

Taming the complex dynamics of scattering events

Pier Monni (CERN)

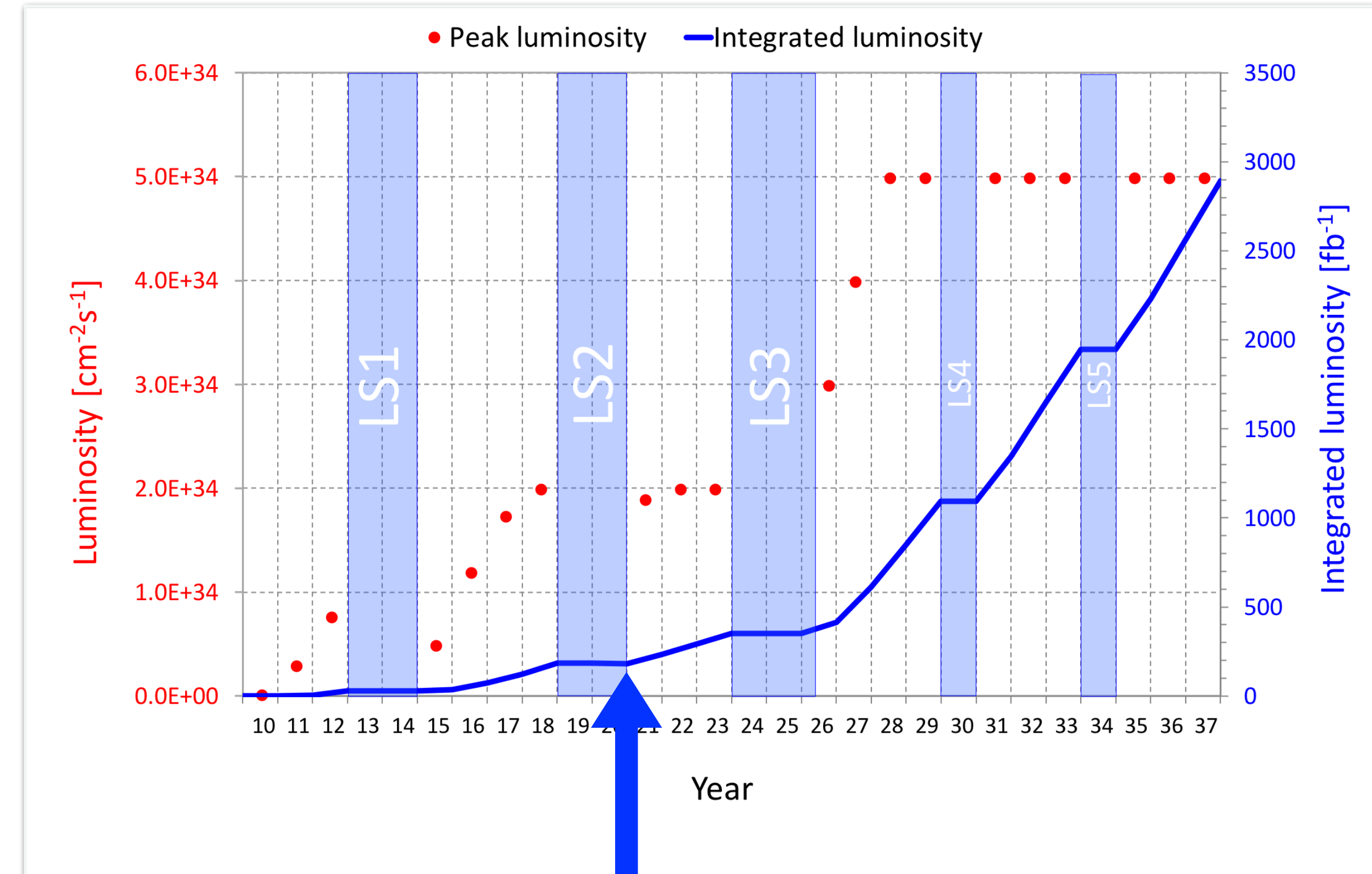
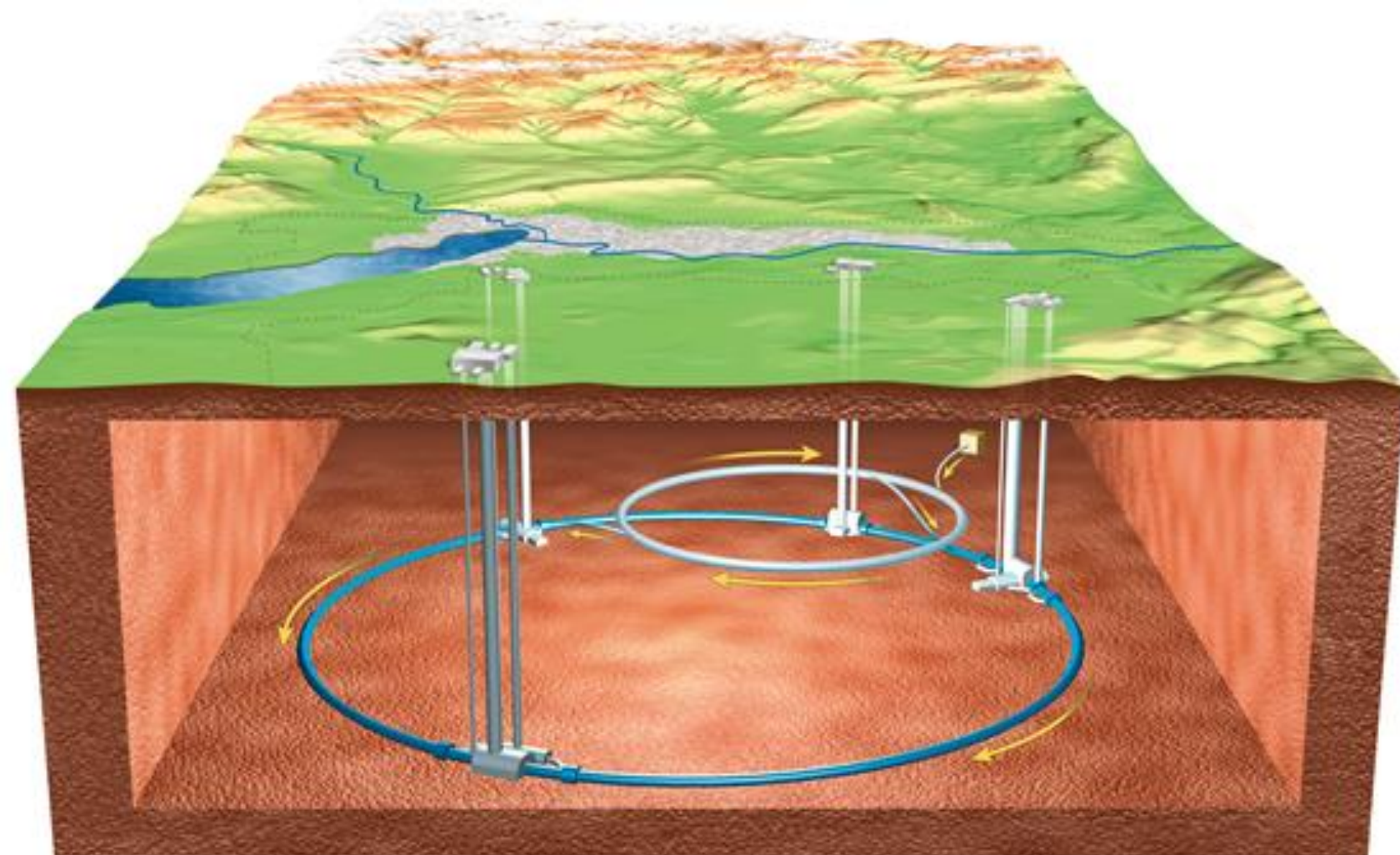
HET Seminar - BNL, June 2022



CERN

Particle physics at the Large Hadron Collider (LHC)

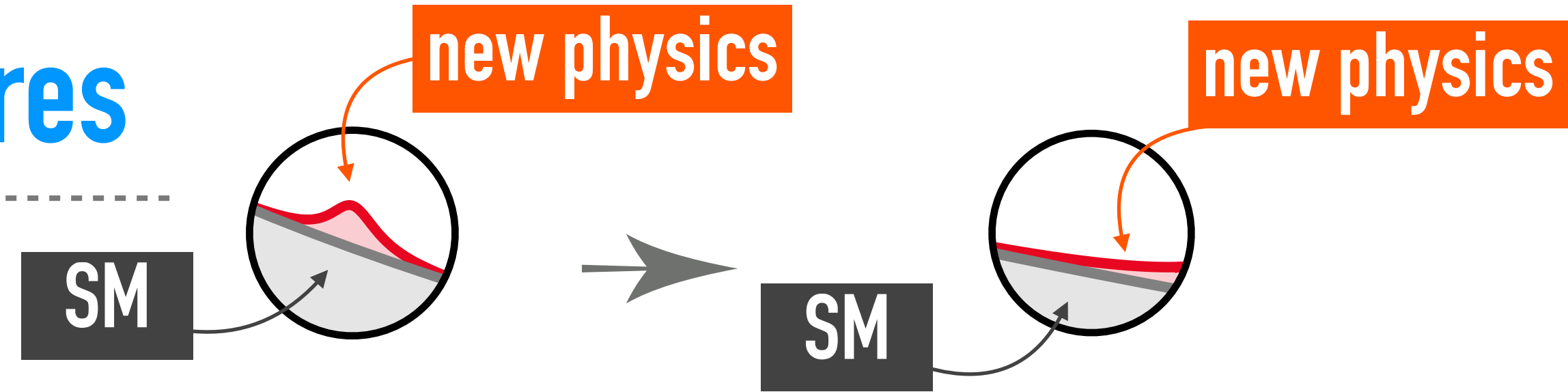
- LHC about to resume operations:
 - Huge boost in experimental precision foreseen (only ~5% of the total luminosity delivered so far)
- Key open questions to be addressed:
 - Establish the Higgs sector
 - Broad searches for New Physics (NP)
 - Stress test of the Standard Model (SM)



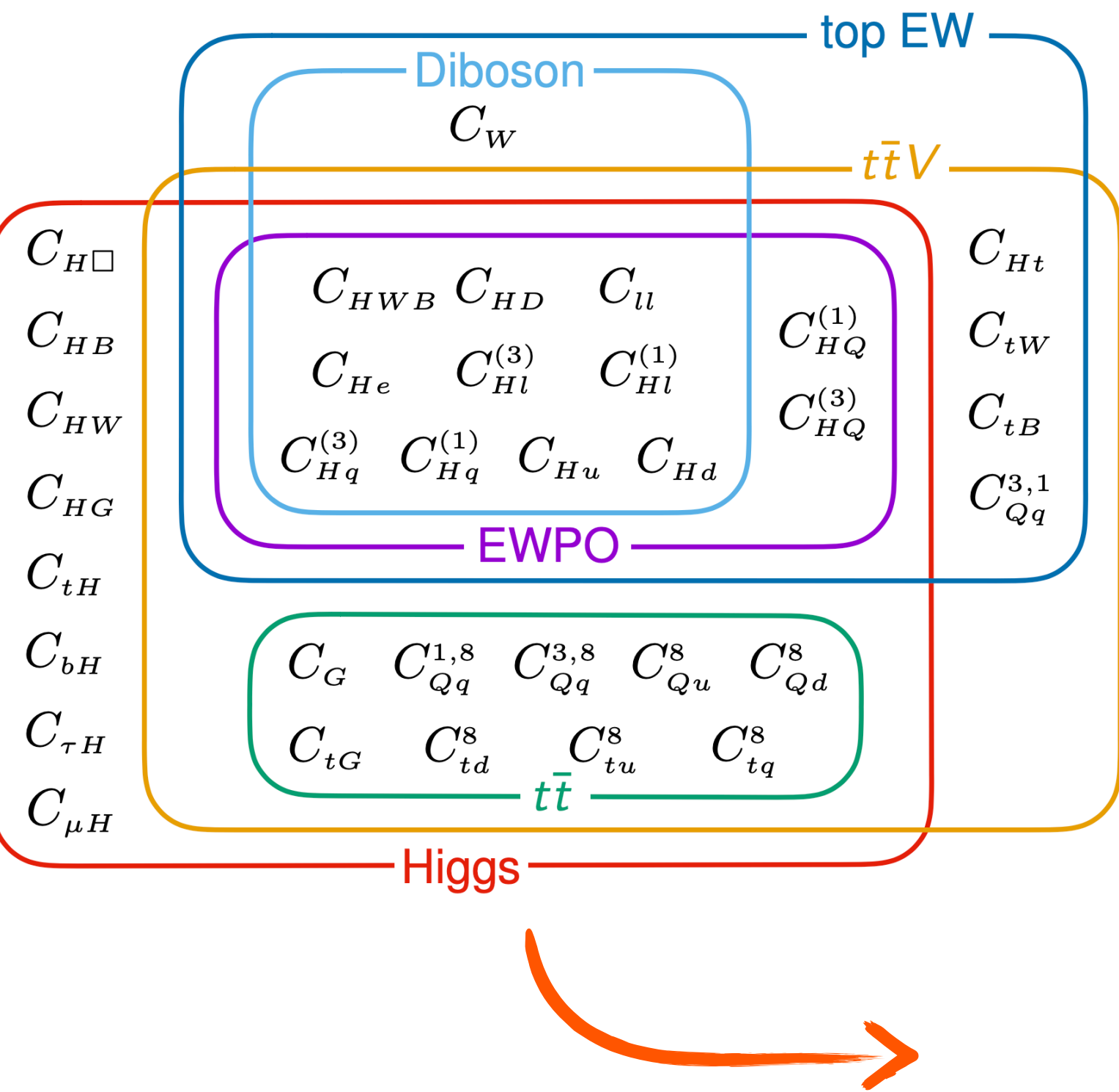
Current integrated luminosity

Broad spectrum searches for NP signatures

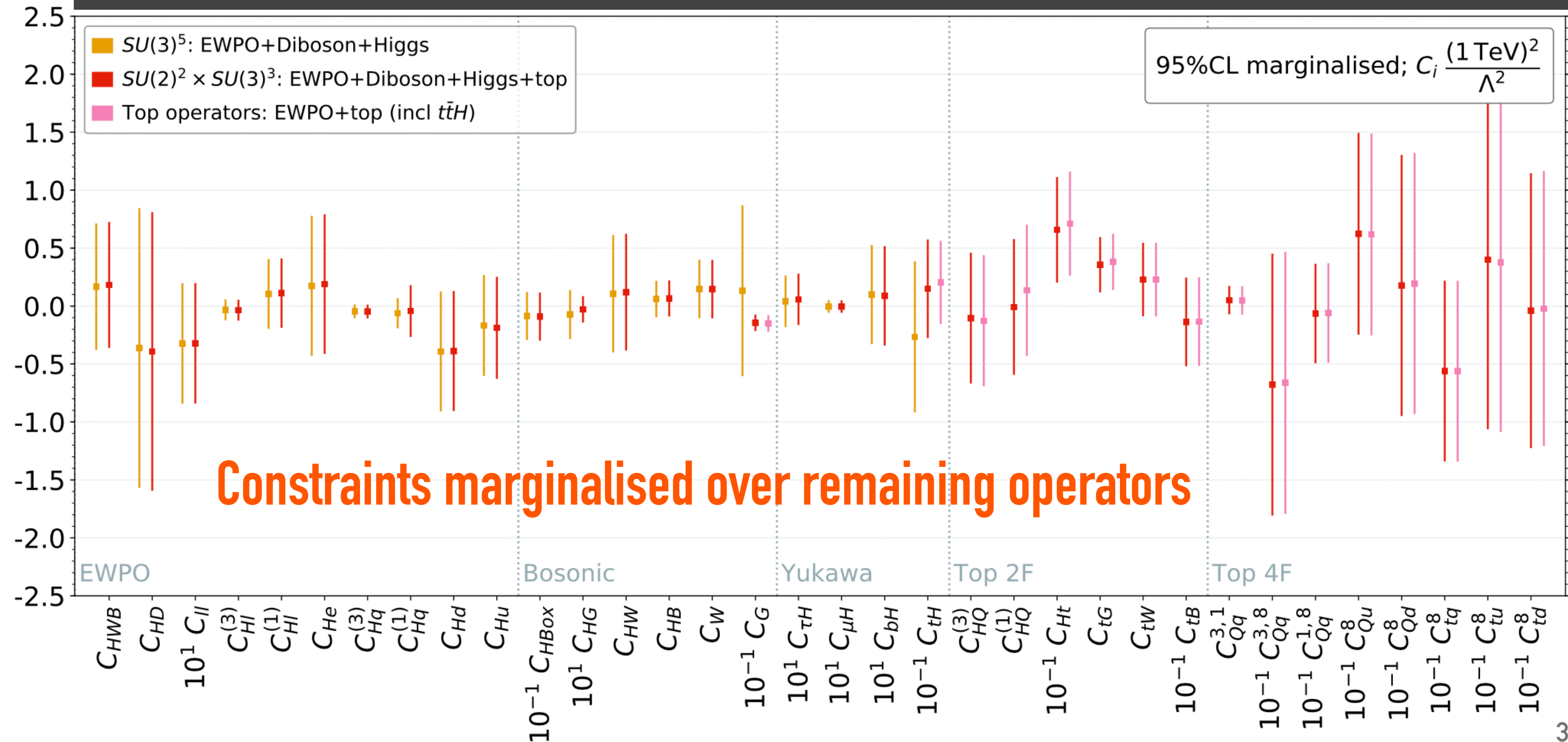
- Detailed scan of accessible regions parameter space
 - e.g. global EFT fits, dedicated searches & specific NP models
 - test of consistency structure of the theory (op. mixing and correlations)



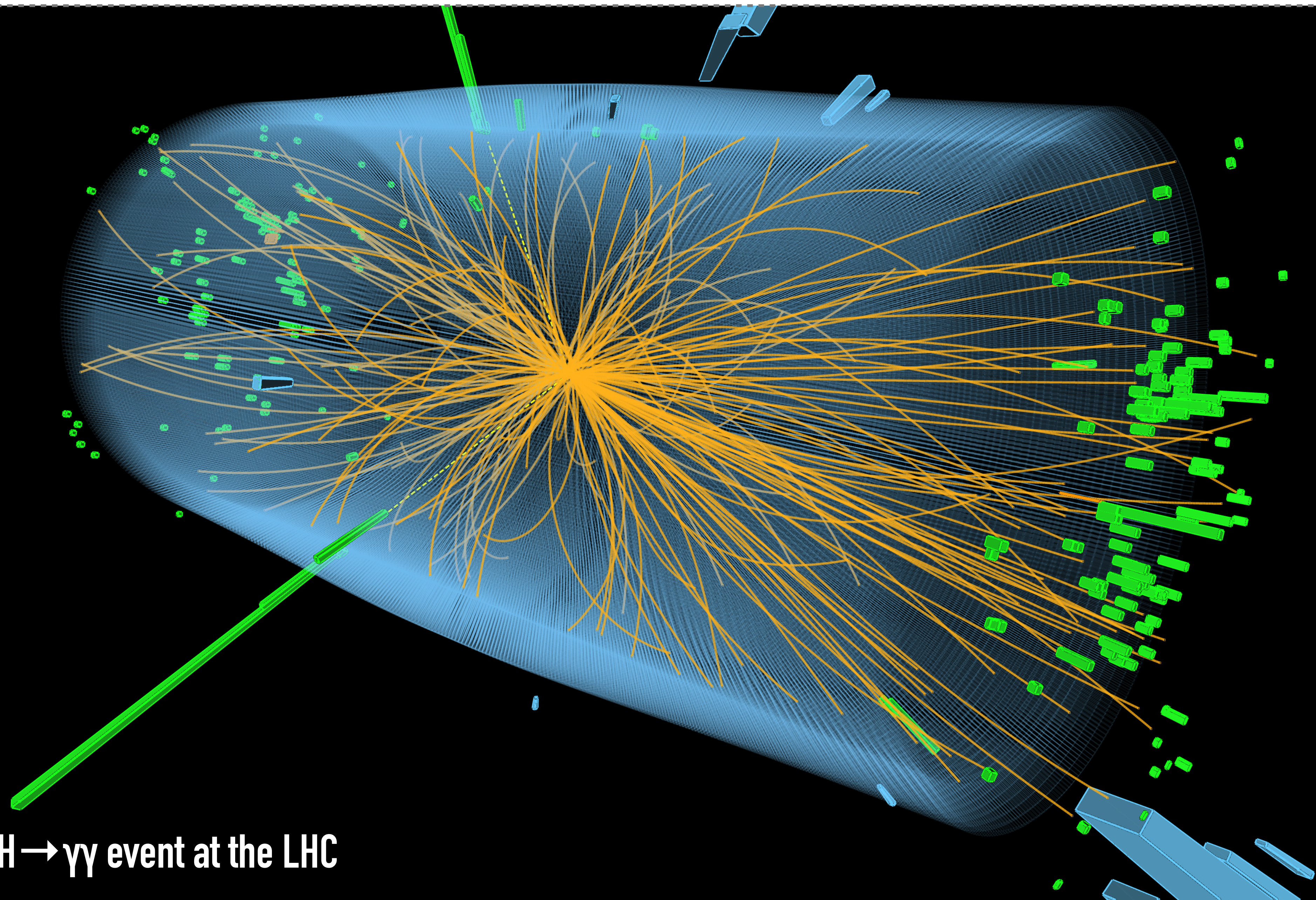
[Ellis, Madigan, Mimasu, Sanz, You '21]



e.g. SMEFT global fit using LEP+SLC+Tevatron+LHC



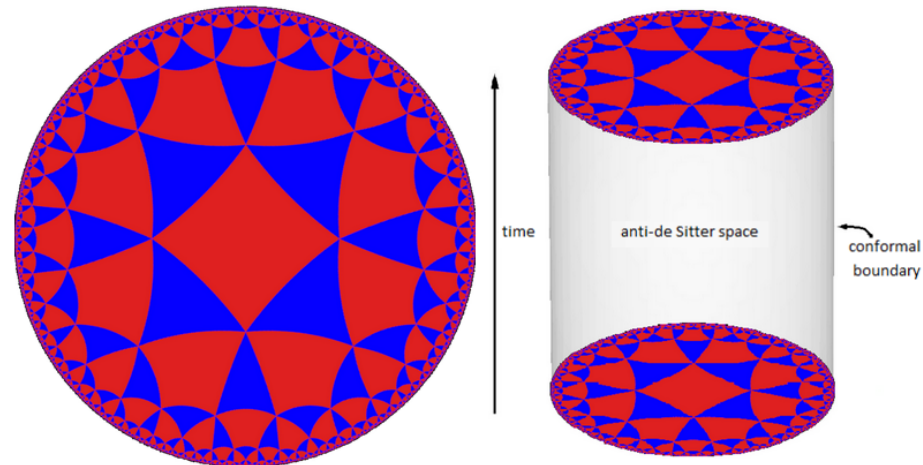
Main challenge: controlling the fine structure of collider events



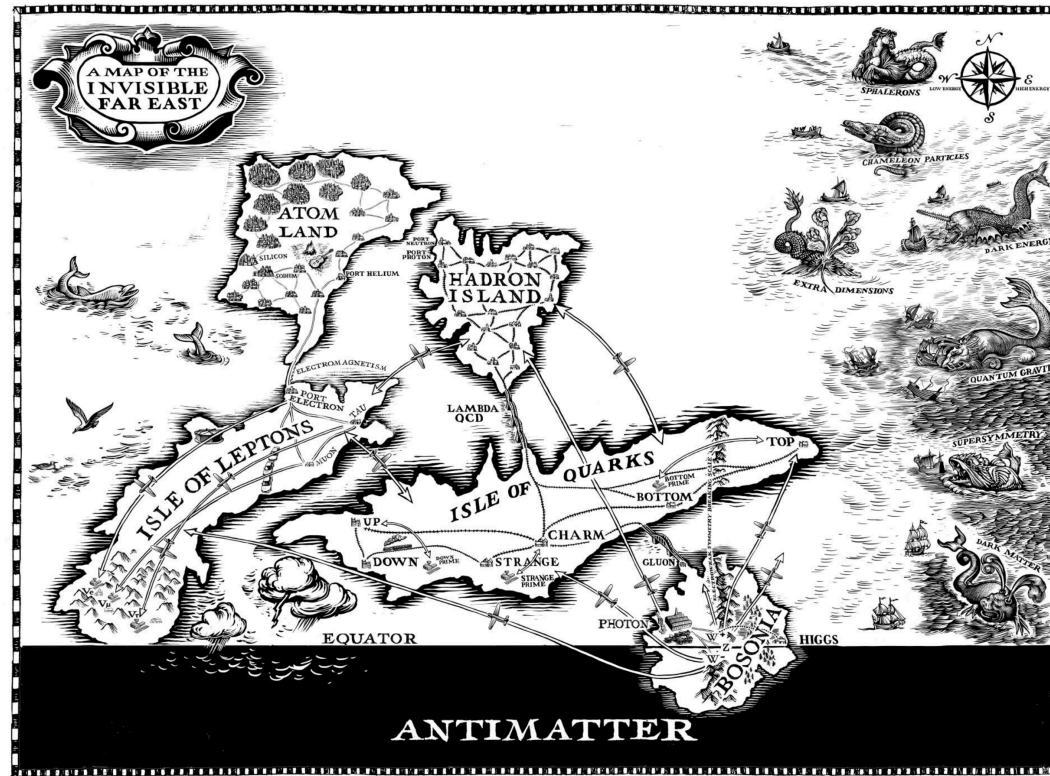
Candidate $H \rightarrow \gamma\gamma$ event at the LHC

Vast technological progress (jointly Theory ⊗ Experiment)

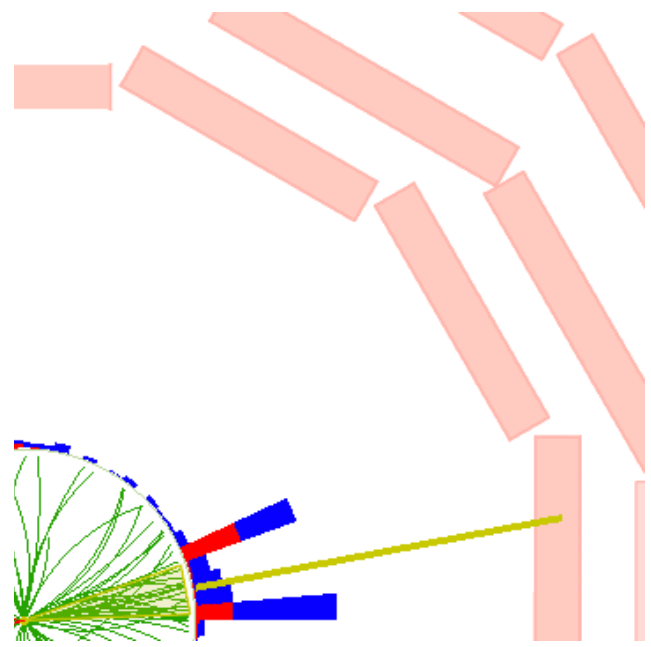
formal developments



landscape of NP models

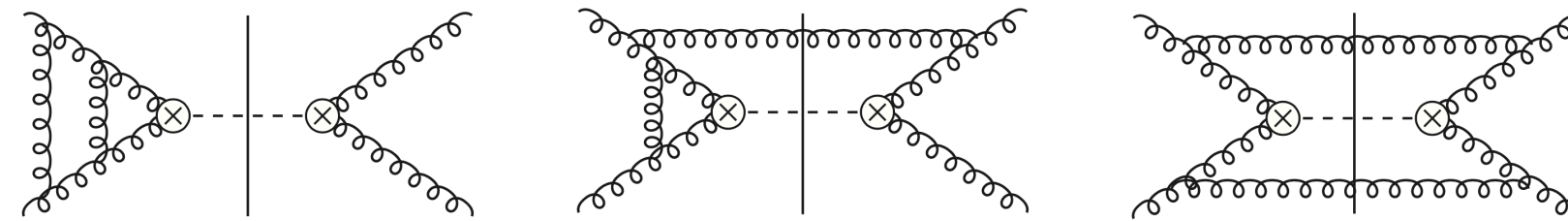


exp. systematics



Understanding of QFTs

perturbative methods

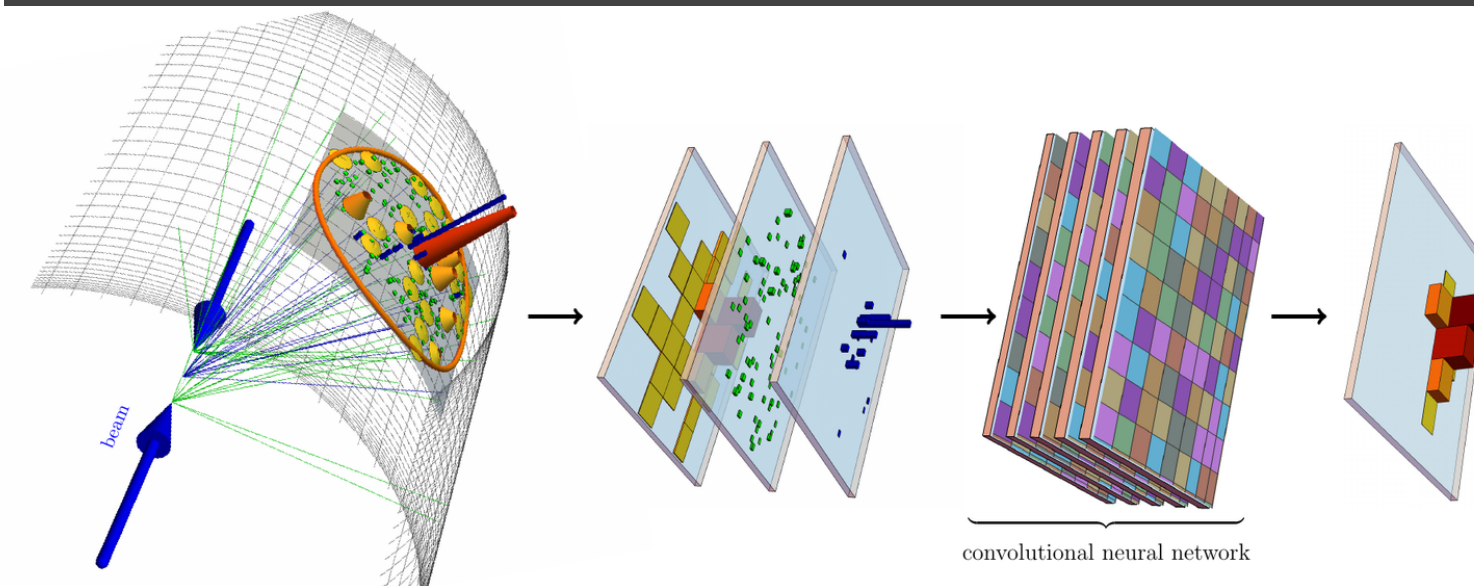


Experiment & pheno

event generators



novel strategies (e.g. ML, new observables)

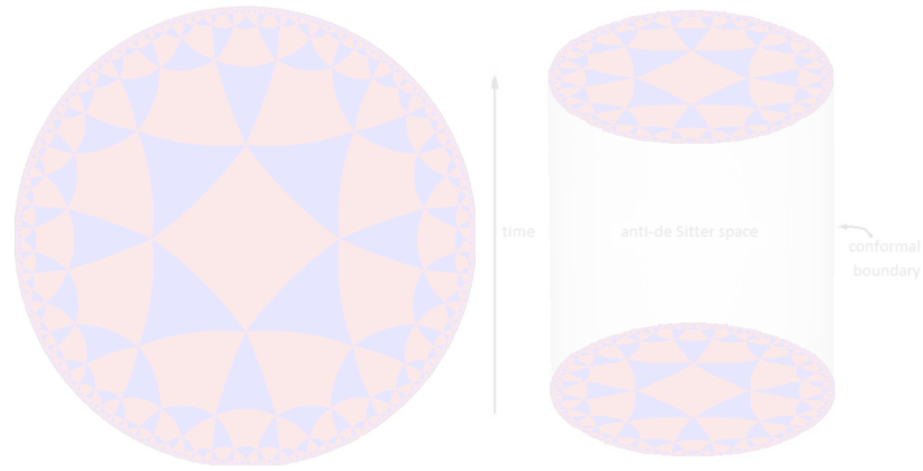


non-perturbative (QCD) corrections

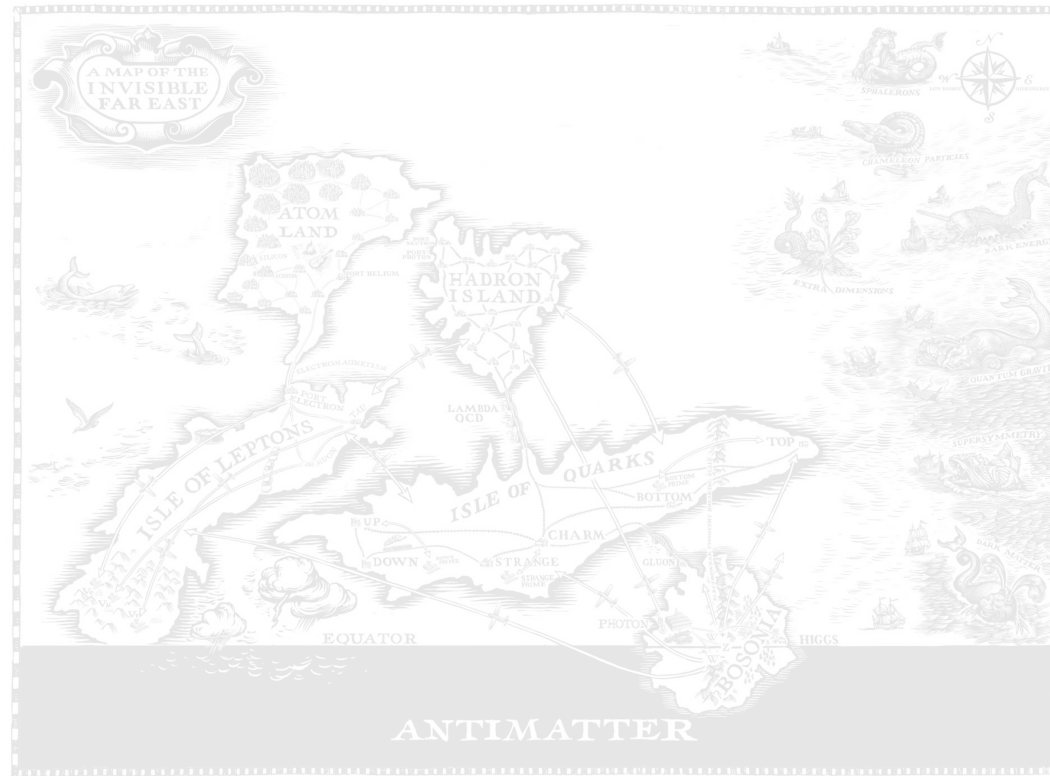
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E.g. Impressive progress in theoretical calculations

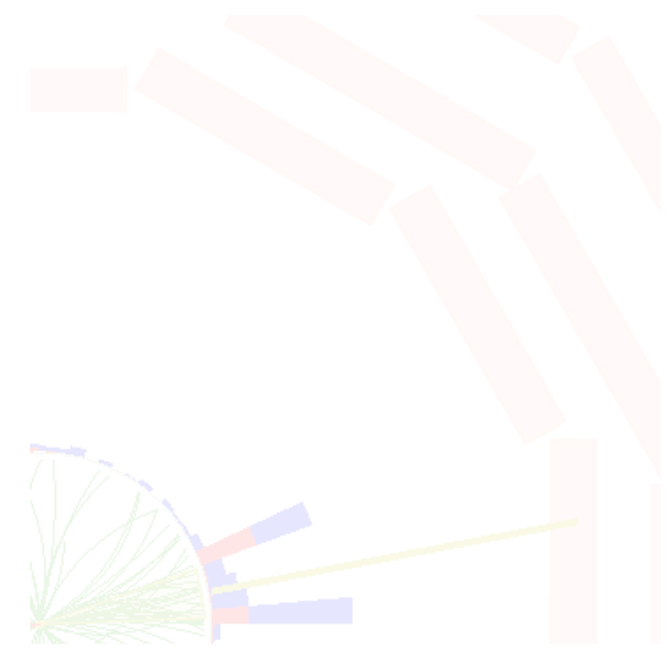
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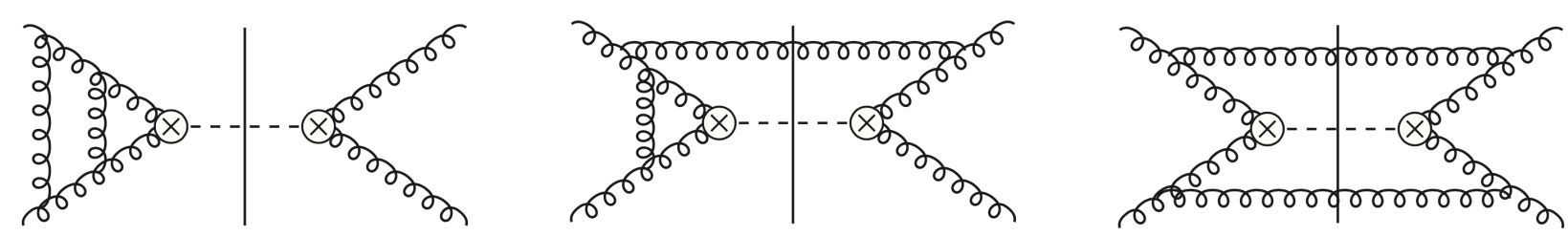


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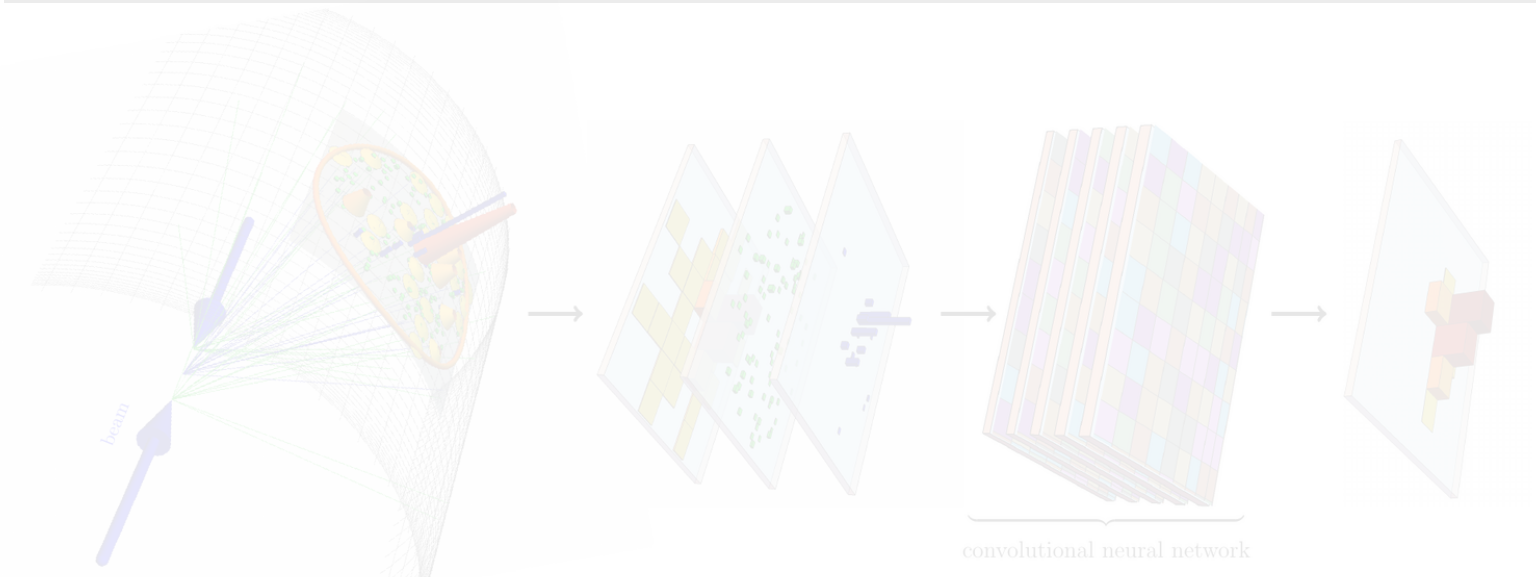


Experiment & pheno

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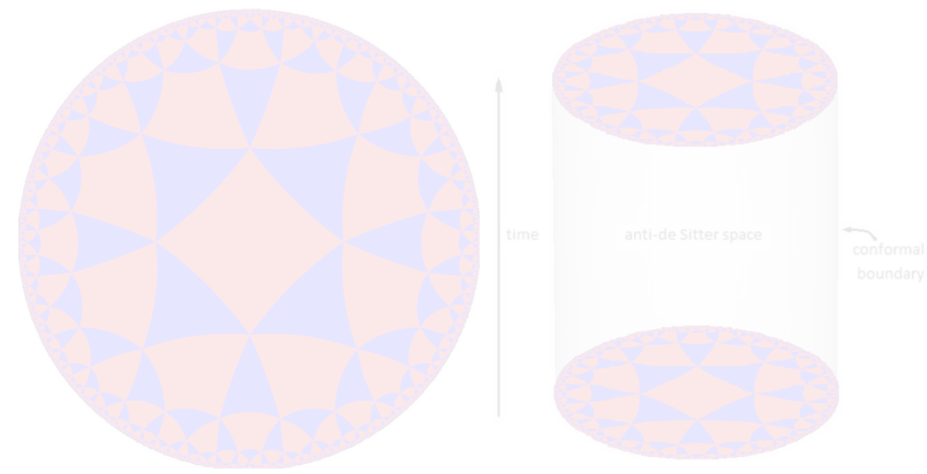


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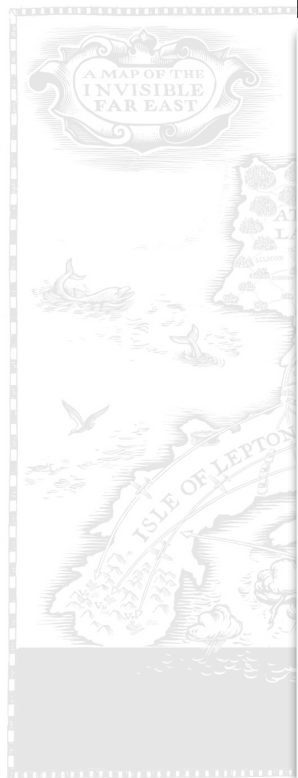
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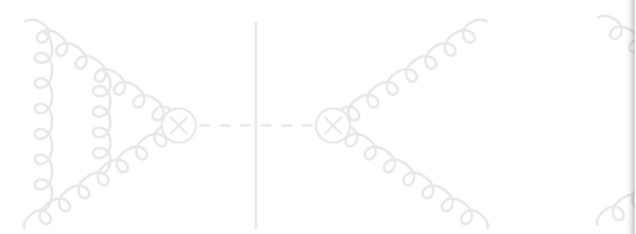


Understanding of QFTs

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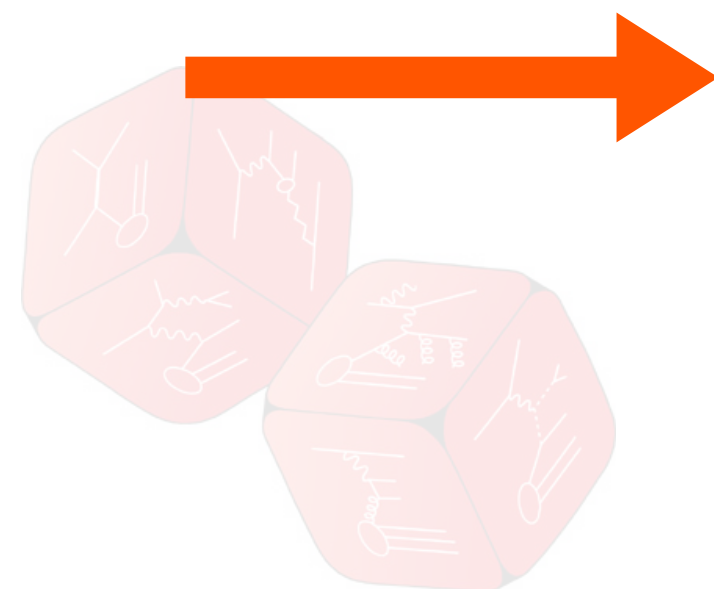
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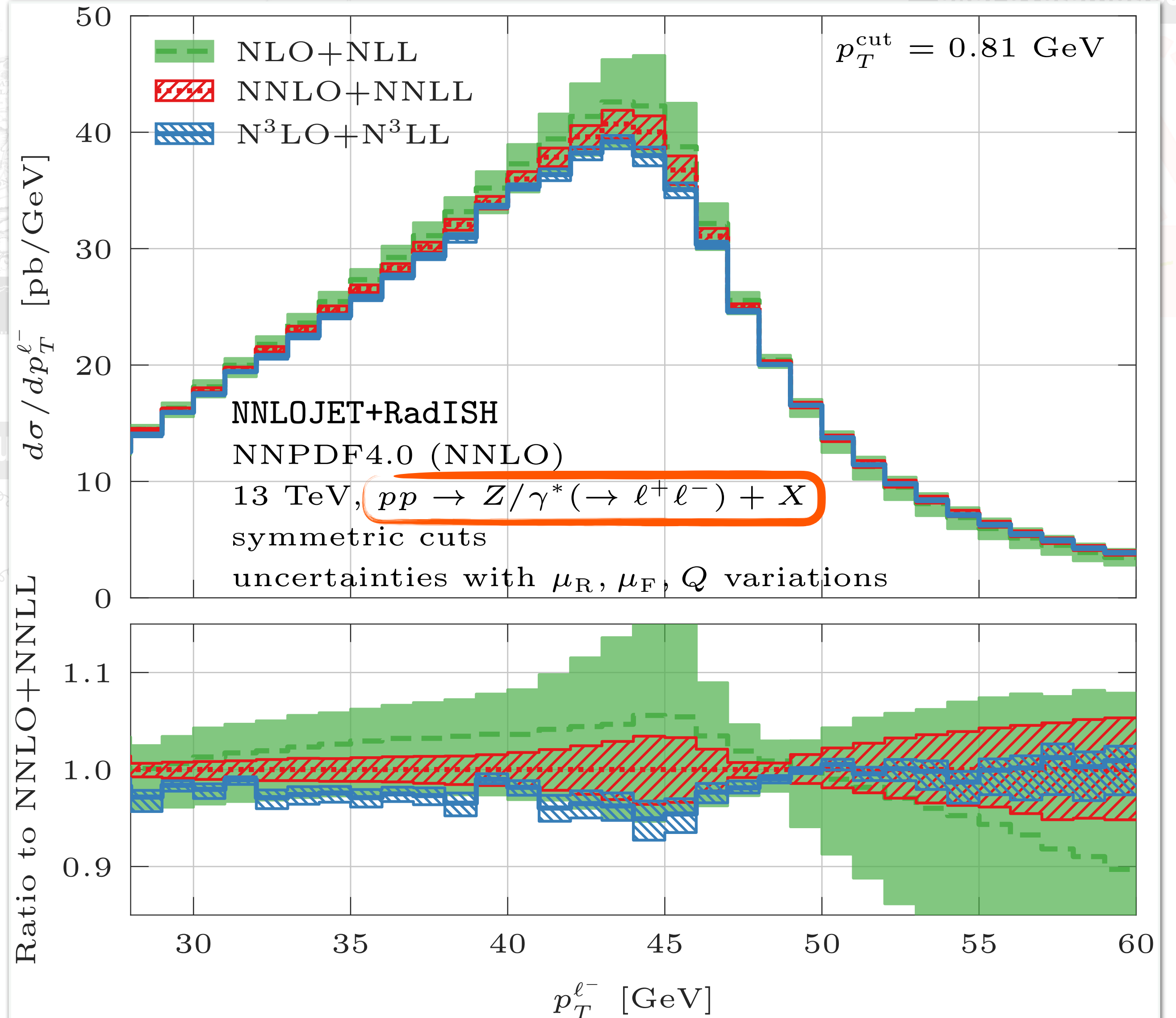
[Chen, Gehrmann, Glover, Huss, [PM](#), Re, Rottoli, Torrielli '22]

non-perturbative (QCD) corrections

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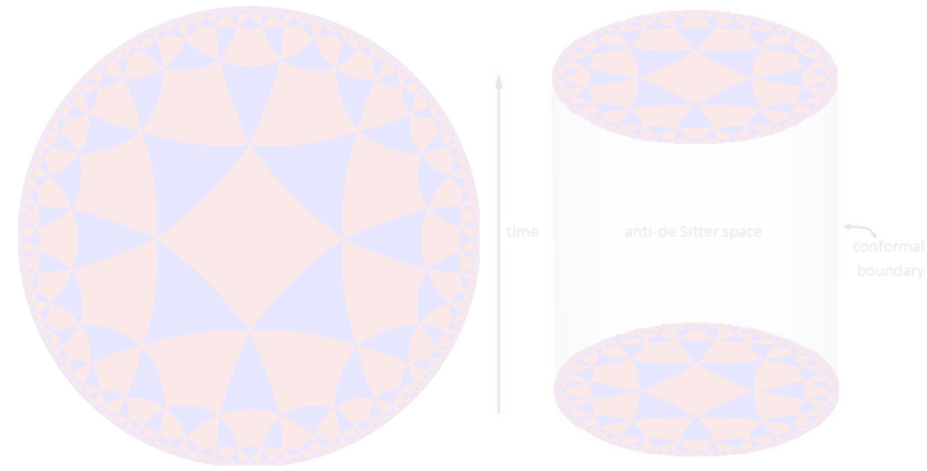


Fiducial Drell-Yan distributions at N³LO

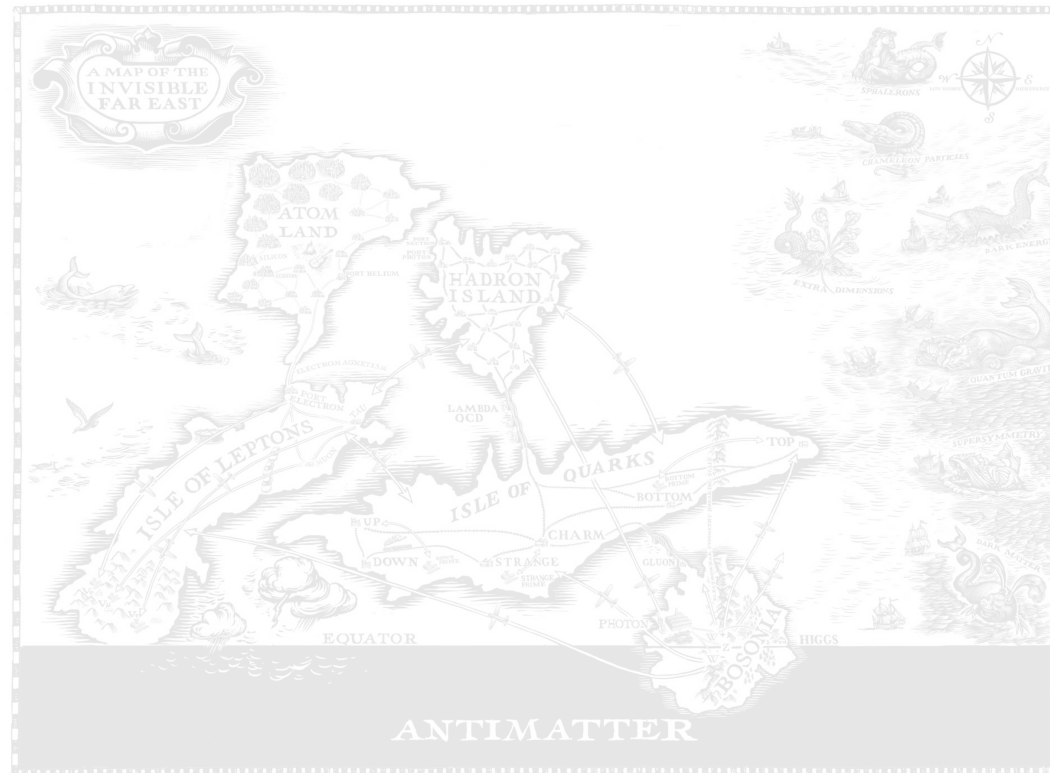


This talk focuses on another crucial aspect: Event Generators

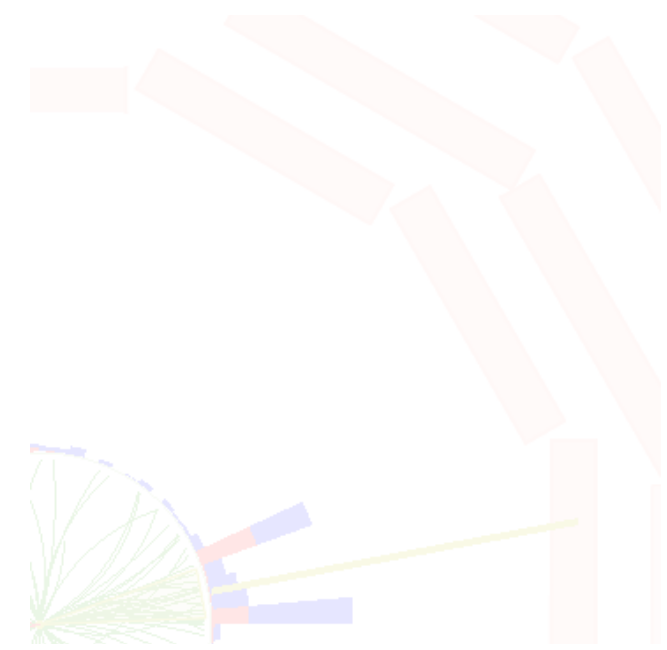
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landscape of NP models



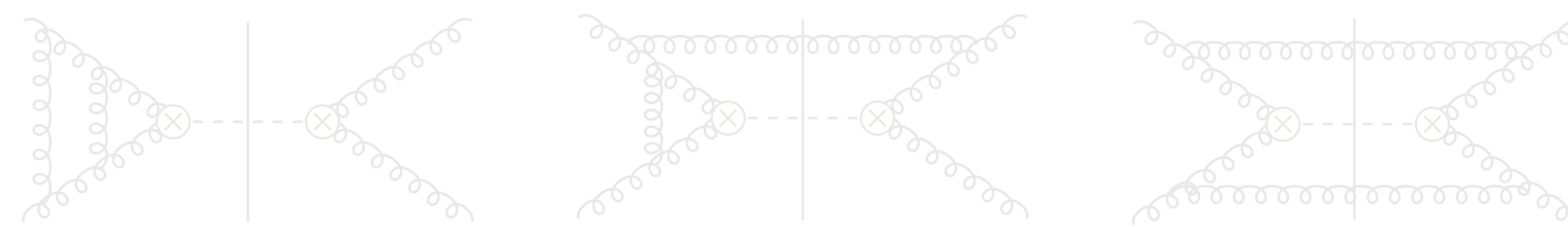
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Understanding of QFTs

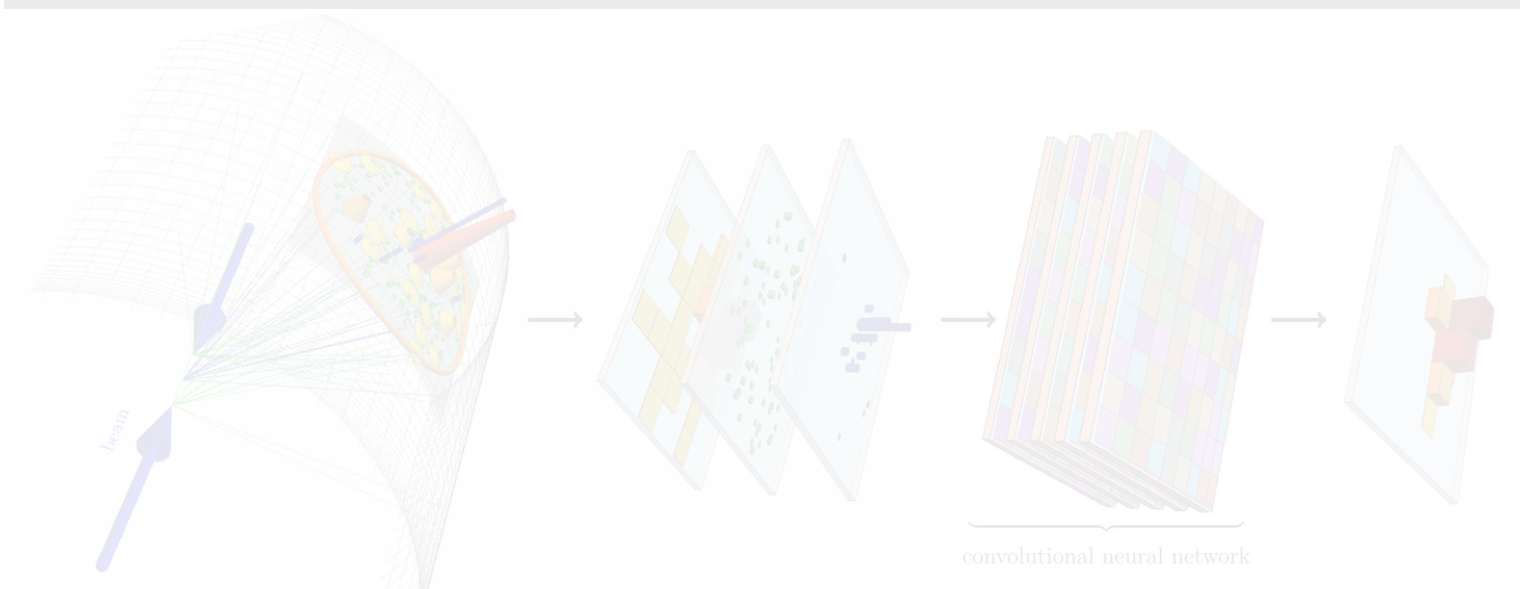
Experiment & pheno

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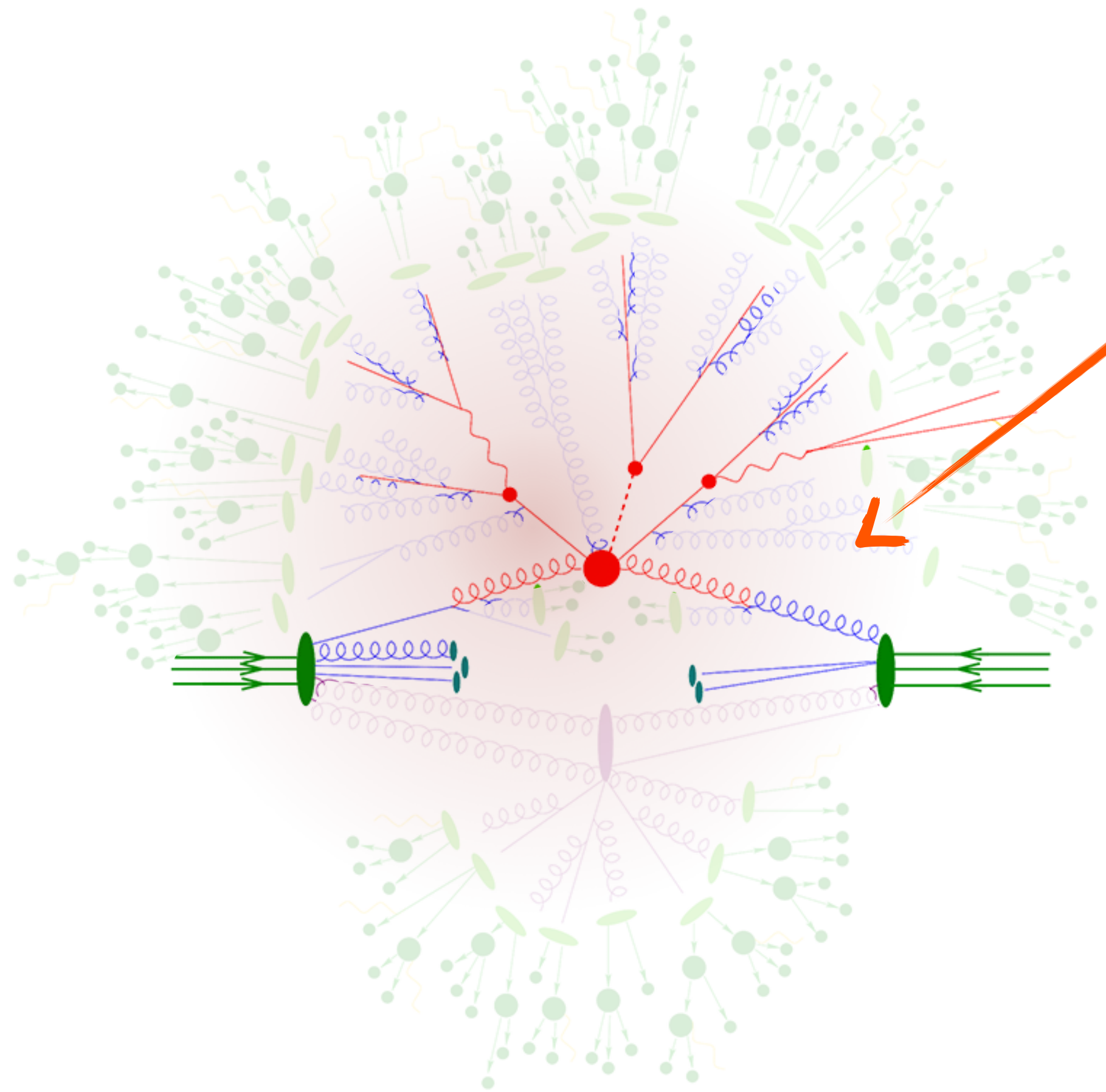
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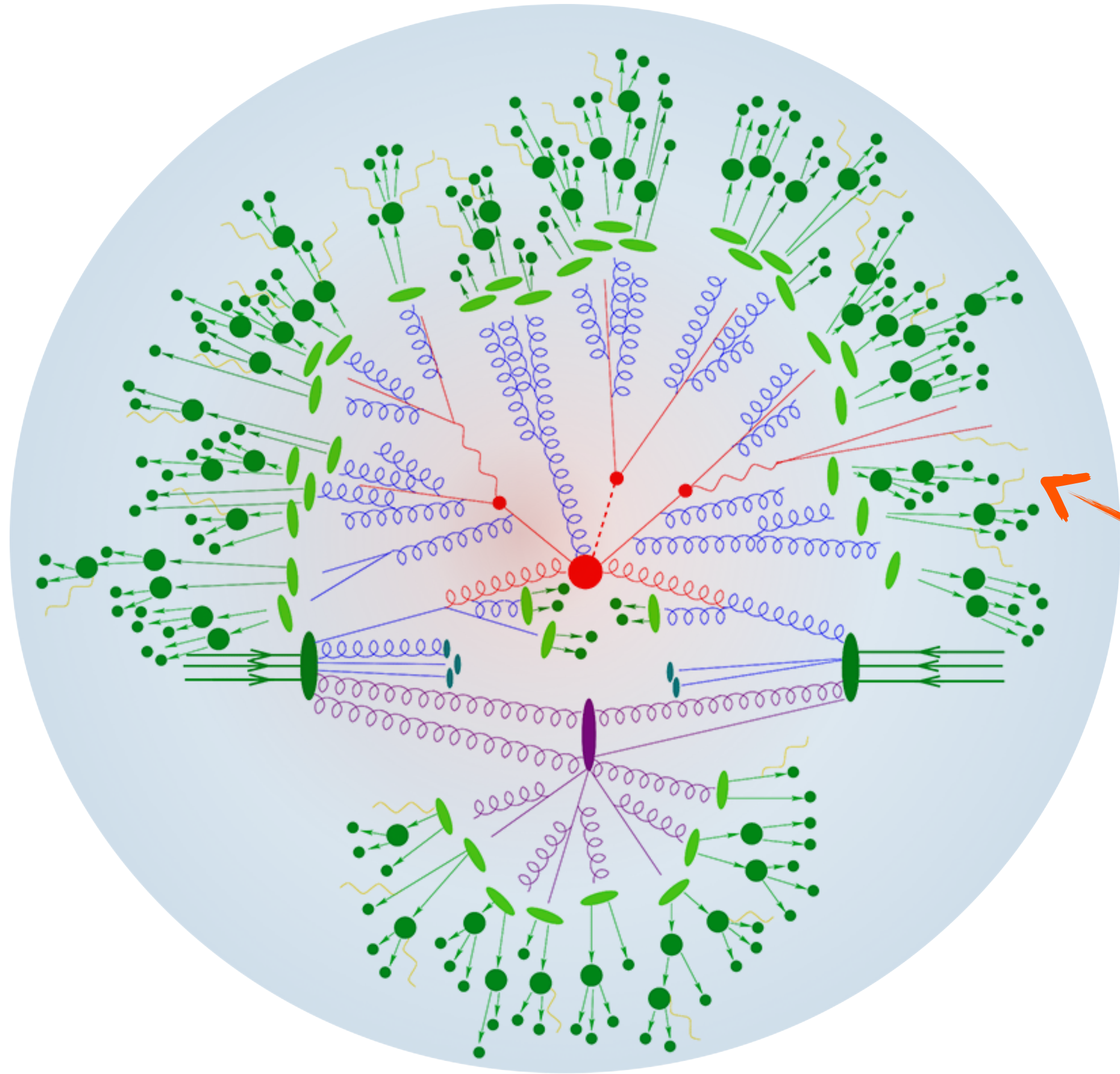
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Anatomy of a scattering reaction at the LHC



- Short distance (hard)
 - scales probed: $O(10^2)$ - $O(10^3)$ GeV
 - stage sensitive to NP

Anatomy of a scattering reaction at the LHC



- Short distance (hard)

→ scales probed: $O(10^2)$ - $O(10^3)$ GeV

→ stage sensitive to NP



*evolution towards a
physical observable state*

(mainly QCD)

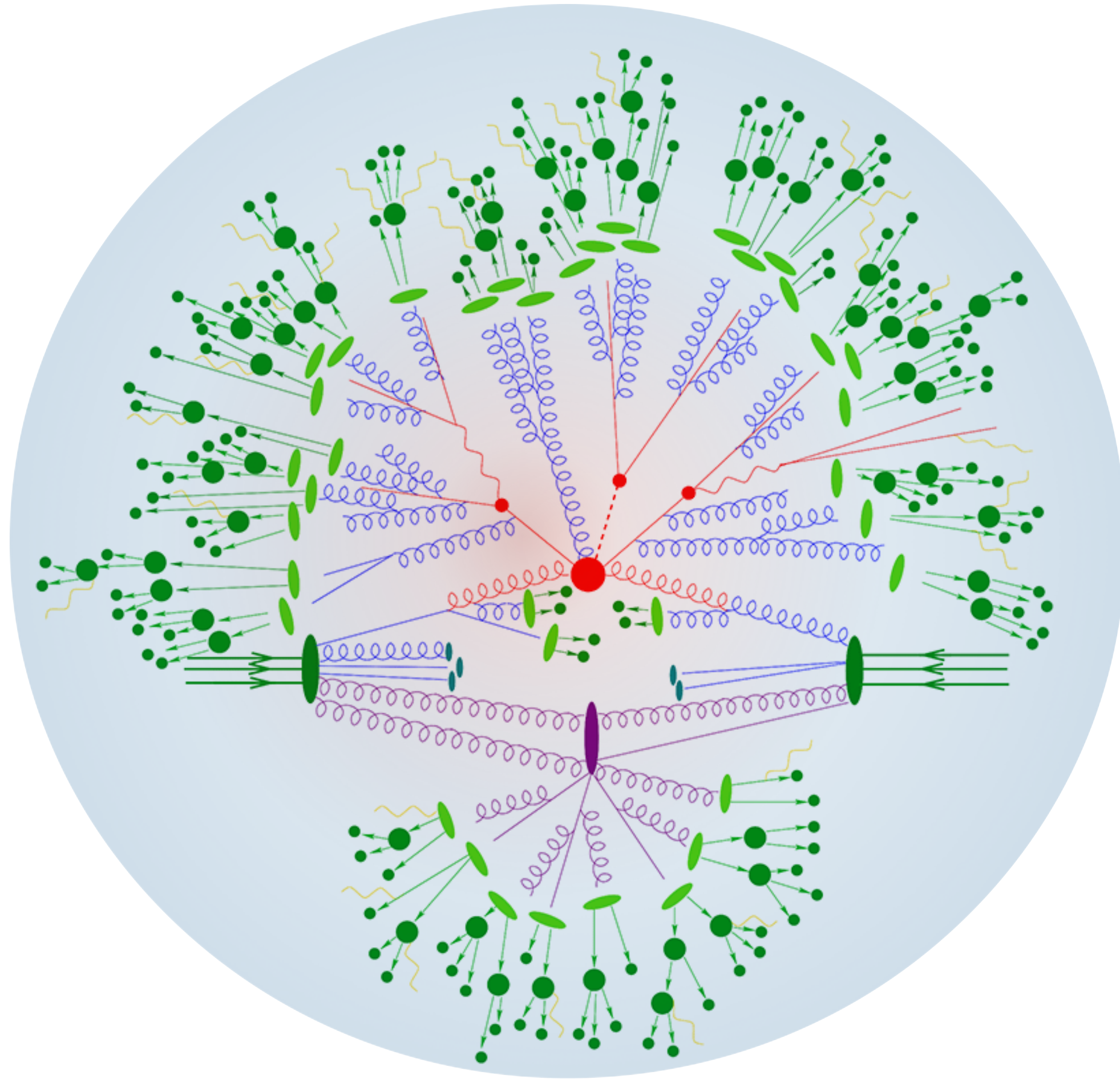
- Long distance (soft)

→ transition from $O(10^2)$ - $O(10^3)$ GeV to $O(1)$ GeV

→ hard scattering gets "showered"
with soft [and/or collinear] radiation

→ Output: what is actually measured

Event generators simulate all stages of the event formation



- ◉ Not a standard theory calculation:
 - **return events**, i.e. particle momenta with a physical probability distribution
 - **allow the computation of many (~any) observables at once**, as opposed to a few of them in perturbative calculations
 - **deeply different mathematical formulation**, difficult to exploit state of the art QFT technology
- ◉ **Crucial pillar of modern collider physics**, e.g. full simulation of experimental analysis, phase-space extrapolation, training of tools (e.g. Machine Learning)

Strength: Back bone of nearly all LHC analyses



Herwig



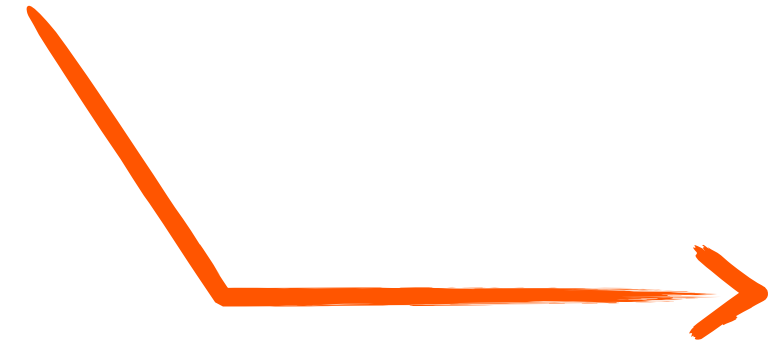
Pythia



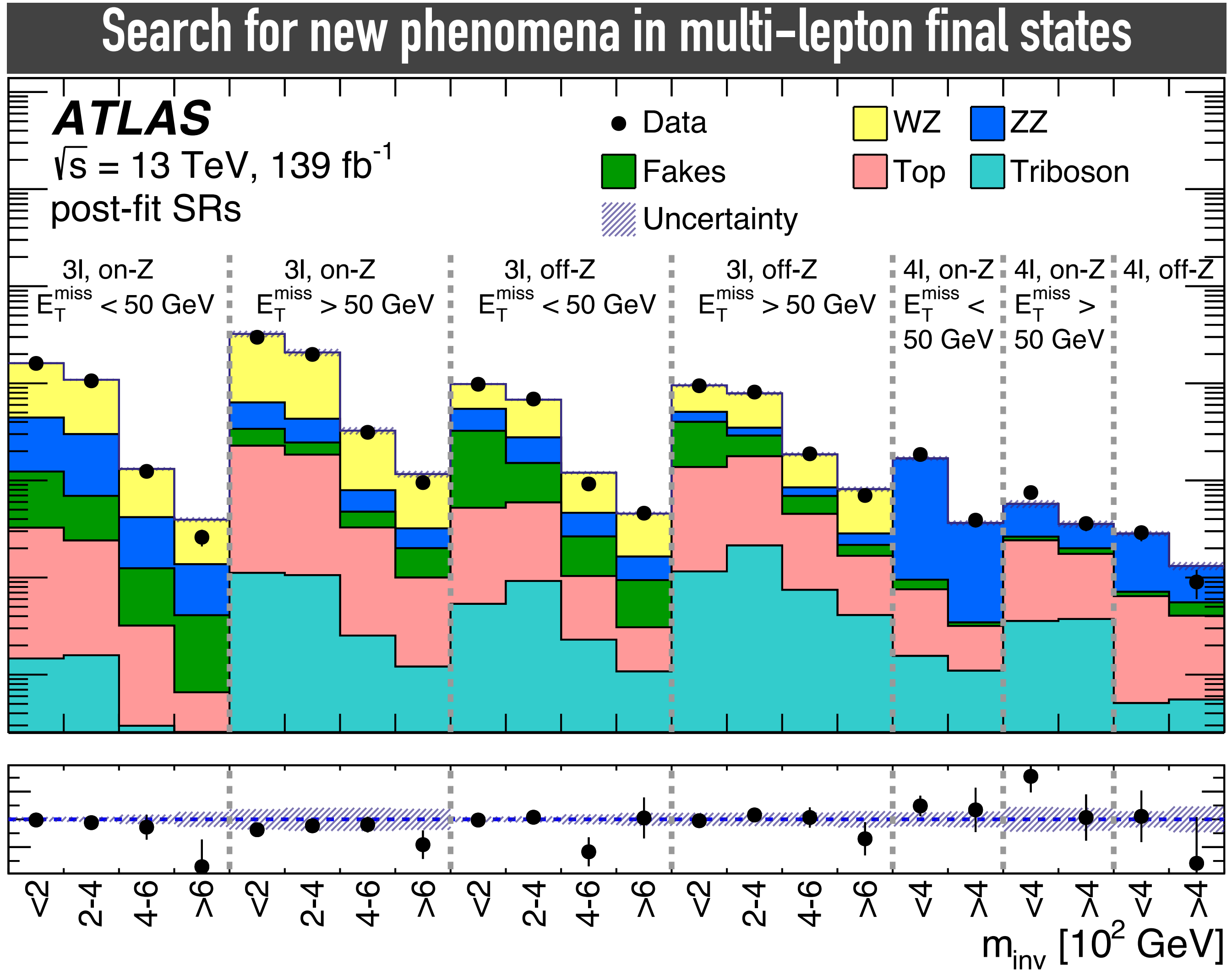
Sherpa

[+ POWHEG & MC@NLO]

Astonishing description of broad classes of collider reactions, with good accuracy (e.g. searches with rich background)

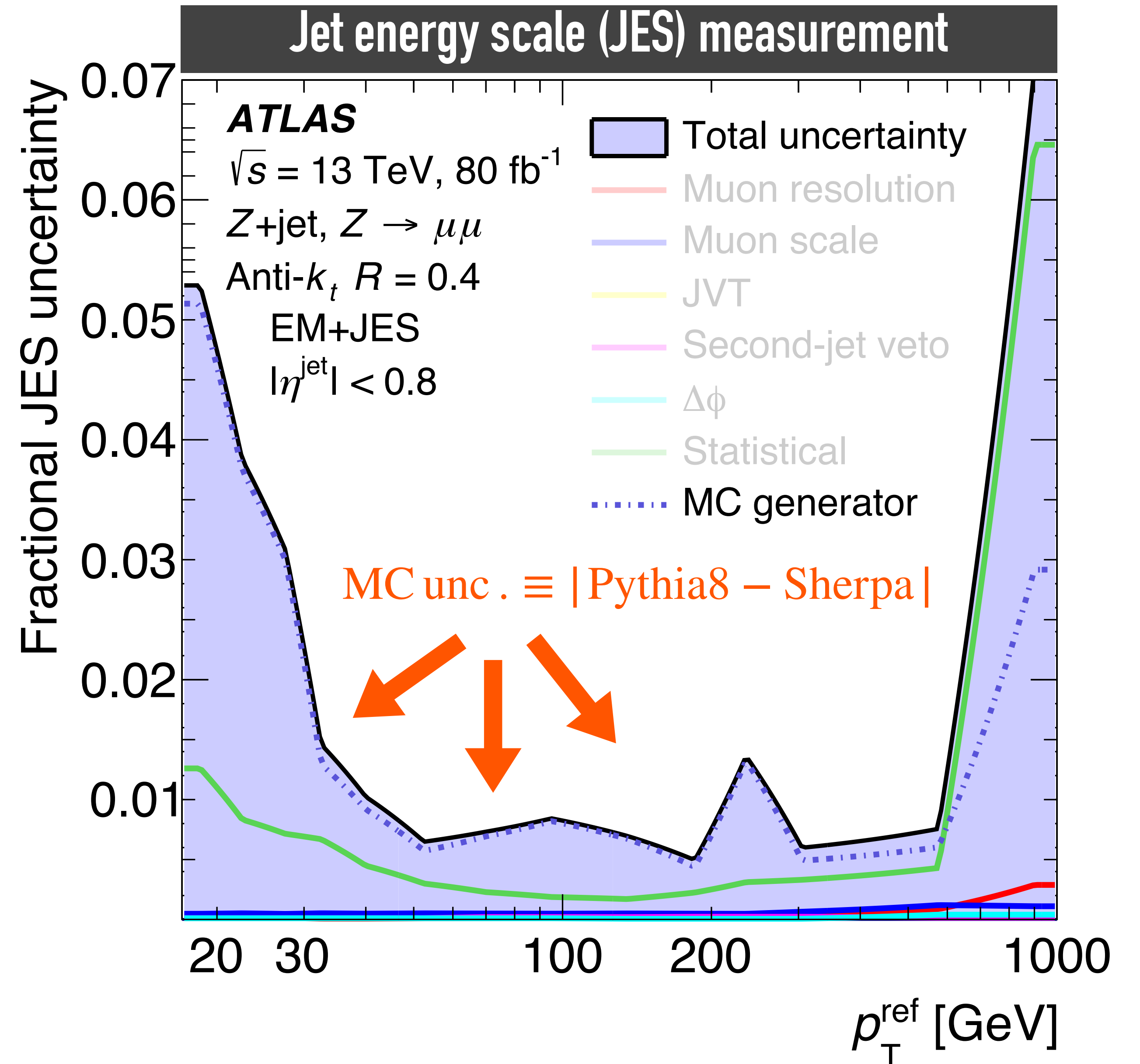


[ATLAS '22]



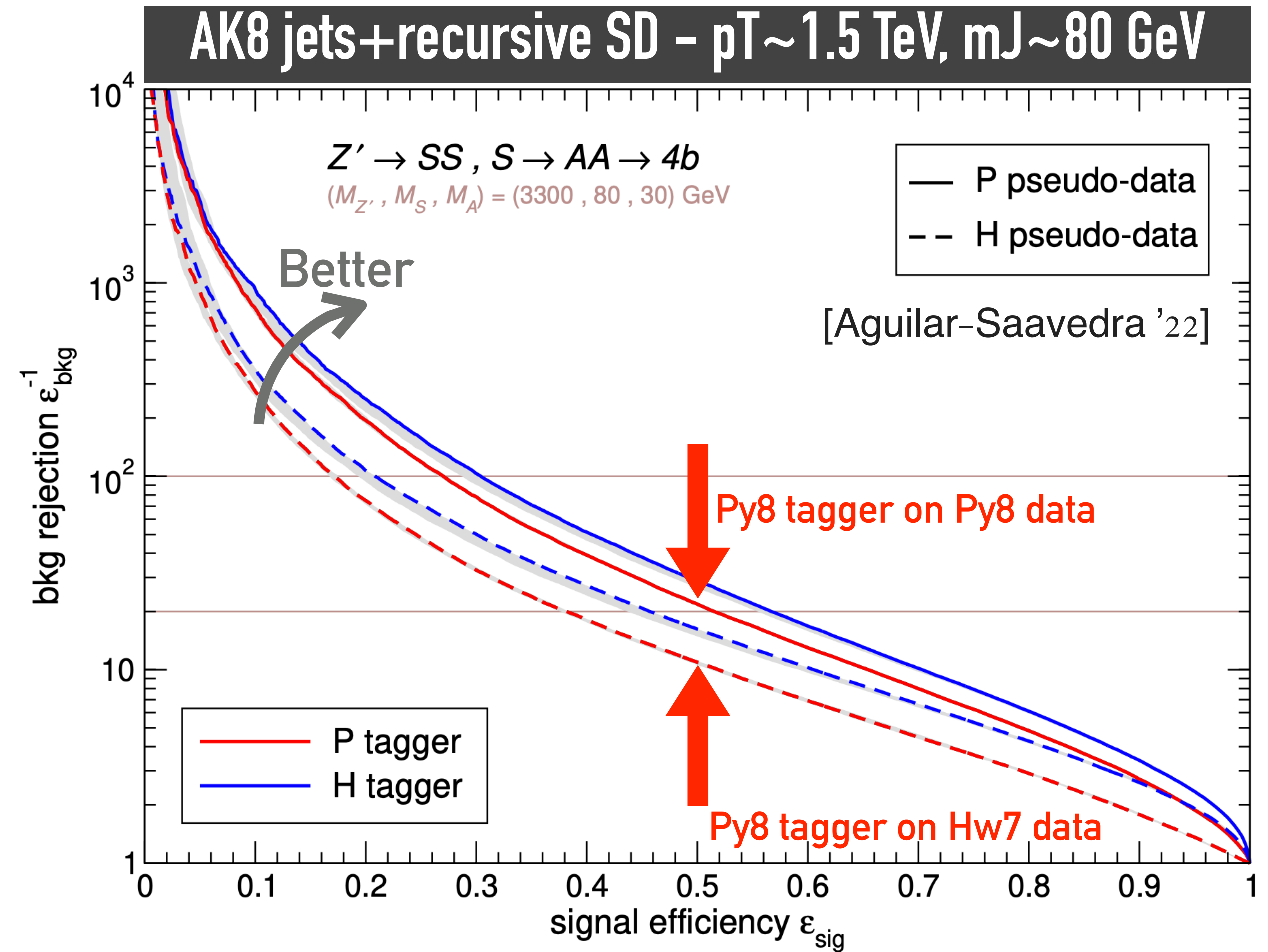
Weakness: Inaccuracies are now often the leading systematics

- The improving experimental performance highlights **limitations of event generators**
- Soon to be the bottleneck of LHC physics programme
 - Jet Energy Scale uncertainty (→ affecting many measurements)
 - ... this is but one example



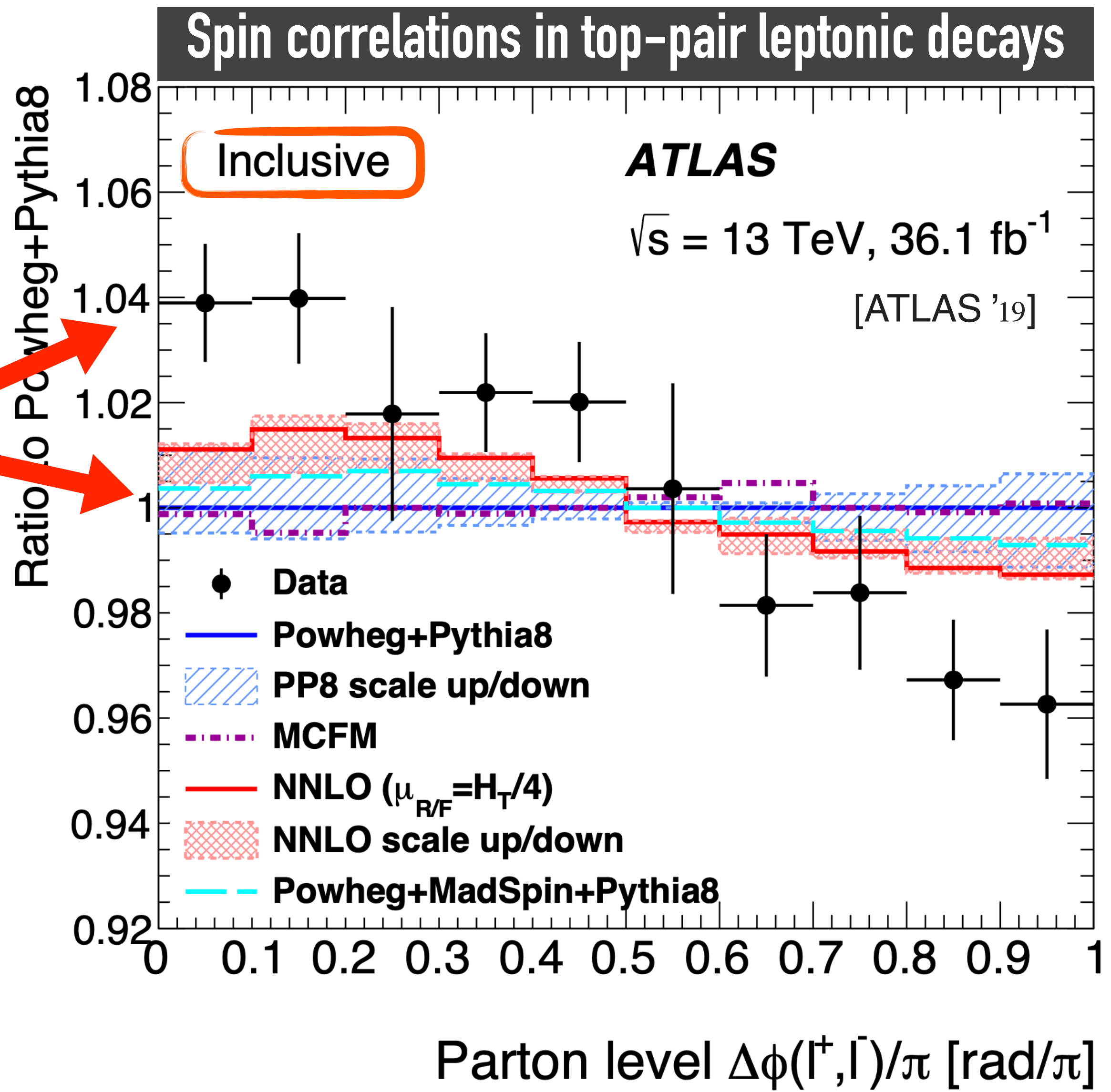
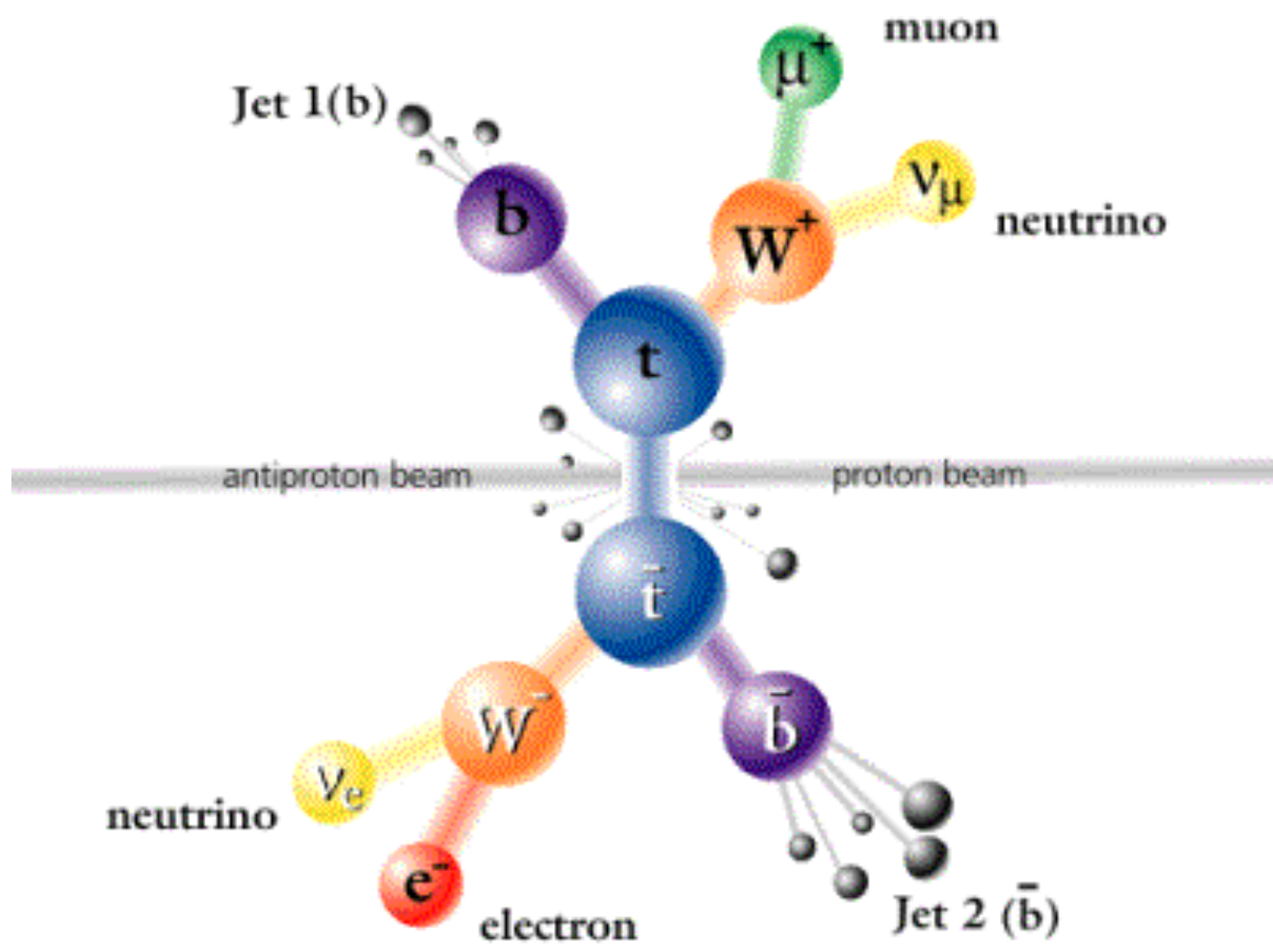
Robust training of Machine Learning (ML) algorithms

- ML technology provides a great boost in sensitivity w.r.t. orthodox analysis techniques
- However, this comes often with a dependence on the modelling, i.e. Monte Carlo generator, raising the question of **accuracy**
 - e.g. dependence of 4-pronged tagger on training model & pseudo-data
- New generation of tools paramount** to push this technology in the precision era of LHC



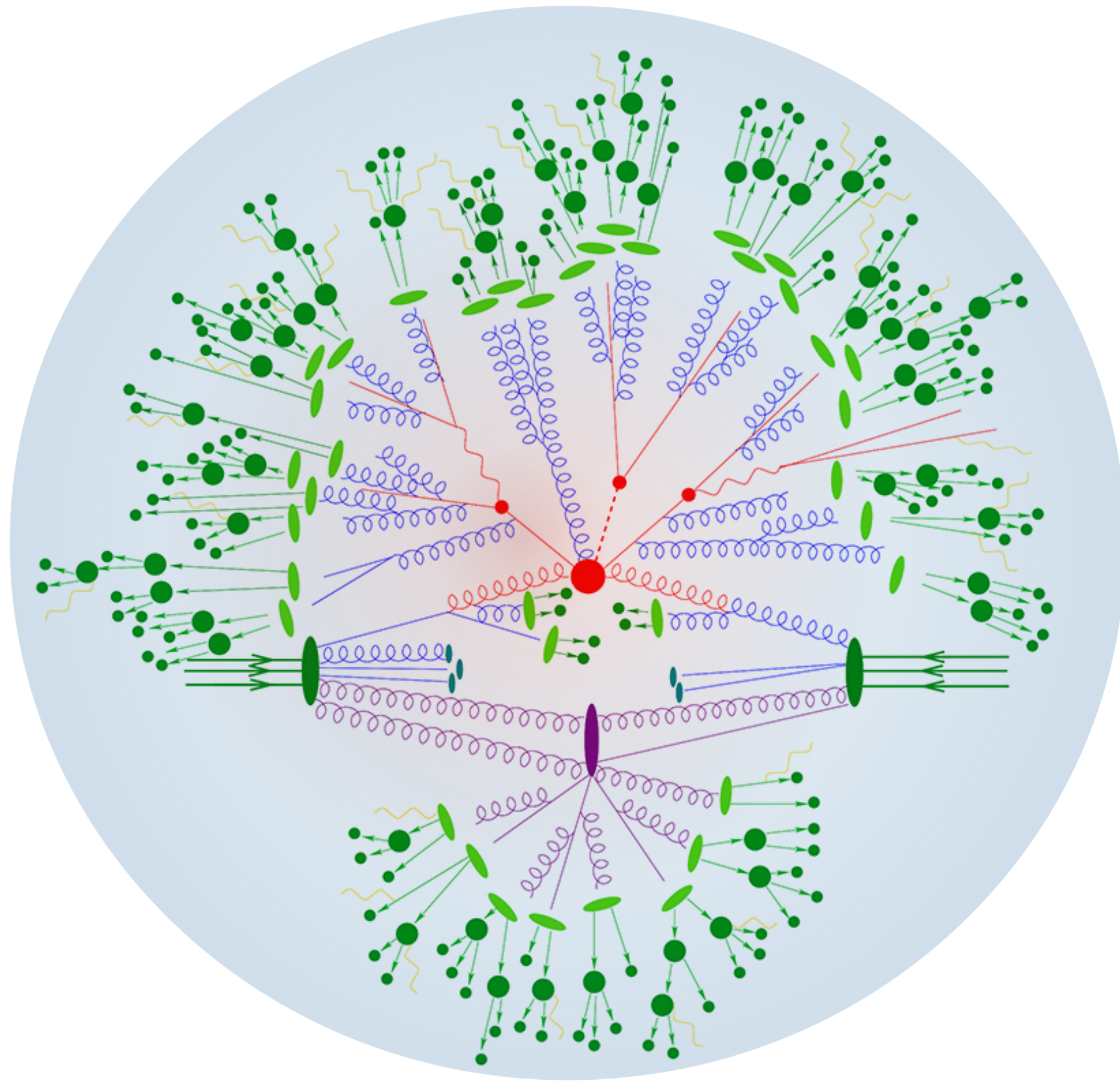
Extrapolation of experimental measurements

- MC generators used to extrapolate experimental data from fiducial to inclusive phase space (easy comparison with theory and interpretation)
- Inaccuracies may lead to dangerous biases
 - e.g. discrepancy in $t\bar{t}$ spin-correlations: new physics or mis-modelling? (more later)



**The overarching question:
Can we do better?**

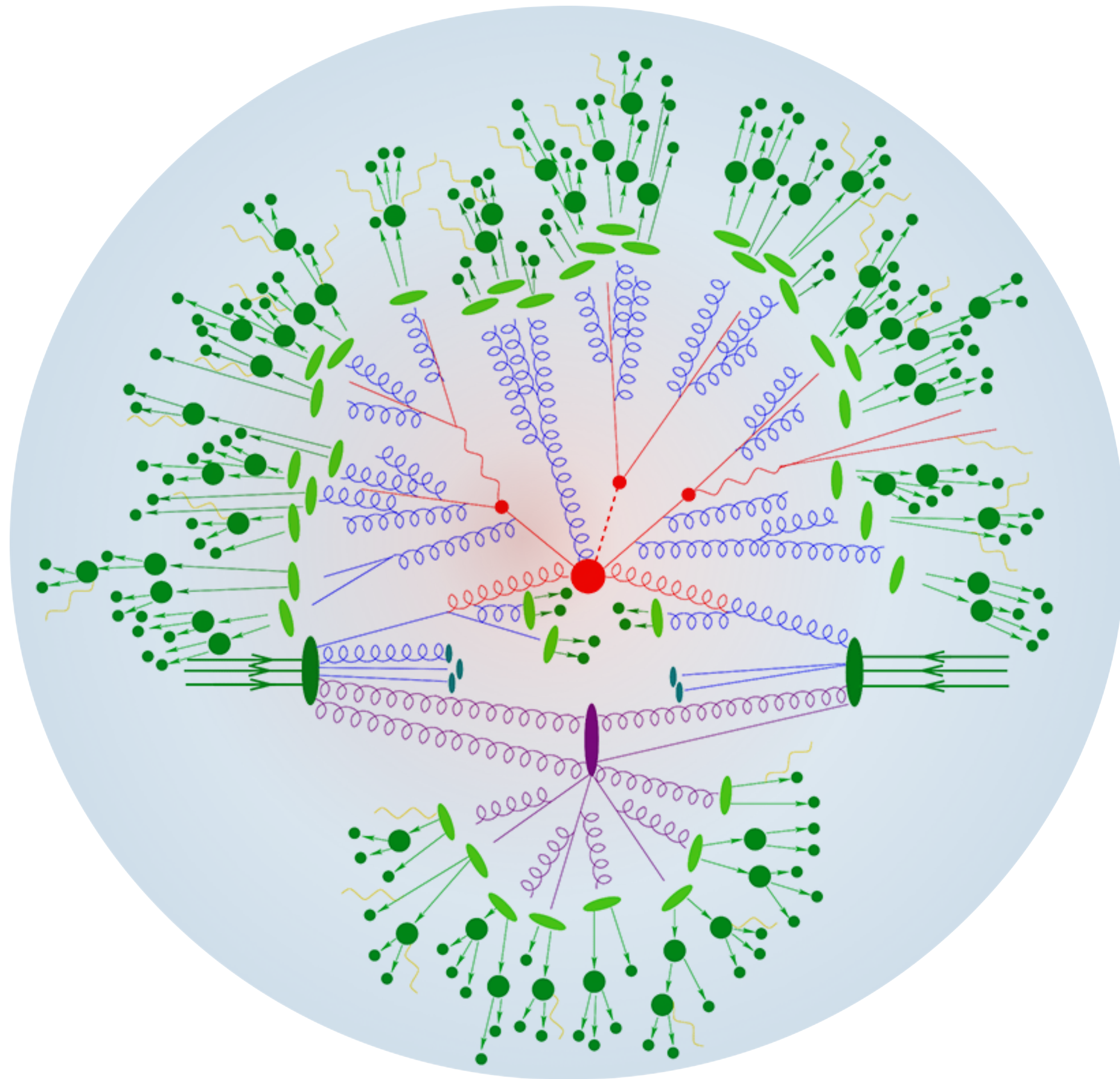
How do we even define the accuracy of event generators?



- Evolution spans several orders of magnitude in energy scale
 - Different perturbation theories needed in different regimes (e.g. fixed-order, logarithmic power counting, subleading power corr.^{ns})
 - We should demand that event generators reproduce these limits correctly
 - **This talk addresses the two main elements: the hard scattering & the parton shower**

The parton shower stage

The parton shower component



- Large hierarchy of scales ($\mu_{\text{hard}} \gg \mu_{\text{soft}}$)
 - Yet, fully perturbative regime ($\mu_{\text{soft}} \gg \Lambda_{\text{QCD}}$)
 - Initial conditions for hadronisation
- Several successful public tools:



Herwig



Pythia



Sherpa

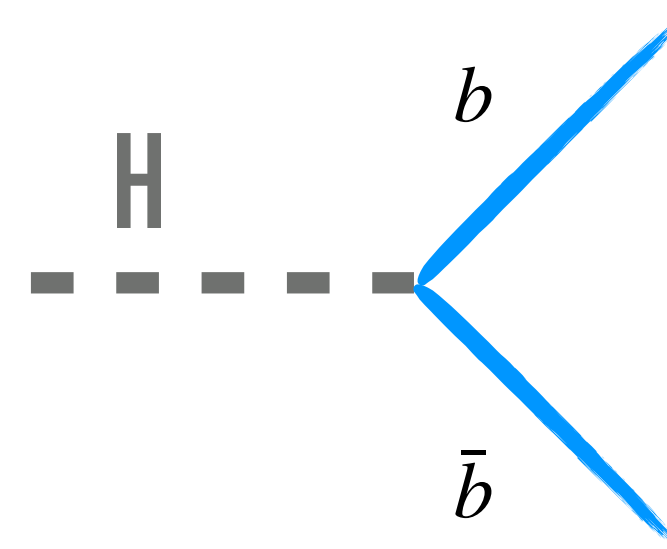
[also DiRe, Deductor]

How do they work? [dipole shower case]

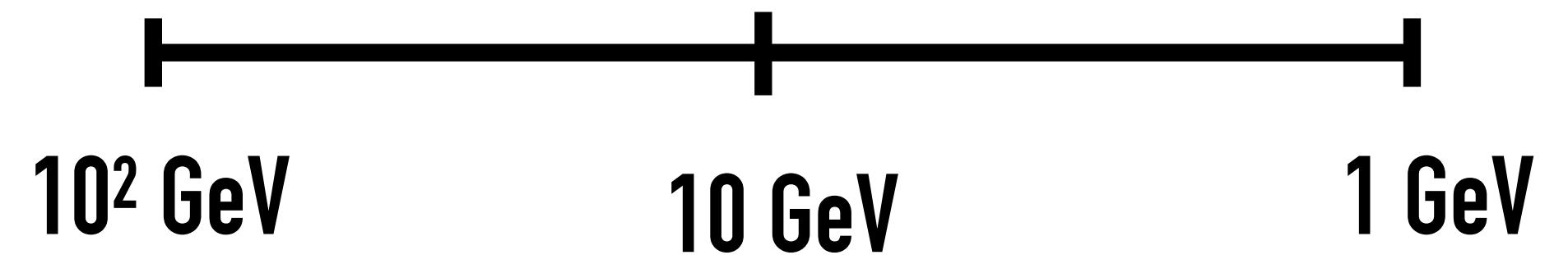
- Algorithms based on concepts invented in the mid '80s. Many variants built across the years
e. g. [Sjostrand '85; Marchesini, Webber '88; Lonnblad '89]

e.g. $H \rightarrow b\bar{b}$ decay

- Schematically [non-linear evolution]:
 - Recursive iteration of 2→3 branching probabilities [i.e. LO splitting functions]
 - Evolve towards smaller values of a resolution variable [e.g. dipole transverse momentum]
 - Kinematic map to restore on-shellness [i.e. recoil scheme]
 - Iterate until hadronisation scale is reached



Energy scale

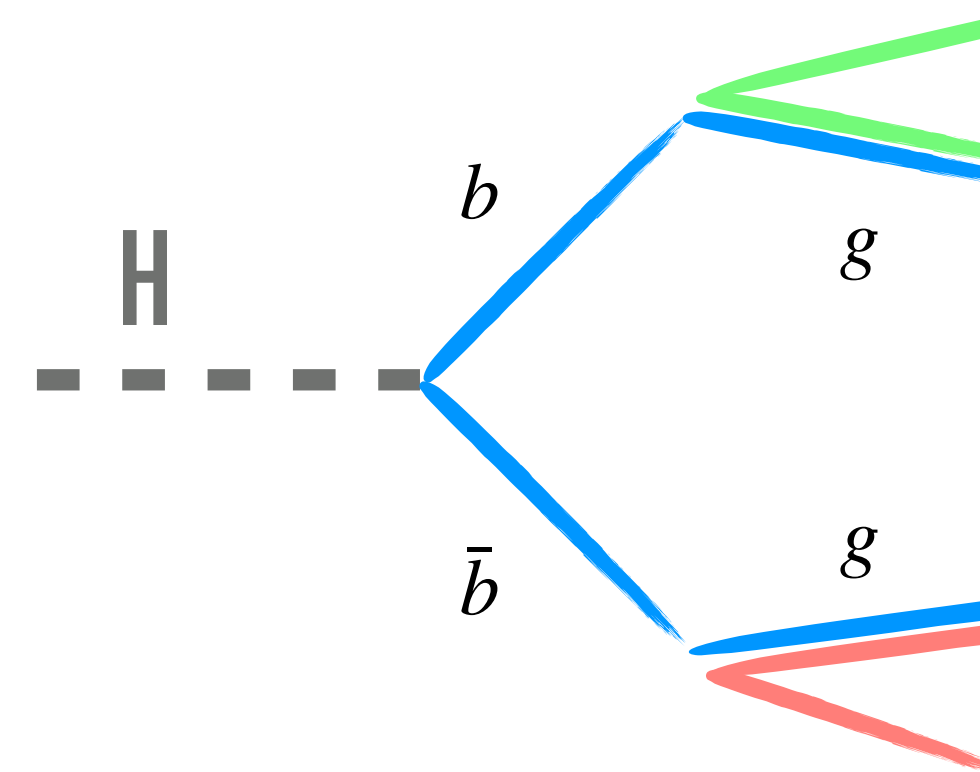


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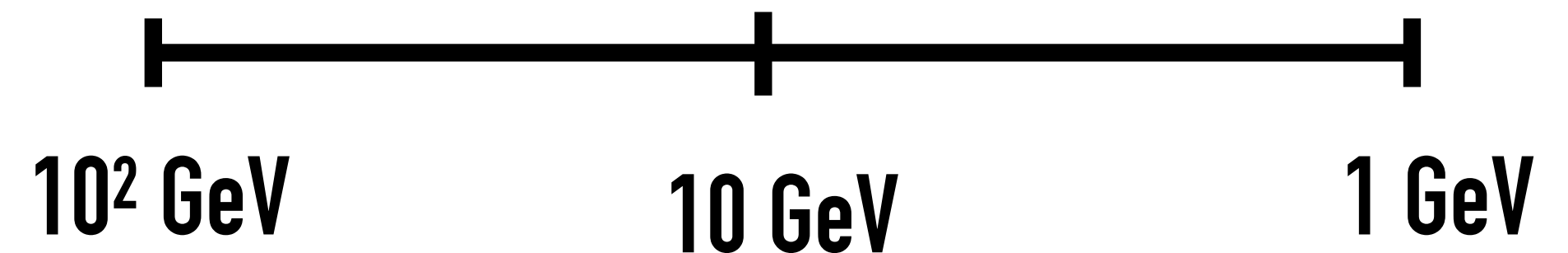
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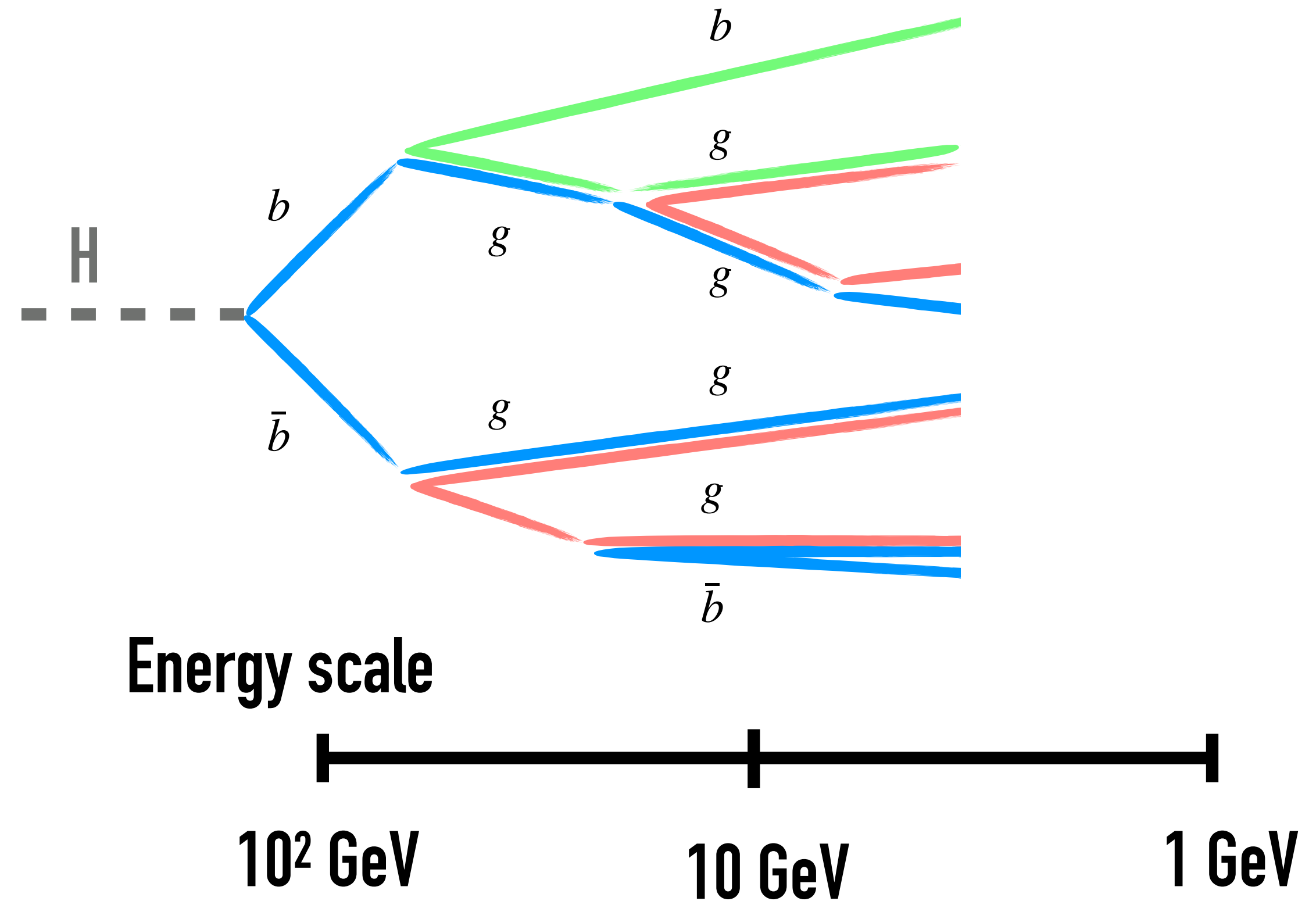


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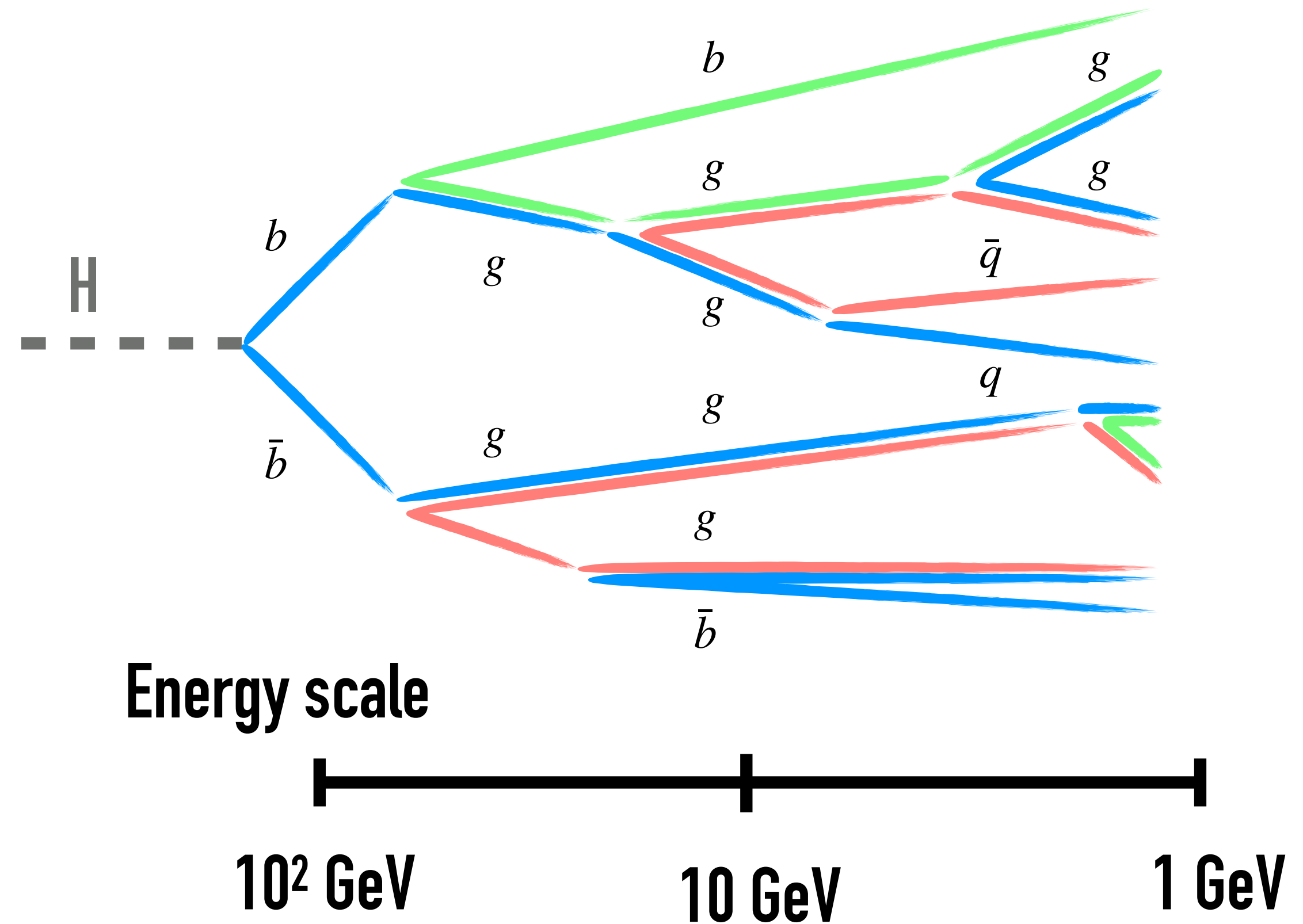


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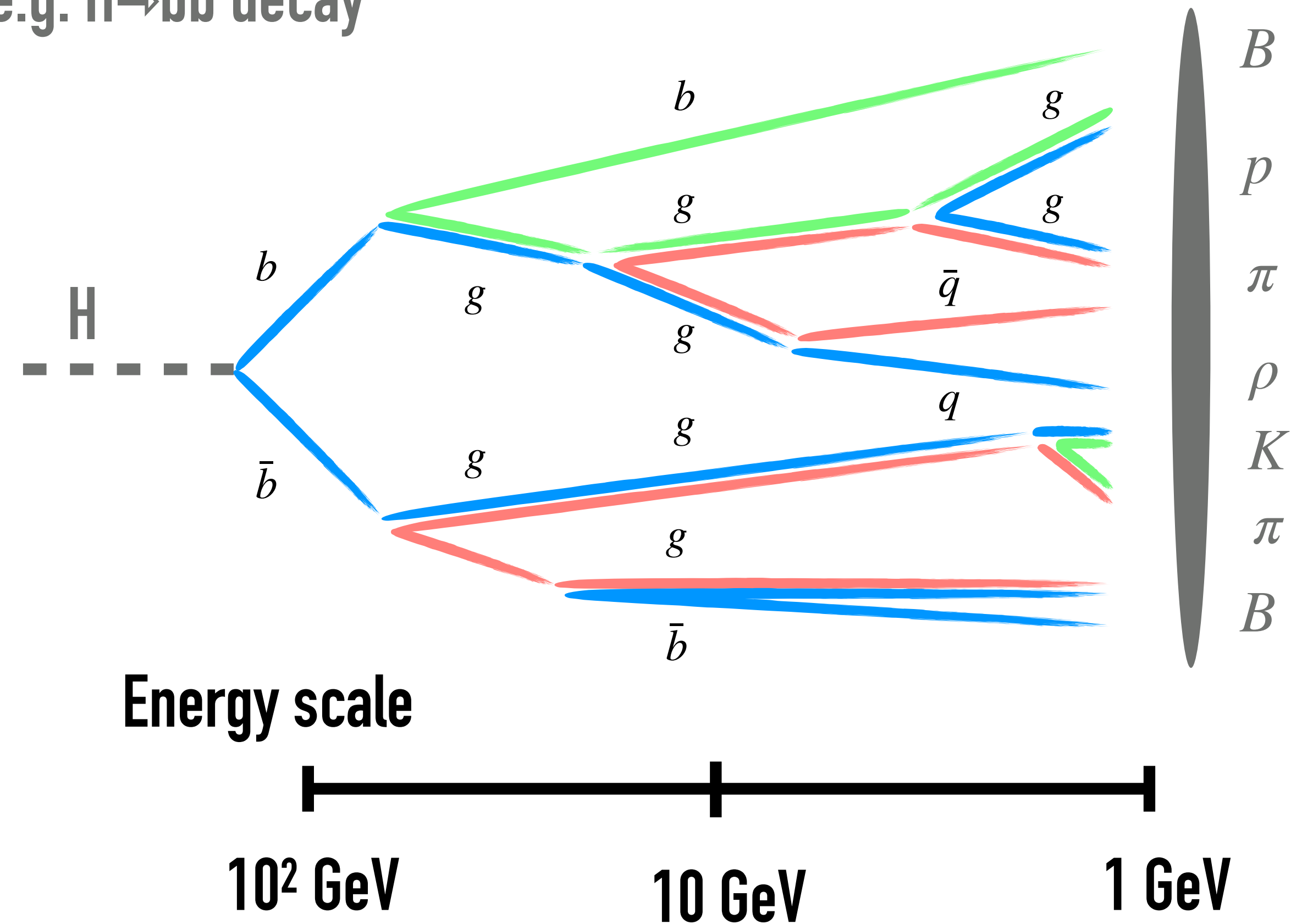


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e.g. $H \rightarrow bb$ decay



What's the logarithmic accuracy of a PS?

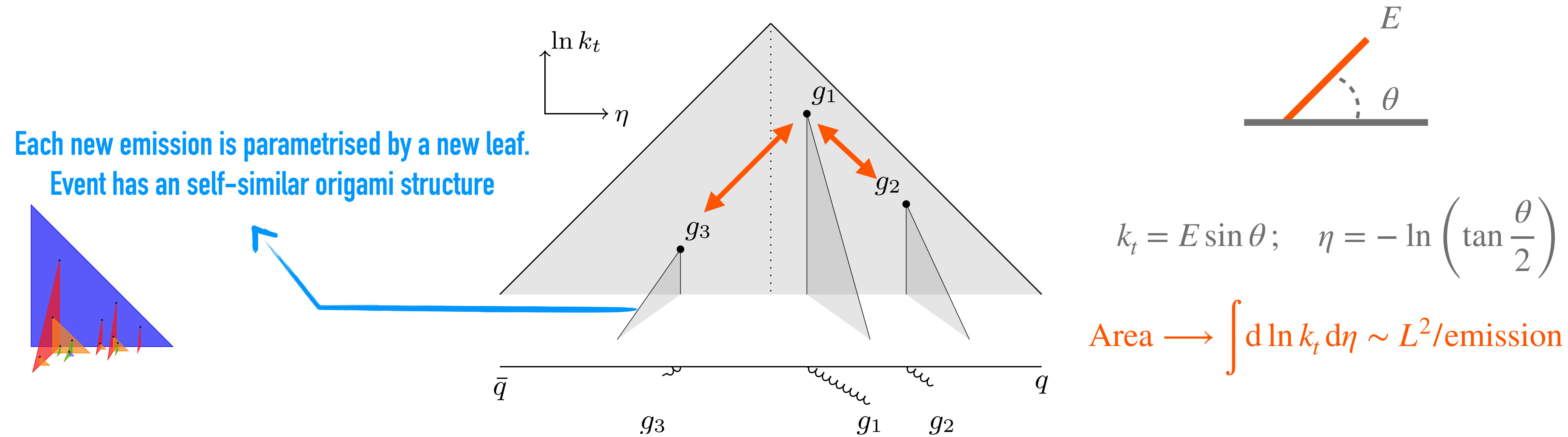
- Identify the appropriate QCD perturbative expansion in the multi-scale regime

Perturbation theory: small coupling, large scale hierarchy [logarithmic counting]

- How can we formulate the concept of accuracy for **whole classes of observables at once**? e.g. for
 - fraction of events passing a **jet veto in a rapidity window**?
 - **azimuthal correlation** between two sub-jets?
 - **event shapes**?
 - ...

A geometric definition of leading-logarithmic (LL) accuracy

- Radiation phase space conveniently organised in the Lund Plane (LP)
[Anderson, Gustafson, Lonnblad, Pettersson '89]
- LL \rightarrow emissions widely separated in both directions of the LP $\rightarrow \mathcal{O}(50 - 100\%)$ uncertainties

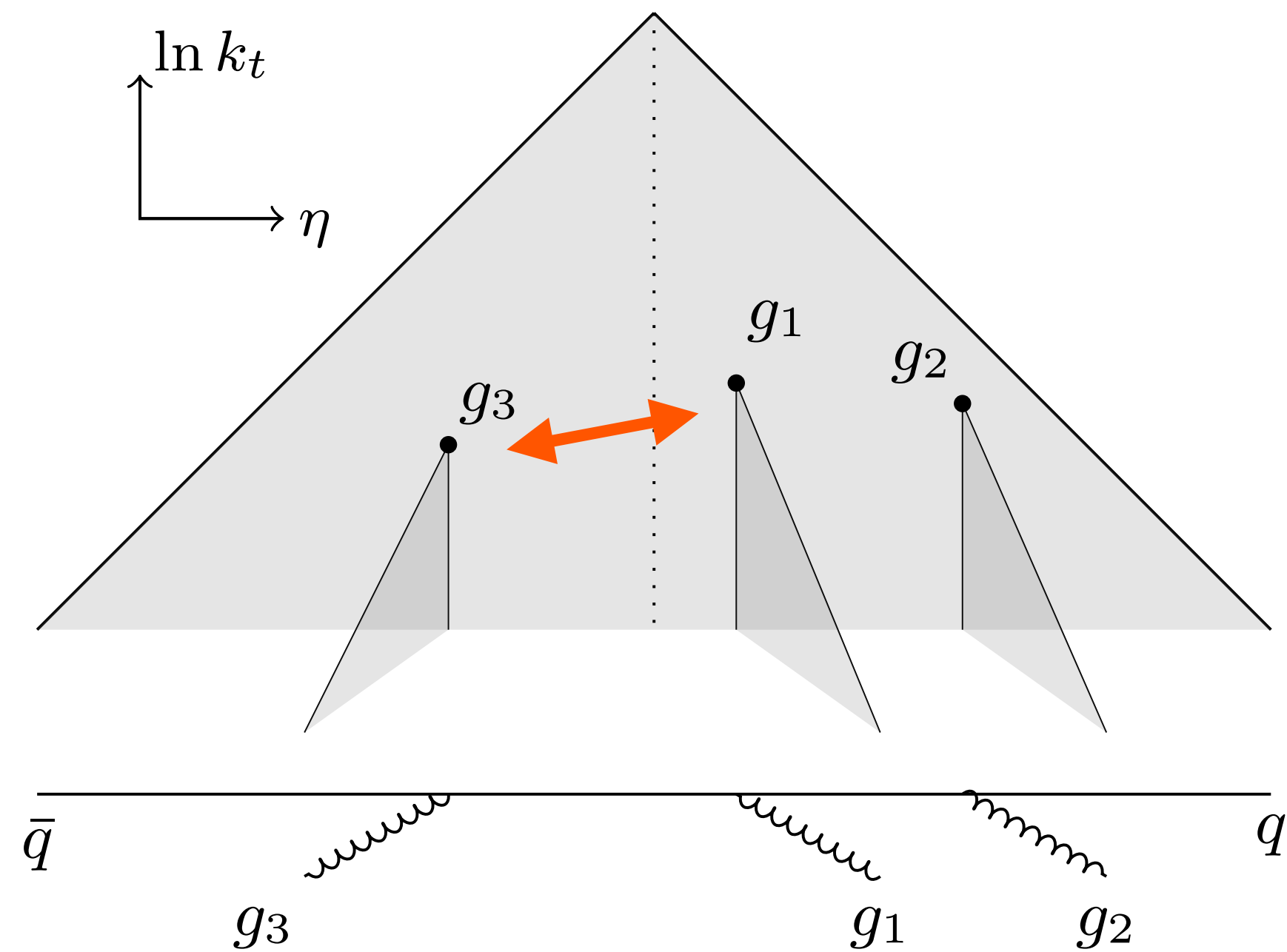


Definition used in QCD resummations, e. g.
[Banfi, Salam, Zanderighi '04; Banfi, McAslan, [PM](#), Zanderighi (JHEP 2015)]

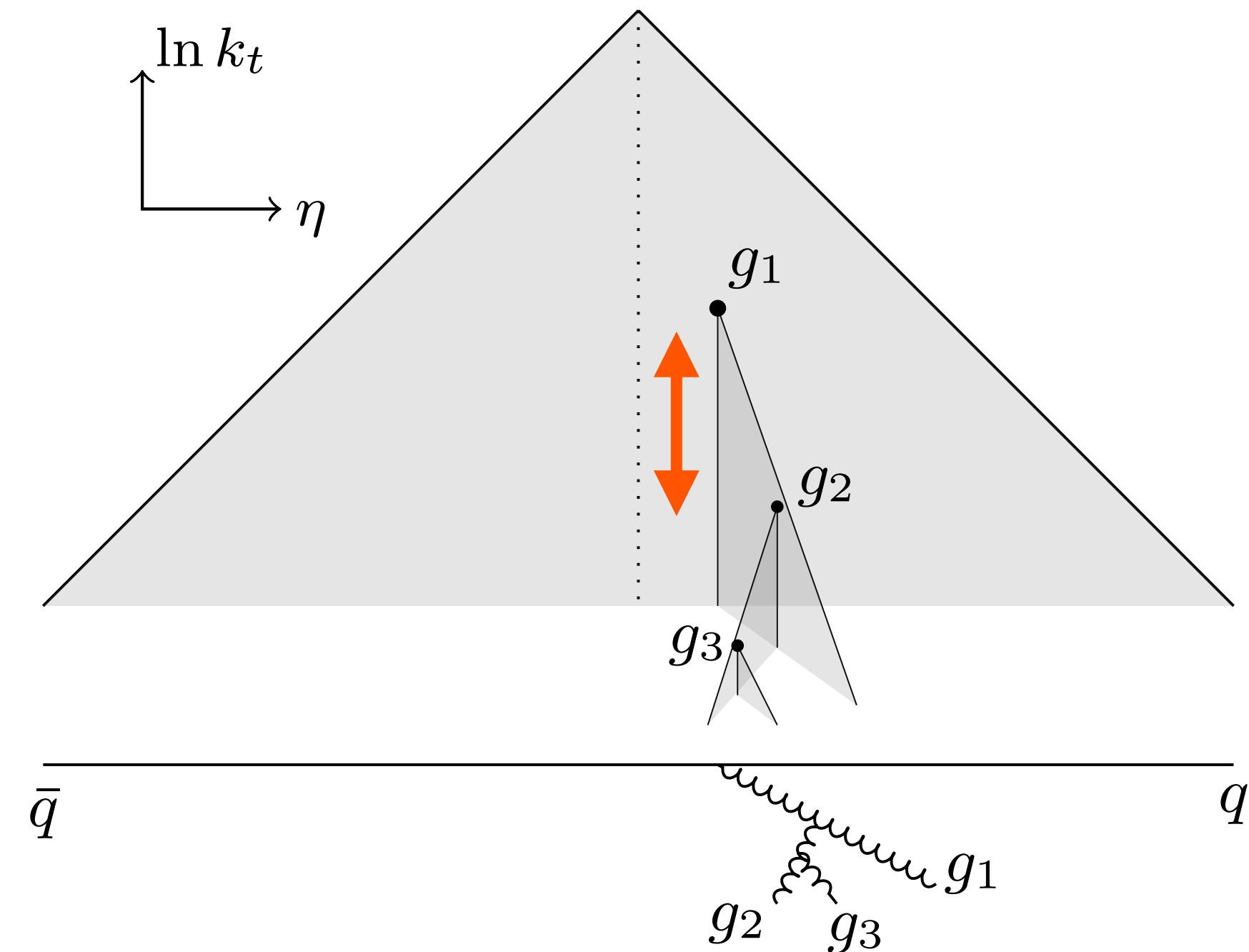
A geometric definition of NLL accuracy

- NLL \rightarrow emissions **strongly separated in a single direction of the LP** $\rightarrow \mathcal{O}(10\%)$ uncertainties

e.g. in rapidity at similar transverse momentum



e.g. in transverse momentum at similar rapidities



$$\text{Line} \longrightarrow \int d \ln k_t \sim \int d\eta \sim L/\text{emission}$$

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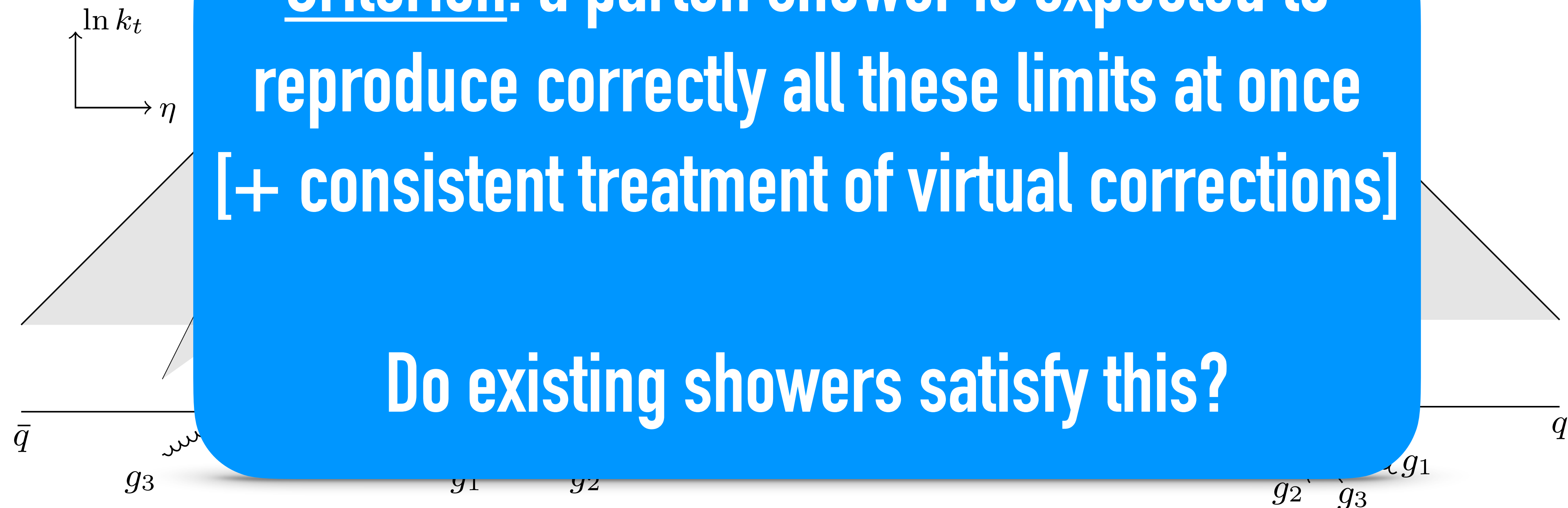
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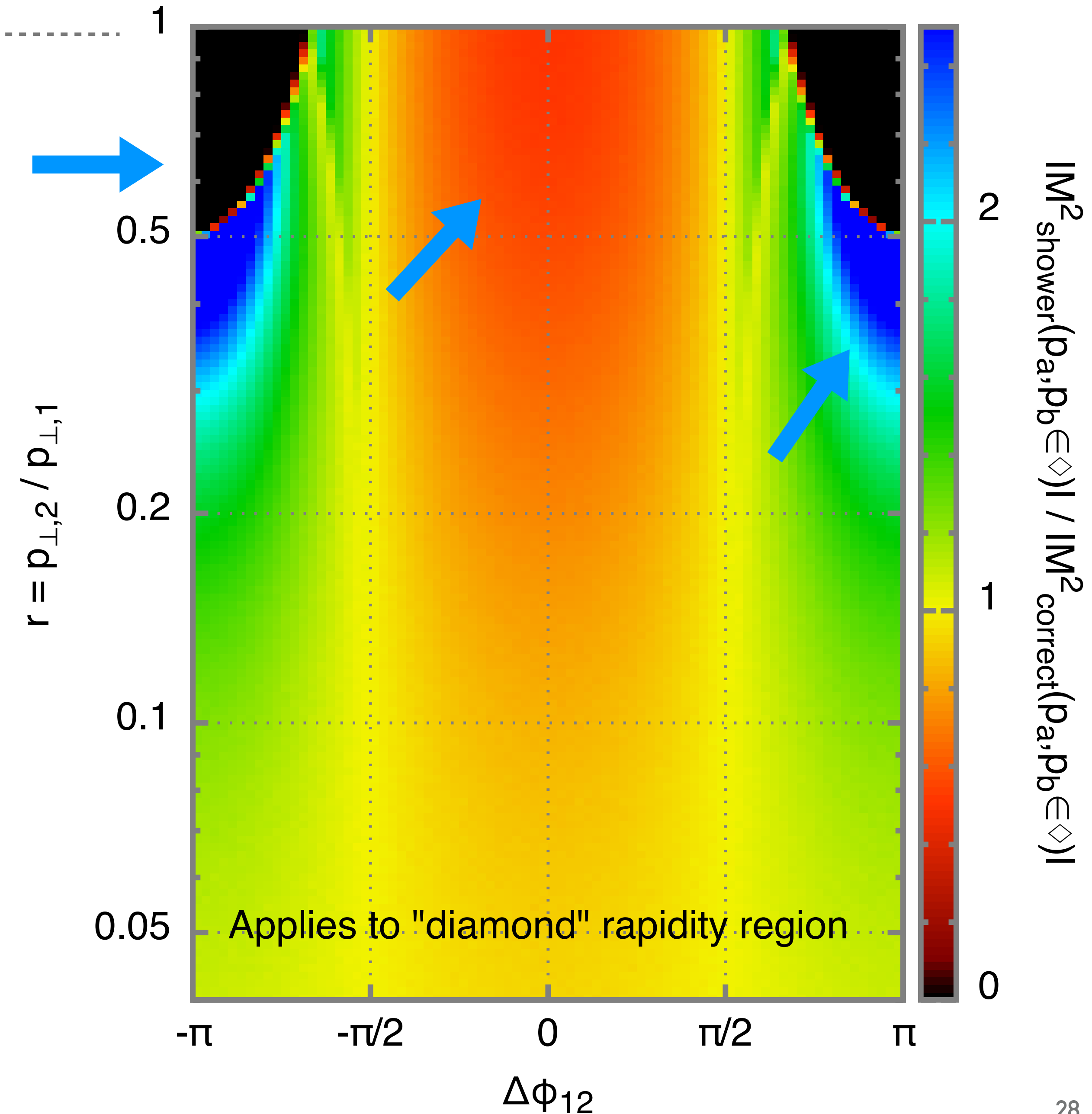
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The double-emission matrix element

- Simplest check: **probability density for radiating two (soft) gluons**
- Compare the result of common showers (e.g. Pythia8, DiRE) to that of a QCD calculation
- Ratio is expected to be = 1** for NLL showers
[Dasgupta, Dreyer, Hamilton, [PM](#), Salam (JHEP 2018)]

Ratio of PS (Py8) radiation pattern to QCD result @ NLL



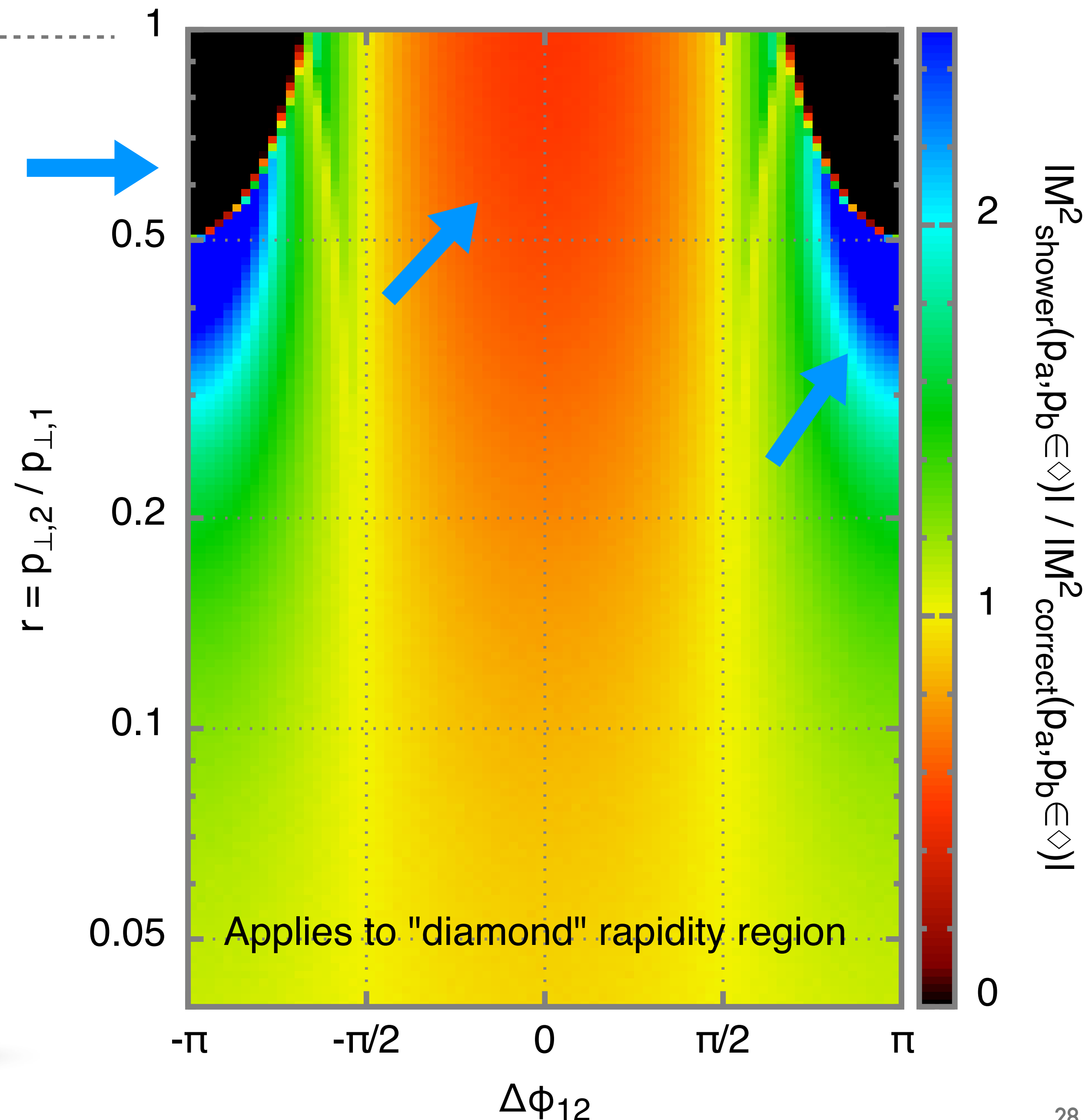
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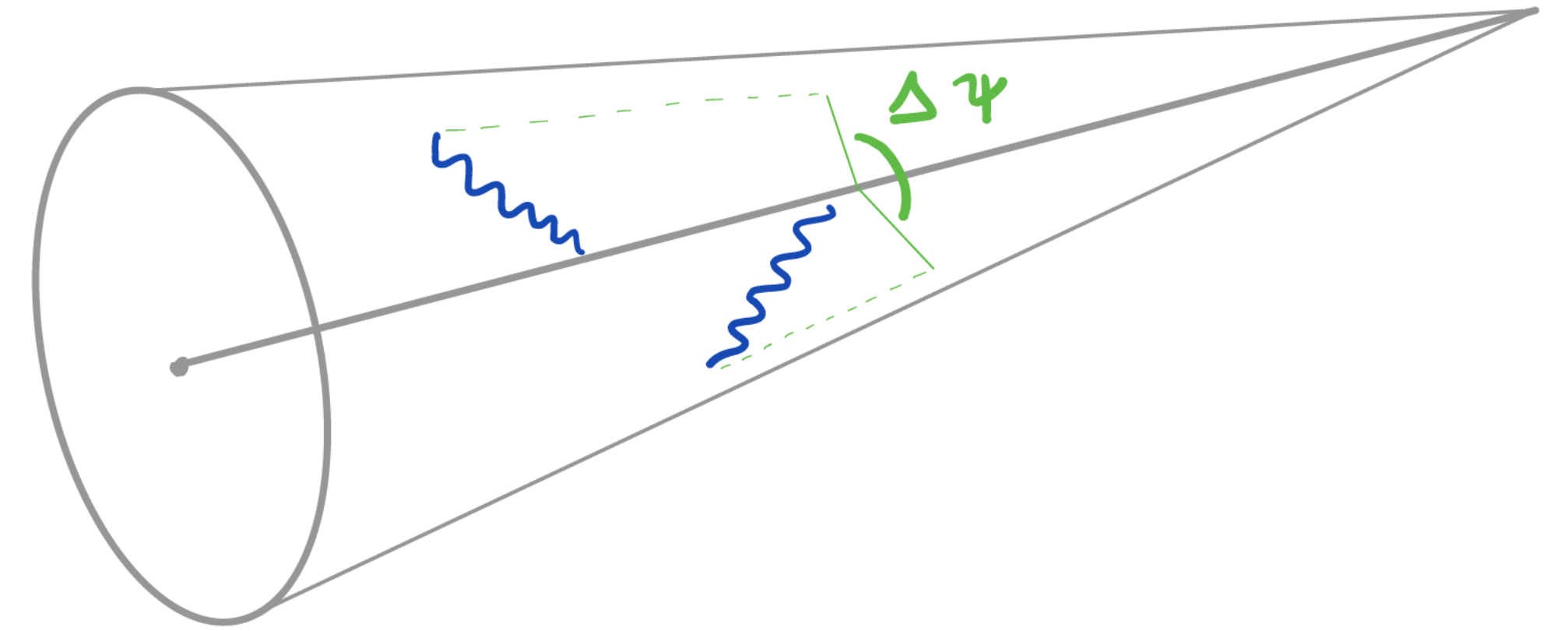
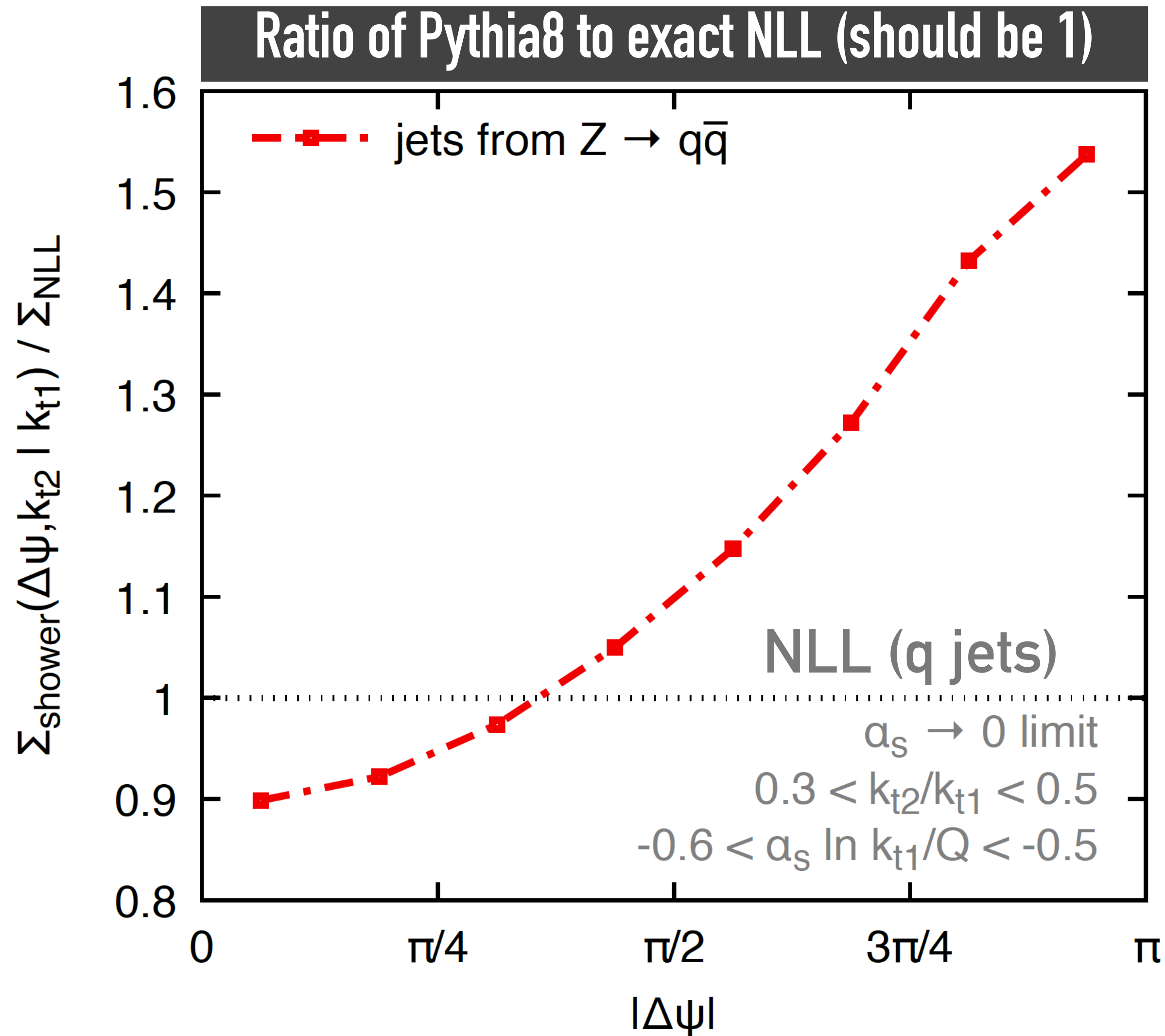
Common parton showers are
only LL accurate

→ large uncertainties $\mathcal{O}(50 - 100\%)$

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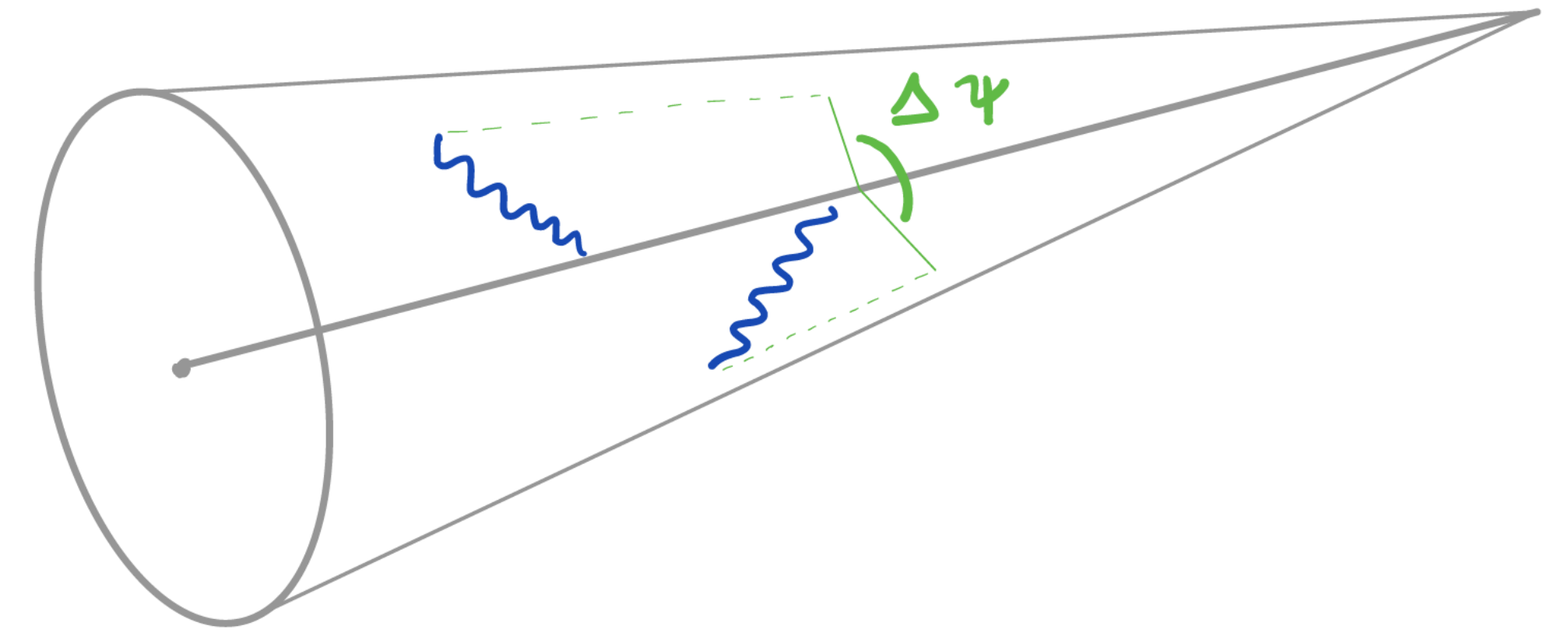
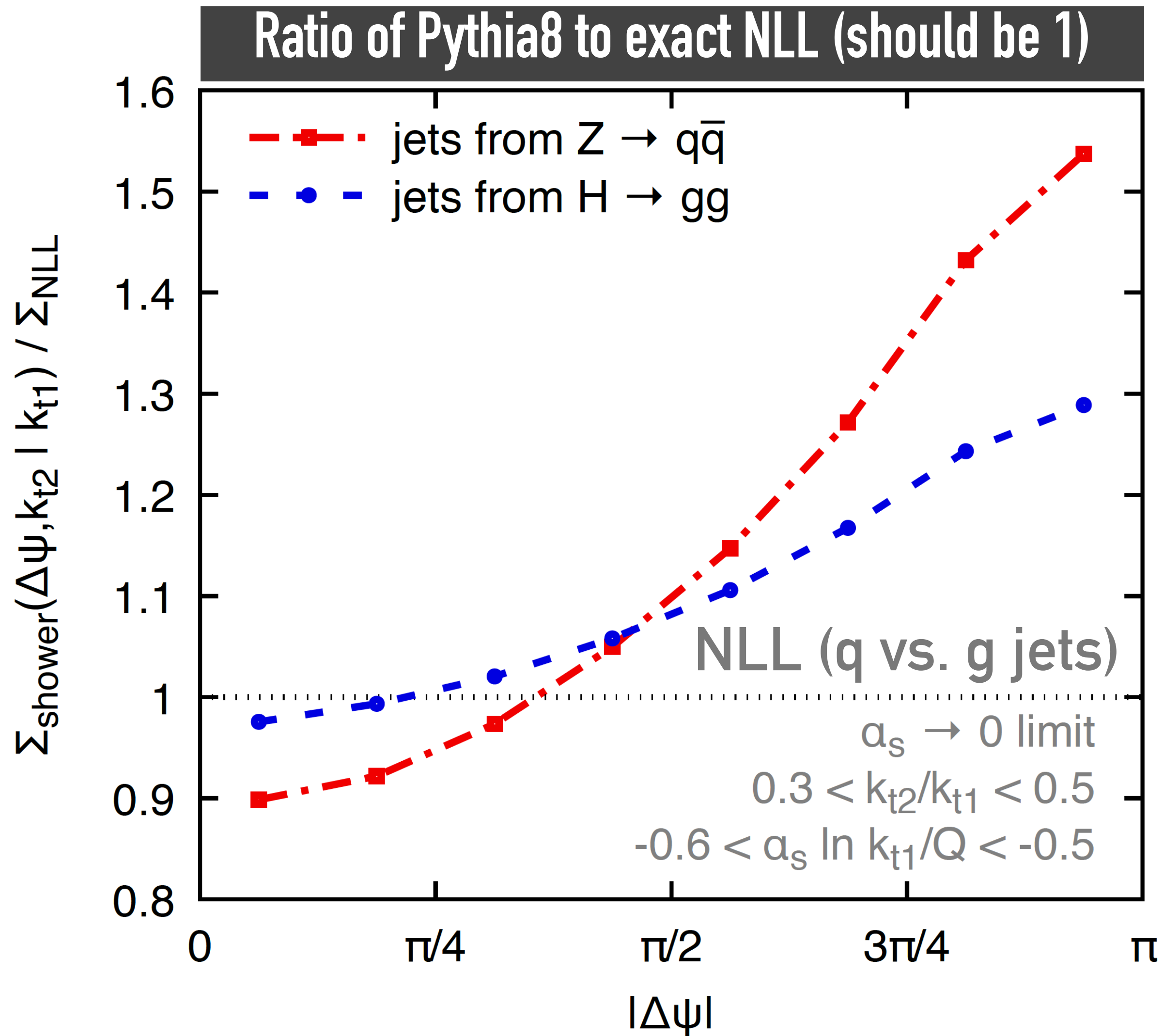


Consequences for accuracy: a jet substructure example



- Consider azimuthal distance between two hardest sub-jets
 - e.g. Z-boson decay: “quark” jets
 - 0(60%) differences with NLL result** (large theory uncertainty)

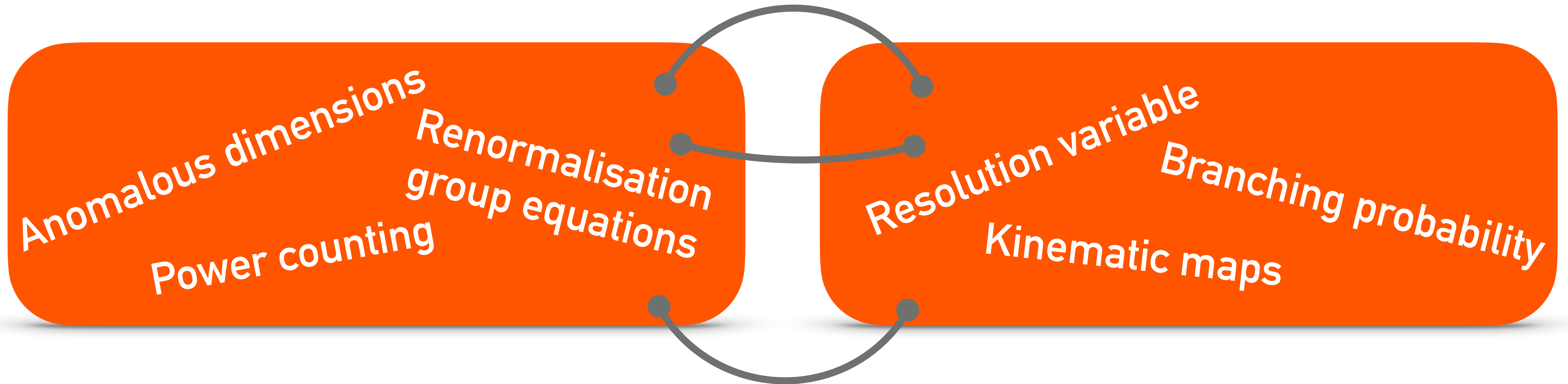
Consequences for accuracy: a jet substructure example



- Consider azimuthal distance between two hardest sub-jets
 - e.g. H-boson decay: “gluon” jets
 - unphysical dependence on jet flavour** (potential bias for machine learning)

Formulating NLL parton showers

- Connection between parton showers and perturbative calculations (i.e. **resummation**) has been an open problem for the past 30 years, i.e. different mathematical language

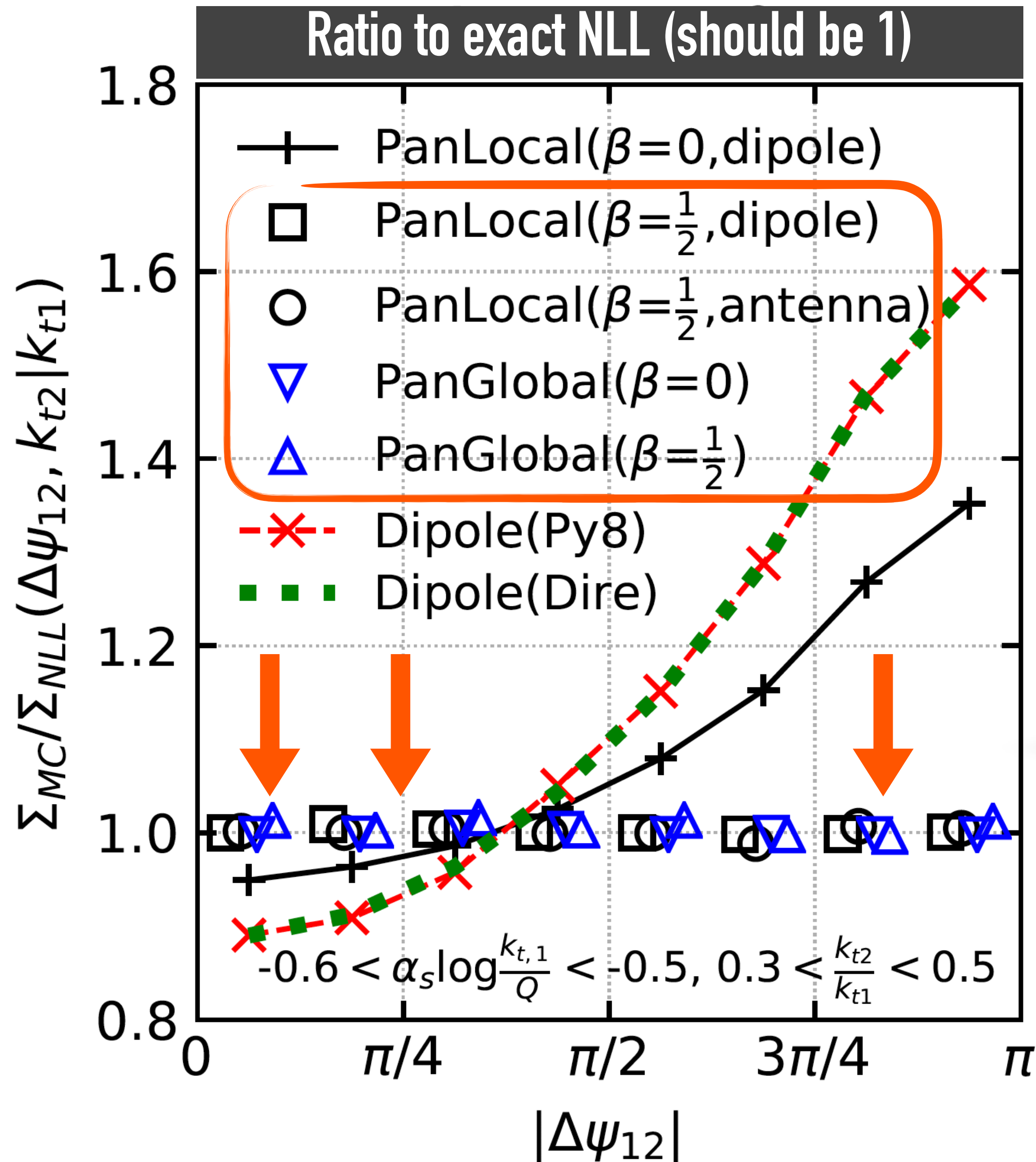


- Mapping of one field into another leads to **criteria (e.g. backup) for the building blocks of a PS[‡]**
- **Methods to create novel algorithms** with higher formal accuracy: the PanScales showers

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

[‡] QCD resummation provides guidelines, **more than one architecture is possible**

Back to sub-jet's azimuthal correlations

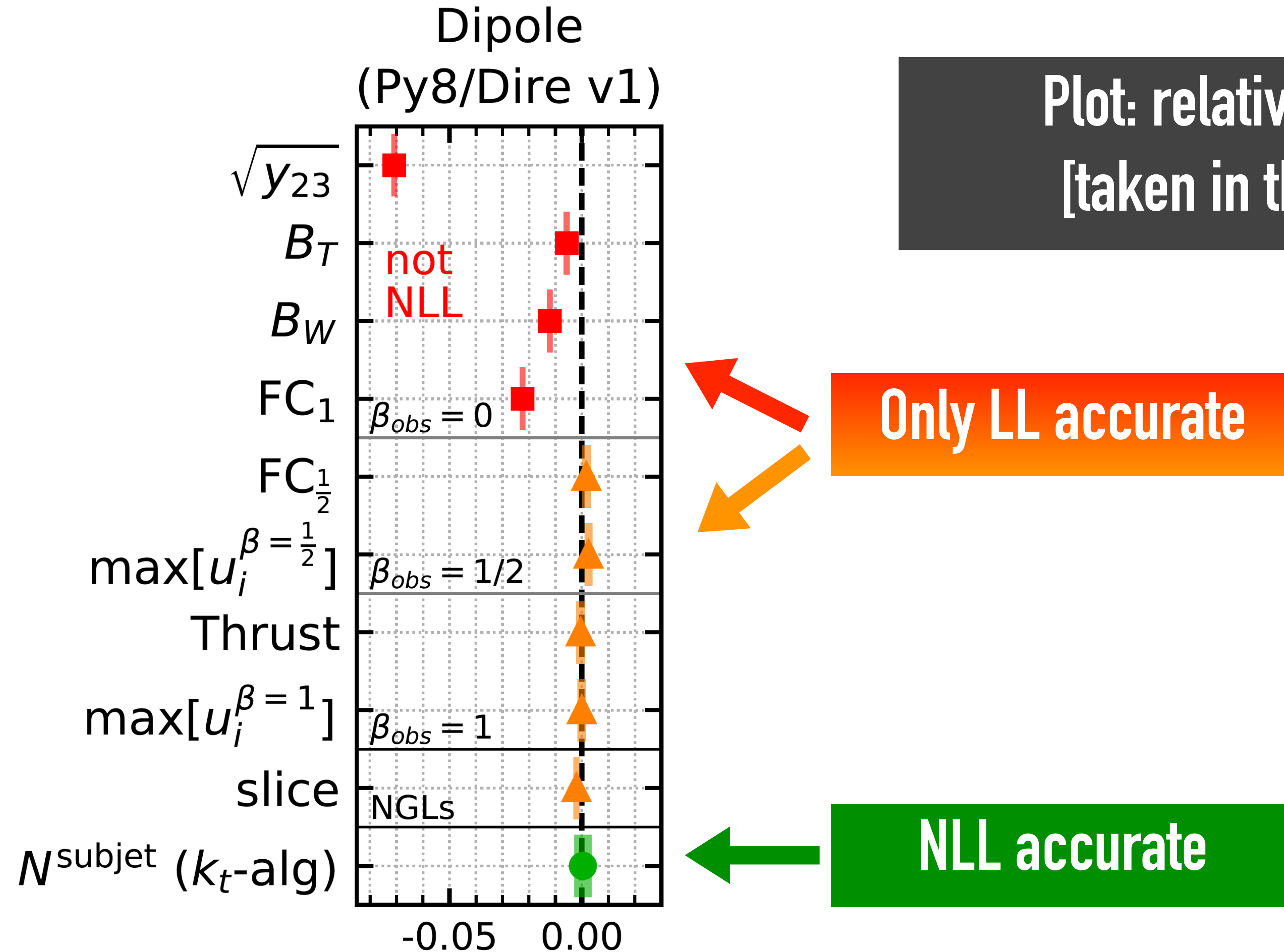


PanScales showers perfectly agree with NLL, while Pythia/Dire do not

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

Repeat the test across several collider observables (e^+e^- collider case)

[Sjostrand et al. (2020); Hoeche, Prestel (2015)]

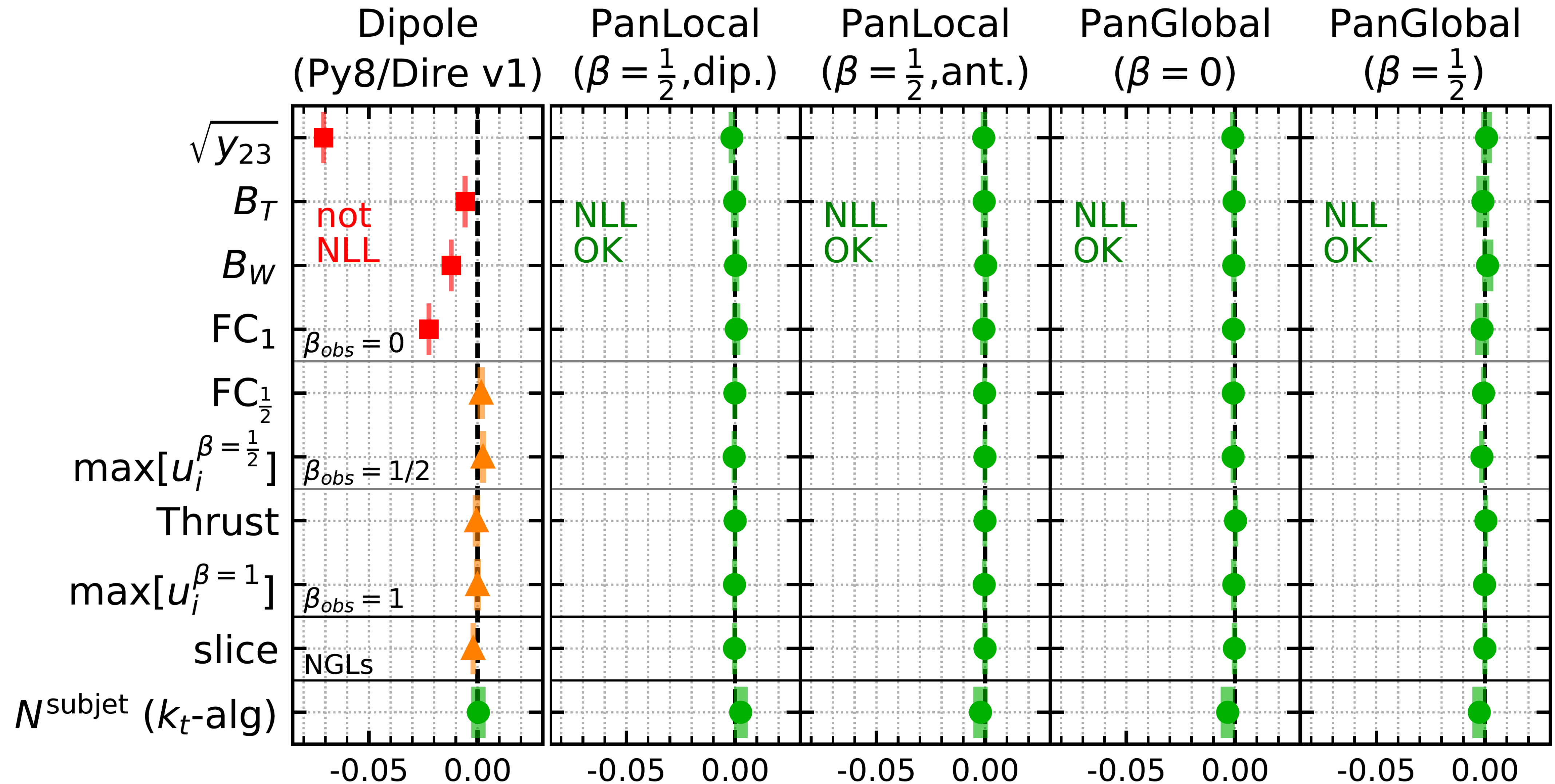


Plot: relative deviations from exact NLL
[taken in the relevant kinematic limit]

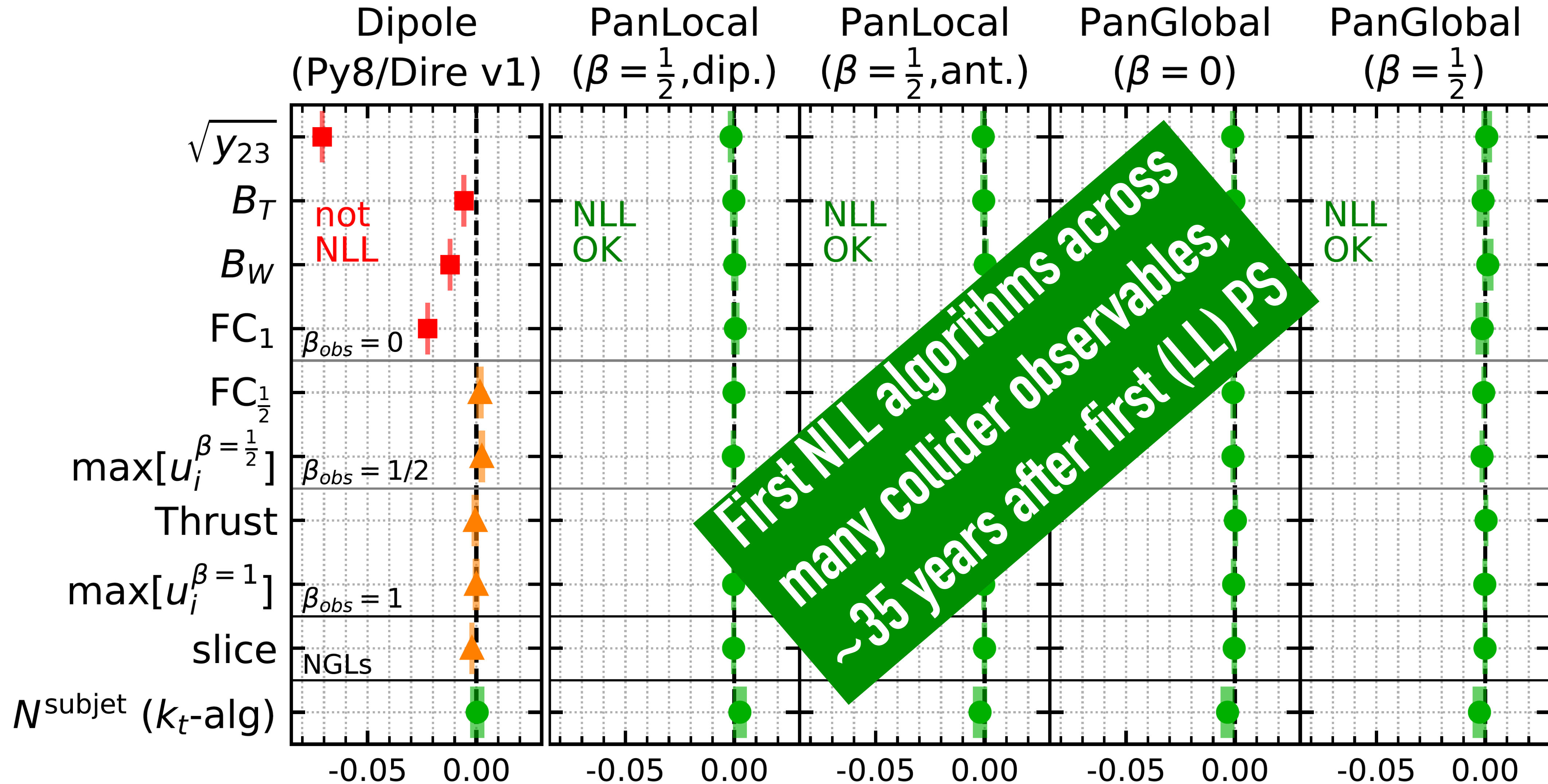
Only LL accurate

NLL accurate

A new generation of NLL showers: PanScales

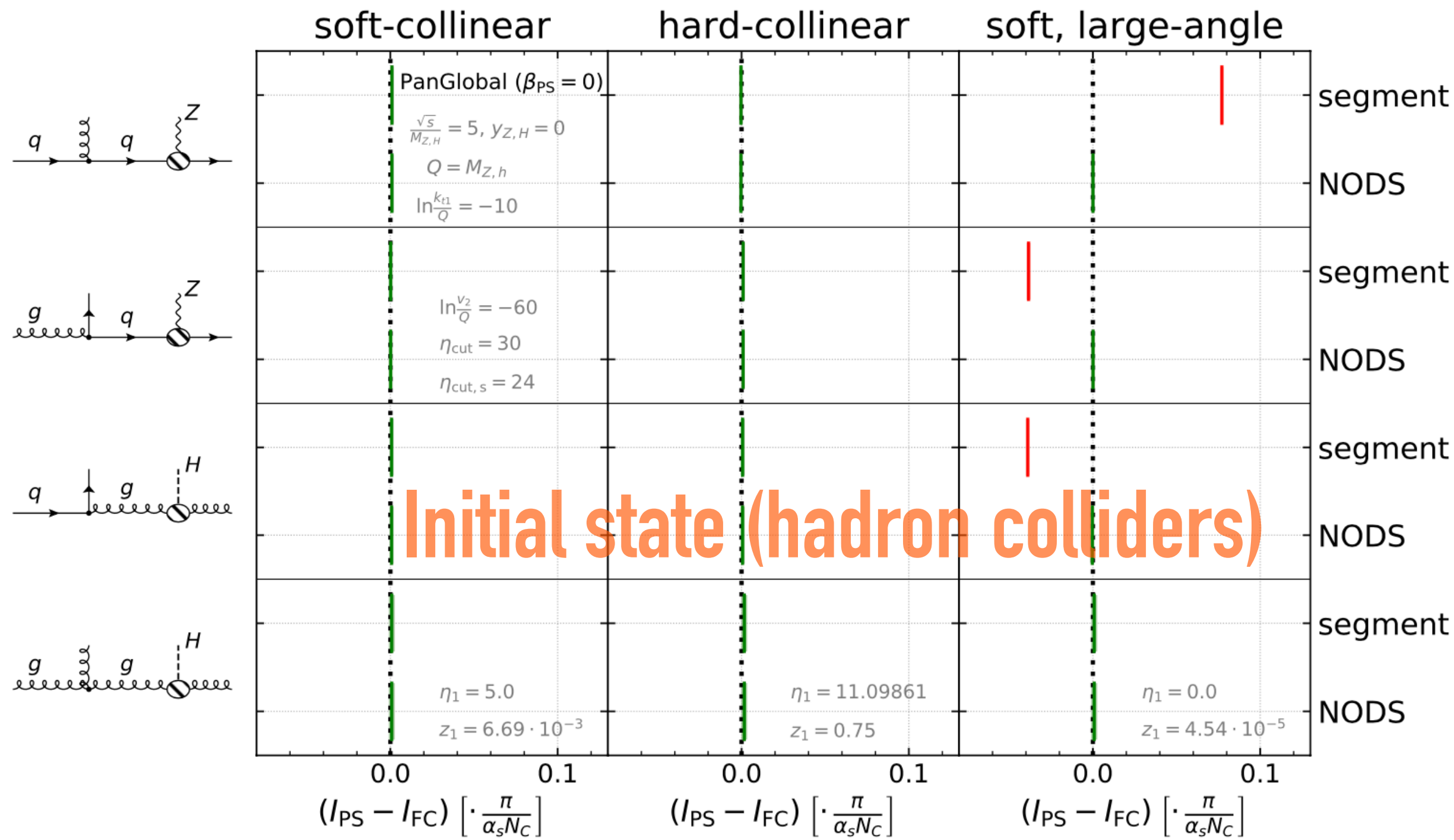


A new generation of NLL showers: PanScales



Further developments: towards a full NLL PanScales shower

[Hamilton, Medves, Salam, Scyboz, Soyez (2020)] NLL accuracy tests — NODS method

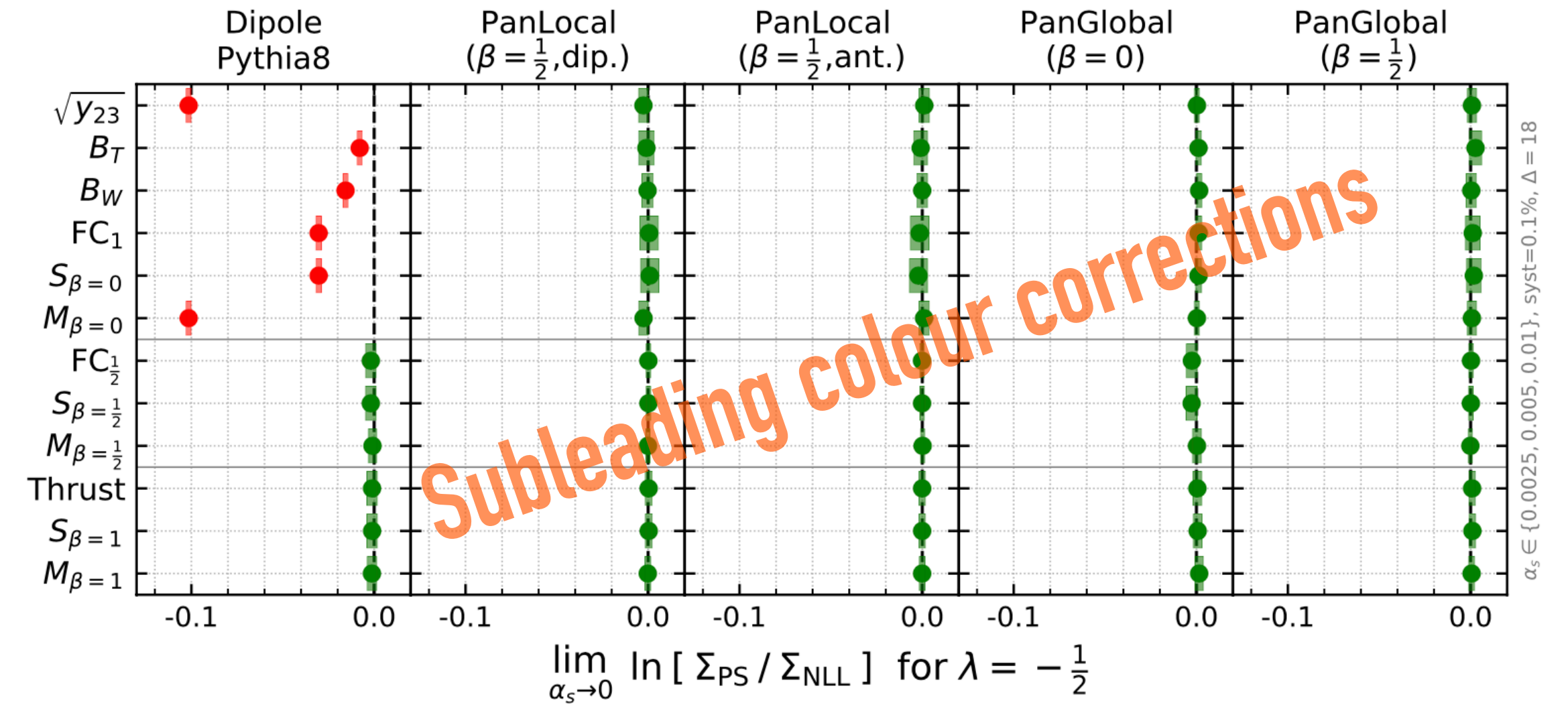


[van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez (2022)]

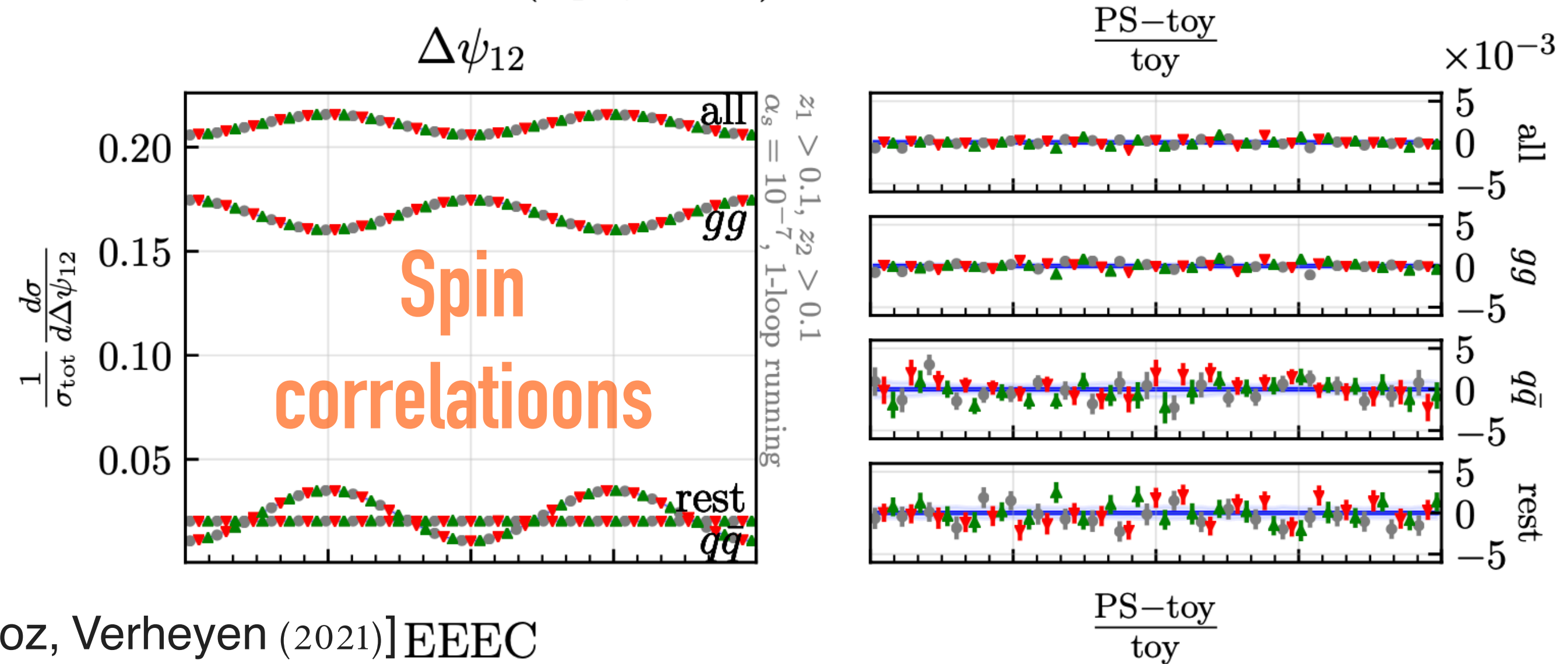
Related work on log accuracy in:

[Bewick, Ferrario Ravasio, Richardson, Seymour (2019);

Forshaw, Holguin, Plaetzer (2020); Nagy, Soper (2020)]



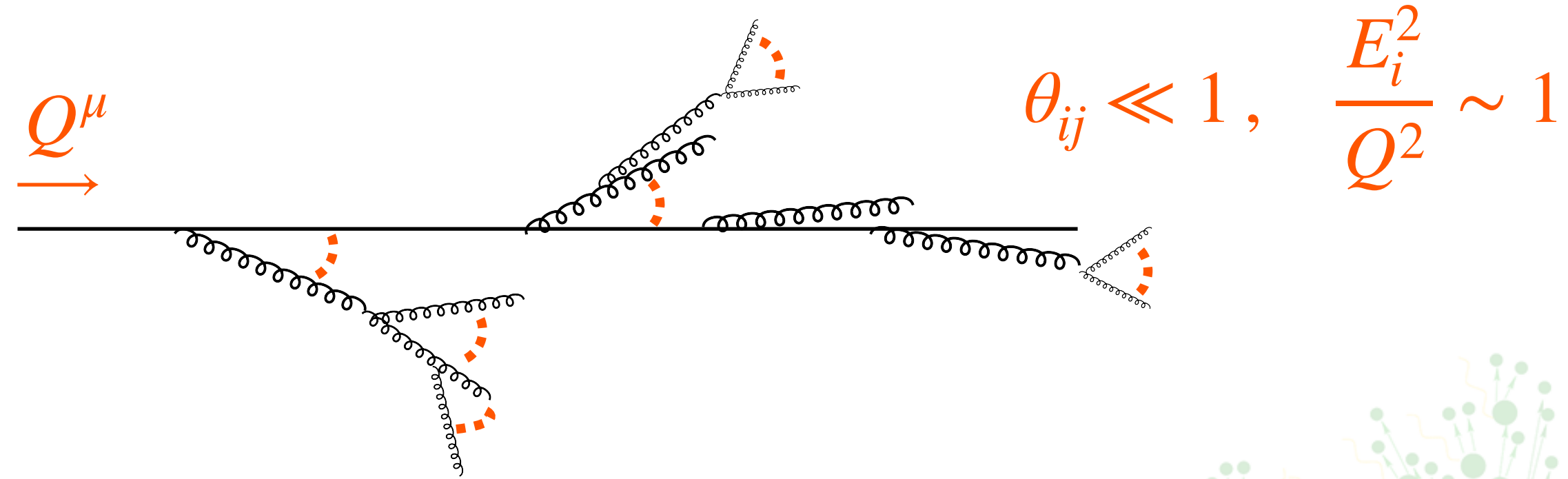
█ PanGlobal ($\beta = 0$)
 █ PanLocal (ant. $\beta = 0.5$)
 █ Toy shower



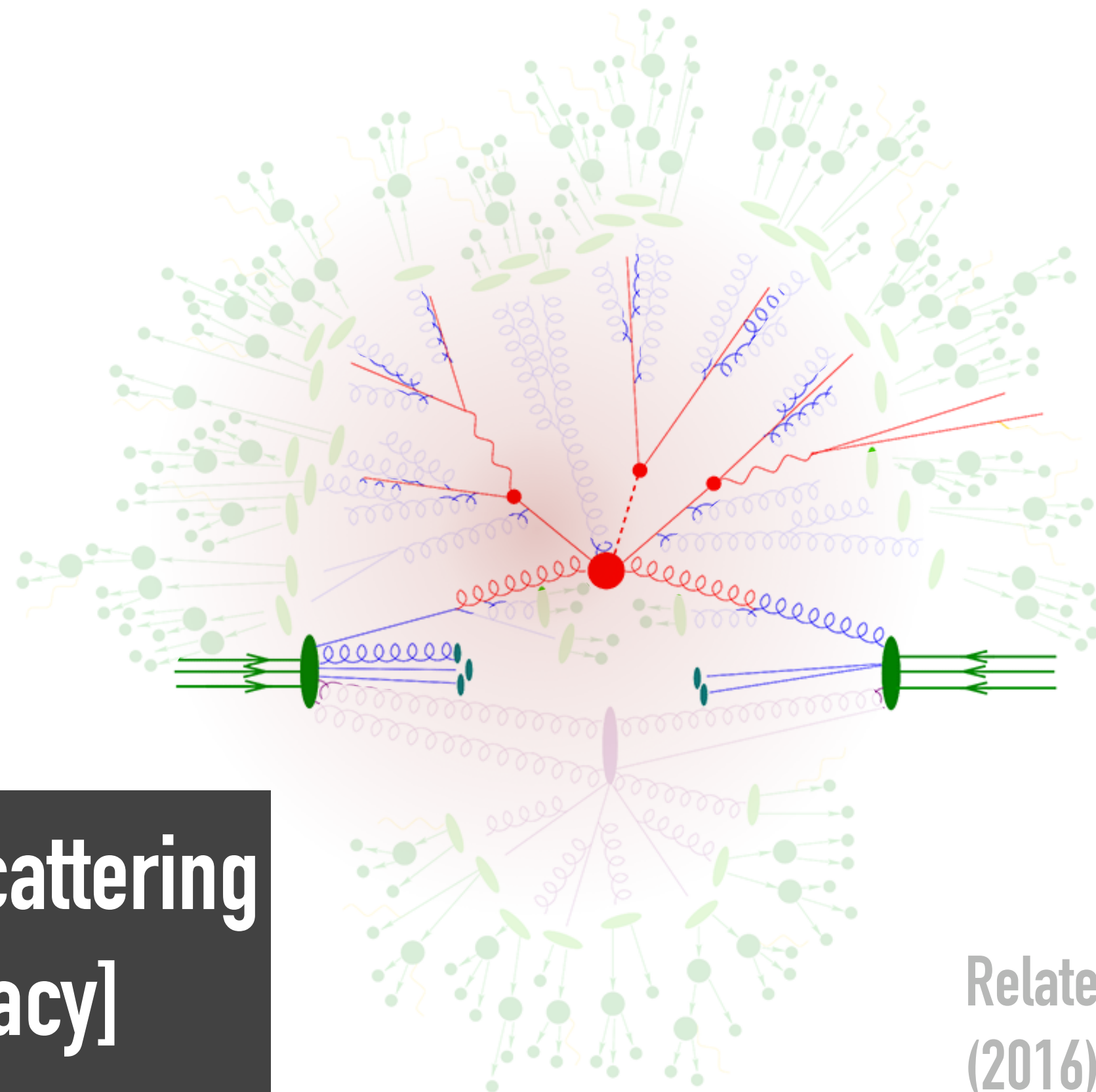
[Karlberg, Salam, Scyboz, Verheyen (2021)] EEEEC

Towards few-percent accuracy: NNLL building blocks

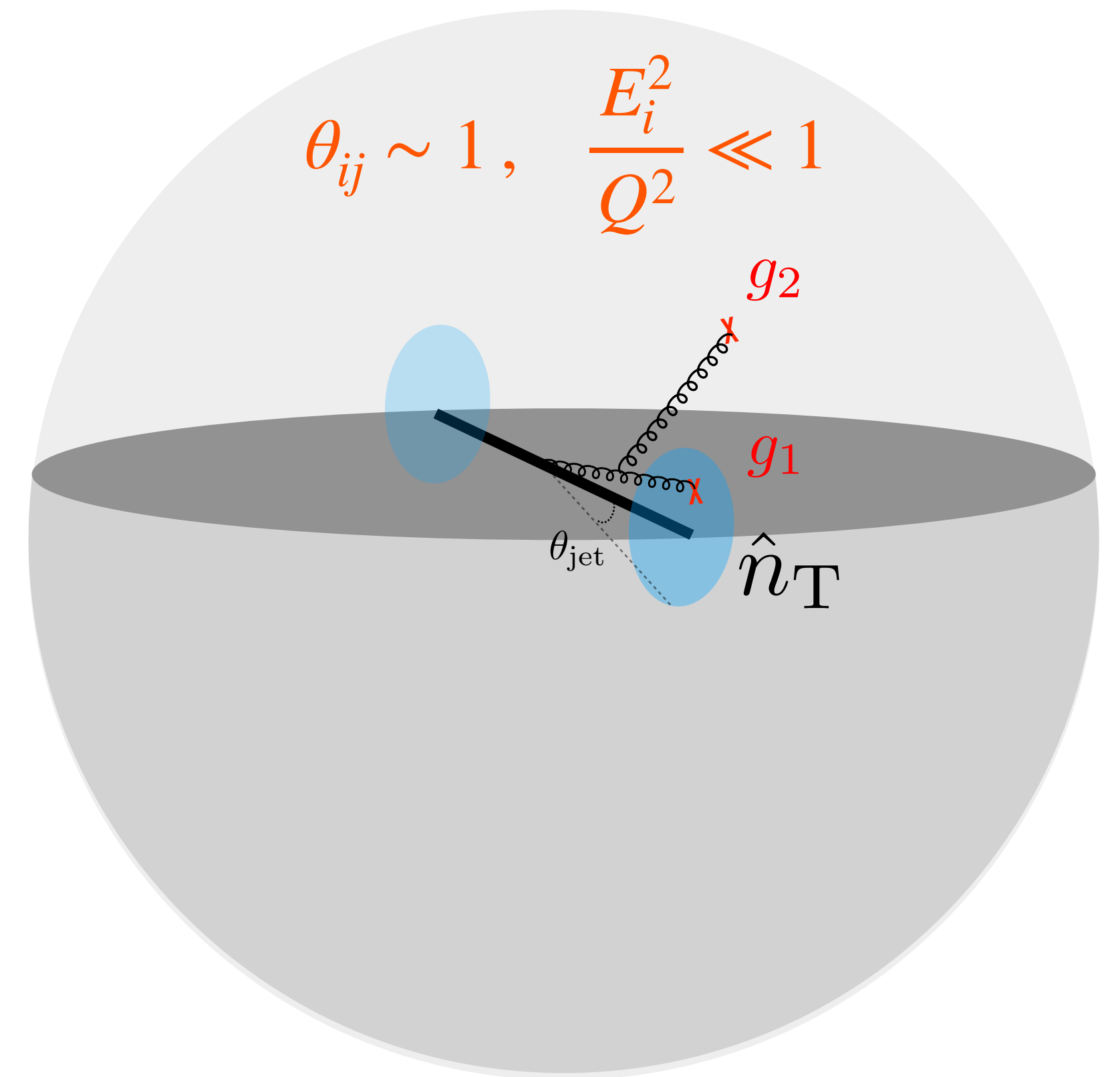
differential collinear fragmentation



[Dasgupta, El-Menoufi (2021);
van Beekveld, Dasgupta, El-
Menoufi, [PM](#), Salam (in progress)]



NNLL soft (non-global) evolution [a 20 years old problem]



[Banfi, Dreyer, [PM](#) (JHEP 2021 + JHEP 2022)]

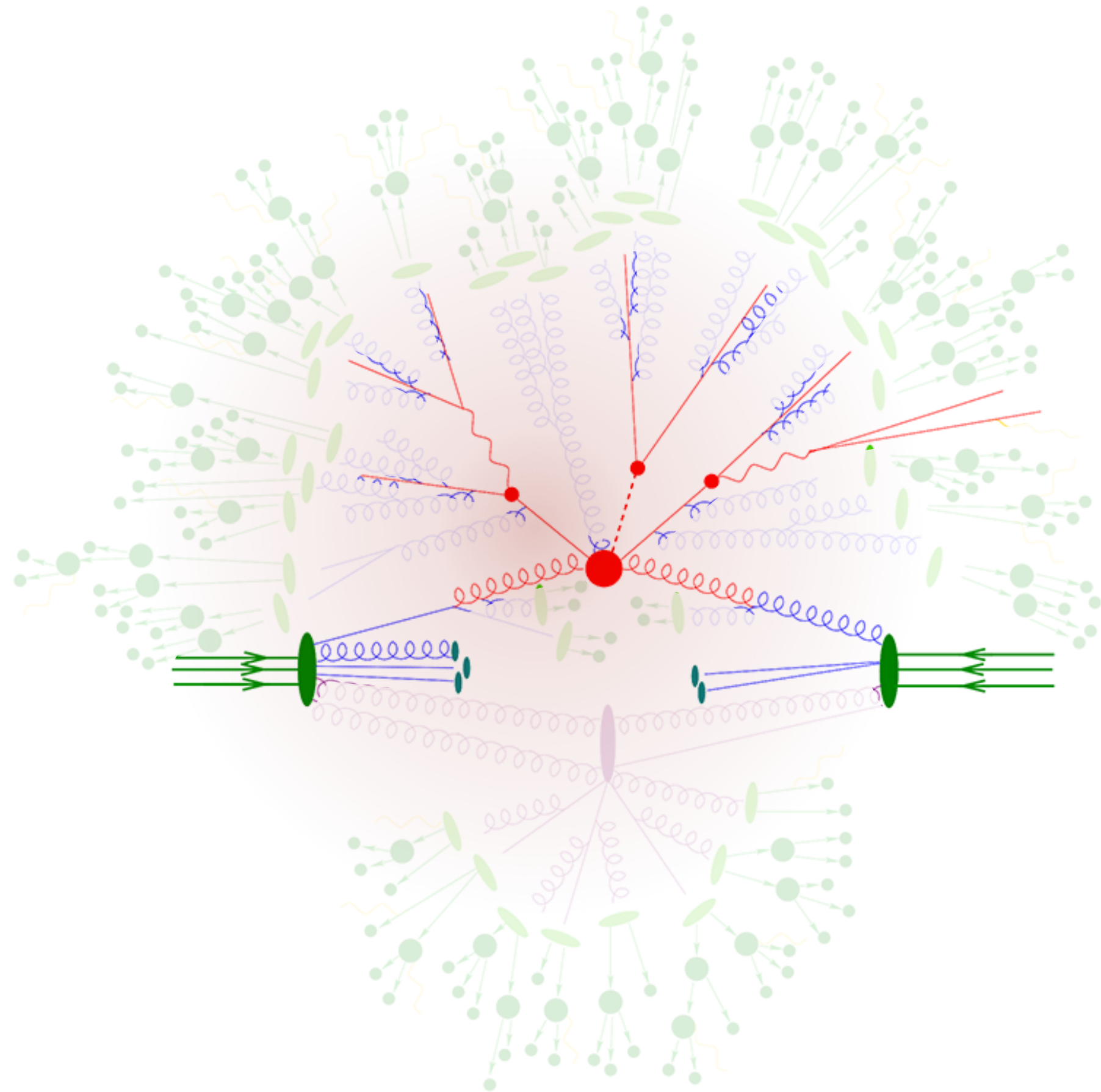
Related work by several groups: [Jadach et al. (2015); Li, Skands (2016); Hoeche, Prestel+Krauss+Dulat+Gellersen (2017-2021)]

**Radiative corr^{ns} to hard scattering
[preserving PS accuracy]**

The hard scattering

The hard partonic scattering

- QCD well described by the radiation of a fixed number of partons (quarks & gluons) [and corresponding virtual corrections]

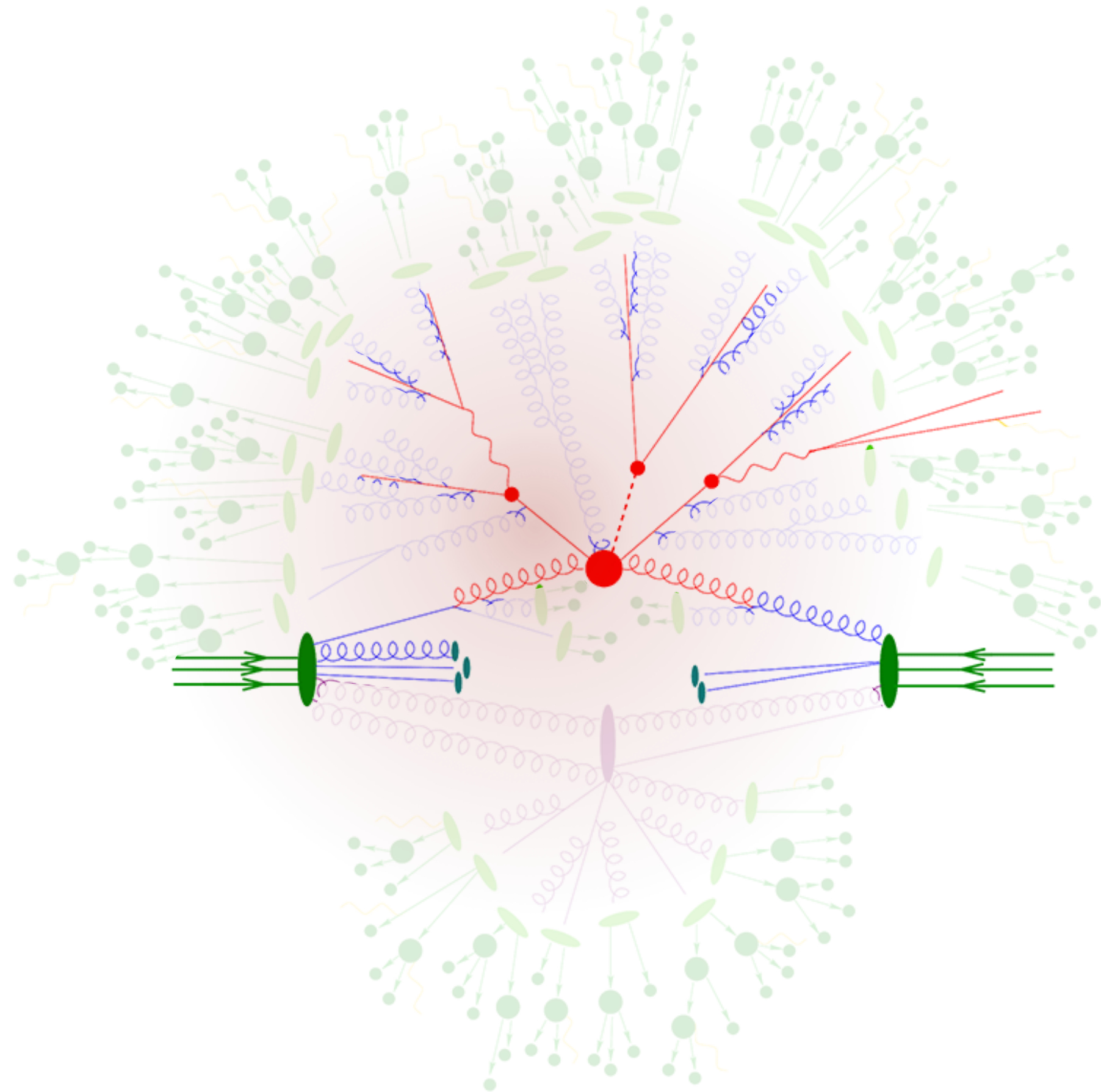


Perturbation theory: [fixed-order]

$$\alpha_s(M_Z) \sim 0.118 \longrightarrow \begin{cases} \text{NLO} & \sim 10\% \\ \text{NNLO} & \sim 1\% \\ \dots & \end{cases}$$

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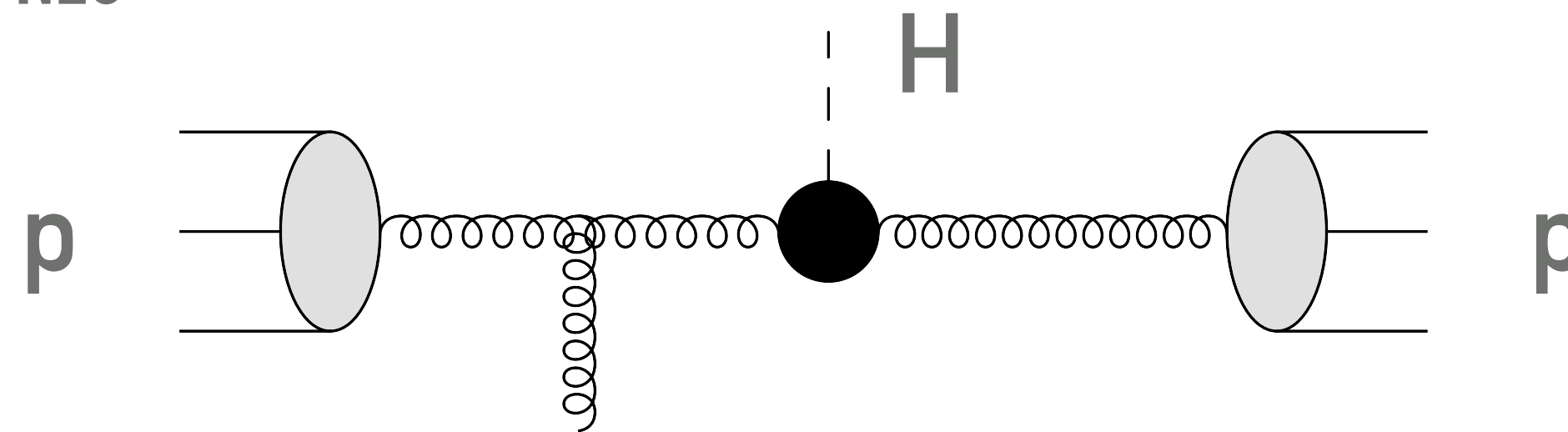
Target: (At least!) Next-to-next-to-leading order corrections with respect to Born approximation: up to two extra emissions

Radiative corrections & interplay with parton showering

- Computation of radiative corrections to the hard process, while
 - Avoiding double-counting with parton shower [PS emits further radiation]

e.g. illustration for Higgs+jet production at NLO

(one order less than our target)

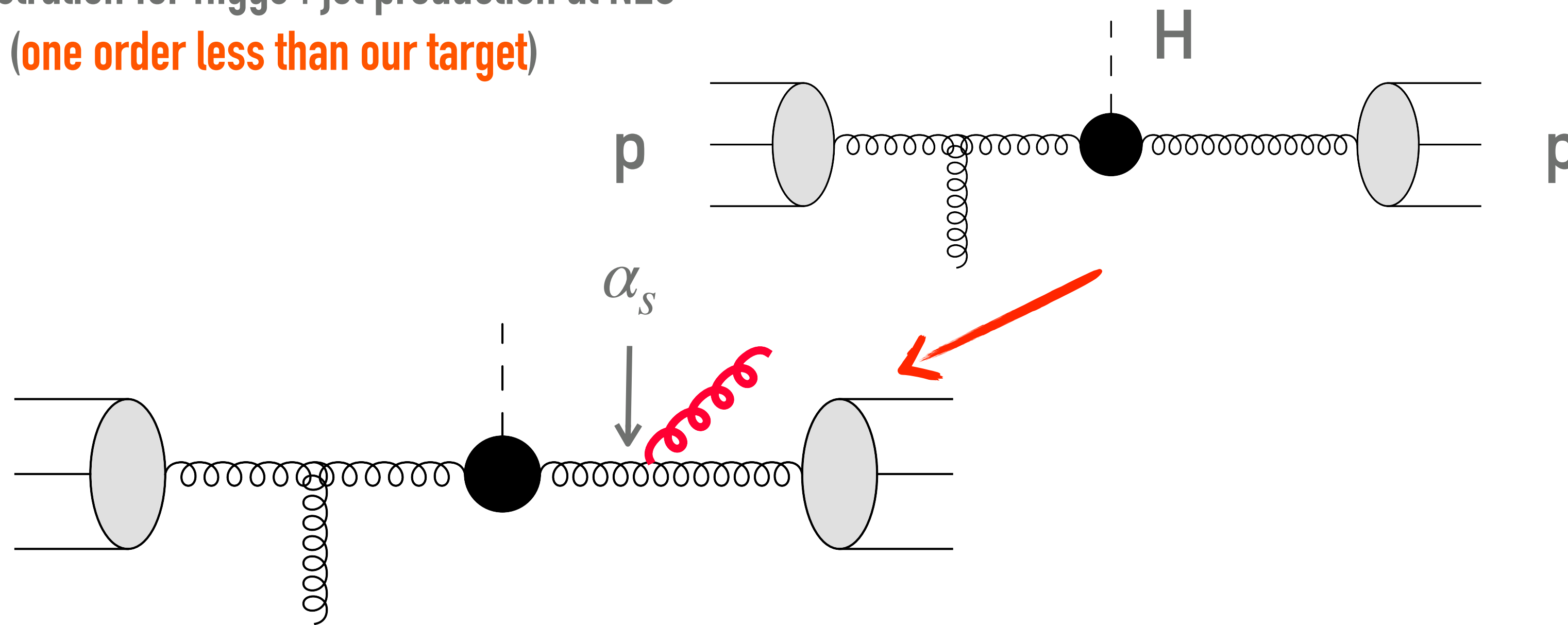


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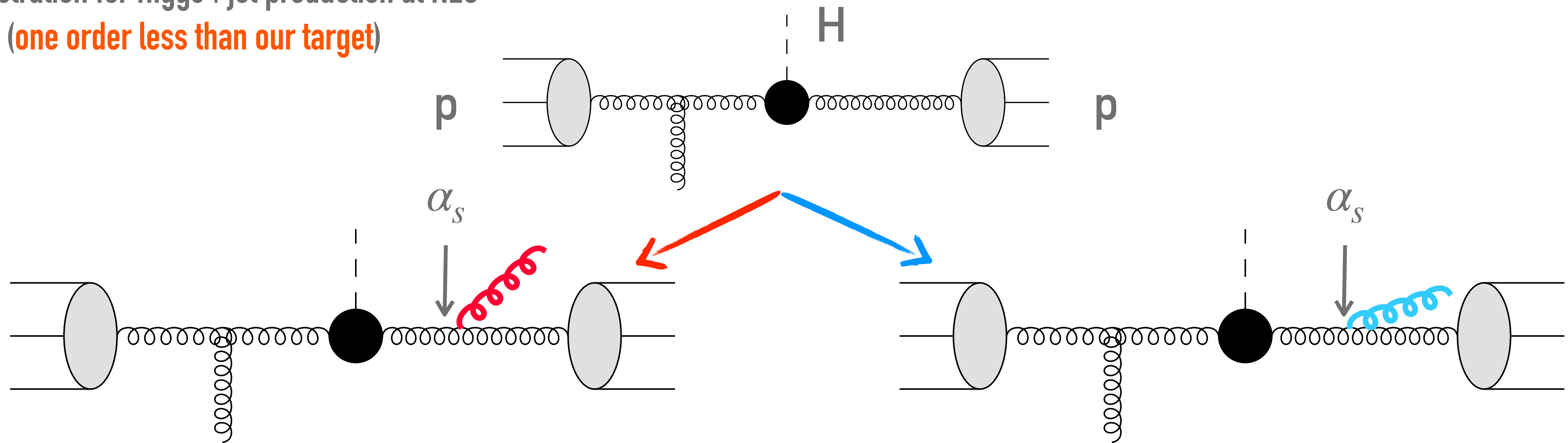
**Radiation included by
radiative corrections**

Radiative corrections & interplay with parton showering

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e.g. illustration for Higgs+jet production at NLO

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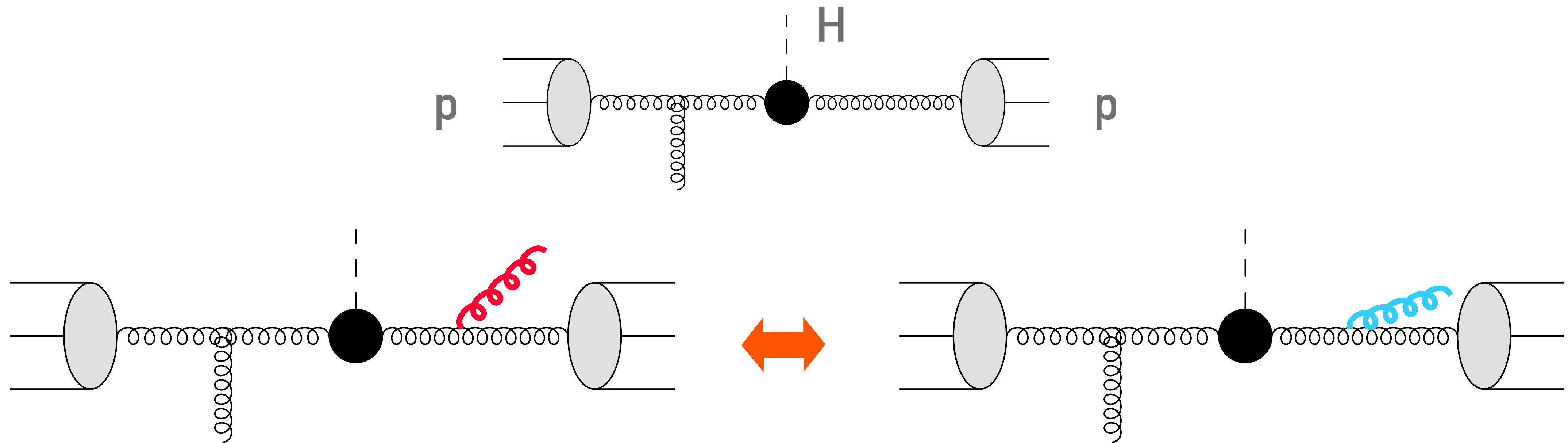


Radiation included by
radiative corrections

Radiation included by parton showering
(soft/collinear approximation)

Radiative corrections & interplay with parton showering

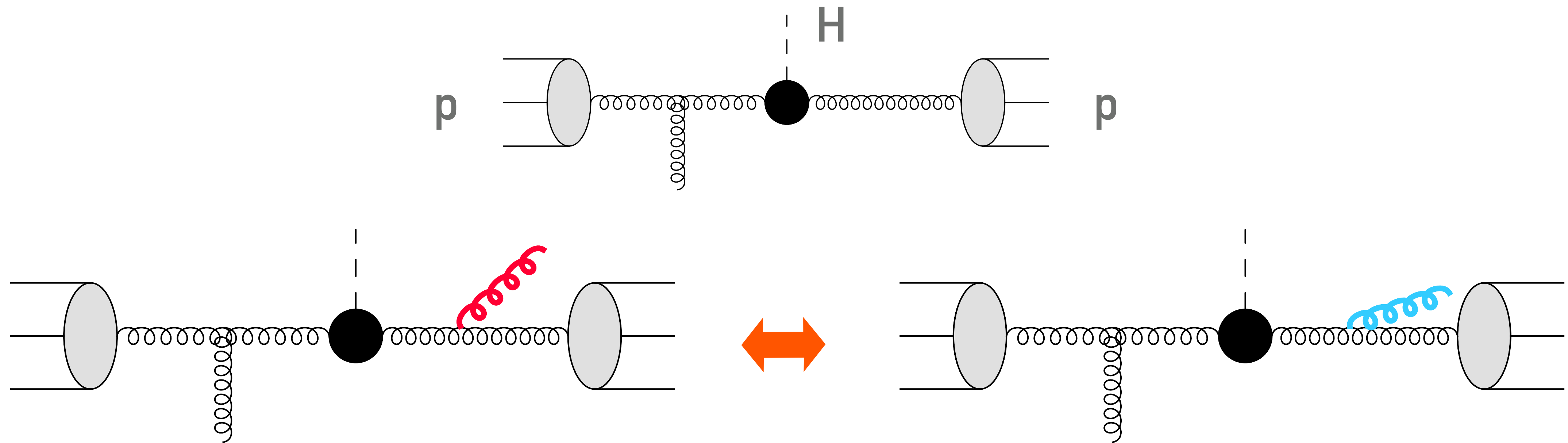
- Computation of radiative corrections to the hard process, while
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**Accuracy broken by double counting
across radiation phase space [and virtual corrections]**

Radiative corrections & interplay with parton showering

- Computation of radiative corrections to the hard process, while
 - Avoiding double-counting with parton shower [PS emits further radiation]



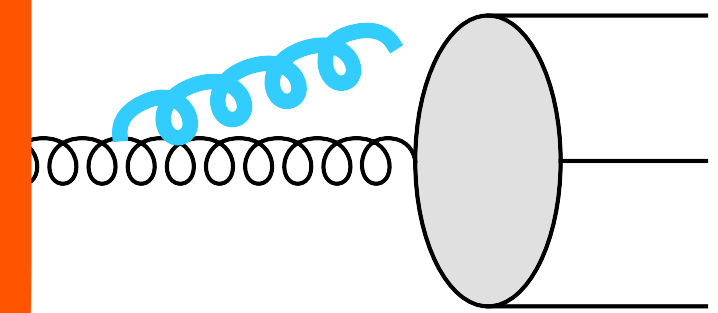
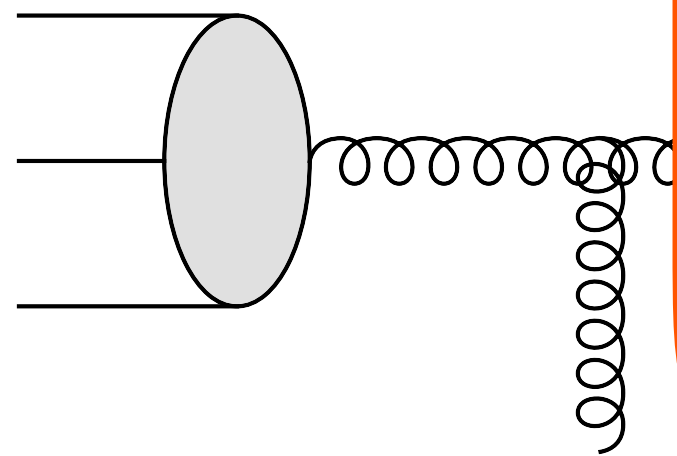
- Not tampering “too much” with the parton shower [i.e. without spoiling its accuracy, so far LL]

At the same time, we want to keep computational aspects under control [e.g. fraction of negative weights, stability, ...]

Radiative corrections & interplay with parton showering

- Computation of radiative corrections to the hard process, while
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**Explosion of complexity at NNLO
[many contributions/configurations,
double counting more convoluted]**



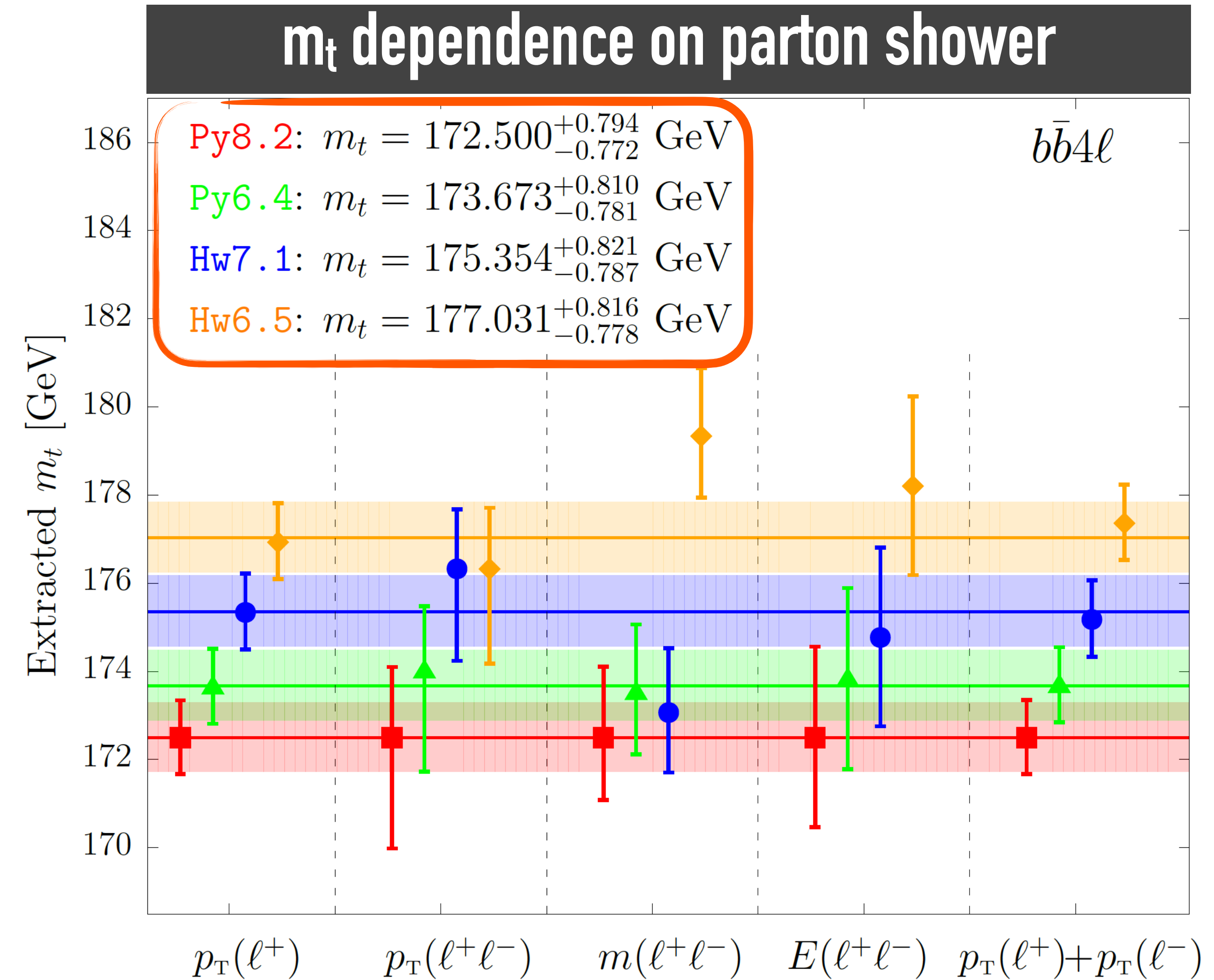
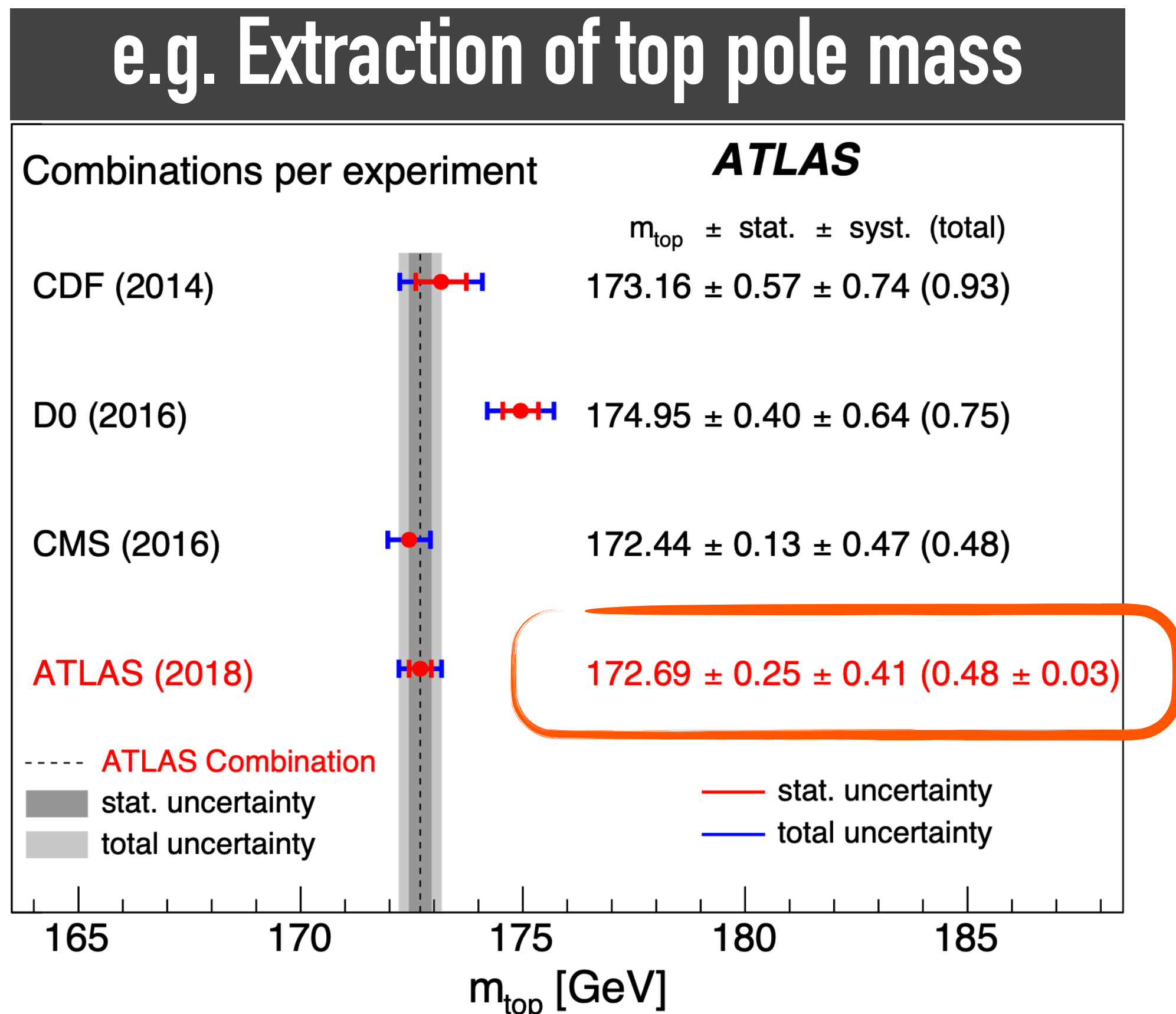
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At the same time, we want to keep computational aspects under control [e.g. fraction of negative weights, stability, ...]

An LHC example: top-quark pair production

- Main top-quark production mechanism at LHC
 - Several NP scenarios couple to top quark. Important ingredient of EFT fits
- Inaccuracy of generators already a nuisance

[Ferrario Ravasio, Jezo, Nason, Oleari '18]



NNLO event generation

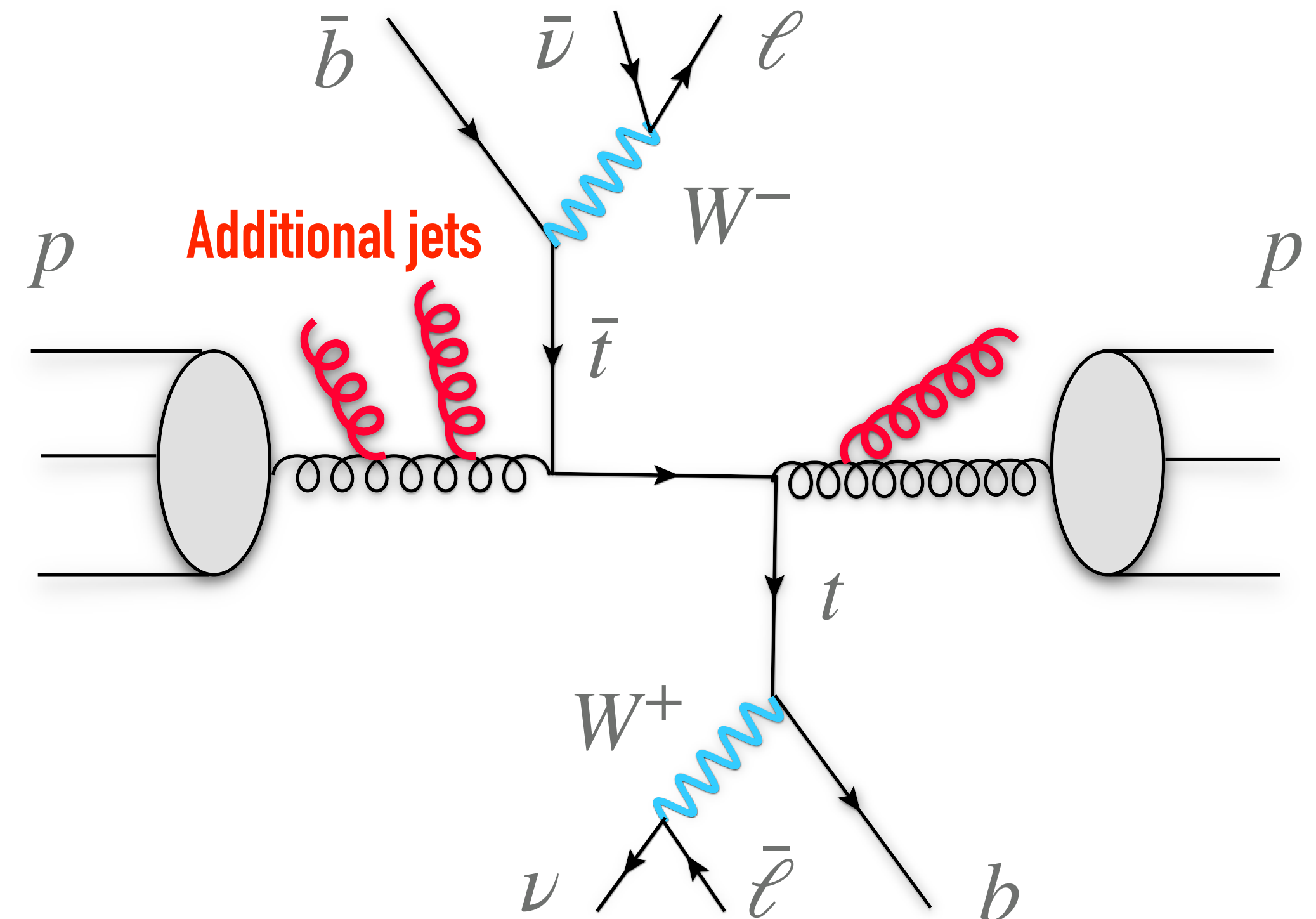
- Variety of methods to handle the production of **colourless systems** (e.g. EW bosons, Higgs boson)

[Hamilton, Nason, Re, Zanderighi '13; Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi '13; Hoeche, Li, Prestel '14; PM, Nason, Re, Wiesemann, Zanderighi '19; PM, Re, Wiesemann '20; Campbell, Hoeche, Li, Preuss, Skands '21]

- NNLO event generator for top-pair production has remained a challenge for many years

- **Colour charges in initial and final state**: involved quantum interference
- Interplay with parton shower highly non trivial
- **Many body decays**: computationally hard

E.g. first **NLO generator for $t\bar{t}$** formulated in **2003**, it took more than 17 years to achieve NNLO!



The MiNNLO_{PS} method

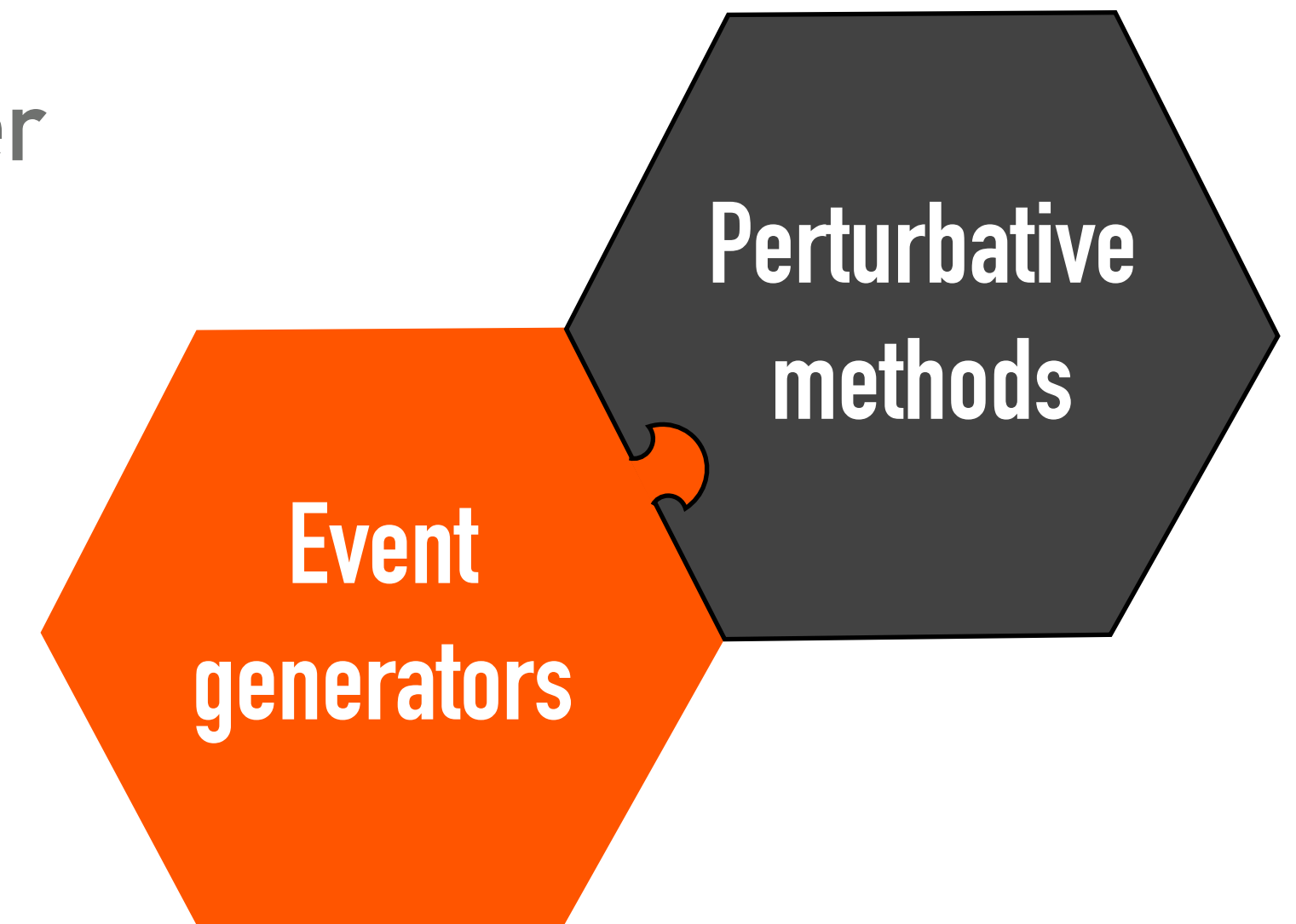
[[PM](#), Nason, Re, Wiesemann, Zanderighi (JHEP 2019); [PM](#), Re, Wiesemann (EPJC 2020)]

- Main observation: exploit link between perturbative methods and Monte Carlo language

- Recast NNLO calculation as the first two steps of a parton shower [i.e. radiation ordered in resolution variable, Sudakov factors]
- Fix d.o.f. by matching it to a NNLO perturbative calculation [i.e. resummation properties of q_T as a resolution variable]

- Advantages:

- ✓ **Accurate**: Fully differential NNLO QCD
- ✓ **Fast**: Marginal loss in complexity w.r.t. NLO computation
- ✓ **Flexible**: Possible to tackle complex reactions



MiNNLO_{PS}: NNLO generator for $t\bar{t}$ production

[Mazzitelli, [PM](#), Nason, Re, Wiesemann, Zanderighi (PRL 2021 + JHEP 2022)]

- Validation: verify agreement with perturbative QCD calculations for inclusive observables (i.e. without experimental selection cuts)

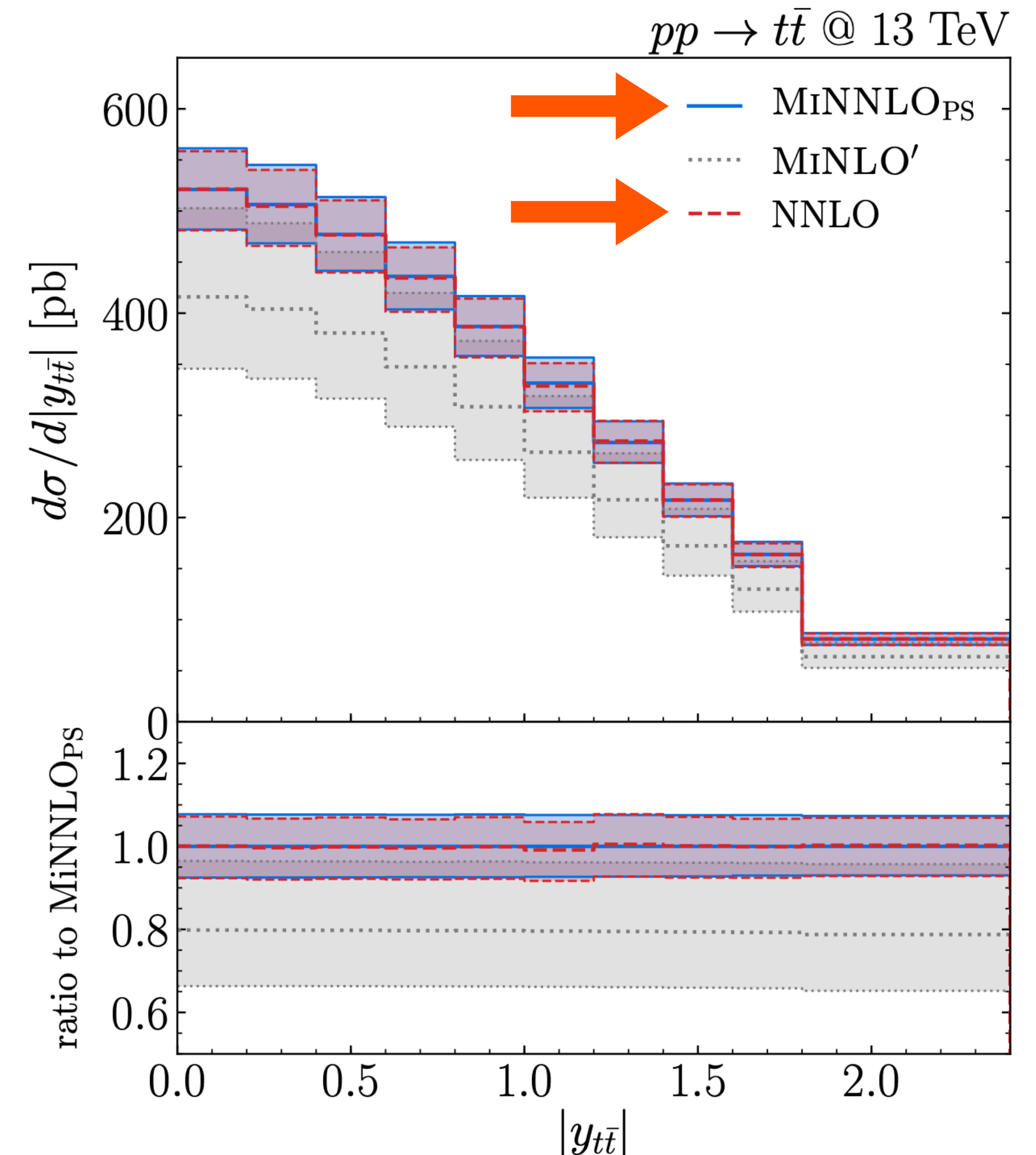
→ total cross section:

MINLO'	NNLO	MiNNLO _{PS}
$572.9(2)_{-17\%}^{+21\%}$ pb	$719.1(8)_{-7.6\%}^{+7.0\%}$ pb	$719.8(2)_{-7.4\%}^{+7.6\%}$ pb



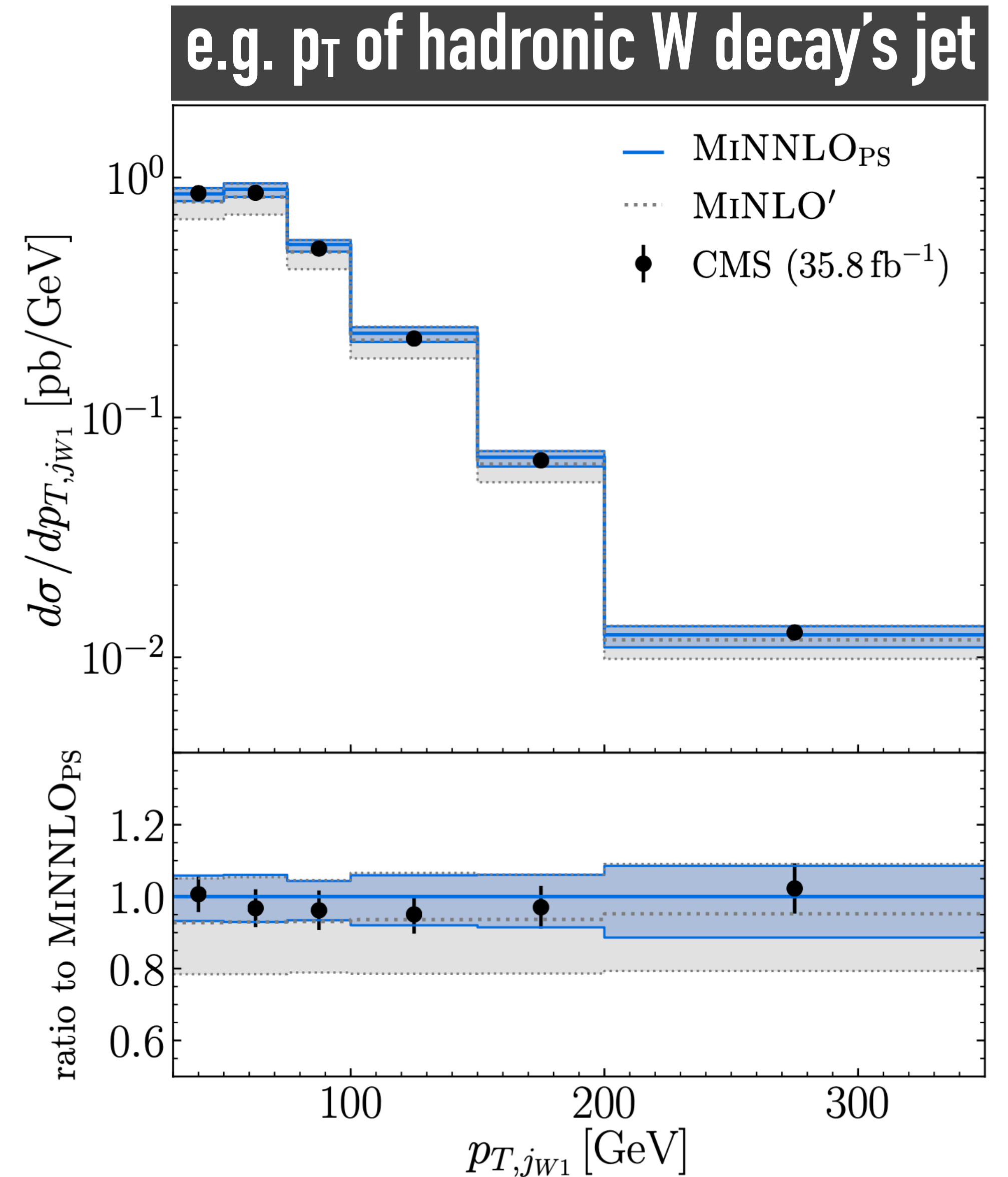
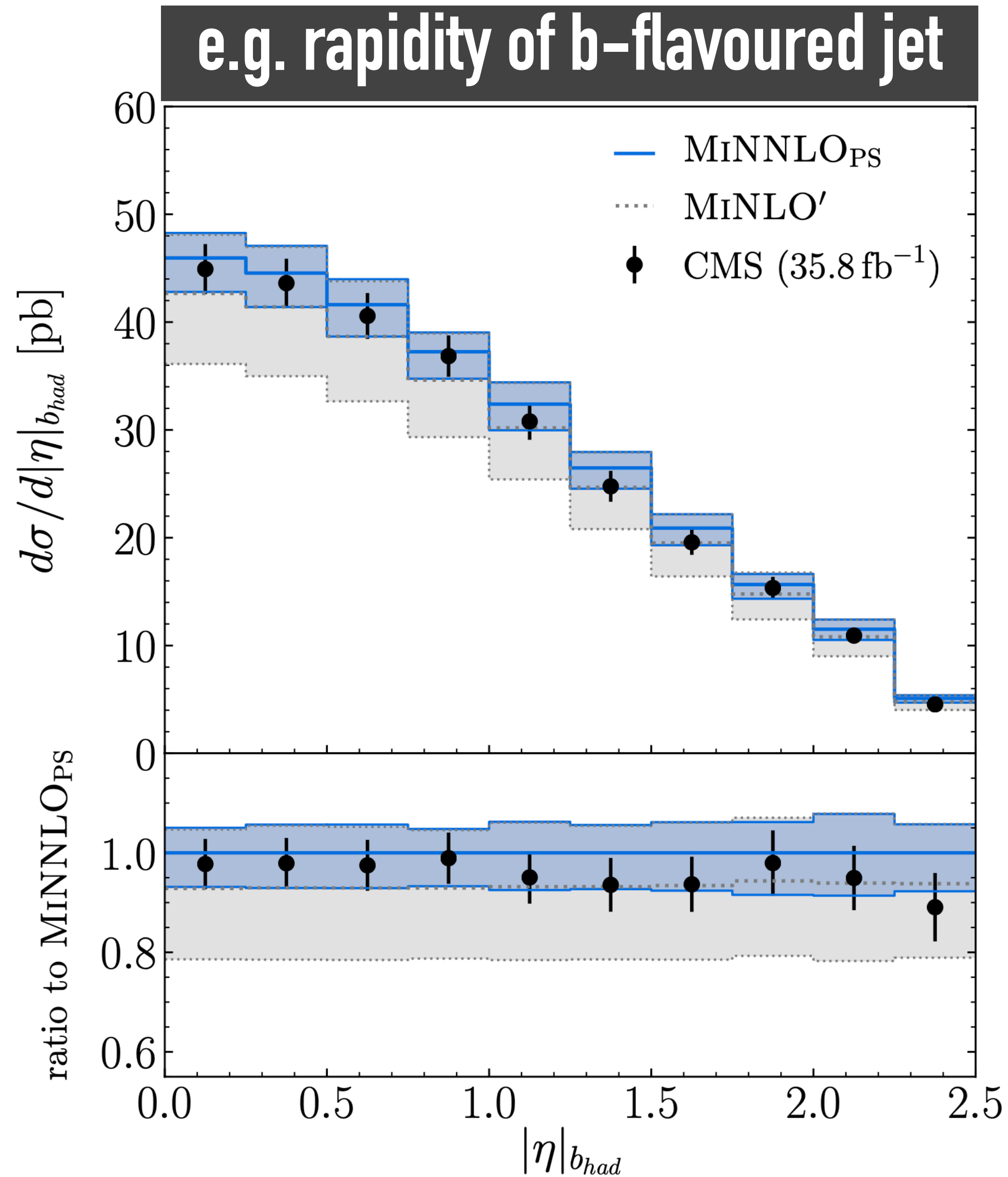
→ rapidity distribution of the top pair

Excellent agreement with pQCD, drastic reduction of theory uncertainties w.r.t. NLO!



MiNNLO_{PS}: broad comparison to experimental data

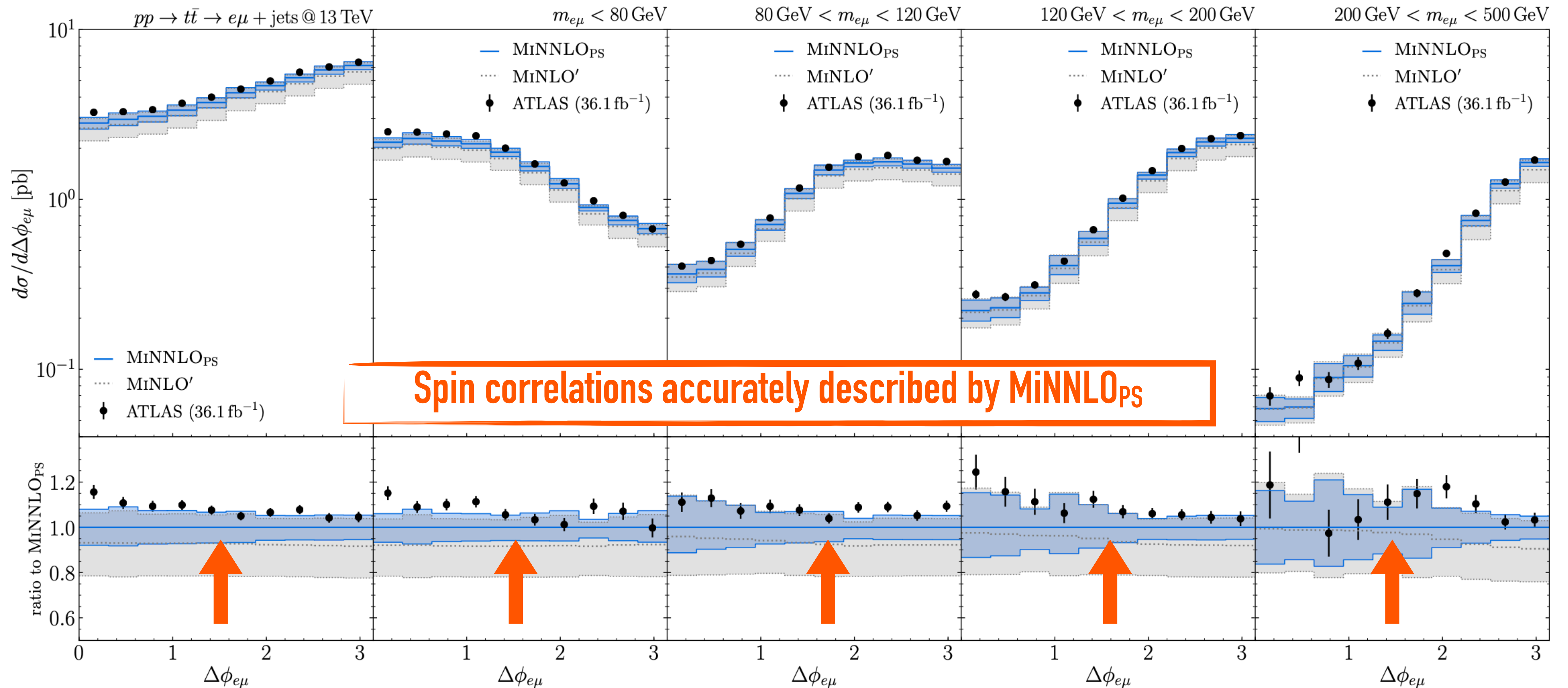
[Mazzitelli, [PM](#), Nason, Re, Wiesemann, Zanderighi (JHEP 2022)]



Possible resolution of a long-standing tension in spin correlations?

- Ongoing studies show good theory/data agreement for correlations

[Mazzitelli, [PM](#), Nason, Re, Wiesemann, Zanderighi (JHEP 2022)]



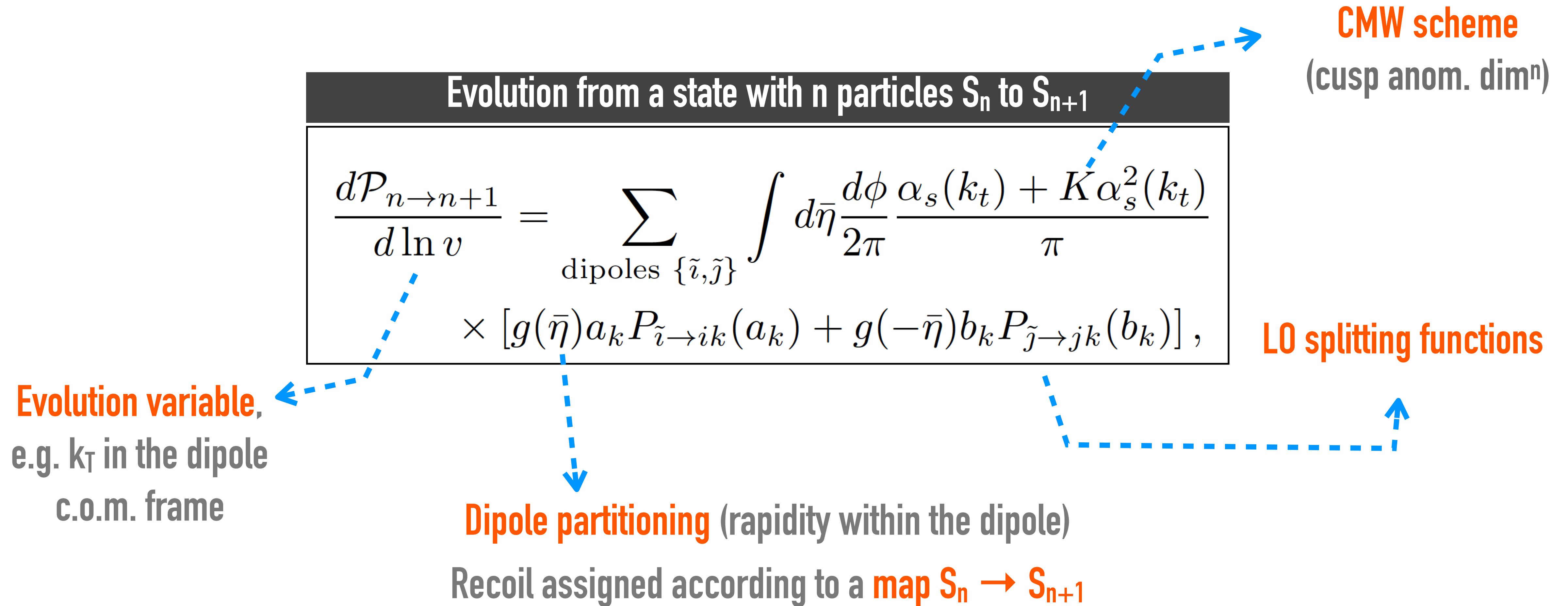
Conclusions and Outlook

- Modern problems in collider physics demand rethinking the approach to **a crucial bridge between theory and experiment: Monte Carlo event generators**
- Novel ideas are paving the way to a new generation of tools with a **higher and controllable formal accuracy**, led by powerful techniques in connection with perturbative QCD:
 - New methods to **diagnose parton-shower (PS) accuracy and design NLL algorithms**
 - PS@NLL is today a nearly-solved problem, accessible via public tools in the future. **Gearing up for higher orders (NNLL) corrections** requires tackling many intriguing conceptual challenges
- Considerable progress in the **matching of PS to NNLO calcⁿ for coloured final states**. Open problems ahead:
 - Consistently **preserve higher-order PS accuracy** (e.g. matching to PanScales showers)
 - First considerations about higher (N³L0) orders matching have started to emerge

Backup

An example: local-recoil dipole showers

- (planar) **squared amplitudes built recursively** via a Markovian chain of emissions (& virtuals via unitarity)



An example: local-recoil dipole showers

- Keep the recoil **local**, i.e. for each new emission use the map

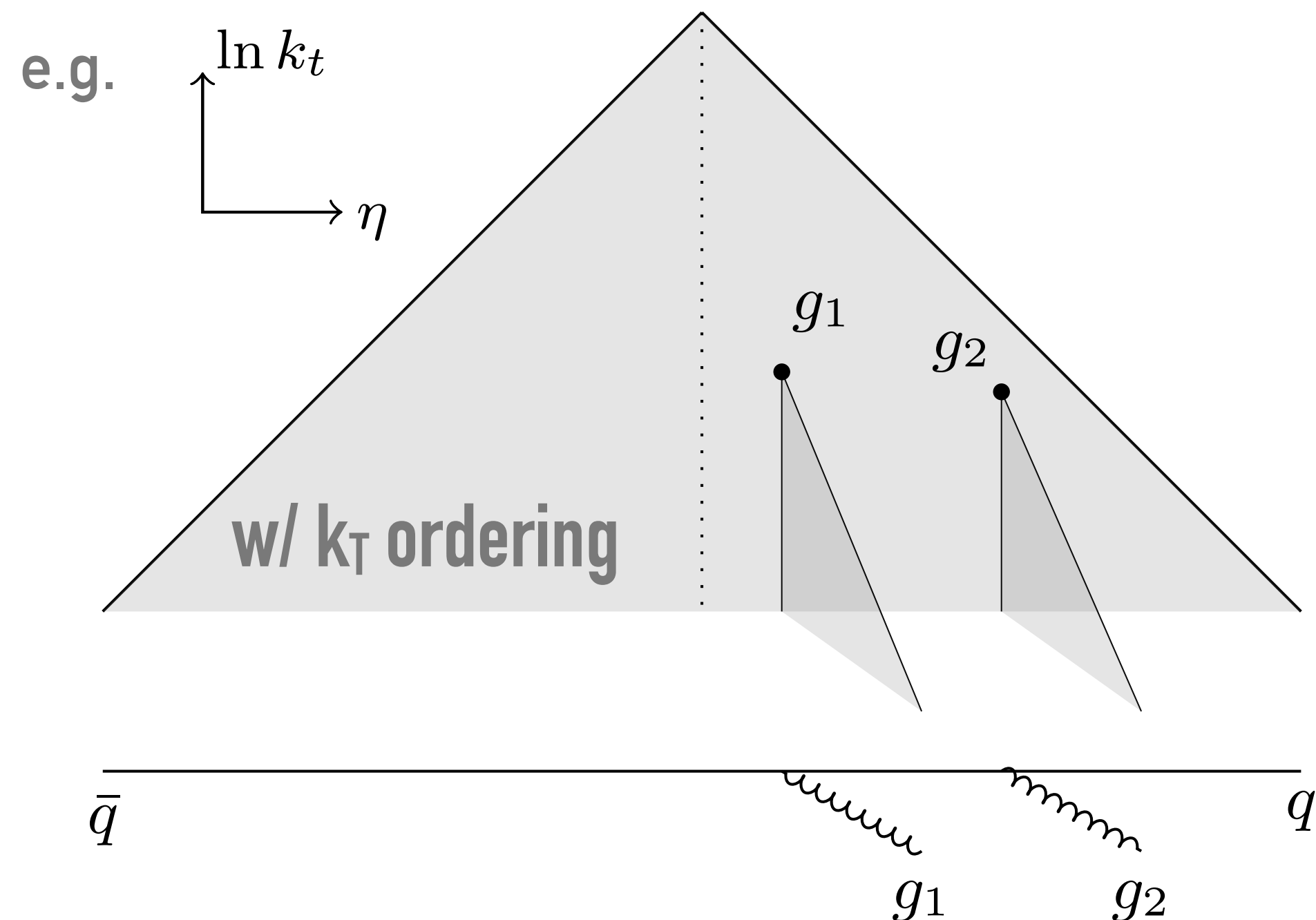
dipole $\{\tilde{p}_i, \tilde{p}_j\}$ \longrightarrow

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_{\perp},$$

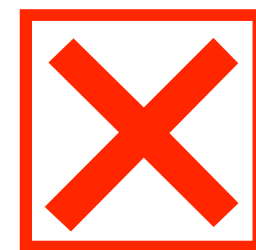
$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - f k_{\perp},$$

$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_{\perp}$$

- Typical problem (source of the issues in the heat plot): **dipole partitioned in the dipole c.o.m. frame**



e. g. Pythias / DiRE



- In the limit of strong angular ordering and commensurate k_T 's, **g_2 can still take the recoil from g_1**

\rightarrow **i.e. violation of locality in the LP**

The PanLocal algorithm

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

- Keep the recoil **local**, i.e. for each new emission use the map

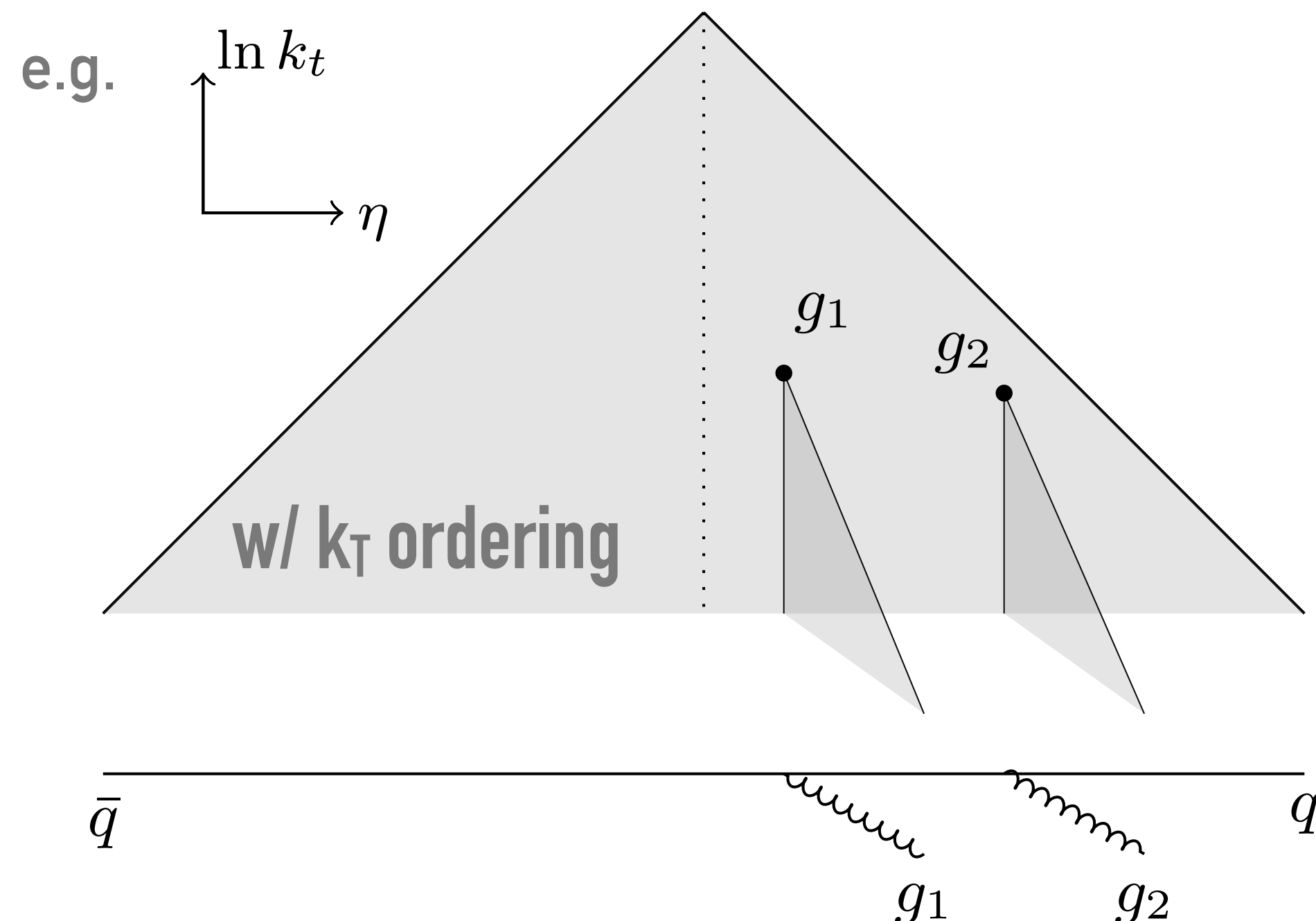
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- Key element #1**: partitioning ($\bar{\eta}=0$) occurs at equal angles to the dipole ends in the **event c.o.m. frame**



- In the limit of strong angular ordering and commensurate k_T 's, **g_2 takes the recoil from the hard quark**

The PanLocal algorithm

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

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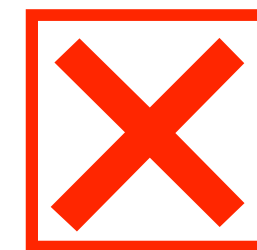
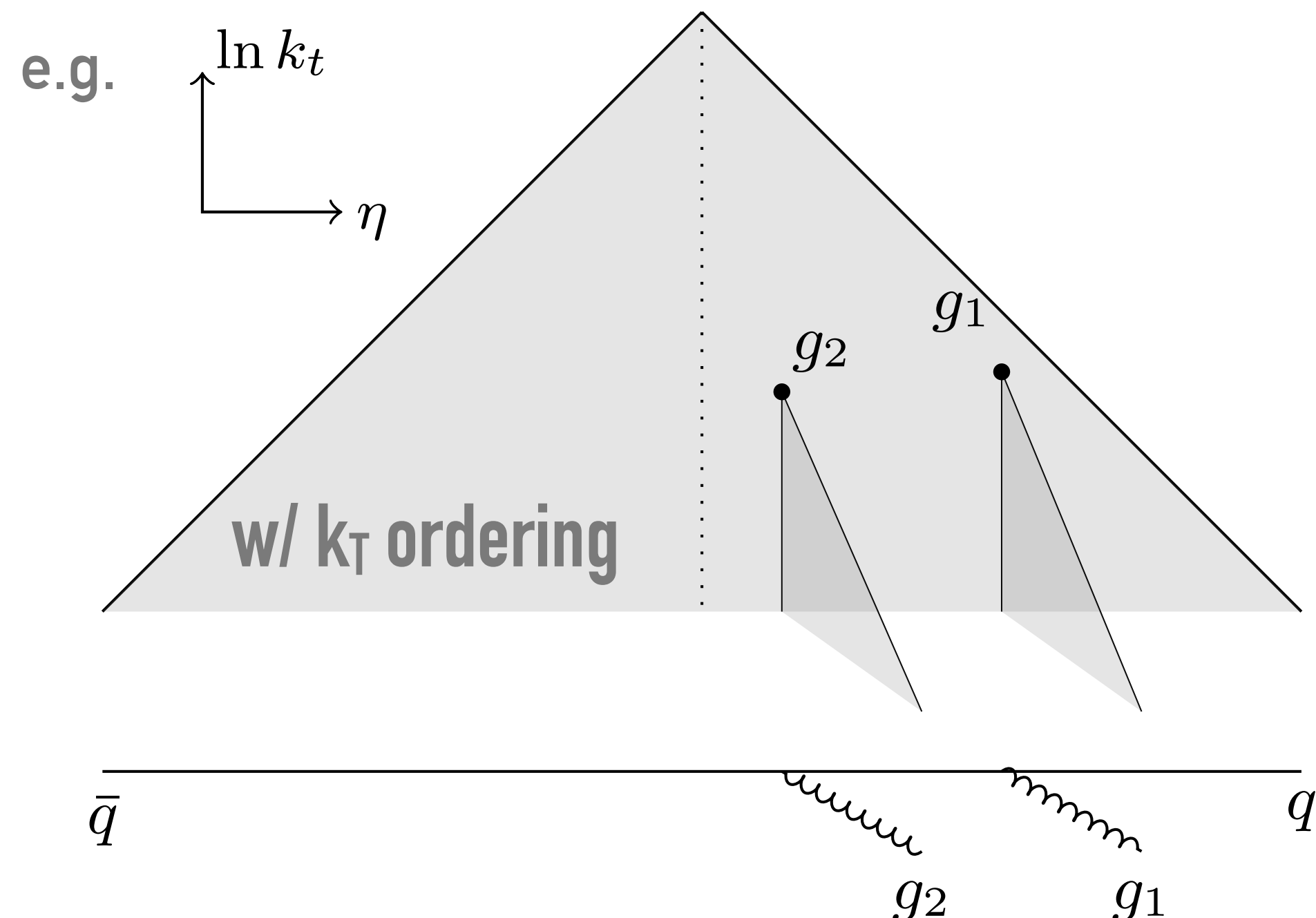
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- Key element #1**: partitioning ($\bar{\eta}=0$) occurs at equal angles to the dipole ends in the **event c.o.m. frame**



- However, if g_2 is produced at larger angles than g_1 , the recoil is still taken from g_1 in a logarithmic (NLL) region of phase space

The PanLocal algorithm

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

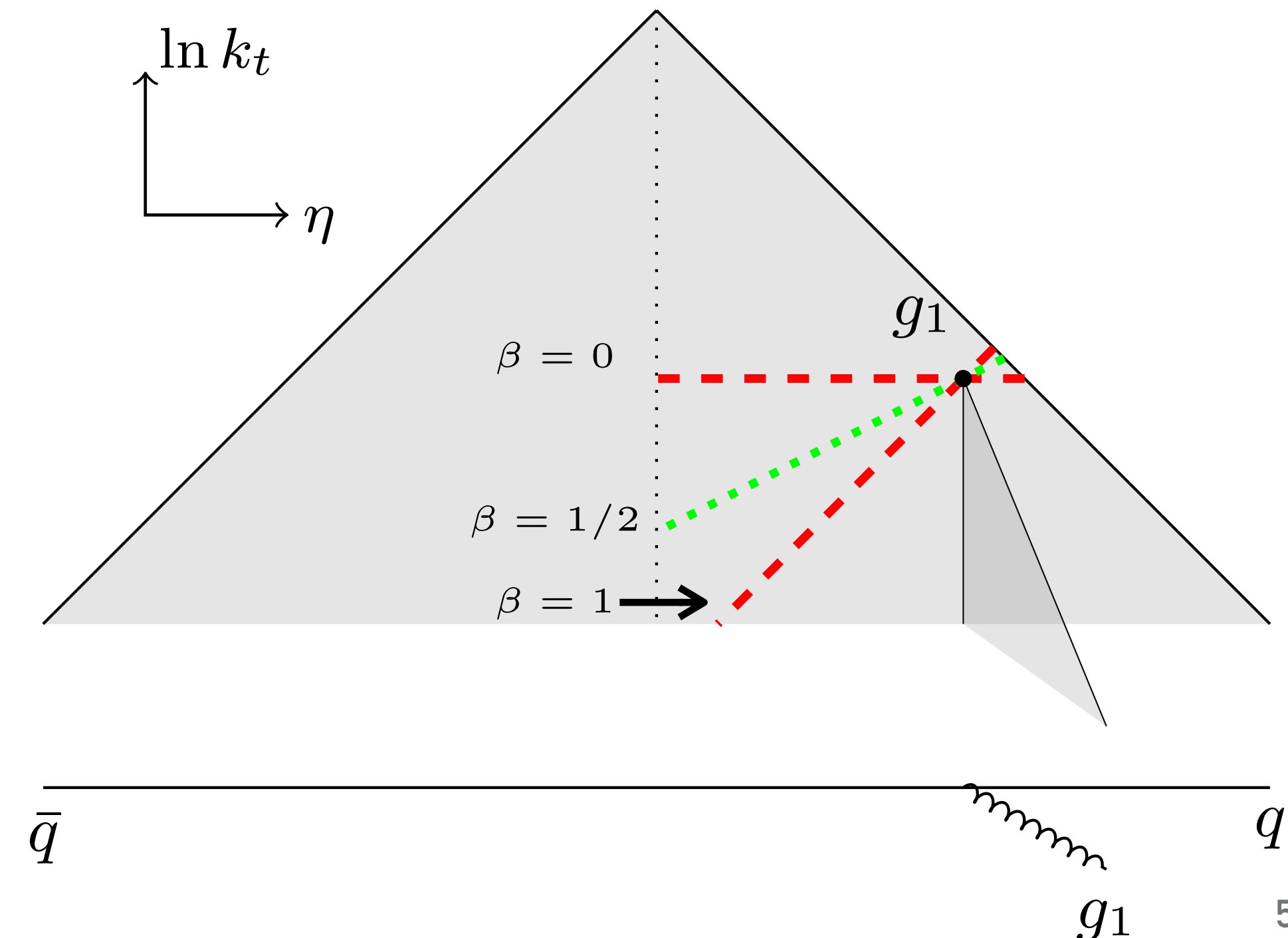
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$$\text{dipole } \{\tilde{p}_i, \tilde{p}_j\} \longrightarrow \begin{aligned} p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\ p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp, \\ p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_\perp \end{aligned}$$

- Key element #2:** modify the evolution variable (instead of dipole k_T)

$$k_t = \rho v e^{\beta |\bar{\eta}|} \sim v e^{\beta |\eta|} \text{ w.r.t. emitter}$$

$$\rho = \left(\frac{s_{\tilde{i}\tilde{j}}}{Q^2 s_{\tilde{i}\tilde{j}}} \right)^{\frac{\beta}{2}}$$



k_T ordering corresponds to $\beta=0$

The PanLocal algorithm

[Dasgupta, Dreyer, Hamilton, [PM](#), Salam, Soyez (PRL 2020)]

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- Key element #2**: modify the evolution variable (instead of dipole k_T)

- Ordering in v now implies that $k_{t2} \ll k_{t1}$ [i.e. no recoil]
- Interplay of partition \oplus ordering ensures that the recoil is always taken from the hard extremities **[OK at NLL]**

