Coulomb Gauge QCD on the Lattice

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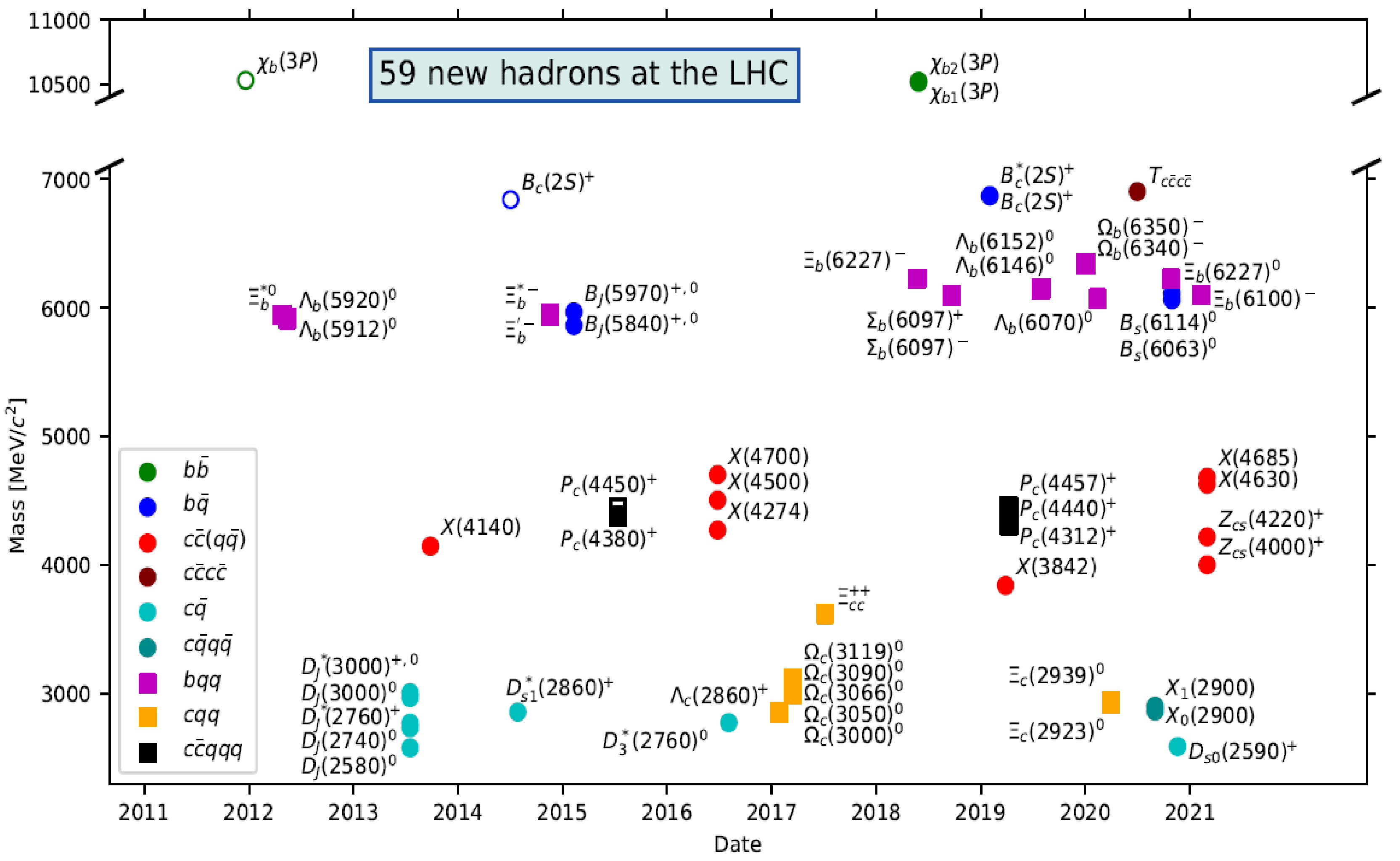
Hadron spectrum

- QCD hard to solve
- Exotic hadrons
- Mechanisms of confinement unknown

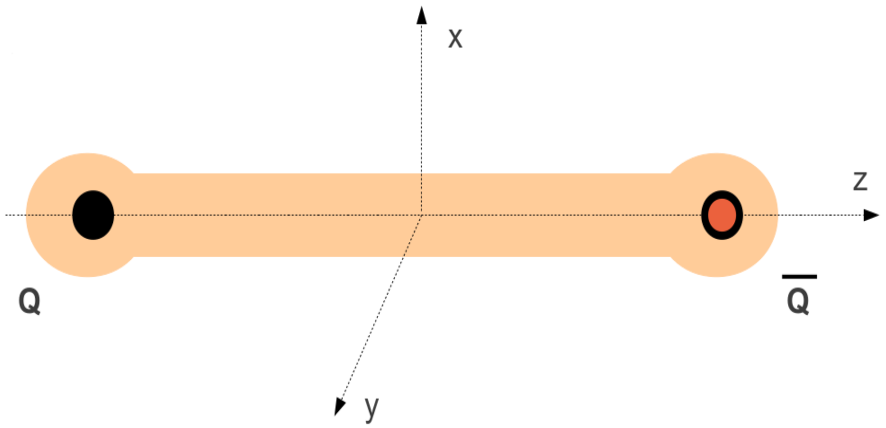


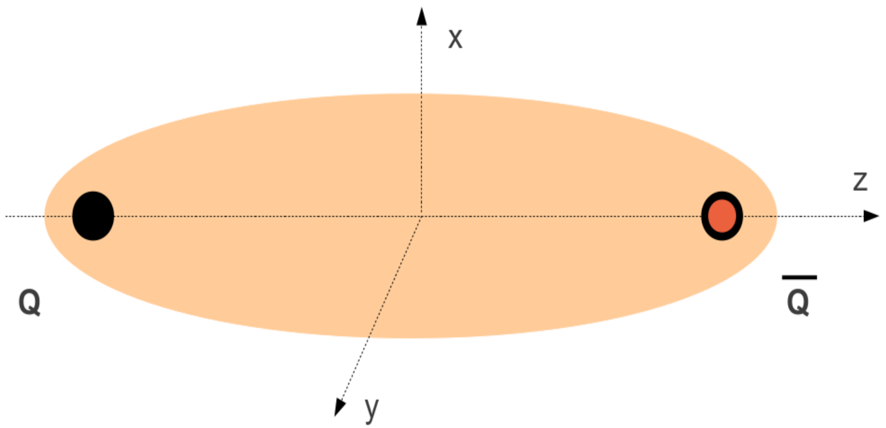
Plot from https://home.cern/news/news/physics/59-new-hadrons-and-counting

Why Coulomb Gauge Lattice QCD?

- LQCD is the only way to probe quark-level interactions currently
- LQCD allows for gluonic probes of hadronic structure, support for EIC
- Can understand QCD through analogy to QED in Coulomb Gauge
- Some questions remain about specifics of origin of Cornell potential, and flux tubes [1 2 3] on the Lattice

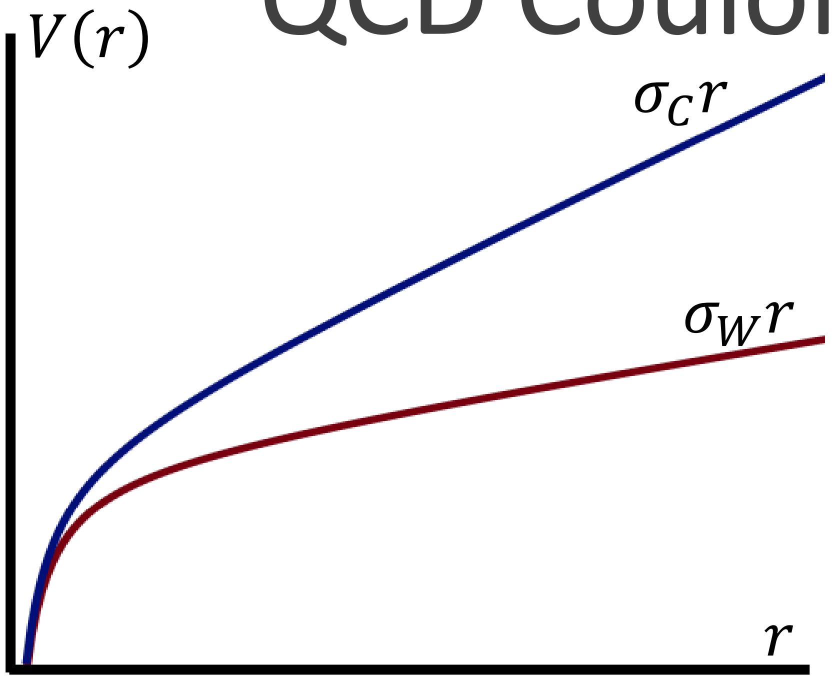
$$V(r) = A + \frac{B}{r} + \sigma r$$

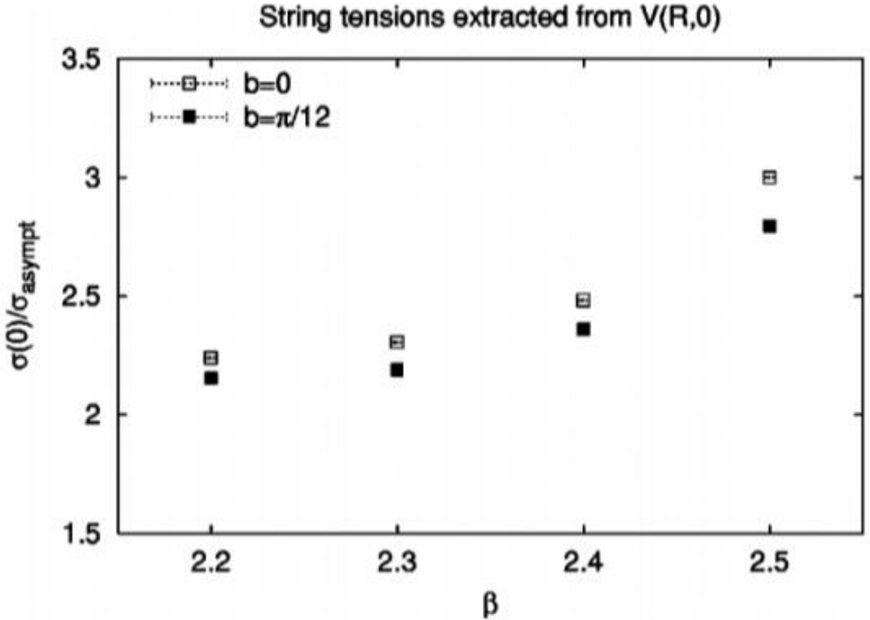




[1] P. O. Bowman and A. P. Szczepaniak, Phys. Rev.D70, 016002 (2004), arXiv:hep-ph/0403075[hep-ph].
[2] K. Chung and J. Greensite, Phys. Rev.D96, 034512 (2017), arXiv:1704.08995 [hep-lat].
[3] S. Dawid and A. P. Szczepaniak , Phys. Rev.D100, 074508 (2019)

QCD Coulomb Potential vs Wilson Potential





- Wilson potential = potential of static quark antiquark pair in ground state
- Coulomb potential = potential of static quark antiquark pair interacting *instantaneously* in Coulomb gauge
- Both potentials parameterized by Cornell potential

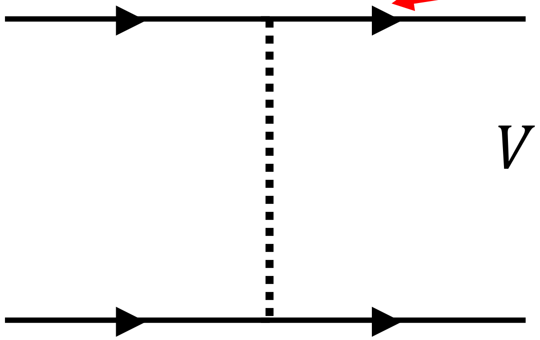
$$V(r) = A + \frac{B}{r} + \sigma r$$

- Confining behavior of Coulomb potential is *necessary* for Wilson confinement[4]

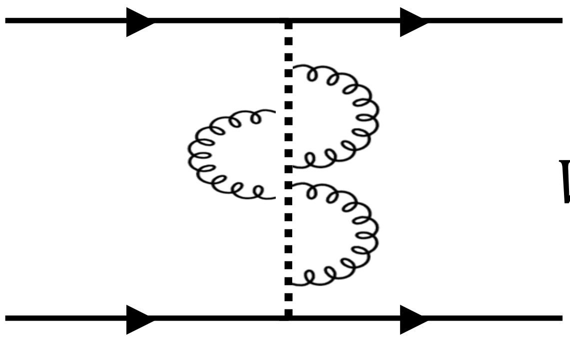
[4] D. Zwanziger, Phys. Rev. Lett.90, 102001 (2003), arXiv:hep-lat/0209105 [hep-lat].
Plot from: J. Greensite and A. P. Szczepaniak,Phys. Rev. D91, 034503(2015).

Coulomb Gauge Hamiltonian:

$$H_{QCD} = H_q + H_g + H_{qg} + H_C$$



$$V \to \frac{1}{r}$$



$$V \to \sigma_C r$$

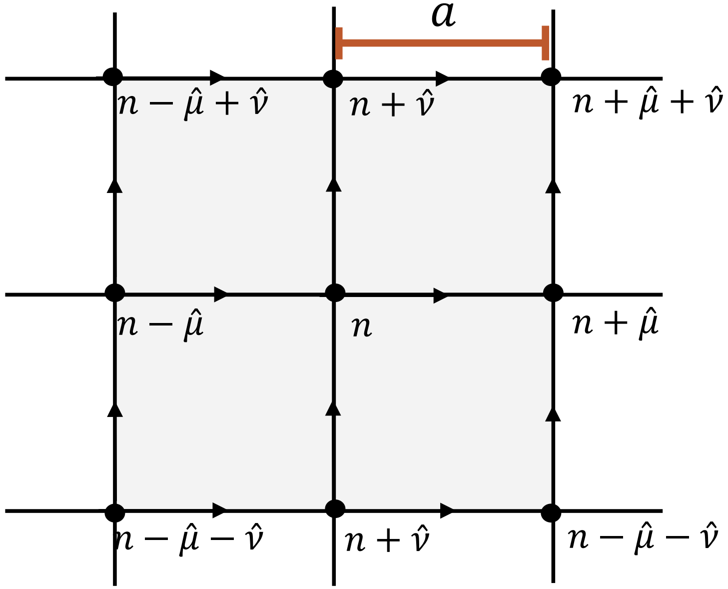
$$\langle q\bar{q}|H_{QCD}|q\bar{q}\rangle = \sigma_C r$$

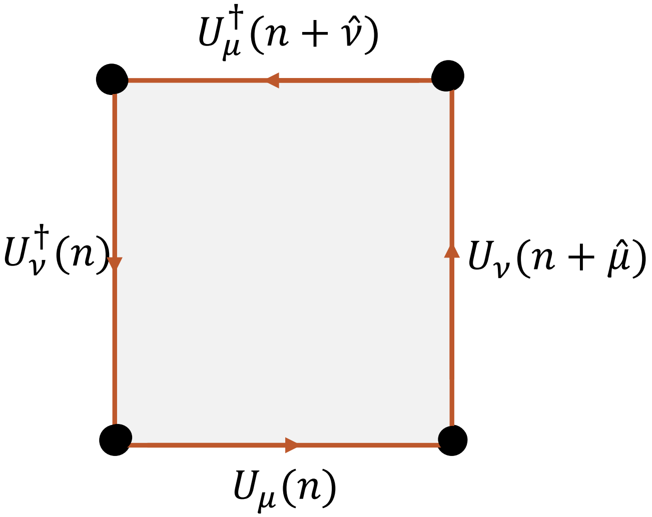
- The static quark-antiquark state which produces the coulomb potential is *not* the ground state!

$$H_{QCD}|q\bar{q}_{true}\rangle = \sigma_W r|q\bar{q}_{true}\rangle$$

$$|q\bar{q}_{true}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots$$

SU(N) Lattice QCD





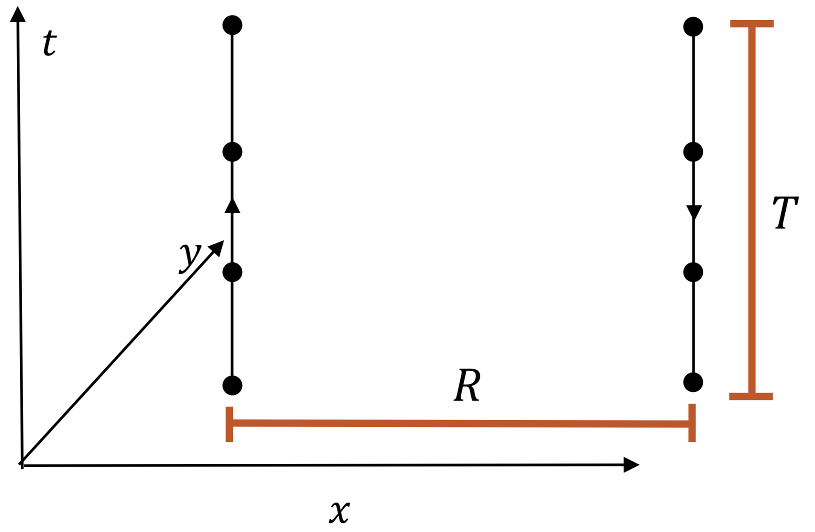
- Links are SU(N) matrices representing gauge transporters between lattice sites

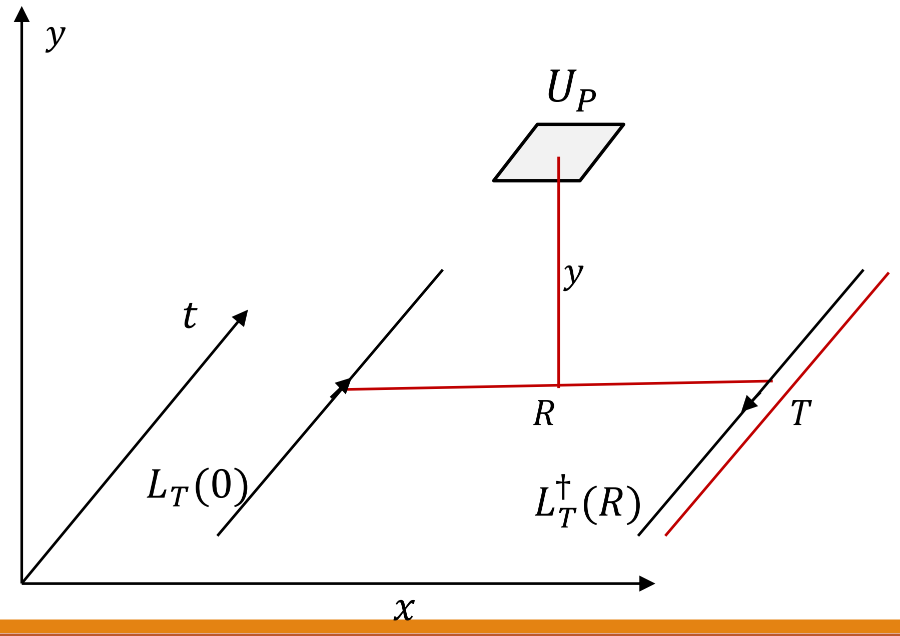
$$U_\mu(n) = e^{iaA_\mu(n)}$$

- Wilson action for SU(N) LQCD:

$$S = \frac{\beta}{N}\sum_n \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - U_{\mu\nu}(n)\right] \qquad \beta = 2N/g^2$$

$$U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})\,U_\nu^\dagger(n)$$





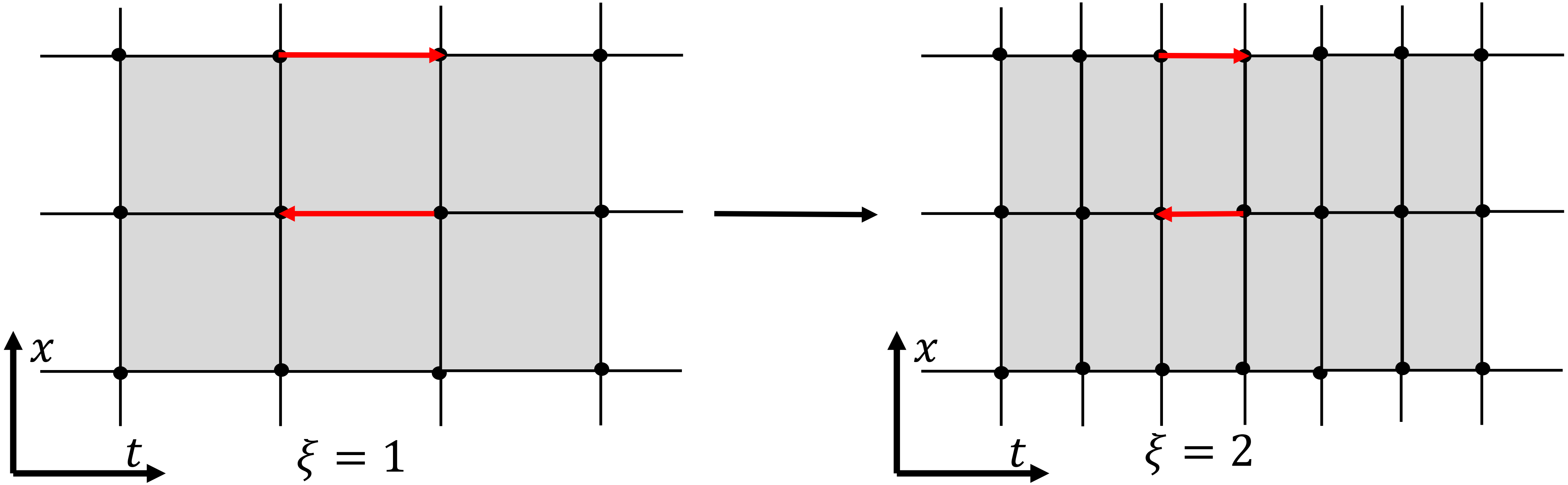
- In Coulomb gauge ($\partial_i A^i = 0$), calculate the potential from correlation of two time-like Wilson lines

$$V(r) = A + \frac{B}{r} + \sigma_C r$$

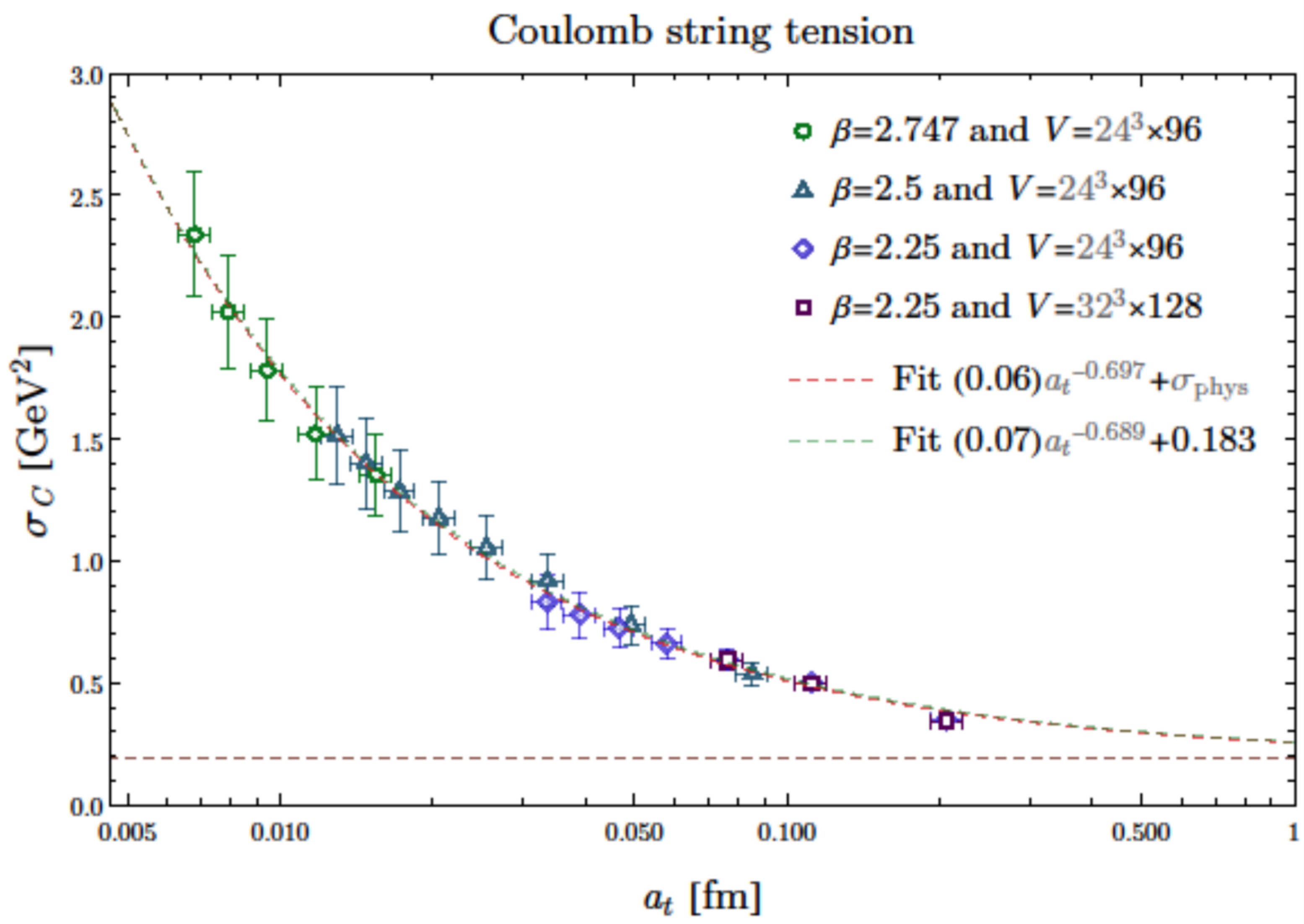
- $T \to \infty$ should recover the (minimal) Wilson Potential.
- $T \to 0$ gives the lattice version of the Coulomb potential
- Can calculate energy density by inserting "probe" above the Wilson lines

Lattice Setup

- Use anisotropic lattice to access $T \to 0$: Different couplings for spatial/time directions
- Quenched Lattice QCD: $N_f = 0$, no fermion determinant (pure gauge action)
- SU(2) and SU(3) (in progress)

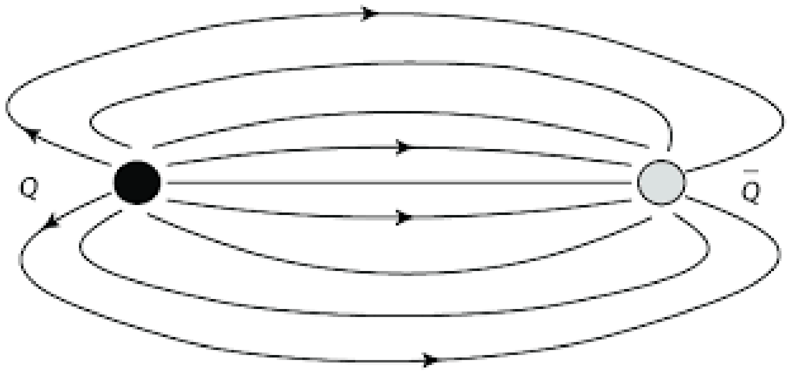


Preliminary Results: SU(2)



Summary

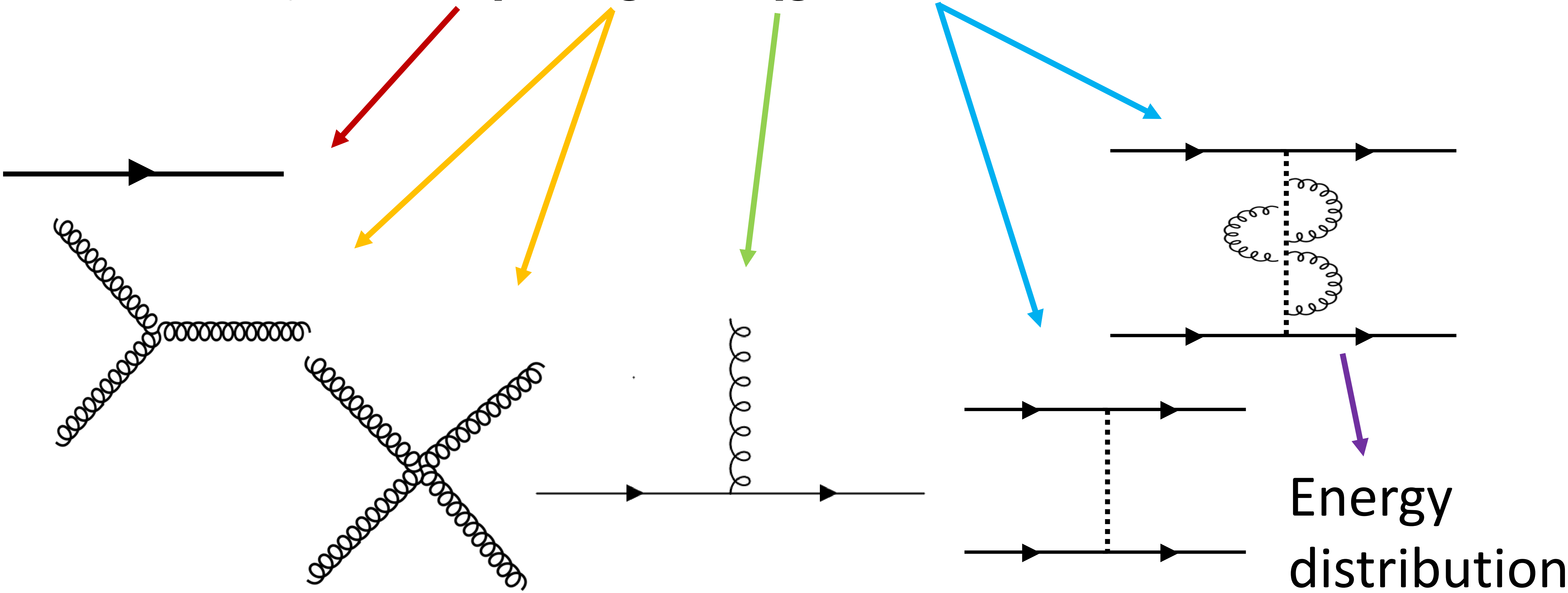
- Improvements in methods/algorithms and theoretical calculations necessary for Coulomb Gauge LQCD
- Coulomb Gauge Physics is important for understanding hadron spectrum, confinement
- LQCD will continue to work in tandem with EIC, probing mesonic and hadronic structure



Backup Slides

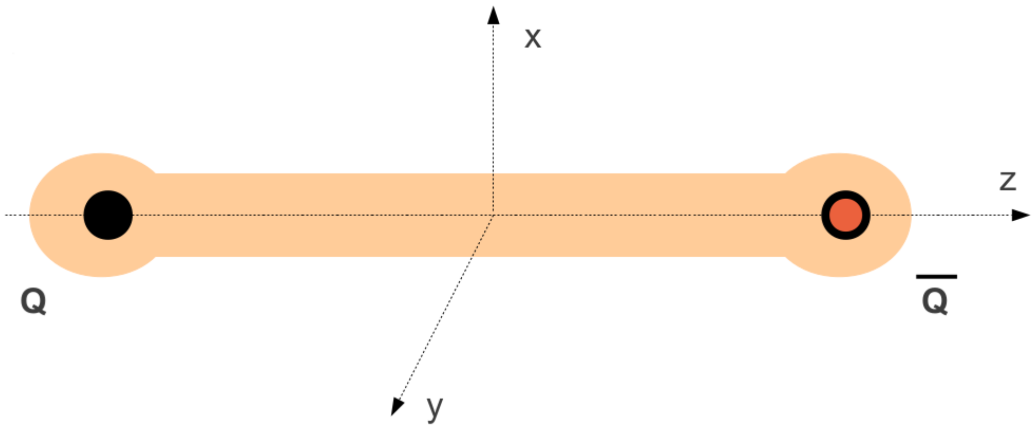
Coulomb Gauge Hamiltonian:

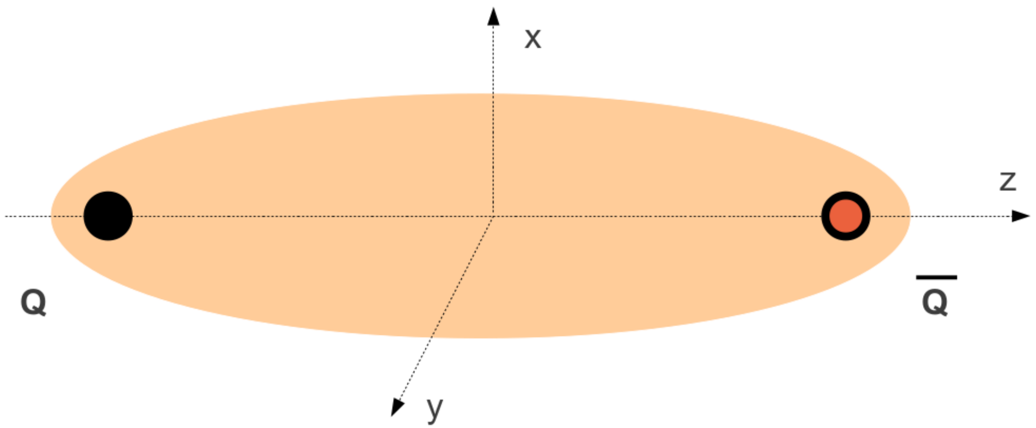
$$H_{QCD} = H_q + H_g + H_{qg} + H_C \text{ (+ counter−terms)}$$



Shape of the Electric Field's Energy Distribution

- Bowman, Szczepaniak prediction: The Energy distribution has a power-law fall off in the transverse direction[1]
- Greensite, Chung calculation: The distribution decays exponentially in the transverse direction ("Flux tube")[2]
- Dawid, Szczepaniak calculation: There might be some evolution between the two with increasing coupling strength[3]
- How does it really decay?





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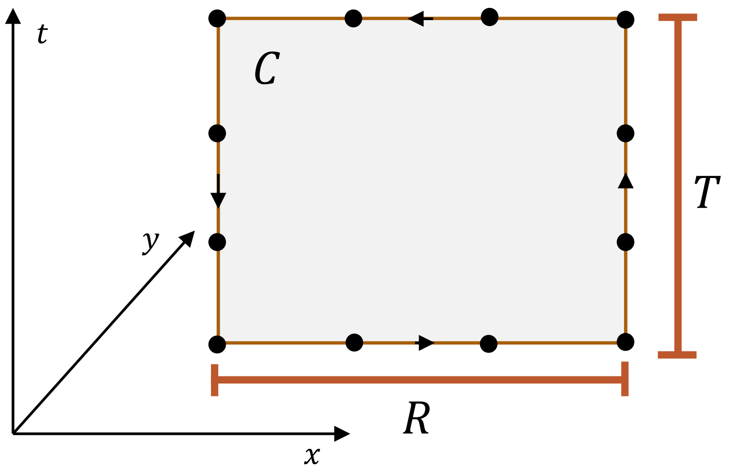
[3] S. Dawid and A. P. Szczepaniak , Phys. Rev.D100, 074508 (2019)

- Basic idea: Equilibrate a 4D matrix of link variables according to the QCD action and calculate observables from link variables

- "Wilson loops" are oriented closed loops on the lattice from which we can extract the potential between heavy static quarks

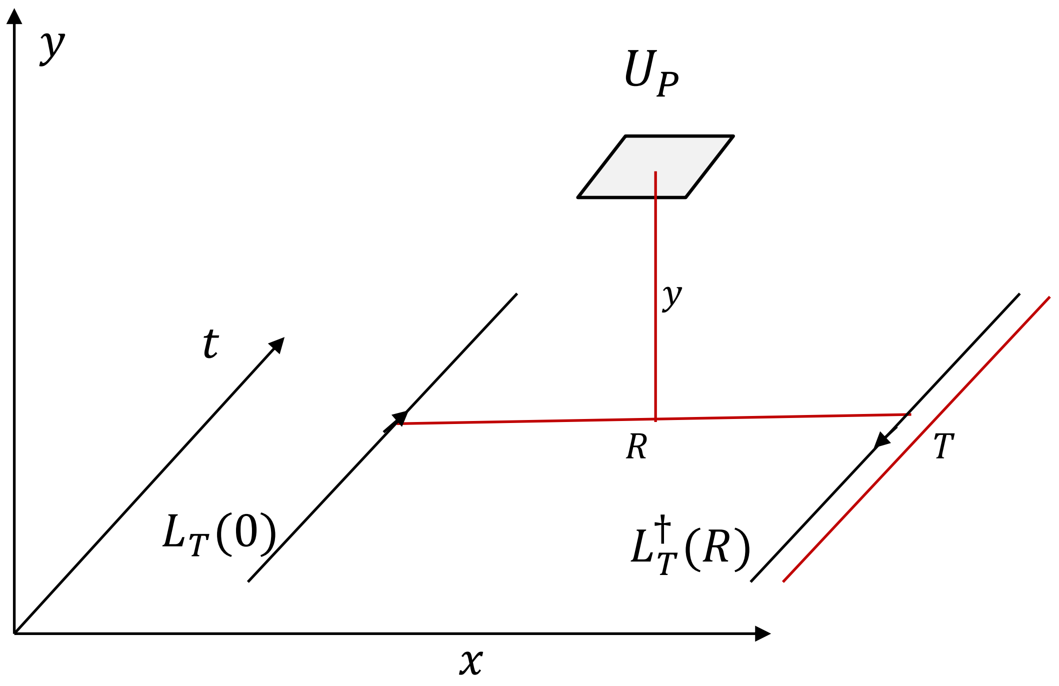


$$W(R,T) = \mathrm{Tr} \prod_{(n,\mu)\in C} U_\mu(n) \qquad V(R,T) = \ln \frac{\langle W(R,T)\rangle}{\langle W(R,T+1)\rangle}$$

- In the limit $T \to \infty$ we identify the static quark potential

$$V(r) = A + \frac{B}{r} + \sigma r$$

- σ is the "string tension"

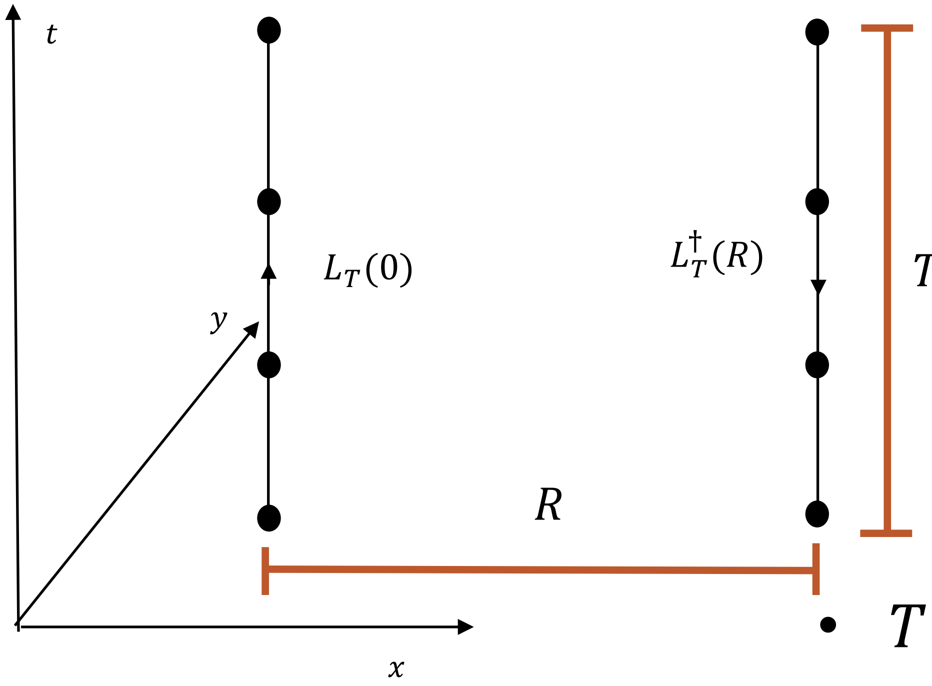


- One def of energy density observable:

$$Q_T(R,Y) = \frac{\left\langle \mathrm{Tr}\left[L_T(0)L_T^\dagger(R)\right]\frac{1}{2}\mathrm{Tr}[U_P(y,T)]\right\rangle}{\left\langle \mathrm{Tr}\left[L_T(0)L_T^\dagger(R)\right]\right\rangle} - \frac{1}{2}\langle \mathrm{Tr}U_P\rangle$$

- Extra plaquette acts as a probe for E_x^2

Coulomb Potential Observable

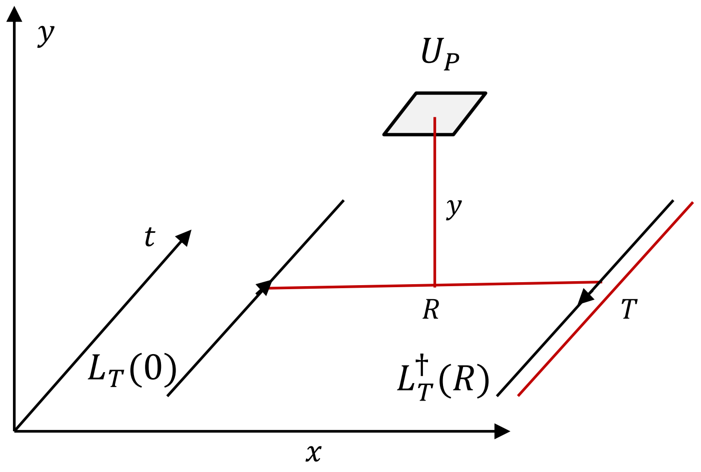


$$aV(R,T) = \log \frac{\langle \mathrm{Tr}[L_T(\mathbf{0}) L_T^\dagger(\boldsymbol{R})] \rangle}{\langle \mathrm{Tr}[L_{T+1}(\mathbf{0}) L_{T+1}^\dagger(\boldsymbol{R})] \rangle}$$

$$aV_C(r) = aV(r,0) = -\log \left\langle \frac{1}{N} \mathrm{Tr}\left[U_0(0,\mathbf{0}) U_0^\dagger(0,\boldsymbol{R})\right] \right\rangle$$

- $T \to \infty$ should recover the Wilson Potential.
- $T \to 0$ gives the lattice version of the Coulomb potential, an instantaneous “chromoelectric” interaction (“bare” state)

Chromo-electric Energy Density



- The energy density profile of this chromoelectric interaction in the bare state corresponds to observable

$$Q_T(R,Y) = \frac{\left\langle \mathrm{Tr}\left[L_T(0)L_T^\dagger(R)\right]\frac{1}{2}\mathrm{Tr}[U_P(y,T)]\right\rangle}{\left\langle \mathrm{Tr}\left[L_T(0)L_T^\dagger(R)\right]\right\rangle} - \frac{1}{2}\langle \mathrm{Tr}U_P\rangle$$

- Theory predicts a power-law fall off.

- Chung and Greensite found bare state has flux-tube characteristics (exponential fall-off) [ref]
- Dawid and Szczepaniak found power-law fall off with increasing β with issues at small y [ref]

Lattice Setup

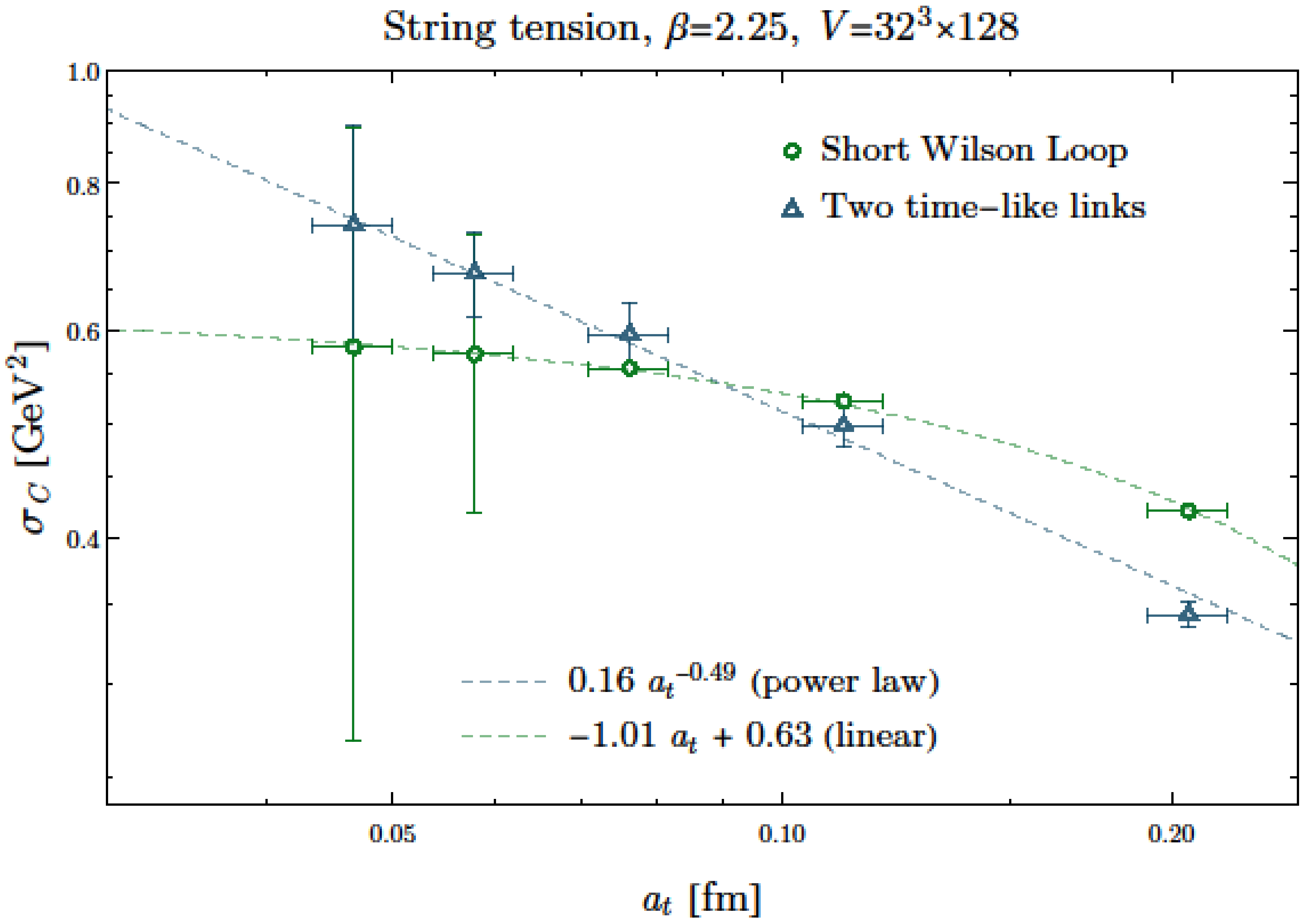
- Forced to use an anisotropic lattice to access $T \to 0$. Must introduce β_s, β_t: different couplings for spatial/time directions

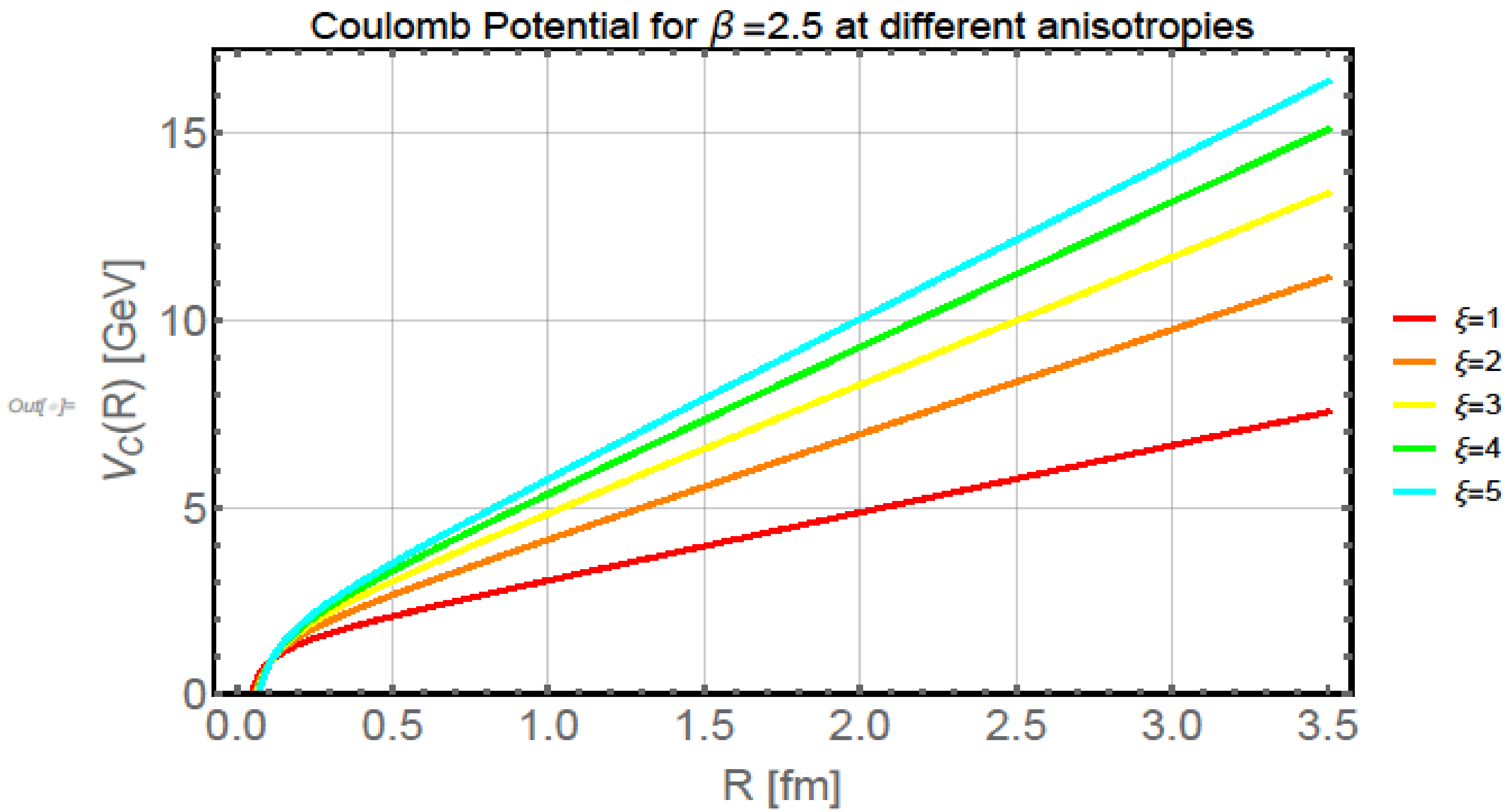
$$S = \sum_n \left[\beta_s \sum_{j>i=1}^{3} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{ij}(n) \right) + \beta_t \sum_{i=1}^{3} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{0i}(n) \right) \right]$$

- Quenched Lattice QCD: $N_f = 0$, no fermion determinant (pure gluodynamics, infinitely heavy quarks)

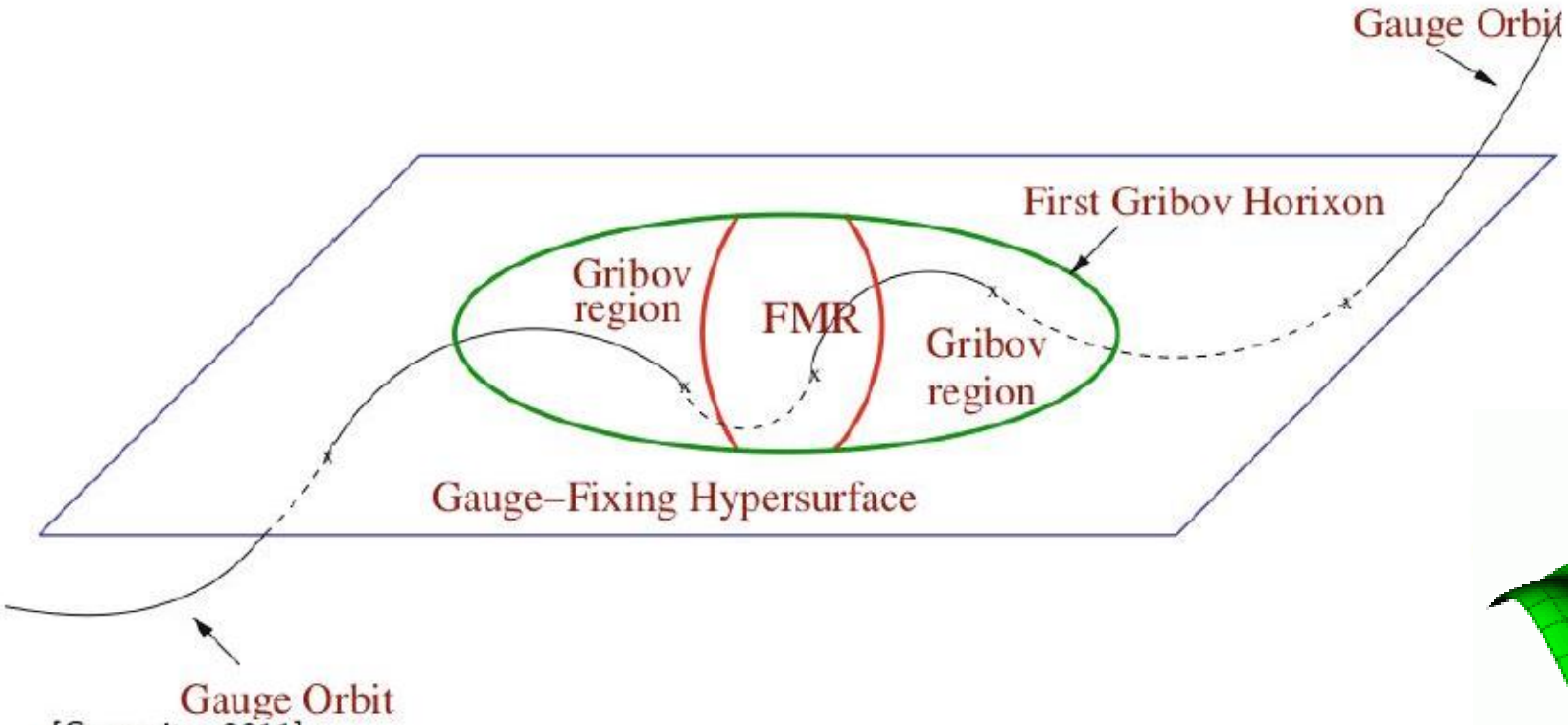
- SU(2) Lattices: $\beta = 2.25, 2.5, 2.7, 3.249,\ \ \xi = 1, \ldots, 8$, $N^3 \times T = 24^3 \times\ 96$, $32^3 \times 128$

- SU(3) Lattices: $\beta = 6.0 \quad \xi = 1, \ldots, 4\ \ N^3 \times T = 24^3 \times\ 96$ (in progress)


String tension, β=2.25, V=32³×128
1.0
0.8
0.6
0.4
σ_C [GeV²]
Short Wilson Loop
Two time-like links
0.16 a_t^−0.49 (power law)
−1.01 a_t + 0.63 (linear)
0.05
0.10
0.20
a_t [fm]


Coulomb Potential for β =2.5 at different anisotropies
Out[ ]=
V_C(R) [GeV]
R [fm]
0
5
10
15
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
ξ=1
ξ=2
ξ=3
ξ=4
ξ=5

Gauge-Fixing



[Greensite, 2011]