ and molecule.
- Electroproduction of $\chi_{c 1}(3872)$ in the Coulomb field of a heavy nucleus may give access to form factor $F_{\text {LT }}\left(Q^{2}, 0\right)$. This is additional information on the structure. We know how to calculate it for $c \bar{c}$, or possibly tetraquark states.
- What about the molecule? Can one calculate its reduced width to $\gamma_{L}^{*} \gamma$ ?
- Beyond exotics: these quantities haven't been measured even for $\chi_{c 1}(1 P)$.
- Production of fully heavy tetraquark potentially has large double parton scattering contribution.
- These complications make a clean environment such as $\gamma \gamma$ fusion in UPC very desirable.

