Coulomb Gauge QCD on the Lattice

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Hadron spectrum

- QCD hard to solve
- Exotic hadrons
- Mechanisms of confinement unknown
Why Coulomb Gauge Lattice QCD?

- LQCD is the only way to probe quark-level interactions currently
- LQCD allows for gluonic probes of hadronic structure, support for EIC
- Can understand QCD through analogy to QED in Coulomb Gauge
- Some questions remain about specifics of origin of Cornell potential, and flux tubes\(^{1,2,3}\) on the Lattice

\[ V(r) = A + \frac{B}{r} + \sigma r \]

Wilson potential = potential of static quark antiquark pair in ground state

Coulomb potential = potential of static quark antiquark pair interacting \textit{instantaneously} in Coulomb gauge

Both potentials parameterized by Cornell potential

\[ V(r) = A + \frac{B}{r} + \sigma r \]

Confining behavior of Coulomb potential is \textit{necessary} for Wilson confinement

Coulomb Gauge Hamiltonian:

\[ H_{QCD} = H_q + H_g + H_{qg} + H_C \]

\[ V \rightarrow \frac{1}{r} \]

\[ V \rightarrow \sigma_C r \]

\[ \langle q\bar{q}|H_{QCD}|q\bar{q}\rangle = \sigma_C r \]

- The static quark-antiquark state which produces the coulomb potential is not the ground state!

\[ H_{QCD}|q\bar{q}_{true}\rangle = \sigma_W r|q\bar{q}_{true}\rangle \]

\[ |q\bar{q}_{true}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots \]
SU(N) Lattice QCD

• Links are SU(N) matrices representing gauge transporters between lattice sites
  \[ U_\mu(n) = e^{i\alpha A_\mu(n)} \]

• Wilson action for SU(N) LQCD:
  \[ S = \frac{\beta}{N} \sum_n \sum_{\mu < \nu} \text{Re Tr}[1 - U_{\mu\nu}(n)] \]
  \[ \beta = 2N/g^2 \]
  \[ U_{\mu\nu}(n) = U_\mu(n)U_\nu(n + \hat{\mu})U_\mu^\dagger(n + \hat{\nu})U_\nu^\dagger(n) \]
In Coulomb gauge ($\partial_i A^i = 0$), calculate the potential from correlation of two time-like Wilson lines

$$V(r) = A + \frac{B}{r} + \sigma_C r$$

- $T \to \infty$ should recover the (minimal) Wilson Potential.
- $T \to 0$ gives the lattice version of the Coulomb potential
- Can calculate energy density by inserting “probe” above the Wilson lines
Lattice Setup

• Use anisotropic lattice to access $T \to 0$: Different couplings for spatial/time directions
• Quenched Lattice QCD: $N_f = 0$, no fermion determinant (pure gauge action)
• SU(2) and SU(3) (in progress)
Preliminary Results: SU(2)
Summary

• Improvements in methods/algorithms and theoretical calculations necessary for Coulomb Gauge LQCD

• Coulomb Gauge Physics is important for understanding hadron spectrum, confinement

• LQCD will continue to work in tandem with EIC, probing mesonic and hadronic structure
Backup Slides
Coulomb Gauge Hamiltonian:

\[ H_{QCD} = H_q + H_g + H_{qg} + H_C \ (\text{+ counter–terms}) \]
Shape of the Electric Field’s Energy Distribution

• Bowman, Szczepaniak prediction: The Energy distribution has a power-law fall off in the transverse direction\(^1\)

• Greensite, Chung calculation: The distribution decays exponentially in the transverse direction ("Flux tube")\(^2\)

• Dawid, Szczepaniak calculation: There might be some evolution between the two with increasing coupling strength\(^3\)

• How does it really decay?

\(^{3}\) S. Dawid and A. P. Szczepaniak, Phys. Rev.D100, 074508 (2019)
• Basic idea: Equilibrate a 4D matrix of link variables according to the QCD action and calculate observables from link variables

• “Wilson loops” are oriented closed loops on the lattice from which we can extract the potential between heavy static quarks

$$ W(R,T) = \text{Tr} \prod_{(n,\mu) \in C} U_\mu(n) $$

$$ V(R,T) = \ln \frac{\langle W(R,T) \rangle}{\langle W(R,T+1) \rangle} $$

• In the limit $T \to \infty$ we identify the static quark potential

$$ V(r) = A + \frac{B}{r} + \sigma r $$

• $\sigma$ is the “string tension”
• One def of energy density observable:

\[ Q_T(R, Y) = \frac{\langle \text{Tr}[L_T(0)L_T^\dagger(R)] \frac{1}{2} \text{Tr}[U_P(y, T)] \rangle}{\langle \text{Tr}[L_T(0)L_T^\dagger(R)] \rangle} - \frac{1}{2} \langle \text{Tr} U_P \rangle \]

• Extra plaquette acts as a probe for \( E_x^2 \)
\[ aV(R, T) = \log \frac{\langle \text{Tr}[L_T(0)L_T^\dagger(R)] \rangle}{\langle \text{Tr}[L_{T+1}(0)L_{T+1}^\dagger(R)] \rangle} \]

\[ aV_C(r) = aV(r, 0) = -\log \left( \frac{1}{N} \text{Tr}[U_0(0, 0)U_0^\dagger(0, R)] \right) \]

- \( T \to \infty \) should recover the Wilson Potential.

- \( T \to 0 \) gives the lattice version of the Coulomb potential, an instantaneous “chromoelectric” interaction (“bare” state)
Chromo-electric Energy Density

- The energy density profile of this chromoelectric interaction in the bare state corresponds to observable

\[ Q_T(R,Y) = \frac{\langle \text{Tr}[L_T(0)L_T^\dagger(R)] \frac{1}{2} \text{Tr}[U_P(y,T)] \rangle}{\langle \text{Tr}[L_T(0)L_T^\dagger(R)] \rangle} - \frac{1}{2} \langle \text{Tr}U_P \rangle \]

- Theory predicts a power-law fall off.

- Chung and Greensite found bare state has flux-tube characteristics (exponential fall-off) [ref]

- Dawid and Szczepaniak found power-law fall off with increasing \( \beta \) with issues at small \( y \) [ref]
Lattice Setup

- Forced to use an anisotropic lattice to access $T \to 0$. Must introduce $\beta_s, \beta_t$: different couplings for spatial/time directions

\[ S = \sum_{n} \left[ \beta_s \sum_{j>i=1}^{3} \left( 1 - \frac{1}{2} \text{Tr} \, U_{ij}(n) \right) + \beta_t \sum_{i=1}^{3} \left( 1 - \frac{1}{2} \text{Tr} \, U_{0i}(n) \right) \right] \]

- Quenched Lattice QCD: $N_f = 0$, no fermion determinant (pure gluodynamics, infinitely heavy quarks)

- SU(2) Lattices: $\beta = 2.25, 2.5, 2.7, 3.249, \; \xi = 1, \ldots, 8 \; , N^3 \times T = 24^3 \times 96 \; , 32^3 \times 128$

- SU(3) Lattices: $\beta = 6.0 \; \; \xi = 1, \ldots, 4 \; N^3 \times T = 24^3 \times 96$ (in progress)
Gauge-Fixing

[Greensite, 2011]