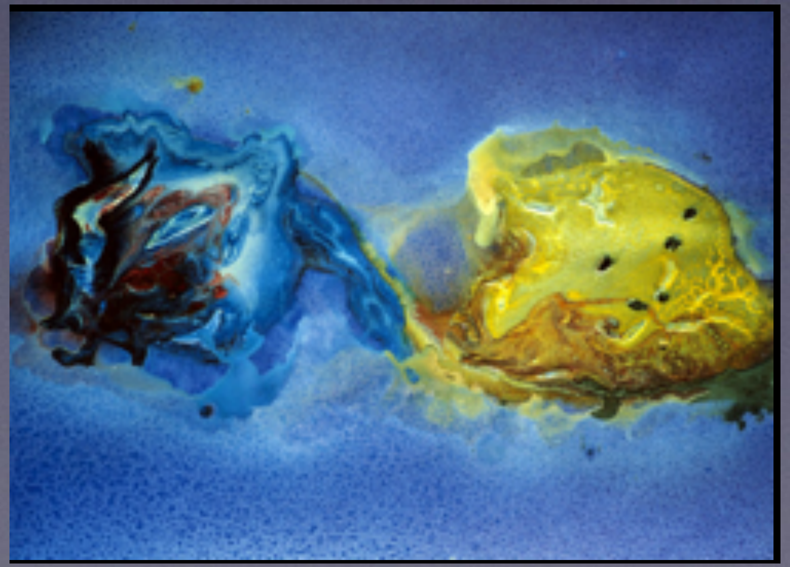
Nonrelativistic Effective Fields Theories for X Y Z states



Physik Department TUM
T30f

Nora Brambilla

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- We have now the tools to describe quarkonium in Quantum Field Theory — Nonrelativistic Effective Quantum Field Theories (NREFT) plus Lattice ; —which Is what renders quarkonium a golden probe of strong interactions

- I will show how we can address X Y Z states on the basis of an EFT called BOEFT and some lattice input i.e. directly in QCD

Plan of the talk

Quarkonium: multiscale system -> hierarchy of scales/hierarchy of NREFTs based on factorization which makes apparent symmetries hidden in QCD and increase model independent predictivity

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The same framework can be used to describe production (see Wang's talk) andX Y Z evolution in medium -in heavy ions ion the basis of BOEFT and open quantum system

Material for discussion/references

Heavy quarkonium: progress, puzzles, and opportunities
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Published in **Eur.Phys.J. C71 (2011) 1534**
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Rev.Mod.Phys. 77 (2005) 1423
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Nora Brambilla , Gastão Krein, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D97 (2018) no.1, 016016
e-Print: **arXiv:1707.09647**

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QCD spin effects in the heavy hybrid potentials and spectra Nora Brambilla, Wai Kin Lai, J. Segovia, J. Tarrus *Phys.Rev.D* 101 (2020) 5, 054040 • e-Print: 1908.11699

Effective Field Theories and Lattice QCD for the X Y Z frontier , N. Brambilla, PosLattice 2021

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Nonrelativistic effective field theory for heavy exotic hadrons
J. Soto, J. Tarrus Published in: *Phys.Rev.D* 102 (2020) 1, 014012 •

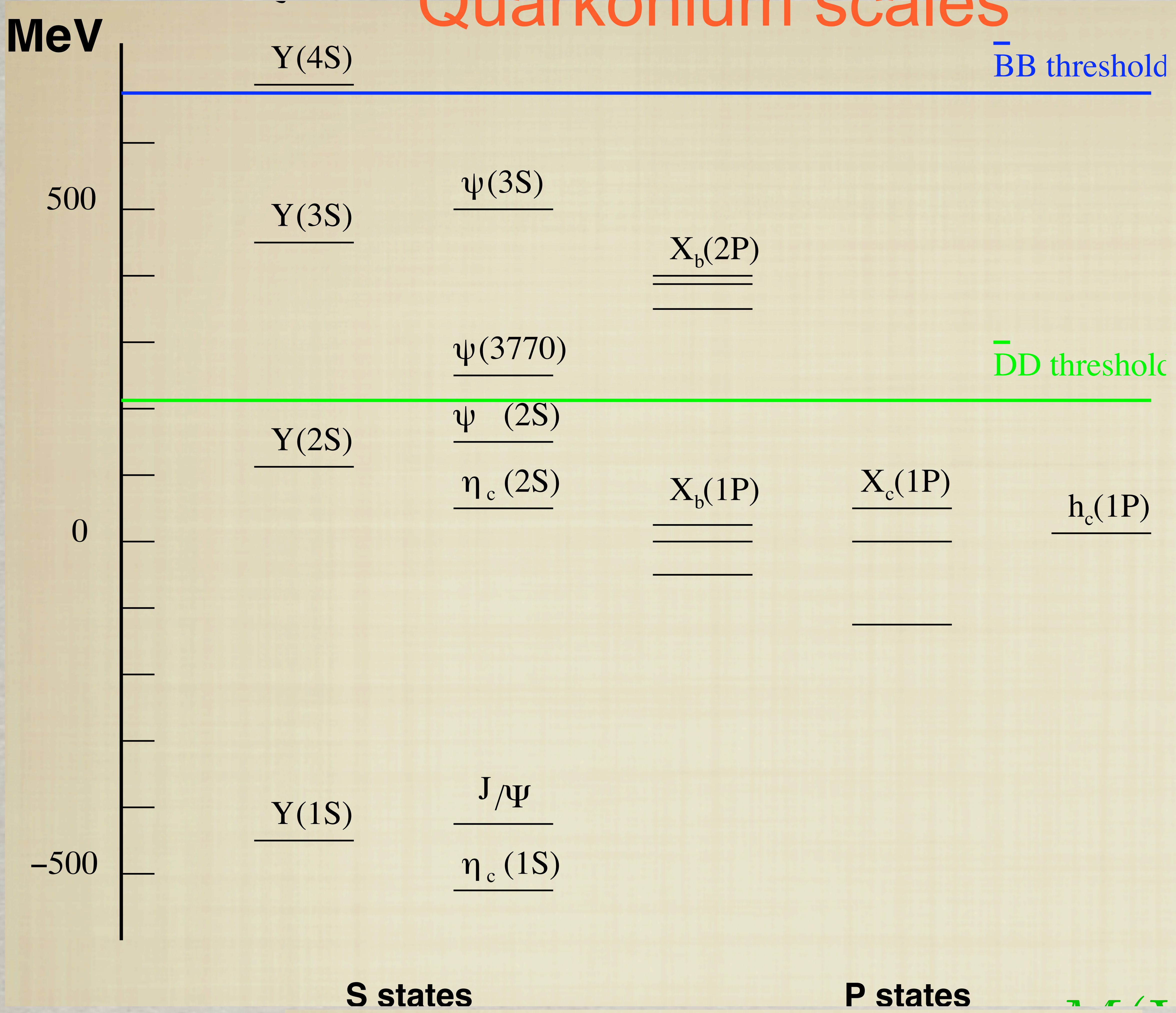
Oncala and Soto
Heavy hybrids: spectrum, decay and mixing
Phys.Rev.D 96 (2017) 1, 014004 •

J. Tarrus, Heavy mesons thresholds in BOEFT, 2207.09365

Systems with two heavy quarks: physical scales and physical significance

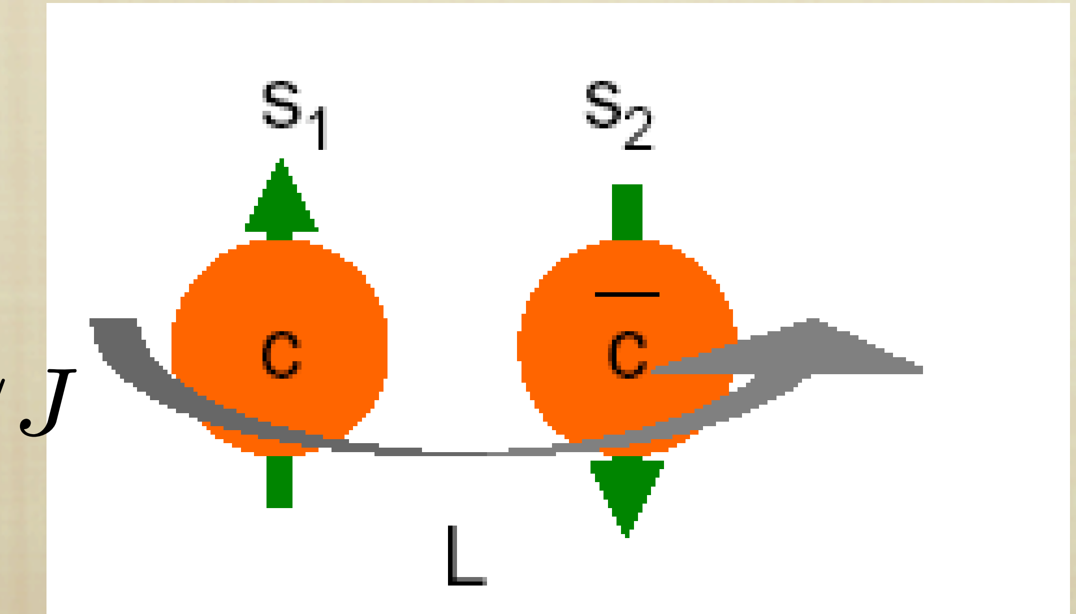
consider QQbar (quarkonium) but things are similar for QQ, QQQ etc

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$

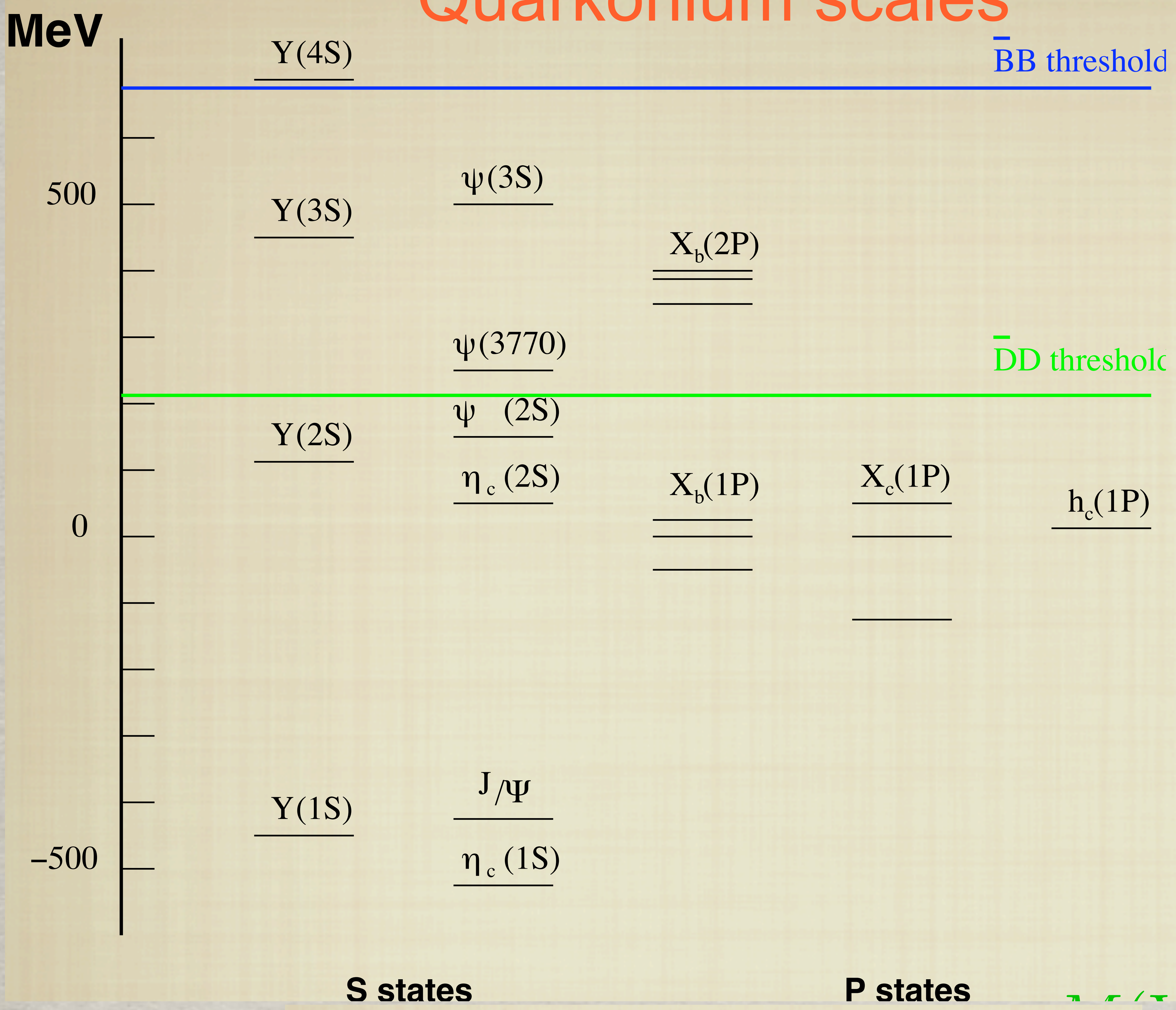


THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV};\, m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium scales



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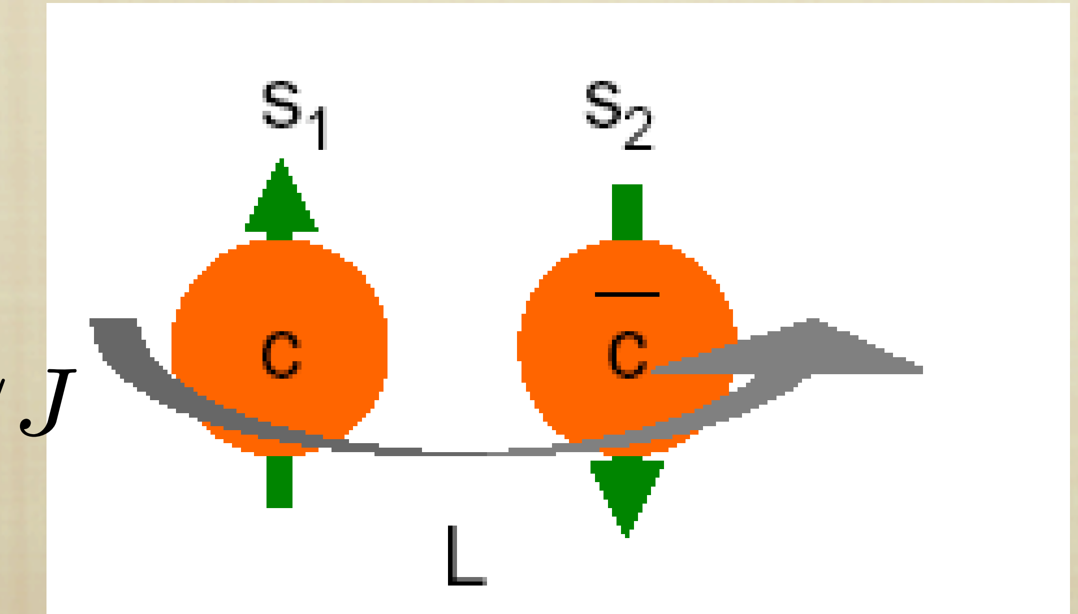

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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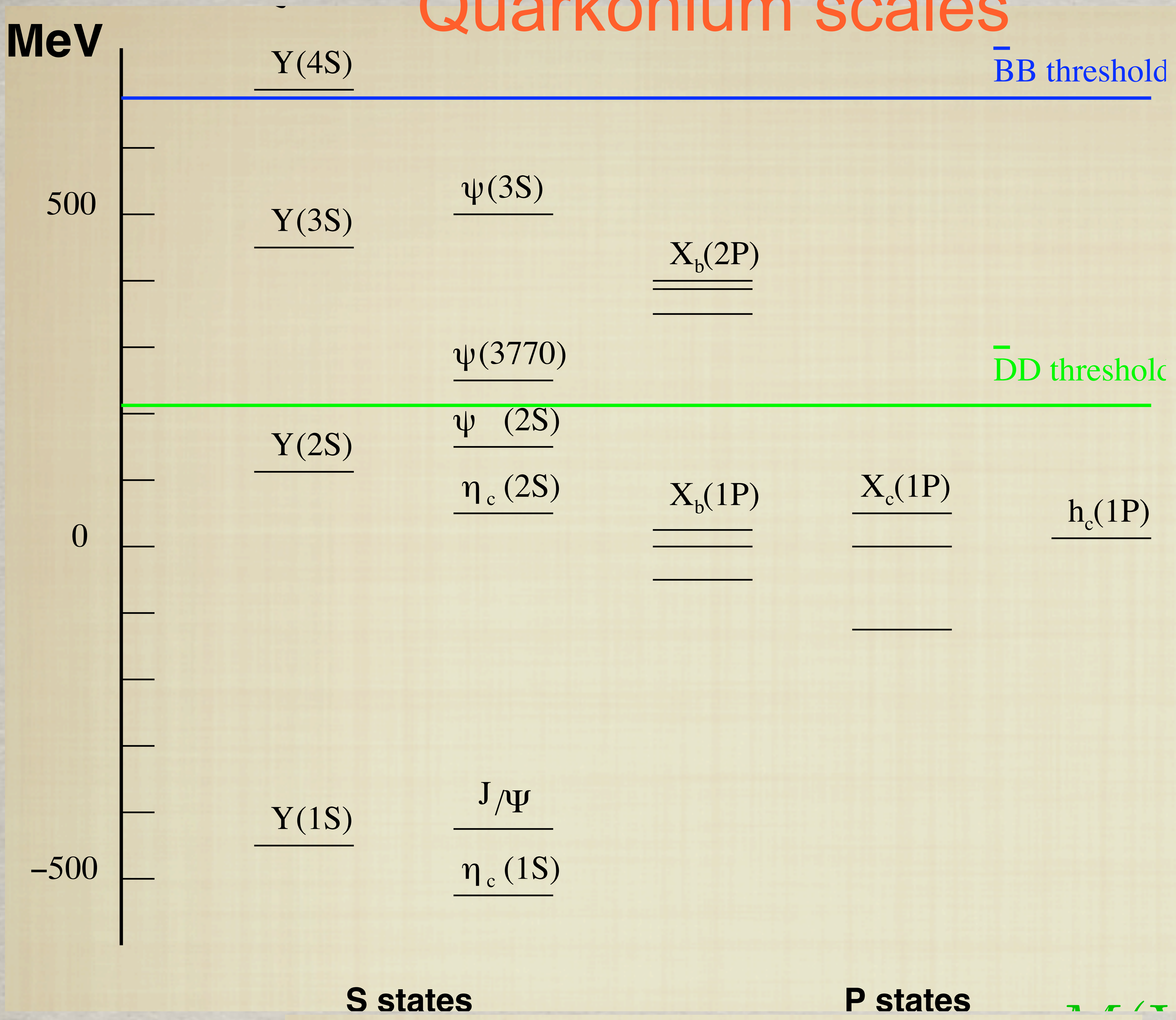

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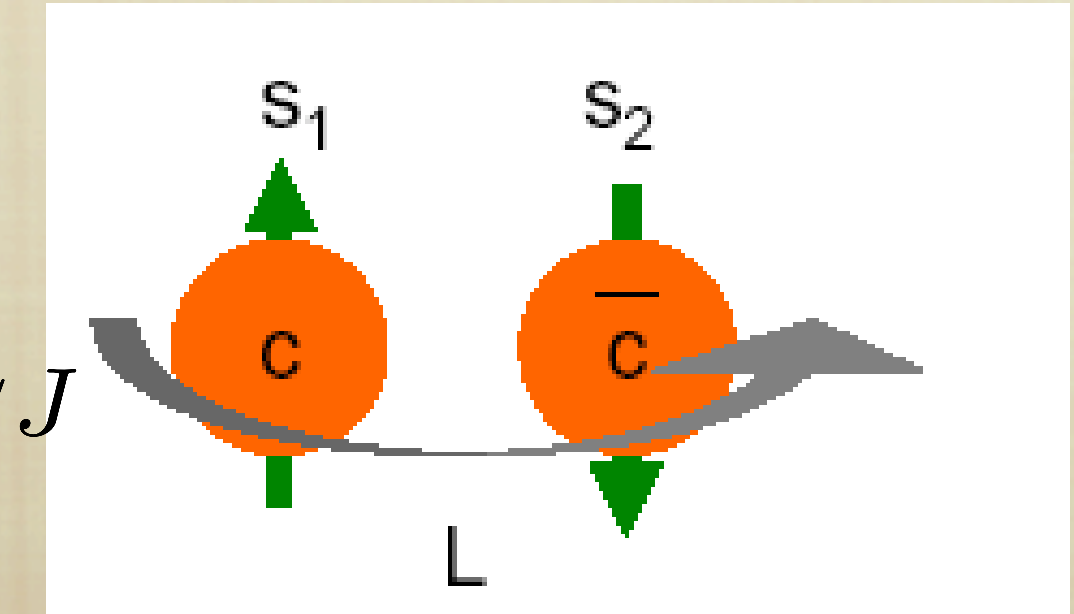

NR BOUND STATES HAVE AT LEAST 3 SCALES

$$m \gg mv \gg mv^2 \quad v \ll 1$$

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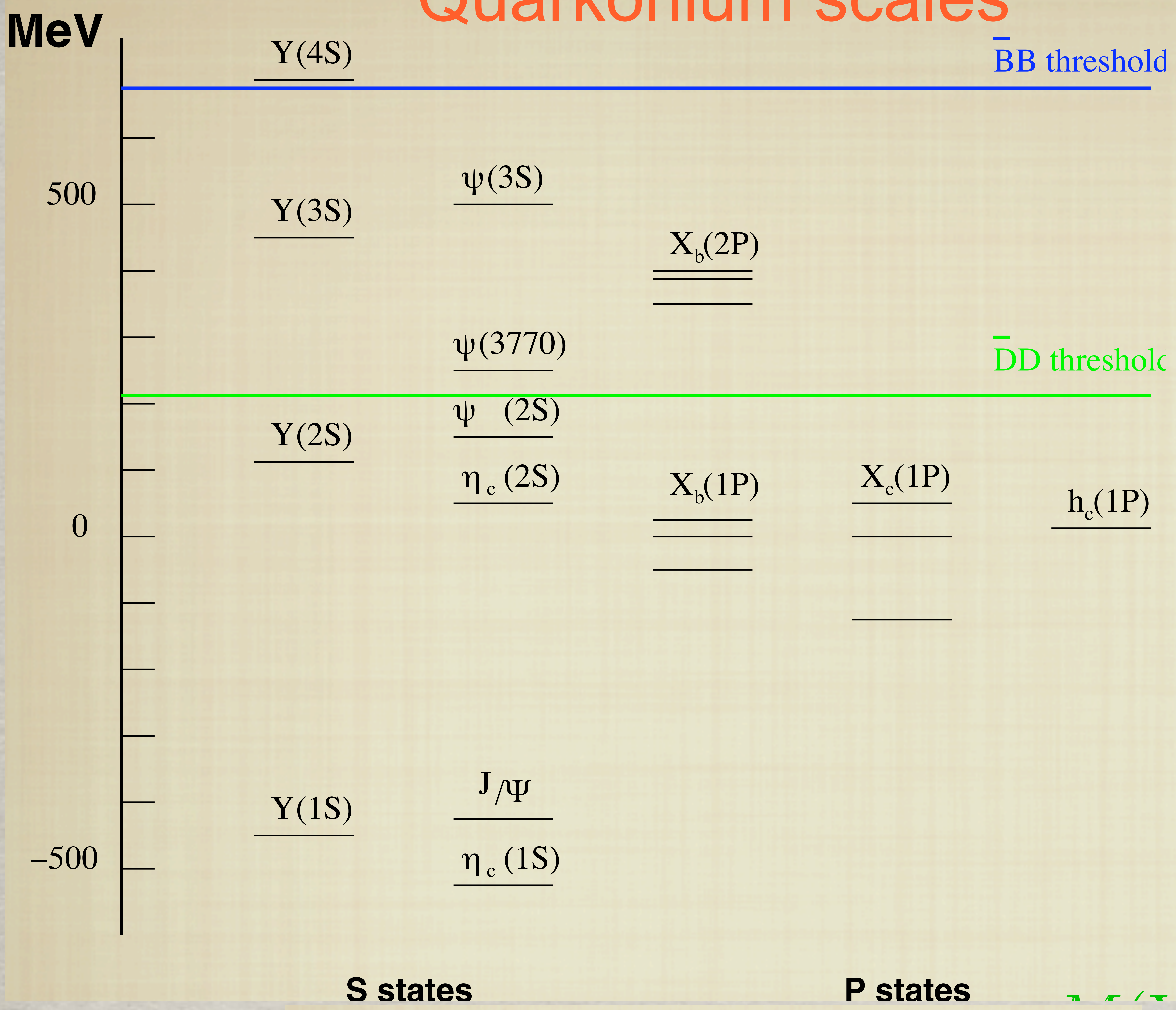

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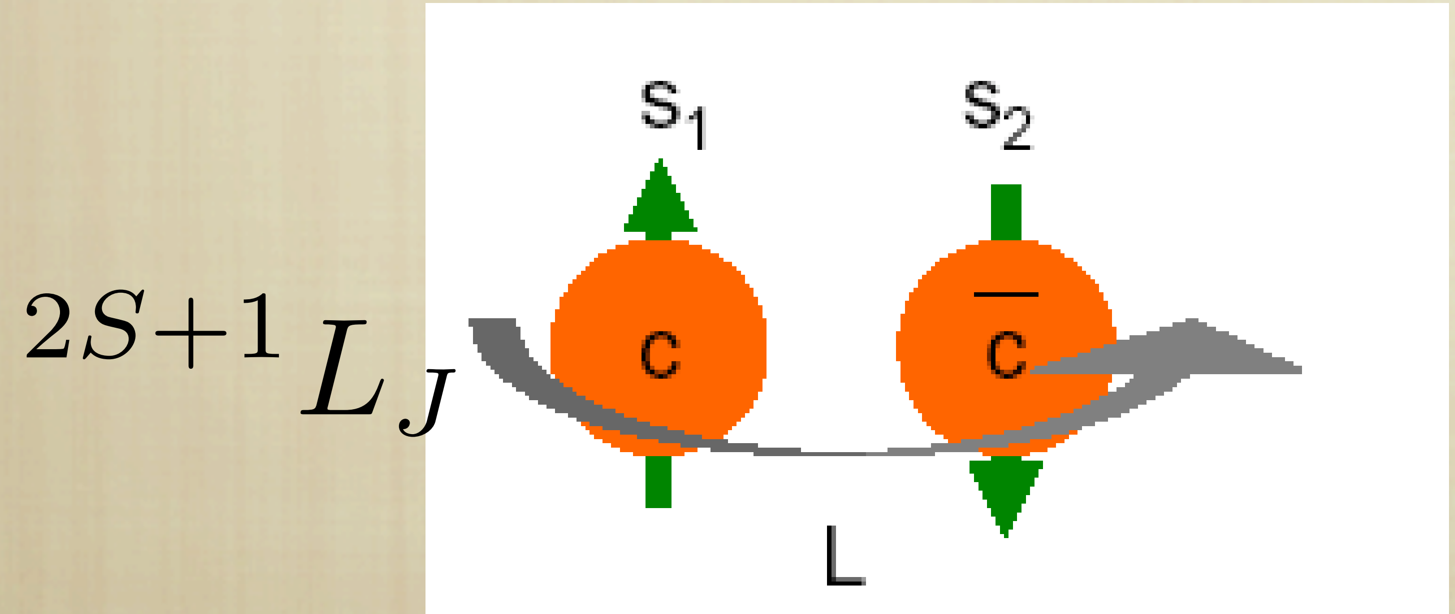

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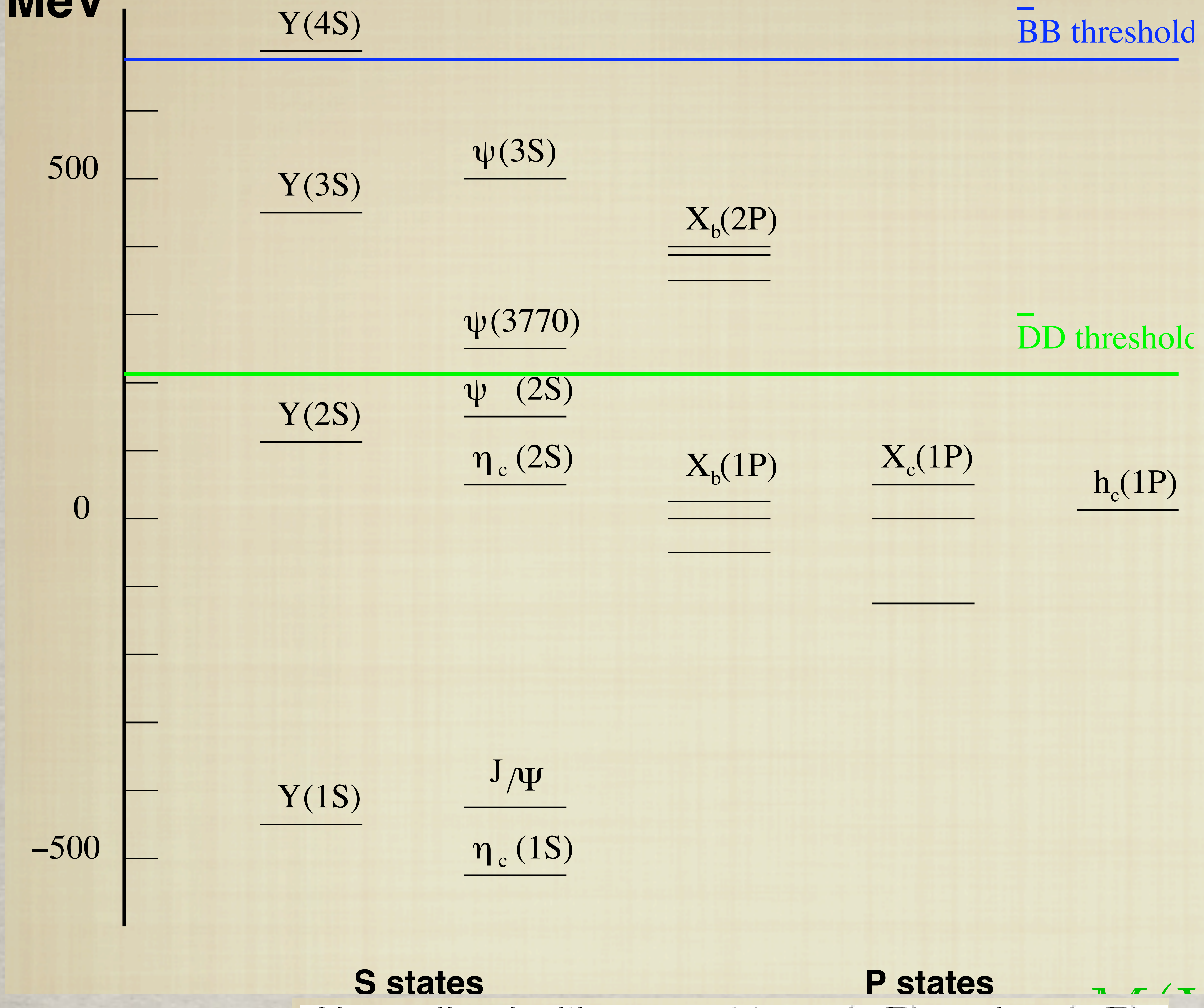

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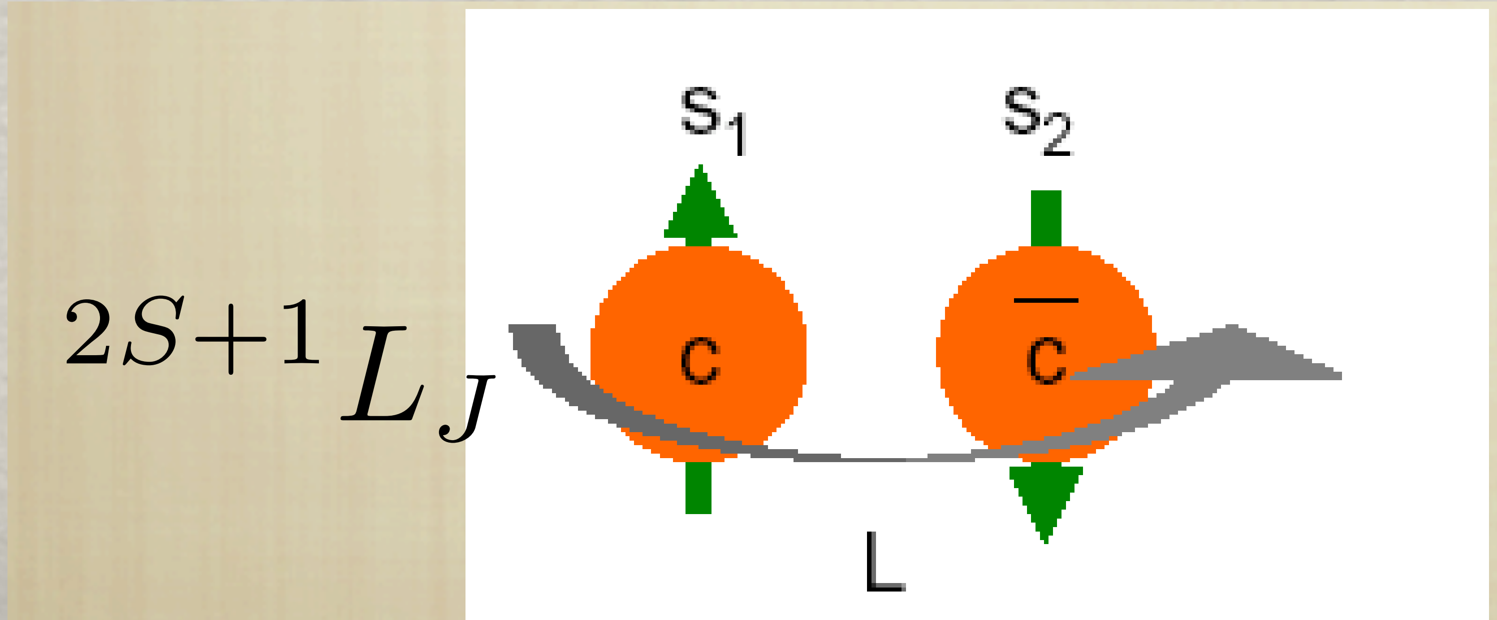

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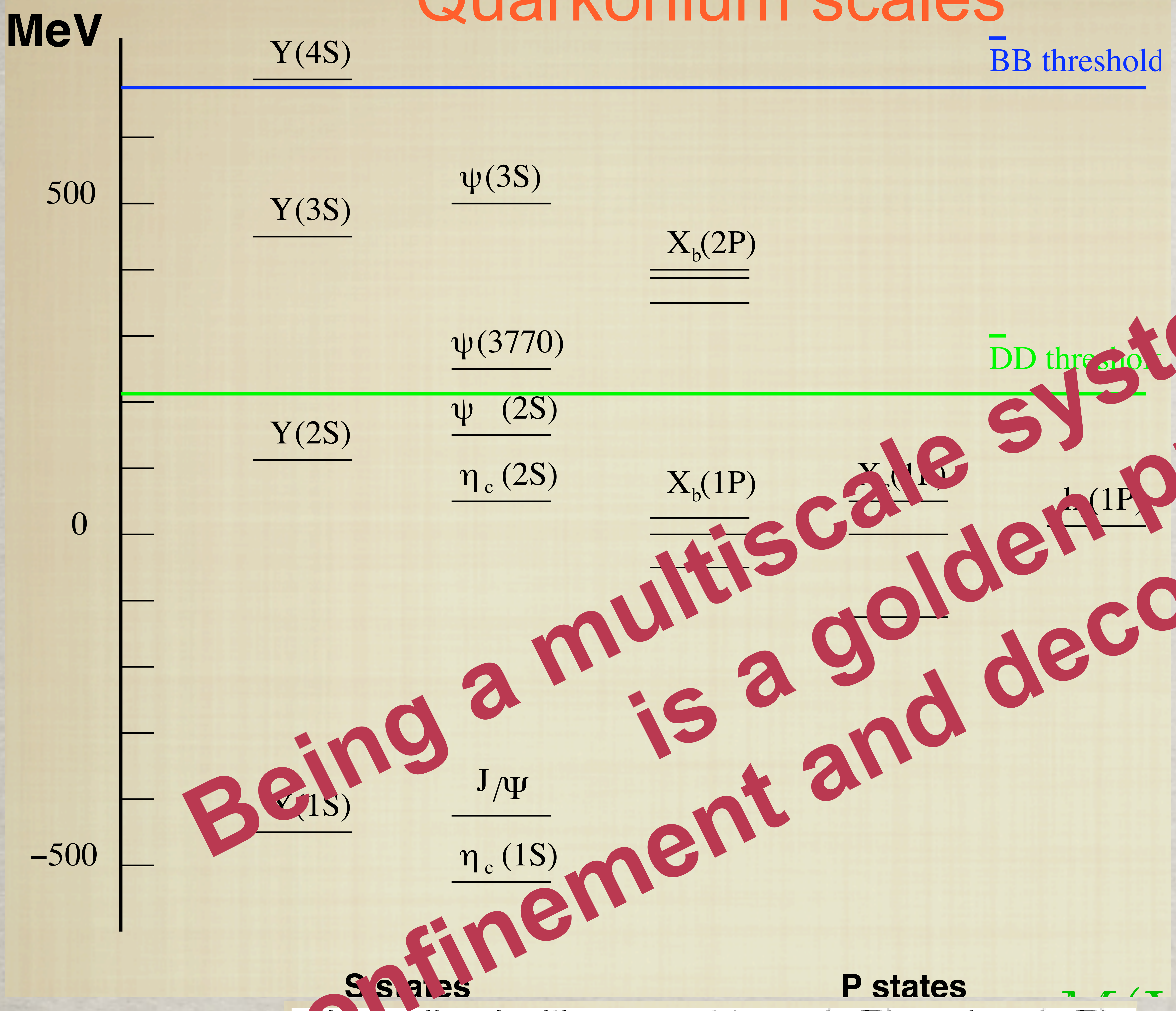

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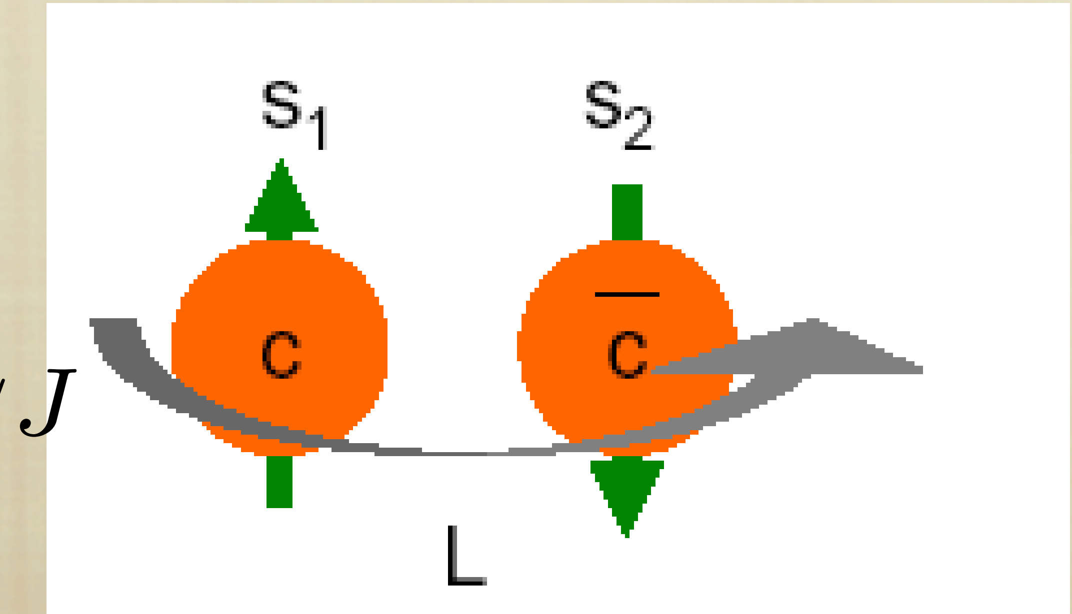

Being a multiscale system quarkonium is a golden probe of confinement and deconfinement physics

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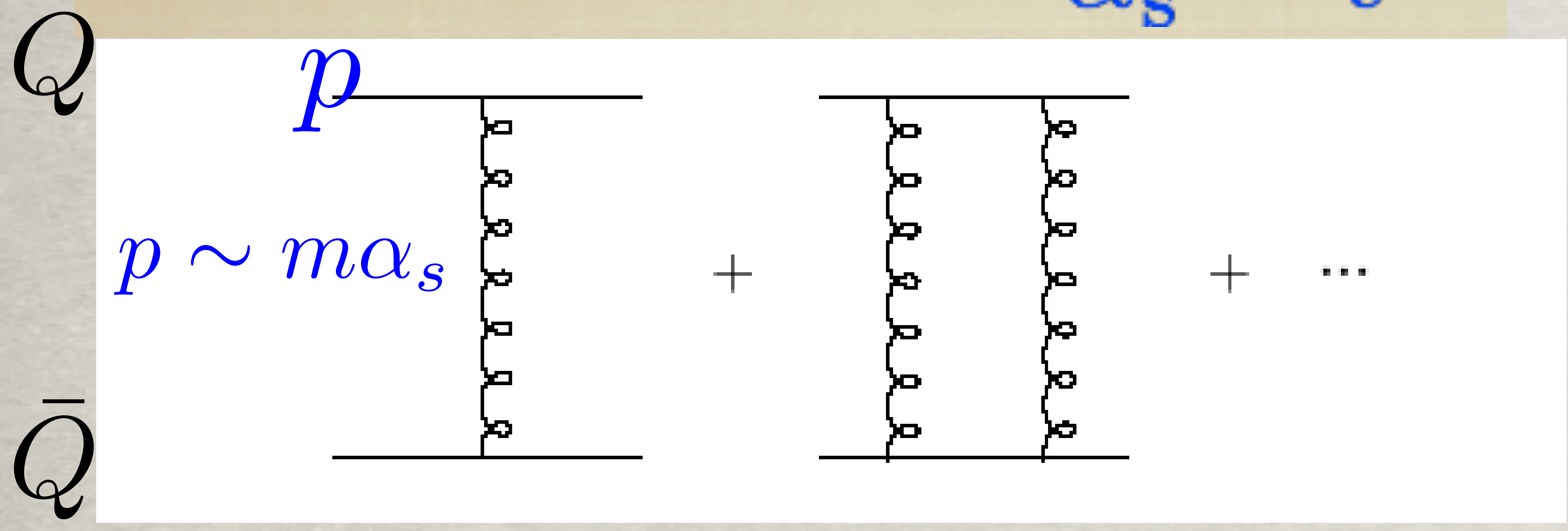

QCD theory of quarkonium: a very challenging problem

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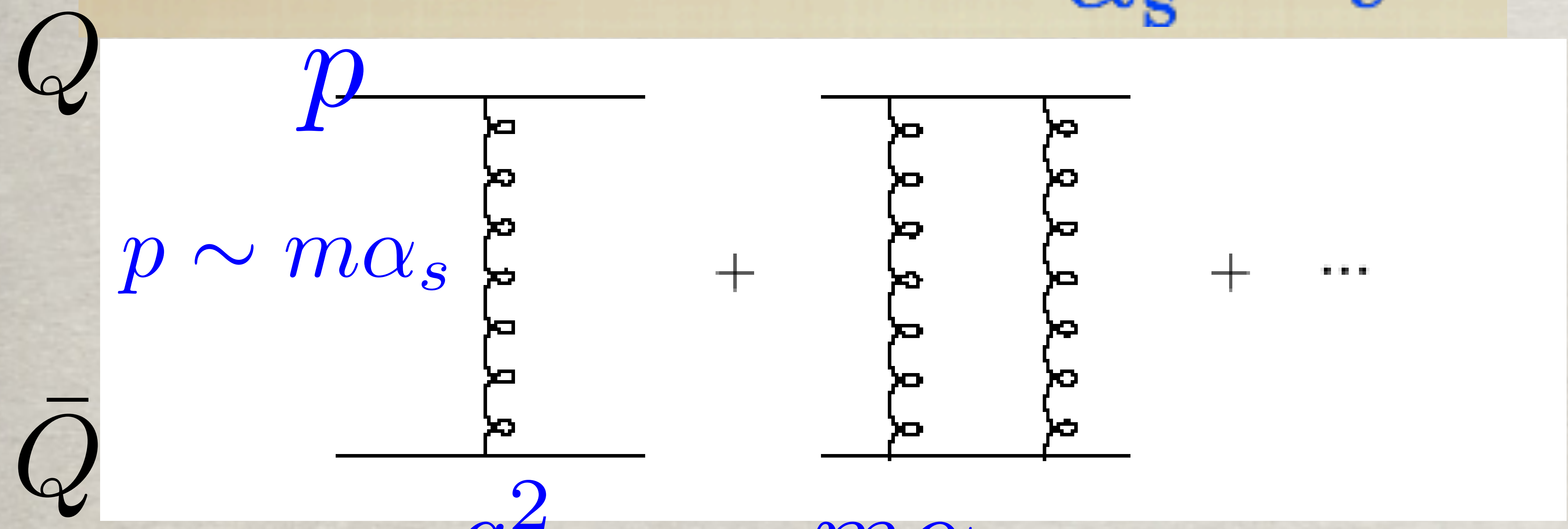

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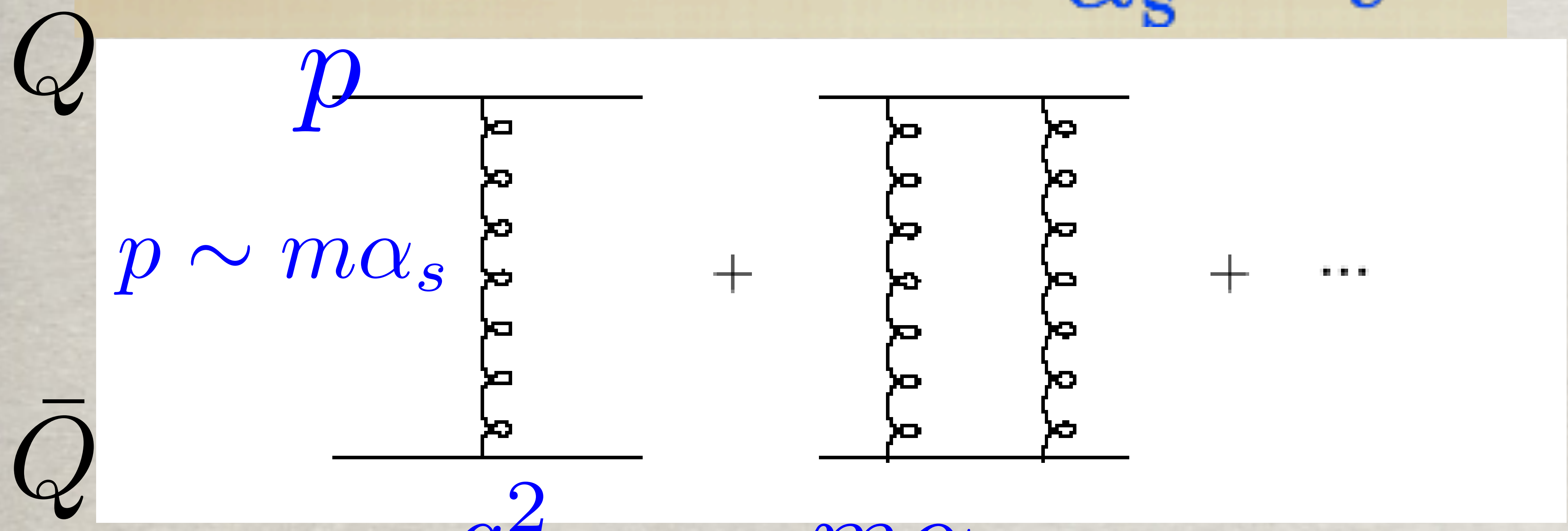

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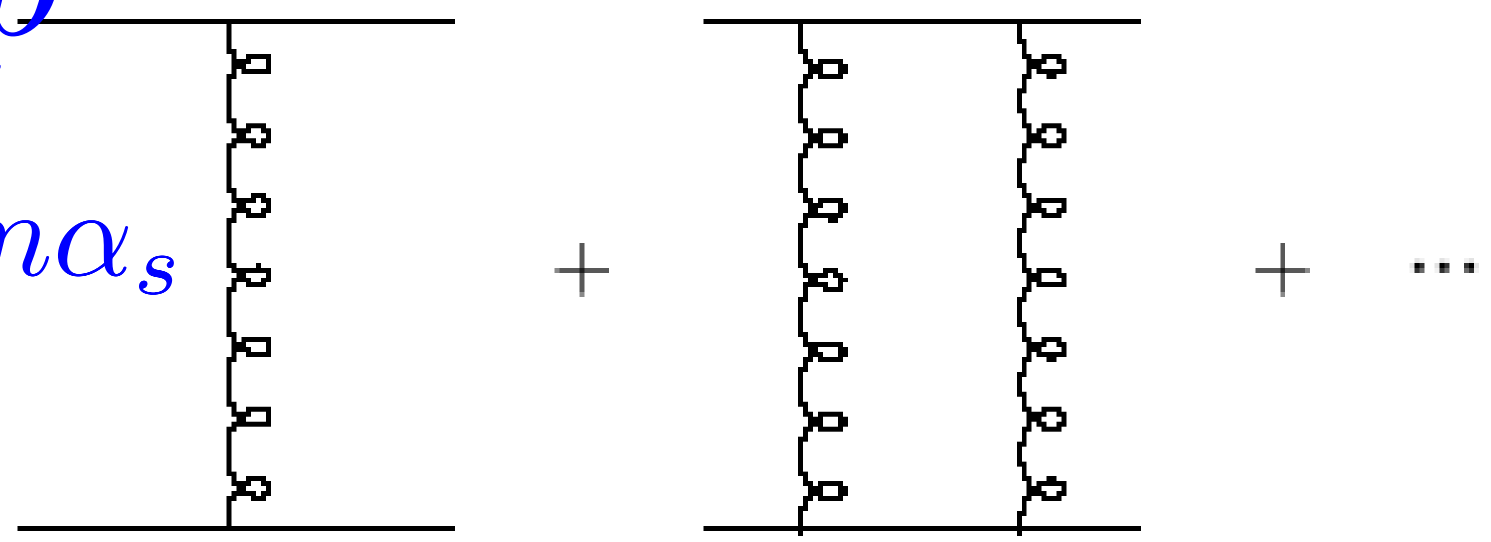
Q p

$p \sim m\alpha_s$



$\bar{Q}$

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- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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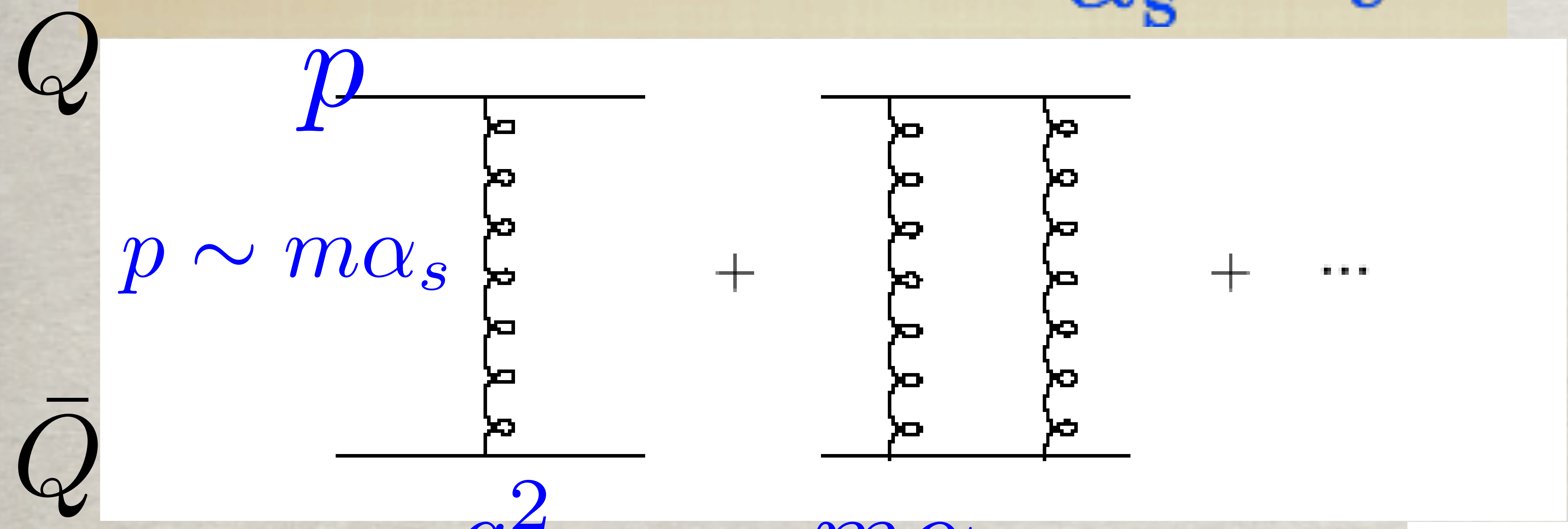

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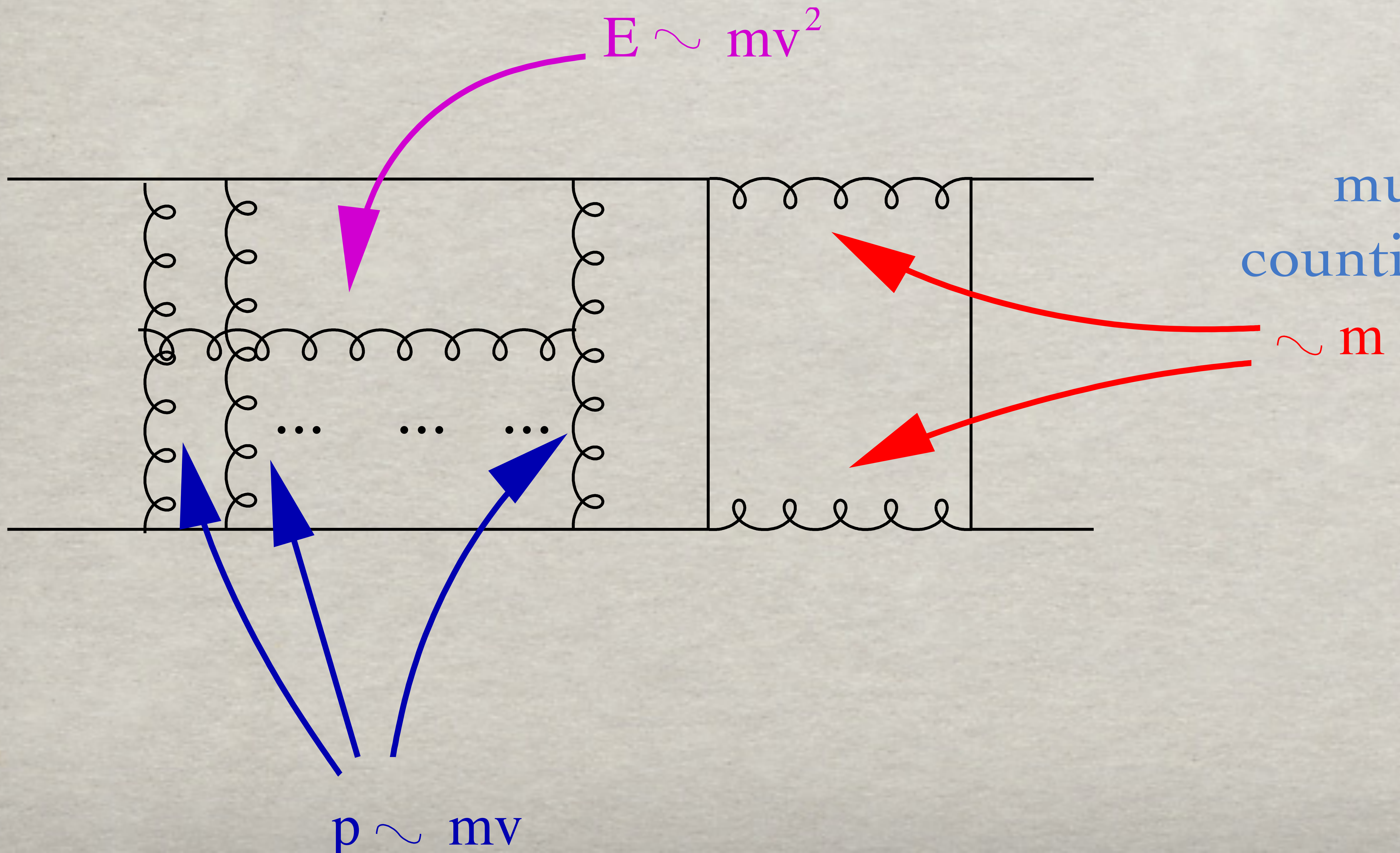

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

QCD theory of quarkonium: a very challenging problem

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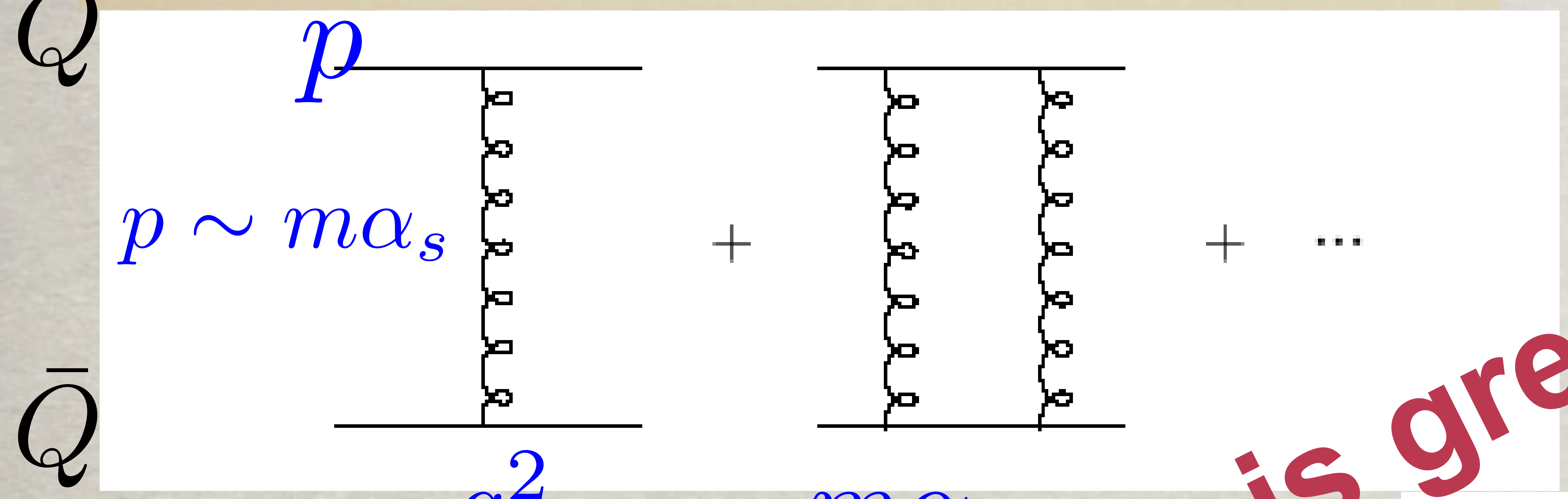
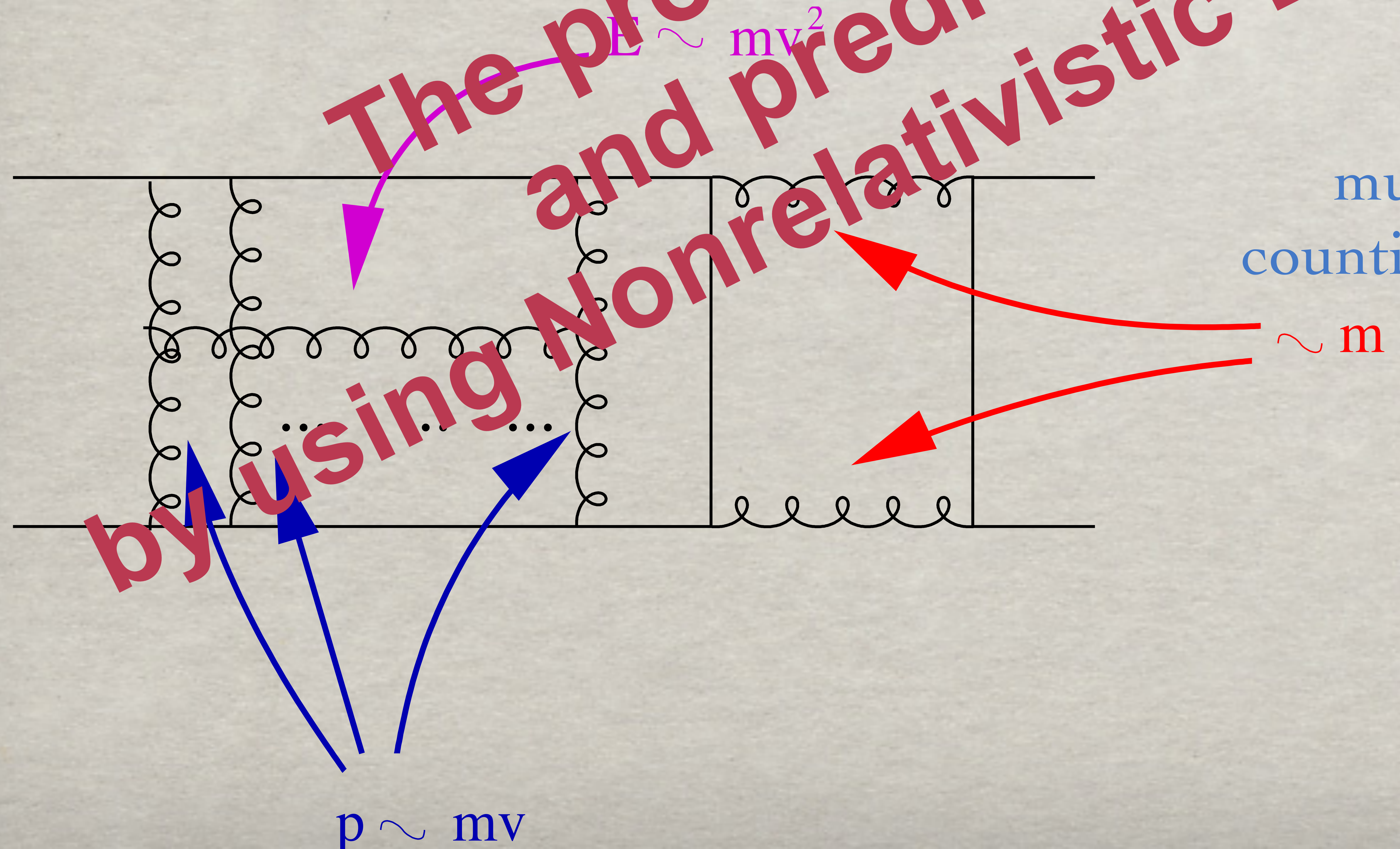


$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p} + \cdots\right) \sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

From $\left(\frac{p^2}{m} + V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories



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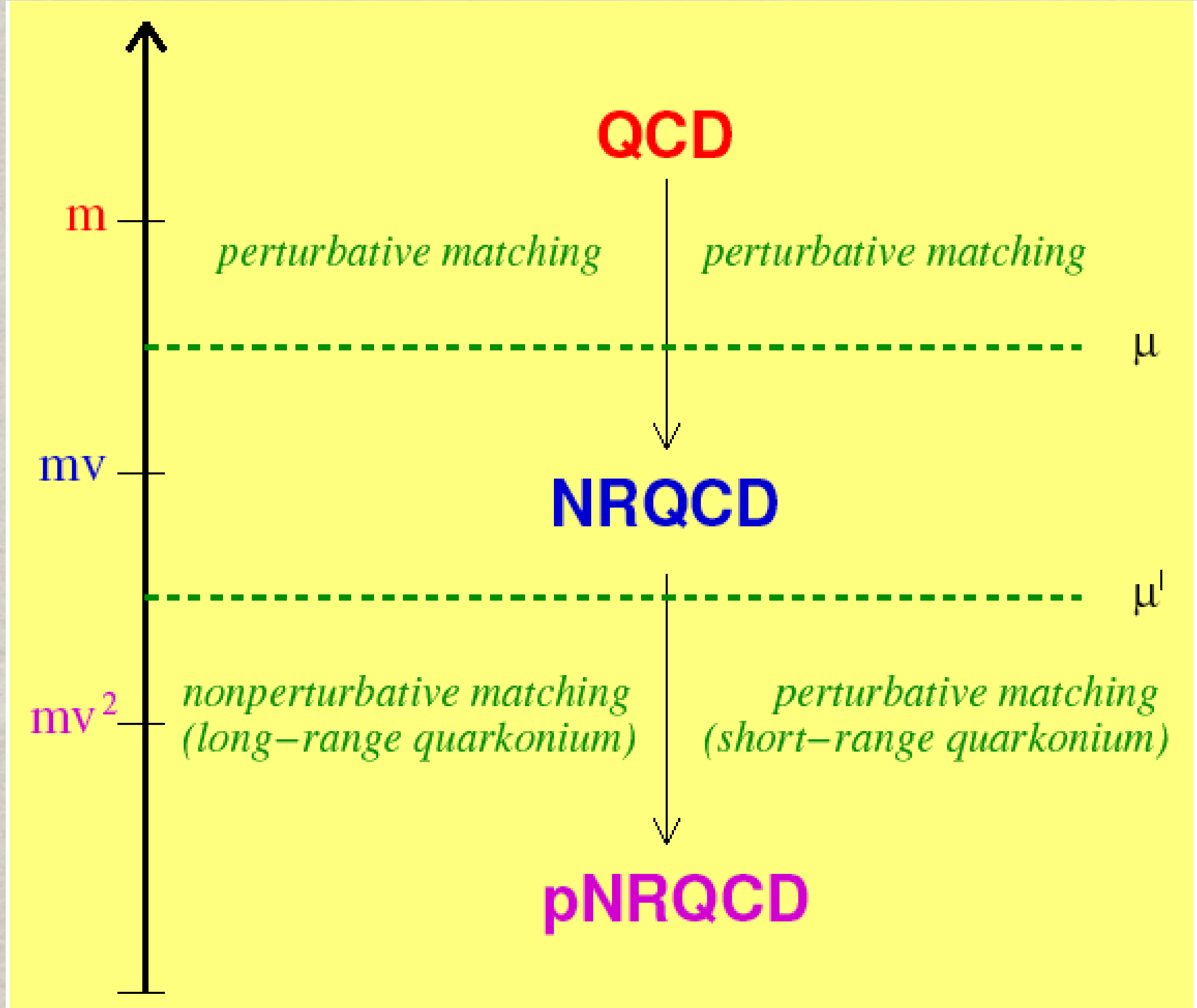
NREFTs for quarkonium

Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

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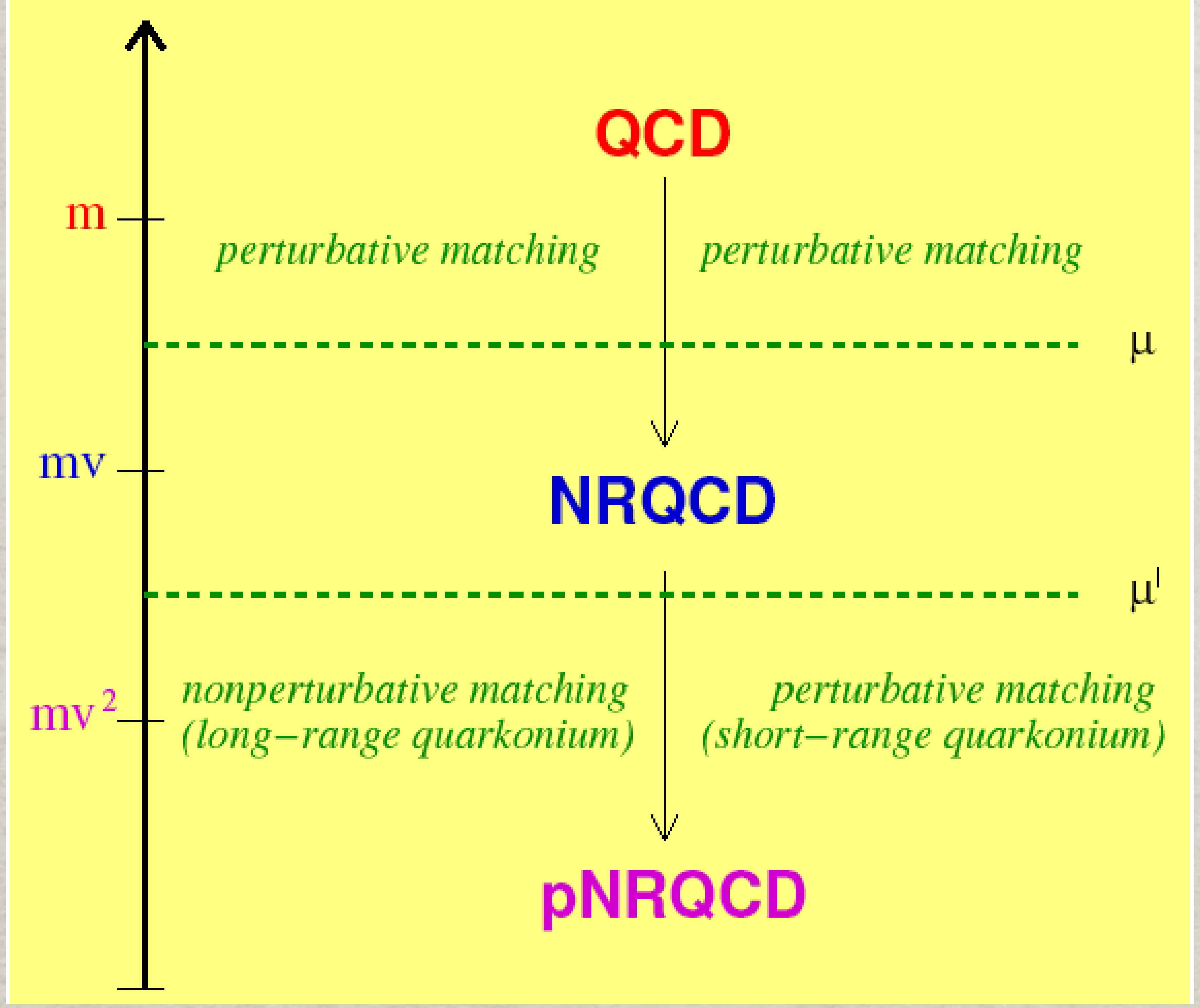

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$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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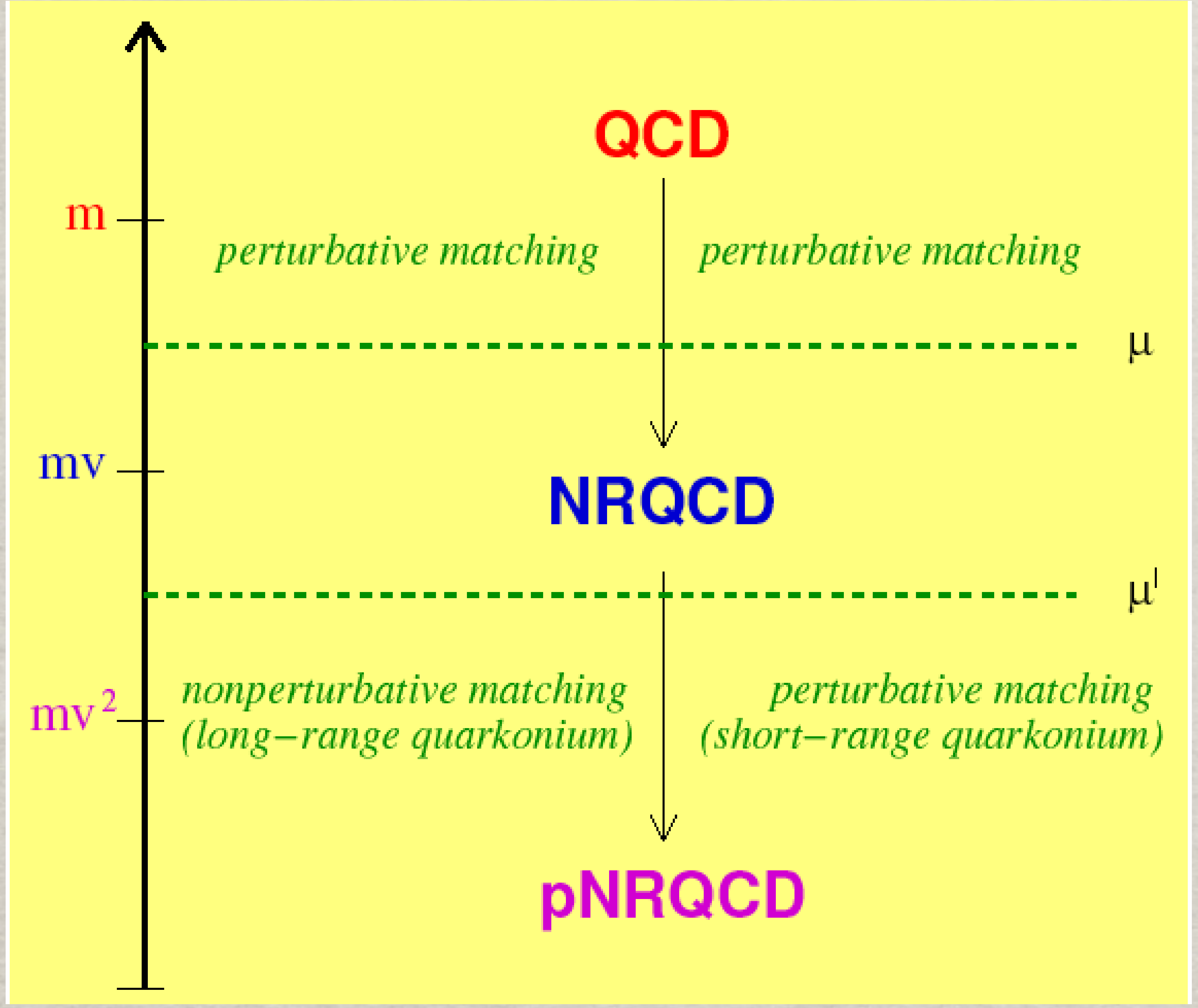

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$$\langle O_n \rangle \sim E_\lambda^n$$

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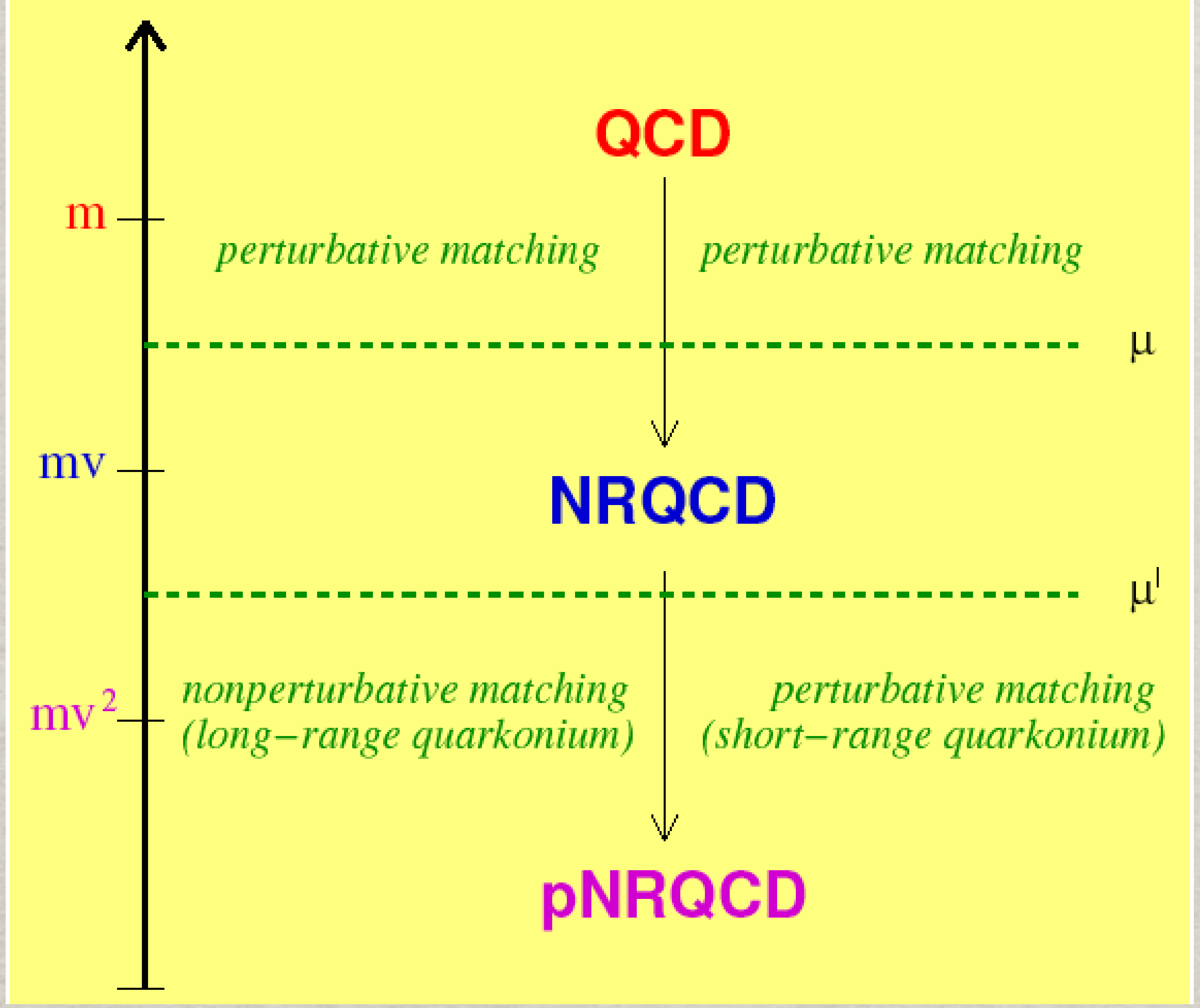

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

Ultrasoft
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$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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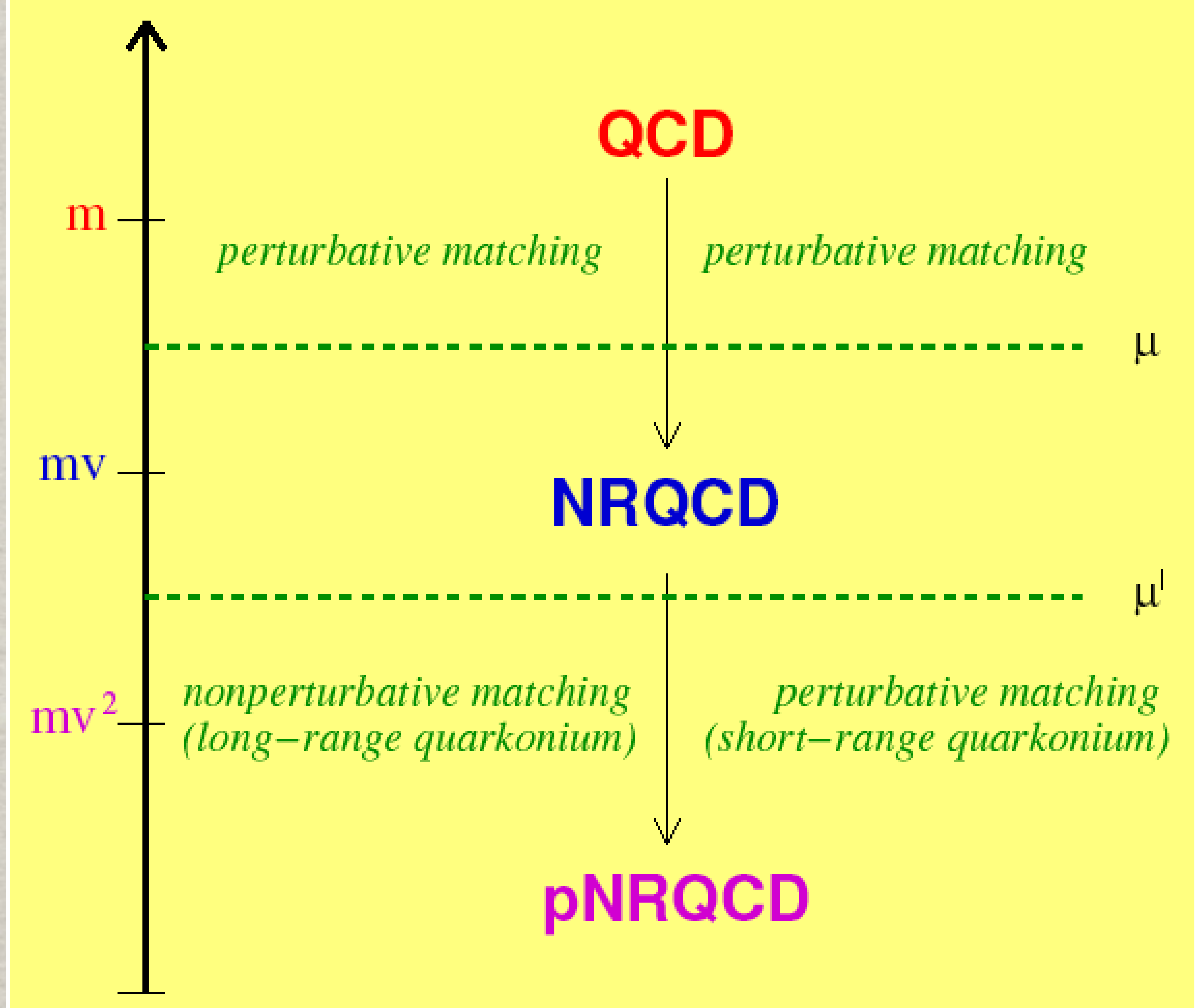

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
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$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

Ultrasoft
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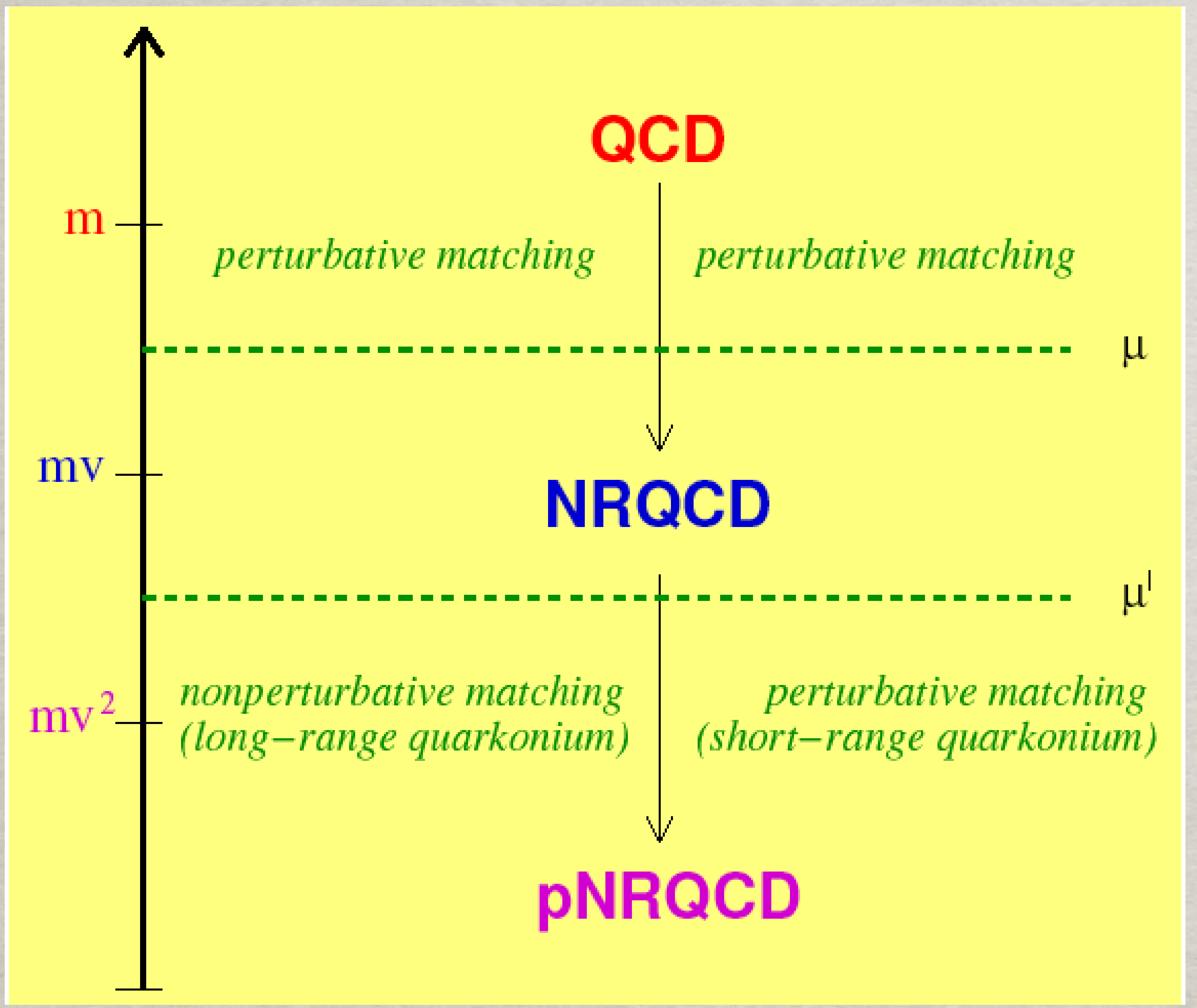
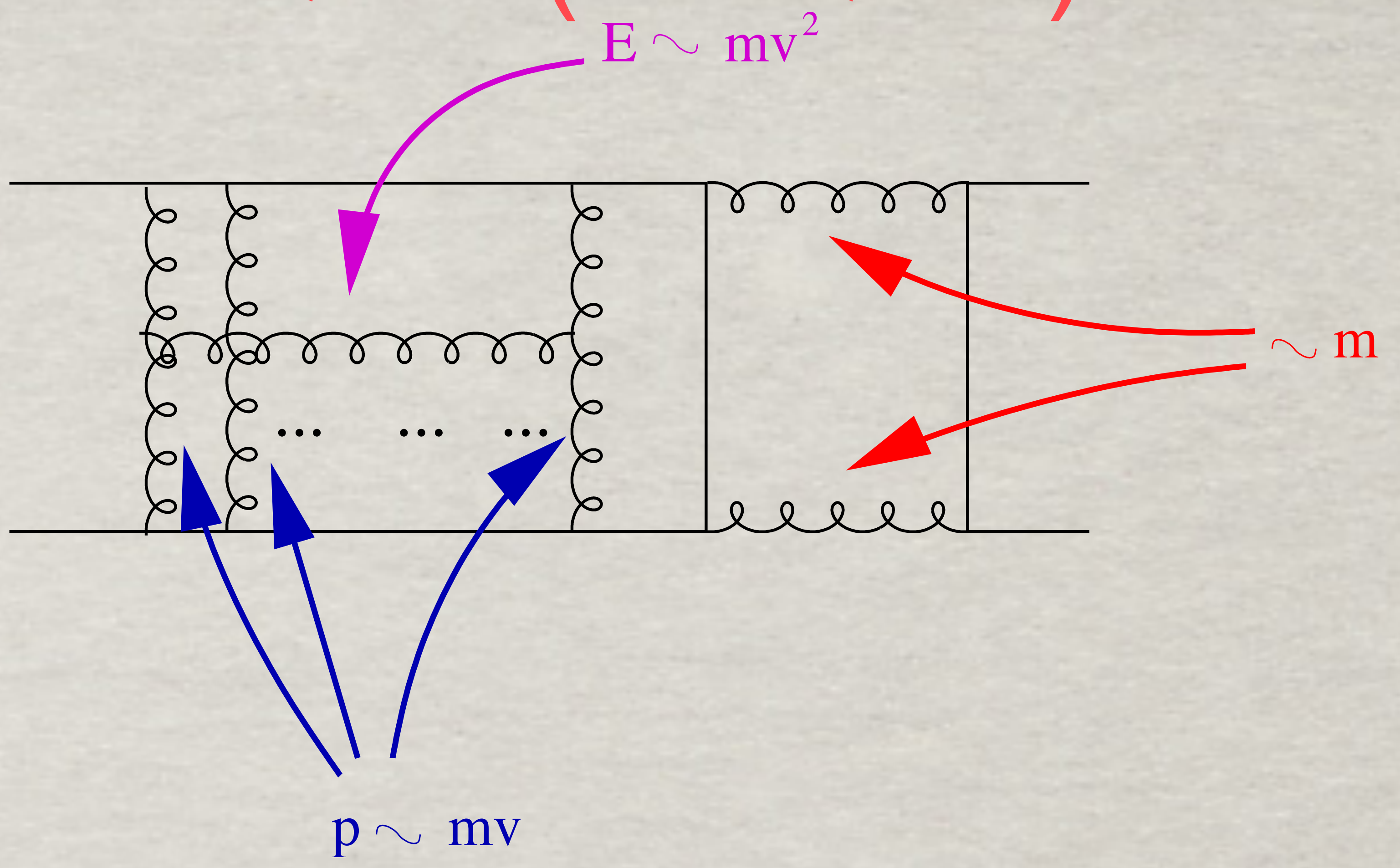
Quarkonium with NREFTs: Non Relativistic QCD (NRQCD)





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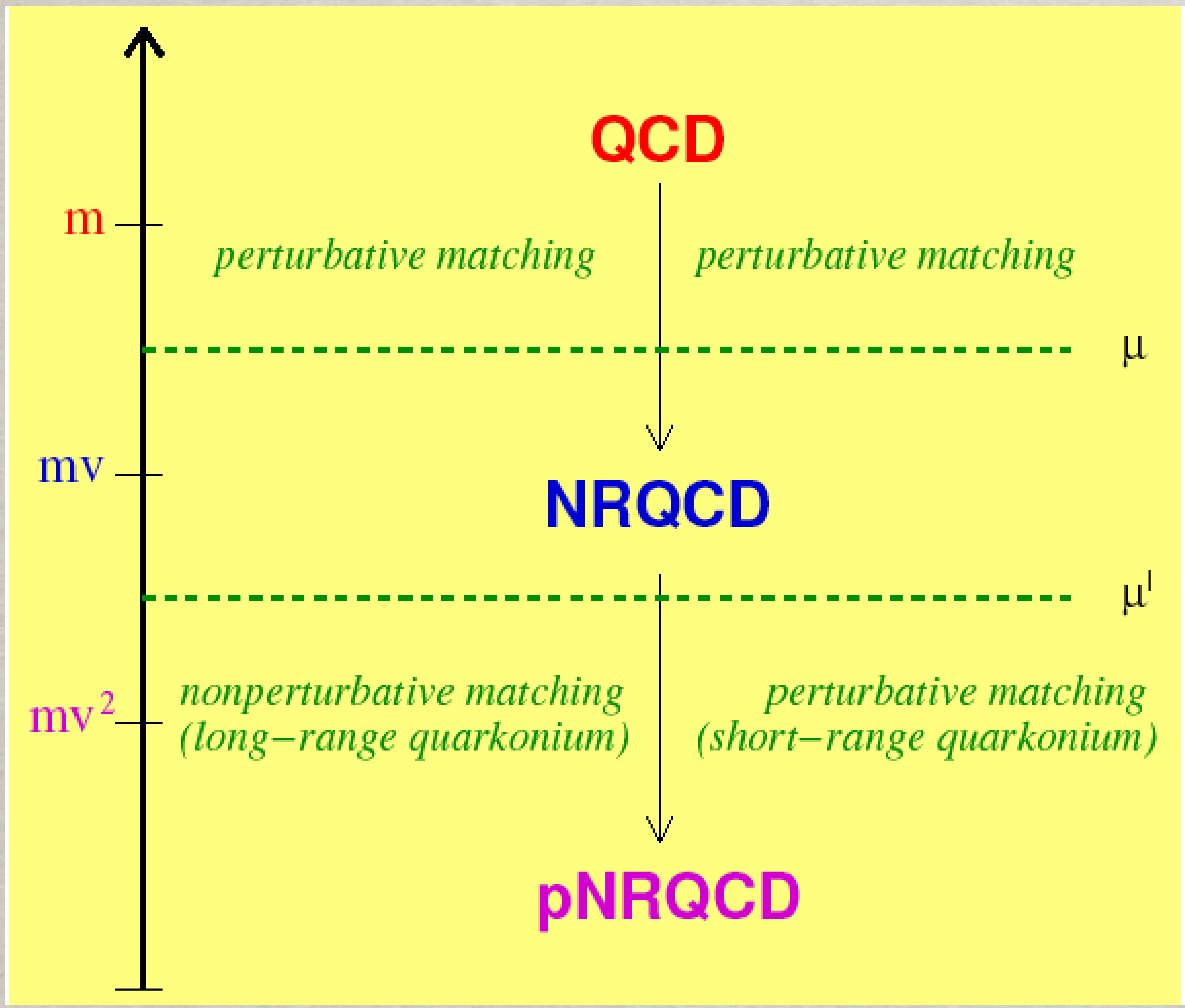

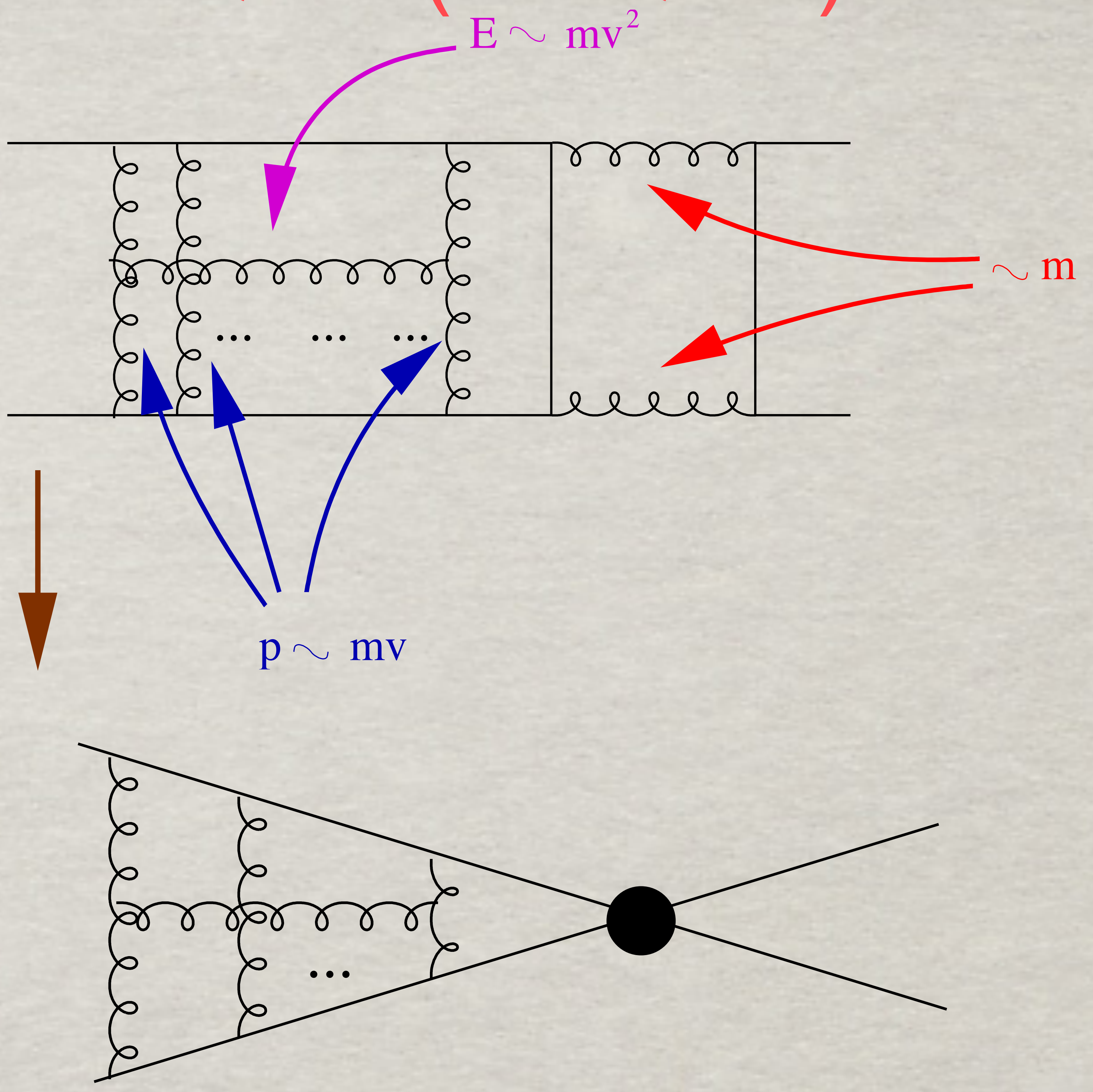

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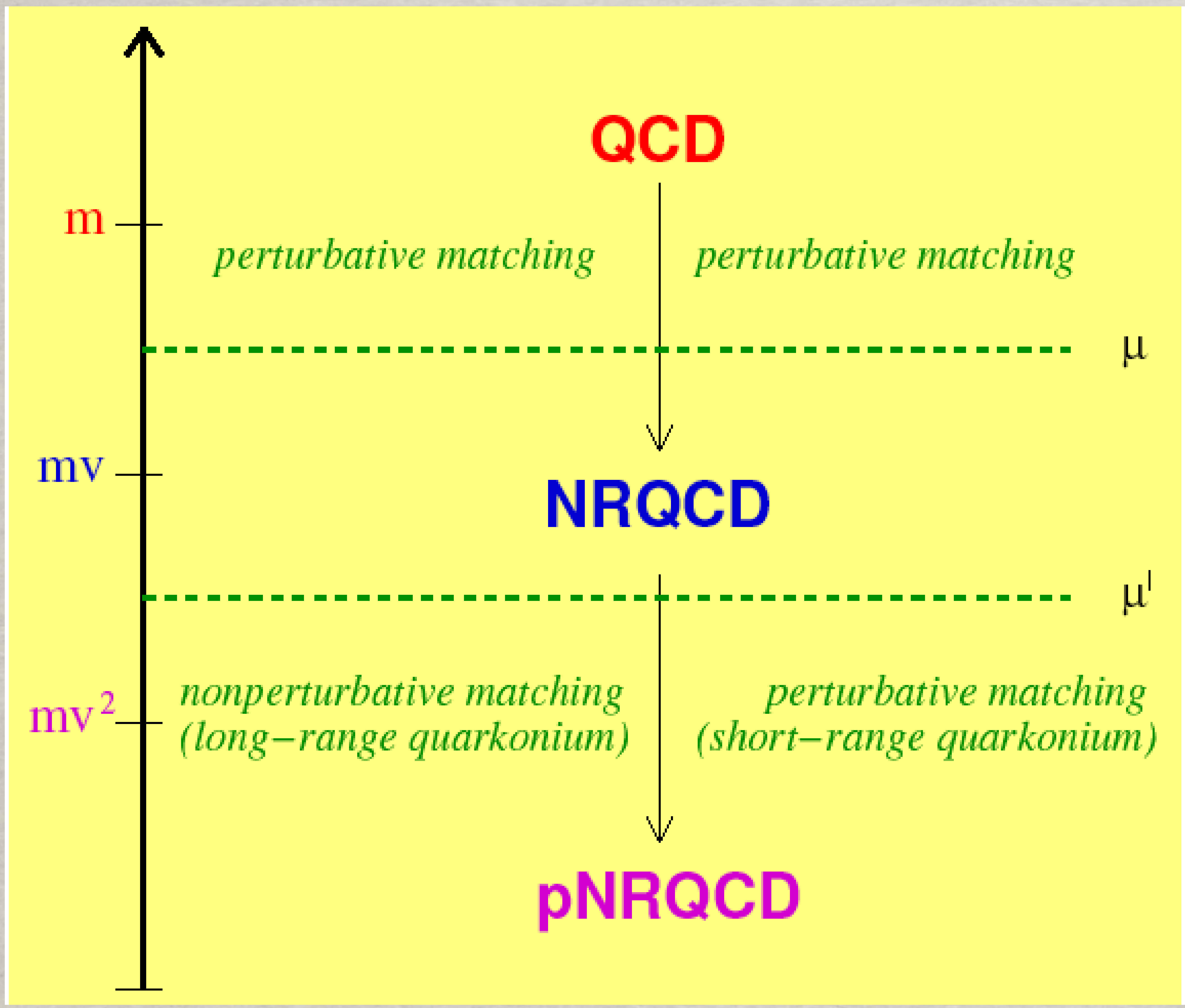
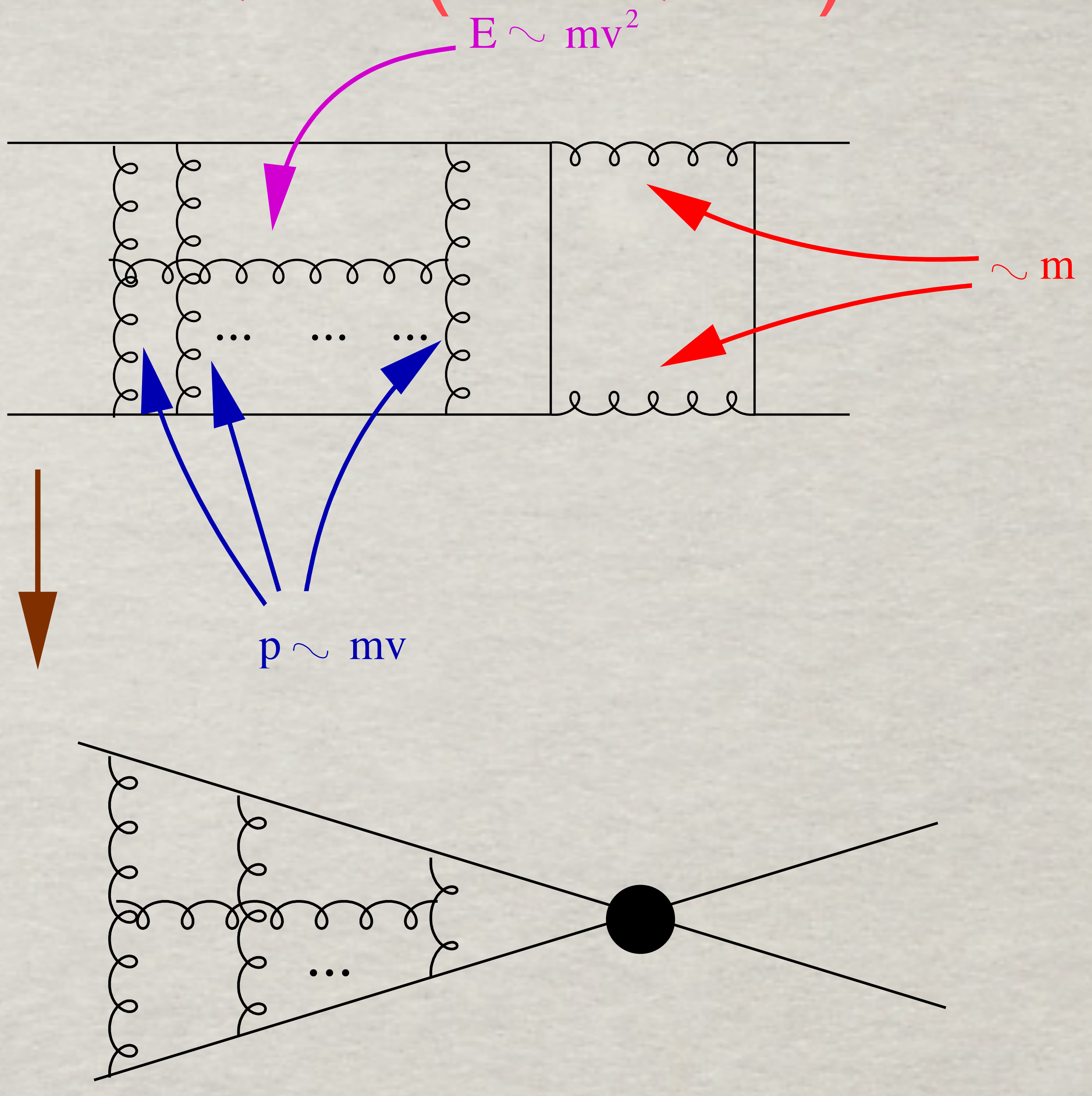
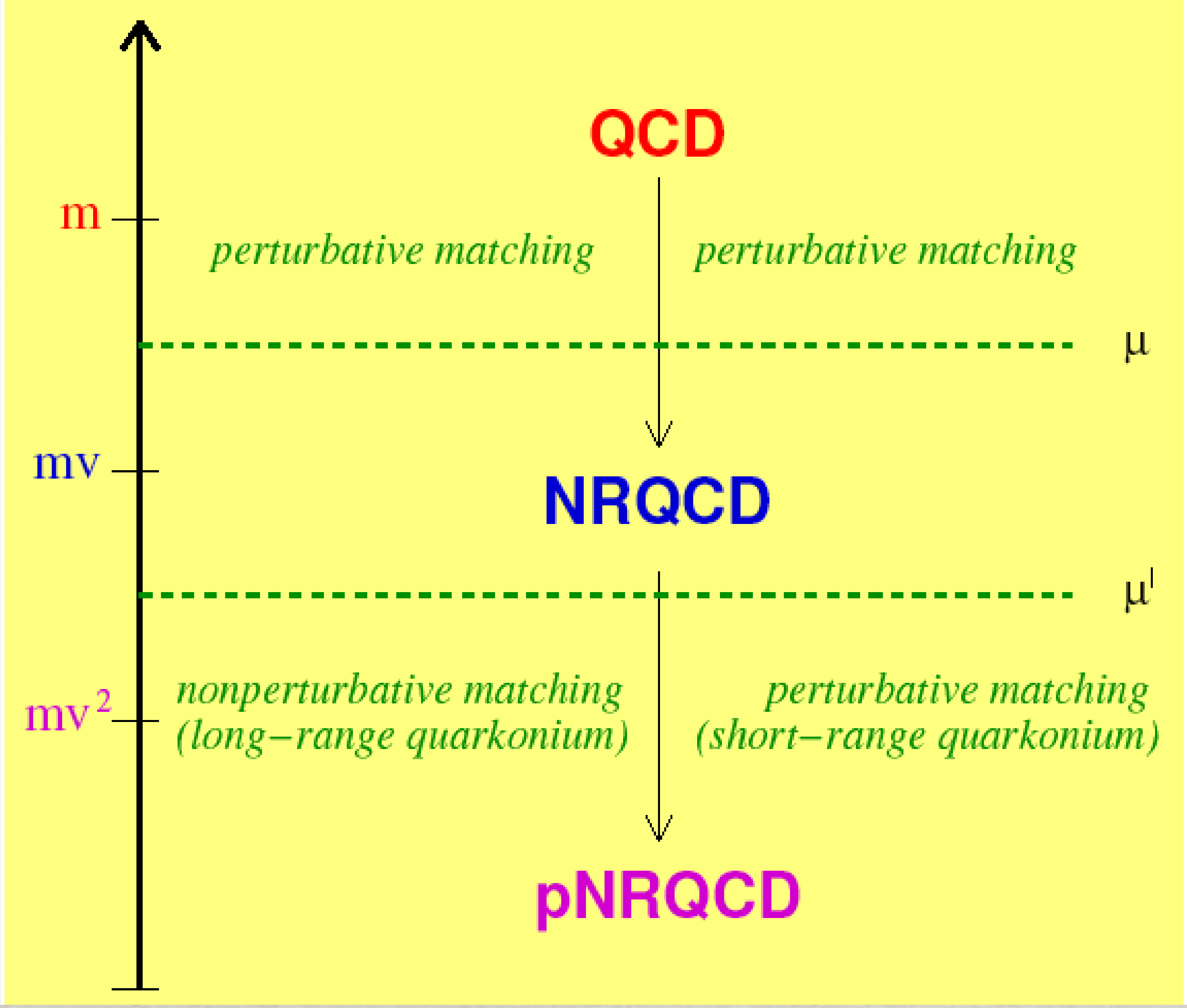
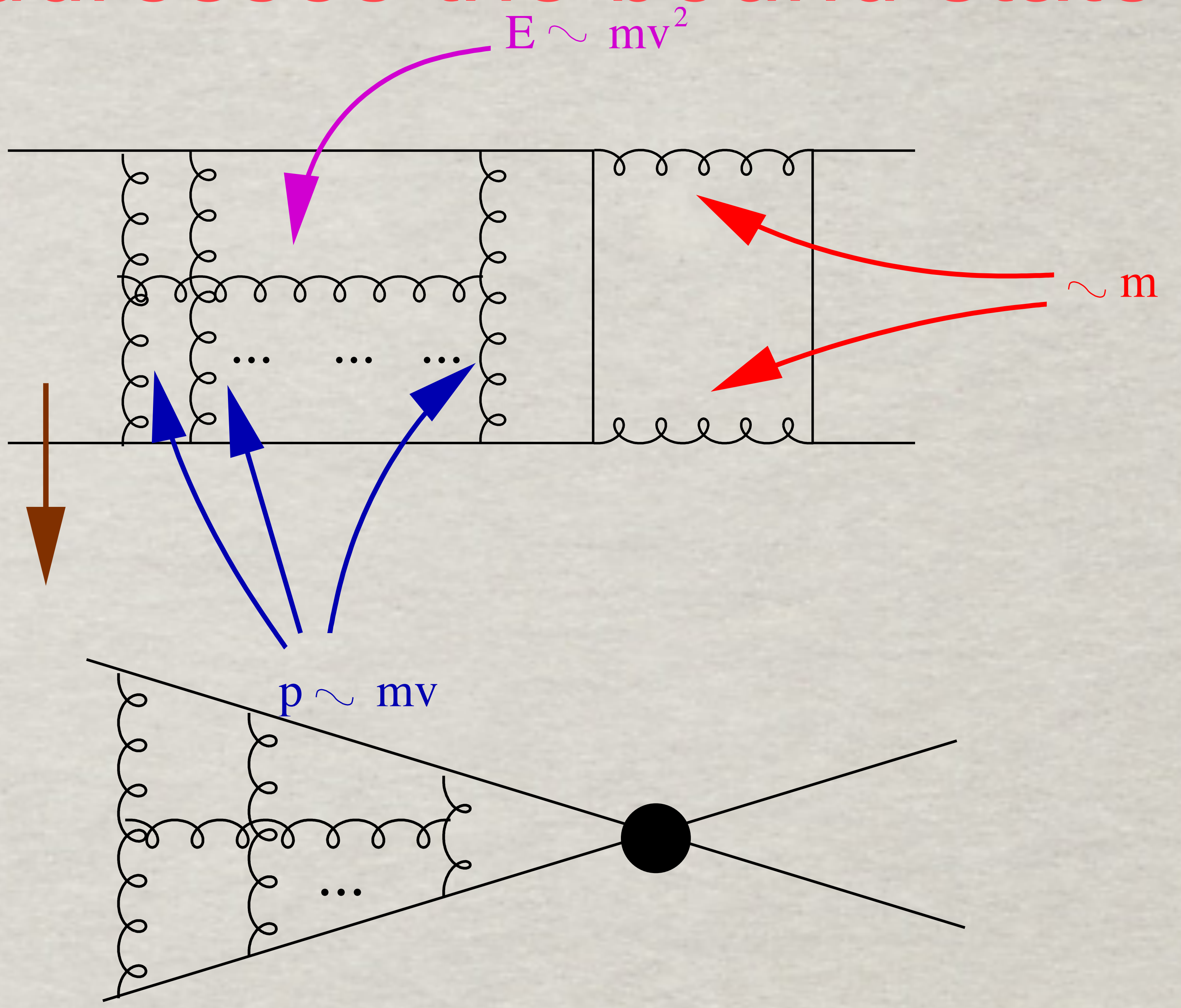




$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

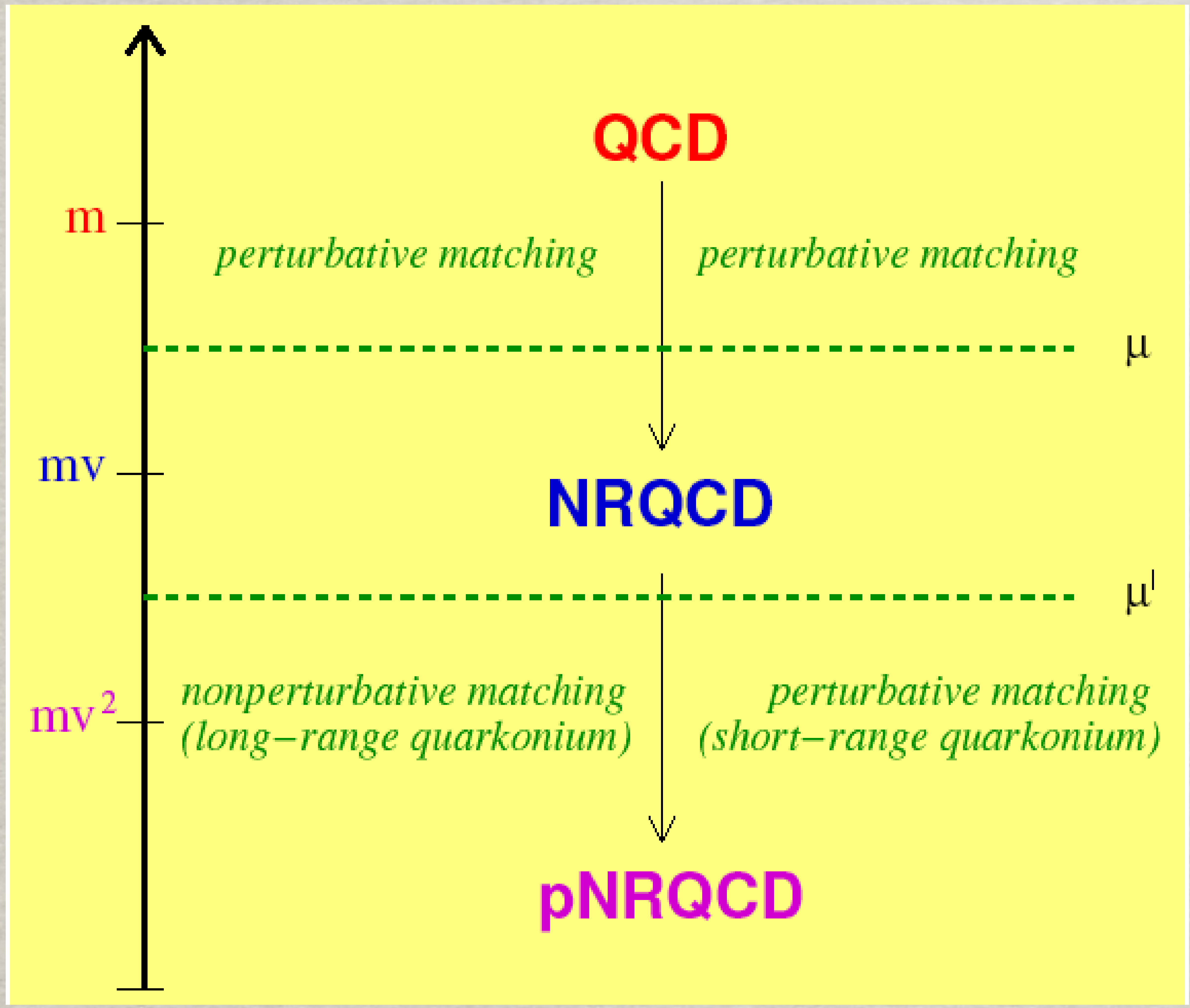
potential NonRelativistic QCD (pNRQCD): addresses the bound state formation

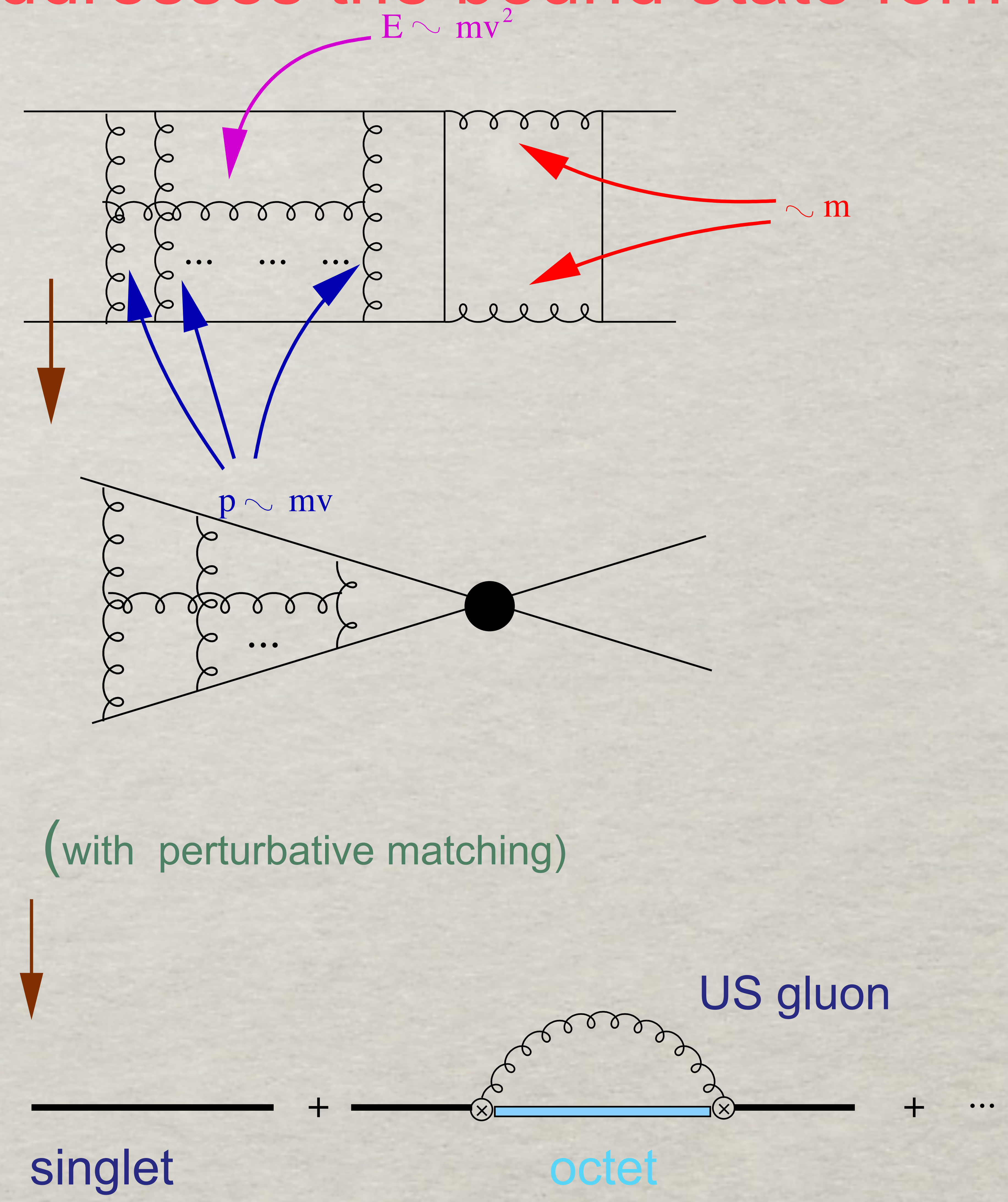




(with perturbative matching)

potential NonRelativistic QCD (pNRQCD): addresses the bound state formation





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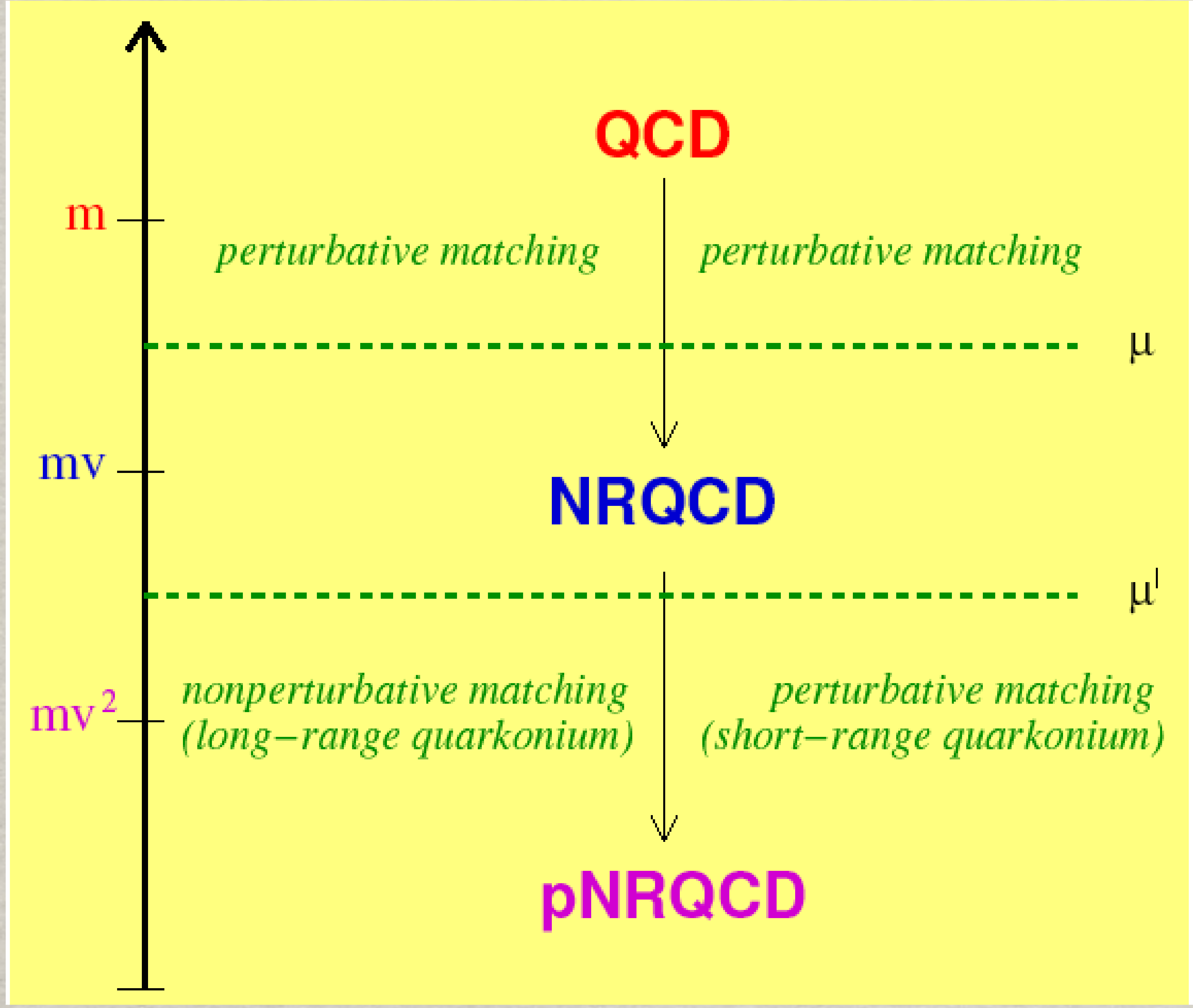

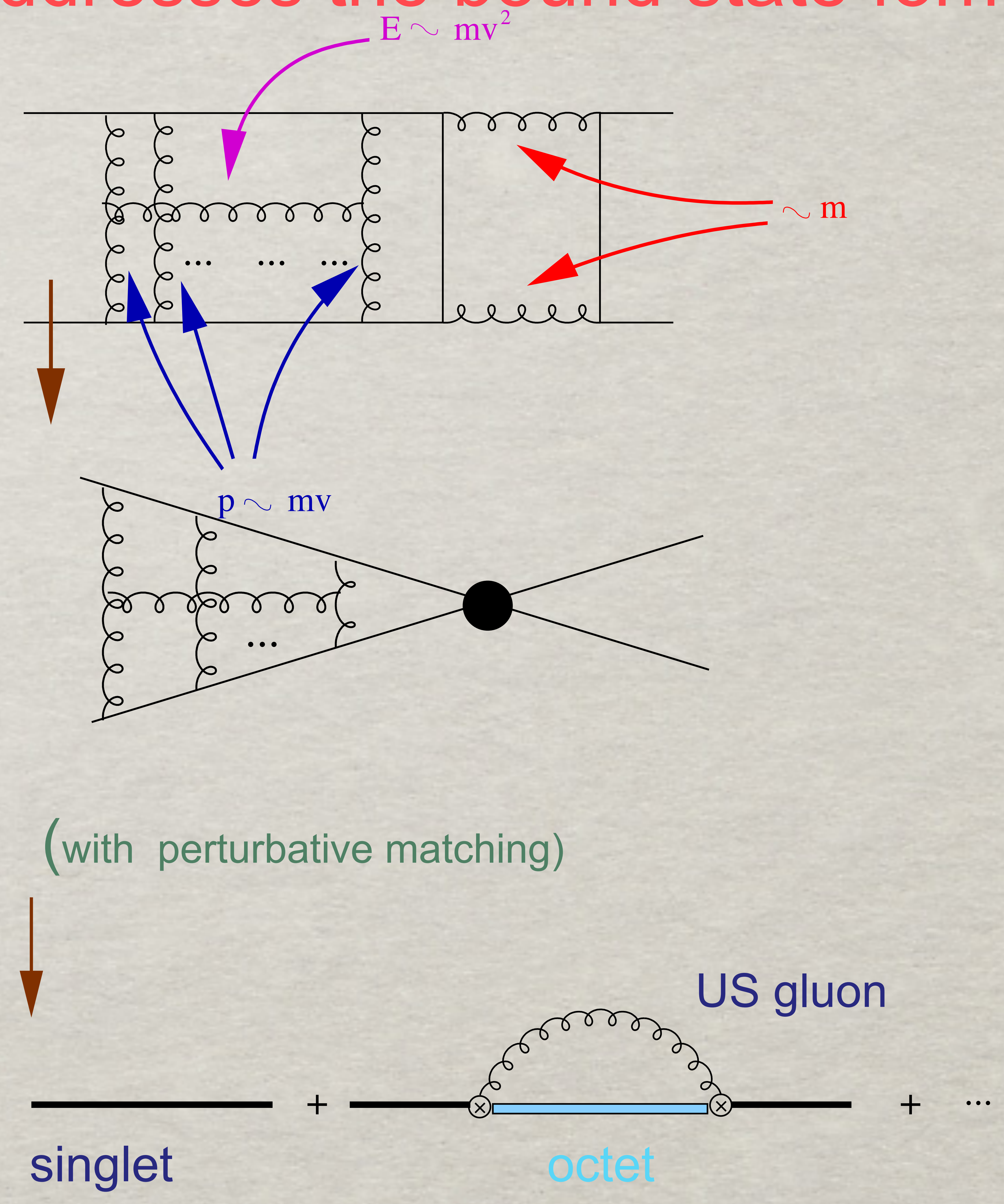

$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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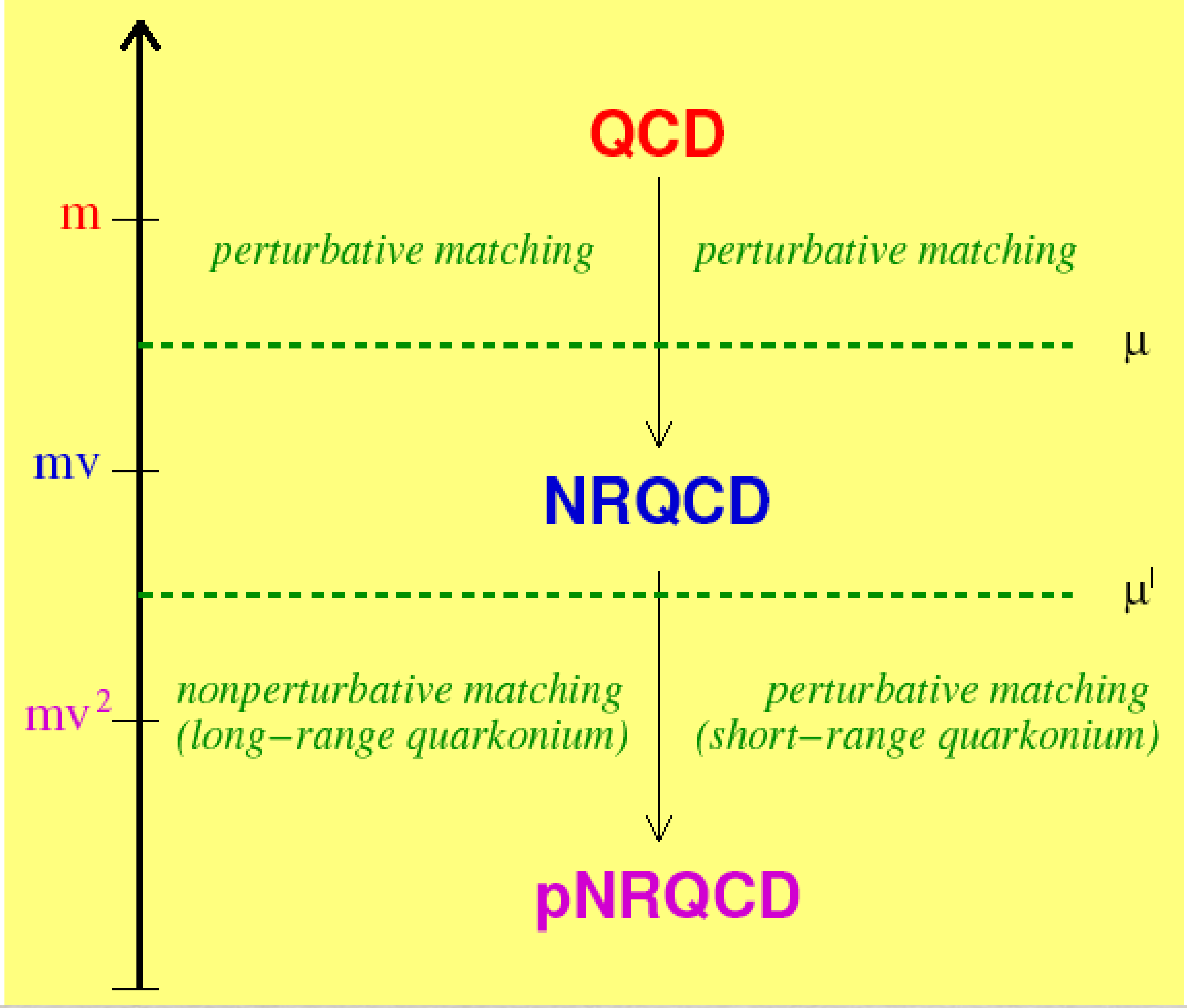

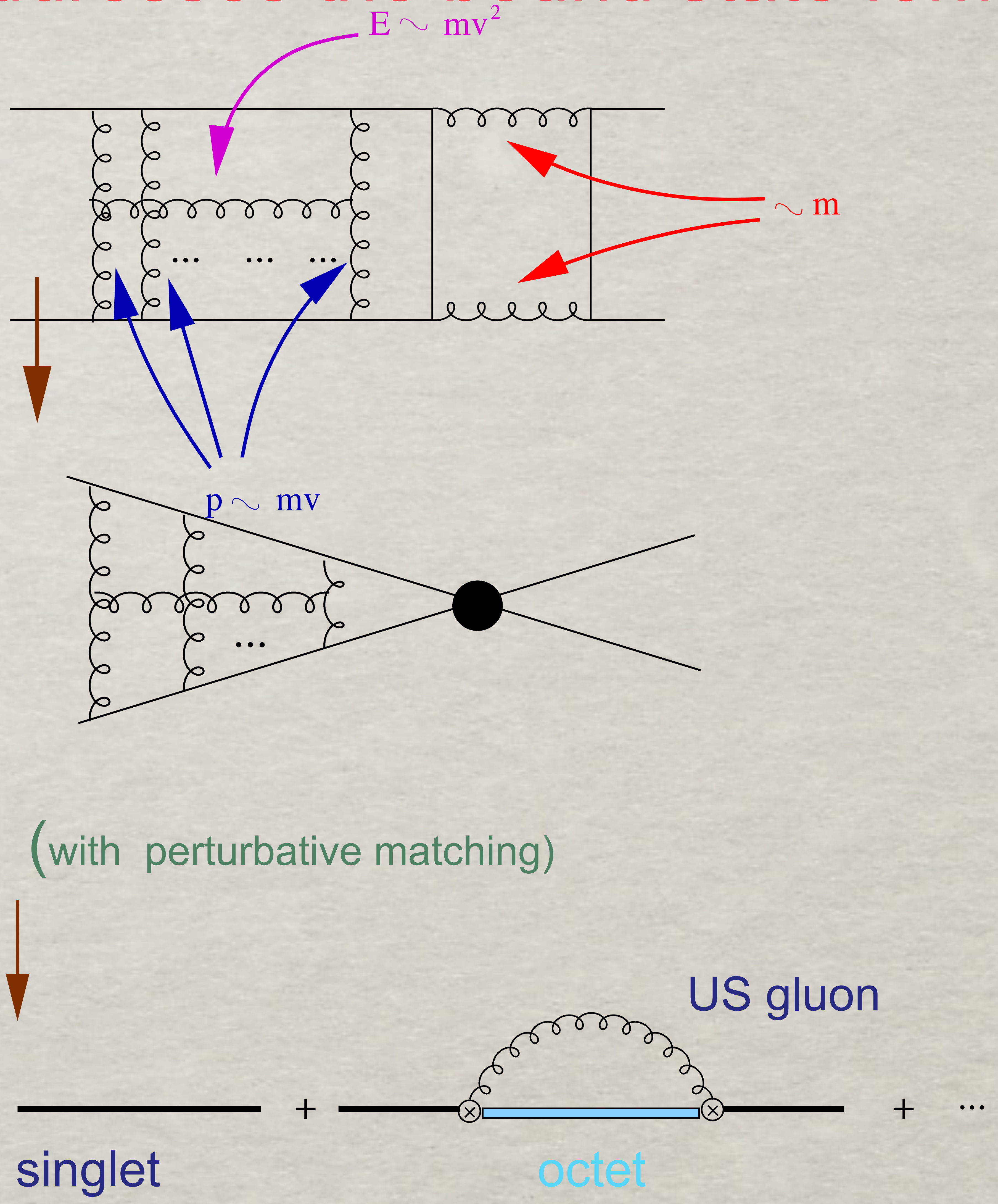

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pNRQCD addresses the bound state dynamics

$$\mathcal{L}_{\text{pNREFT}} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

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$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$
- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.

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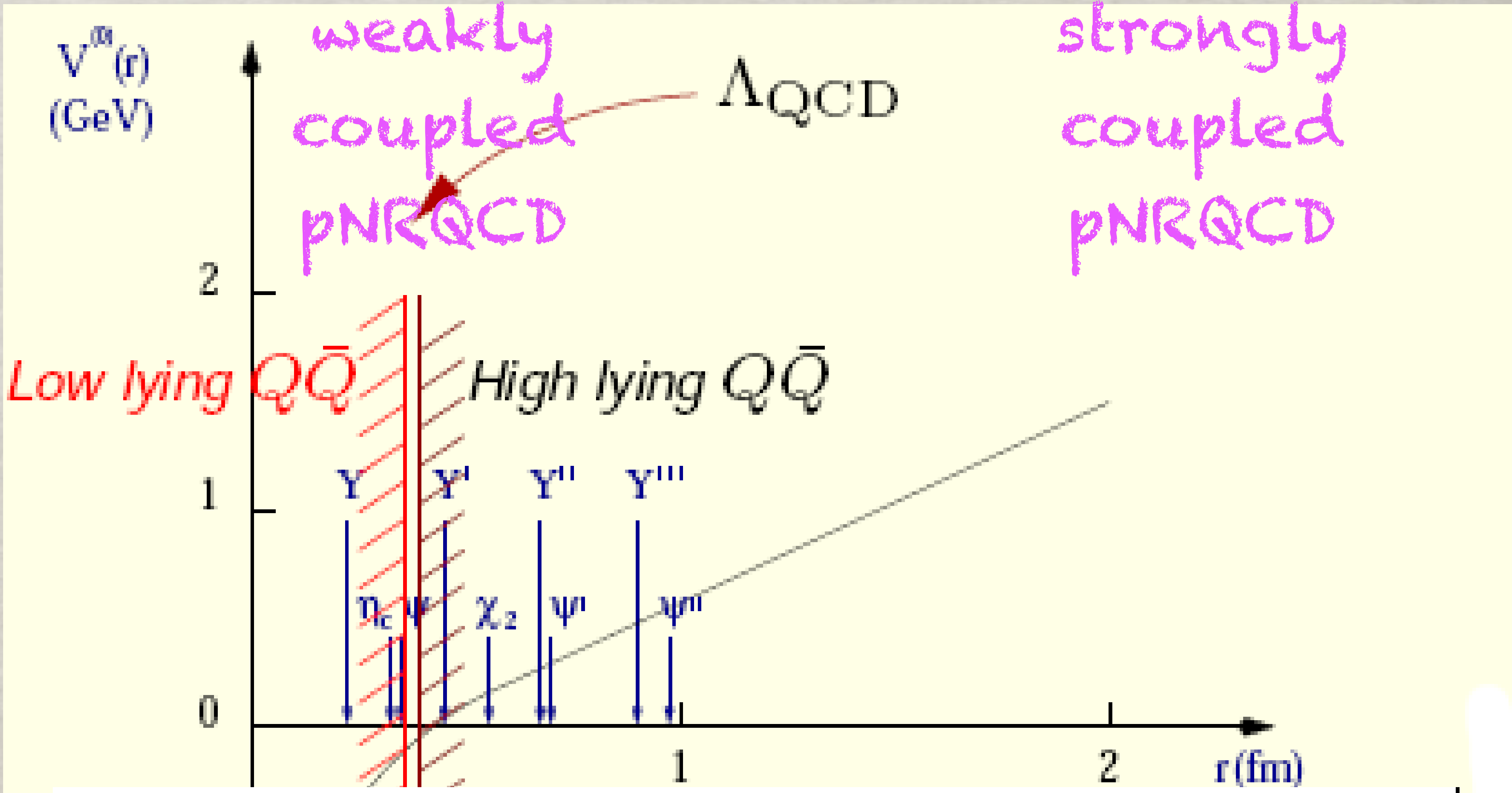
in QCD another scale is relevant $\Lambda_{\rm QCD}$



A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{\rm QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{\rm QCD}$

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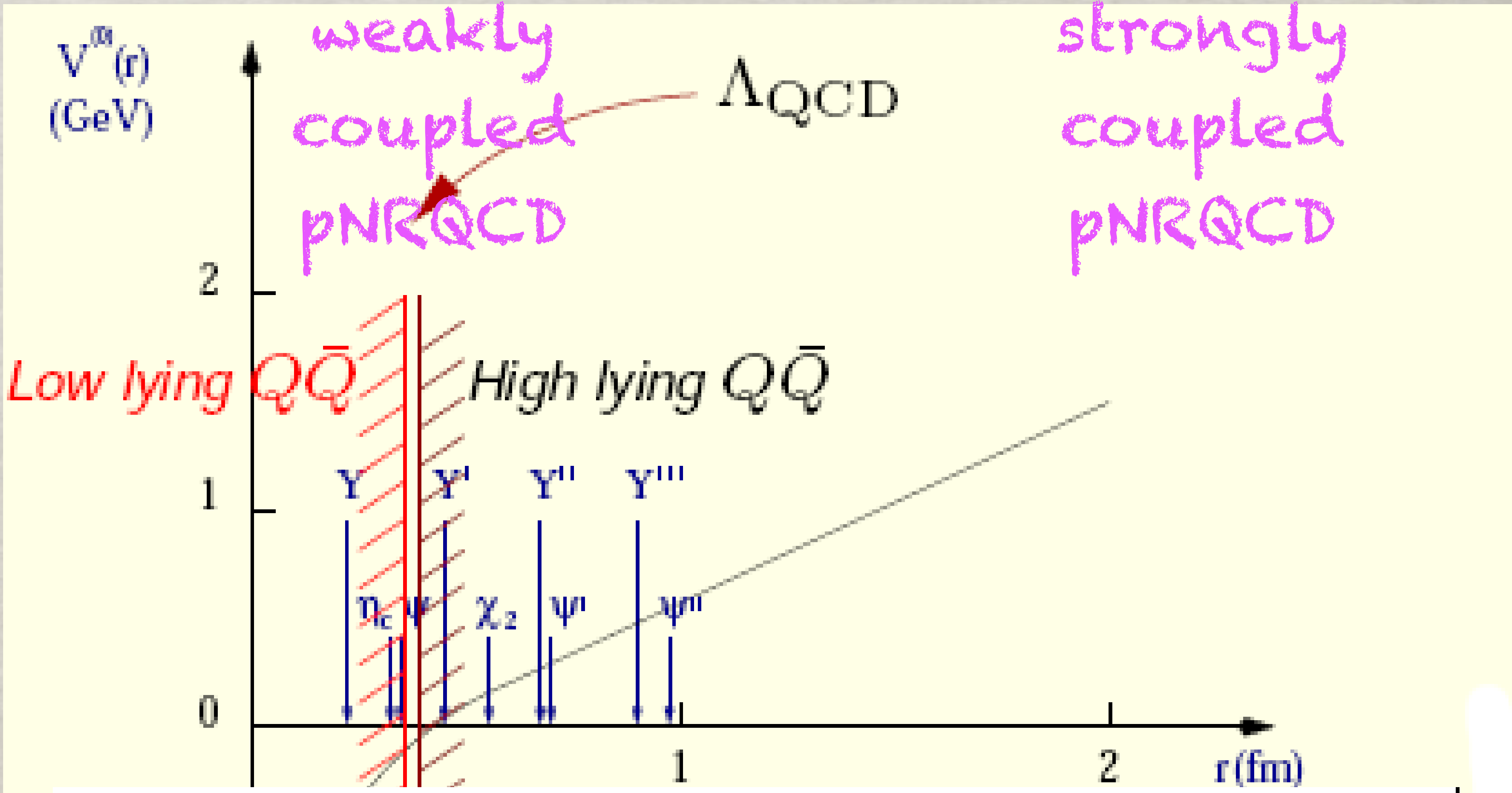
in QCD another scale is relevant $\Lambda_{\rm QCD}$

- The leading picture is Schoedinger eq.,the potentials appear once all scales above the energy have been been integrated out
- non potential effects appear as correction to the leading picture and are nonperturbative
- Any prediction of pNRQCD is a prediction of QCD at the given order of expansion
- Effects at the nonperturbative scale are carried by gauge invariant purely glue dependent correlators to be calculated on the lattice or in QCD vacuum models



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Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R,t) + \ldots$

$\mathbf{R}$ = center of mass
$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r \, {\rm Tr}\, \{ S^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots) S + O^\dagger (iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots) O +$$

LO in r

$$+ V_A (S^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger \mathbf{r} \cdot g\mathbf{E} S) + \frac{V_B}{2} (O^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger O \mathbf{r} \cdot g\mathbf{E}) \} + \ldots$$

NLO in r

$$-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} q_i$$

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NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

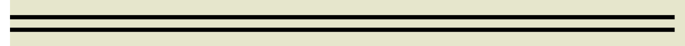
The matching coefficients are the Coulomb potential

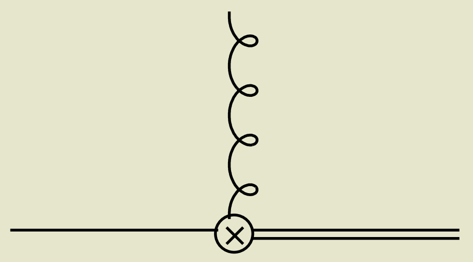
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \dots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \dots,$$

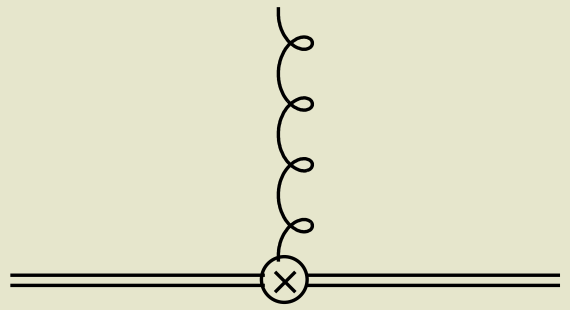
$$V_A = 1 + \mathcal{O}(\alpha_s^2),\ V_B = 1 + \mathcal{O}(\alpha_s^2).$$

Feynman rules

 $= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$

 $= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$

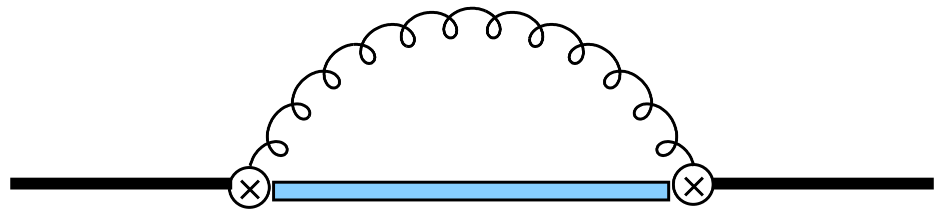
 $= {\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm S}$

 $= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02   NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

$\sim e^{i\Lambda_{\rm QCD} t}$

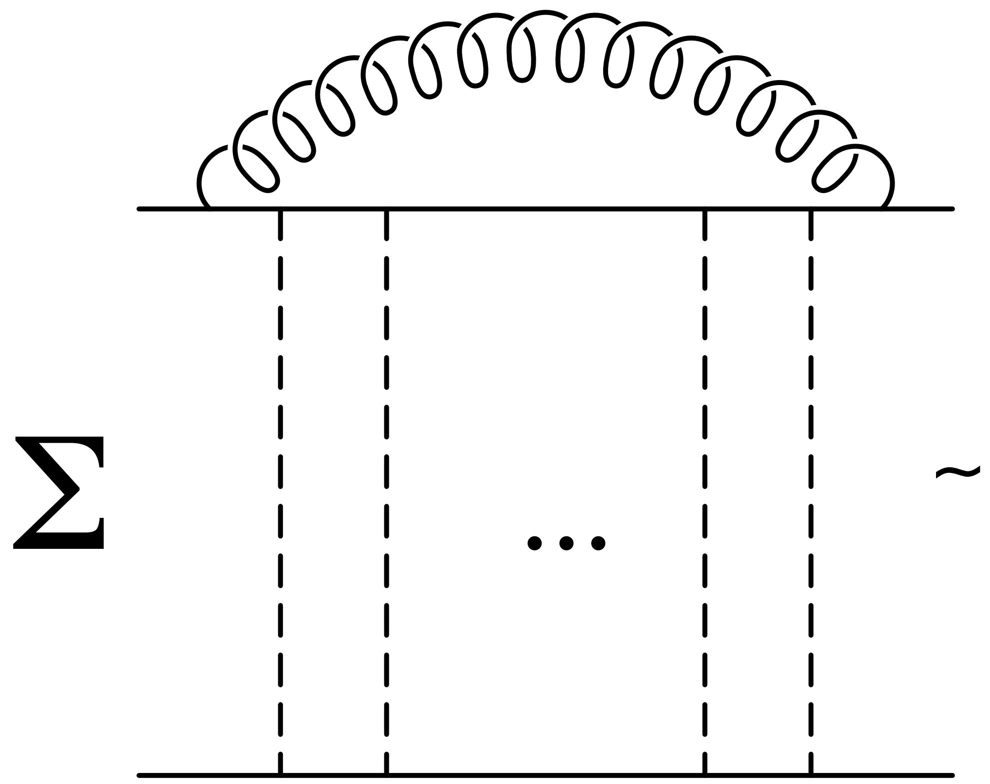
$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\, \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

—>used to extract precise (NNNLO) determination of m_c and m_b

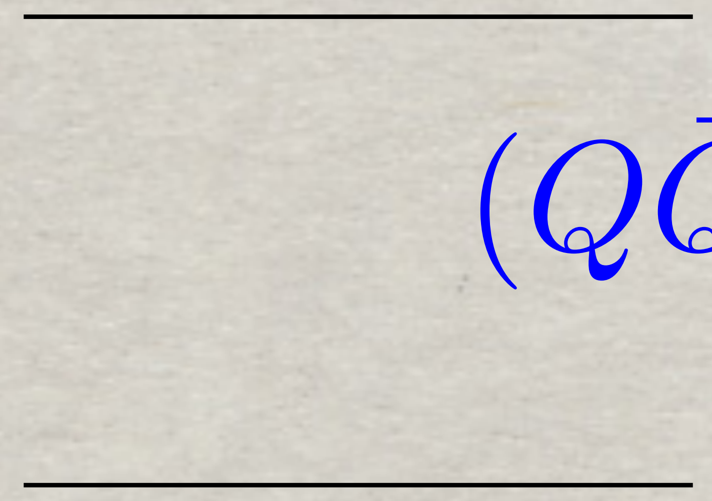


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

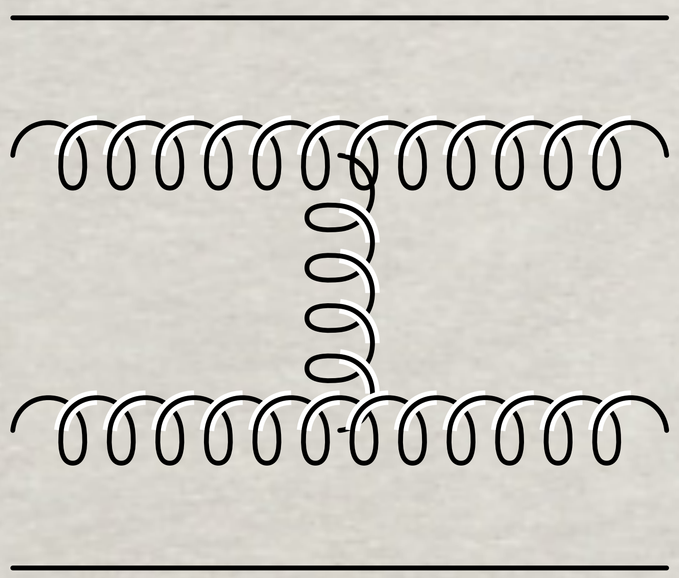
Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Strongly coupled pNRQCD: Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

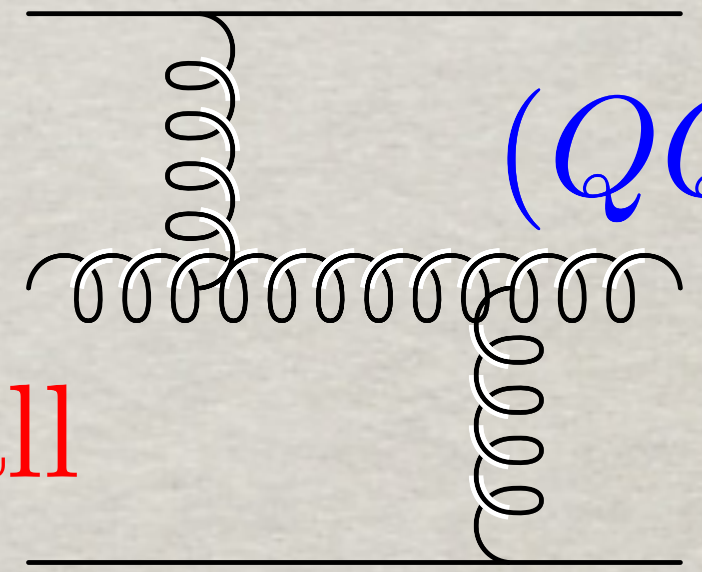
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1$ + Glueball



$(Q\bar{Q})_8 G$

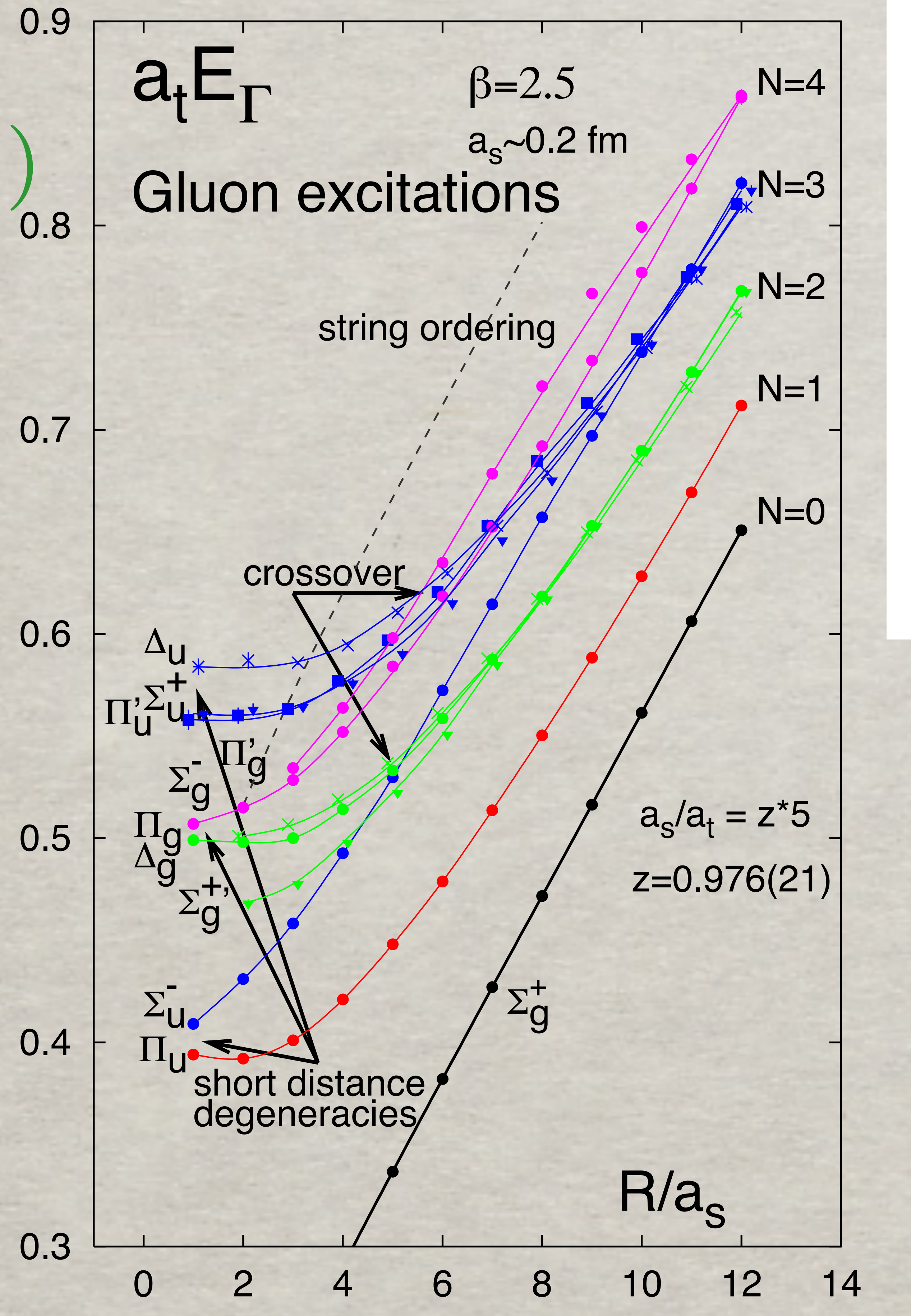
hybrid

with gluons at the scale $\Lambda_{\rm QCD}$ —>nonperturbative problem, use lattice

Strongly coupled pNRQCD: Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

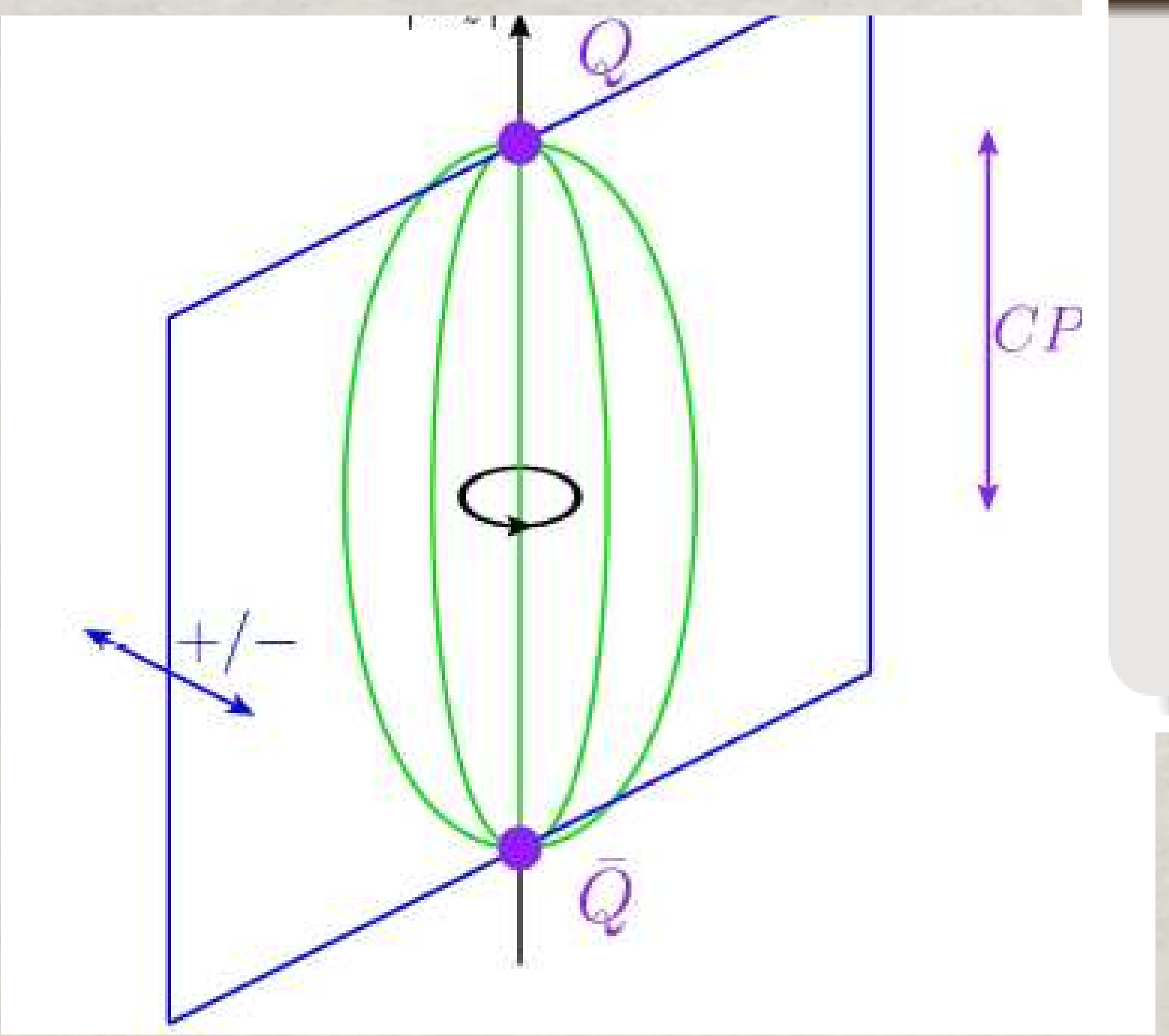
Spectrum of NRQCD static energies E^0_n from Lattice

$E_n^{(0)}$



$r = |\mathbf{x}_1 - \mathbf{x}_2|$

Λ



Irreducible representations of $D_{\infty h}$ Λ_η^σ

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), \; -1\,(u)$
- σ: eigenvalue of relfection about a plane containin

K is the angular momentum of the light degrees of freedom;same symmetry as the diatomic molecule

NRQCD

$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$\Lambda_\eta^\sigma \;{-}{-}{>}\; n$

NRQCD states

$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -> \quad |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

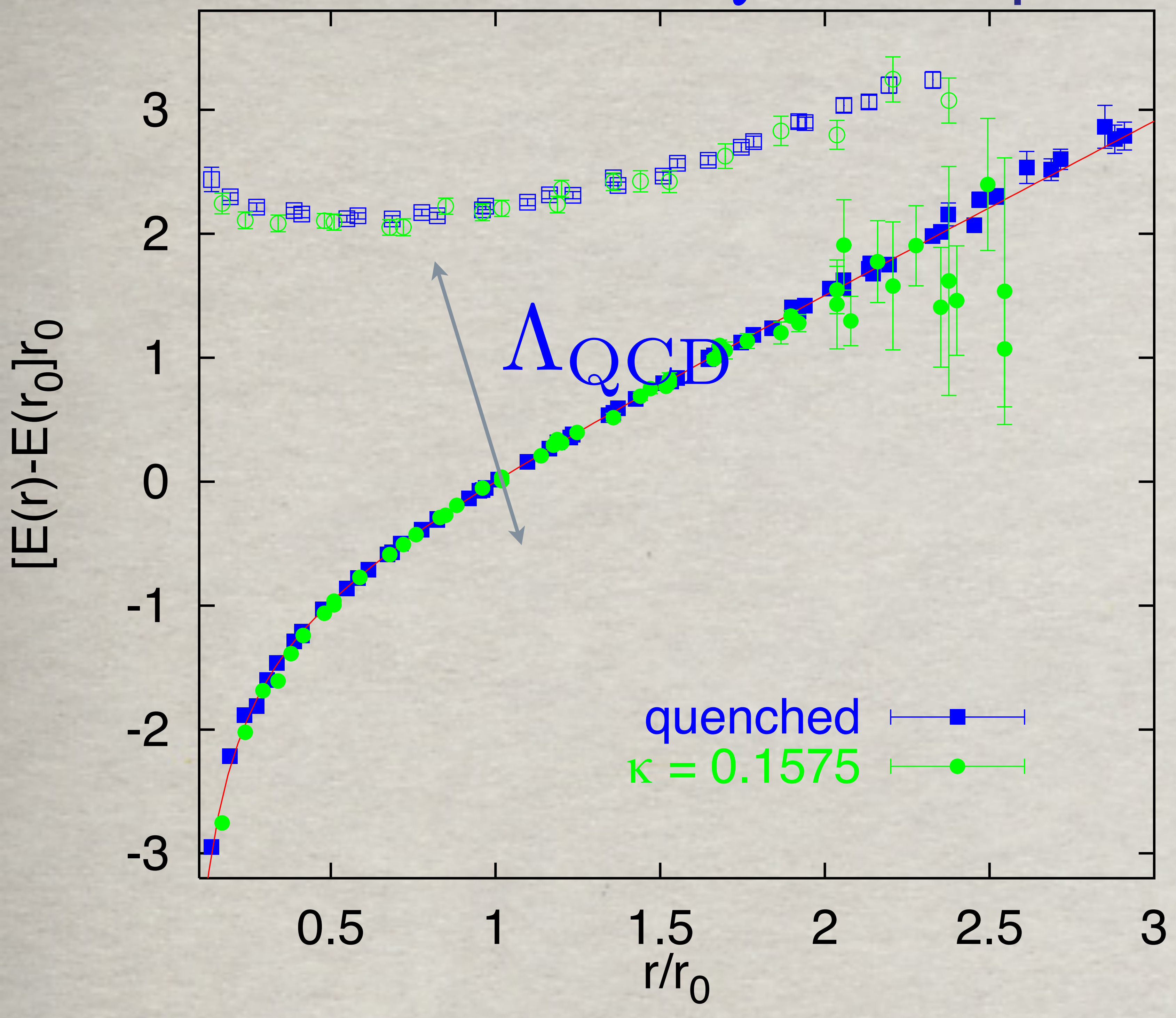
pNRQCD states

$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -> \quad |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

Juge Kuti Mornigstar 98-06

Bali et al. 98 $mv \sim \Lambda_{QCD}$

- pNRQCD and the potentials come from integrating out all scales up to mv^2



- gluonic excitations develop a gap Λ_{QCD} and are integrated out

Brambilla Pineda Soto Vairo 00

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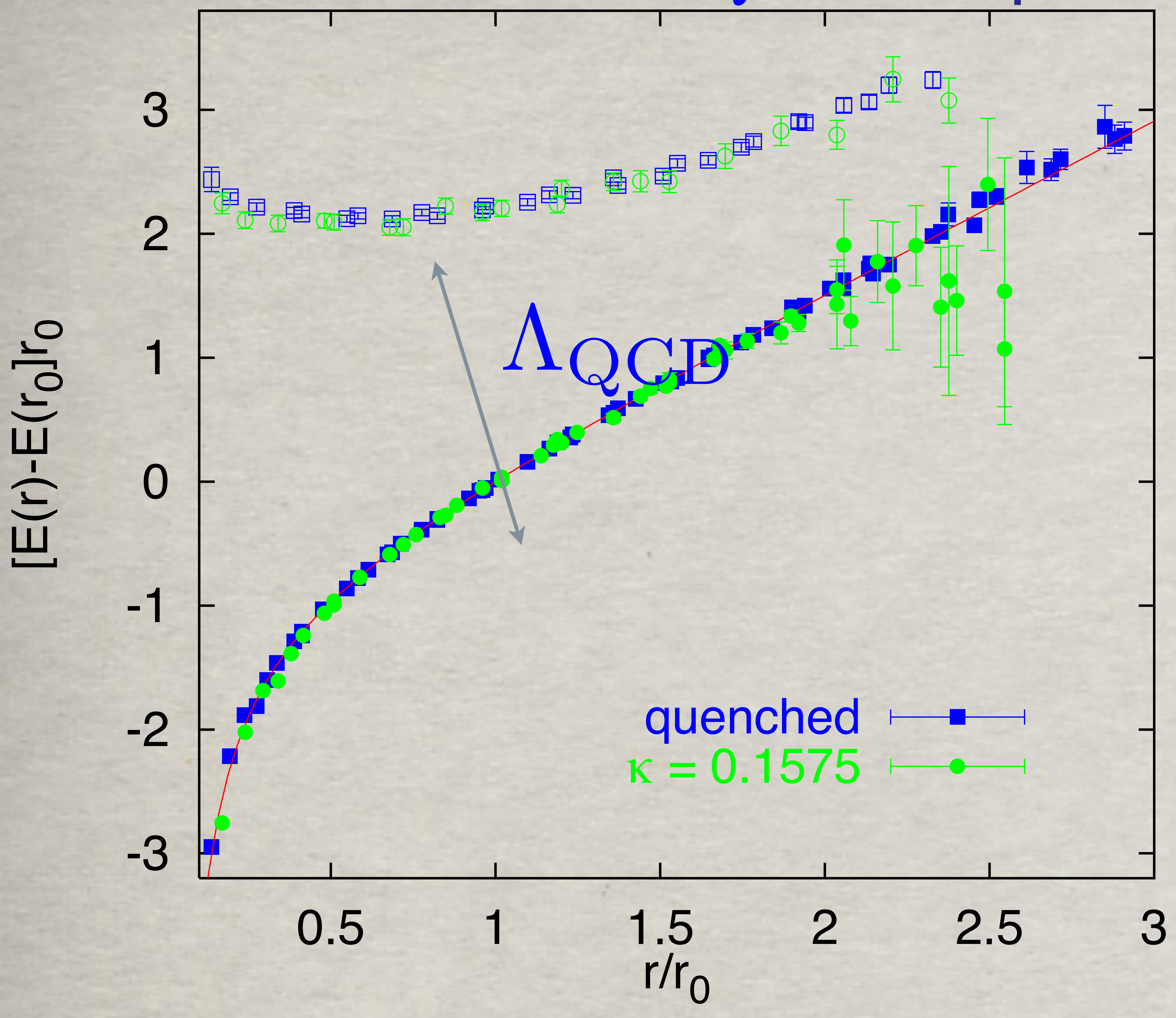

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⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

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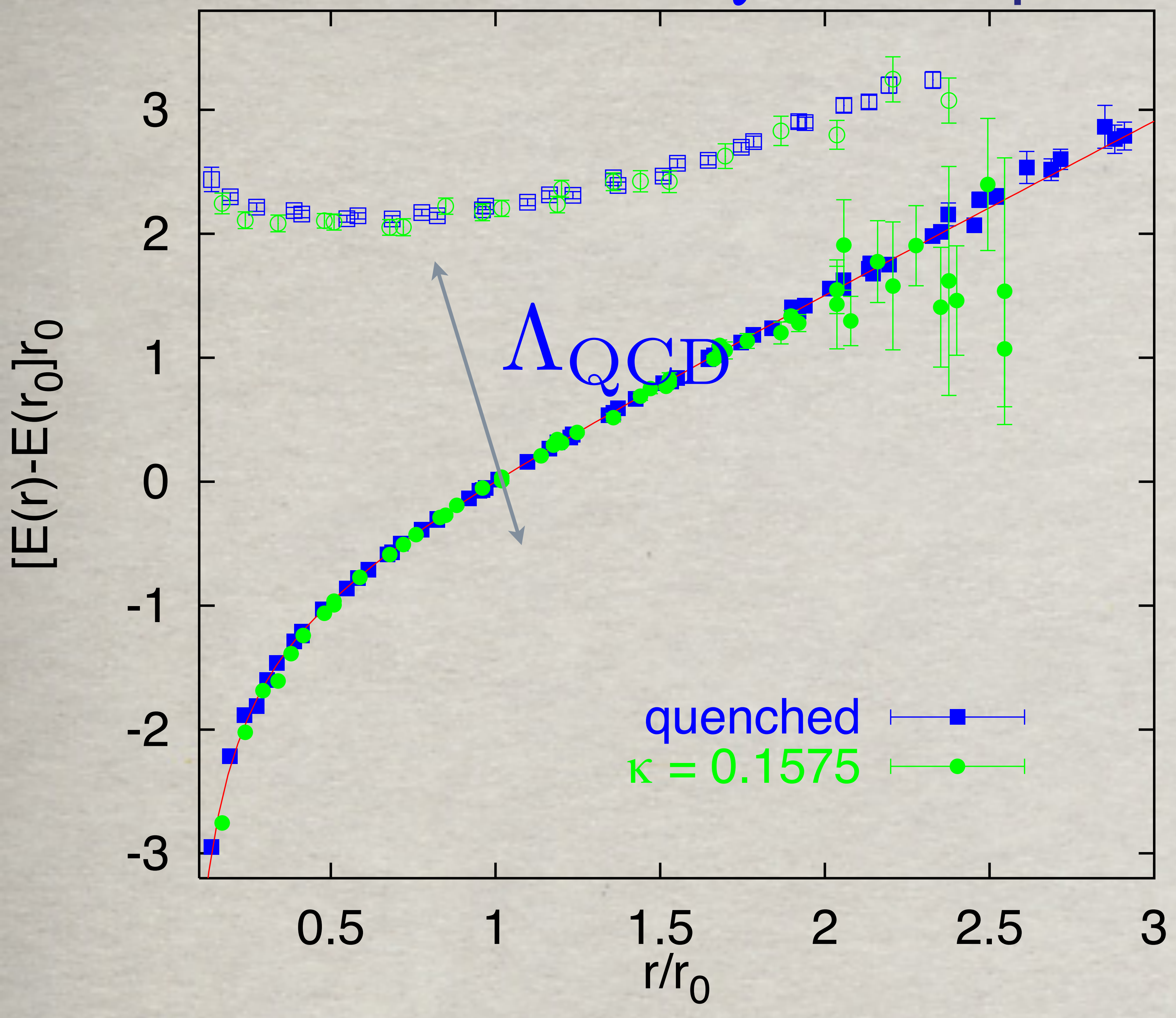

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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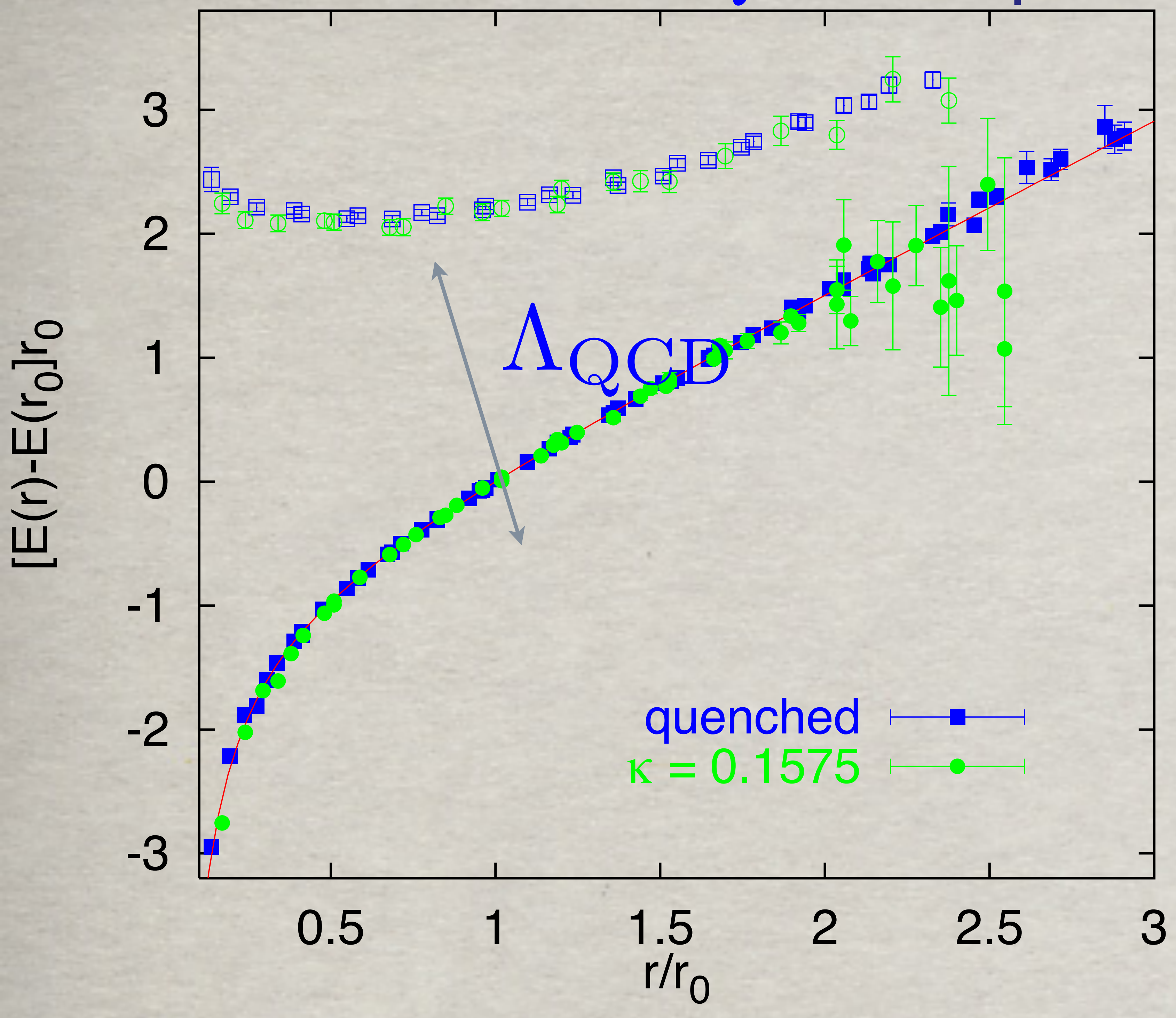

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- A pure potential description emerges from the EFT **however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters**

- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays

- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t\, \langle \square_{E(t)\,\bullet\,\bullet} \rangle$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

$$V_{\rm SD}^{(2)} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \square_{i\ j}^{E(t),\,B} \rangle\ \mathbf{L}_1\cdot\mathbf{S}_2 + (1\leftrightarrow 2)\ |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle \square_{i\ j} \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1\cdot\mathbf{S}_1 + (1\leftrightarrow 2)\ |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square_{i\ j} \rangle - \frac{\delta_{ij}}{3} \langle \square \rangle \right) \left(\mathbf{S}_1\cdot\mathbf{S}_2 - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}})\right)\ |V_T$$

$$+\left(\frac{2}{3}c_F^2 i \int_0^\infty dt \langle \square \rangle - 4\left(d_{sv} + \frac{4}{3}d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1\cdot\mathbf{S}_2\ |V_S$$

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2\ln m/\mu) + \ldots),\ d_{sv,vv} = O(\alpha_s^2)$ from NRQCD.

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

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E(t)

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Pineda Vairo PRD 63 (2001) 054007

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$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2}c_F\,\epsilon^{kij}\, i\int_0^\infty dt\, t \langle \boxed{i\quad j} \rangle\, \mathbf{L}_1\cdot\mathbf{S}_2 + (1\leftrightarrow 2)\ |V^{(2)}_{LS}$$

E(t), B

$$-\frac{r^k}{r^2}\left(c_F\,\epsilon^{kij}\, i\int_0^\infty dt\, t\, \langle \boxed{i\quad j} \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1\cdot\mathbf{S}_1 + (1\leftrightarrow 2)\ |V^{(1)}_{LS}$$

$$-c_F^2\hat{r}_i\hat{r}_j i\int_0^\infty dt\left(\langle \boxed{i\quad j} \rangle - \frac{\delta_{ij}}{3}\langle \boxed{} \rangle\right)\left(\mathbf{S}_1\cdot\mathbf{S}_2 - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}})\right)\ |V_T$$

$$+\left(\frac{2}{3}c_F^2 i\int_0^\infty dt\langle \boxed{} \rangle - 4\left(d_{sv} + \frac{4}{3}d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1\cdot\mathbf{S}_2\ |V_S$$

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2\ln m/\mu) + \ldots)$, $d_{sv,vv} = O(\alpha_s^2)$ from NRQCD.

- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

N. B., Hee Sok Chung, A. Vairo 2106.09417, 2007.10078, see talk Wang

pNRQCD can describe also quarkonium production and, together with open quantum systems, the nonequilibrium evolution of quarkonium in medium (in heavy ions if the medium is characterised by a temperature or in a nuclear medium)

N. B., M. Escobedo, M. Strickland, A. Vairo, P. Vandergriend,J. weber 2012.01240

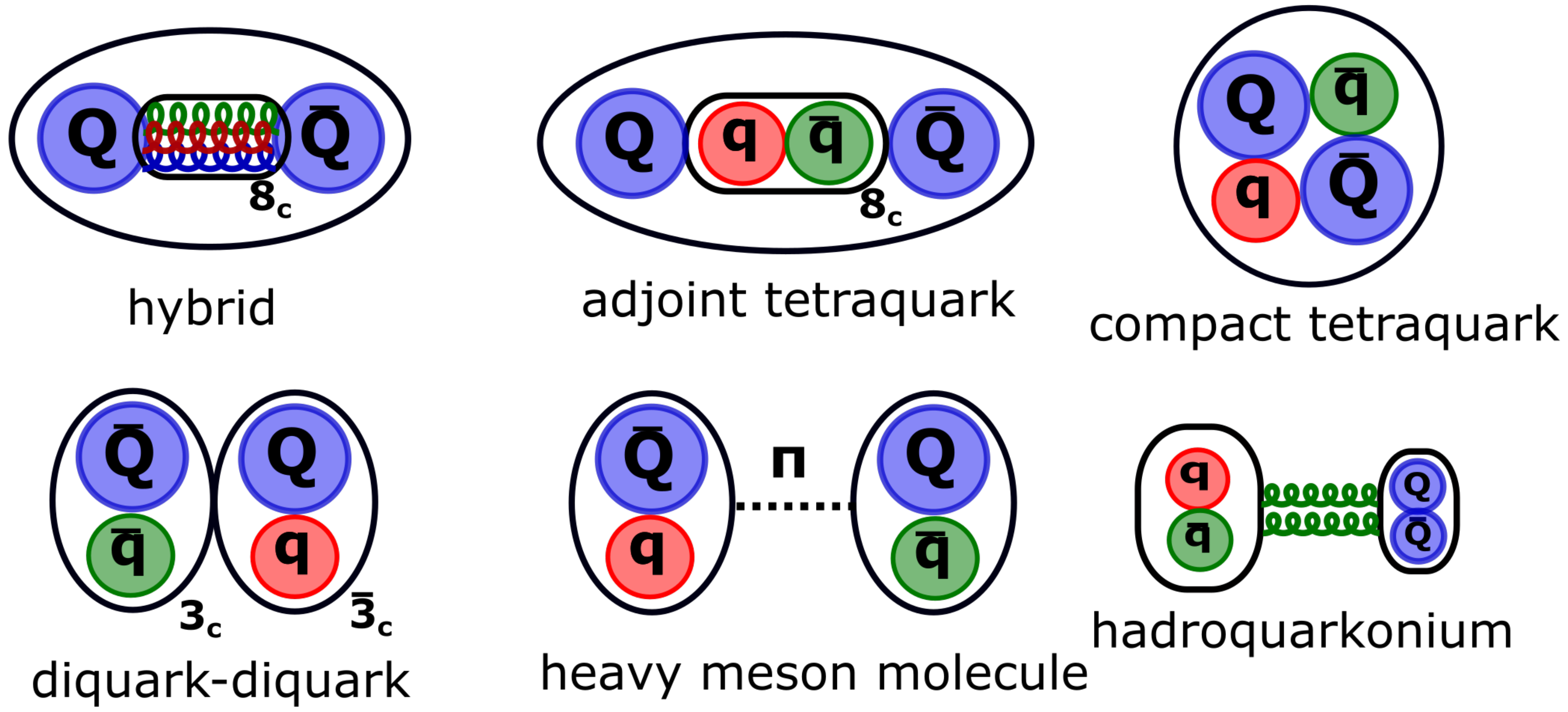
—> **which has implications on the fact that BOEFT could do the same**

X Y Z : close or above the quarkonium strong decay threshold

the situation is much more complicate

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple, nor to gluon excitations and many additional states built on the light quark quantum numbers may appear

many different configurations may appear



depending on the underlying QCD dynamics

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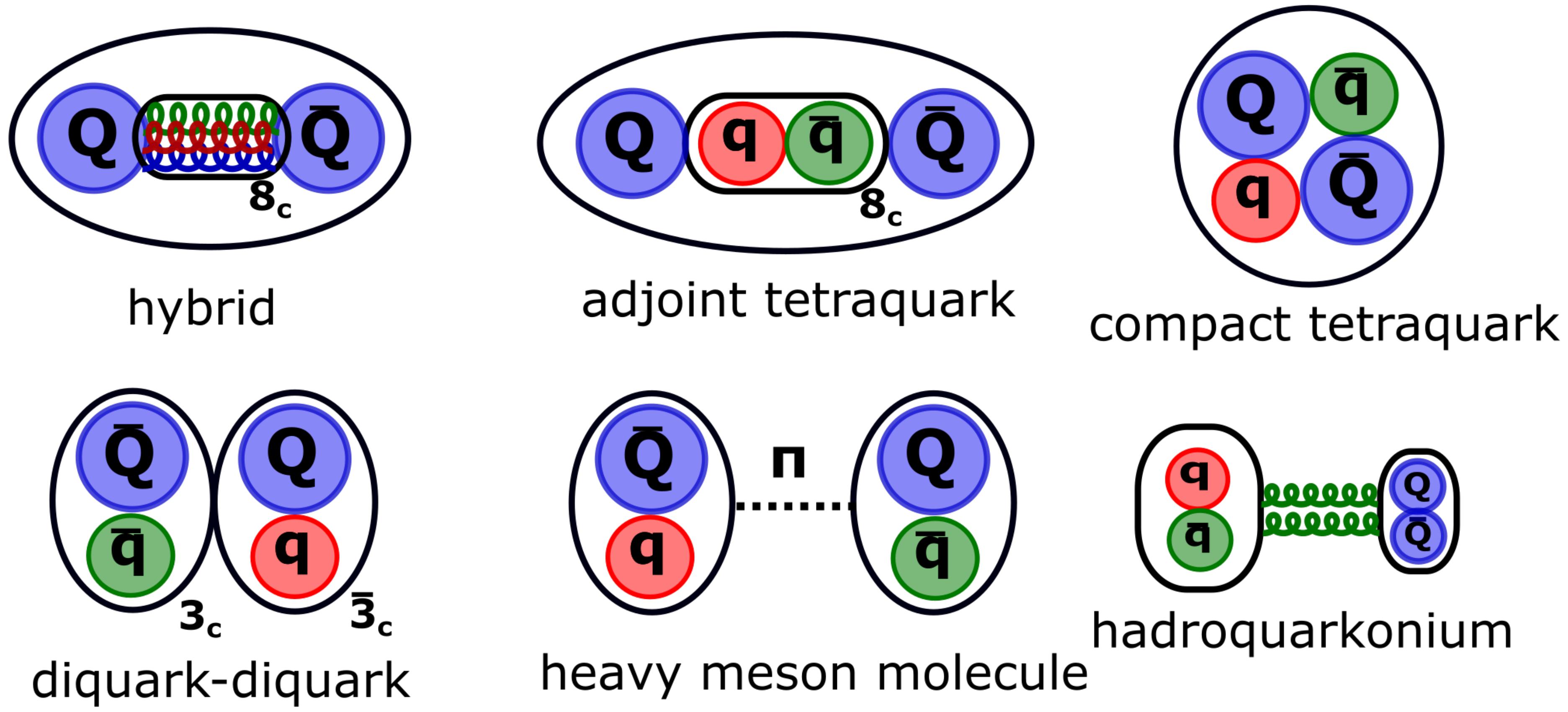


depending on the underlying QCD dynamics

Still: m is the bigger scale —> NRQCD is still valid

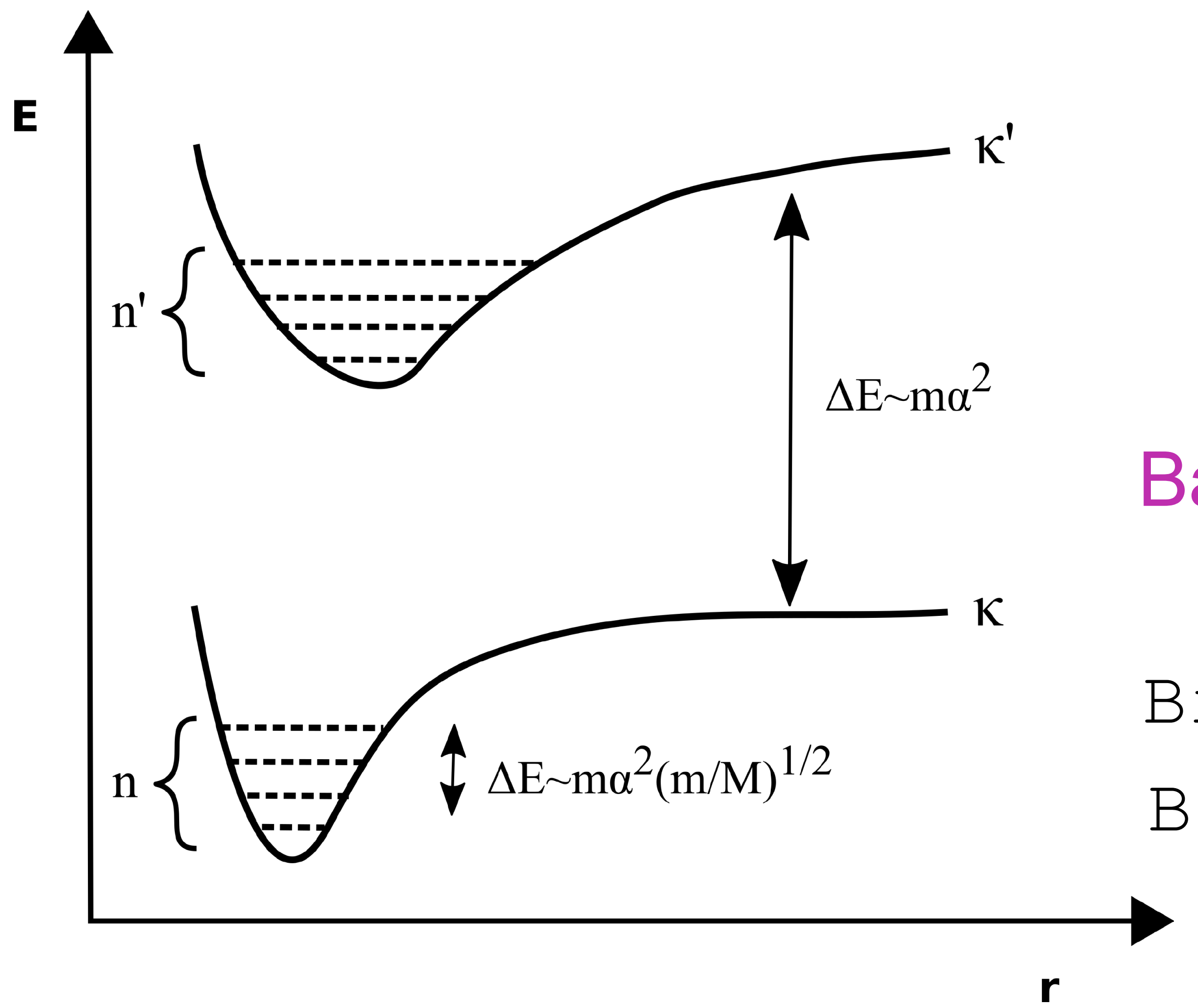
another separation of scales allows to construct an EFT —> BOEFT

BOEFT: EFT for nonrelativistic pairs and light d.o.f.

Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ($Q\bar{Q}g$ states)or tetraquarks ($Q\bar{Q}q\bar{q}$ states):

- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential V_κ between static sources, where κ labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials V_κ by solving the corresponding Schrödinger equation.

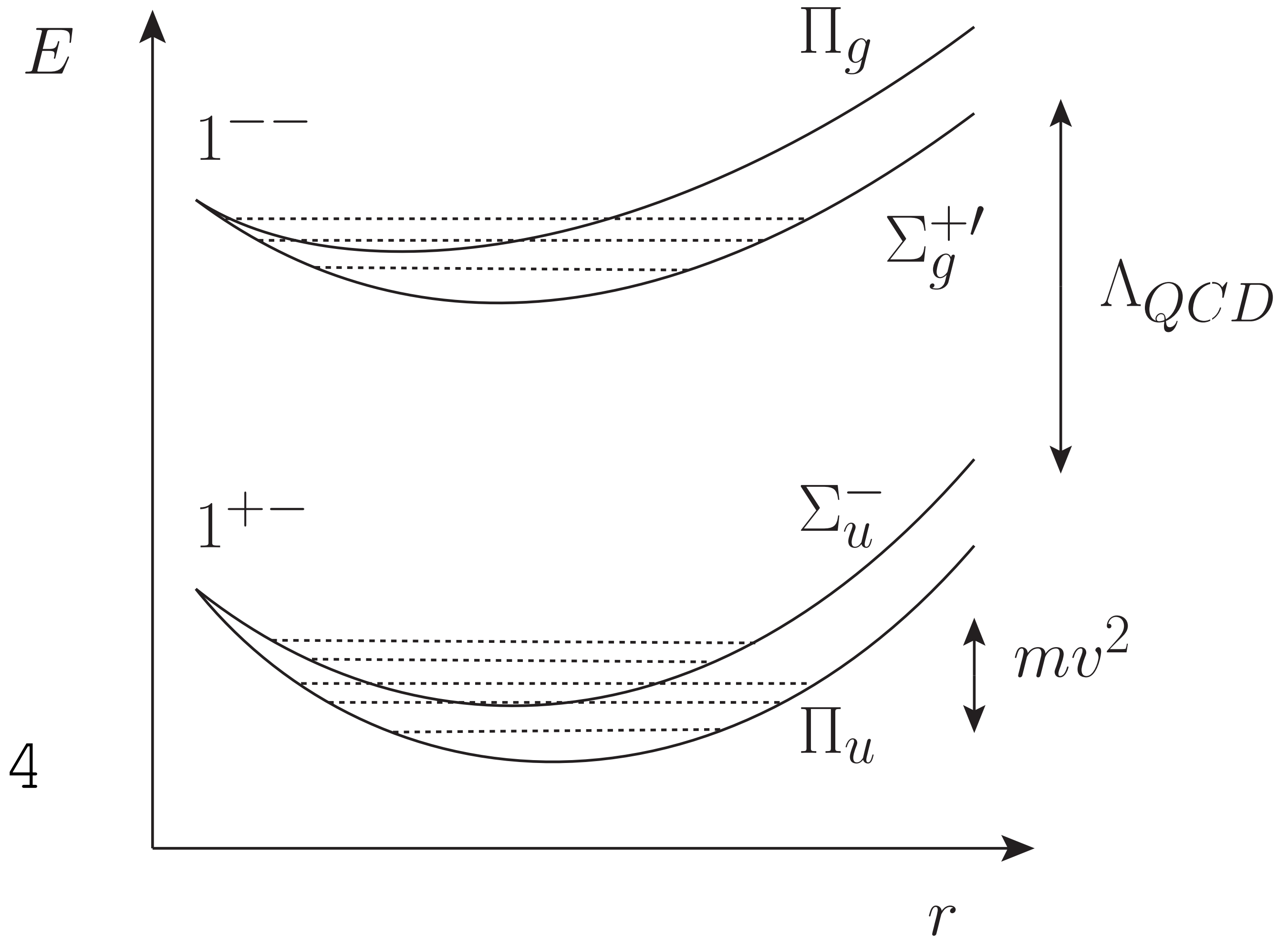
This picture goes also under the name of Born-Oppenheimer approximation. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called Born–Oppenheimer EFT (BOEFT).



Lattice evaluation of the QCD static energies:
Michael et al. 1983,
Juge, Kuti, Morningstar 1997, 1998,
Bali Pineda 2004, Capitani, Philipsen, Reisinger, Riehl, Wagner 2018

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044



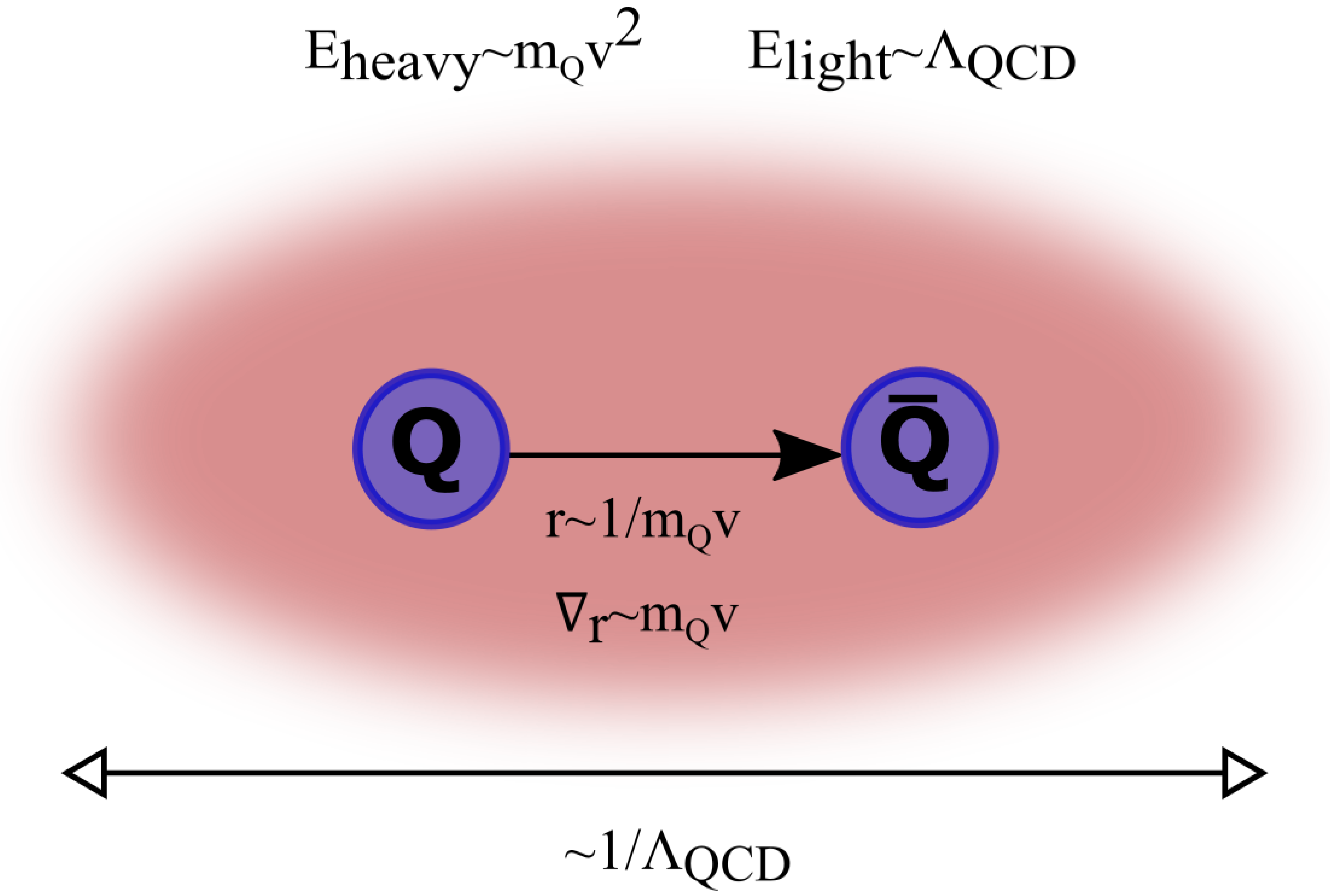
Focus on hybrids

two different scales

$\Lambda_{\rm QCD} \gg mv^2$

we proceed to integrate out 1/r and then $\Lambda_{\rm QCD}$
(or simultaneously see Soto, Tarrus)

- 2005.00552



analogous to

$E_{electrons} \gg E_{nuclei}$

in QED

$\Lambda_{\rm QCD}$

Focus on hybrids

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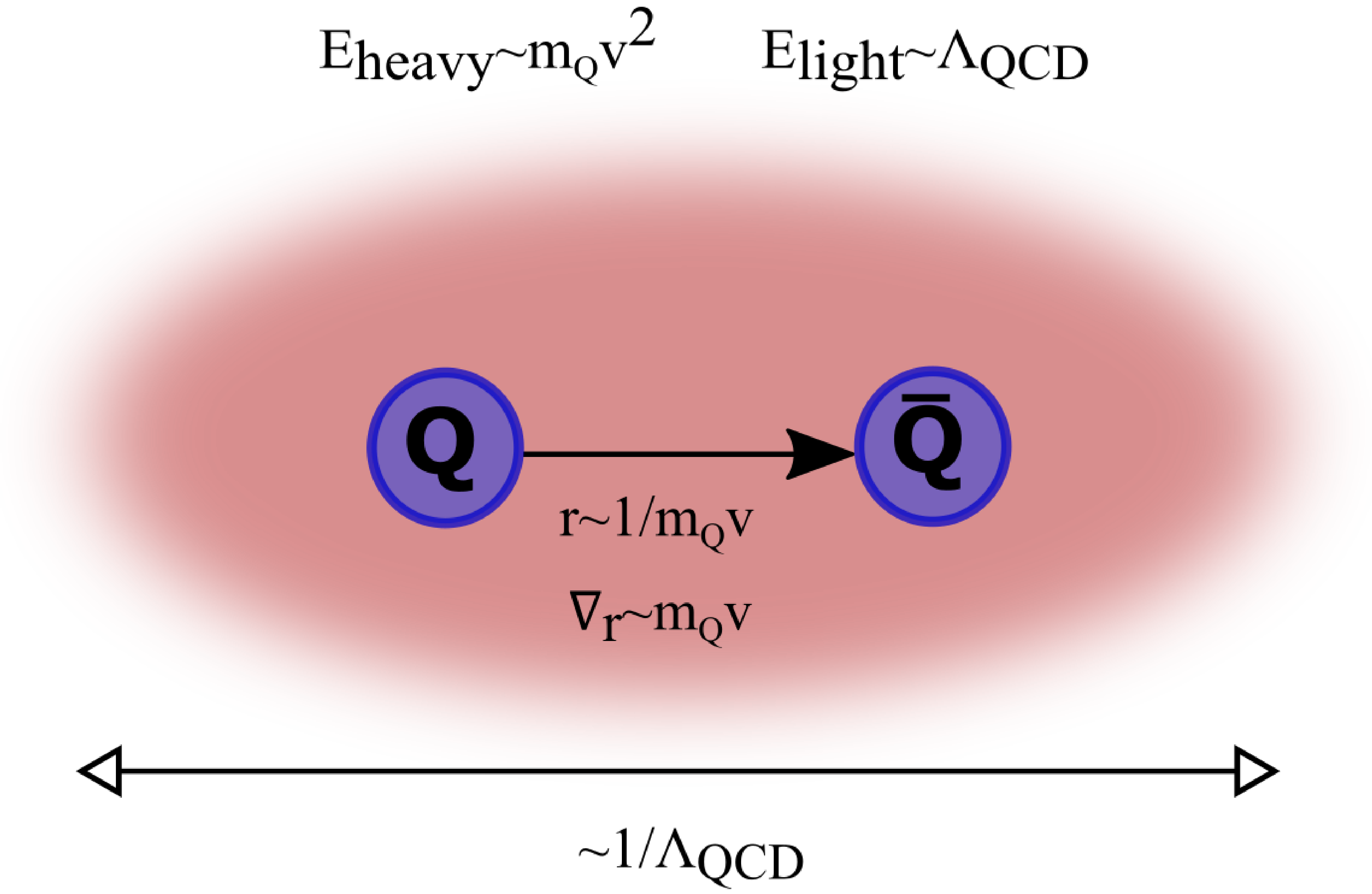


analogous to

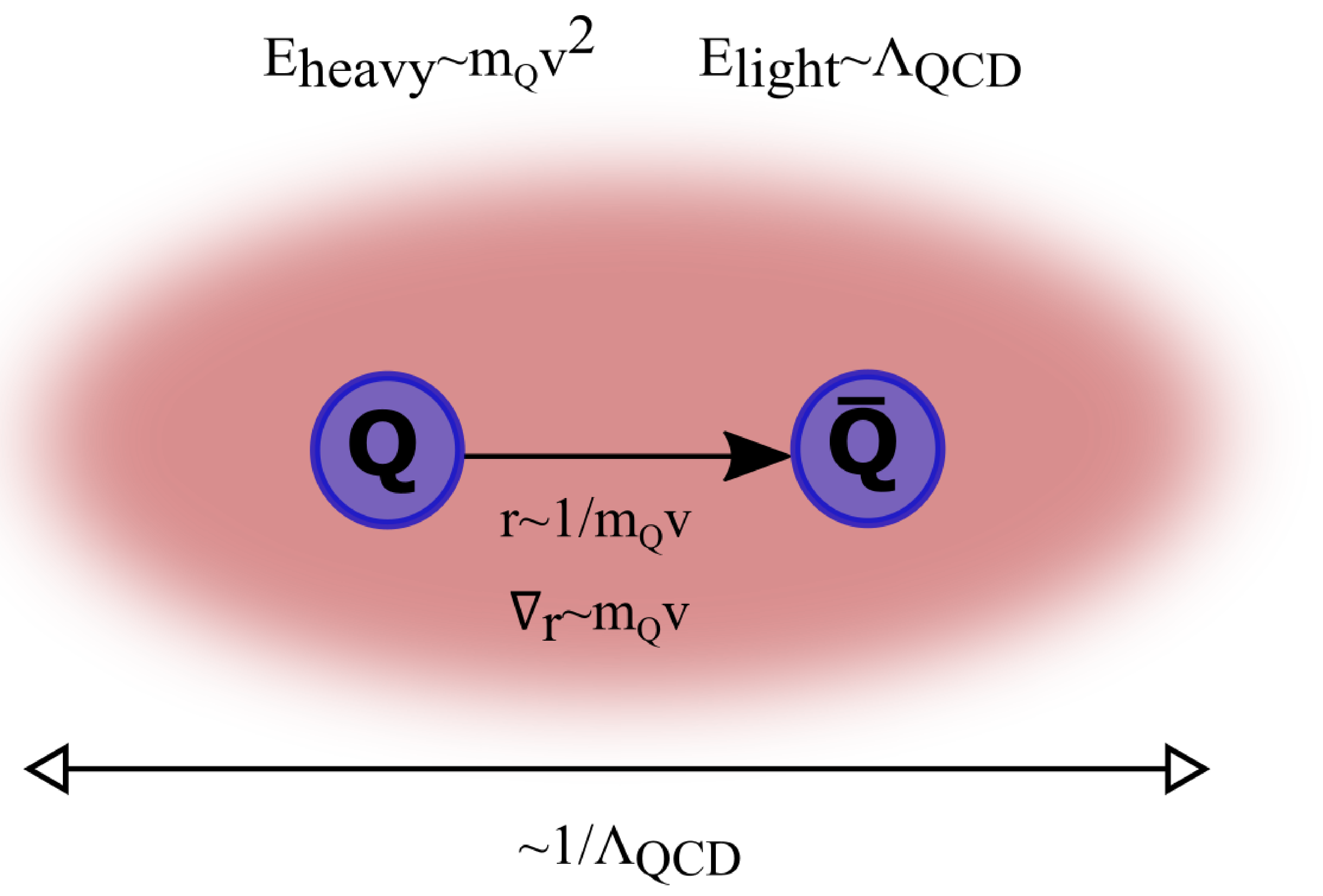
$E_{electrons} \gg E_{nuclei}$

in QED

$\Lambda_{\rm QCD}$ is nonperturbative but we can use the lattice to calculate the appropriate gluonic static energies (corresponding in molecular physics to the electronic static energies)

Focus on hybrids

We need the static energies for the lattice

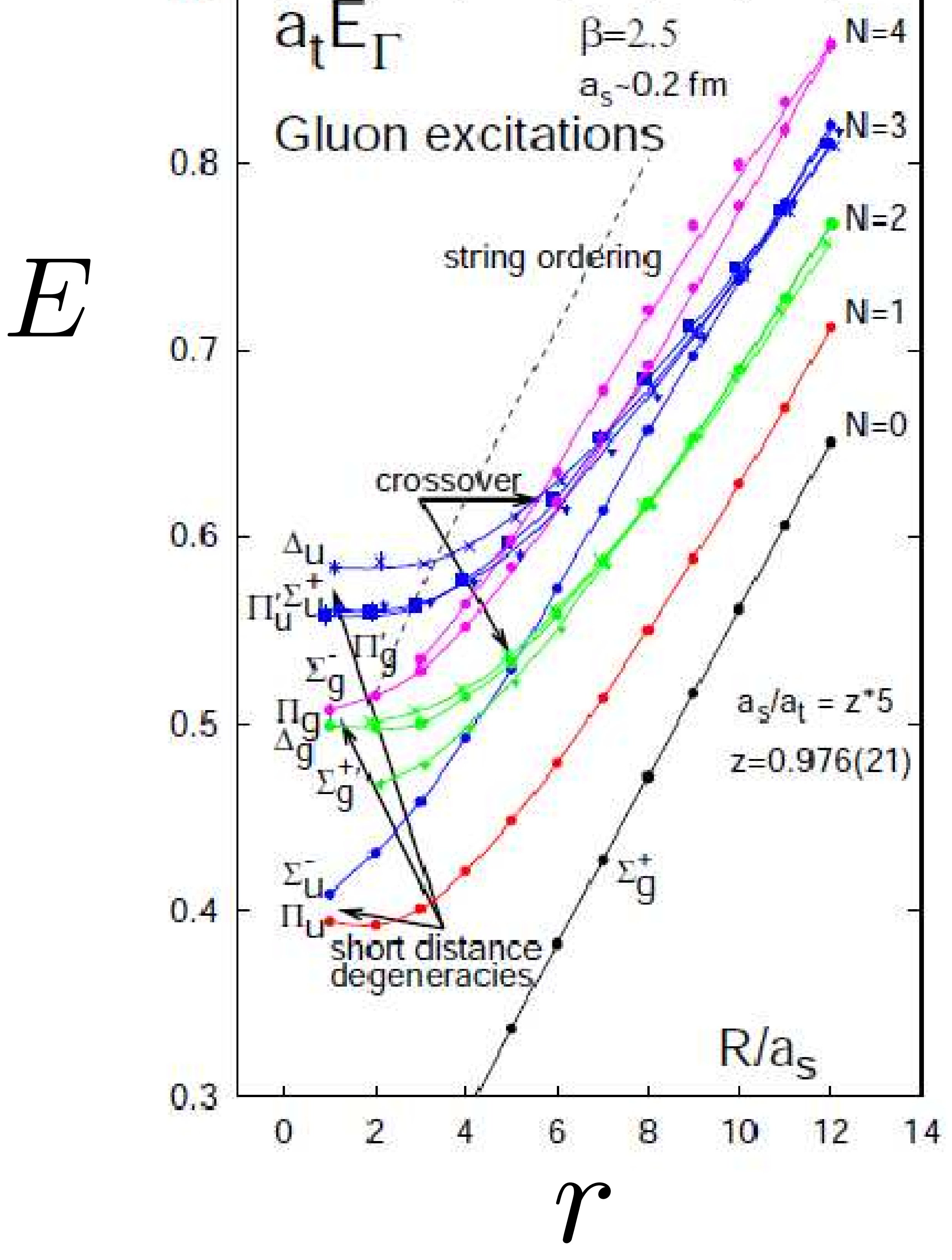


$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n,\, T/2 | X_n,\, -T/2\rangle$$

→ wilson loop

$$|X_n\rangle = \chi(\mathbf{x_2})\phi(\mathbf{x_2},\mathbf{R})T^a H^a(\mathbf{R})\phi(\mathbf{R},\mathbf{x}_1)\psi^\dagger(\mathbf{x_1})|\mathrm{vac}\rangle$$

Phi wilson lines and H gluonic operator with the correct quantum numbers



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

Bali Pineda PRD69 (2004) 094001

- Juge Kuti Morningstar PRL 90 (2003) 161601

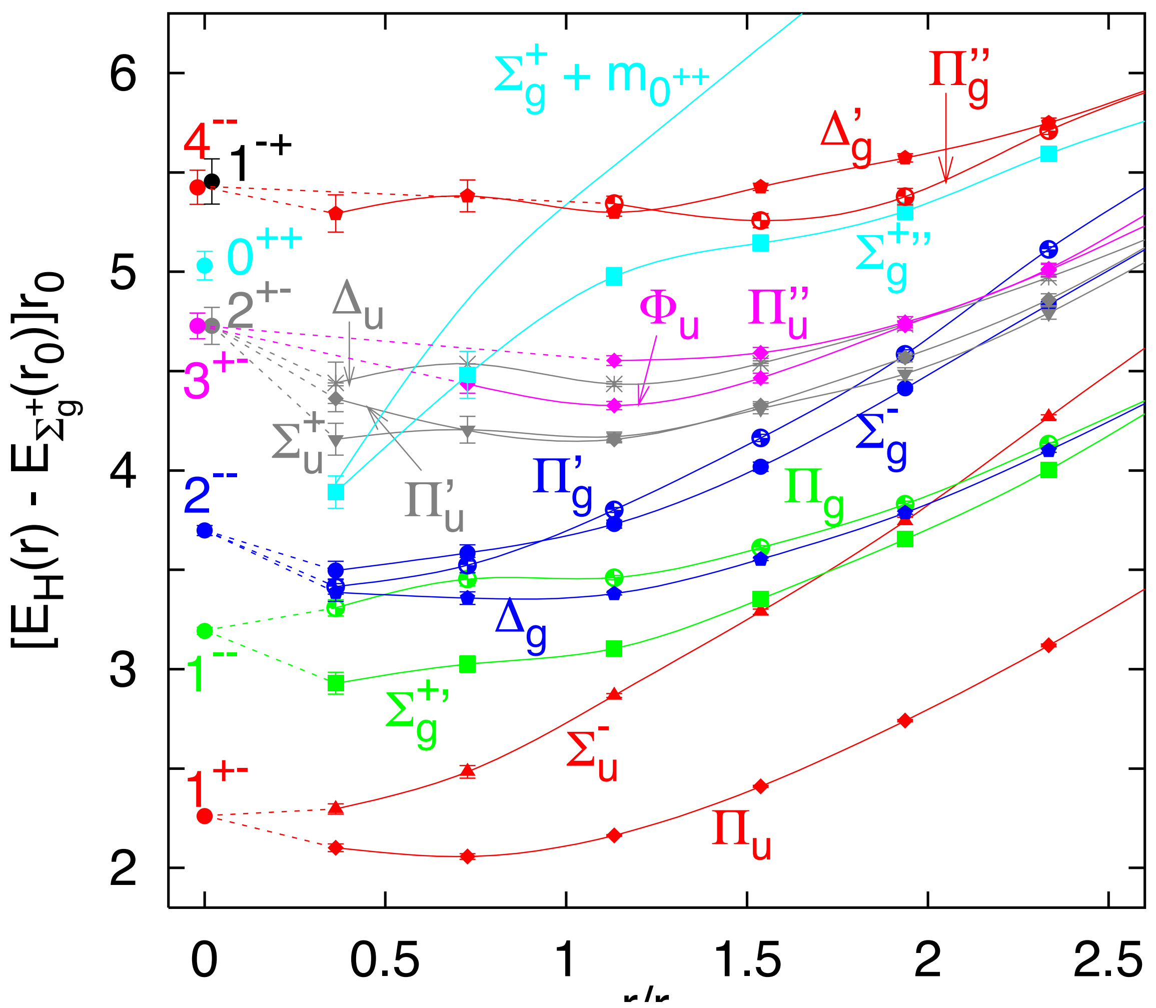
Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 03450

Schlosser, Wagner 2111.00741

We understand the static energies —>

The BOEFT characterises the hybrids static energy for short distance

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.



the hybrid static energy can be written as a (multipole) expansion in r:

$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

octet potential

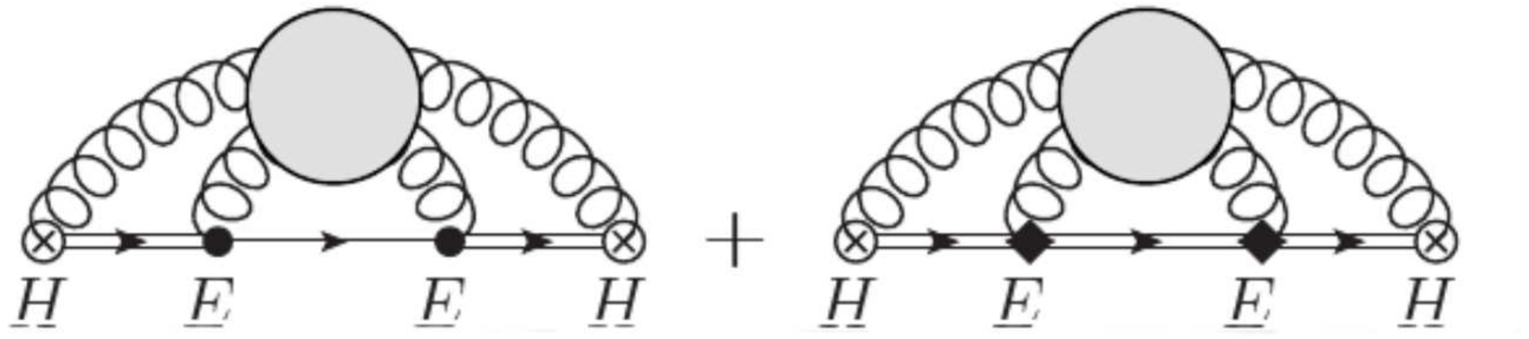
non perturbative coefficient

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi_{ab}^{\text{adj}}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

The BOEFT gives the set of coupled Schroedinger equation and the recipe to construct multiplets

$(\Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u)$

Hybrids Multiplets

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-. In the $r \to 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}, E_{\Pi_u}$
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}, E_{\Pi_u}$

the J^PCquantum numbers come from the properties of the solution of the coupled Schroedinger eqs. in BOEFT

We do not consider the quark spin here so S=0 and 1 are degenerated

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

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models may have a different multiplets structure: for example constituent gluon picture: quantum numbers obtained adding gluon and heavy quarks angular momentum—-> larger multiplets

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$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_\kappa(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

we use $\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$ as degree of freedom in BOEFT

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

$$\mathcal{L}_{\text{BOEFT for } 1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^{\dagger}_{1^{+-}\lambda}\left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_{\lambda}\frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^{i}_{\lambda'}\right)\Psi_{1^{+-}\lambda'}\right\}$$

- $\lambda = \pm 1, 0; \quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1^{+-}\lambda\lambda'} = V^{(0)}_{1^{+-}\lambda\lambda'} + \frac{V^{(1)}_{1^{+-}\lambda\lambda'}}{m} + \frac{V^{(2)}_{1^{+-}\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

$$\mathcal{L}_{\text{BOEFT for }1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$

fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda \left[\frac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\nabla^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \dfrac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

$$\left[-\frac{1}{mr^2}\,\partial_r r^2\partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E^{(0)}_\Sigma & 0 \\ 0 & E^{(0)}_\Pi\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix} = \mathcal{E}_N\begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_\Pi\right]\psi^{(N)}_{+\Pi} = \mathcal{E}_N\,\psi^{(N)}_{+\Pi}$$

Mixing remove the degeneration among opposite parity states: ->Lambda doubling existing also in molecular physics

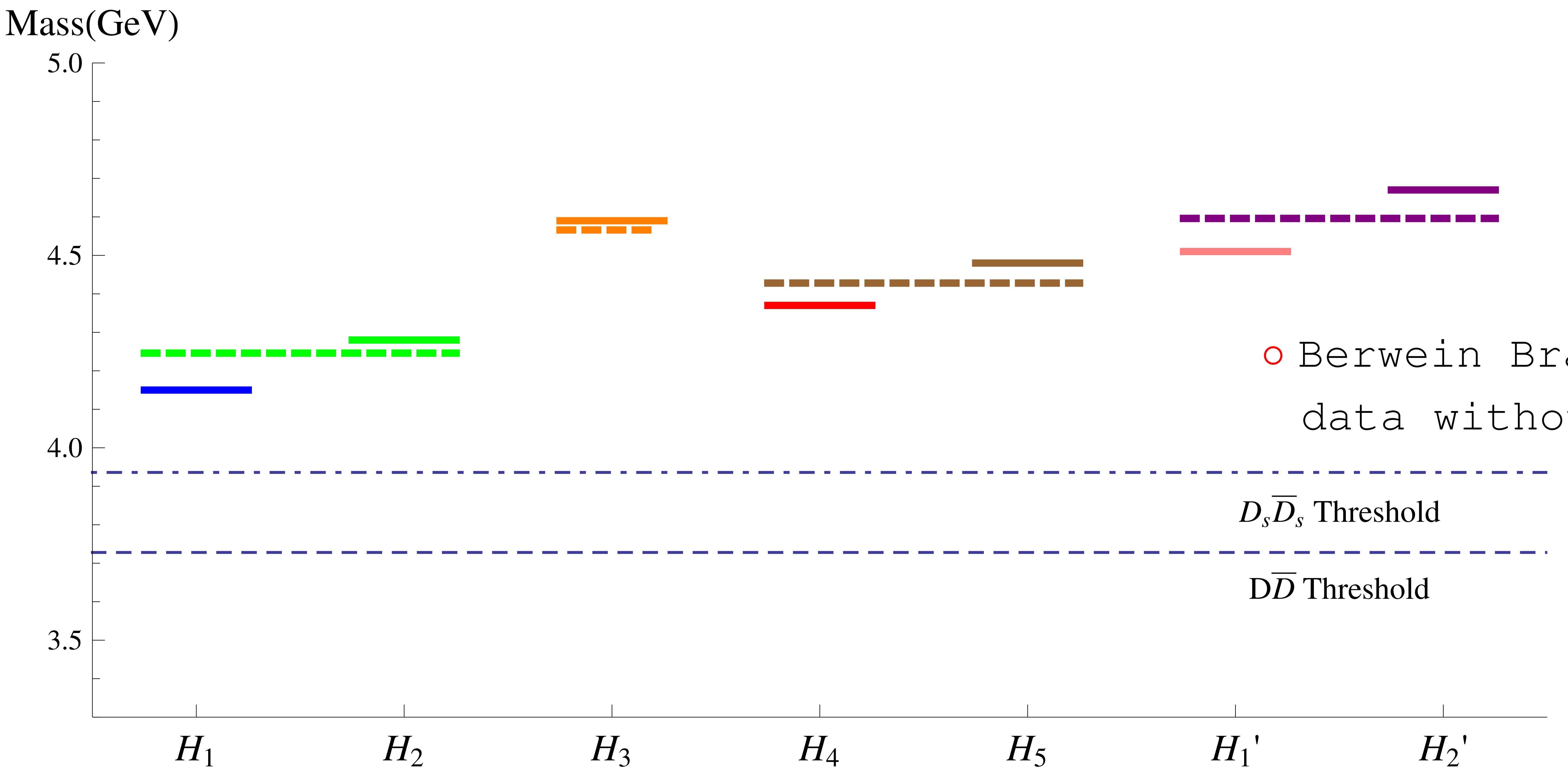
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Spectrum: general consideration

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

  The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of ± 0.15 GeV, which is the largest source of uncertainty in the hybrid masses.

Spectrum: with mixing and Λ-doubling

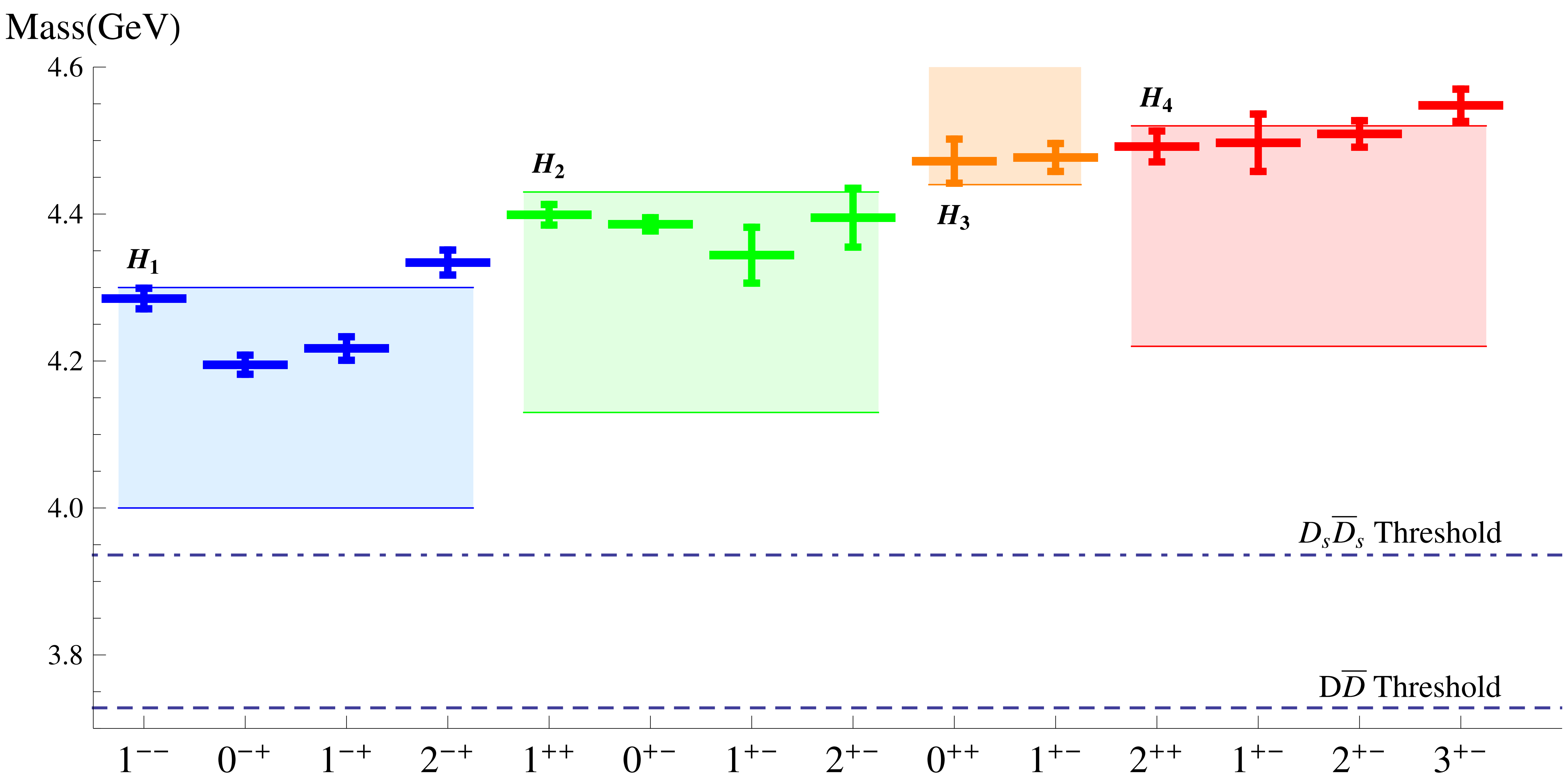


charmonium hybrids

○ Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
data without mixing (dashed) from Braaten et al PRD 90 (2014)

in BO papers
without the BOEFT
masses of opposite parity
states are degenerate

Charmonium hybrid states vs direct lattice data

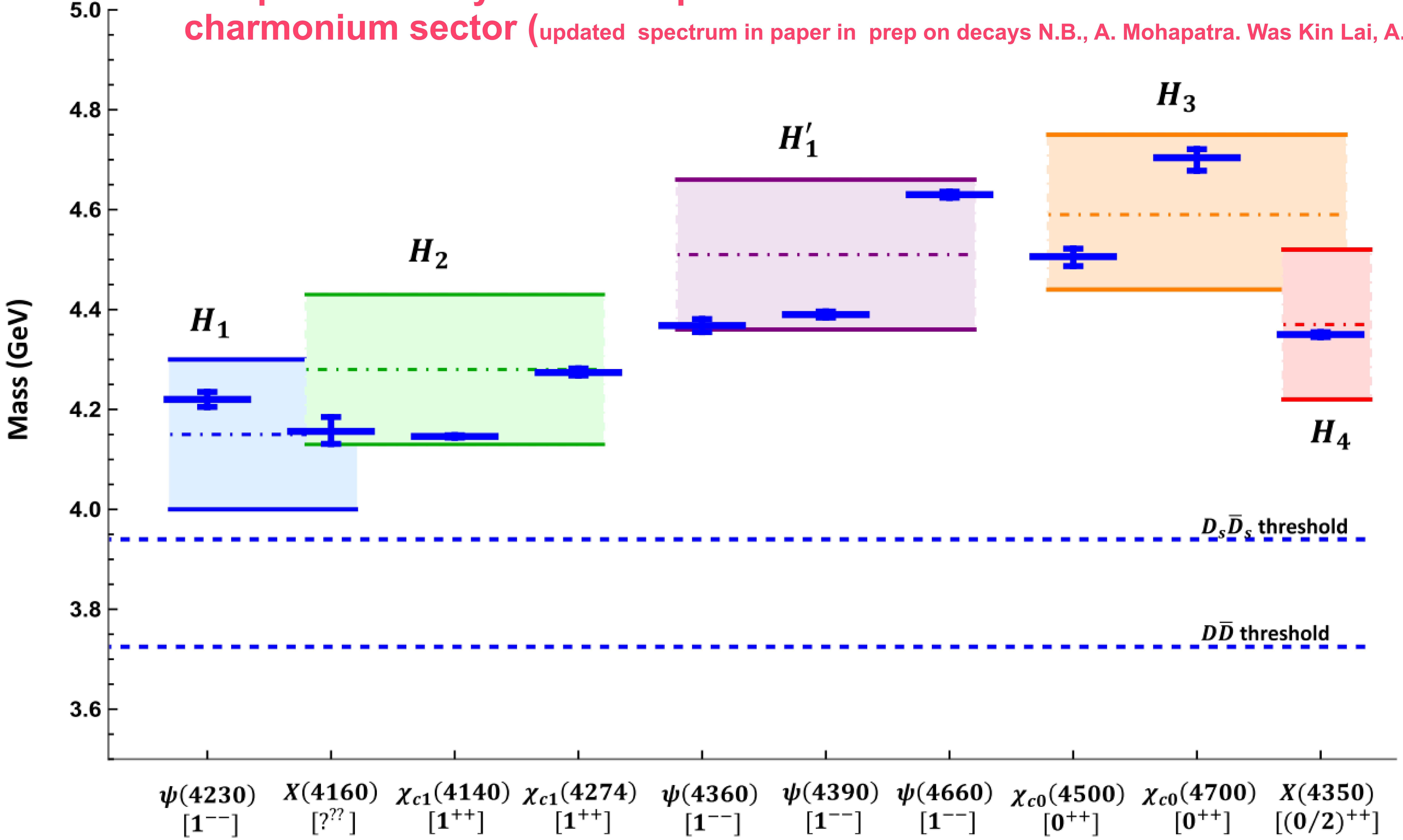


Lattice (crosses) confirms Lambda doubling (H_1 not degenerate with H_2)

Bands BOEFT predict -uncertainty comes from the uncertainty on the mass of the gluelump

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from the Hadron Spectrum coll JHEP 1207 (2012) 126
  [2+1 flavors, $m_\pi = 400$ MeV]

Comparison of hybrids multiplets with neutral isoscalar states observed in the charmonium sector (updated spectrum in paper in prep on decays N.B., A. Mohapatra. Was Kin Lai, A. Vairo 2022)



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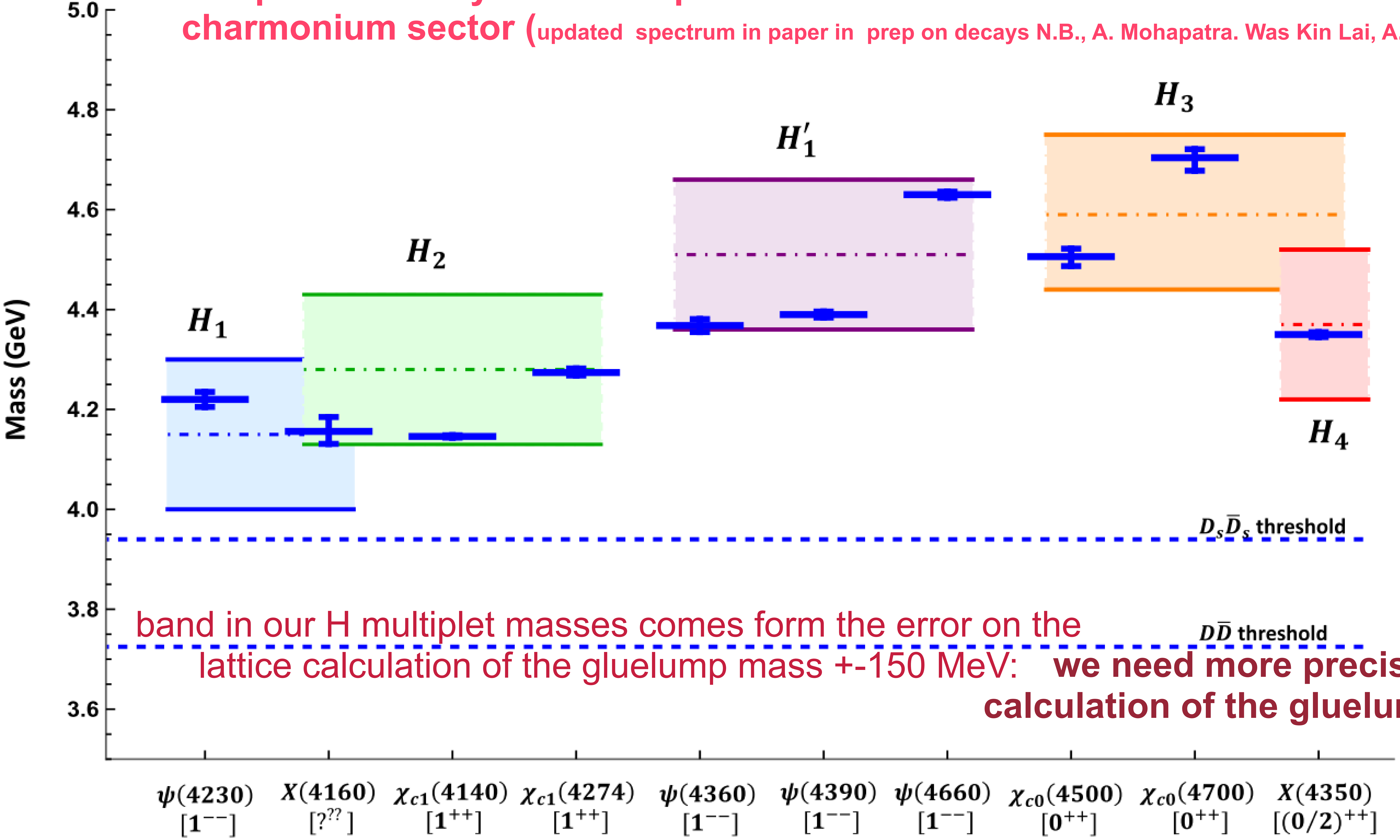
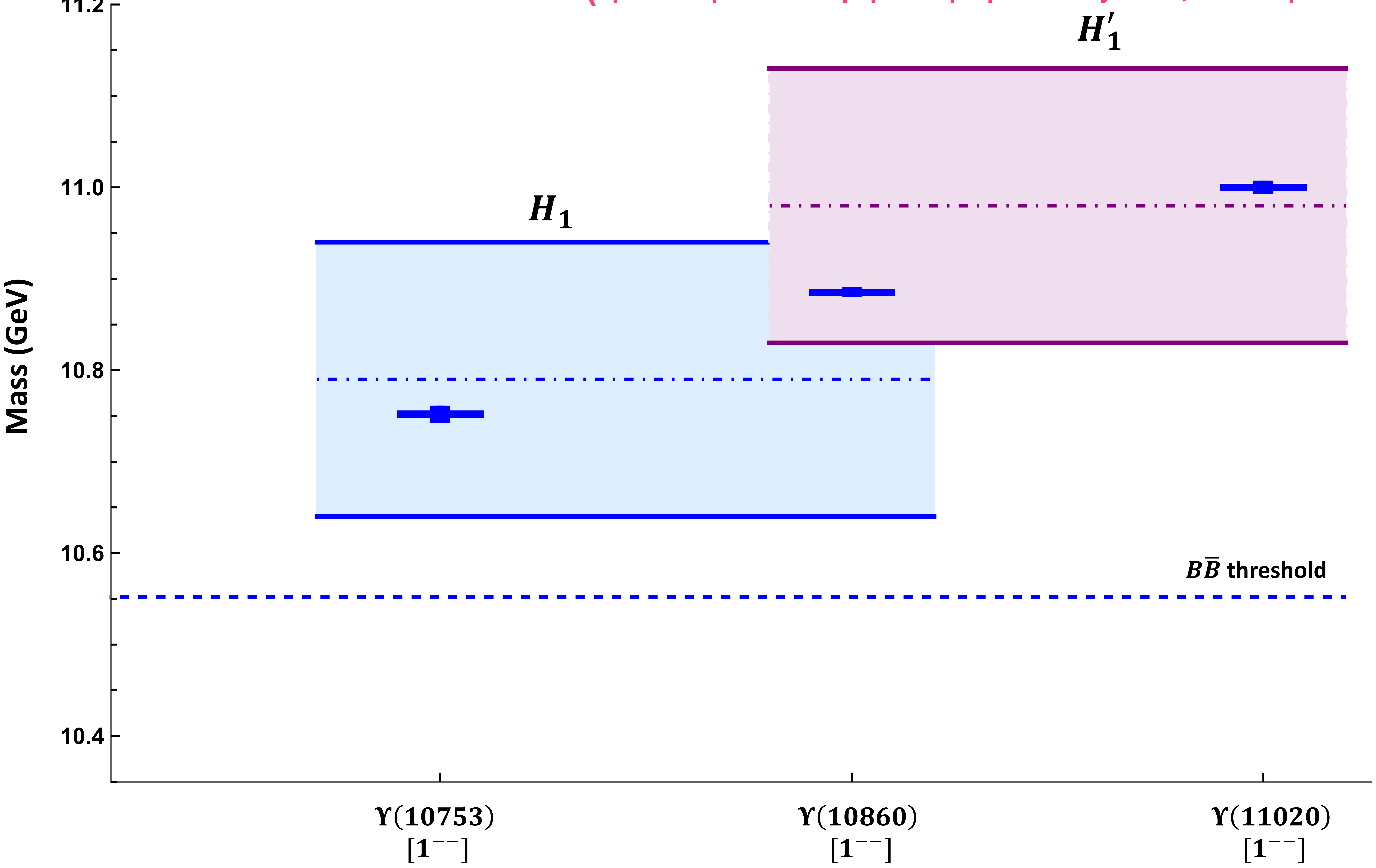


band in our H multiplet masses comes form the error on the lattice calculation of the gluelump mass +-150 MeV: **we need more precise lattice calculation of the gluelump masses !**

Comparison of hybrids multiplets with neutral isoscalar states observed in the bottomonium sector (updated spectrum in paper in prep on decays N.B., A. Mohapatra. Was Kin Lai, A. Vairo 2022)



to identify states besides the spectrum we need:

- relativistic corrections, especially **spin dependent potentials**
- mixing with quarkonium, decays and transitions: what is the width of these states? **—> calculation of hybrids to quarkonium decays**
- production
- nonequilibrium evolution of X Y Z in medium

BOEFT gives or has the potential to give all of that to us!

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

1/m^2

$$V^{(2)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

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Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

1/m^2

$$V^{(2)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Mixing with quarkonium via spin may also enhanced and decay to different spin states may be enhanced

Hybrid spin dependent potentials at order 1/m and 1/m^2

1/m

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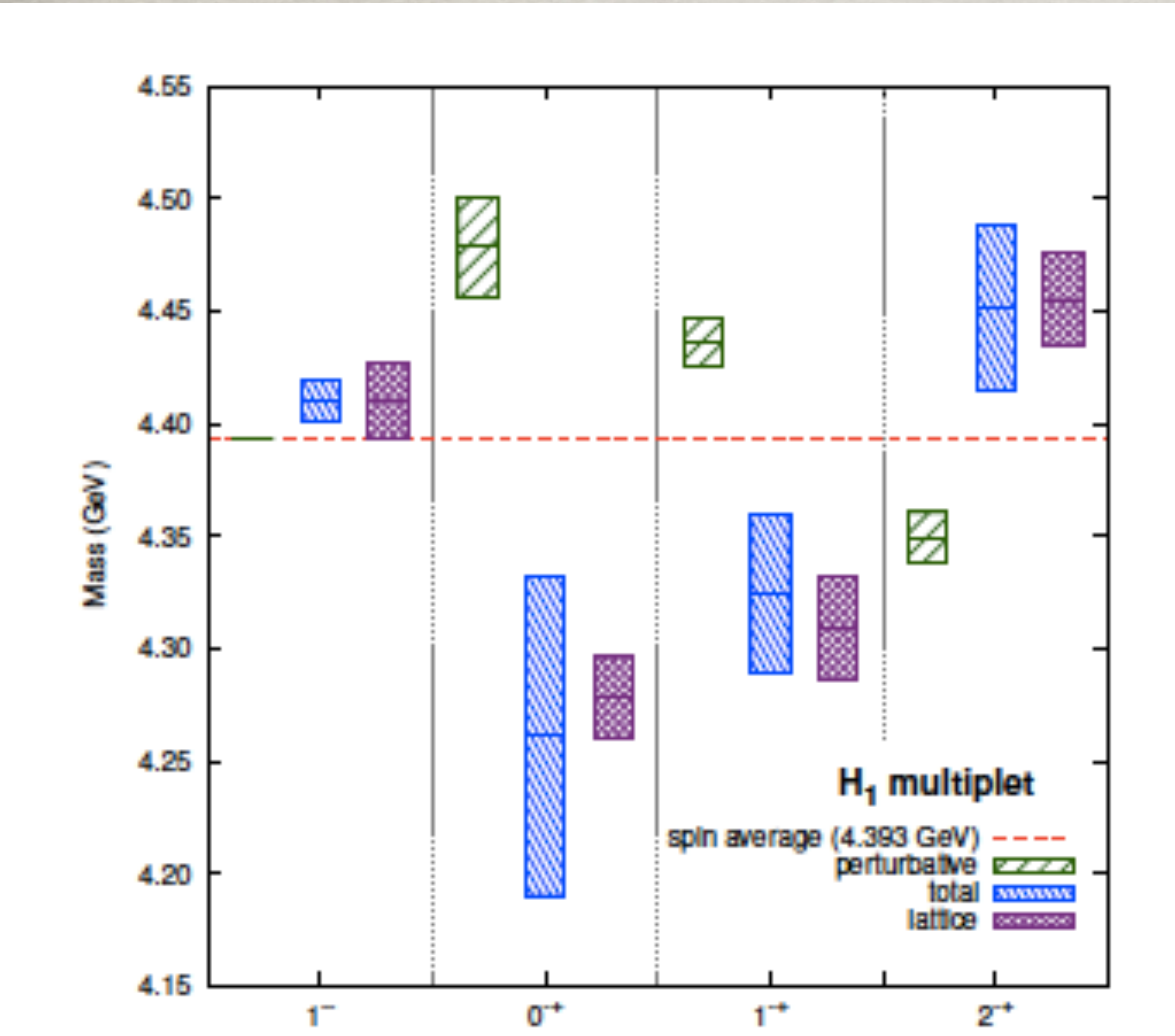
USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

Charmonium Hybrids Multiplets H_1



lattice data from (violet) from

G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

with a pion of about 240 MeV

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

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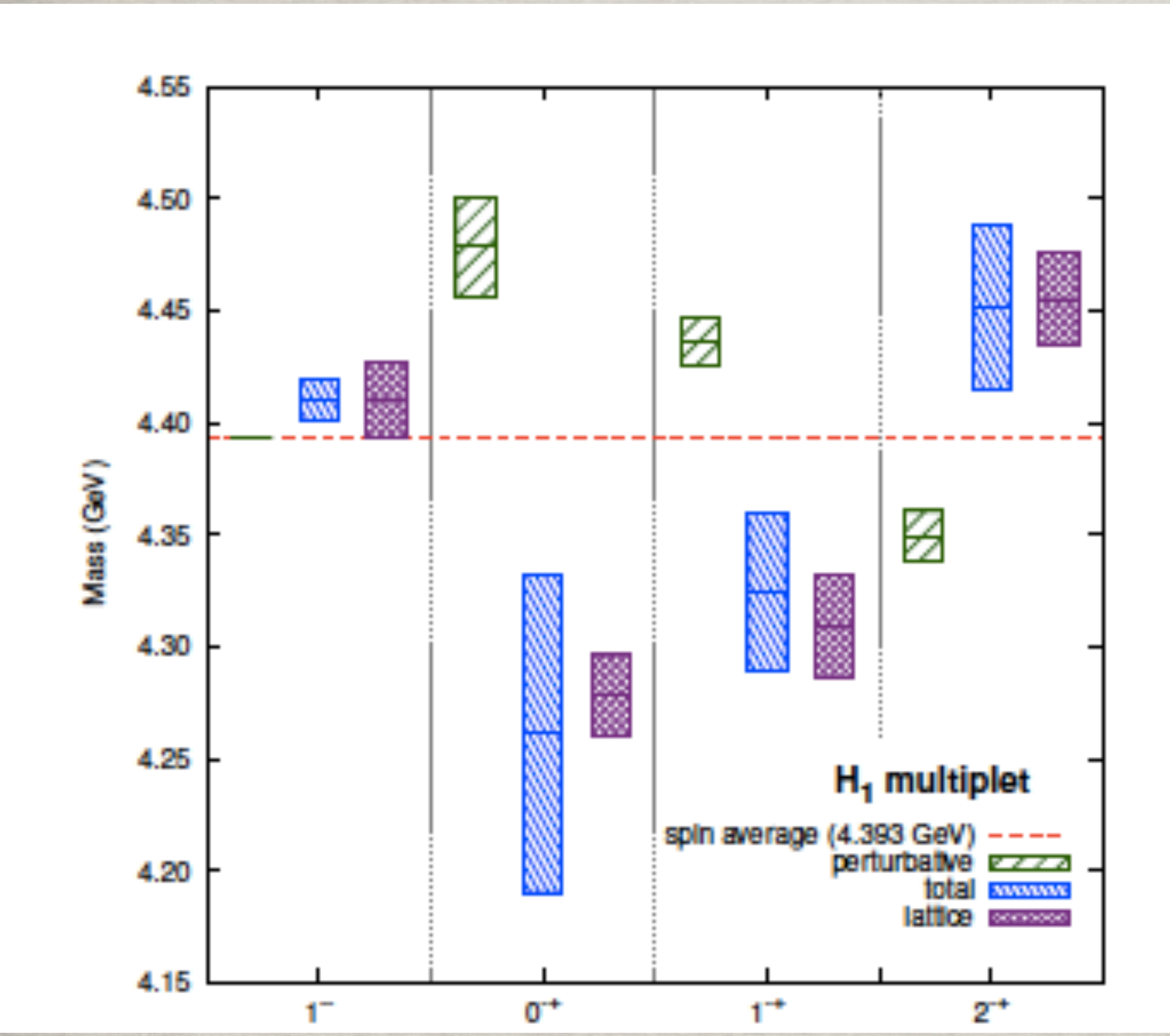

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with a pion of about 240 MeV

height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

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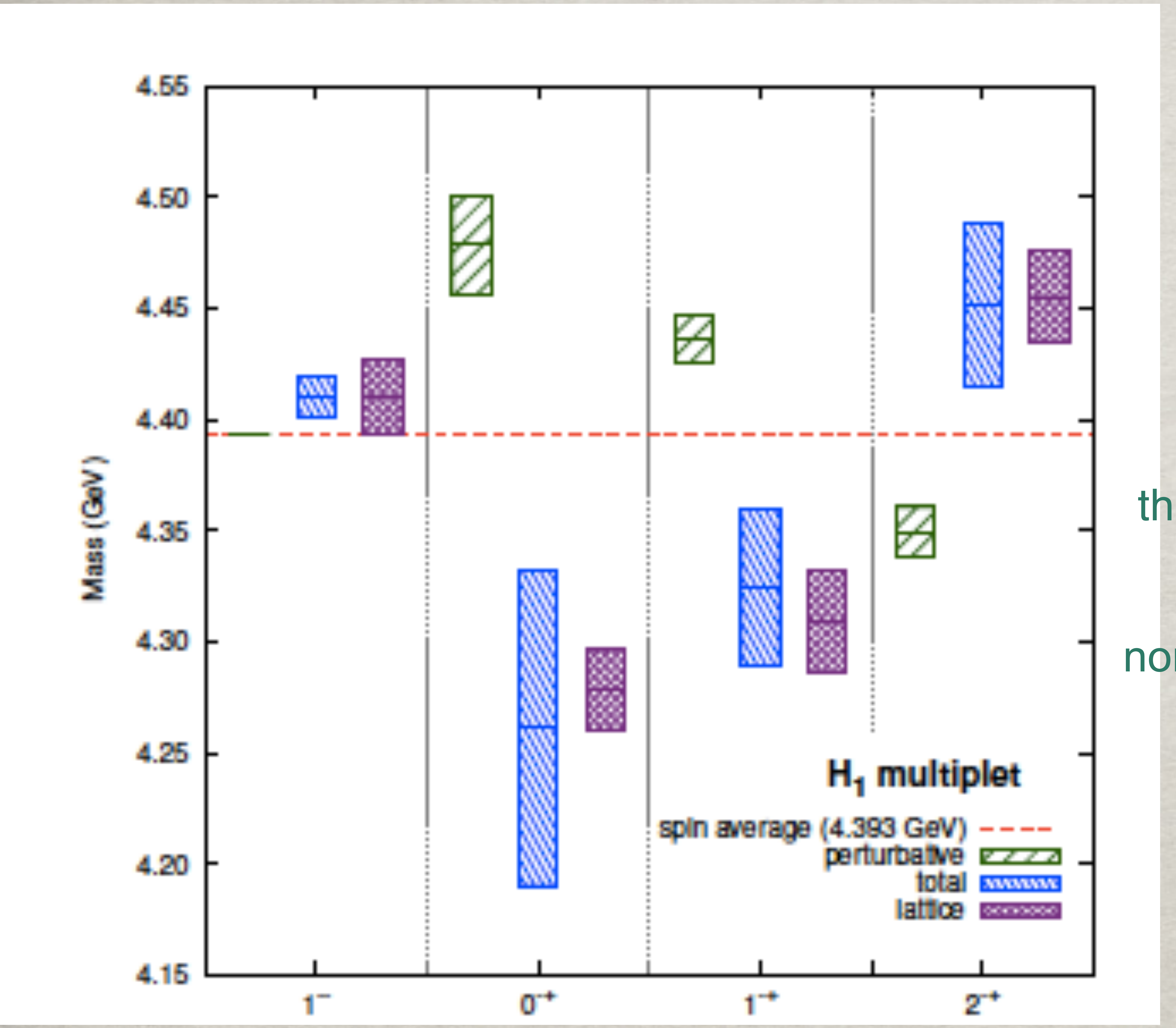

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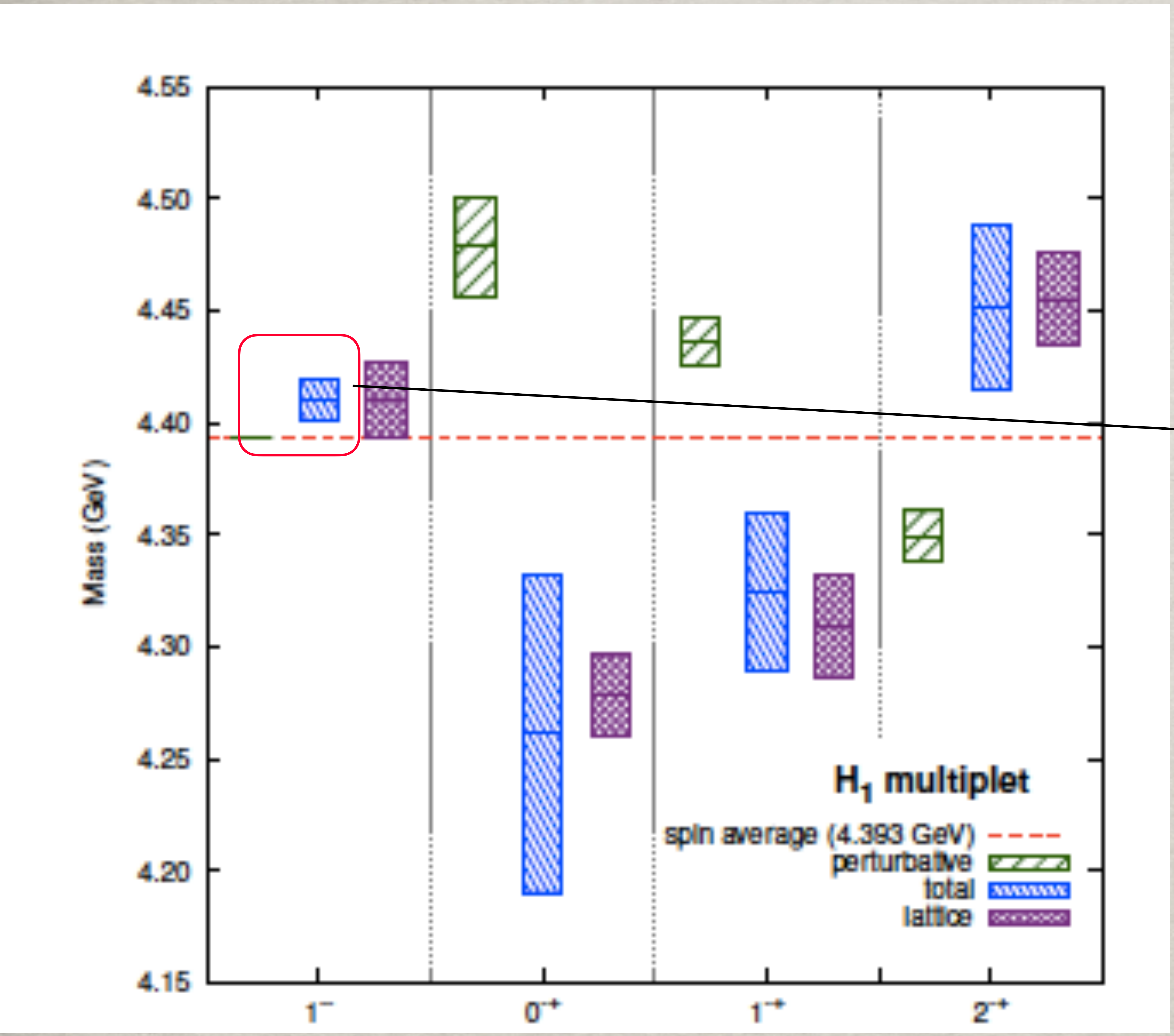
the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

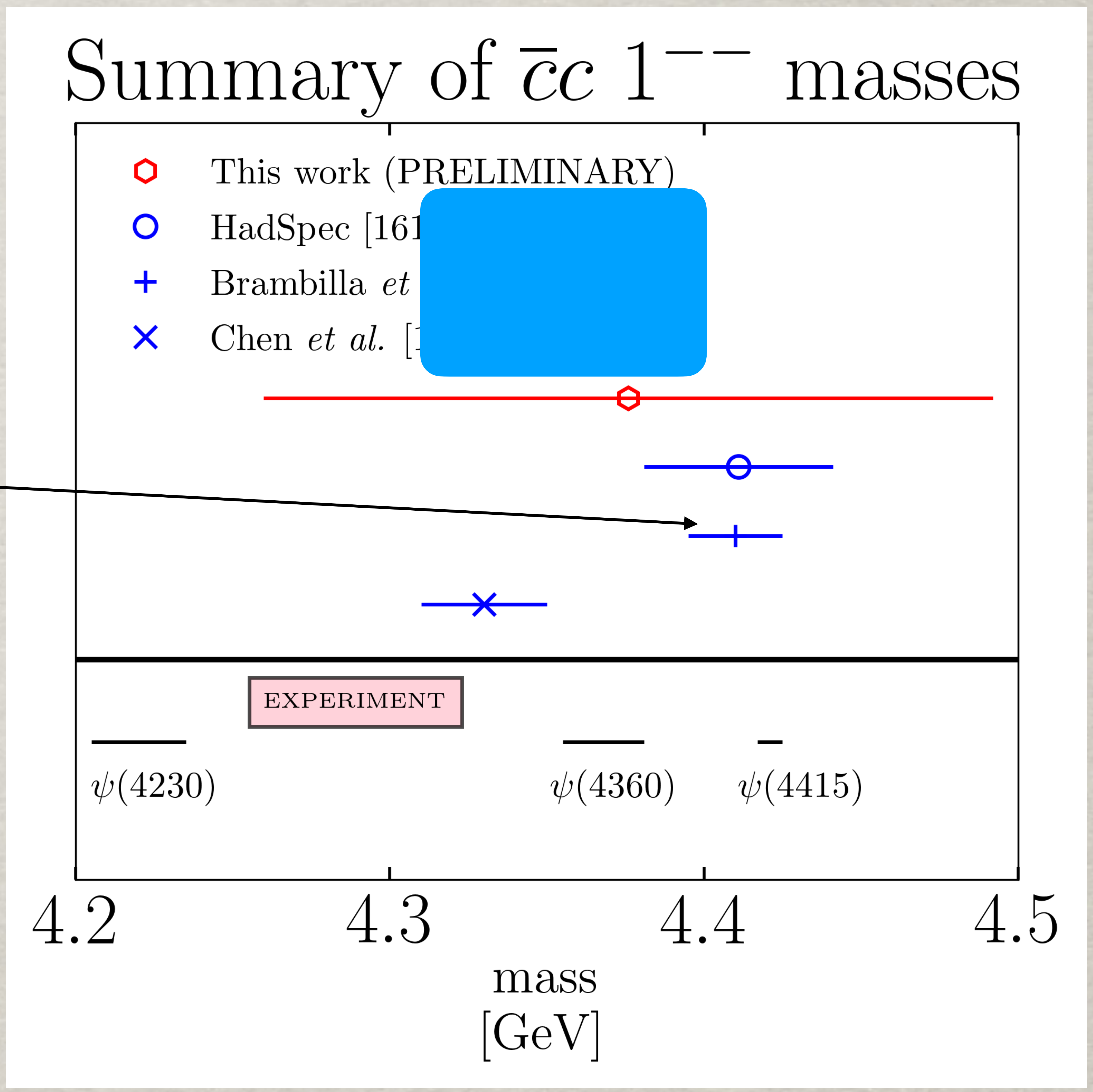
which is interesting as some models are taking the spin interaction from perturbation theory with a constituent gluon

Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1

HISQ lattice action with 2+1+1 sea quarks

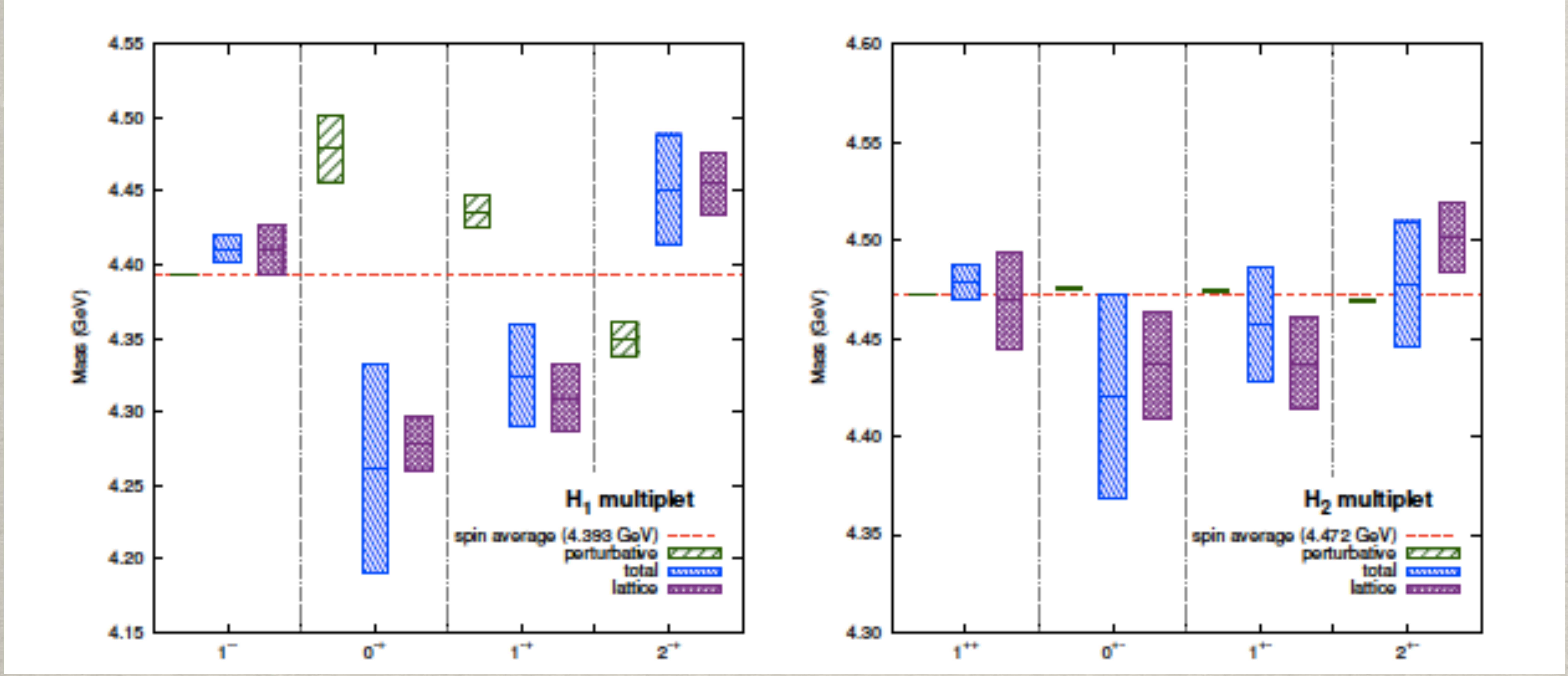




G. Ray, C. McNeile, 2110.14101

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1 and H_2

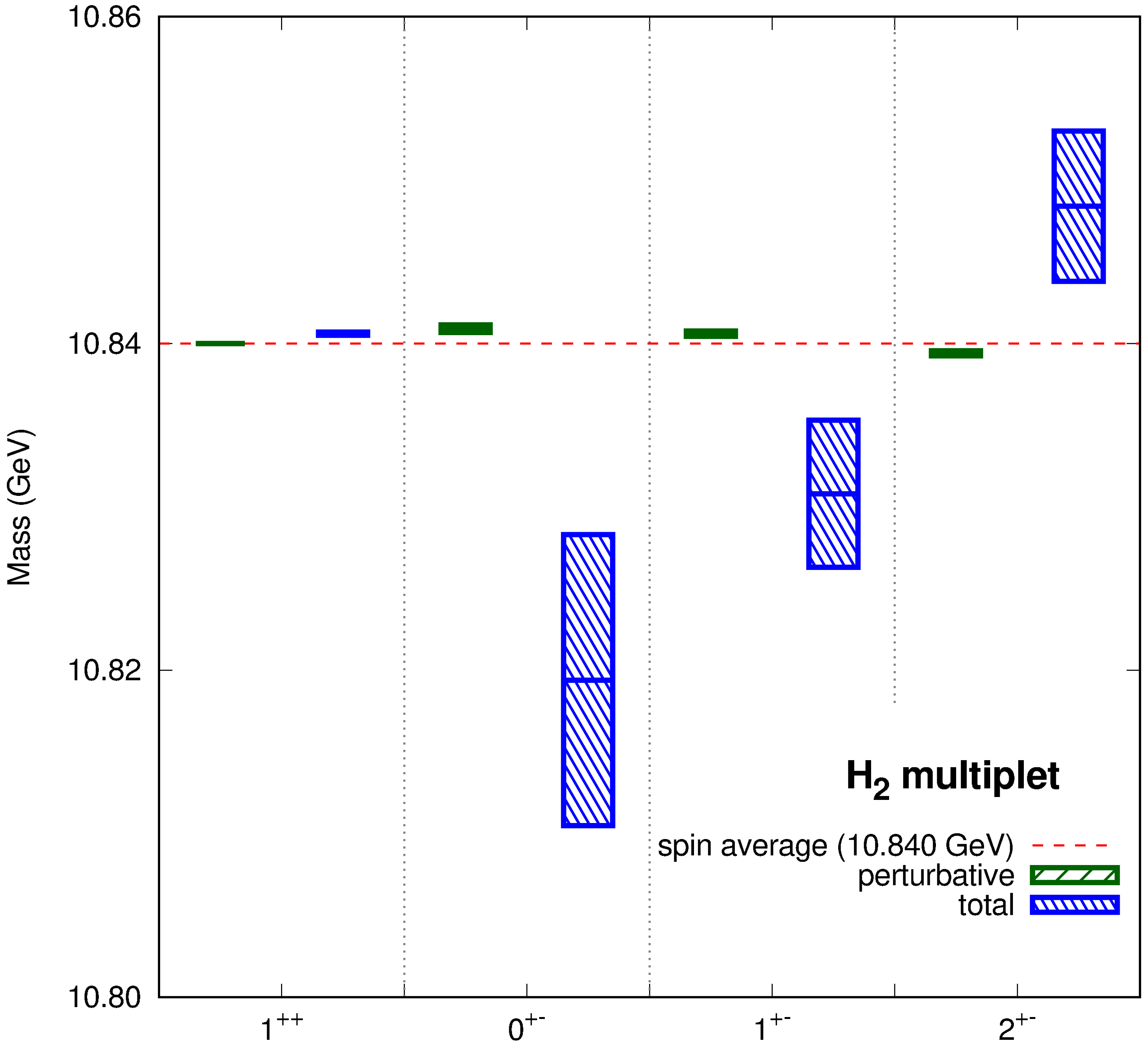


H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

H_3 and H_4 are also calculated

Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings



and also the other H multiplets

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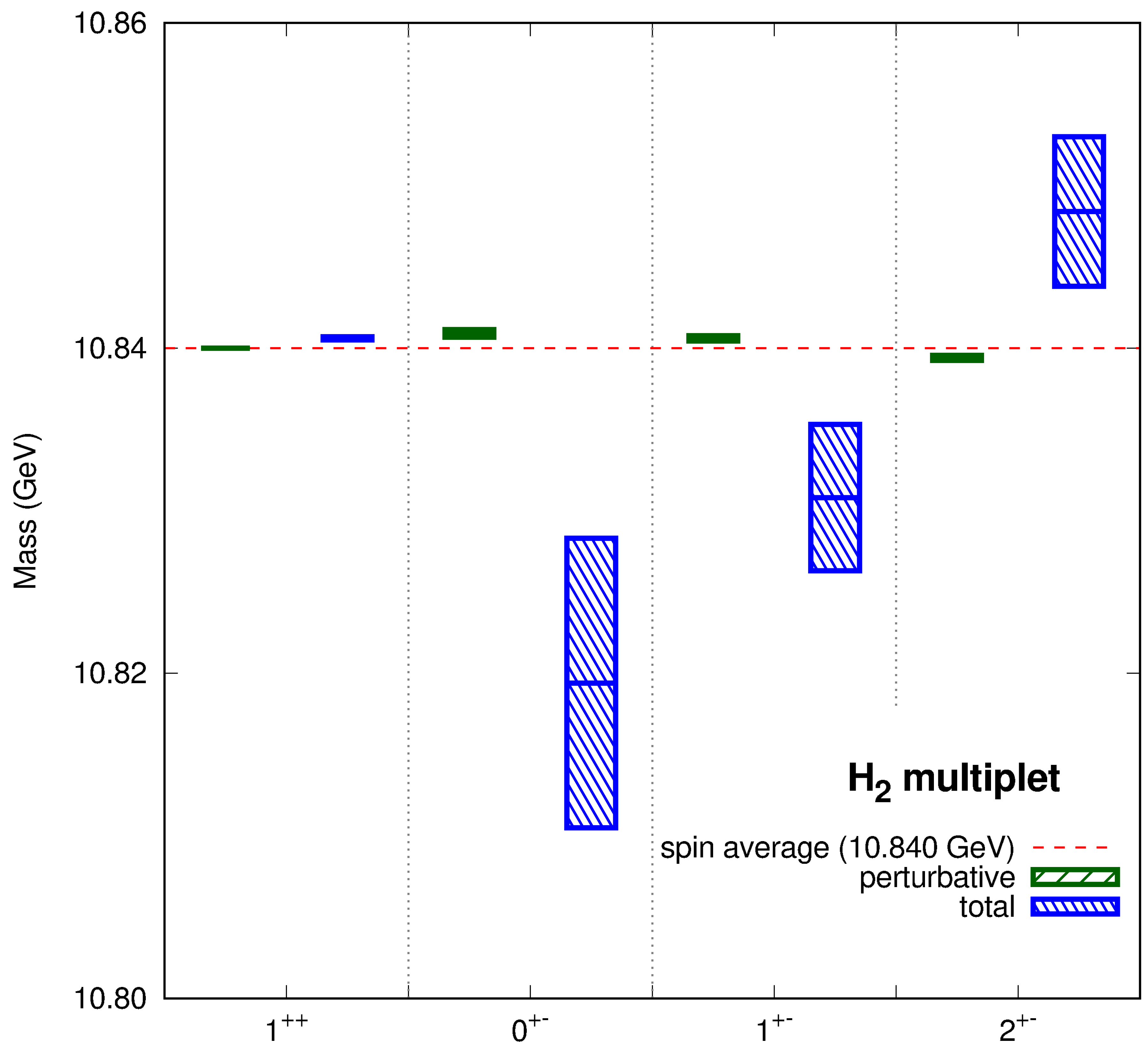
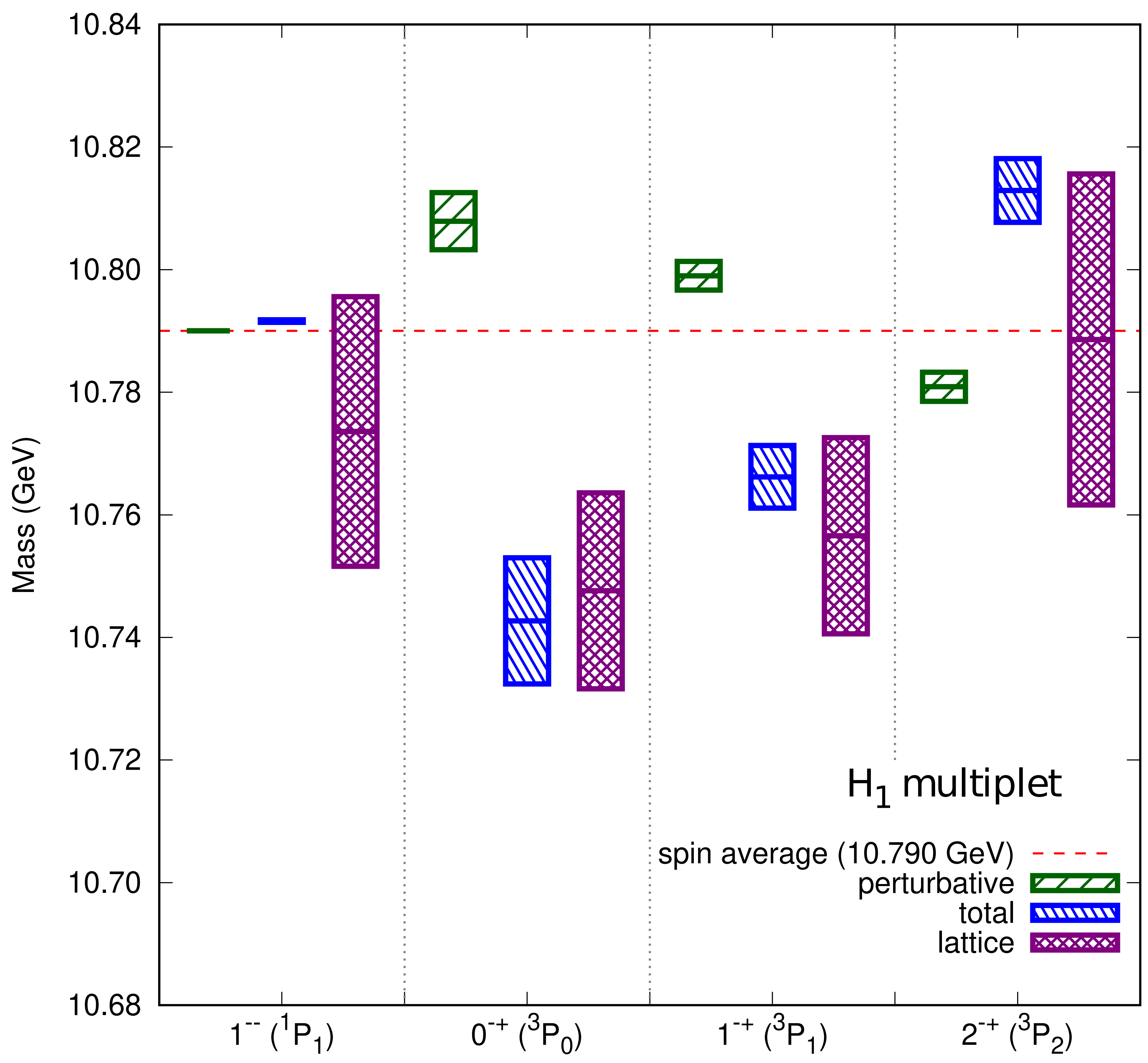


and also the other H multiplets

Comparison of our prediction to the existing lattice data on H1

Bottomonium H_1 hybrid spin splittings



blue BOEFT predictions (more precise), violet actual lattice calculation

- Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV] unpublished plot by J. Segovia and J. Tarrus

Inclusive and semi inclusive hybrids decay to quarkonia

N. B., A. Mohapatra, W.K. Lai, A. Vairo 2022

BOEFT allows to calculate these decays as

$$\Gamma_{H\to S} = -2\,\langle H|\mathrm{Im}\Delta V|H\rangle.$$

We are currently calculating all spin conserving and spin violating decays from hybrids to charmonia and bottomonia, semiinclusive (quarkonium plus X) or inclusive

Tetraquarks and pentaquarks

BOEFT may be used to describe any system made by two heavy quarks bound adiabatically with some light degrees of freedom: glue (hybrids) or light quarks (tetraquarks, pentaquark)

in case of light quarks isospin quantum numbers should be added
steps go as before:

identify the symmetries, identify the interpolating operators O_n and define the static energies

$$\mathcal{O}_n(t,\,\boldsymbol{r},\,\boldsymbol{R}) = \chi(t,\,\boldsymbol{R}-\boldsymbol{r}/2)\phi(t,\,\boldsymbol{R}-\boldsymbol{r}/2,\boldsymbol{R})H_n(t,\,\boldsymbol{R})\phi(t,\,\boldsymbol{R},\boldsymbol{R}+\boldsymbol{r}/2)\psi^\dagger(t,\,\boldsymbol{R}+\boldsymbol{r}/2)$$

$$E_n^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\mathcal{O}_n(T,\,\boldsymbol{r},\,\boldsymbol{R})|\mathcal{O}_n(0,\,\boldsymbol{r},\,\boldsymbol{R})\rangle$$

.. Examples of gluonic operators and light-quark operators for quarkonium hybrids and tetraquarks respectively, $\boldsymbol{q} = (u,\,d)$ and τ^a are isospin Pauli matrices.

Λ_η^σ	κ	H	$H = H^aT^a(I=0,\,I=1)$
Σ_g^+	0^{++}	$\mathbb{1}$	$\bar{\boldsymbol{q}}T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	1^{+-}	$\hat{\mathbf{r}}\cdot\mathbf{B}$	$\bar{\boldsymbol{q}}\,[(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\cdot,\,\boldsymbol{\gamma}]\,T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
Π_u	1^{+-}	$\hat{\mathbf{r}}\times\mathbf{B}$	$\bar{\boldsymbol{q}}\,[\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma},\,\boldsymbol{\gamma}]\,T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\mathbf{r}}\cdot\mathbf{E}$	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma})T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{--}	$\hat{\mathbf{r}}\times\mathbf{E}$	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$

needs lattice calculations of tetraquarks static energies

The direct use of the $I=1$ BO effective Lagrangian is limited by the fact that the potentials have not, even in their static limit, been measured on the lattice. Hence, the situation is different from the hybrid case, where static hybrid energies are known since long time.

I=1 S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

BOEFT for $I = 1$ tetraquarks

o Tarrus arXiv:1901.09761

$$\Gamma_\mu = \left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)/2 \text{ and } u = \exp(i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/(2$$

$$D_\mu \boldsymbol{Z} = \partial_\mu + [\Gamma_\mu, \boldsymbol{Z}]$$

$$\mathcal{L}_{\text{BOEFT for } I=1} = \int d^3r \, \text{Tr}\left\{ Z^\dagger_{0+-} \left(iD_0 - V^{\text{tetra}}_{\Sigma_g^+}(r) + \frac{\boldsymbol{\nabla}_r^2}{m_h} \right) Z_{0+-} \right\}$$
$$+ \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{ Z^\dagger_{1+-\lambda} \left(iD_0 - V^{\text{tetra}}_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h} \hat{r}^i_{\lambda'} \right) Z_{1+-\lambda'} \right\}$$
$$+ \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{ Z^\dagger_{1--\lambda} \left(iD_0 - V^{\text{tetra}}_{1--\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h} \hat{r}^i_{\lambda'} \right) Z_{1--\lambda'} \right\}$$

$+$ terms with higher orbital momentum and mixing of states

with the isovector field

$$Z_\kappa = Z^i_\kappa \sigma^i = \begin{pmatrix} Z^0_\kappa & \sqrt{2} Z^+_\kappa \\ \sqrt{2} Z^-_\kappa & -Z^0_\kappa \end{pmatrix}$$

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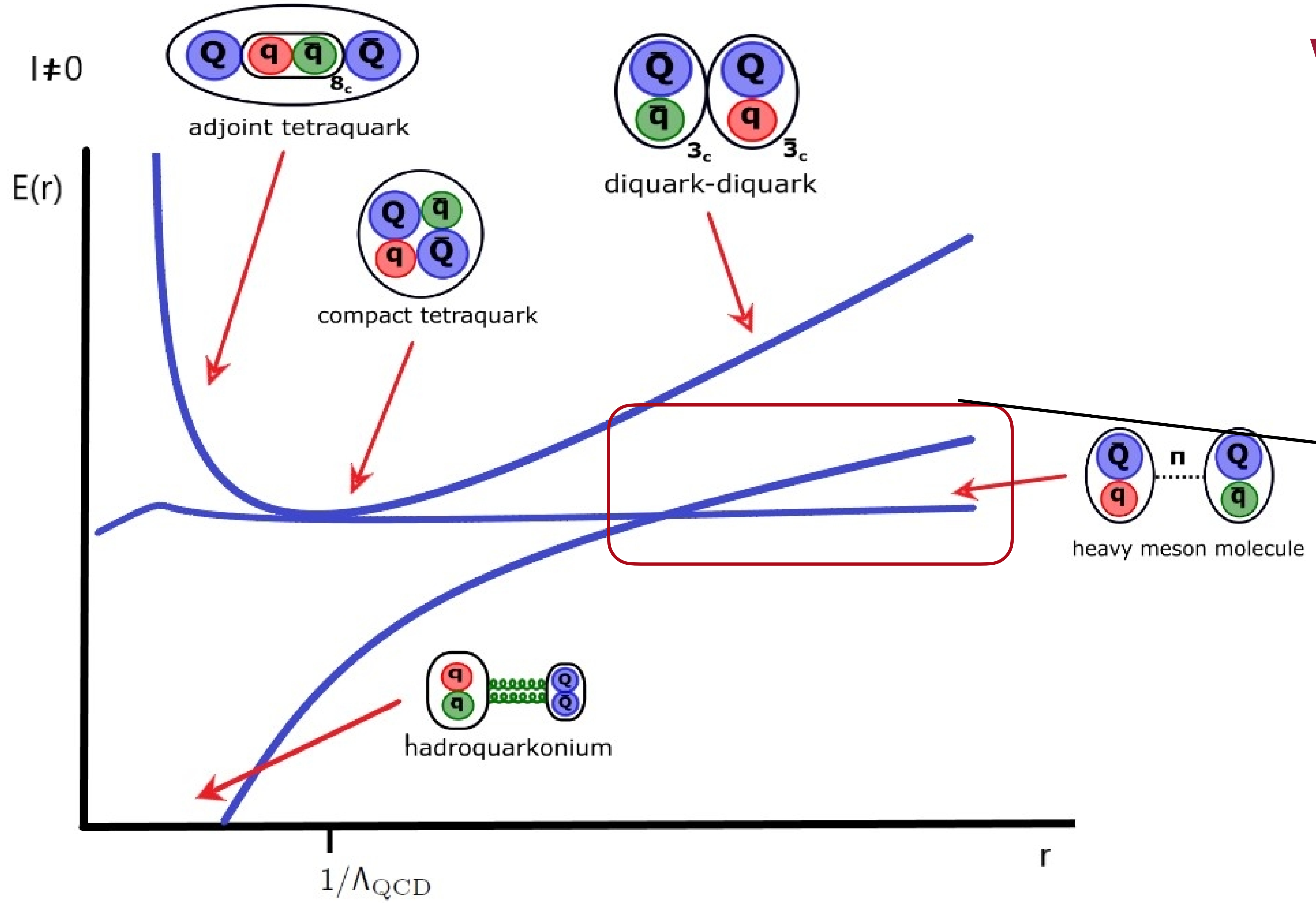
I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

We expect too get static energy in presence of qqbar of this type

Static energies for $I \neq 0$ (schematic):



The BOEFT contains all configurations: what dominates and where depends on the QCD dynamics

avoided crossing of the energy levels, mixing with open flavour meson-meson configurations

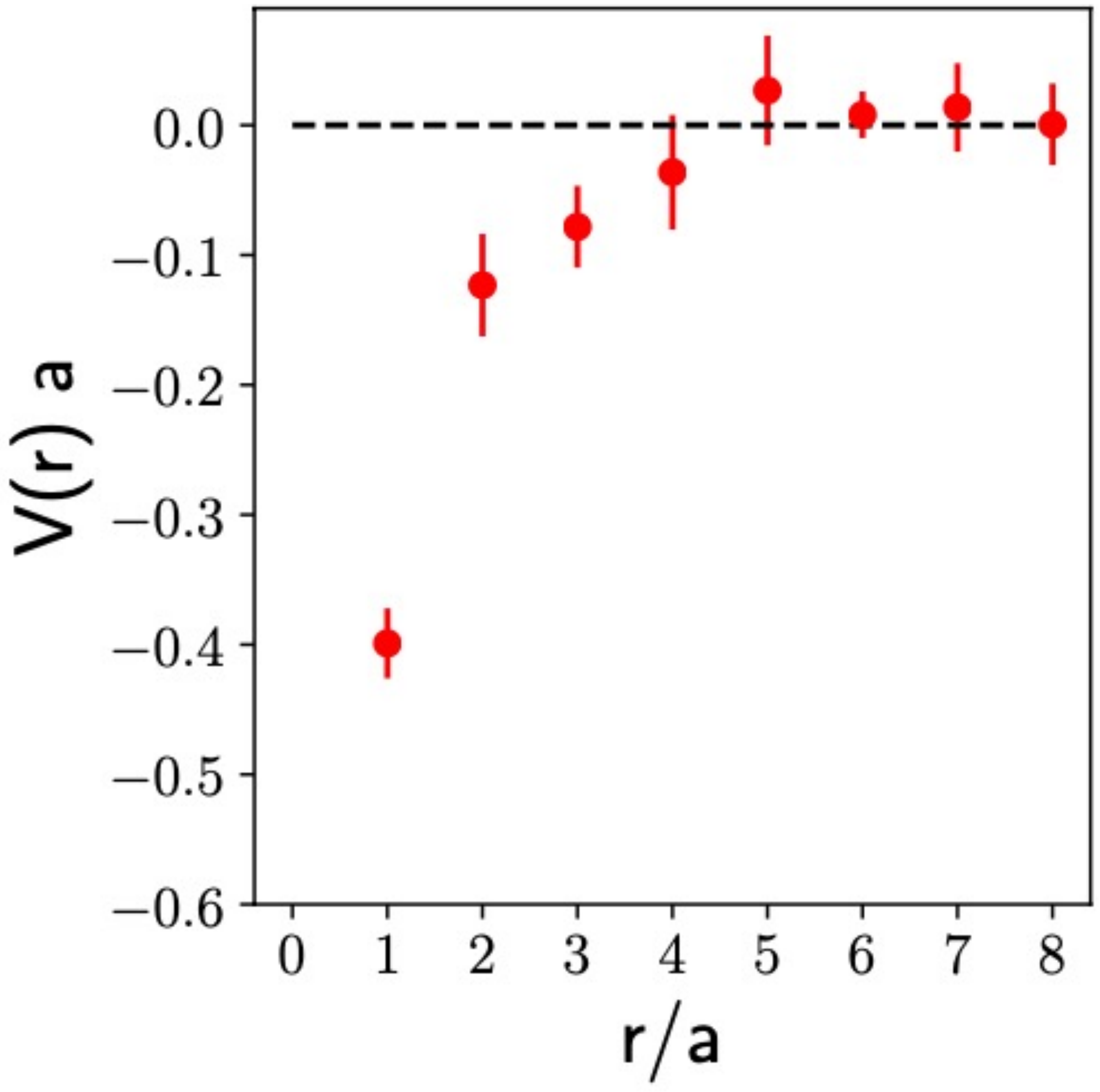
Bruschini, Gonzalez 2021

Jaume Tarrus 2207.09365

The static energies are defined in BOEFT that gives the appropriate set of operators to be used and could describe the short distance limit. Being nonperturbative objects E(r) should be calculated on the lattice (or in QCD vacuum models)

Lattice computation of the tetraquark static energies

Binding configuration found on the lattice



S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

$\bar{b}\, b\, \bar{d}\, u$ Z_b channel $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

Eigen-energies $E_n(r)$: channel $S_h=1$, CP=-1, ε=-1

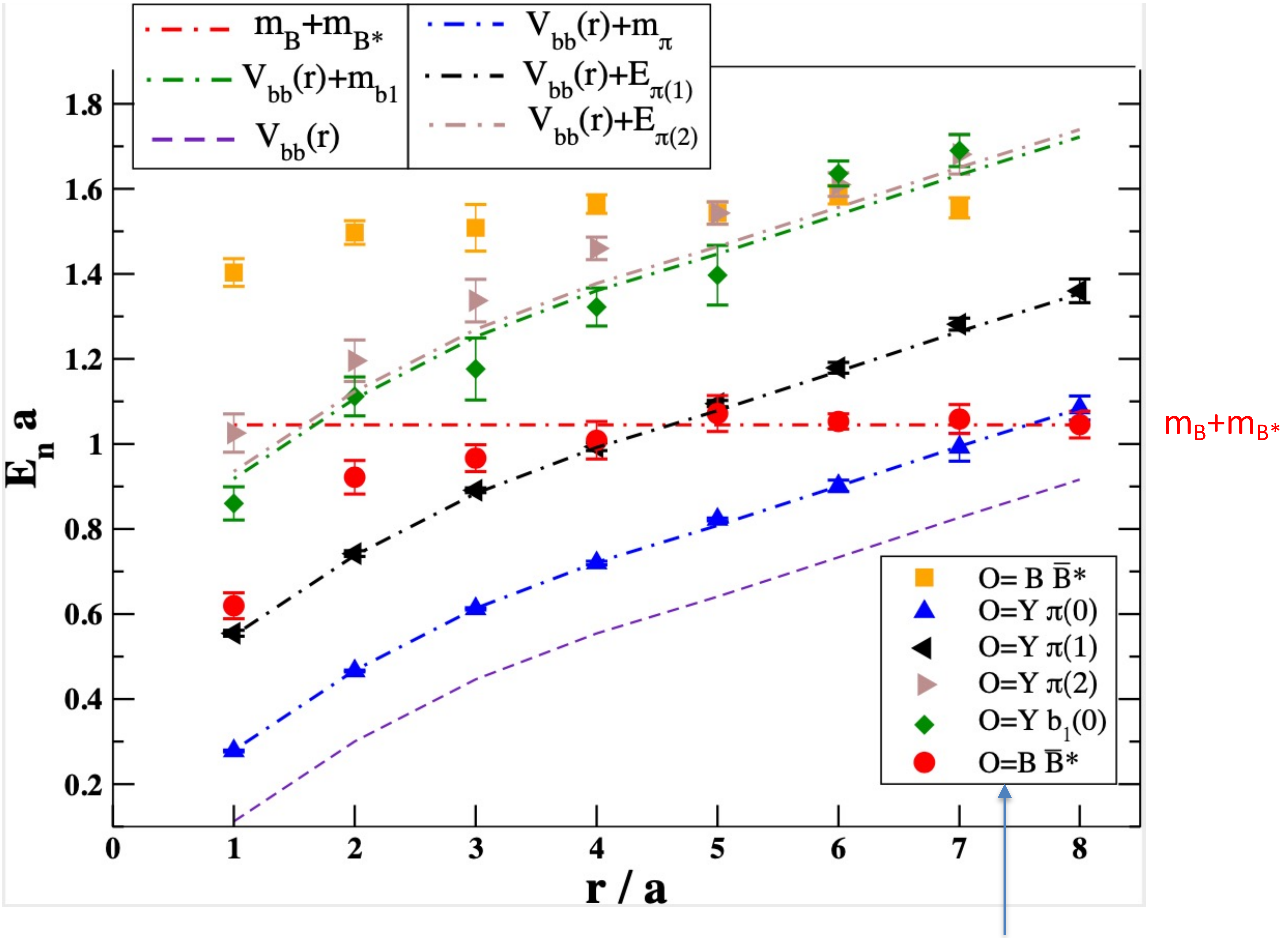


Figure from S. Prevlosek

To increase the predicitivity of the EFT we need a nonperturbative calculation of low energy chomoelectric and chromomagnetic correlators

We need to calculate several gauge invariant correlators:

- for X Y Z states: static energies for hybrids and tetraquarks, i.e. generalized Wilson loops, time nonlocal correlators of several E and B field (for the spin stricture), three point functions of a magnetic field between singletand hybrid (mixing between quarkonium and hybrids), gluelump masses

NOTICE that all these objects are not depending on flavour, they are low energy objects, we need even the quenched determinations—> so they are simple

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BUT…..

- they are noisy
- bad convergence to continuum

The force as a Wilson loop with a chromoelectric field

Example of the QCD force

A direct computation of the force that avoids interpolating the static energy and taking numerically the derivative is possible from the expression of a rectangular Wilson loop, $W_{r\times T}$, with a chromoelectric field insertion on a quark line:

$$F(r) = \frac{d}{dr}E_0(r) = \lim_{T\to\infty} -i\frac{\langle \mathrm{Tr}\{\mathrm{P}\, W_{r\times T}\, \hat{\mathbf{r}}\cdot g\mathbf{E}(\mathbf{r},t^*)\}\rangle}{\langle \mathrm{Tr}\{\mathrm{P}\, W_{r\times T}\}\rangle}$$

insertion of a single E into a static Wilson loop

physical object

An equivalent expression can be written using a Polyakov loop instead of a Wilson loop.
At fixed t^* for $T\to\infty$, the rhs is independent of t^*.
The force is mass renormalon free and finite after charge renormalization.

- Brambilla Pineda Soto Vairo PRD 63 (2001) 014023
  Vairo MPLA 31 (2016) 34, 1630039

Lattice analysis of 2111.07916

For a study of concept, we have computed the Wilson loop and Polyakov loop with a chromoelectric field on three quenched QCD ($n_f = 0$) ensembles.

ensemble	β	$(L/a)^3 \times T/a$	r_0/a	a in fm
A	6.284	$20^3 \times 40$	8.333	0.060
B	6.451	$26^3 \times 50$	10.417	0.048
C	6.594	$30^3 \times 60$	12.500	0.040

- TUMQCD coll. 2111.07916

These correlators depending on electric and magnetic field are better calculated with gradient flow

- better convergence, less noisy

- no need of change of scheme, all the calculation regarding that can be done in msbar in continuum

R. Harlander , F. Lange 2201.08618 , A. 1808.09837

E. Mereghetti,C. Monahan, M. Rizik, A. shindler, P Stoffer 2111.11449

- requires dedicated perturbative calculations

we plan to go ahead this way to obtain the input that are needed for BOEFTs calculations:
we started to calculate gluelump masses to reduce the error

Outlook

NREFTs and lattice allows us to describe the physics of quarkonium away from the strong decay threshold in quantum field theory: higher order perturbative calculation can be performed and quarkonium can be used for precision physics/ factorisation allows to systematically studyconfinement

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The same picture can be extended to tetraquarks and pentaquarks, once some lattice input on relevant correlators will be available.

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NREFTs and lattice and open quantum system allows us to describe the nonequilibrium evolution quarkonium in the quark gluon plasma and production processes

BOEFT allows to describe hybrids: new unexpected features are found (Lambda doubling, Spin structure) that have important impact on the phenomenology

BOEFT allows to describe hybrids and calculate multiplets, mixing and decays: on going work

The same picture can be extended to tetraquarks and pentaquarks, once some lattice input on relevant correlators will be available.

NOTICE that the needed lattice calculations are simpler than the direct calculations of the X Y Z properties on the lattice, the knowledge of few correlators together with the BOEFTwill allow to obtain many phenomenological information

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Combining BOEFT + open quantum systems one can attempt to study the X Y Z in heavy ion collisions