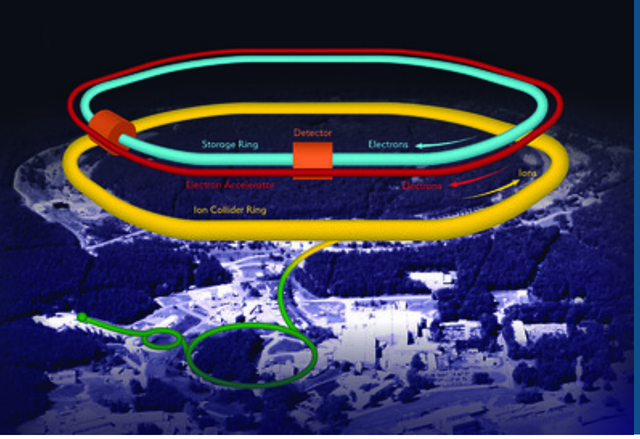

The 2022 CFNS Summer School on the Physics of the Electron Ion Collider

THE 3D STRUCTURE OF THE NUCLEON

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INFN Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

2020

How can we built up
a multidimensional picture
of the nucleon ❓

Charges

$$\boxed{\frac{1}{2P^+}\langle p^+,\vec{0}_\perp,\Lambda'|\bar{\psi}(0)\Gamma\psi(0)|p^+,\vec{0}_\perp,\Lambda\rangle}$$

Light-front coordinates:

$$a^+ = \frac{a^0 + a^3}{\sqrt{2}} \qquad a^- = \frac{a^0 - a^3}{\sqrt{2}}$$

$$a_\perp = (a^1, a^2)$$

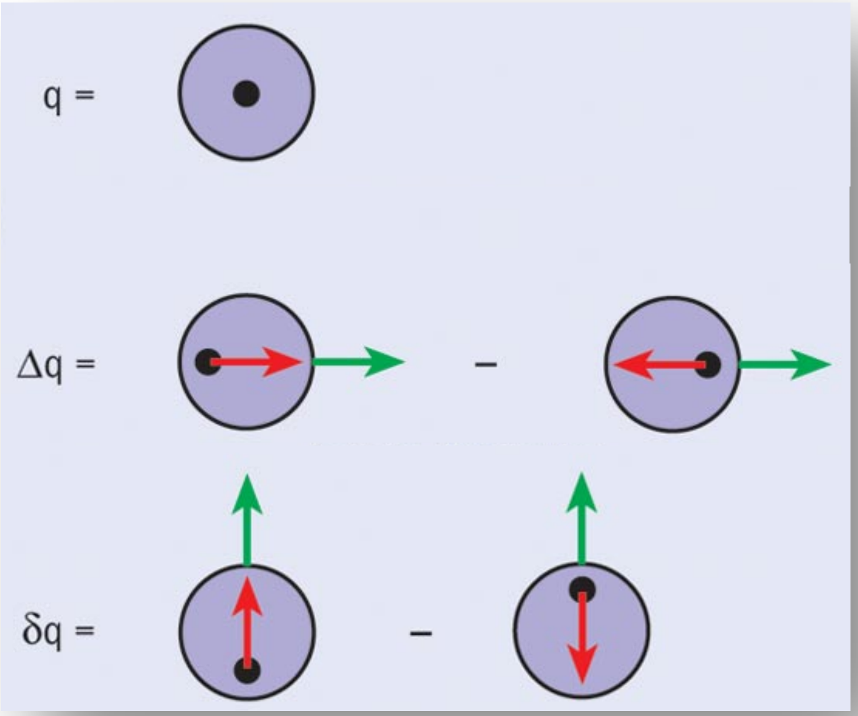
Depend on

Λ, Λ', Γ: nucleon and quark polarizations

Vector: $\Gamma = \gamma^+$
Parton number

Axial: $\Gamma = \gamma^+\gamma_5$
Parton helicity

Tensor: $\Gamma = i\sigma^{+i}\gamma_5$
Parton transversity



Charges

Form Factors (FFs)

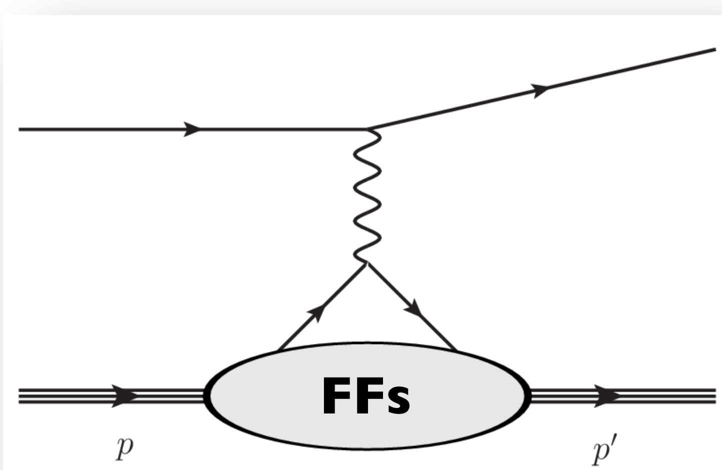
$$\boxed{\frac{1}{2P^+}\langle p^+,\, \frac{\vec{\Delta}_\perp}{2},\, \Lambda'|\bar{\psi}(0)\,\Gamma\,\psi(0)|p^+,\, -\frac{\vec{\Delta}_\perp}{2},\, \Lambda\rangle}$$

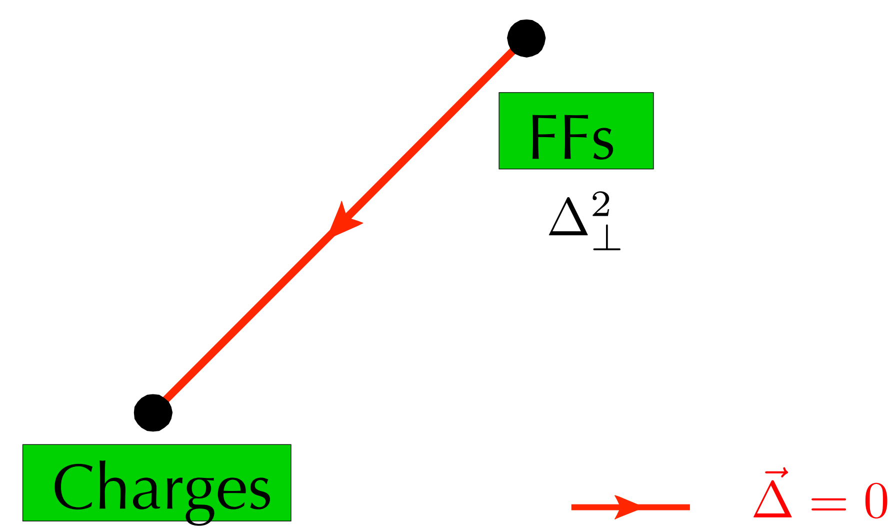
Depend on

$\Lambda,\, \Lambda',\, \Gamma$: nucleon and quark polarizations

Δ : momentum transfer $\qquad \vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

Elastic Scattering

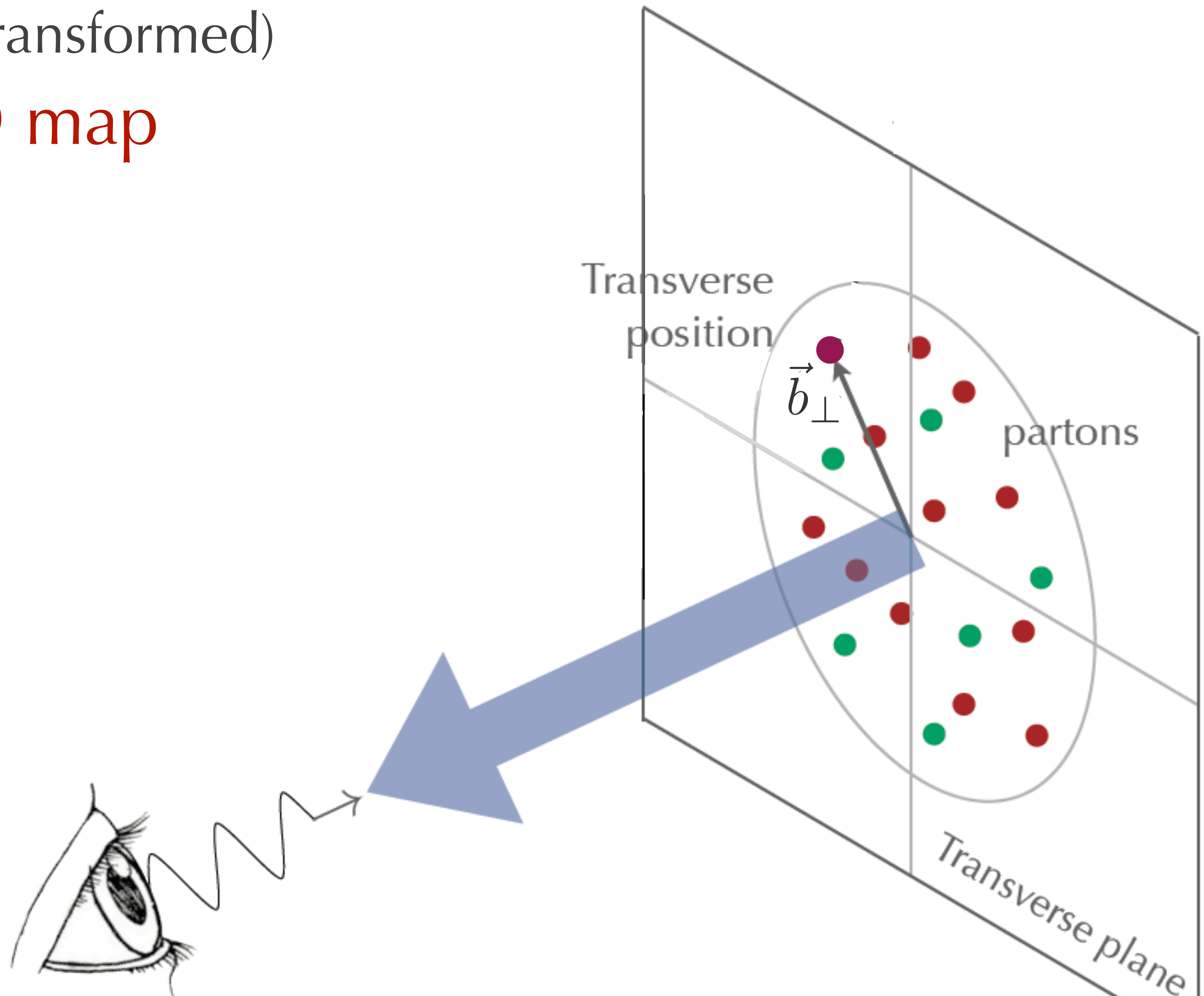




Form Factors:

(Fourier transformed)

0D+2D map



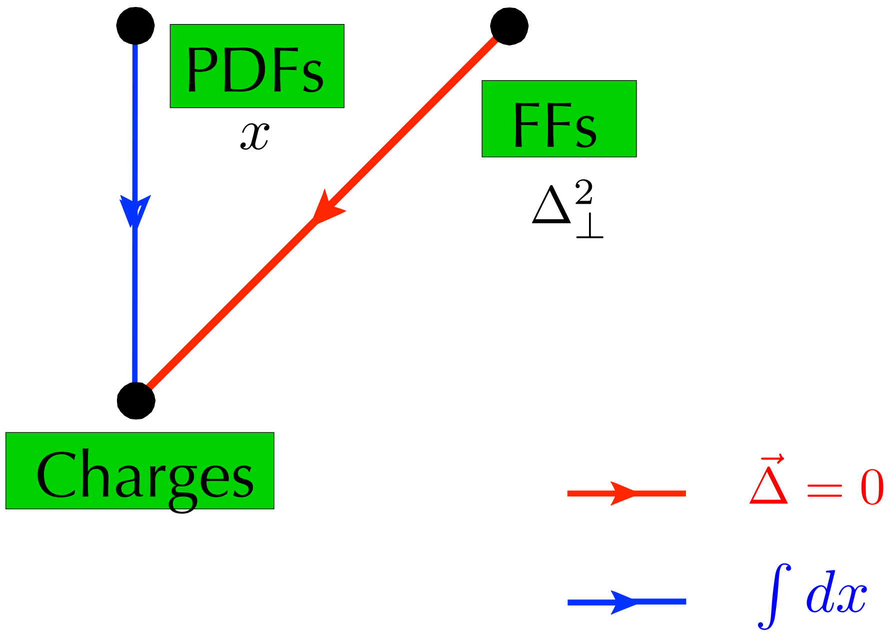
Parton Distribution Functions (PDFs)

$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}e^{ik^+z^-}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0,z_\perp=0}}$$

Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction



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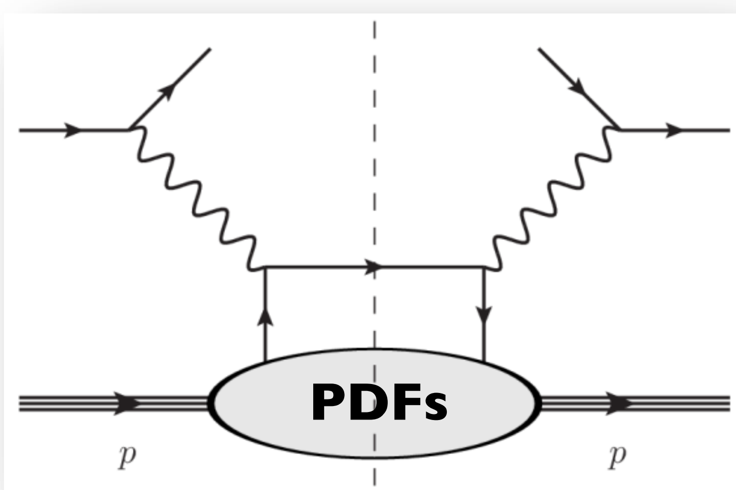
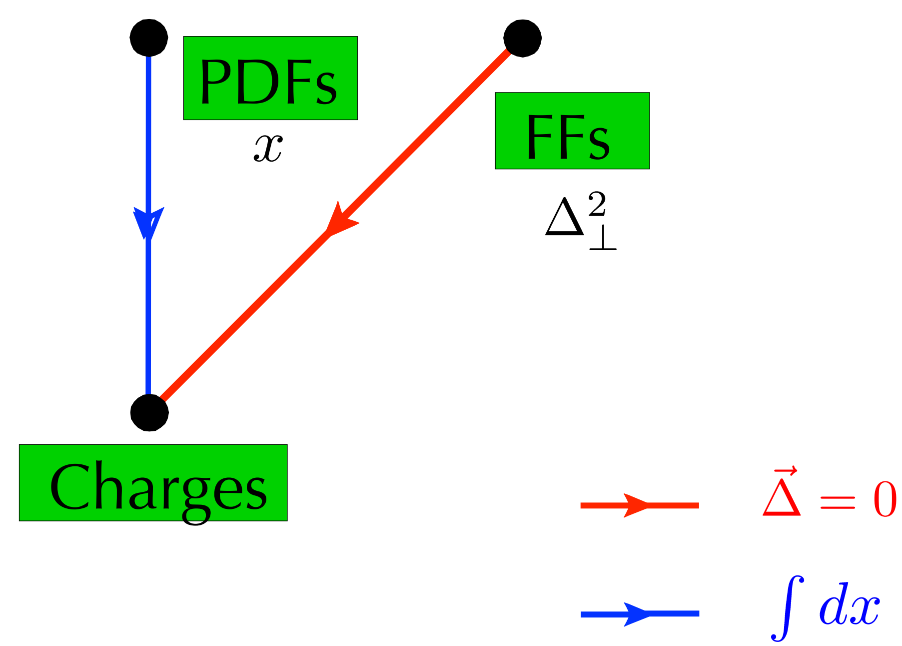
Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

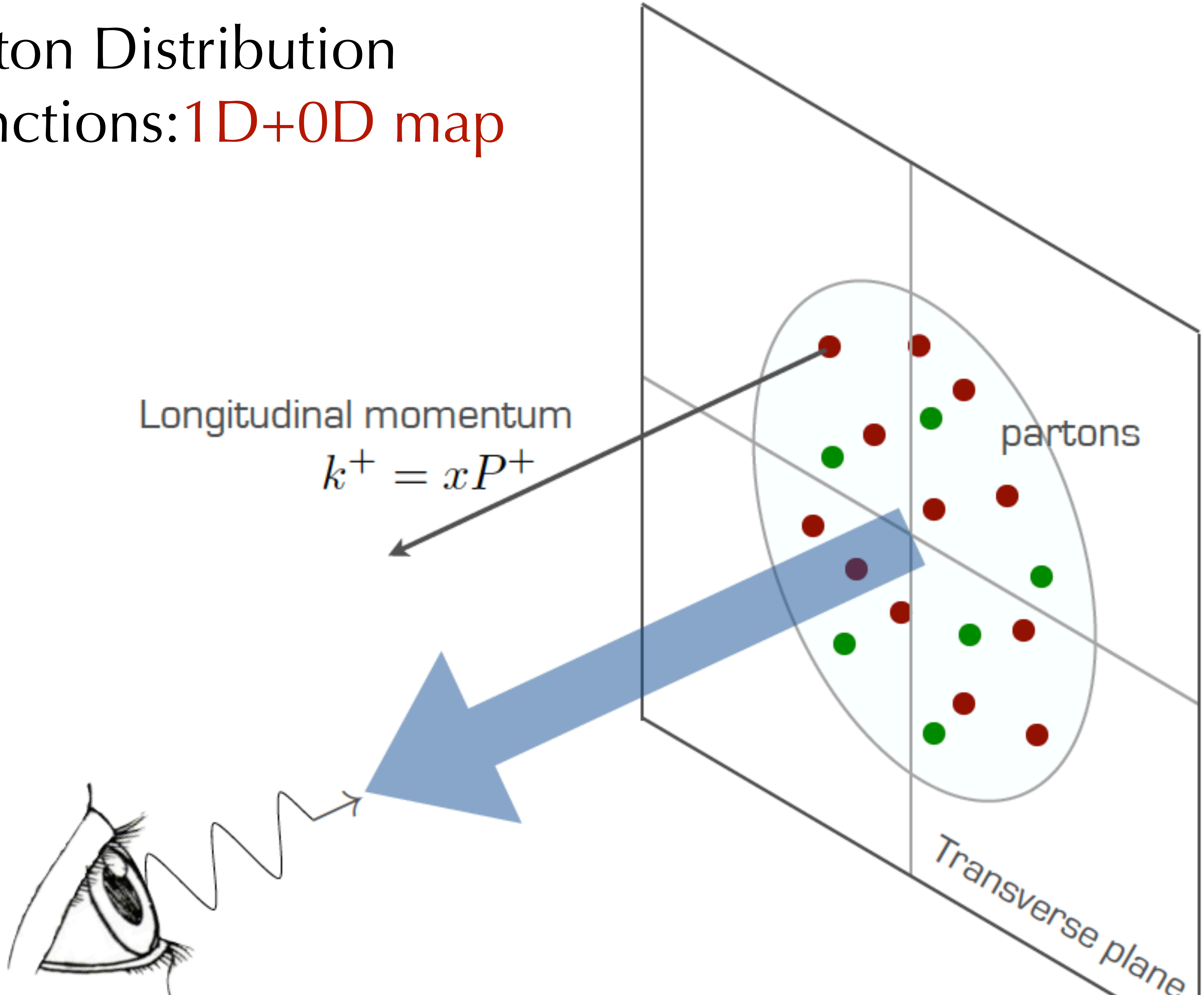
$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction

Deep Inelastic Scattering





Parton Distribution Functions: 1D+0D map



Generalized Parton Distributions (GPDs)

$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}e^{ik^+z^-}\langle p'^+,\,-\frac{\vec{\Delta}_\perp}{2},\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\frac{\vec{\Delta}_\perp}{2},\,\Lambda\rangle_{z^+=0,z_\perp=0}}$$

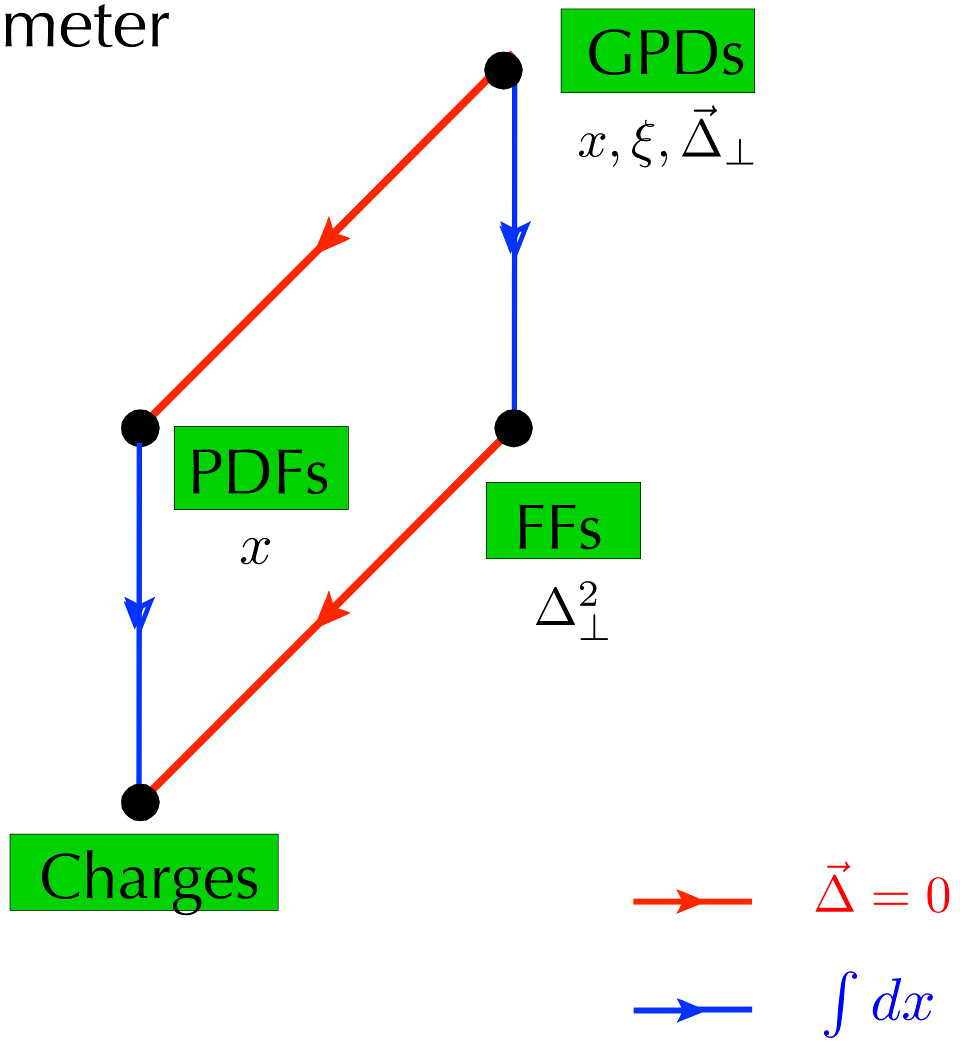
non-diagonal matrix elements

Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction

Δ : momentum transfer $\vec{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{b}_\perp$: impact parameter



Generalized Parton Distributions (GPDs)

$$\boxed{\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ik^+z^-} \langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0, z_\perp=0}}$$

non-diagonal matrix elements

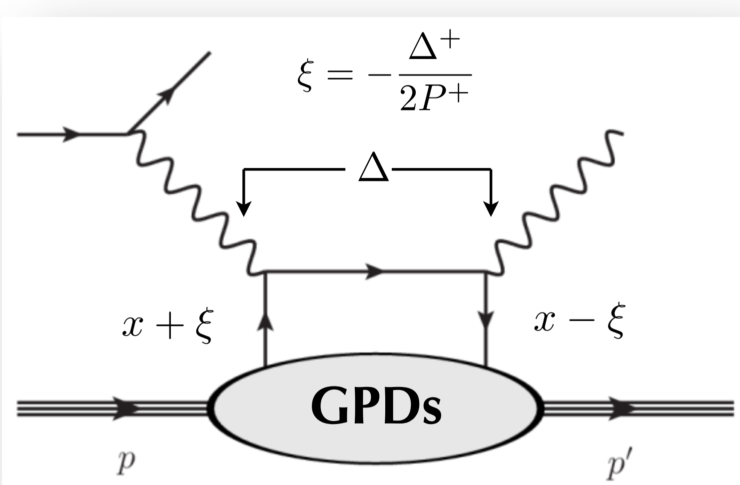
Depend on

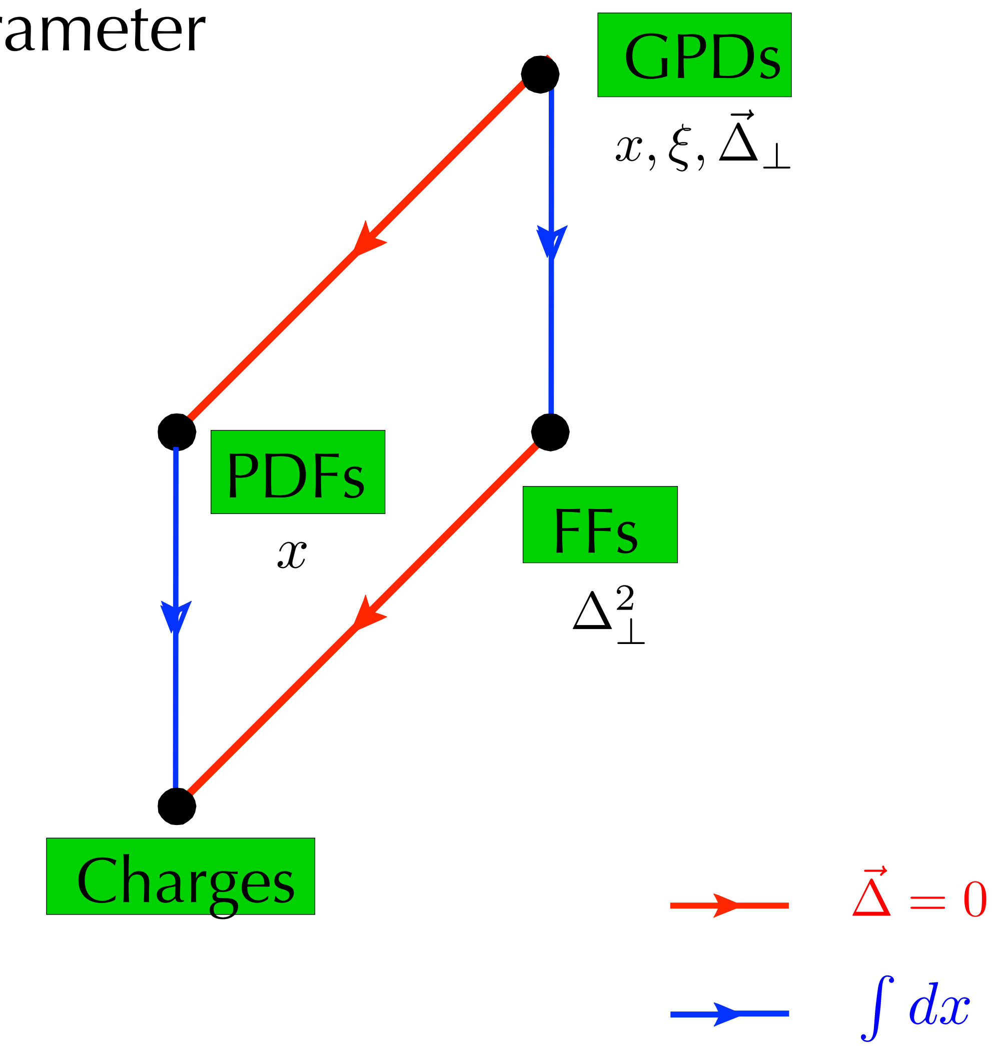
$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

$x = \dfrac{k^+}{p^+}$: longitudinal momentum fraction

Δ : momentum transfer $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

Deeply Virtual Compton Scattering

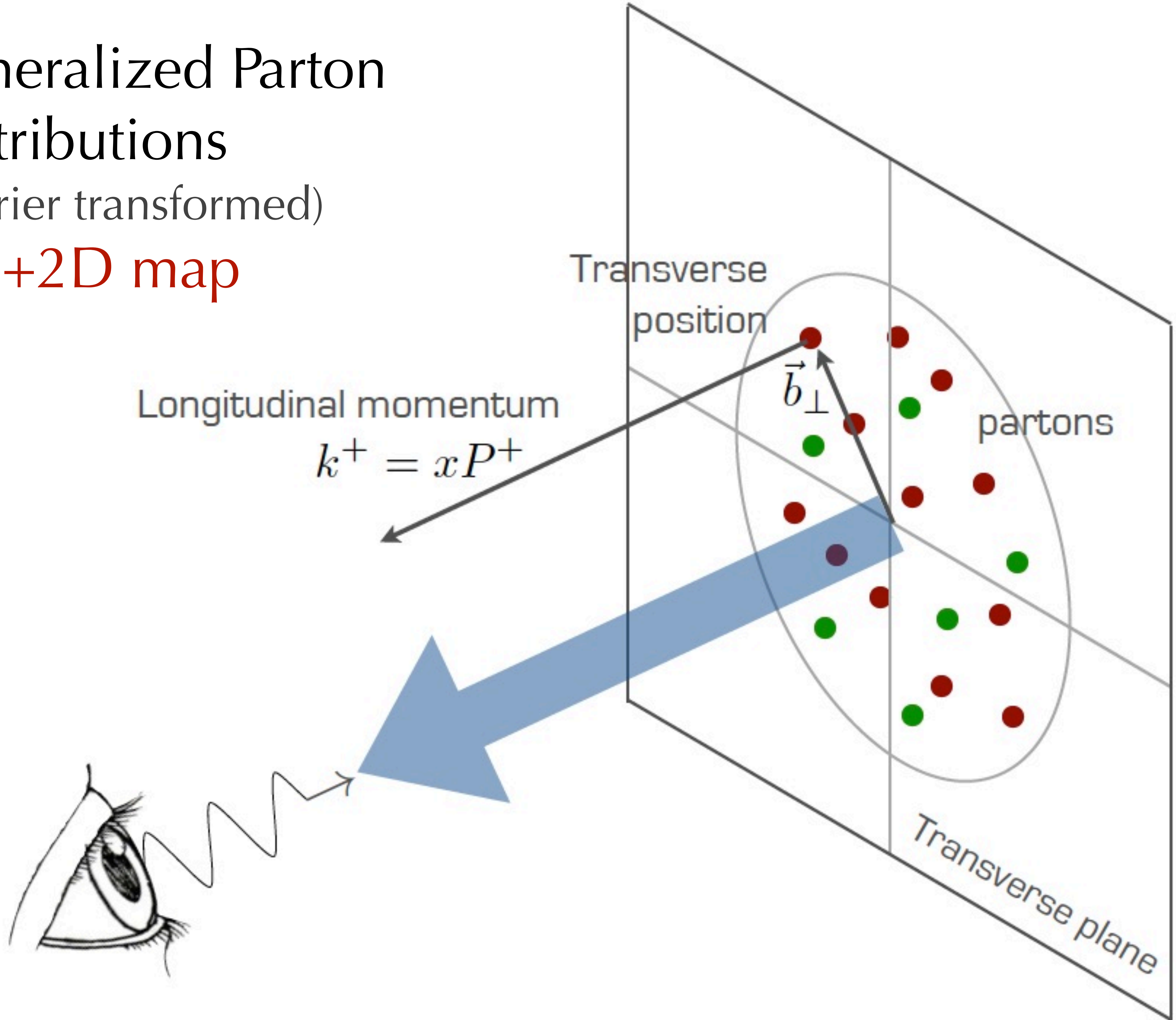




Generalized Parton Distributions

(Fourier transformed)

1D+2D map



Key information from GPDs

- Multidimensional picture of the proton in the 1+2D
- Decomposition of Form Factors as function of x
- Sum rule for Angular Momentum
- Access to Form Factors of Energy Momentum Tensor

$\longrightarrow$ "mechanical" properties of the nucleon

Recent Review

Eur. Phys. J A52 (2016)

The European Physical Journal A
All Volumes & Issues

The 3-D Structure of the Nucleon

ISSN: 1434-6001 (Print) 1434-601X (Online)

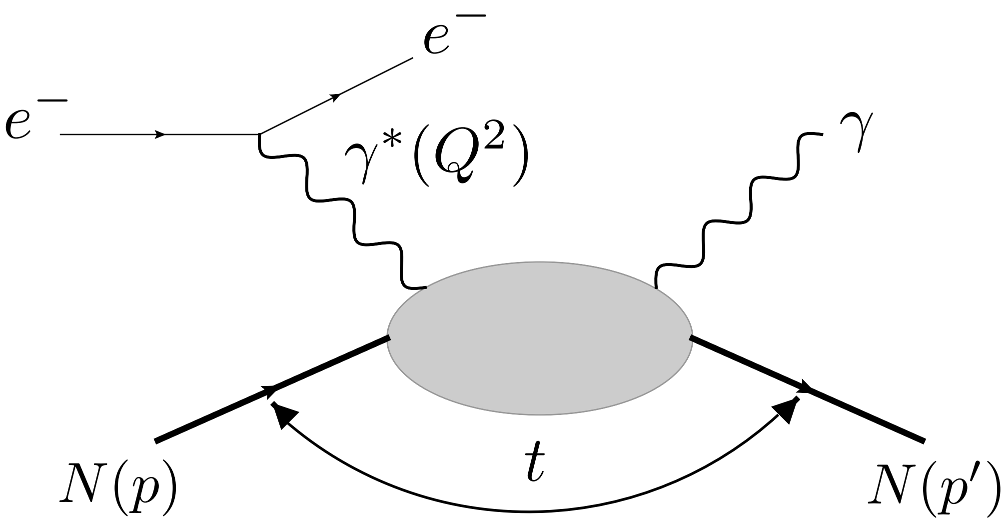
In this topical collection (17 articles)

Editors: M. Anselmino, M. Guidal, P. Rossi

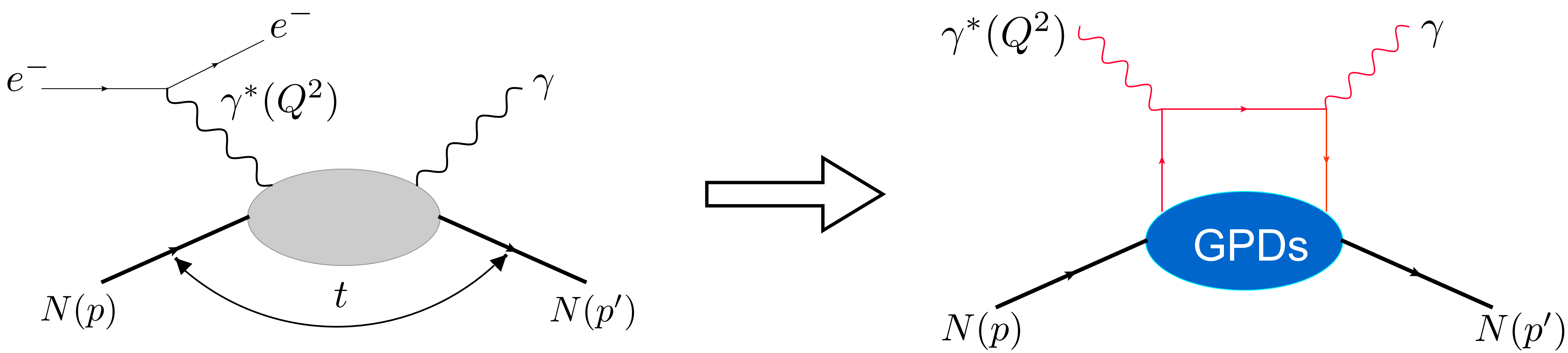
A few references on GPDs

- Overviews with full bibliography:
  - M. Diehl, Phys. Rep. 388 (2003) 41
  - K. Goeke, M. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401
  - X. Ji, Ann. Rev. Nucl. Part. Sci. 54 (2004) 413
  - A.V. Belitsky, A.V. Radyushkin, Phys. Rept. 418 (2005)
  - S. Boffi, B. Pasquini, Riv. Nuovo Cim. 30 (2007) 387
  - M. Guidal, H. Moutarde, M. Vanderhaeghen, Rept. Prog. Phys. 76 (2013) 066202
  - K. Kumericki, S. Liuti, H. Moutarde, Eur. Phys. J. A52 (2016) 157

How to access GPDs



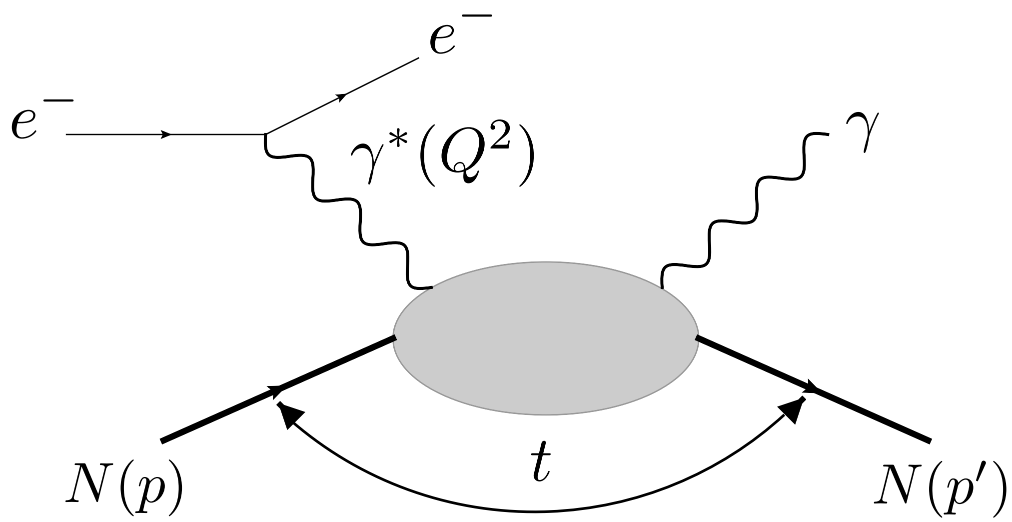
How to access GPDs

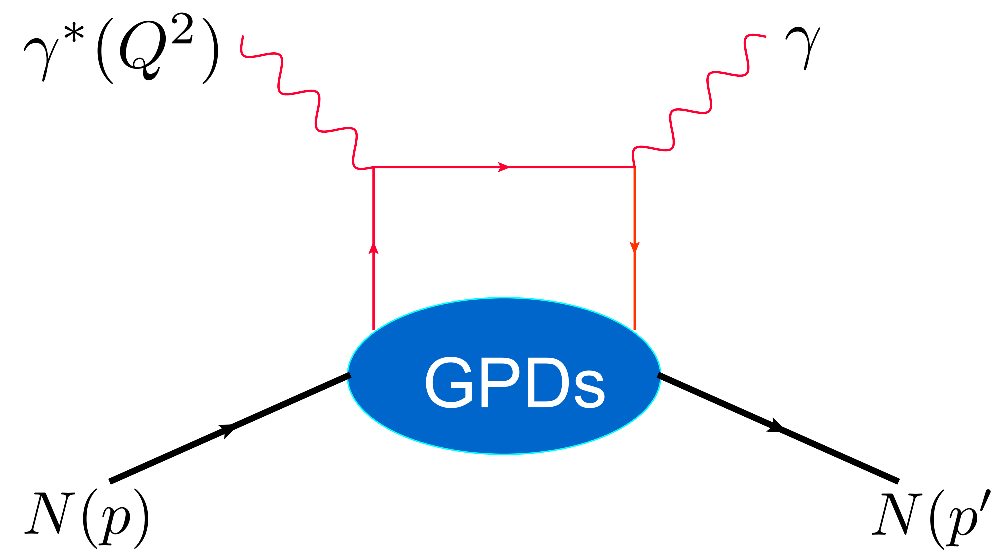


factorization for large Q², |t|<< Q² , s

$\mathcal{M} = [\text{parton Ampl.}] \otimes [\text{GPDs}]$

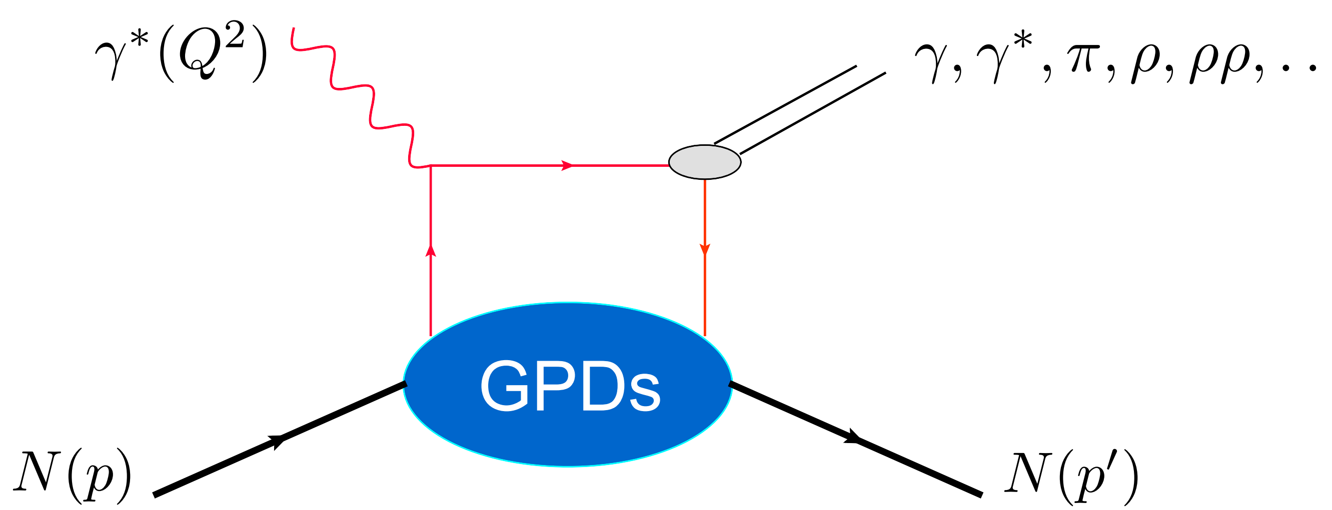
How to access GPDs





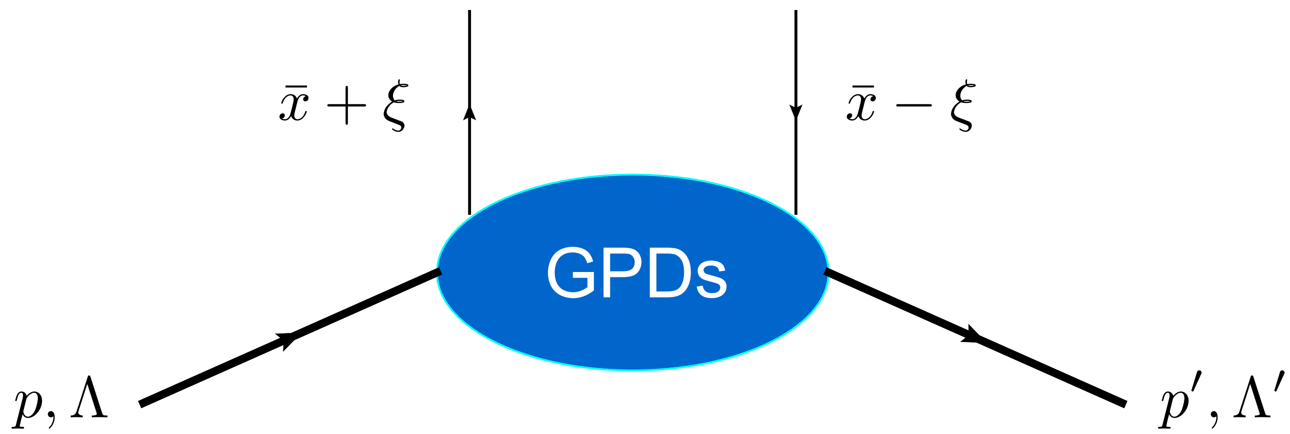
factorization for large Q^2, $|t| << Q^2$, s

$\mathcal{M} =$ [parton Ampl.] $\otimes$ [GPDs]



universality: the same GPDs enter a variety of exclusive reactions

Leading-Twist GPDs



$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \psi\left(\frac{z}{2}\right) |p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$\Gamma = \begin{cases} \gamma^+ & H^q,\ E^q & \text{unpol.} \\ \gamma^+\gamma^5 & \tilde{H}^q,\ \tilde{E}^q & \text{long. pol.} \\ i\sigma^{+i}\gamma^5 & H_T^q,\ E_T^q,\ \tilde{H}_T^q,\ \tilde{E}_T^q & \text{transv. pol.} \end{cases}$$

- $p \neq p' \Rightarrow$ GPDs depend on two momentum fractions $\bar{x}$, ξ, and t

$$\bar{x} = \frac{(k+k')^+}{(p+p')^+} = \frac{\bar{k}^+}{P^+}$$

average fraction of the longitudinal momentum carried by partons

$$\xi = \frac{(p-p')^+}{(p+p')^+} = -\frac{\Delta^+}{2P^+}$$

skewness parameter: fraction of longitudinal momentum transfer

$$t = (p-p')^2 \equiv \Delta^2$$

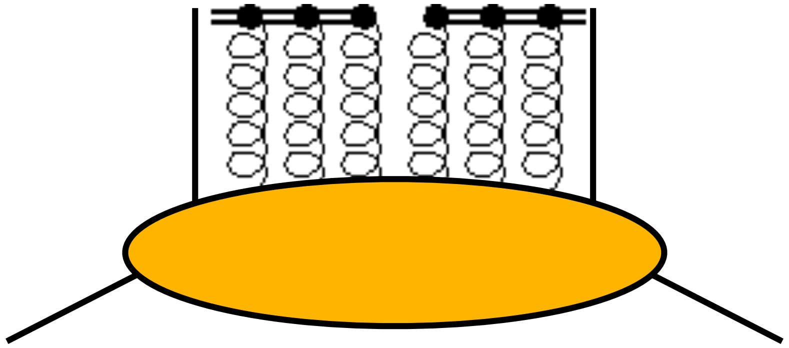
t-channel momentum transfer squared

Need of a gauge link

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

not invariant under $\psi(z) \to e^{i\alpha(z)}\psi(z)$

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]}\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$



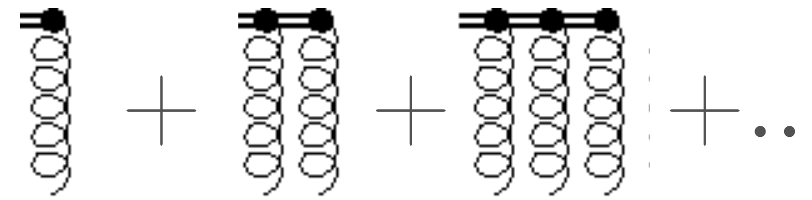
$$\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2}) \to e^{i\alpha(-\frac{z}{2})}\,\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2})\,e^{-i\alpha(\frac{z}{2})}$$

$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^\mu\, A_\mu(\eta)\right]$$

Wilson line definition of GPDs

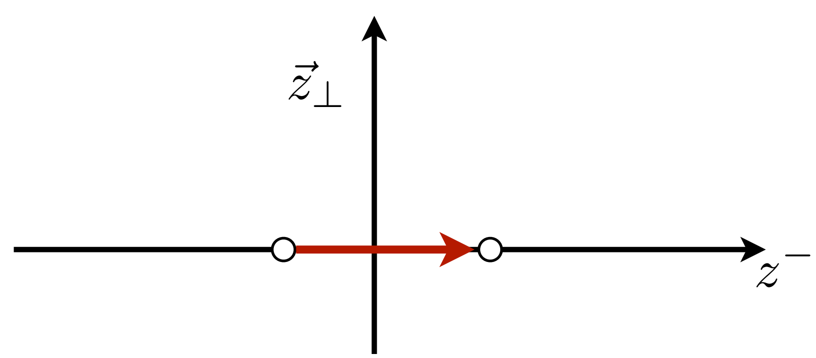
$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]} \psi\left(\frac{z}{2}\right) |p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right] = (1 + \ldots$$



exchange of more than 2 partons between hard scattering process (H) and soft amplitude (A) is suppressed except for gluons with polarization A^+

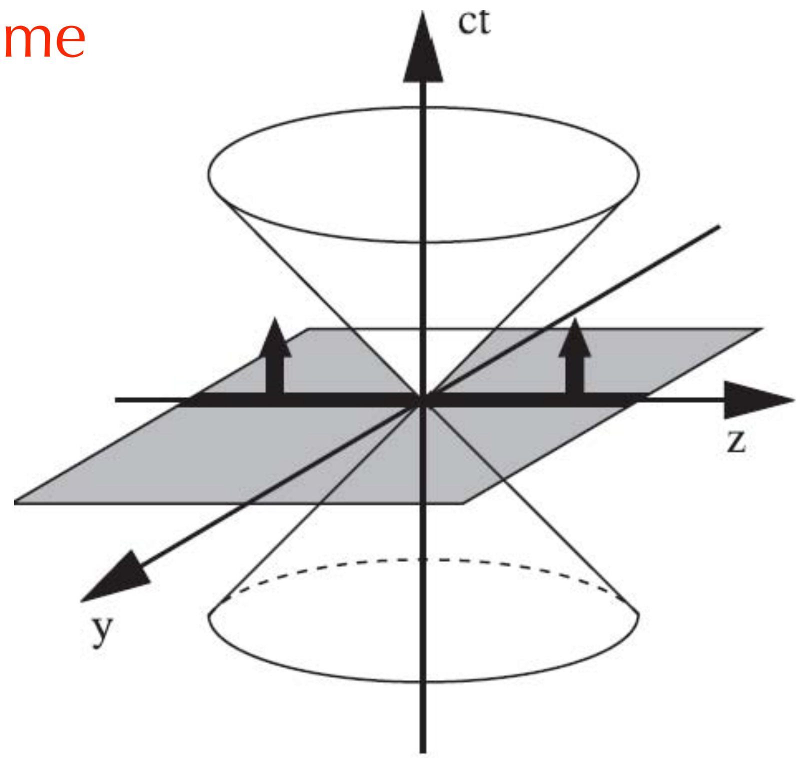
$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^- \ A^+(\eta)\right]_{z^+=0,z_\perp=0}$$



for convenience, choose light-cone gauge: $A^+ = 0$ in which $U = 1$

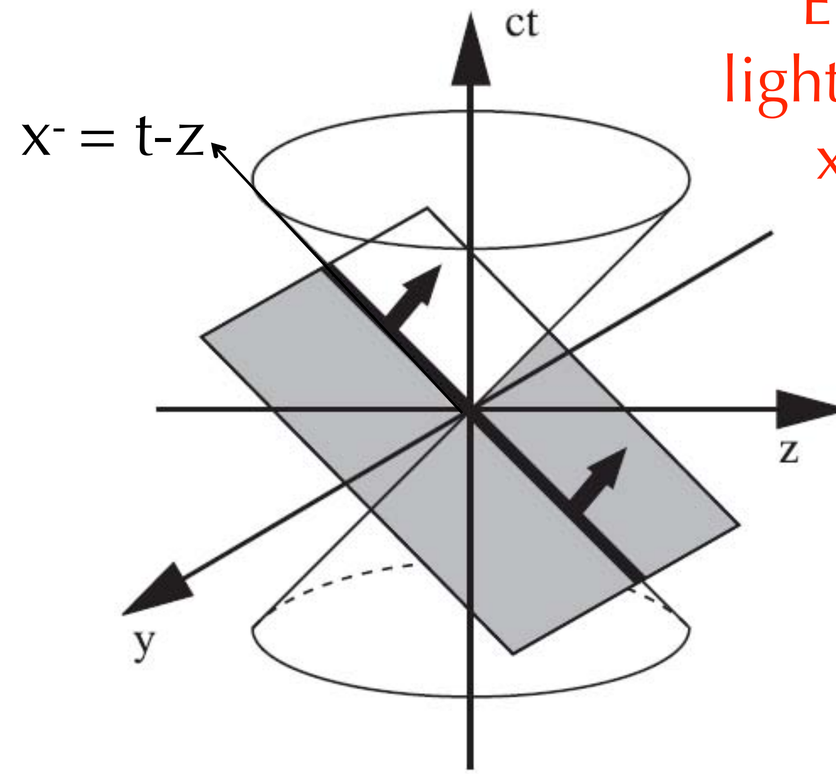
Partonic interpretation

Evolve in
ordinary time



Instant Form

Evolve in
light-front time
$x^+ = t+z$



Light-Front Form

coordinates

Instant Form		Light-Front Form	
x^0	time	$\dfrac{x^0 + x^3}{\sqrt{2}}$	time
x^1, x^2, x^3	space	$\dfrac{x^0 - x^3}{\sqrt{2}},\ x_\perp = (x^1, x^2)$	space

Hamiltonian

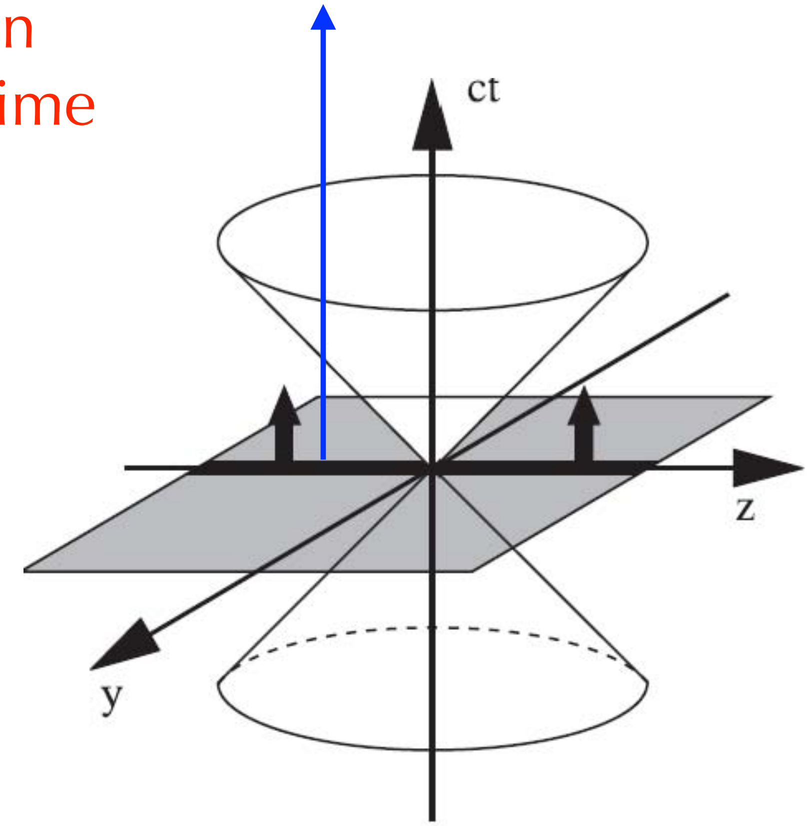
$$H = \sqrt{P^2 + M_0^2} + V \qquad\qquad P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

generators of Poincare′ group interaction independent

6 7

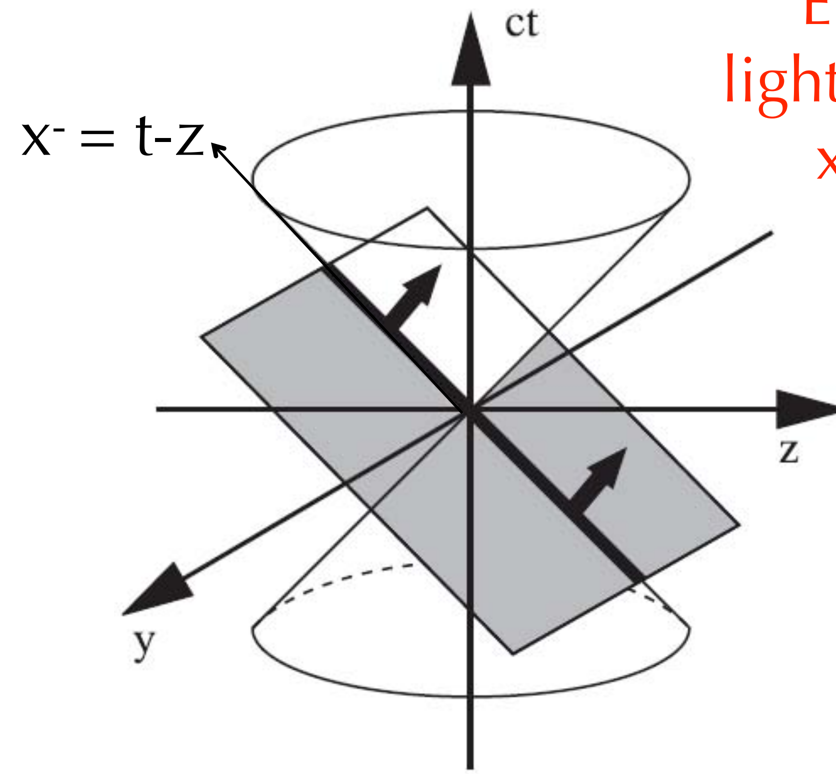
P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time



Instant Form

Evolve in light-front time $x^+ = t+z$



Light-Front Form

coordinates

x^0	time	$\dfrac{x^0 + x^3}{\sqrt{2}}$	time
$x^1,\, x^2,\, x^3$	space	$\dfrac{x^0 - x^3}{\sqrt{2}},\; x_\perp = (x^1,\, x^2)$	space

Hamiltonian

$$H = \sqrt{P^2 + M_0^2} + V \qquad\qquad P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

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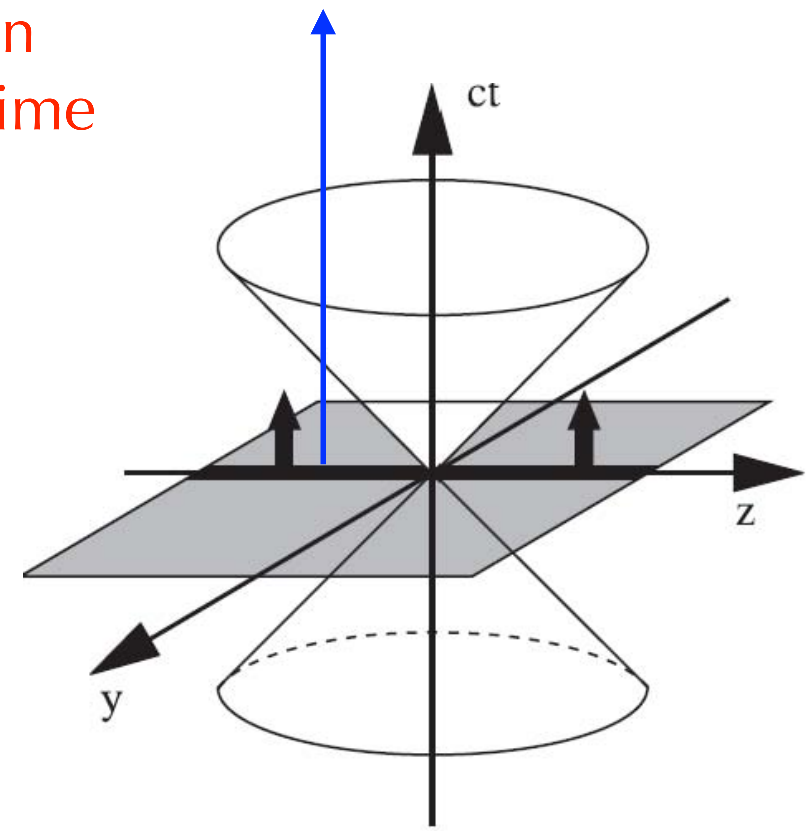
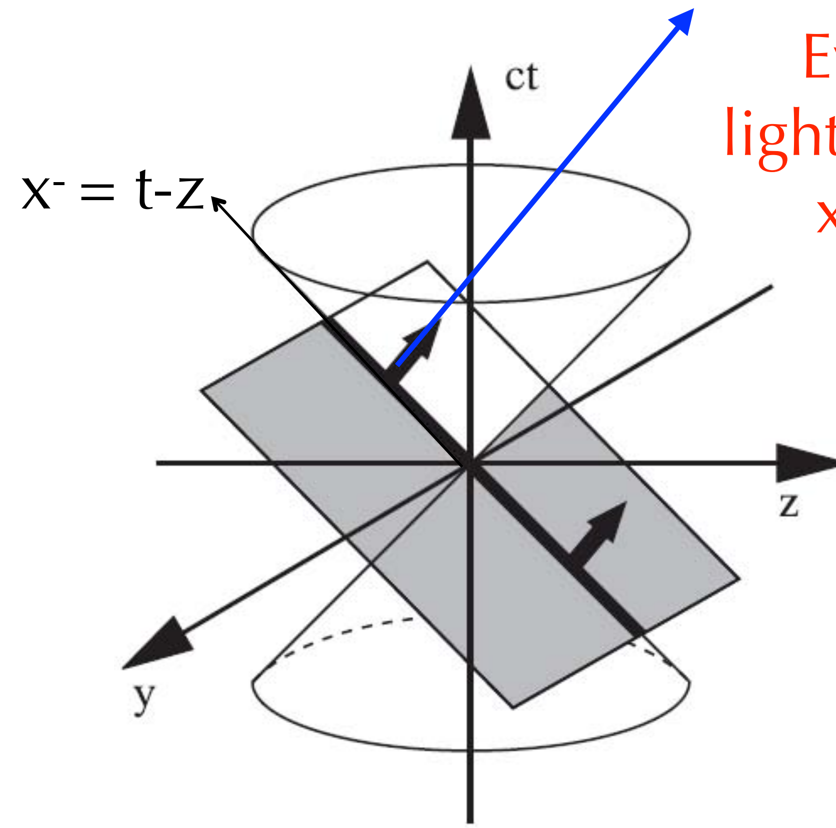


Instant Form



Evolve in
light-front time
$x^+ = t+z$

Light-Front Form

coordinates

x^0	time	$\dfrac{x^0 + x^3}{\sqrt{2}}$	time
$x^1,\, x^2,\, x^3$	space	$\dfrac{x^0 - x^3}{\sqrt{2}},\; x_\perp = (x^1,\, x^2)$	space

Hamiltonian

$$H = \sqrt{P^2 + M_0^2} + V \qquad\qquad P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

generators of Poincare′ group interaction independent

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P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Good and bad components

- Decompose the four-component fermion field in bad (-) and good (+) components

$$\psi = \psi^+ + \psi^- \qquad \text{with } \psi^+ = P_+\psi \text{ and } \psi^- = P_-\psi$$

- Properties of projector operators: $P_+ = \frac{1}{2}\gamma^-\gamma^+ \quad P_- = \frac{1}{2}\gamma^+\gamma^-$

$$P_+ + P_- = I \qquad (P_+)^2 = P_+ \quad (P_-)^2 = P_- \qquad P_+P_- = P_-P_+ = 0$$

- Projecting the Dirac equation and using the light-cone gauge $A^+ = 0$

$$i\gamma^-\frac{\partial}{\partial x^-}\psi_- = -\vec{\gamma}_\perp \cdot \vec{D}_\perp\psi_+ + m\psi_+ \qquad i\gamma^+ D_+\psi_+ = -\vec{\gamma}_\perp \cdot \vec{D}_\perp\psi_- + m\psi_-$$

constrained field　　　　independent dynamical degree of freedom

R. Jaffe, hep-ph/9602236

Light-Cone Dirac Spinor

$$u(k,\lambda=+1/2)=\frac{1}{\sqrt{2^{3/2}k^+}}\begin{pmatrix}\sqrt{2}k^+ + m\\ k_R\\ \sqrt{2}k^+ - m\\ k_R\end{pmatrix}\qquad P_+u(k,1/2)=u_+(k,1/2)=\frac{k^+}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

with $k_R = k^x + ik^y$

$$u(k,\lambda=-1/2)=\frac{1}{\sqrt{2^{3/2}k^+}}\begin{pmatrix}-k_L\\ \sqrt{2}k^+ + m\\ k_L\\ -\sqrt{2}k^+ + m\end{pmatrix}\qquad P_+u(k,-1/2)=u_+(k,-1/2)=\frac{k^+}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

with $k_L = k^x - ik^y$

Partonic interpretation of GPDs

- Unpolarized GPDs

$$\Phi^{[\gamma^+]}(\bar{x},\xi,t) = \langle P',\Lambda'| \int \frac{\mathrm{d}z^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\psi(\tfrac{z}{2})|P,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$\Longrightarrow$ $\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\psi(\tfrac{z}{2}) = \psi_+^\dagger(-\tfrac{z}{2})\psi_+(\tfrac{z}{2})$ good components of the quark fields

$$\Longrightarrow \quad \psi_+(z^-,\mathbf{z}_\perp) = \int \frac{\mathrm{d}k^+\mathrm{d}\mathbf{k}_\perp}{2k^+(2\pi)^3}\theta(k^+)\sum_\mu \big[b_q(w)u_+(w)\exp[-ik^+z^- + i\mathbf{k}_\perp\cdot\mathbf{z}_\perp]$$

$$+d_q^\dagger(w)v_+(w)\exp[ik^+z^- - i\mathbf{k}_\perp\cdot\mathbf{z}_\perp]\big]$$ with $w=(k^+,\mathbf{k}_\perp,\mu)$

$b_q,\, b_q^\dagger$ annihilation and creation operator of quark

$d_q,\, d_q^\dagger$ annihilation and creation operator of antiquark

$\Longrightarrow$ apply momentum conservation

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+>0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

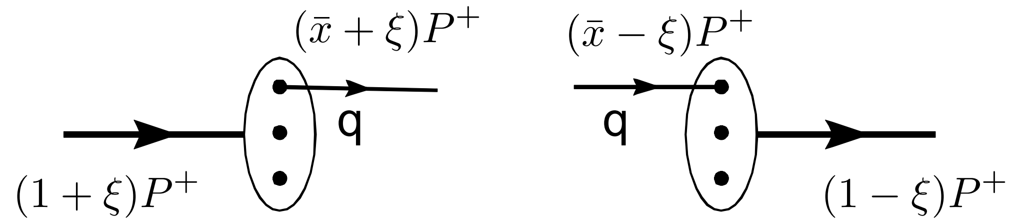
non-diagonal matrix elements of momentum-density matrix

⇩

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

DGLAP region $\xi \leq \bar{x} \leq 1$



$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+ \rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+ > 0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

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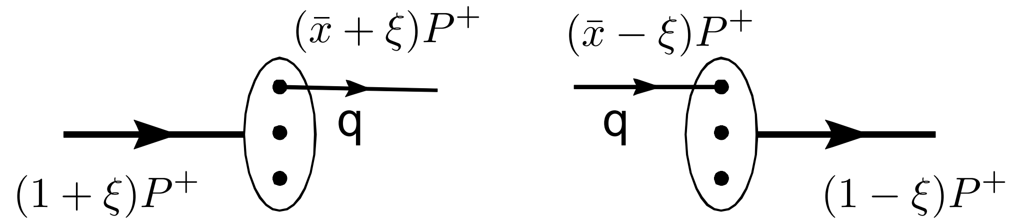
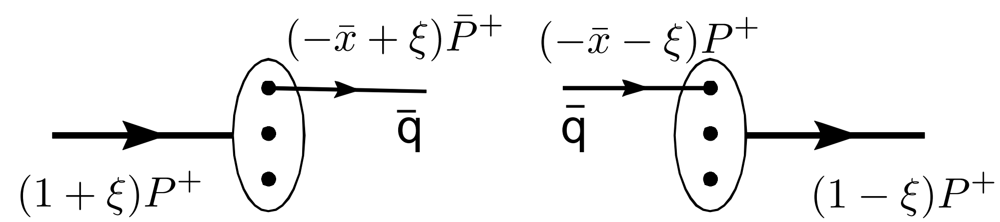


$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \le \bar{x} \le -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+ > 0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

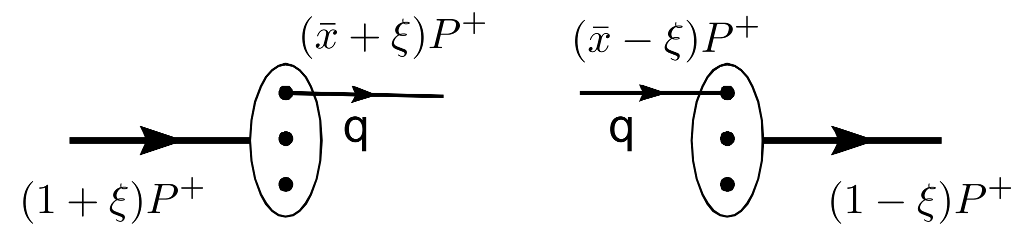
non-diagonal matrix elements of momentum-density matrix

⇩

we loose the probabilistic interpretation of the PDF

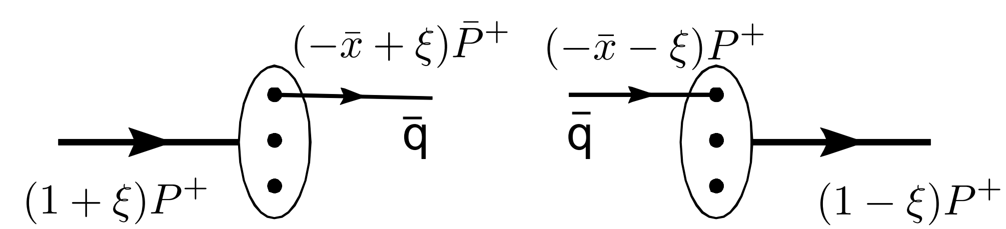
we gain information on the quark-momentum correlation

DGLAP region $\xi \le \bar{x} \le 1$



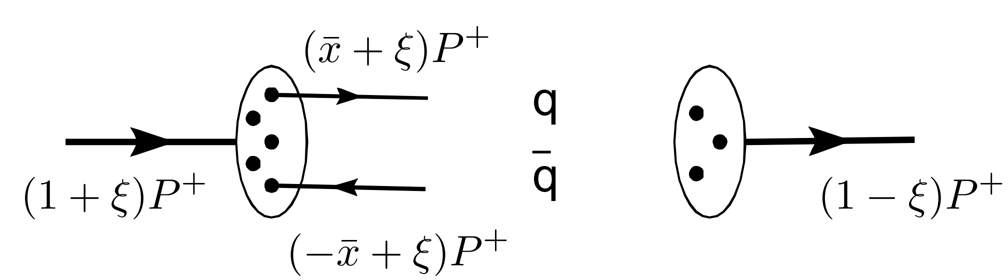
$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \le \bar{x} \le -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

ERBL region $-\xi \le \bar{x} \le \xi$



$$\langle N, (1-\xi)\bar{P}^+ | b_{\lambda'}[(\bar{x}+\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition k′+,k+>0 and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

non-diagonal matrix elements of momentum-density matrix

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

Spin projection

helicity space

$b_\uparrow^\dagger b_\uparrow + b_\downarrow^\dagger b_\downarrow$

$H^q,\ E^q$

$b_\uparrow^\dagger b_\uparrow - b_\downarrow^\dagger b_\downarrow$

$\tilde{H}^q,\ \tilde{E}^q$

transverse-spin space

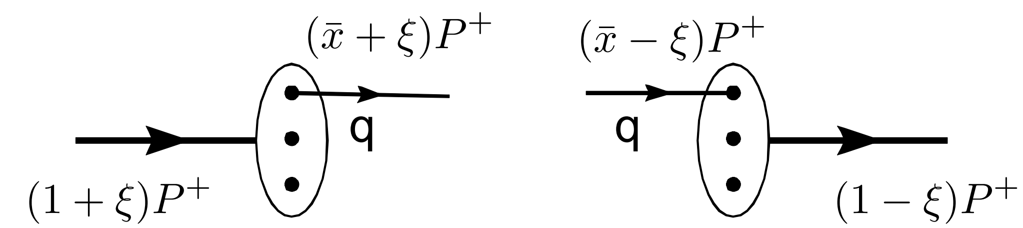
$b_\rightarrow^\dagger b_\rightarrow + b_\leftarrow^\dagger b_\leftarrow$

$H_T^q,\ E_T^q$

$b_\rightarrow^\dagger b_\leftarrow - b_\leftarrow^\dagger b_\rightarrow$

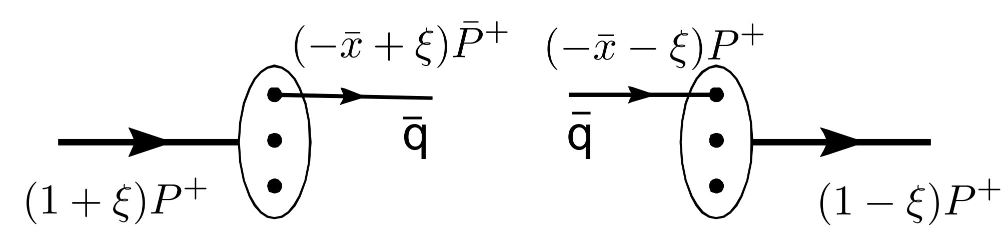
$\tilde{H}_T^q,\ \tilde{E}_T^q$

DGLAP region $\xi \le \bar{x} \le 1$



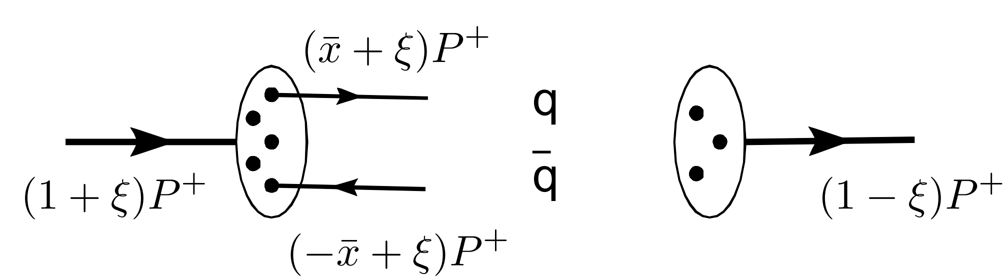
$$\langle N, (1-\xi)\bar{P}^+ | b_{\lambda'}^\dagger[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \le \bar{x} \le -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d_{\lambda'}^\dagger[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

ERBL region $-\xi \le \bar{x} \le \xi$



$$\langle N, (1-\xi)\bar{P}^+ | b_{\lambda'}[(\bar{x}+\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

Quark polarization

Nucleon pol.

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

ξ**-odd**

$$\mathcal{H} = \sqrt{1-\xi^2}\left(H - \frac{\xi^2}{1-\xi^2}E\right) \qquad \mathcal{E} = \frac{E}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}} = \sqrt{1-\xi^2}\left(\tilde{H} - \frac{\xi^2}{\sqrt{1-\xi^2}}\tilde{E}\right) \qquad \tilde{\mathcal{E}} = \frac{\xi\tilde{E}}{\sqrt{1-\xi^2}}$$

$$\mathcal{H}_T = \sqrt{1-\xi^2}\left(H_T - \frac{\vec{\Delta}_\perp^2}{2M^2}\frac{\tilde{H}_T}{\sqrt{1-\xi^2}} + \frac{\xi\tilde{\mathcal{E}}_T}{\sqrt{1-\xi^2}}\right) \qquad \mathcal{E}_T = \frac{2\tilde{H}_T + E_T - \xi\tilde{E}_T}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}}_T = -\frac{\tilde{H}_T}{2\sqrt{1-\xi^2}} \qquad \tilde{\mathcal{E}}_T = \frac{\tilde{E}_T - \xi E_T}{\sqrt{1-\xi^2}}$$

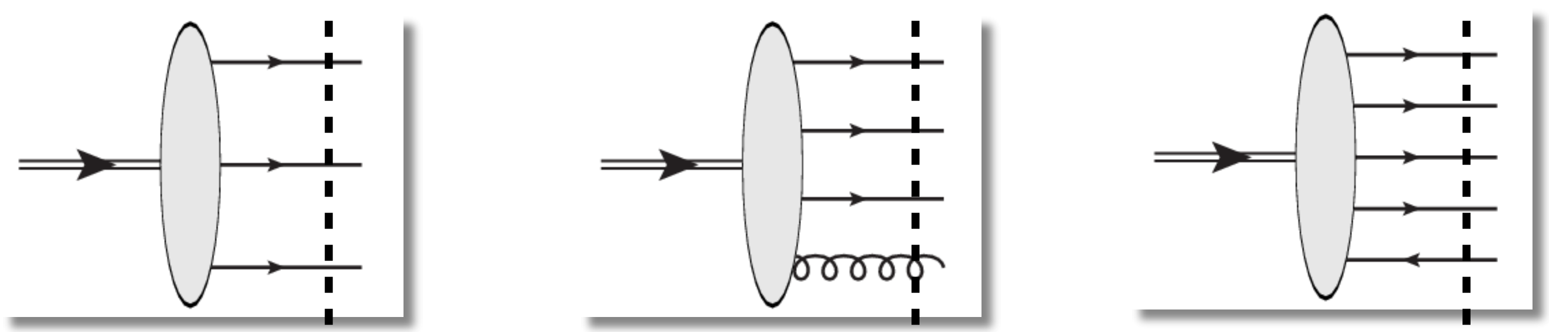
✦ Helicity structure: nucleon (Λ', Λ) quark (λ', λ)

$$U = (++) + (--) \quad T_x = (-+) + (+-) \quad T_y = i[(-+) - (+-)] \quad L = (++) - (--)$$

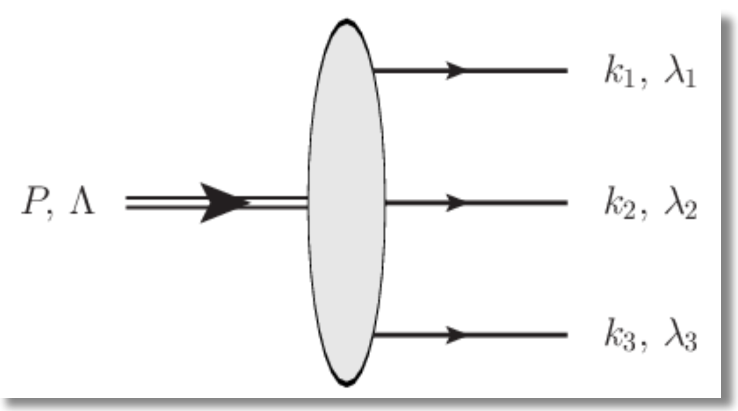
Light-cone Fock expansion

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots$$

Fixed light-cone time x^{+}=0



Fock states



Simultaneous eigenstates of

$$\left.\begin{aligned} P^{+} &= \textstyle\sum_i^N p_i^{+} \\ P_{\perp} &= \textstyle\sum_i^N p_{\perp i} \end{aligned}\right\} \quad \textbf{Momentum}$$

λ_i **Light-front helicity**

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Proton state

Probability Amplitude for the N,β Fock state

$$|(P^+,\vec{P}_\perp),\Lambda\rangle = \sum_{N,\beta}[dx]_N[d\vec{k}_\perp]_N\Psi^{\Lambda}_{N,\beta}(x_i,\vec{k}_{\perp i})|N,\beta;(x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp i}),\lambda_i\rangle$$

Light-front wave functions

Internal variables: $x_i = \frac{p_i^+}{P^+}$ $\sum_{i=1}^{N} x_i = 1$ $\sum_{i=1}^{N}\vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz}\Psi^{\Lambda}_{\lambda_1\dots\lambda_N} = \lambda_i\Psi^{\Lambda}_{\lambda_1\dots\lambda_N}$$

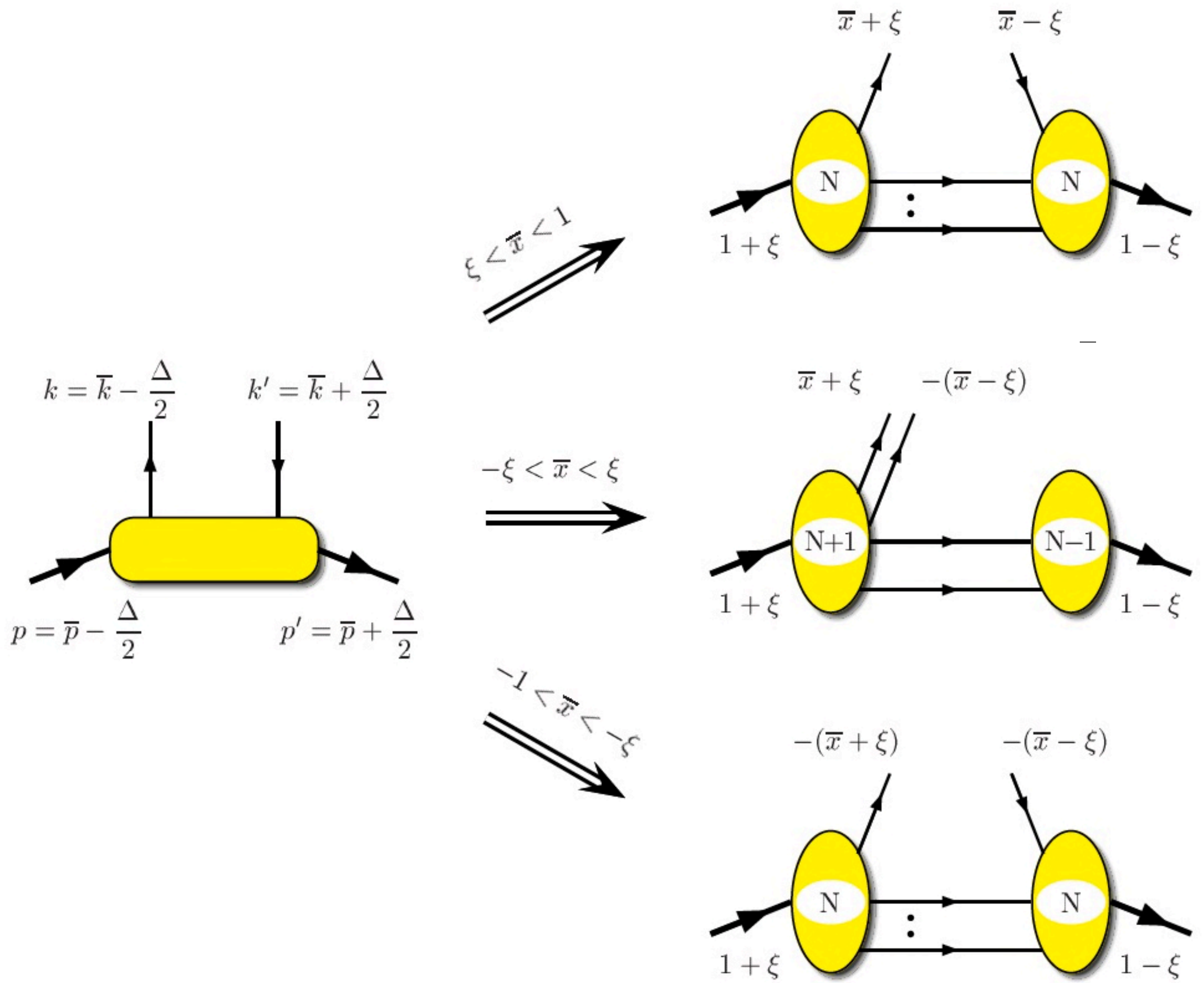
Eigenstates of total OAM

$$\hat{L}_z\Psi^{\Lambda}_{\lambda_1\dots\lambda_N} = \ell_z\Psi^{\Lambda}_{\lambda_1\dots\lambda_N}$$

$$\boxed{\Lambda = \sum_{i=1}^{N}\lambda_i + \ell_z}$$

$A^+ = 0$ **gauge**

Light-Front Wave Function Overlap Representation



DGLAP region

ERBL region

DGLAP region

$$\text{GPDs} \sim \sum_N \int [d^3k]_N\, \Psi^*_N(k'_N)\Psi_N(k_N)\delta(\dots)$$ interference of probability amplitudes

$$\text{PDFs} \sim \sum_N \int [d^3k]_N\, |\Psi_N(k_N)|^2\delta(\dots)$$ probability density

Diehl, Feldmann, Jakob, Kroll, NPB596, 2001
Diehl, Hwang, Brodsky, NPB596, 2001
Boffi, Pasquini, NPB649, 2003

Properties of GPDs

- Forward limit: ordinary parton distributions

$H^q(x, \xi = 0, t = 0) = q(x)$ unpolarized quark distributions

$\widetilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$ long. polarized quark distributions

$H_T^q(x, \xi = 0, t = 0) = h_1(x)$ transv. polarized quark distributions

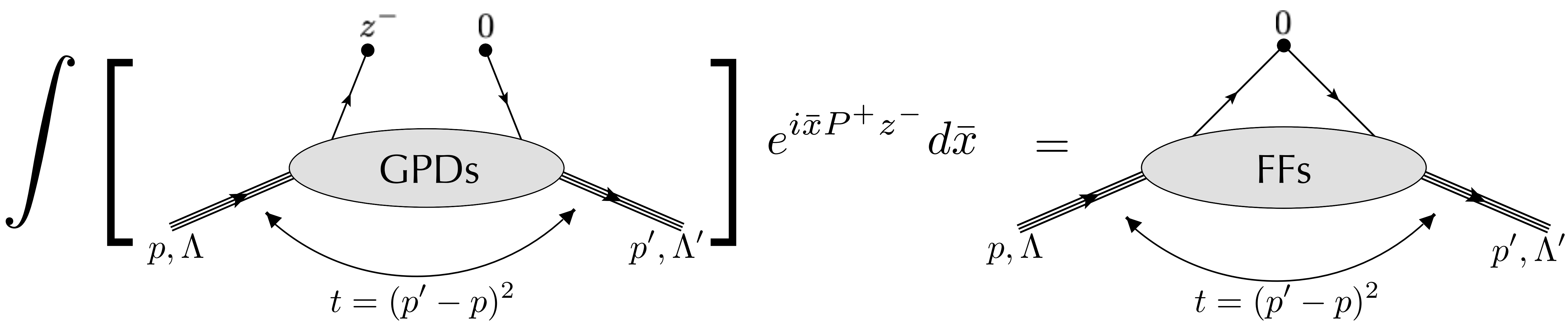
$x > 0$: quarks $x < 0$: antiquarks

analogous relations for gluons, except for transversity distribution

- all the other GPDs do NOT appear in inclusive DIS $\Longrightarrow$ new information

- They all depend on the renormalisation scale $(\mu^2 = Q^2)$
with different evolution equations in the DGLAP and ERBL regions

Properties of GPDs



$$\int_{-1}^{1} d\bar{x}\, H^q(\bar{x}, \xi, t) = F_1^q(t)$$ Dirac Form Factor

$$\int_{-1}^{1} d\bar{x}\, E^q(\bar{x}, \xi, t) = F_2^q(t)$$ Pauli Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{H}^q(\bar{x}, \xi, t) = G_A^q(t)$$ Axial Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{E}^q(\bar{x}, \xi, t) = G_P^q(t)$$ Pseudoscalar Form Factor

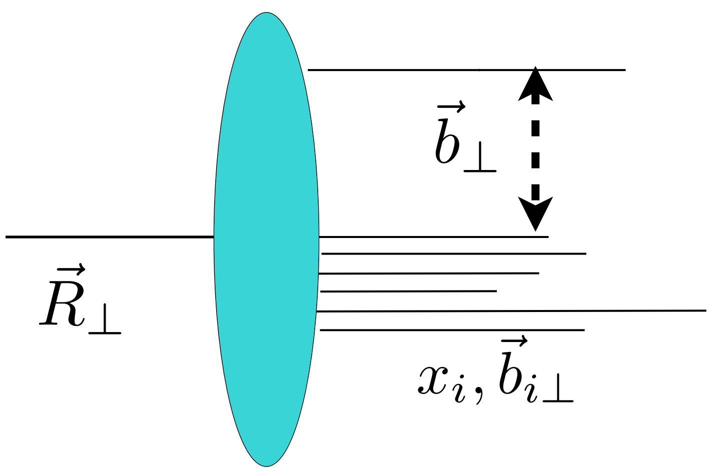
- matrix elements of local operators

$\longrightarrow$ can be calculated on the lattice

- renormalisation scale independent

- ξ independence: Lorentz invariance

Impact Parameter Space



- Center of momentum of the partons in the proton

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

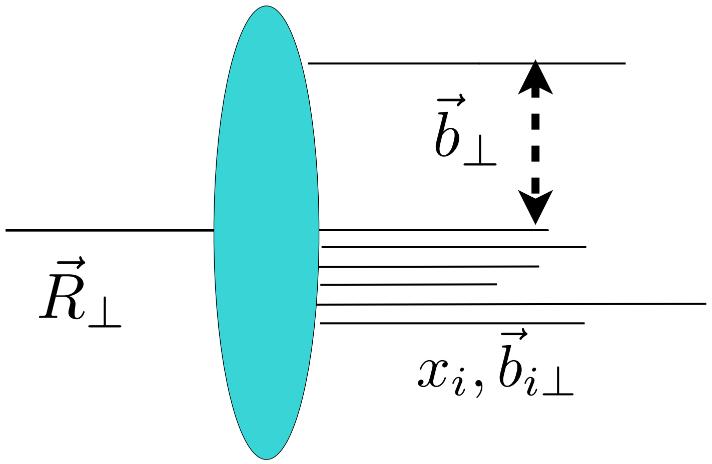
- $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

Isomorphism between Galilei and subgroup of Light-Front operators

Galilei transformation:	Transverse boost:
$m_i \to m_i$ $\quad$ $\vec{p}_i \to \vec{p}_i - m_i \vec{v}$	$p_i^+ \to p_i^+$ $\quad$ $\vec{p}_{\perp i} \to \vec{p}_{\perp i} - p_i^+ \vec{v}$
Center of mass:	Center of plus momentum:
$\vec{r}_* = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$	$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+}$

Impact Parameter Space



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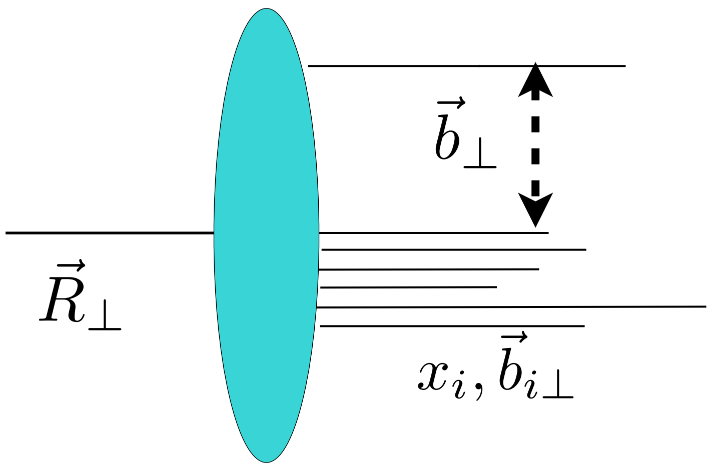
Impact Parameter Space



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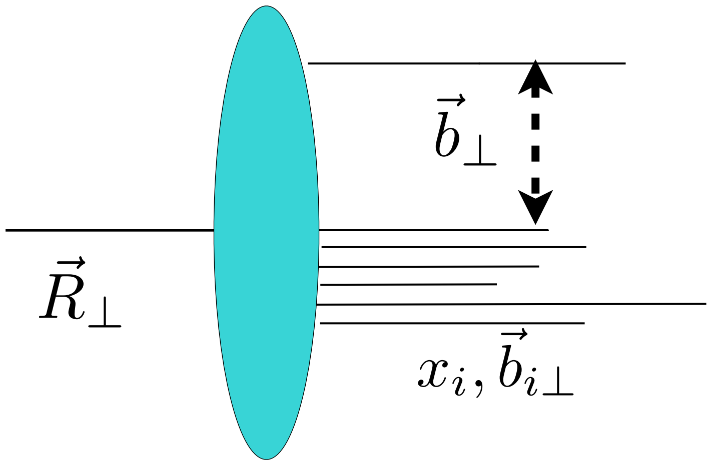
➢ $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

- Localized wave packet in the transverse plane polarized in the X direction in IMF

$$|p^+, S_x\rangle \equiv \frac{1}{\sqrt{2}} \left(|p^+, \vec{R}_\perp = \vec{0}_\perp, \uparrow\rangle + |p^+, \vec{R}_\perp = \vec{0}_\perp, \downarrow\rangle \right)$$

Impact Parameter Space



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- Impact parameter dependent GPD for the $\perp$ pol. state

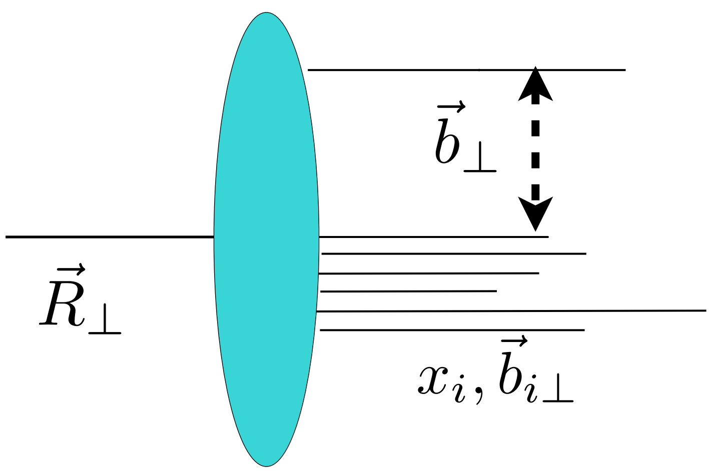
$\Longrightarrow$ quark density in proton state $\perp$ pol.

$$q_x(x, \vec{b}_\perp) = \langle p^+, S_x| \int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}(-\frac{x^-}{2}, \vec{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \vec{b}_\perp)|p^+, S_x\rangle$$

$$q_x(x, \vec{b}_\perp) = H^q(x, \vec{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y} E^q(x, \vec{b}_\perp)$$

$$H^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \qquad E^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} E^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}$$

Impact Parameter Space



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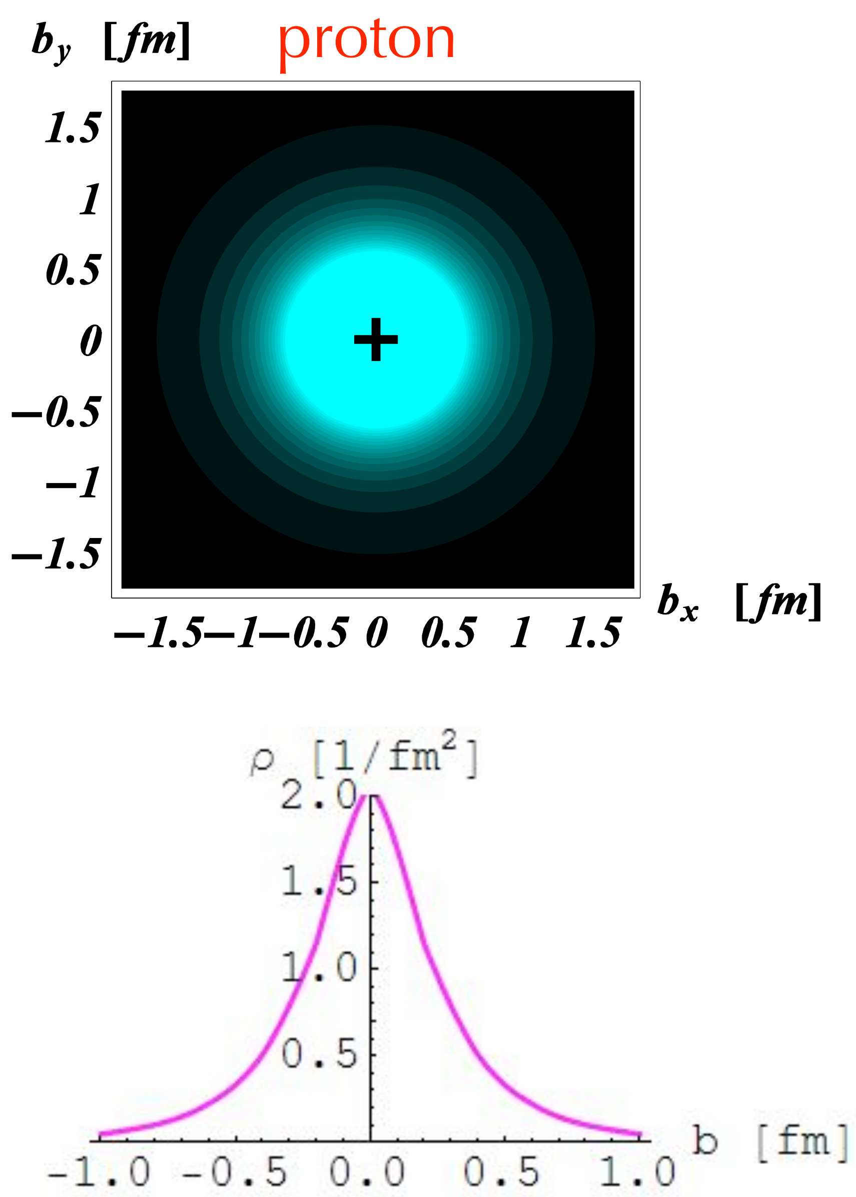
❖ Homework: derive the relation between GPDs and IPDs

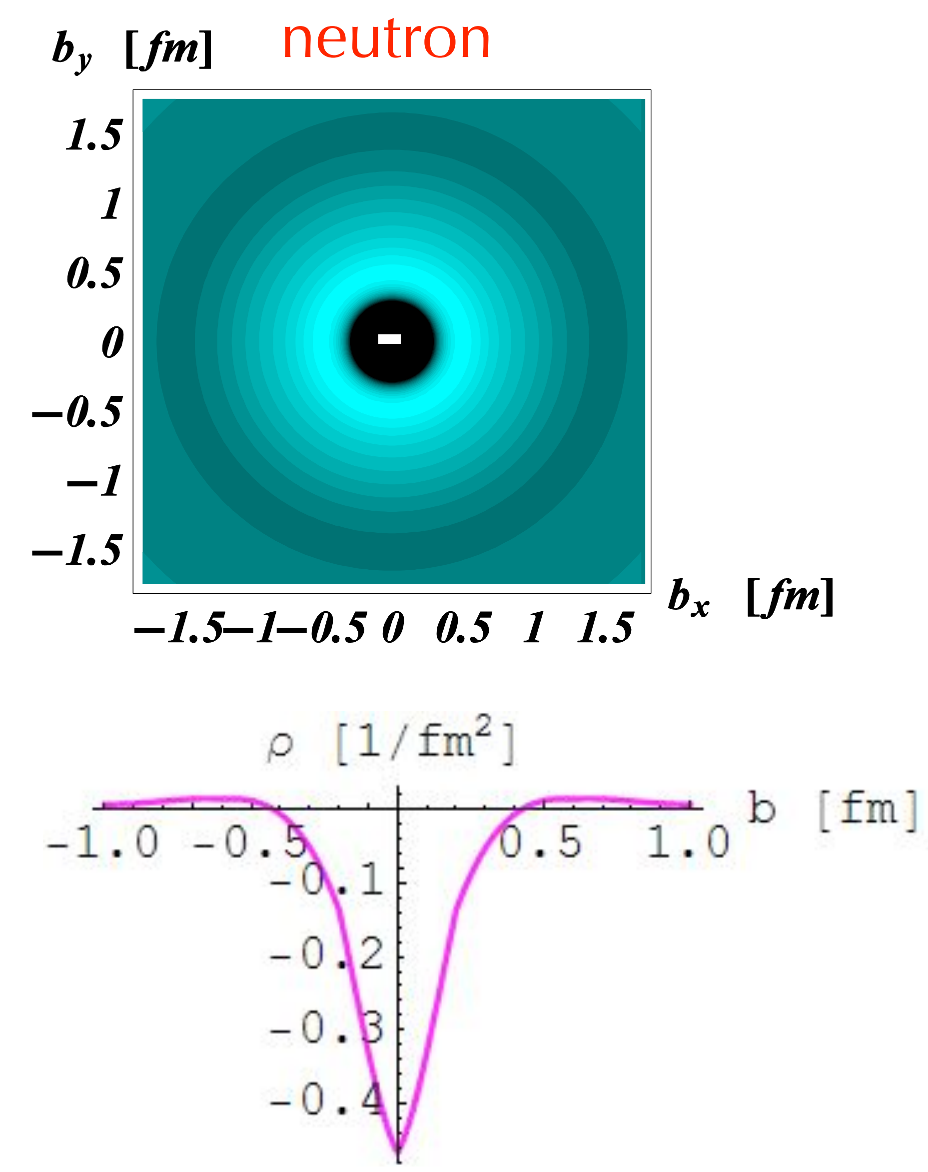
Charge density of partons in the transverse plane

Number density of quark with longitudinal momentum x and transverse position $b_\perp$

$$\rho^q(b_\perp) = e_q \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} \int dx H^q(x, 0, \Delta_\perp^2) = \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} F_1^q(\Delta_\perp^2)$$

Infinite-Momentum-Frame Parton charge density in the transverse plane



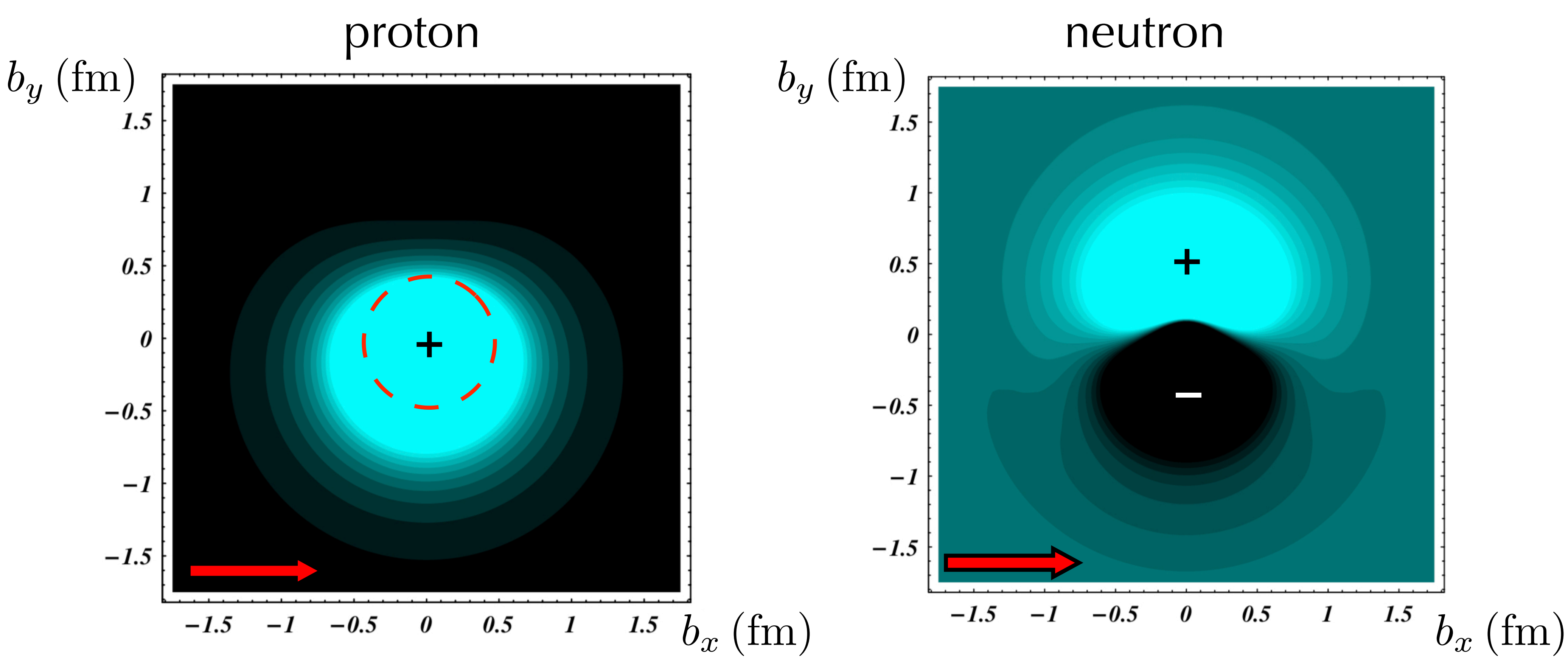


Miller (2007); Burkardt (2007)

Electromagnetic Form Factors

Transversely polarized proton

$$\rho_T(\vec{b}_\perp) = \underbrace{\rho(\vec{b}_\perp)}_{\text{monopole}} + \sin(\phi_b - \phi_s) \int \frac{\mathrm{d}Q}{2\pi} \underbrace{\frac{Q^2}{2M}}_{\text{dipole}} J_1(Qb_\perp) F_2(Q^2)$$



nucleon polarized in the x direction

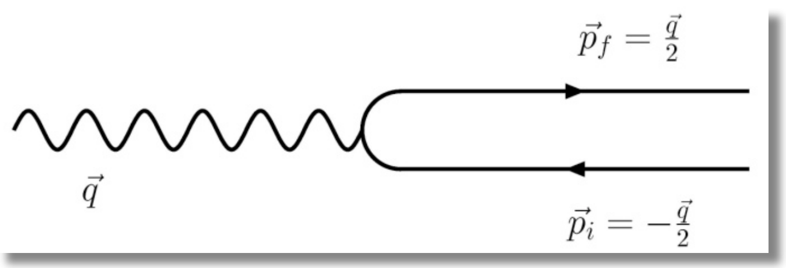
C. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. 100 (2008) 032004

Textbook interpretation

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Breit frame



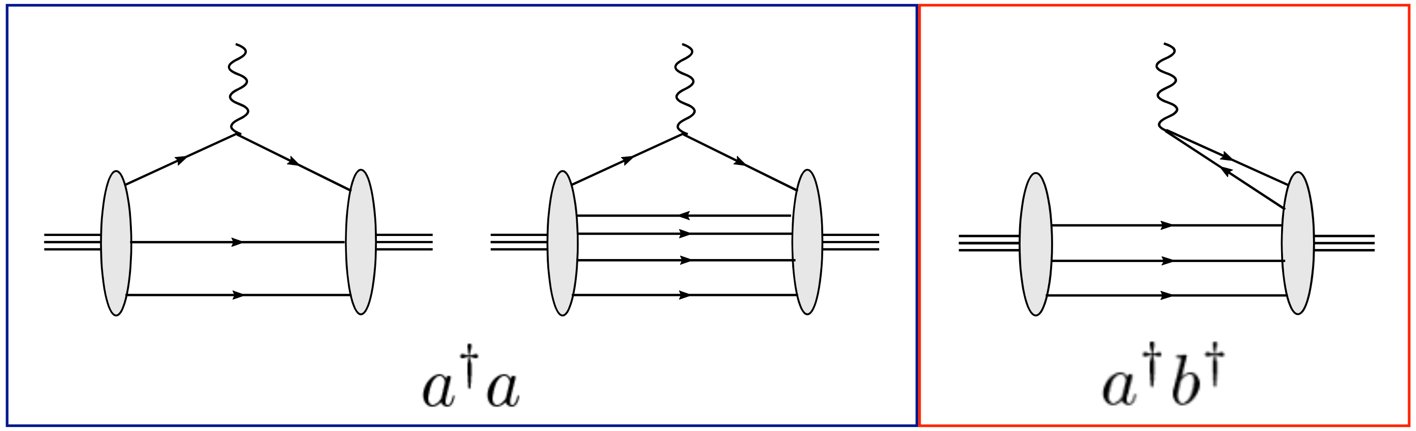
Spatial charge density

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}} G_E(Q^2)$$

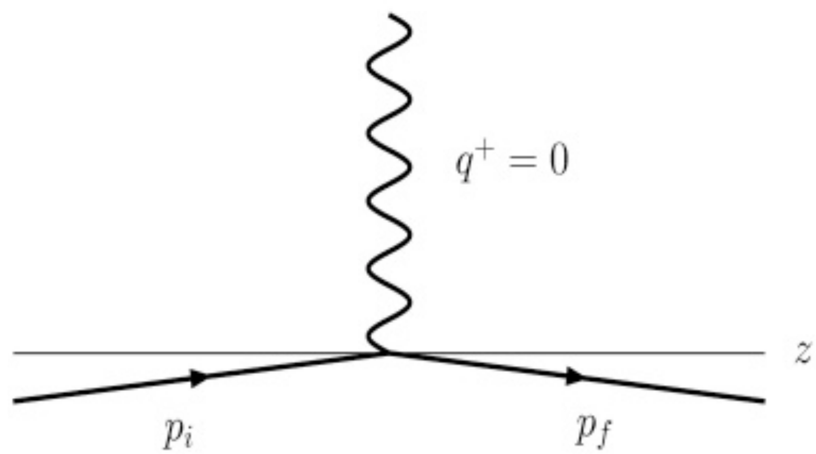
[Ernst, Sachs, Wali (1960)]
[Sachs (1962)]

No probabilistic/charge interpretation

Creation/annihilation of pairs



Drell-Yan-West frame



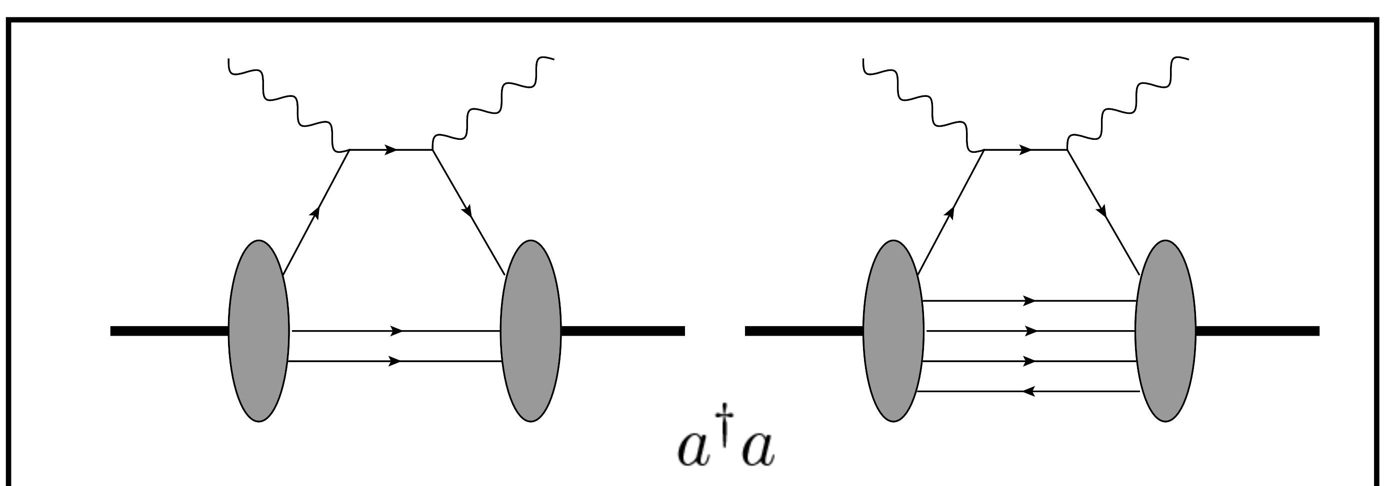
✓ $q^+ = 0$ ⟶ no sensitivity to longitudinal Lorentz contraction

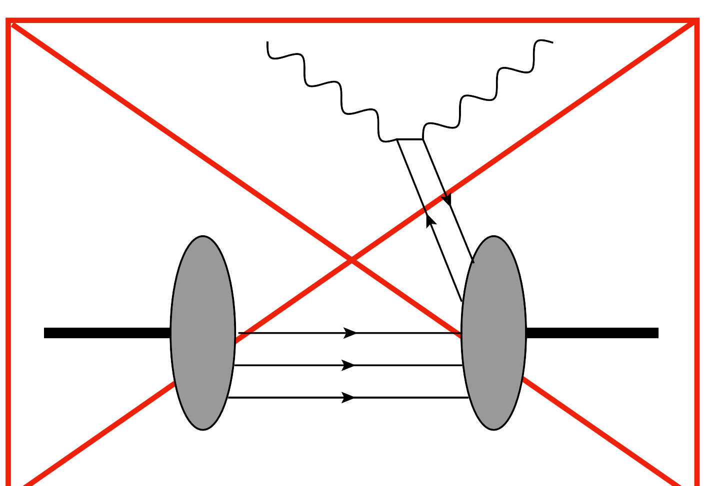
✓ $\vec{q}_\perp \neq 0$: Transverse boosts ⟶ no transverse Lorentz contraction

✓ Particle number is conserved in Drell-Yan frame $\Delta^+ = 0$

$$\rho(\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \frac{e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}}{2P^+} J^+(\vec{\Delta}_\perp)$$

probabilistic/charge interpretation





Operator	Forward matrix element	Non-forward matrix element	Position-space interpretation
$e_q \bar{\psi}_q(0)\gamma^+\psi_q(0)$	Q	$F_1(t)$	$\rho(b_\perp^2)$
$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x}\bar{\psi}_q(-\frac{z}{2})\gamma^+\psi_q(\frac{z}{2})$	$q(x)$	$H_q(x,0,t)$	$q(x,b_\perp^2)$

$\rho(b_\perp^2)$ 2-dim distribution of charge in the transverse plane

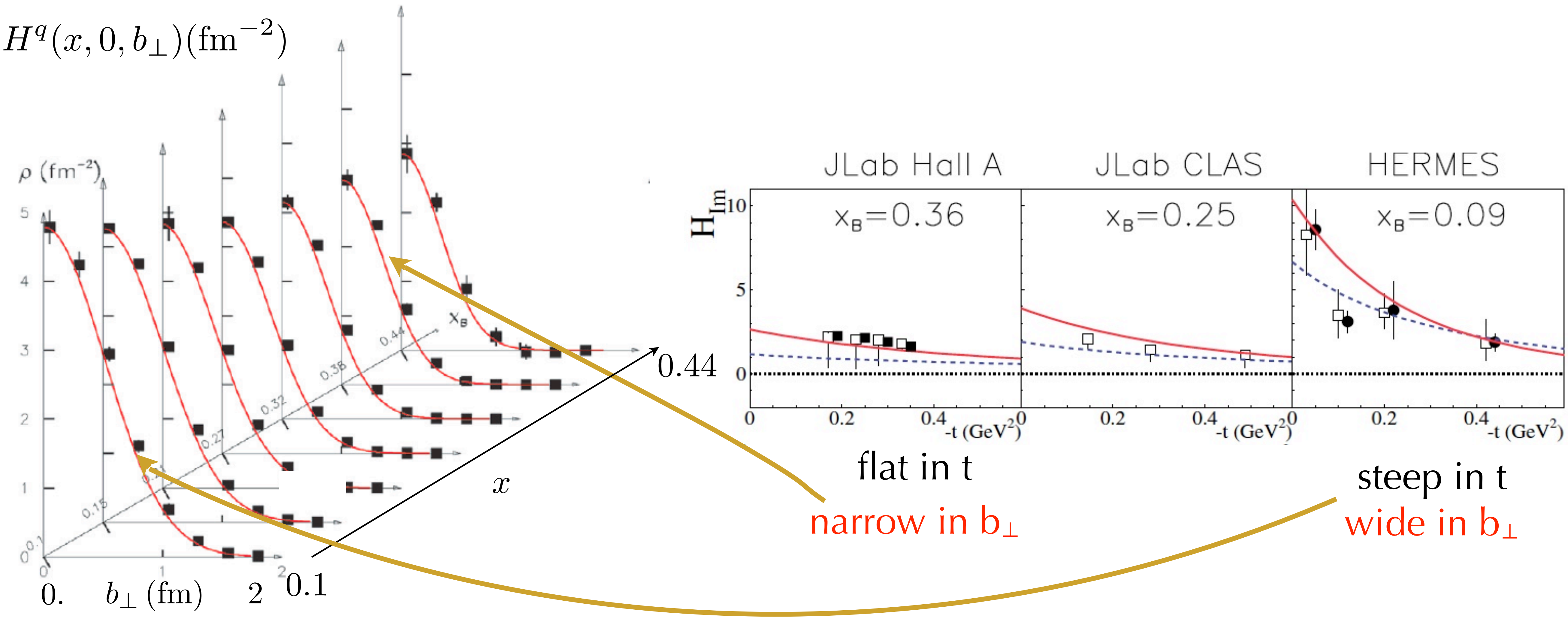
$q(x,b_\perp^2)$ 2-dim. "distribution of the PDF" in the transverse plane

M. Burkardt, Int. J. Mod. Phys. A18 (2003) 173

The unpolarized GPD H

$$H(x,0,\vec{b}_\perp) = \int \mathrm{d}^2\Delta_\perp \, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

extrapolation from data

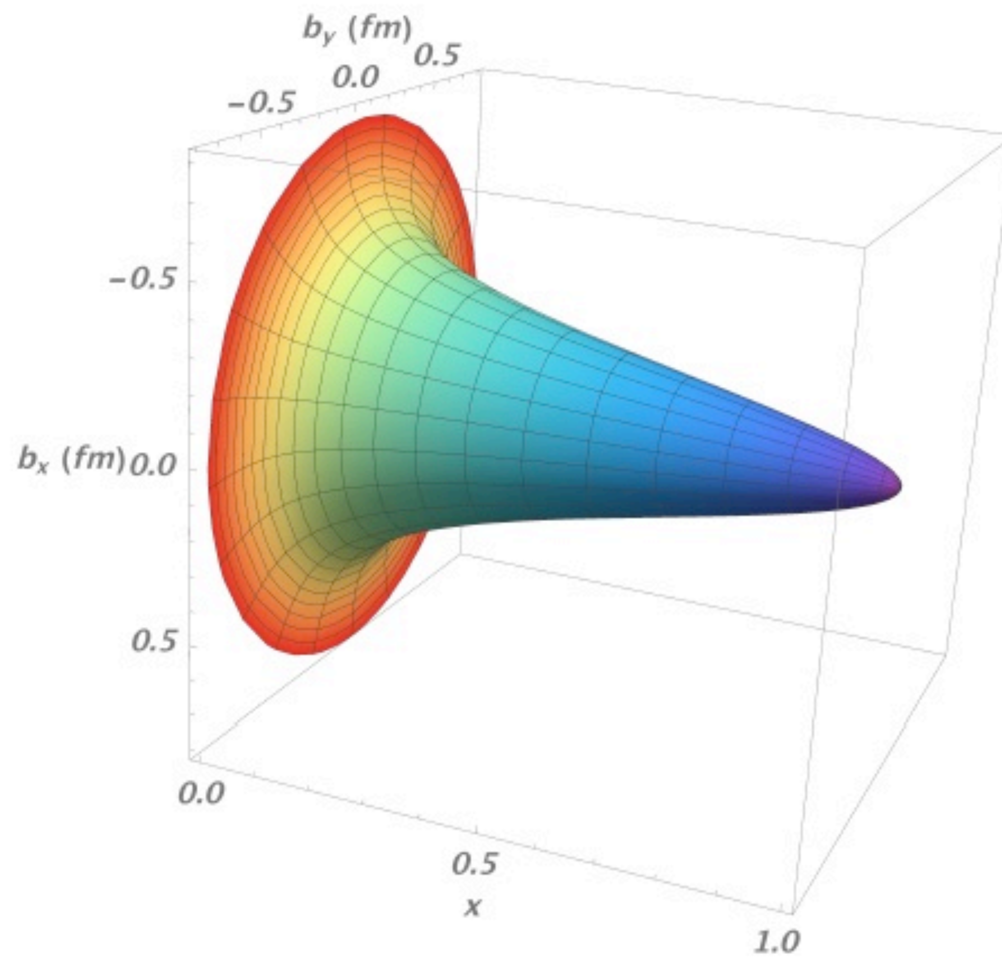


Guidal et al., Rep. Prog. Phys. ***76*** *(2013) 066202*

The unpolarized GPD H

$$H(x,0,\vec{b}_\perp) = \int \mathrm{d}^2\Delta_\perp \, \underbrace{H(x,0,t)}_{\text{extrapolation from data}} \, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

$$\langle \vec{b}_\perp^2(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp \, \vec{b}_\perp^2 \, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

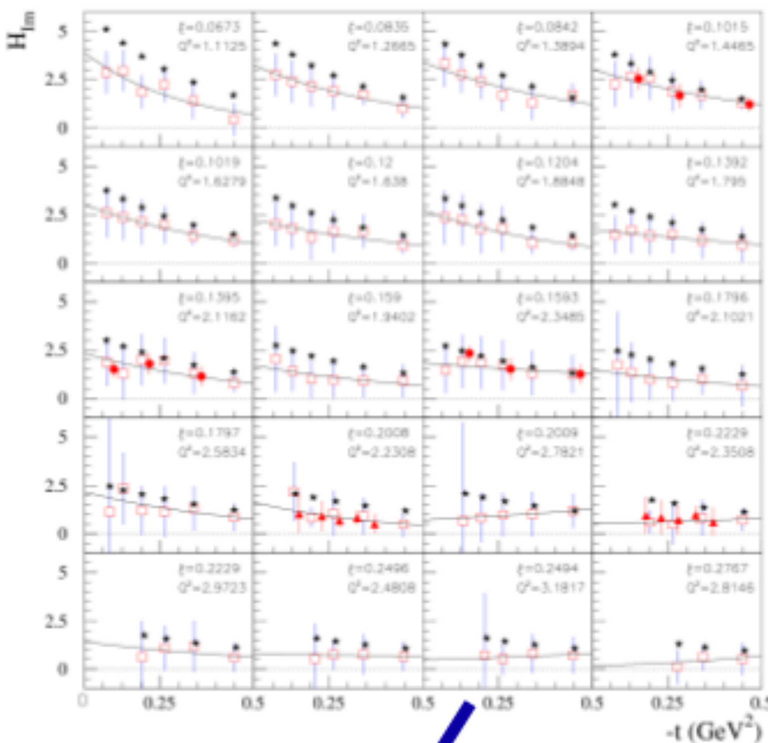


As x ⟶ 1, the active parton carries all the momentum and represents the centre of momentum

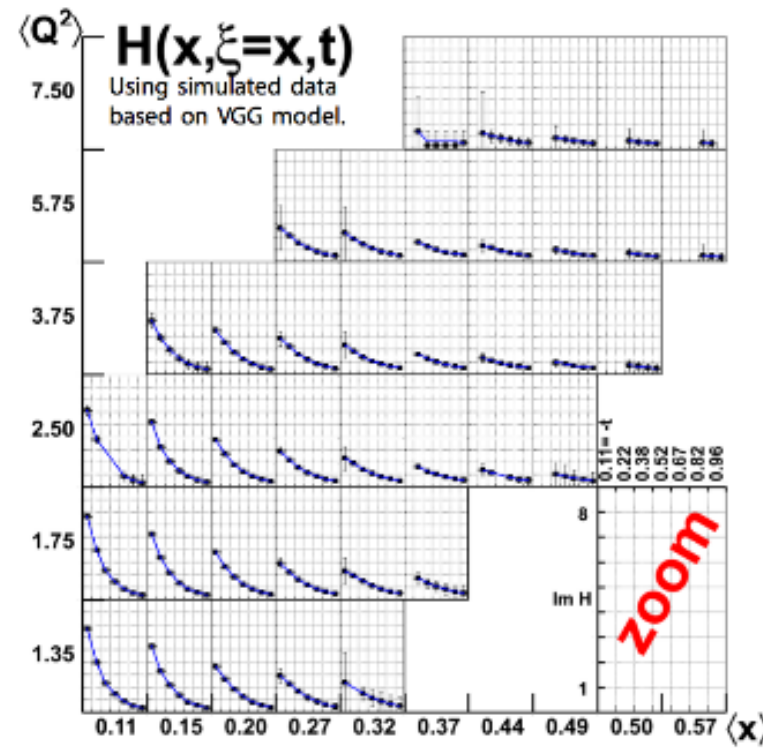
Dupré et al., PRD95(2017)011501

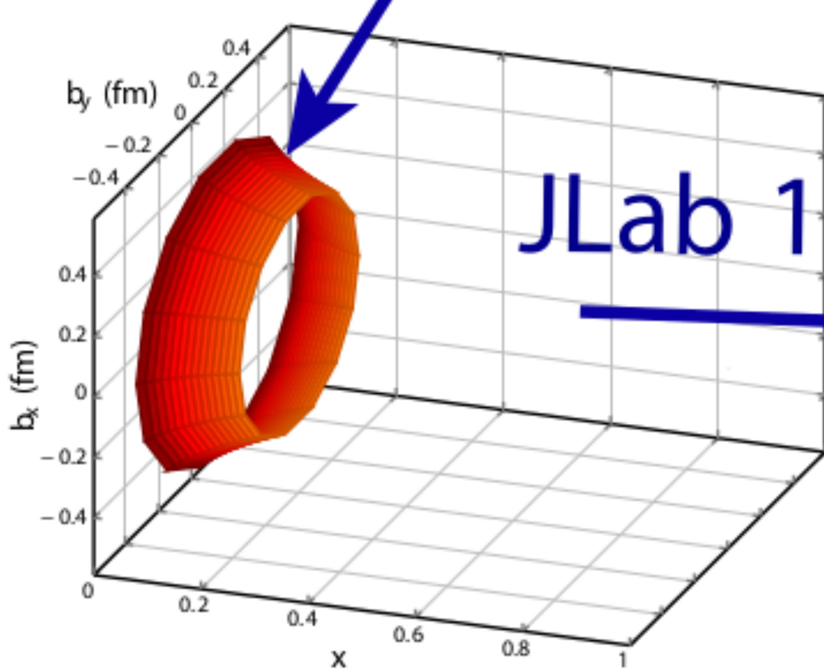
The unpolarized GPD H

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)

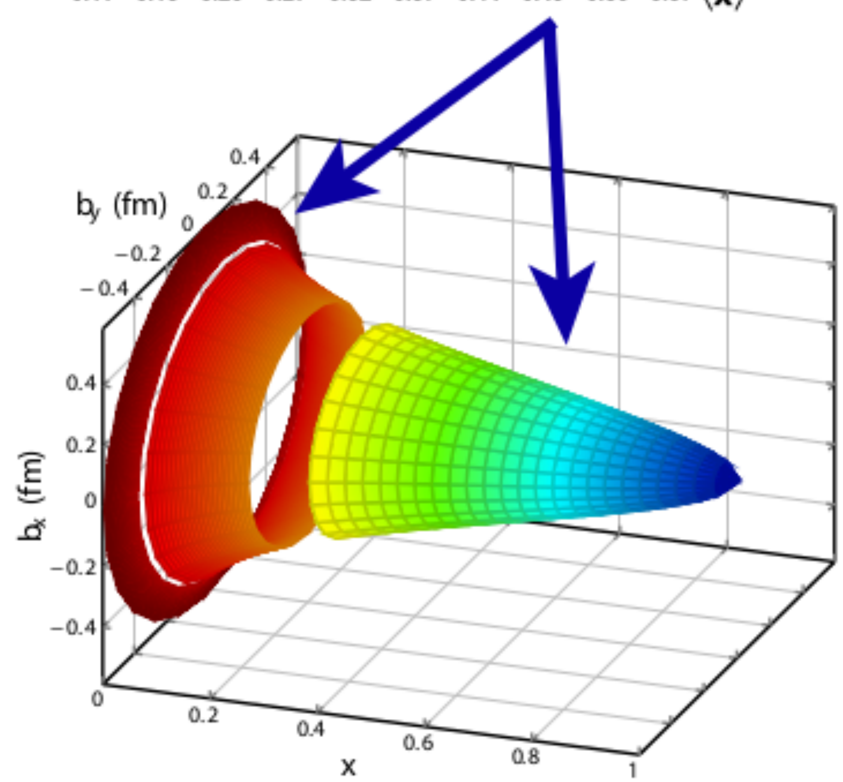


CLAS12 projections E12-06-119 with DVCS A_{UL} and A_{LU}





JLab 12 GeV



Courtesy of R. Dupré, M. Vanderhaeghen and M. Guidal

Energy-momentum tensor and GPDS

The Energy-Momentum Tensor

Energy Density Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

shear forces

pressure

Energy Flux Momentum Flux

$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle$$

- Where does the spin of the proton come from?
- What are the mechanical properties (pressure, shear forces) inside the proton ?
- What is the origin of the proton mass?

Canonical Energy Momentum Tensor

Emmy Noether (1882-1935)

If a system has a continuous symmetry property,
then there are corresponding quantities whose values are conserved in time

Translation invariance $\longrightarrow$ Conservation of the canonical EMT $T_C^{\mu\nu}(x)$

Lorentz invariance $\longrightarrow$ Conservation of the generalized Angular Momentum (AM) density $J_C^{\mu\alpha\beta}(x)$

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta} + S_C^{\mu\alpha\beta} \qquad L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

Space components: $J_C^i(x) = \frac{1}{2}\epsilon^{ijk} J_C^{0jk}(x)$

$$\vec{J}_C = \underbrace{\vec{L}_C}_{\text{Orbital AM}} + \underbrace{\vec{S}_C}_{\text{Spin}}$$

$T_C^{\mu\nu}$ is in general neither gauge-invariant nor symmetric

Belinfante improved EMT

$$T^{\mu\nu}_{\text{Bel}}(x) = T^{\mu\nu}_C(x) + \partial_\lambda G^{\lambda\mu\nu}(x)$$

Belinfante generalized AM

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = J^{\mu\alpha\beta}_C(x) + \partial_\lambda[x^\alpha G^{\lambda\mu\beta}(x) - x^\beta G^{\lambda\mu\alpha}(x)]$$

with the super-potential

$$G^{\lambda\mu\nu}(x) = \frac{1}{2}[S^{\lambda\mu\nu}_C(x) - S^{\mu\nu\lambda}_C(x) - S^{\nu\mu\lambda}_C(x)] = -G^{\mu\lambda\nu}(x)$$

$$\Downarrow$$

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = x^\alpha T^{\mu\beta}_{\text{Bel}}(x) - x^\beta T^{\mu\alpha}_{\text{Bel}}(x)$$

Belinfante, Rosenfeld (1940)

Canonical ⟷ Belinfante

Canonical ⟷ Belinfante

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

symmetric

$$T_{\mathrm{Bel}}^{[\mu\nu]}(x) = 0$$

Canonical ⟷ Belinfante

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

symmetric

$$T_{\text{Bel}}^{[\mu\nu]}(x) = 0$$

clear distinction between OAM and spin at the density level

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta}(x) + S_C^{\mu\alpha\beta}(x)$$

$$L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

purely OAM density

$$J_{\text{Bel}}^{\mu\alpha\beta}(x) = x^\alpha T_{\text{Bel}}^{\mu\beta}(x) - x^\beta T_{\text{Bel}}^{\mu\alpha}(x)$$

Canonical $\longleftrightarrow$ Belinfante

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

symmetric

$$T_{\text{Bel}}^{[\mu\nu]}(x) = 0$$

clear distinction between OAM and spin
at the density level

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta}(x) + S_C^{\mu\alpha\beta}(x)$$

$$L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

purely OAM density

$$J_{\text{Bel}}^{\mu\alpha\beta}(x) = x^\alpha T_{\text{Bel}}^{\mu\beta}(x) - x^\beta T_{\text{Bel}}^{\mu\alpha}(x)$$

The total charge does not change:

$$\int T_C^{0\nu}\, d^3x = \int T_{\text{Bel}}^{0\nu}\, d^3x$$

$$\int J_C^{0\alpha\beta}\, d^3x = \int J_{\text{Bel}}^{0\alpha\beta}\, d^3x$$

Kinetic EMT in QCD

Ji, 1997

$$T^{\mu\nu}_{\rm kin}(x) = T^{\mu\nu}_{{\rm kin},q}(x) + T^{\mu\nu}_{{\rm kin},g}$$

Quark contribution: $T^{\mu\nu}_{{\rm kin},q}(x) = \frac{1}{2}\bar{\psi}(x)\gamma^\mu i\overleftrightarrow{D}^\nu\psi(x)$ $\quad (D^\mu = \partial^\mu + igA^\mu)$

$$\frac{1}{2}T^{\{\mu\nu\}}_{{\rm kin},q}(x) = T^{\mu\nu}_{{\rm Bel},q}(x)$$

$$\frac{1}{2}T^{[\mu\nu]}_{{\rm kin},q}(x) = -\partial_\lambda S^{\lambda\mu\nu}_q(x)$$

$$S^{\lambda\mu\nu}_q(x) = \frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}(x)\gamma_\alpha\gamma_5\psi(x)$$

Gluon contribution: $T^{\mu\nu}_{{\rm kin},g}(x) = T^{\mu\nu}_{{\rm Bel},g}(x)$

$$T^{\mu\nu}_{{\rm kin},g} = -2\,{\rm Tr}[F^{\mu\lambda}(x)F^\nu_{\ \lambda}(x)] + \frac{1}{2}g^{\mu\nu}\,{\rm Tr}[F^{\rho\sigma}(x)F_{\rho\sigma}(x)]$$

Kinetic generalized AM

$$J^{\mu\alpha\beta}_{{\rm kin},q}(x) = L^{\mu\alpha\beta}_{{\rm kin},q}(x) + S^{\mu\alpha\beta}_q(x)$$

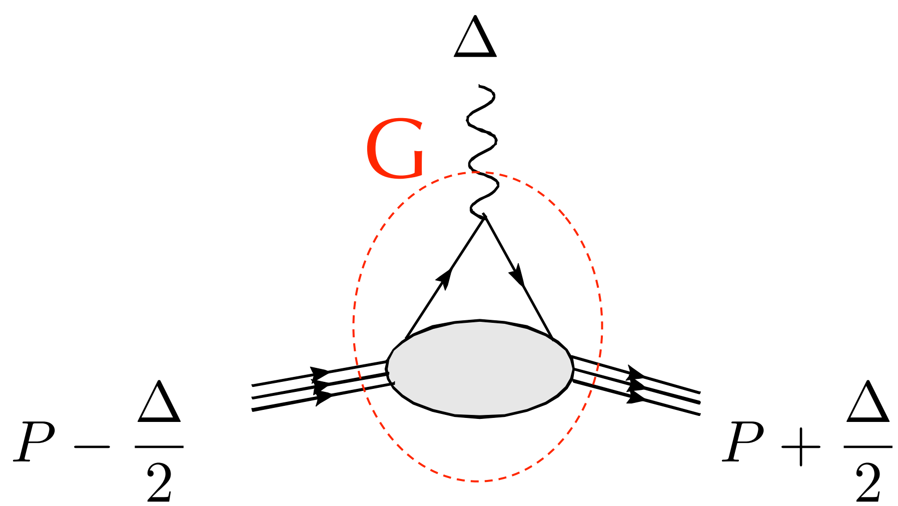
$$J^{\mu\alpha\beta}_{{\rm Bel},q}(x) = J^{\mu\alpha\beta}_{{\rm kin},q}(x) + \frac{1}{2}\partial_\lambda[x^\alpha S^{\lambda\mu\beta}_q(x) - x^\beta S^{\lambda\mu\alpha}_q(x)]$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x J^{0jk}$$

$$\int \vec{J}_{{\rm Bel},q}\,d^3x = \int \vec{J}_{{\rm kin},q}\,d^3x$$

equal total charge

Form Factors of Energy Momentum Tensor



Nucleon in external classical gravitational field

G couples to energy-momentum tensor
(simmetric-Belinfante)

$$(\mu, \nu) \equiv \frac{1}{2}(\mu\nu + \nu\mu)$$

$$\langle P + \frac{\Delta}{2} | T^{\mu\nu} | P - \frac{\Delta}{2} \rangle$$

$$= \bar{u}(P + \frac{\Delta}{2}) \left\{ A(t) \gamma^{(\mu} P^{\nu)} + B(t) P^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2M} \right.$$

$$\left. + D(t)(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{4M} \right\} u(P - \frac{\Delta}{2})$$

Gordon identity

$$\bar{u}' \gamma^\mu u = \bar{u}' \left(\frac{P^\mu}{M} + i\sigma^{\mu\alpha} \frac{\Delta_\alpha}{2M} \right) u$$

$$= \bar{u}(P + \frac{\Delta}{2}) \left\{ A(t) P^\mu P^\nu / M + (A(t) + B(t)) P^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2M} \right.$$

$$\left. + D(t)(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{4M} \right\} u(P - \frac{\Delta}{2})$$

Momentum Sum Rule

$$\langle P|P^\nu|P\rangle \quad = \quad \langle P|\int d^3\vec{x}\, T^{0\nu}(x)|P\rangle$$

use the decomposition of EMT matrix elements in form factors $\longrightarrow$

Momentum Sum Rule

$$
\begin{aligned}
\langle P|P^\nu|P\rangle &= \langle P|\int d^3\vec{x}\, T^{0\nu}(x)|P\rangle \\
&= \lim_{\Delta\to 0}\langle P+\frac{\Delta}{2}|\int d^3\vec{x}\, T^{0\nu}(x)|P-\frac{\Delta}{2}\rangle
\end{aligned}
$$

use the decomposition of EMT
matrix elements in form factors $\longrightarrow$

Momentum Sum Rule

$$\begin{aligned}
\langle P|P^\nu|P\rangle &= \langle P|\int d^3\vec{x}\, T^{0\nu}(x)|P\rangle \\
&= \lim_{\Delta\to 0}\langle P+\frac{\Delta}{2}|\int d^3\vec{x}\, T^{0\nu}(x)|P-\frac{\Delta}{2}\rangle \\
&= \lim_{\Delta\to 0}\int d^3\vec{x}\, e^{-i\vec{x}\cdot\vec{\Delta}}\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle
\end{aligned}$$

use the decomposition of EMT
matrix elements in form factors $\longrightarrow$

Momentum Sum Rule

$$\begin{aligned}
\langle P|P^\nu|P\rangle &= \langle P|\int d^3\vec{x}\,T^{0\nu}(x)|P\rangle \\
&= \lim_{\Delta\to 0}\langle P+\frac{\Delta}{2}|\int d^3\vec{x}\,T^{0\nu}(x)|P-\frac{\Delta}{2}\rangle \\
&= \lim_{\Delta\to 0}\int d^3\vec{x}\,e^{-i\vec{x}\cdot\vec{\Delta}}\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle \\
&= \lim_{\Delta\to 0}(2\pi)^3\delta^3(\vec{\Delta})\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle
\end{aligned}$$

use the decomposition of EMT matrix elements in form factors $\longrightarrow$

Momentum Sum Rule

$$
\begin{aligned}
\langle P|P^\nu|P\rangle &= \langle P|\int d^3\vec{x}\,T^{0\nu}(x)|P\rangle \\
&= \lim_{\Delta\to 0}\langle P+\frac{\Delta}{2}|\int d^3\vec{x}\,T^{0\nu}(x)|P-\frac{\Delta}{2}\rangle \\
&= \lim_{\Delta\to 0}\int d^3\vec{x}\,e^{-i\vec{x}\cdot\vec{\Delta}}\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle \\
&= \lim_{\Delta\to 0}(2\pi)^3\delta^3(\vec{\Delta})\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle
\end{aligned}
$$

use the decomposition of EMT matrix elements in form factors $\longrightarrow$ $= A(0)P^\nu(2P^0)(2\pi)^3\delta^3(0)$

Momentum Sum Rule

$$\langle P|P^\nu|P\rangle = \langle P|\int d^3\vec{x}\,T^{0\nu}(x)|P\rangle$$

$$= \lim_{\Delta\to 0}\langle P+\frac{\Delta}{2}|\int d^3\vec{x}\,T^{0\nu}(x)|P-\frac{\Delta}{2}\rangle$$

$$= \lim_{\Delta\to 0}\int d^3\vec{x}\,e^{-i\vec{x}\cdot\vec{\Delta}}\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle$$

$$= \lim_{\Delta\to 0}(2\pi)^3\delta^3(\vec{\Delta})\langle P+\frac{\Delta}{2}|T^{0\nu}(0)|P-\frac{\Delta}{2}\rangle$$

use the decomposition of EMT matrix elements in form factors $\longrightarrow$

$$= A(0)P^\nu(2P^0)(2\pi)^3\delta^3(0)$$

$$= A(0)P^\nu\langle P|P\rangle$$

Momentum Sum Rule

$$\langle P|P^\nu|P\rangle = A(0)P^\nu\langle P|P\rangle$$

$$\Downarrow$$

$$A(0) = 1$$

- Total system: energy-momentum conservation

- Physical interpretation in terms of:

quarks: $A^q(0)$

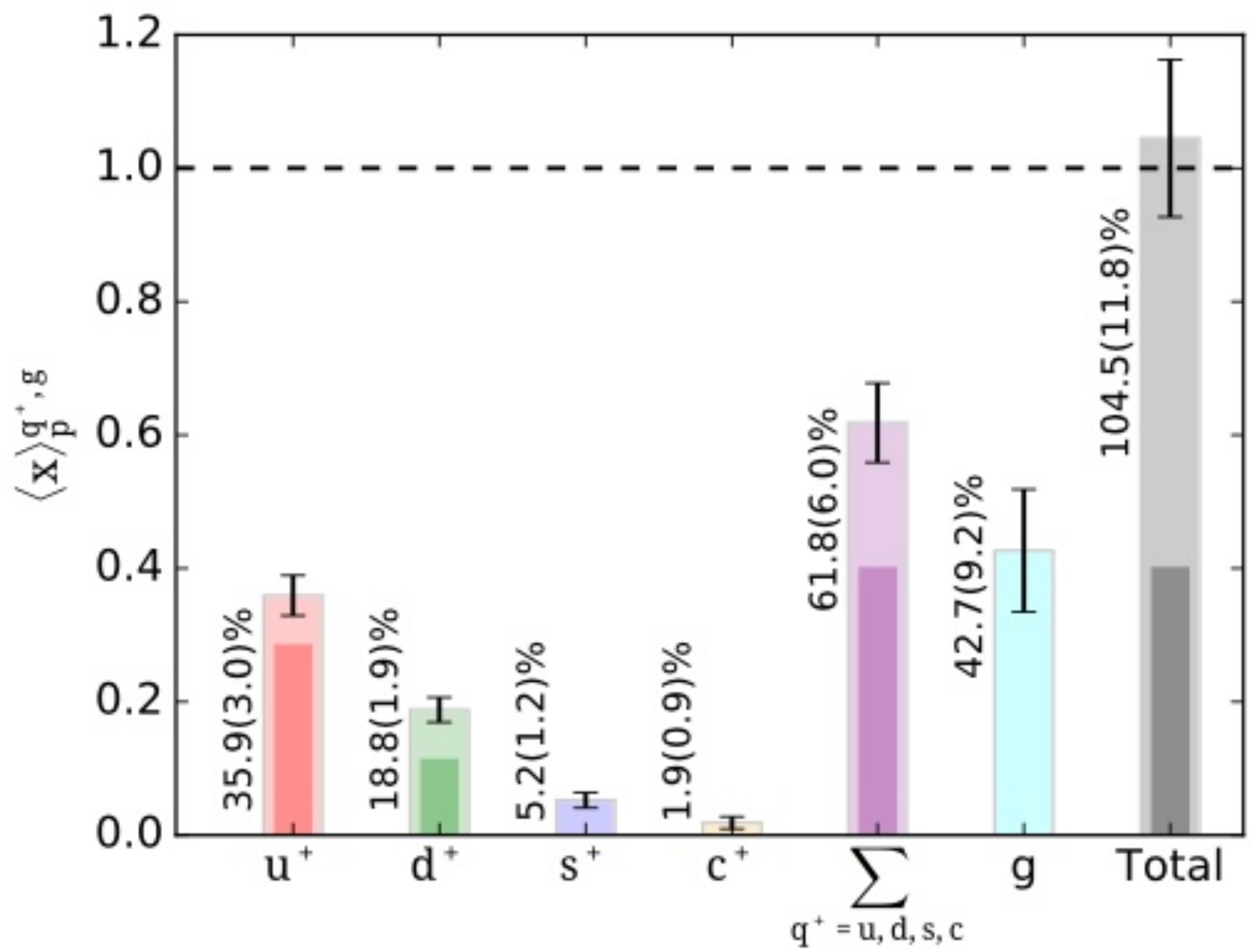
&

gluons: $A^g(0)$

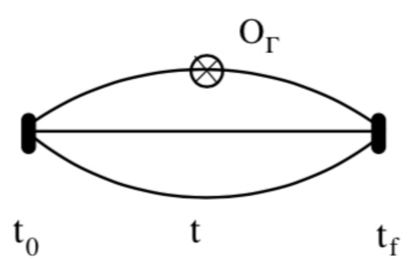
$$\boxed{A^q(0) + A^g(0) = 1}$$

Momentum sum rule from Lattice QCD

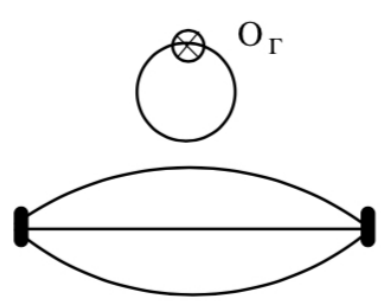
- Results at physical pion mass at the scale of 2 GeV



- dark bars: connected



- light bars: disconnected contributions (quarks & gluons)



Alexandrou et al. [Extended Twist Mass Coll.], Phys.Rev.D 101 (2020), 094513

Angular Momentum Sum Rule

Consider N in the rest frame $P^\mu = (M, 0, 0, 0) \quad S^\mu = (0, 0, 0, 1)$

$$\langle P, +\frac{1}{2}|\hat{J}^{12}|P, +\frac{1}{2}\rangle = J\langle P, +\frac{1}{2}|P, +\frac{1}{2}\rangle$$

Angular Momentum Sum Rule

Consider N in the rest frame $P^\mu = (M, 0, 0, 0) \quad S^\mu = (0, 0, 0, 1)$

$$\langle P, +\frac{1}{2} | \hat{J}^{12} | P, +\frac{1}{2} \rangle = J \langle P, +\frac{1}{2} | P, +\frac{1}{2} \rangle$$

$$= \langle P, +\frac{1}{2} | \int d^3\vec{x} \, \left\{ x^1 T^{02}(x) - x^2 T^{01}(x) \right\} | P, +\frac{1}{2} \rangle$$

Angular Momentum Sum Rule

Consider N in the rest frame $P^\mu = (M, 0, 0, 0) \quad S^\mu = (0, 0, 0, 1)$

$$\langle P, +\frac{1}{2}|\hat{J}^{12}|P, +\frac{1}{2}\rangle = J\langle P, +\frac{1}{2}|P, +\frac{1}{2}\rangle$$

$$= \langle P, +\frac{1}{2}| \int d^3\vec{x} \left\{x^1 T^{02}(x) - x^2 T^{01}(x)\right\} |P, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \langle P + \frac{\Delta}{2}, +\frac{1}{2}| \int d^3\vec{x}\, x^i T^{0j}(x)|P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

Angular Momentum Sum Rule

Consider N in the rest frame $P^\mu = (M, 0, 0, 0) \quad S^\mu = (0, 0, 0, 1)$

$$\langle P, +\frac{1}{2}|\hat{J}^{12}|P, +\frac{1}{2}\rangle = J\langle P, +\frac{1}{2}|P, +\frac{1}{2}\rangle$$

$$= \langle P, +\frac{1}{2}| \int d^3\vec{x}\ \left\{x^1 T^{02}(x) - x^2 T^{01}(x)\right\}|P, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \langle P + \frac{\Delta}{2}, +\frac{1}{2}| \int d^3\vec{x}\, x^i T^{0j}(x)|P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \int d^3\vec{x}\, x^i e^{-i\vec{x}\cdot\vec{\Delta}} \langle P + \frac{\Delta}{2}, +\frac{1}{2}|\ T^{0j}(0)|P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

Angular Momentum Sum Rule

Consider N in the rest frame $P^\mu = (M, 0, 0, 0) \quad S^\mu = (0, 0, 0, 1)$

$$\langle P, +\frac{1}{2}|\hat{J}^{12}|P, +\frac{1}{2}\rangle = J\langle P, +\frac{1}{2}|P, +\frac{1}{2}\rangle$$

$$= \langle P, +\frac{1}{2}| \int d^3\vec{x} \, \left\{x^1 T^{02}(x) - x^2 T^{01}(x)\right\} |P, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \langle P + \frac{\Delta}{2}, +\frac{1}{2}| \int d^3\vec{x} \, x^i T^{0j}(x) |P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \int d^3\vec{x} \, x^i e^{-i\vec{x}\cdot\vec{\Delta}} \langle P + \frac{\Delta}{2}, +\frac{1}{2}| \, T^{0j}(0) |P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \left[i\frac{\partial}{\partial\Delta^i}(2\pi)^3\delta^3(\vec{\Delta})\right]$$

$$\times \langle P + \frac{\Delta}{2}, +\frac{1}{2}| \, T^{0j}(0) |P - \frac{\Delta}{2}, +\frac{1}{2}\rangle$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i \sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \quad \text{terms independent of } \Delta \; + \quad \text{terms quadratic in } \; \Delta \; \right\}$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta\to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i\frac{\partial}{\partial \Delta^i}\right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i \sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \text{ terms independent of } \Delta + \text{ terms quadratic in } \Delta \right\}$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M}$$

$$\times \bar{u}(P)(-\frac{1}{2})\{P^0 \sigma^{ji} + P^j \sigma^{0i}\} u(P)$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i \sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \ \text{terms independent of } \Delta + \ \text{terms quadratic in } \Delta \right\}$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M}$$

$$\times \bar{u}(P)(-\frac{1}{2})\{\underset{\overset{\big\downarrow}{M}}{P^0}\sigma^{ji} + \underset{\overset{\big\downarrow}{0}}{P^j}\sigma^{0i}\} u(P) \quad \text{in rest frame}$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta\to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i\frac{\partial}{\partial \Delta^i}\right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i\sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \text{ terms independent of } \Delta + \text{ terms quadratic in } \Delta \right\}$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M}$$

$$\times \bar{u}(P)(-\frac{1}{2})\{\underbrace{P^0}_{M} \sigma^{ji} + \underbrace{P^j}_{0} \sigma^{0i}\} u(P)$$

in rest frame

$$= (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M} M \bar{u}(P) \sigma^{12} u(P)$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta\to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i \sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \text{ terms independent of } \Delta + \text{ terms quadratic in } \Delta \right\}$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M}$$

$$\times \bar{u}(P)(-\frac{1}{2})\{\underset{\downarrow \, M}{P^0} \sigma^{ji} + \underset{\downarrow \, 0}{P^j} \sigma^{0i}\} u(P)$$

in rest frame

$$= (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M} M \underbrace{\bar{u}(P) \sigma^{12} u(P)}_{2M \text{ in rest frame}}$$

Angular Momentum Sum Rule

$$= \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right)$$

$$\times \left\{ [A(t) + B(t)] \bar{u}(P + \frac{\Delta}{2}) \left[P^{(0} i \sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} \right] u(P - \frac{\Delta}{2}) \right.$$

$$\left. + \ \text{terms independent of } \Delta \ + \ \text{terms quadratic in } \Delta \right\}$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M}$$

$$\times \bar{u}(P)(-\frac{1}{2})\{\underset{M}{\underset{\downarrow}{P^0}} \sigma^{ji} + \underset{0}{\underset{\downarrow}{P^j}} \sigma^{0i}\} u(P) \quad \text{in rest frame}$$

$$= (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M} M \underbrace{\bar{u}(P) \sigma^{12} u(P)}_{2M \text{ in rest frame}}$$

$$= \frac{1}{2} [A(0) + B(0)] \langle P | P \rangle$$

Angular Momentum Sum Rule

$$J = \frac{1}{2}[A(0) + B(0)]$$

- Total system: angular momentum conservation

$$A(0) + B(0) = 1 \xrightarrow[\text{momentum conservation}]{A(0) = 1} B(0) = 0$$

gravitomagnetic sum rule

- Physical interpretation in terms of quarks & gluons

$$J^{q,g} = \frac{1}{2}[A^{q,g}(0) + B^{q,g}(0)] \qquad \frac{1}{2} = J^q + J^g = \frac{1}{2}\Delta\Sigma + L^q + J^g$$

$$\vec{J}^q = \int d^3\vec{x}\left[\underbrace{\psi^\dagger \frac{\vec{\Sigma}}{2}\psi}_{\frac{1}{2}\Delta\Sigma} + \underbrace{\psi^\dagger \vec{x} \times \left(-i\vec{D}\right)\psi}_{L^q}\right] \qquad \vec{J}^g = \int d^3\vec{x}\,\vec{x} \times \left(\vec{E} \times \vec{B}\right)\psi\Big]$$

Link to generalized parton distributions

- Polinomiality:

$$\int dx\, x H(x,\xi,t) = A(t) + D(t)\,\xi^2$$

$$\int dx\, x E(x,\xi,t) = B(t) - D(t)\,\xi^2$$

Link to generalized parton distributions

- Polinomiality:

$$\int dx\, x H(x,\xi,t) = A(t) + D(t)\,\xi^2 \qquad\qquad \int dx\, x E(x,\xi,t) = B(t) - D(t)\,\xi^2$$

- Momentum sum rule $\quad A^q(0) + A^g(0) = 1 = \int dx\, x\big(H^q(x,0,0) + H^g(x,0,0)\big)$

Link to generalized parton distributions

- Polinomiality:

$$\int dx\, x H(x,\xi,t) = A(t) + D(t)\,\xi^2 \qquad\qquad \int dx\, x E(x,\xi,t) = B(t) - D(t)\,\xi^2$$

- Momentum sum rule $\quad A^q(0) + A^g(0) = 1 = \int dx\, x(H^q(x,0,0) + H^g(x,0,0))$

- Angular momentum sum rule (Ji's relation)

$$J^{q,g} = \frac{1}{2}[A^{q,g}(0) + B^{q,g}(0)] = \frac{1}{2}\int dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right)$$

Link to generalized parton distributions

- Polinomiality:

$$\int dx\, xH(x,\xi,t) = A(t) + D(t)\,\xi^2 \qquad\qquad \int dx\, xE(x,\xi,t) = B(t) - D(t)\,\xi^2$$

- Momentum sum rule $\quad A^q(0) + A^g(0) = 1 = \int dx\, x(H^q(x,0,0) + H^g(x,0,0))$

- Angular momentum sum rule (Ji's relation)

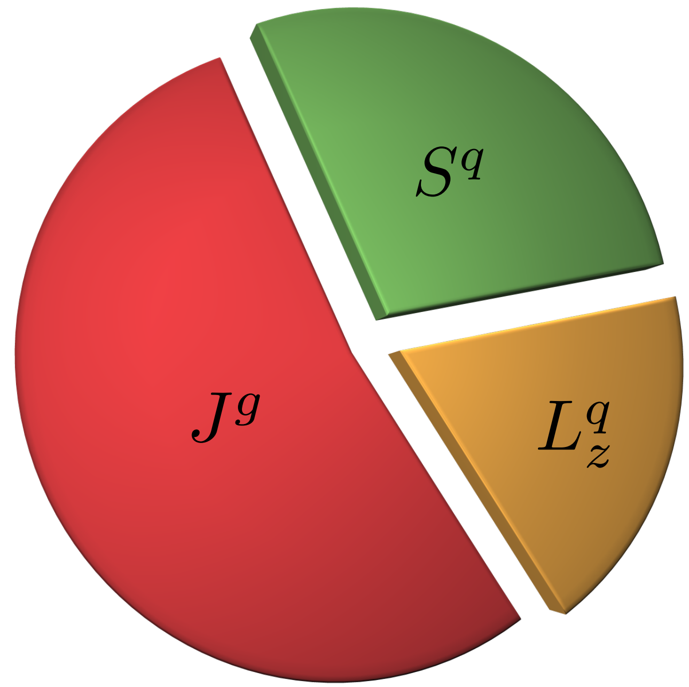
$$J^{q,g} = \frac{1}{2}[A^{q,g}(0) + B^{q,g}(0)] = \frac{1}{2}\int dx\, x\; (H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0))$$

- Gravitomagnetic sum rule

$$J^{\mathrm{TOT}} = J^q + J^g = \frac{1}{2} = \frac{1}{2}[A^q(0) + A^g(0) + B^q(0) + B^g(0)] = \frac{1}{2}[1 + B^q(0) + B^g(0)]$$

$$B^q(0) + B^g(0) = 0 = \int dx\, x(E^q(x,\xi,0) + E^g(x,\xi,0))$$

Link to generalized parton distributions



Ji's sum rule: $J^{q,g}(t=0) = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right)$ $\quad \left(\xi = -\frac{\Delta^+}{2P^+}\right)$

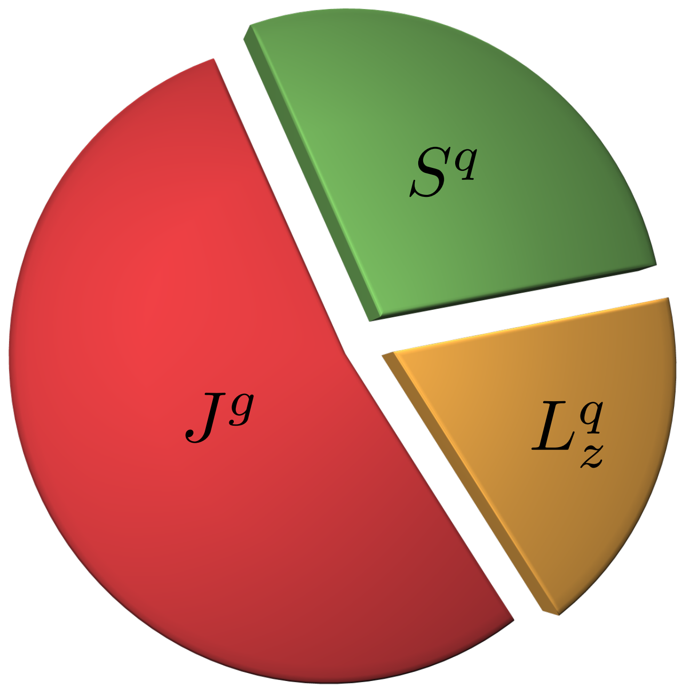
$H^{q,g}$: at $\xi = 0$ unpolarized PDF

$E^{q,g}$: not directly accessible

$$J^q = L^q + S^q \qquad\qquad S^q = \frac{1}{2}\int_0^1 dx\, g_1(x)$$

- Requires extrapolation at t=0
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)
- $J^q(x) \neq \frac{1}{2}x(H^q(x,0,0) + E^q(x,0,0))$ contribution from surface term

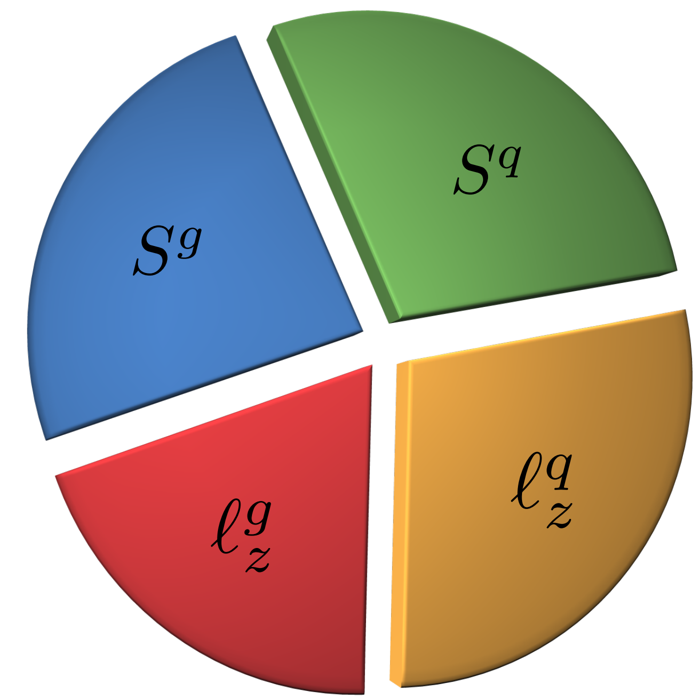
Ji (kinetic EMT) Sum Rule



$$\frac{1}{2} = \underbrace{S^q(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

- each term is gauge invariant
- frame independent
- it works also for the transverse AM in the infinite momentum frame
- J^q and J^g can be obtained from moments of GPDs

Jaffe-Manohar (canonical EMT) Sum Rule



$$\frac{1}{2} = S^q(\mu) + \ell_z^q(\mu) + \ell_z^g(\mu) + S^g(\mu)$$

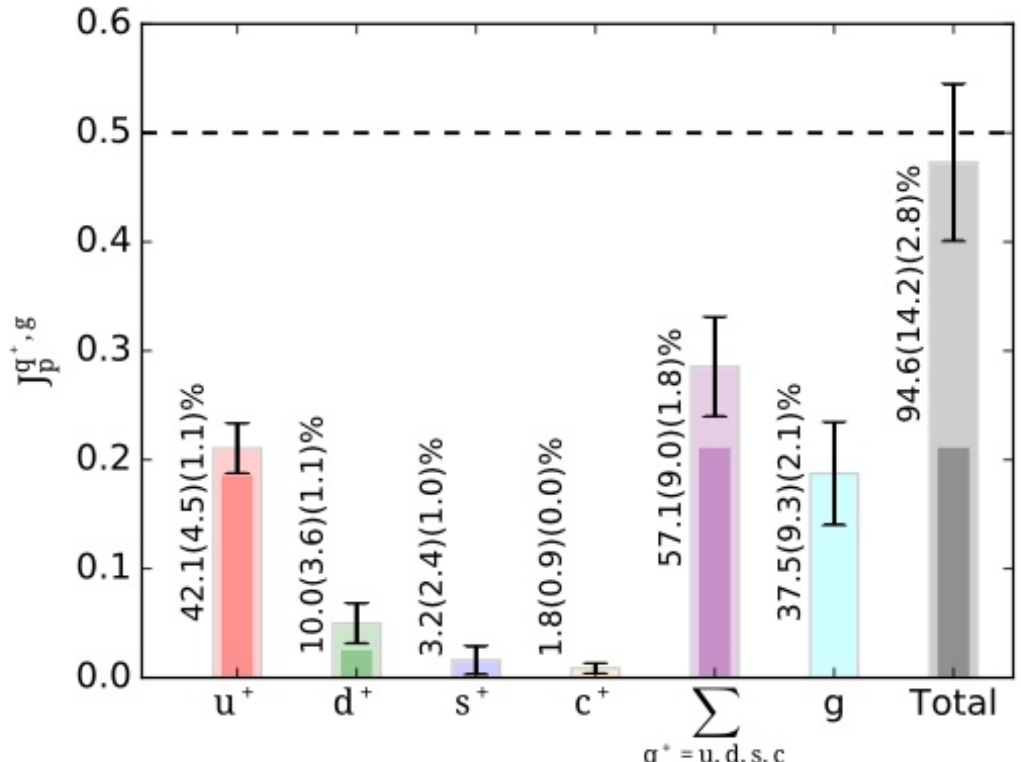
- ℓ_z^q, ℓ_z^g, S^g are gauge dependent, BUT measurable
- S^q, S^g can be obtained from pol. PDFs
- ℓ_z^q, ℓ_z^g can be obtained from twist-3 GPDs and Wigner distributions
- simple partonic interpretation in the IMF

$$\ell_z^q - l_z^{q,\mathrm{pot}} = L_z^q \longrightarrow \ell_z^g + S^g + l_z^{q,\mathrm{pot}} = J^g$$

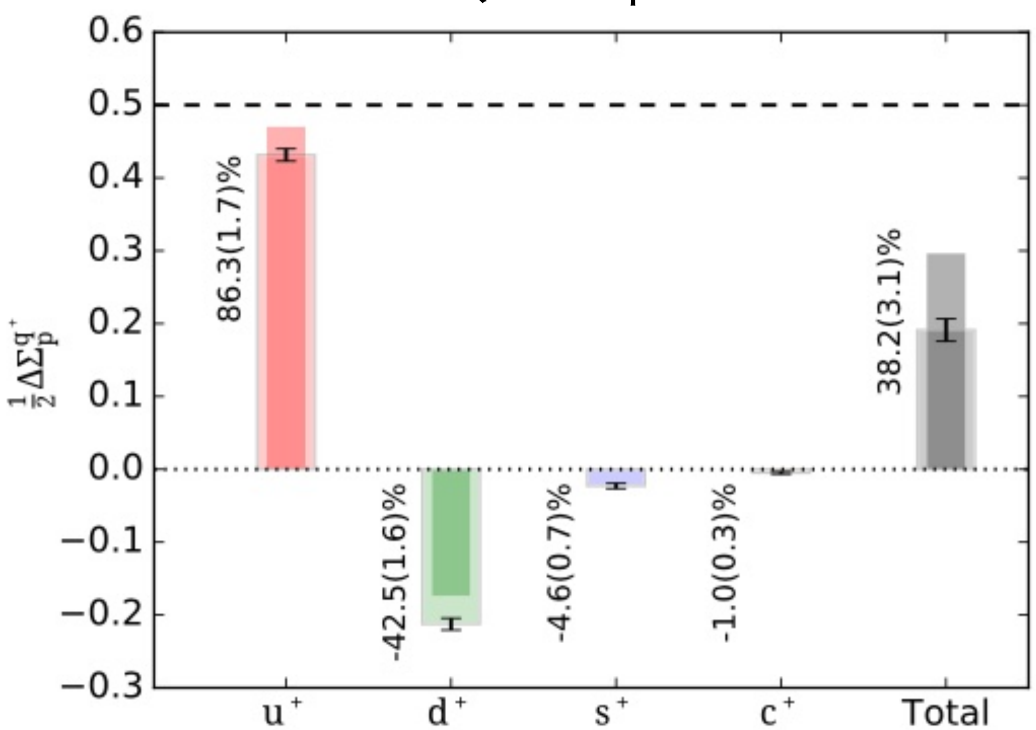
Y. Guo, X. Ji, K. Shiells, NPB 96 (2021) 115440; X. Ji, F. Yuan, Y. Zhao, Nature Rev. Phys. 3 (2021) 1

Sum rules from lattice QCD

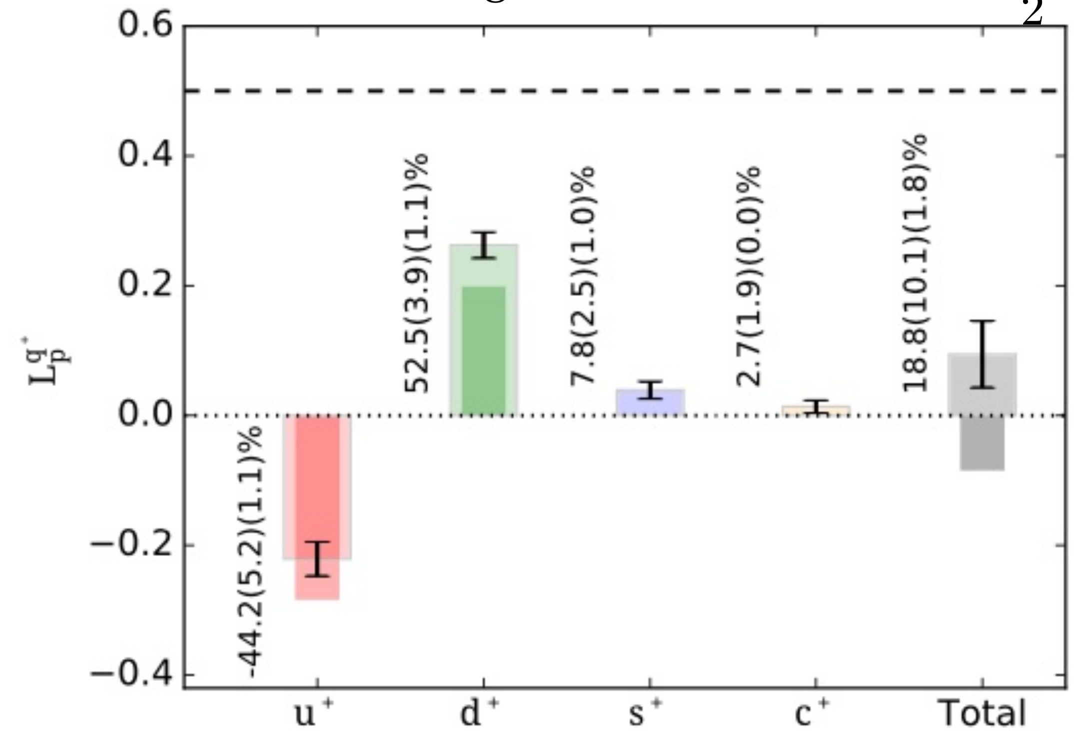
- Results at physical pion mass at the scale of 2 GeV



Quark spin

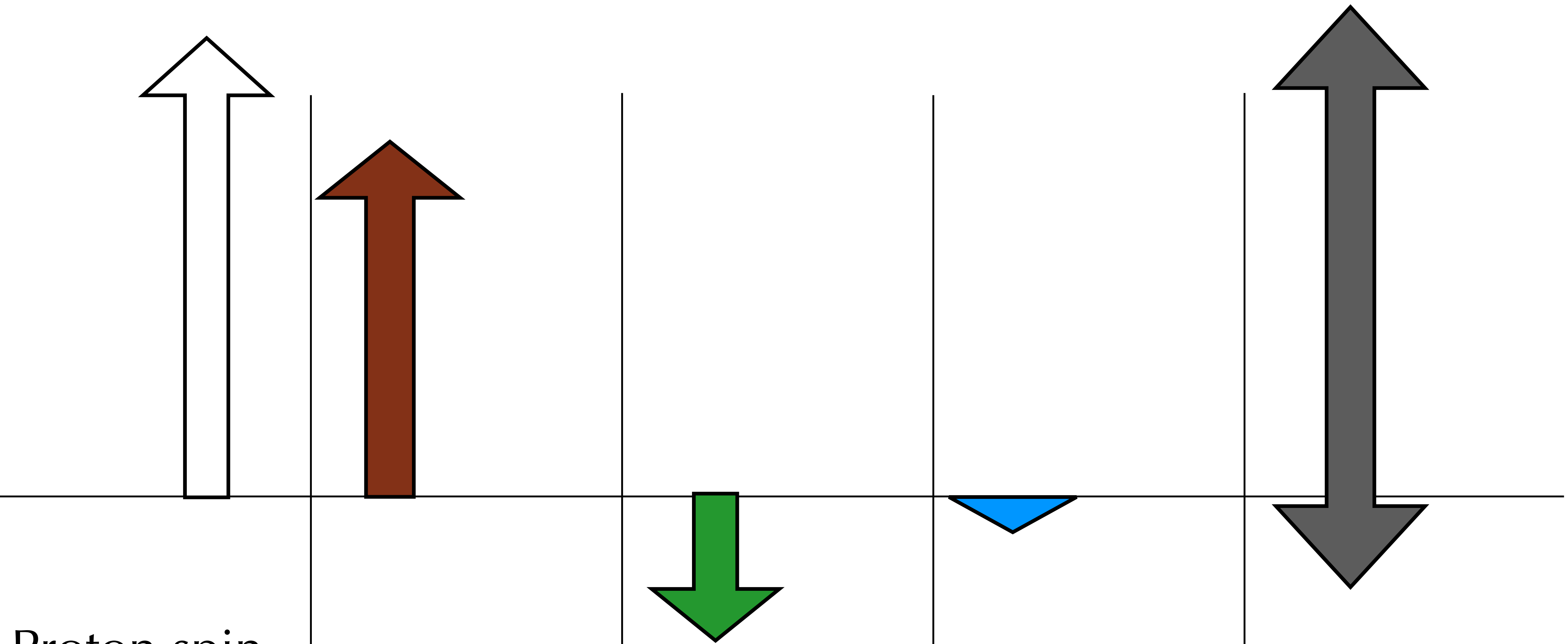


Quark orbital angular momentum $J^q - \frac{1}{2}\Delta\Sigma^q$



Alexandrou et al. [Extended Twist Mass Coll.], Phys.Rev.D 101 (2020), 094513

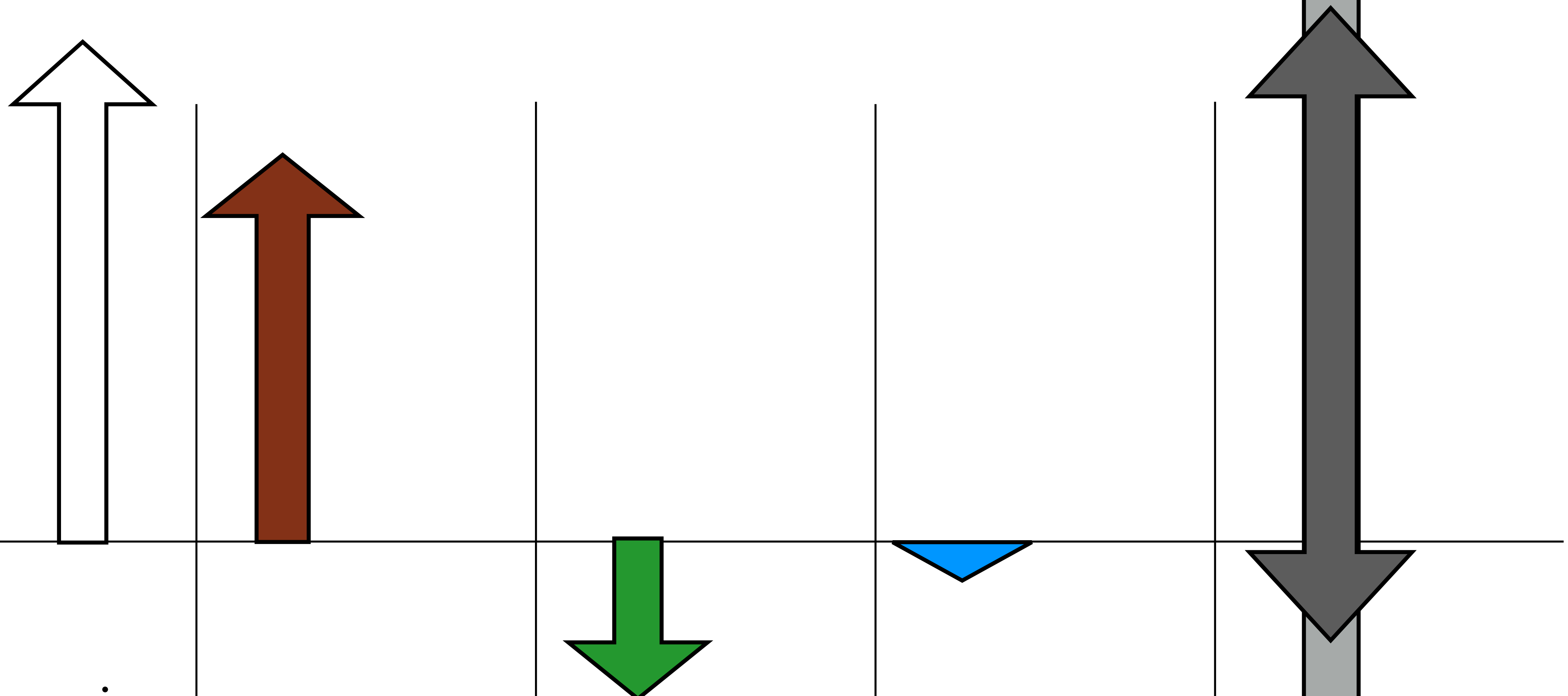
Status of Spin Sum Rule



Proton spin
(Q^2=10 GeV^2)

$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
81 ± 1		-43 ± 1		-11 ± 2		40 ± 60		DSSV14
79 ± 5		-45 ± 5		12 ± 8		3 ± 324		NNPDF

Status of Spin Sum Rule



Proton spin
(Q^2=10 GeV^2)

$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
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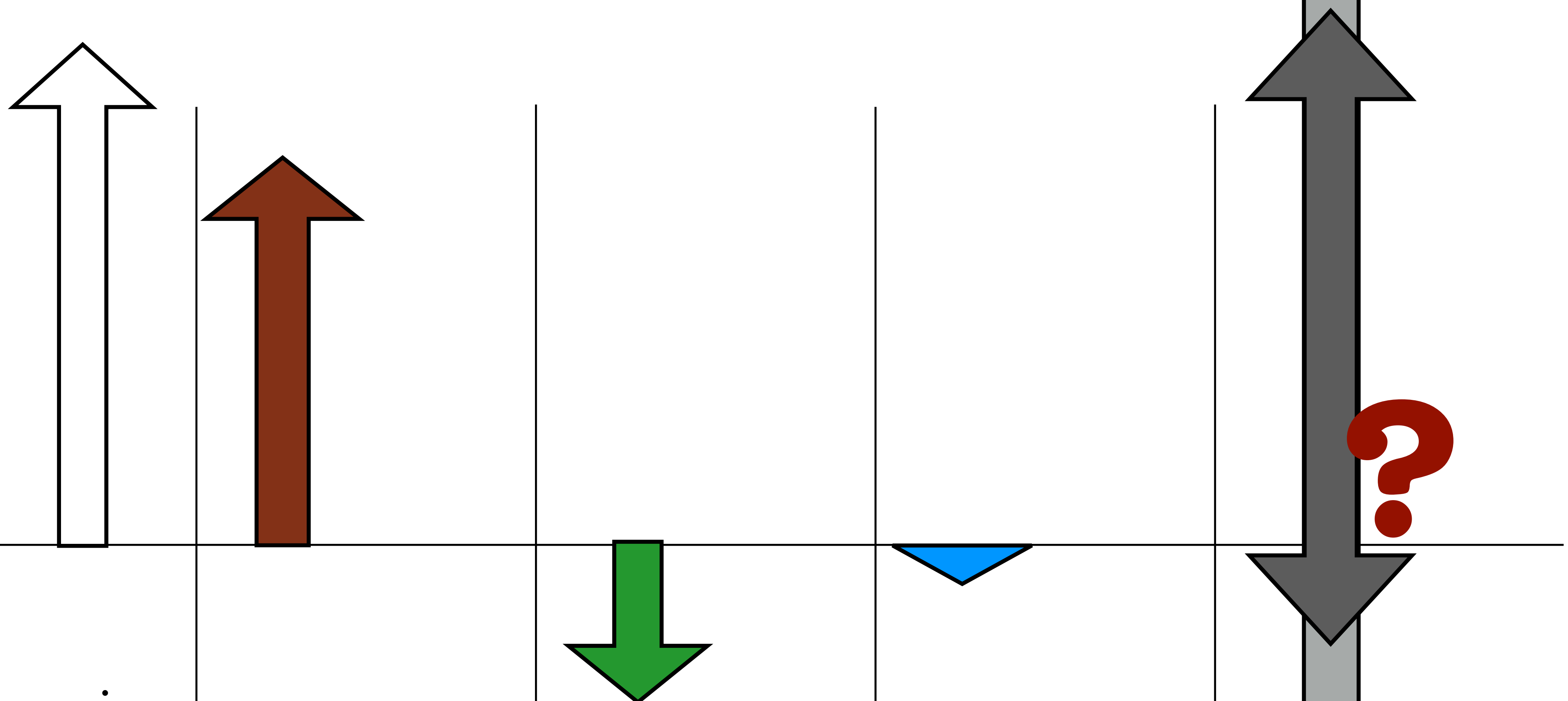
Status of Spin Sum Rule



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$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
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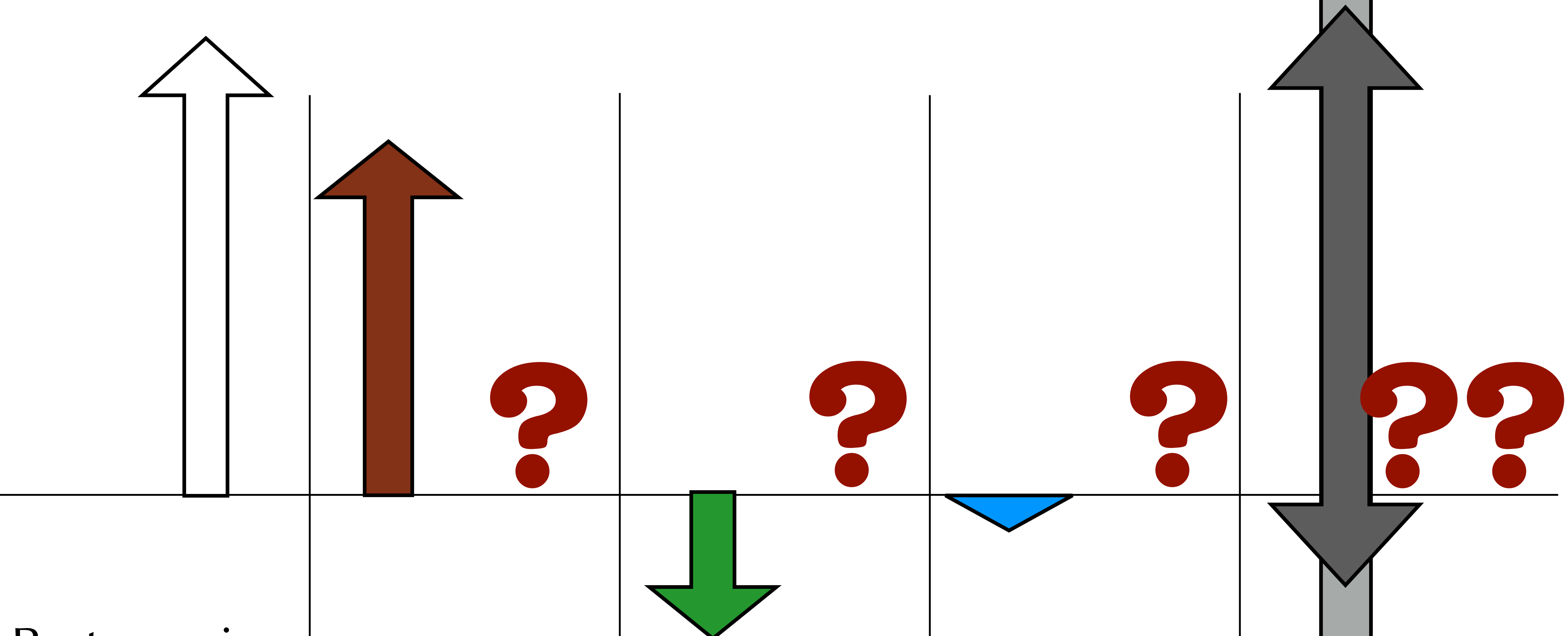
Status of Spin Sum Rule

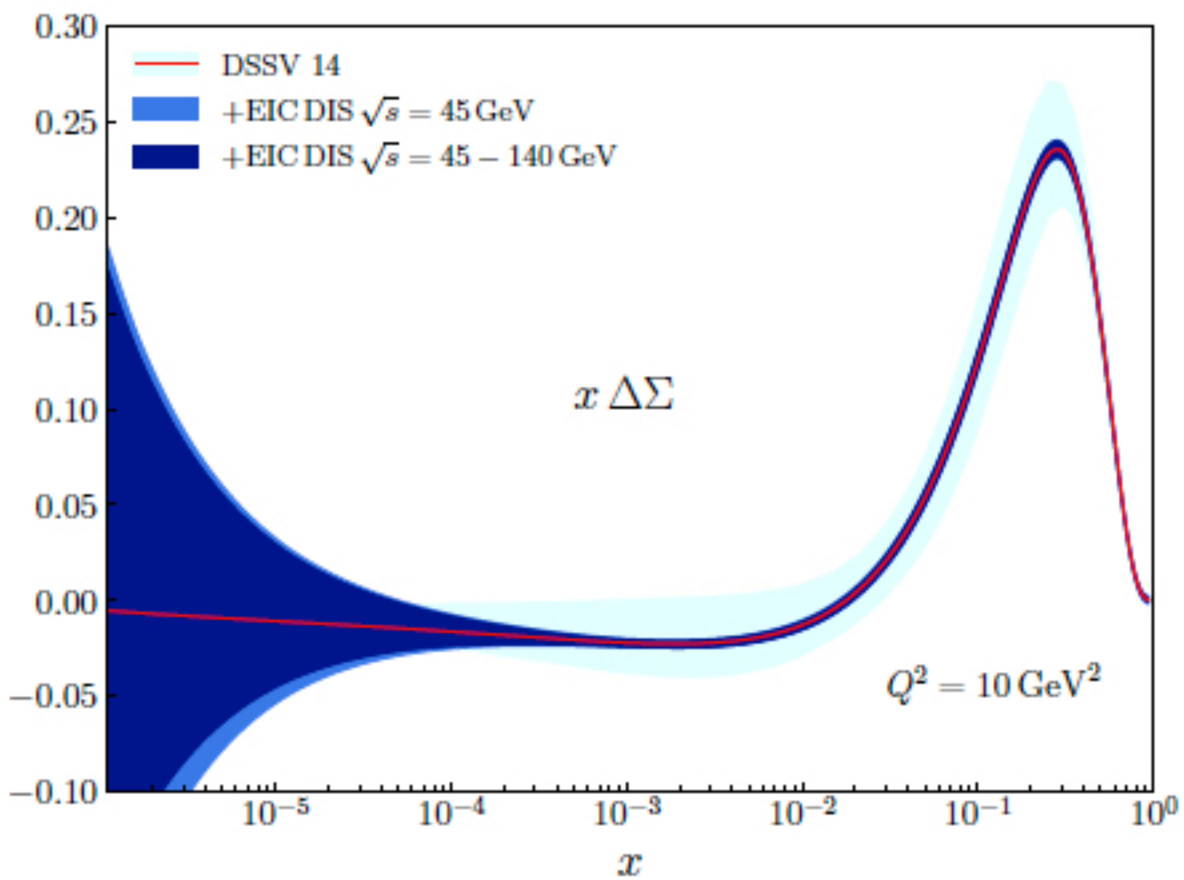


Proton spin
(Q^2=10 GeV^2)

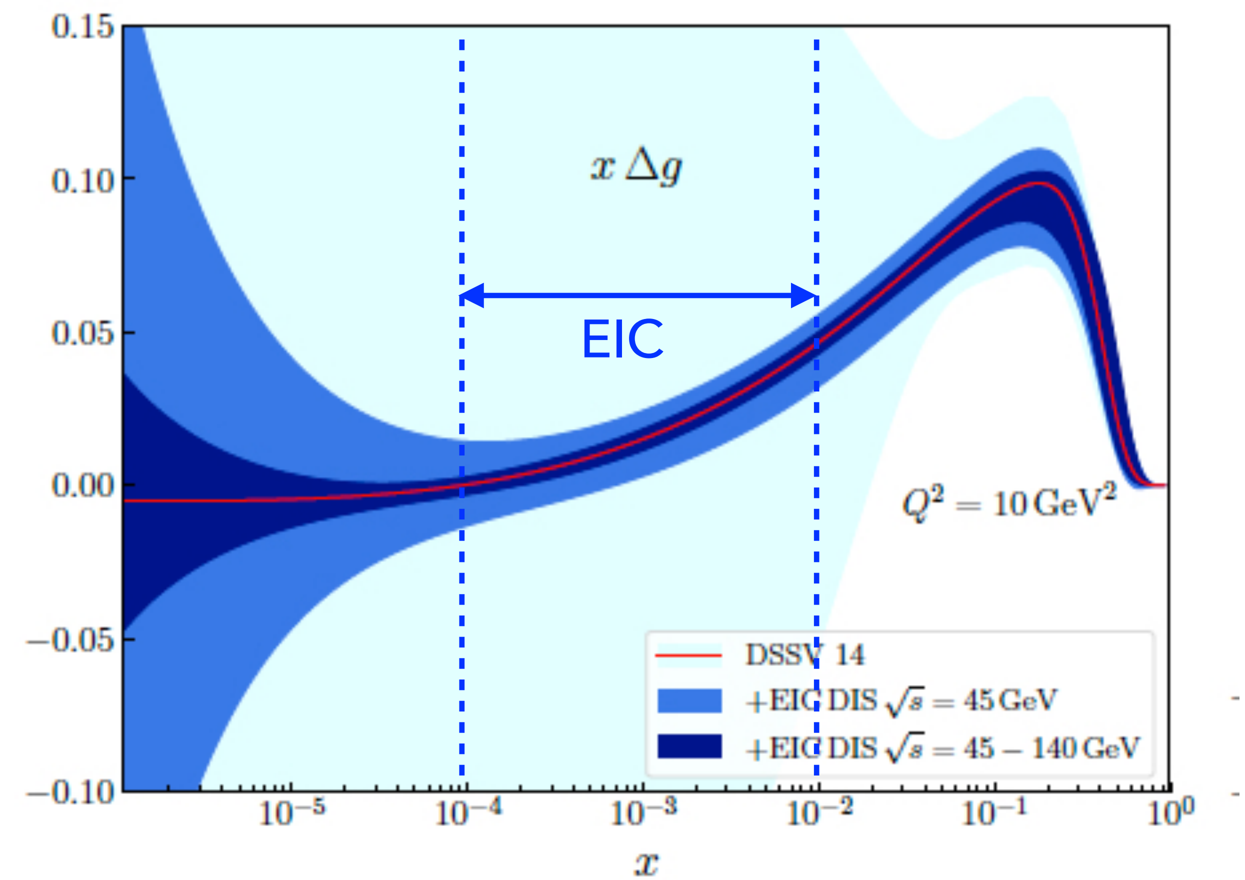
$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
81 ± 1		-43 ± 1		-11 ± 2		40 ± 60		DSSV14
79 ± 5		-45 ± 5		12 ± 8		3 ± 324		NNPDF

Impact of future EIC for quark and gluon spin contributions

Quark Spin



Gluon Spin



EIC Yellow Report: arXiv: 2103.05419

We are constantly improving the knowledge of the contributions to the spin of the nucleon

However the details on the flavor and sea contributions are still sketchy

Orbital Angular momentum of the proton from available GPD measurements

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right) \qquad L^q = J^q - S^q$$

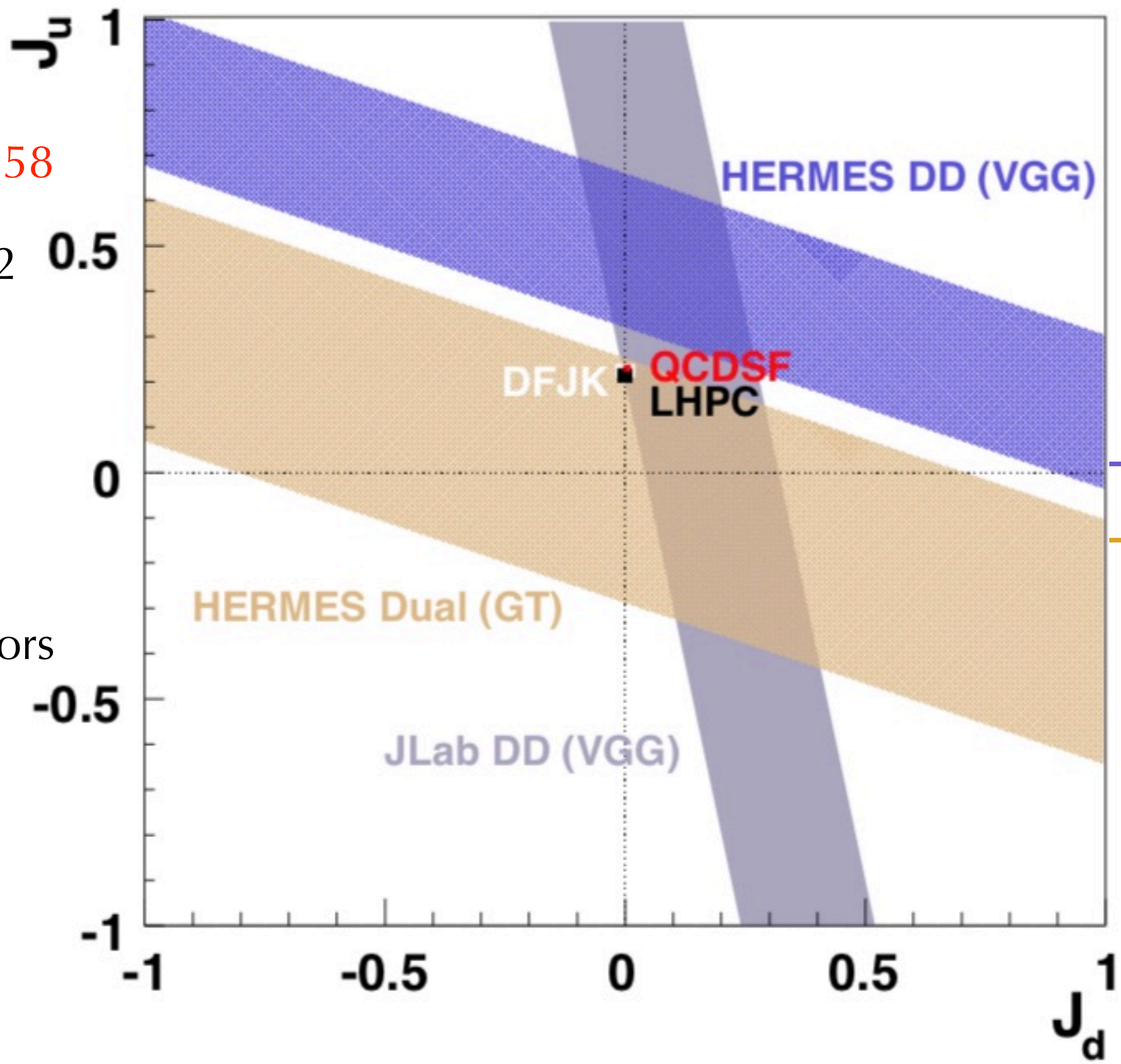
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Improved accuracy with JLab12 and future EIC measurements!

Nucleon Structure Properties

em	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \| J^\mu_{\text{em}} \| N \rangle$	$\longrightarrow$	$Q,\ \mu$
weak	$\partial_\mu J^\mu_{\text{weak}} = 0$	$\langle N' \| J^\mu_{\text{weak}} \| N \rangle$	$\longrightarrow$	$g_A,\ g_p$
gravity	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N' \| T^{\mu\nu}_{\text{grav}} \| N \rangle$	$\longrightarrow$	M_N, J, D

$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\,\text{C}$

$\mu_{\text{prot}} = 2.792847356(23)\mu_N$

$M_{\text{prot}} = 938.272013(23)\,\text{MeV}$

$g_p = 8 - 12$

$g_A = 1.2694(28)$

$J = \frac{1}{2}$

$$D = \frac{4}{5} d_1 = ??$$

can be accessed from GPDs in hard exclusive reactions

M. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

D(t) form factor from data

$$T_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\delta_{ij}$$

shear forces — pressure

neglecting gluon contribution:

$$r^2 p(r) = \frac{1}{3}\frac{d}{dr} r^2 \frac{d}{dr}\tilde{D}(r)$$

D(t) form factor from data

$$T_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\delta_{ij}$$

shear forces

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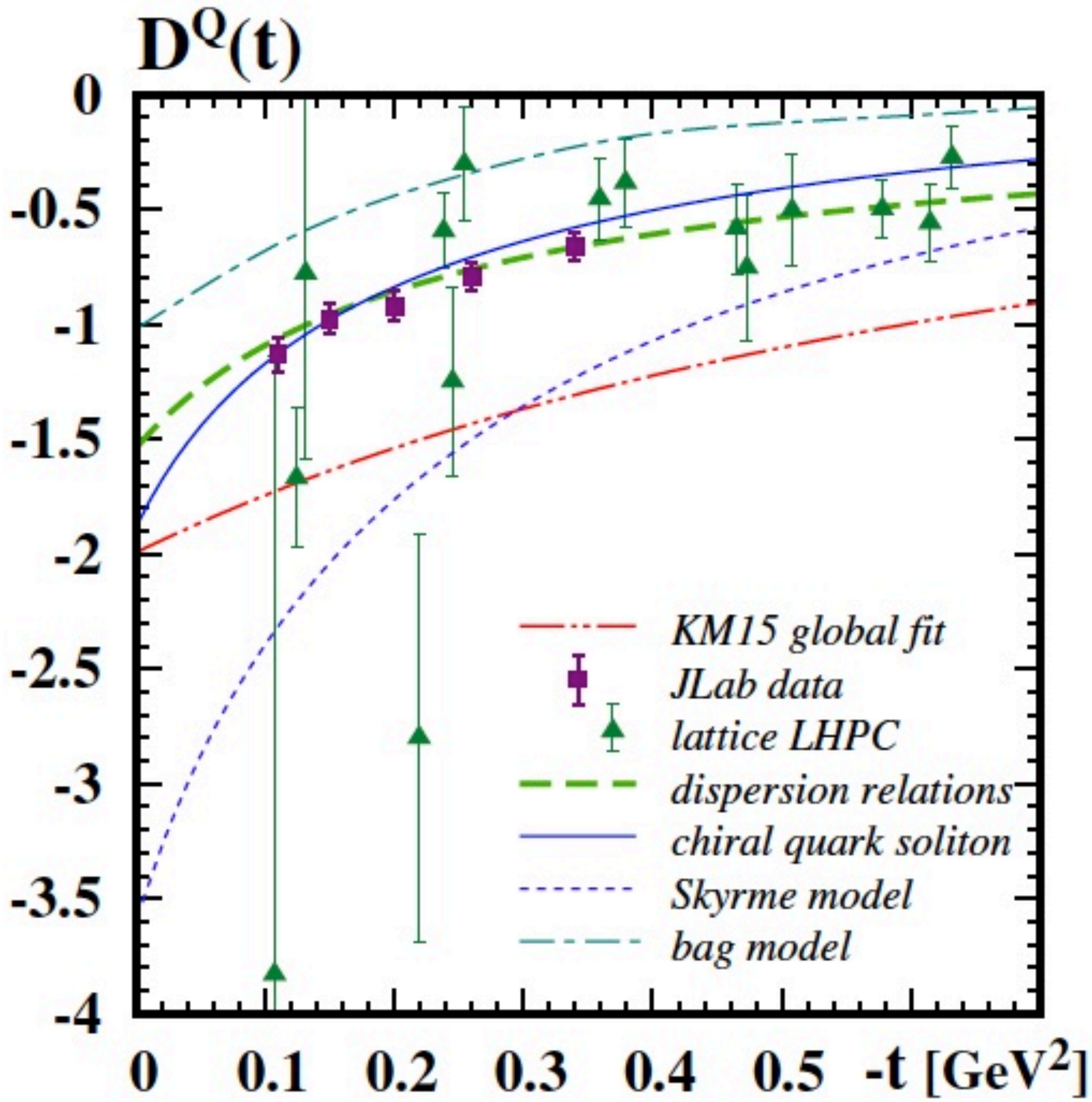


Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705

Polyakov and Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

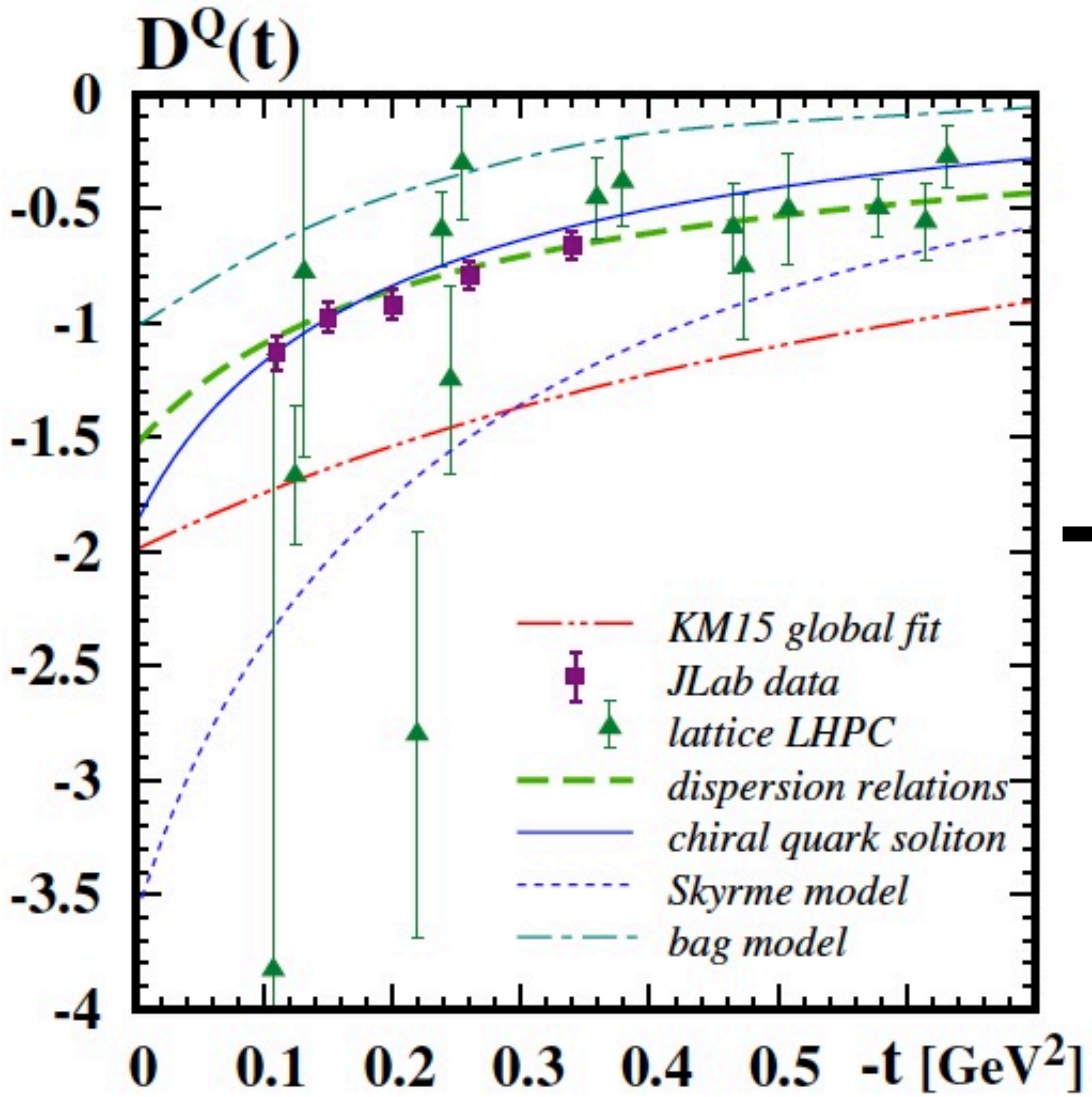
D(t) form factor from data

$$T_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\delta_{ij}$$

$$\underset{\text{shear forces}}{\downarrow} \qquad\qquad \underset{\text{pressure}}{\downarrow}$$

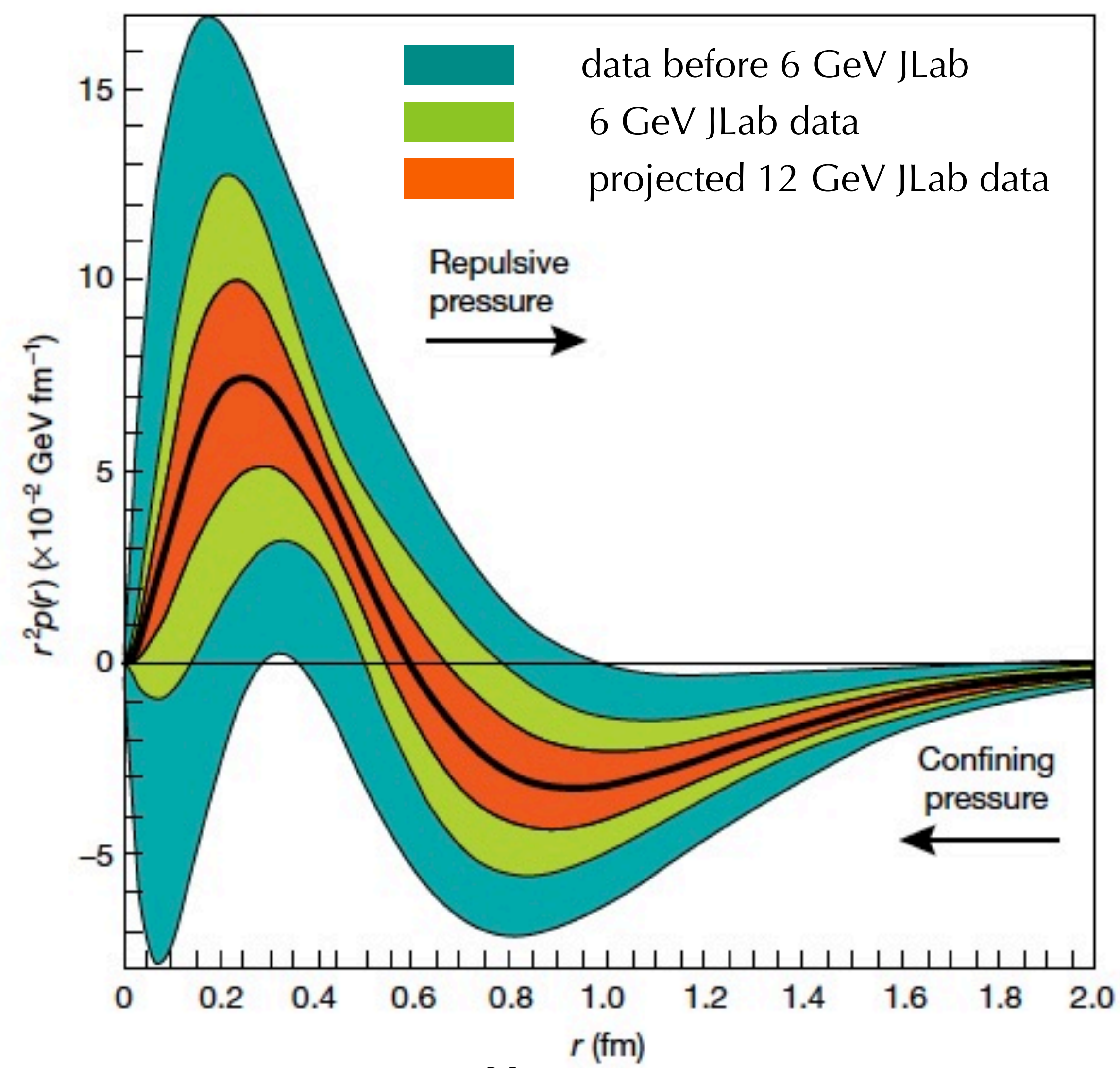
neglecting gluon contribution:

$$r^2 p(r) = \frac{1}{3}\frac{d}{dr}r^2\frac{d}{dr}\tilde{D}(r)$$



FT →

$r^2 p(r)$ radial pressure distribution



$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

stability condition

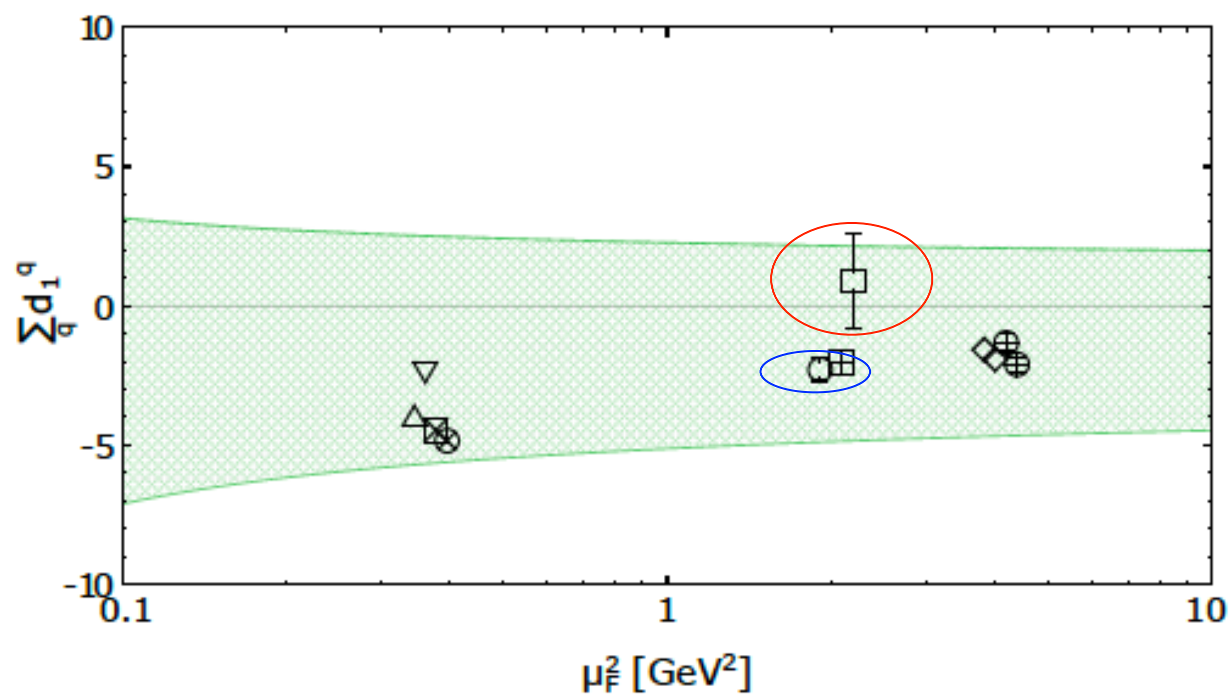
Girod, Elouadrhiri, Burkert, Nature 557 (2018) 7705

Polyakov and Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

Necessary to verify model assumptions in the exp extraction
with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param., Girod et al.

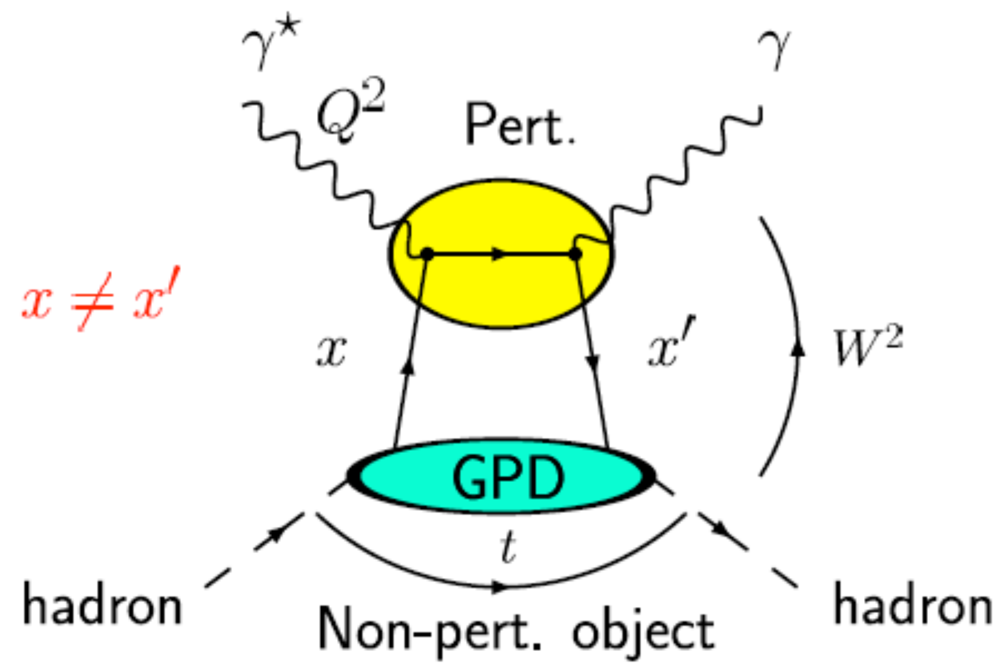
CLAS data, with neural networks Kumericki

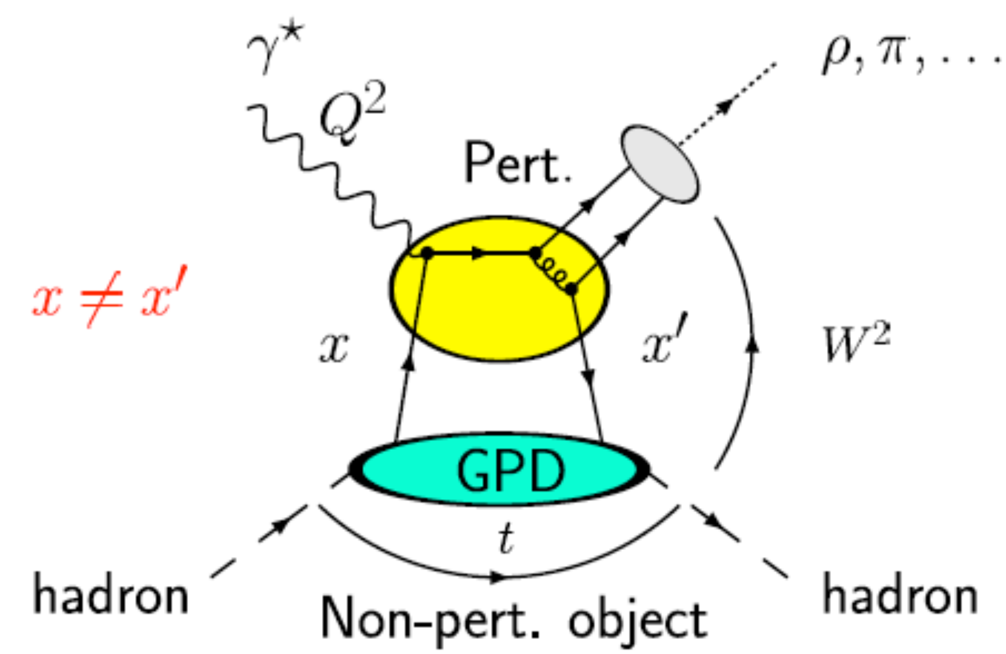
$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	t-channel saturated model
	-1.92	4	2	t-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

How to measure GPDs





- accessible in exclusive reactions
- factorization for large Q^2, $|t| << Q^2$, W^2
- depend on 3 variables: x, ξ, t

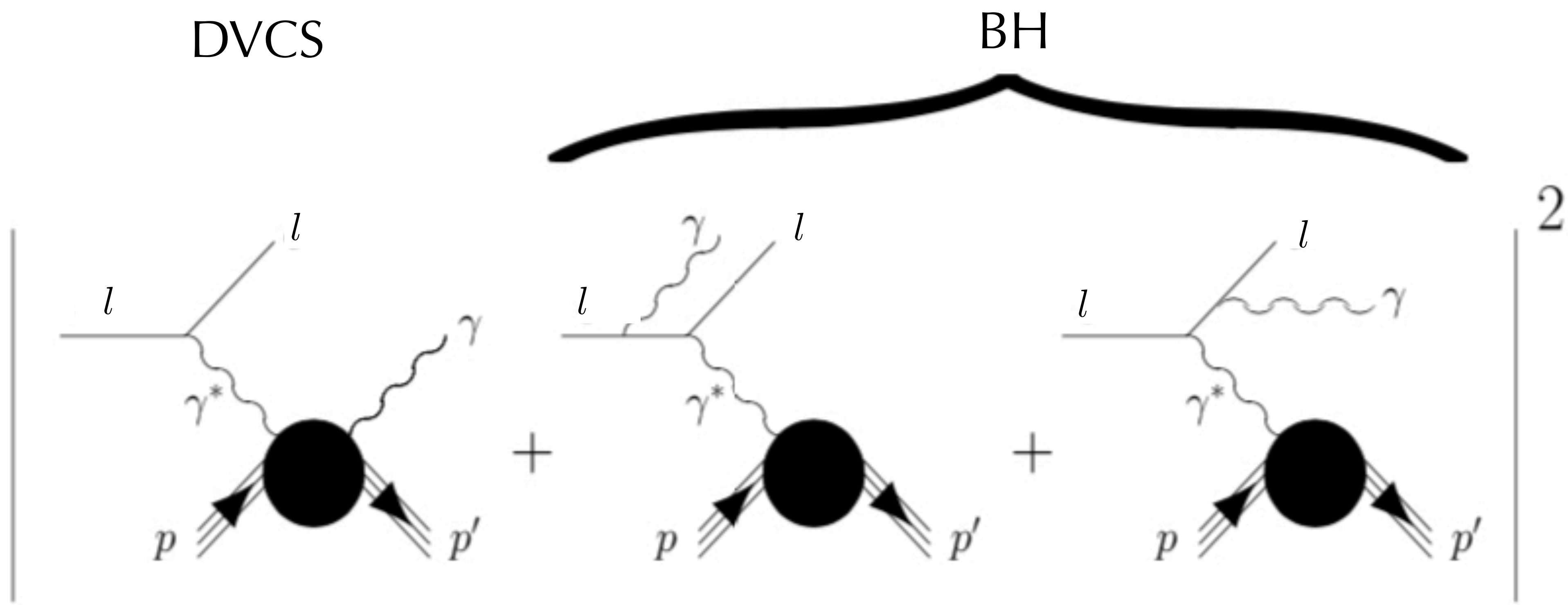
Compton Form Factors

$$\mathrm{Im}\,\mathcal{H}(\xi, t) \stackrel{\mathrm{LO}}{=} H(\xi, \xi, t)$$

$$\mathrm{Re}\,\mathcal{H}(\xi, t) \stackrel{\mathrm{LO}}{=} \mathcal{P} \int_{-1}^{1} \mathrm{d}x\ \mathrm{H}(x, \xi, t) \frac{1}{x-\xi}$$

Golden channel: deeply virtual Compton scattering

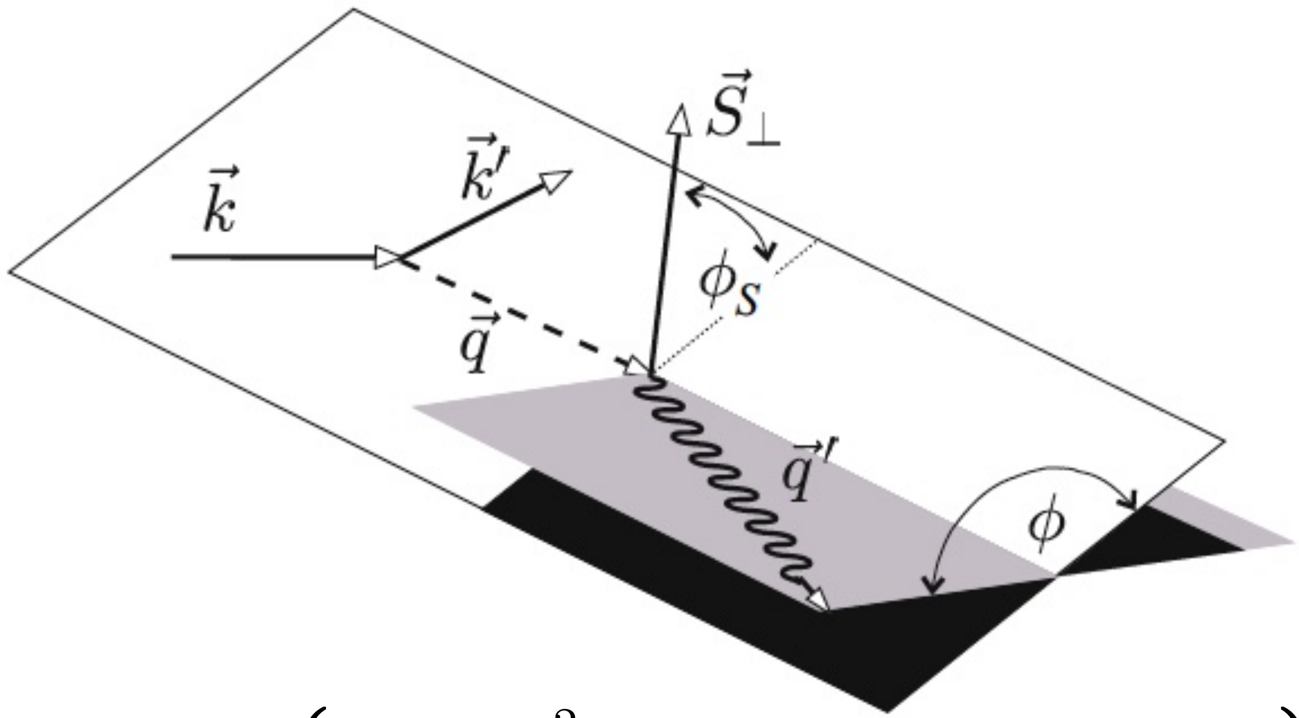
$\sigma(lp \to l\gamma p) \propto$



$$\sigma(lp \to l\gamma p) \propto |\mathcal{T}^{\mathrm{BH}}|^2 + |\mathcal{T}^{\mathrm{DVCS}}|^2 + e_l\,\mathcal{I}$$

- BH: calculable in QED with ~1% knowledge of e.m. at low momentum transfer
- $|\mathrm{DVCS}|^2$: bilinear in GPDs
- $\mathcal{I}(\mathrm{BH}\cdot\mathrm{DVCS})$: linear combination of GPDs

DVCS cross section



Filter out interference term using cross section dependence on

- beam charge
- azimuth
- beam polarization
- target polarization

$$|\mathcal{T}_{\mathrm{BH}}|^2 \propto \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin\phi \right\}$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 \propto \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right\}$$

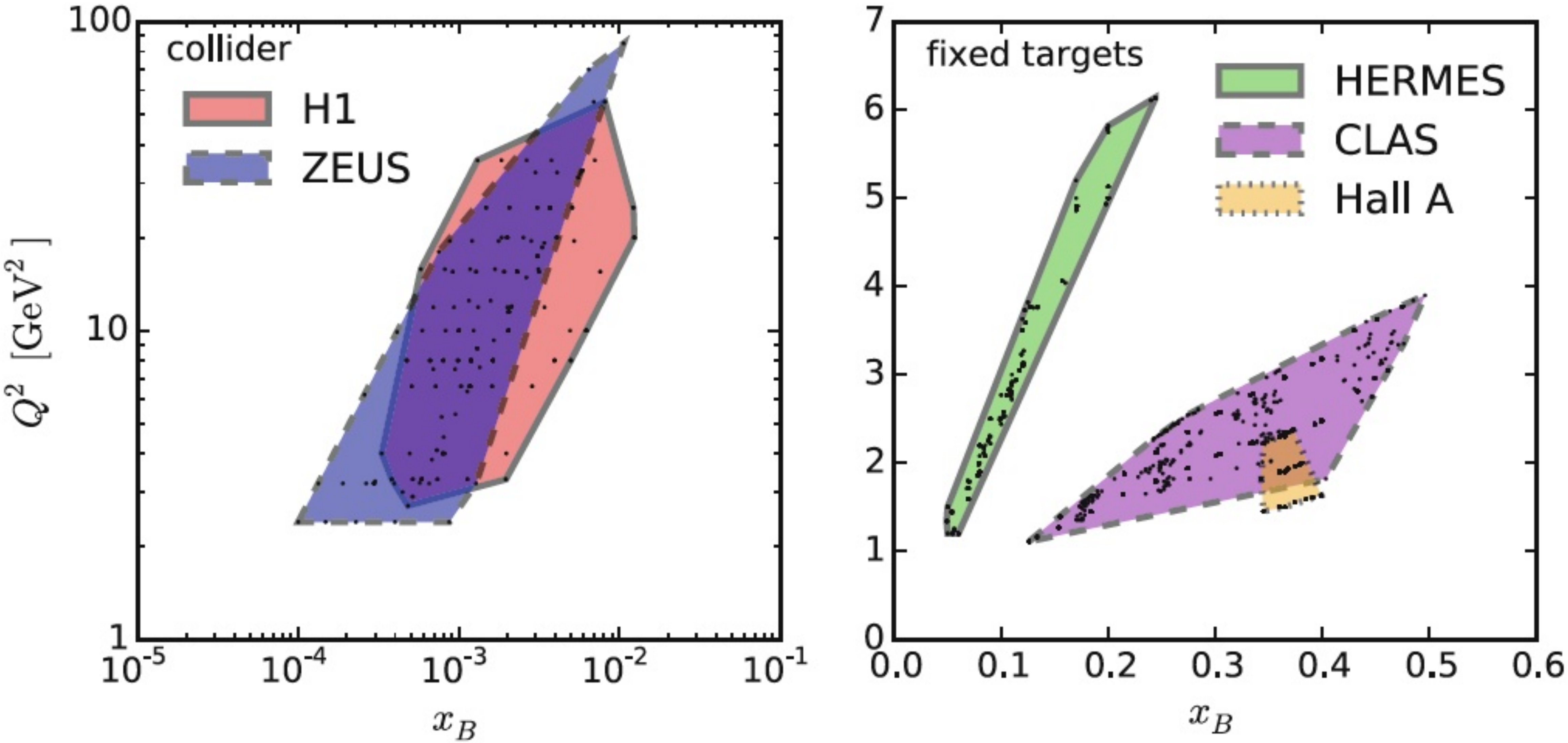
$$\mathcal{I} \propto \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right\}$$

- $c_0^{\mathrm{DVCS},\mathcal{I}}$, $(c,s)_1^{\mathcal{I}}$ dominated by leading-twist GPDs

- Similar decomposition for various polarization states of the target, but different dependence of the coefficients on the CFFs

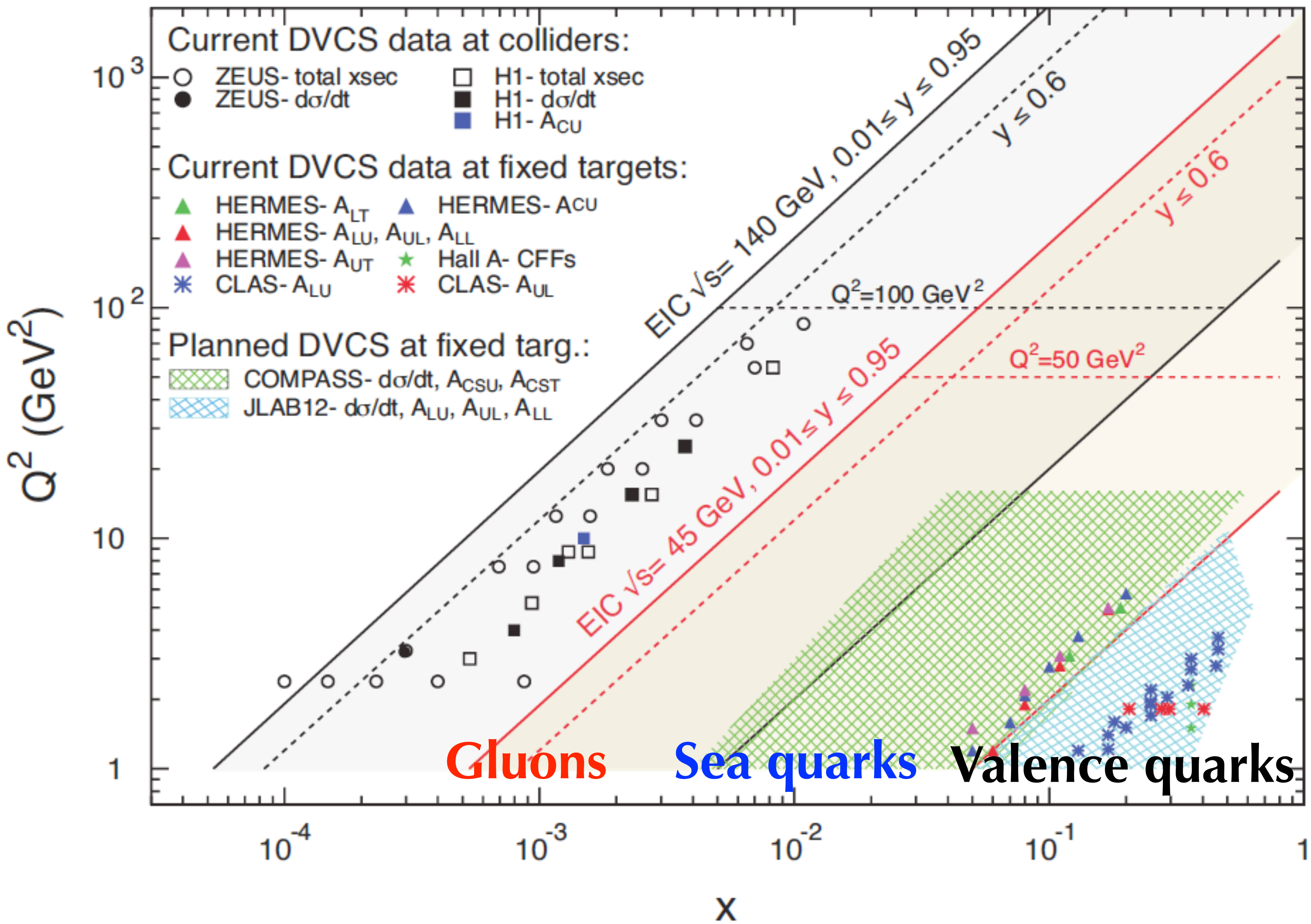
Belitsky, Kirchner, Müller, NPB629, 323 (2001)

World-data kinematic coverage



Kumericki, Liuti, Moutarde, EPJA52, 157 (2016)

Paste, present and future DVCS experiments



DESY

hermes

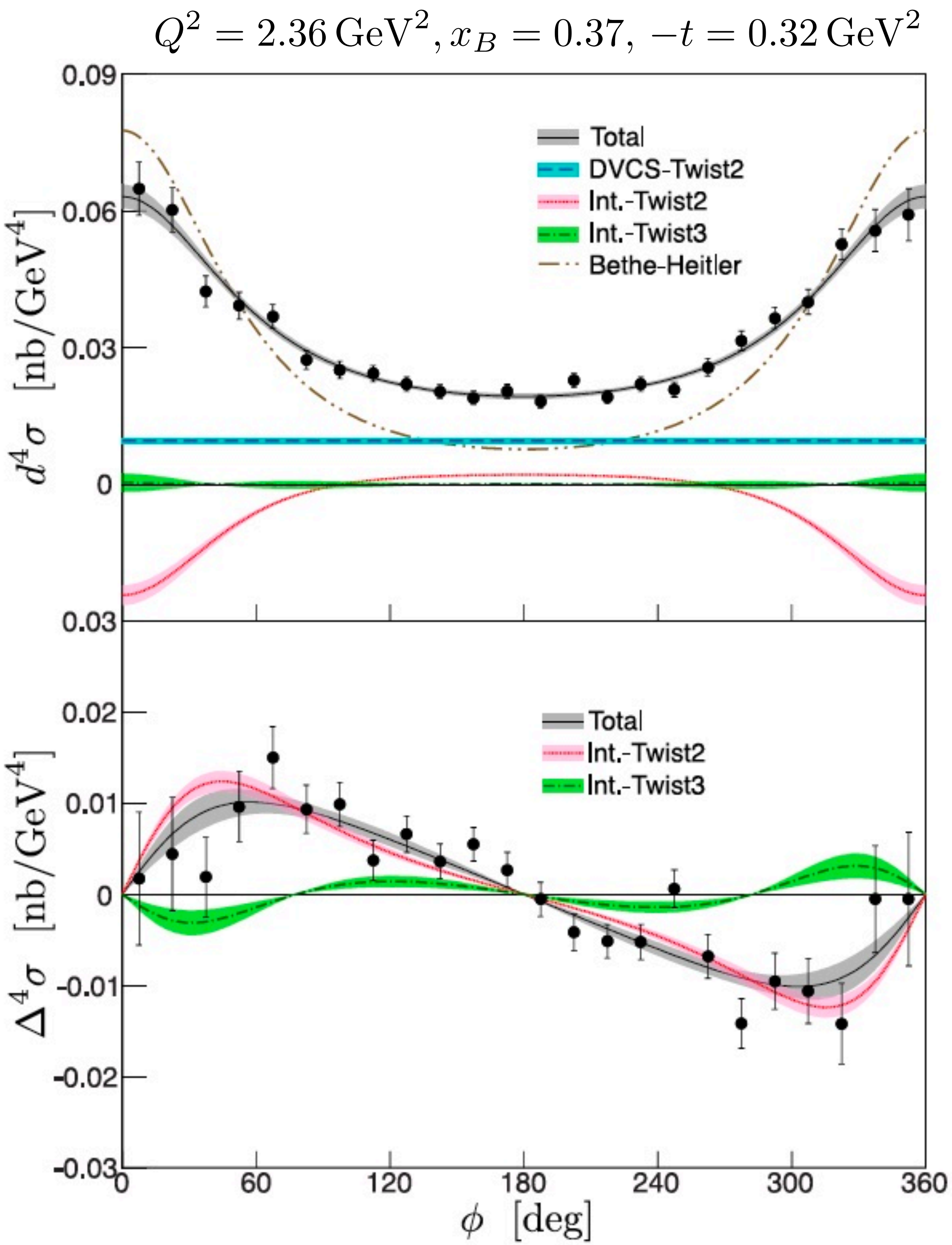
Jefferson Lab

COMPASS

EIC

Accardi et al., The Electron Ion Collider: the next QCD Frontier, Eur. Phys. J., A52, 2016

A sample of typical results



$$d^4\sigma = |\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{T}_{\mathrm{BH}}\,\mathrm{Re}(\mathcal{T}_{\mathrm{DVCS}}) + |\mathcal{T}_{\mathrm{DVCS}}|^2$$

$$\mathrm{Re}(\mathcal{T}_{\mathrm{DVCS}}) \sim c_0^{\mathcal{I}} + c_1^{\mathcal{I}}\cos(\phi) + c_2^{\mathcal{I}}\cos(2\phi)$$

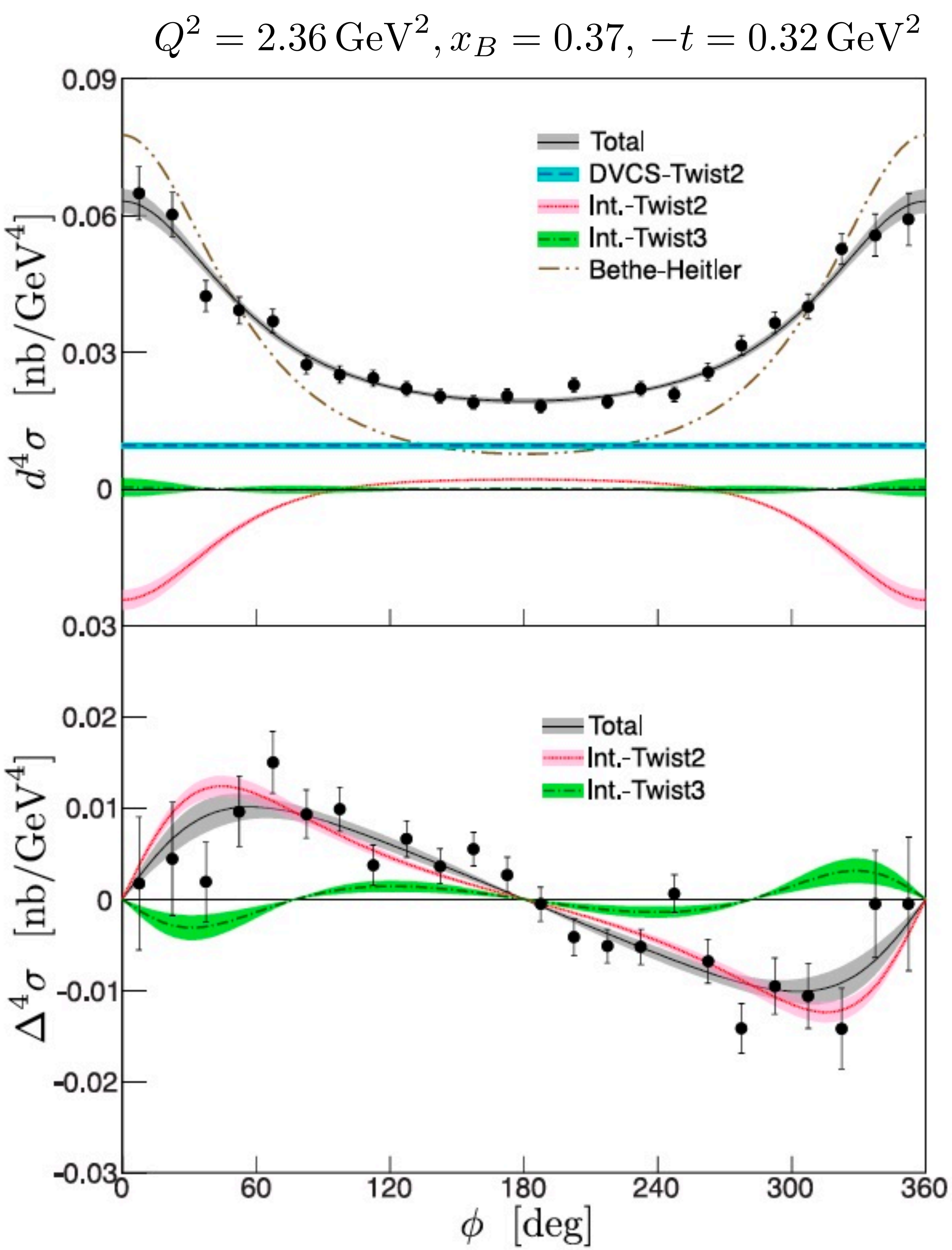
$$|\mathcal{T}_{\mathrm{DVCS}}|^2 \sim c_0^{\mathrm{DVCS}} + c_1^{\mathrm{DVCS}}\cos\phi$$

$$\Delta^4\sigma = \frac{d^4\overrightarrow{\sigma} - d^4\overleftarrow{\sigma}}{2} \sim \mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}})$$

$$\mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}}) \sim s_1^{\mathcal{I}}\sin\phi + s_2^{\mathcal{I}}\sin(2\phi)$$

Defurne, et al., PRDC 92, 055202 (2015)

A sample of typical results



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$$|\mathcal{T}_{\mathrm{DVCS}}|^2 \sim c_0^{\mathrm{DVCS}} + \cancel{c_1^{\mathrm{DVCS}}}\cos\phi$$

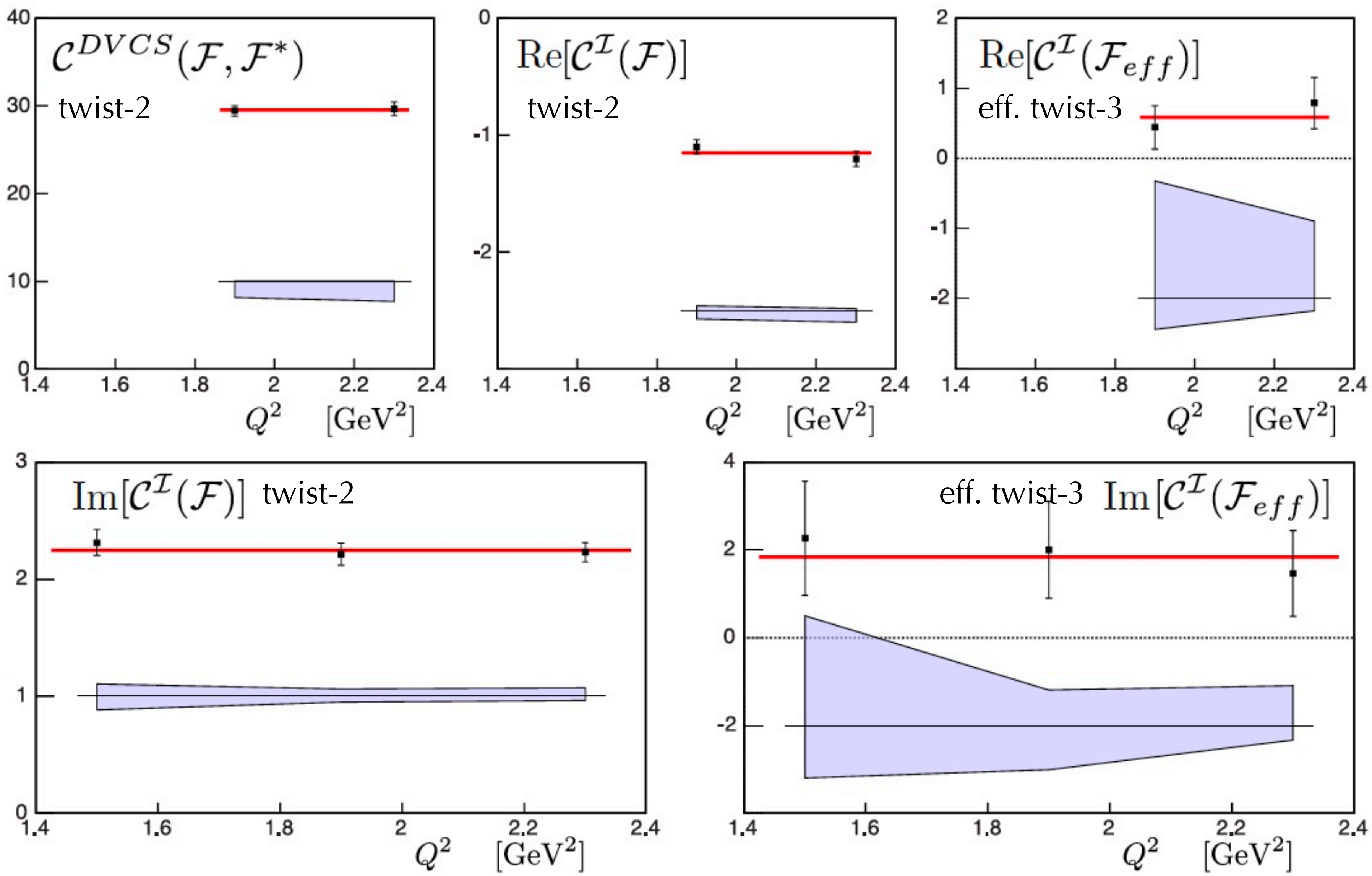
keeping only twist-2 contribution

$$\Delta^4\sigma = \frac{d^4\overrightarrow{\sigma} - d^4\overleftarrow{\sigma}}{2} \sim \mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}})$$

$$\mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}}) \sim s_1^{\mathcal{I}}\sin\phi + s_2^{\mathcal{I}}\sin(2\phi)$$

Defurne, et al., PRDC 92, 055202 (2015)

DVCS cross section: Q^2 dependence



unpolarized cross section

helicity dependent cross section

- Limited range in Q^2
- No Q^2 dependence observed
- Support leading-twist dominance

Defurne, et al., PRDC 92, 055202 (2015)

Extraction of Compton form factors

• Domain space of the unknown functions: 3 dimensions for GPDs (x, ξ, t) vs 1 dimension for PDF (x)

curse of dimensionality: *"It is easy to find a coin lost on a 100 meter line, but difficult to find it on a football field."*
Here we could say that we deal with a haystack, 100 m per side!

• Mapping of GPDs will significantly improve with the release of new data of unprecedented accuracy (JLab) and data in a larger kinematic domain (EIC)

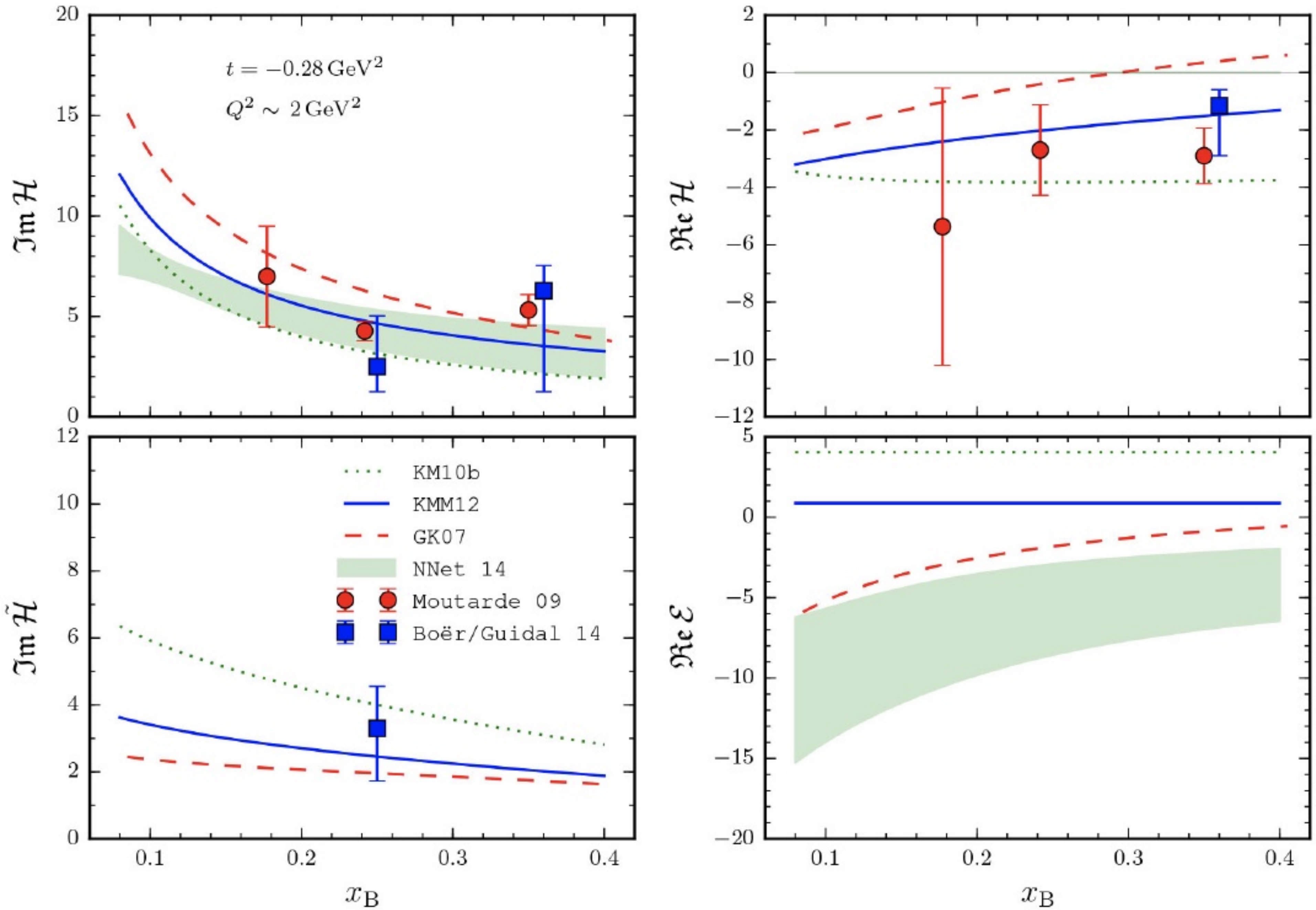
• Fitting strategies:

Global fits: use a parametrization of GPD and consider all kinematic bins at the same time

Local fits: take each kinematic bin independently and fit CCF-value at this point

Artificial neural network: already used for PDF fits. In progress for GPDs.

Comparison of different extraction methods



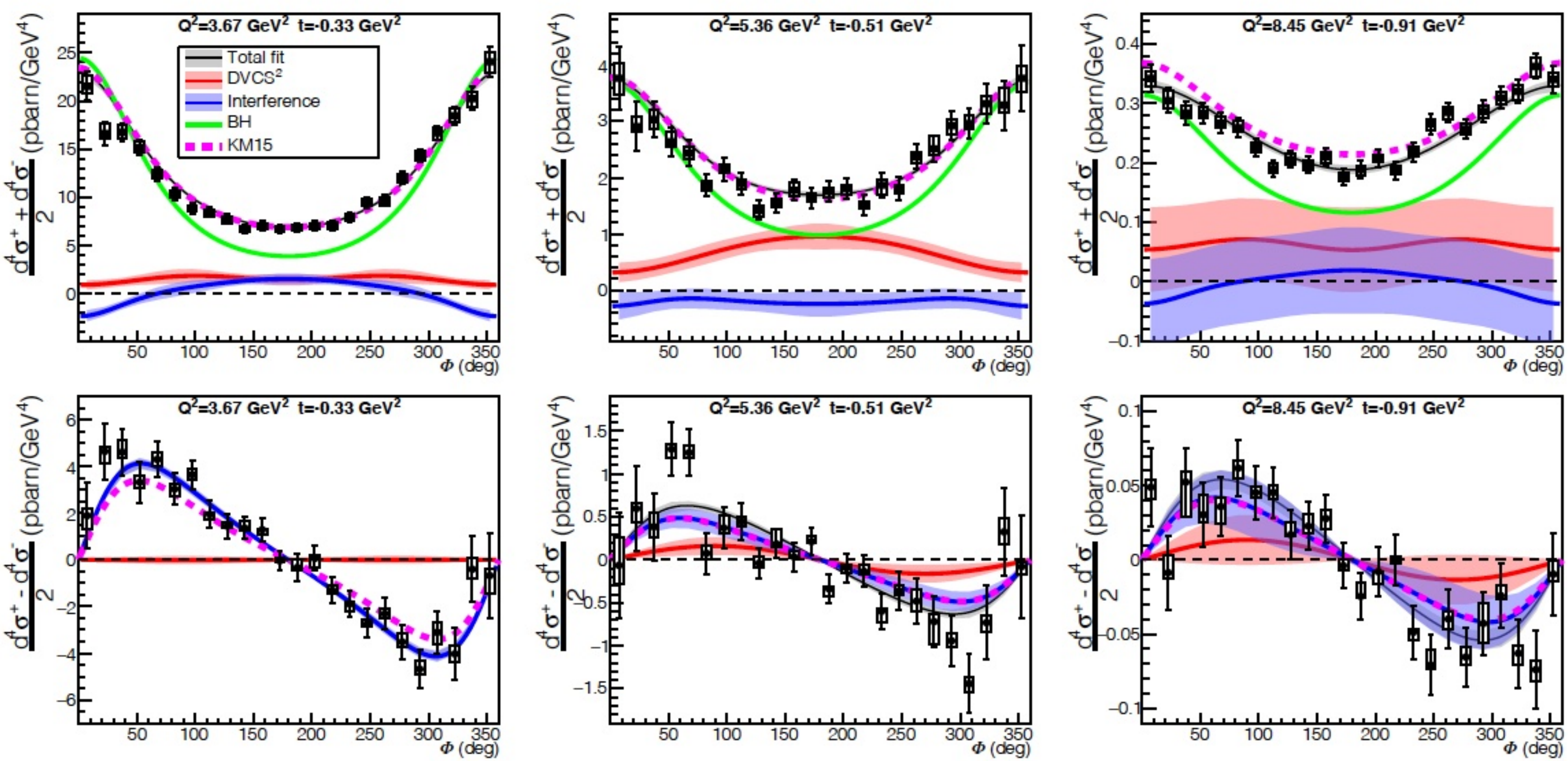
- Continuous curves: global fits based on double distributions or dispersion relations
- Two fit methods are compatible: good consistency check!

K. Kumericki, S. Liuti, H. Moutarde, Eur. Phys. J. A52 (2016) 157

Recent JLab12 results

Exploit energy dependence of cross section to separate $|\mathrm{DVCS}|^2$ and $\mathcal{I}$ contributions

$$|\mathcal{T}^{\mathrm{DVCS}}|^2 \propto 1/y^2 \qquad \mathcal{I} \propto 1/y^3 \quad \text{with} \quad y = E_b/\nu$$

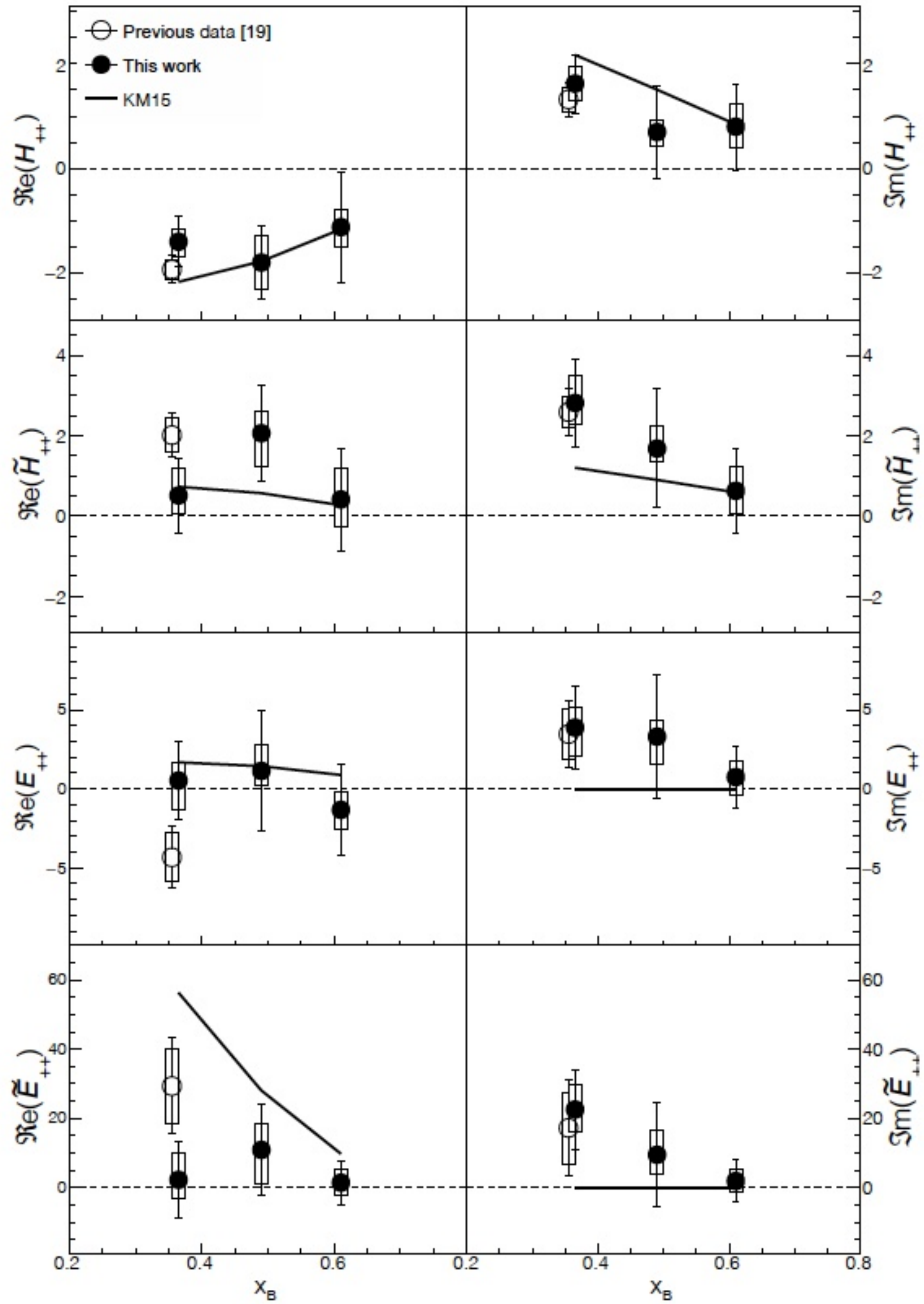


F. Georges et al. (Hall A Coll.), arXiv:2201.03714

Recent JLab12 results

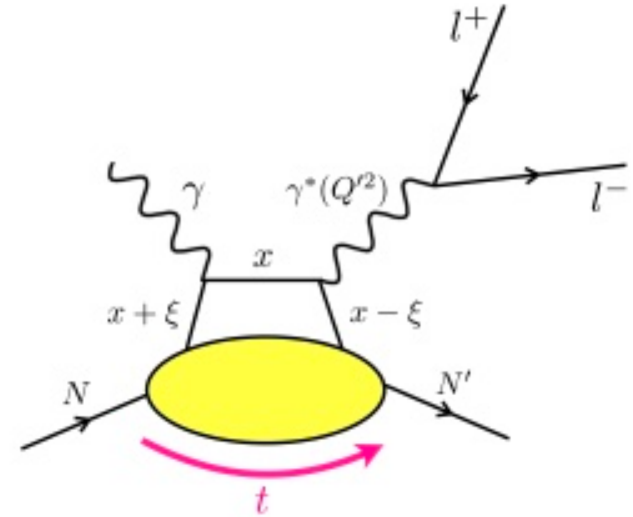
First extraction of all four helicity conserving CFFs!

F. Georges et al. (Hall A Coll.), arXiv:2201.03714



Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)

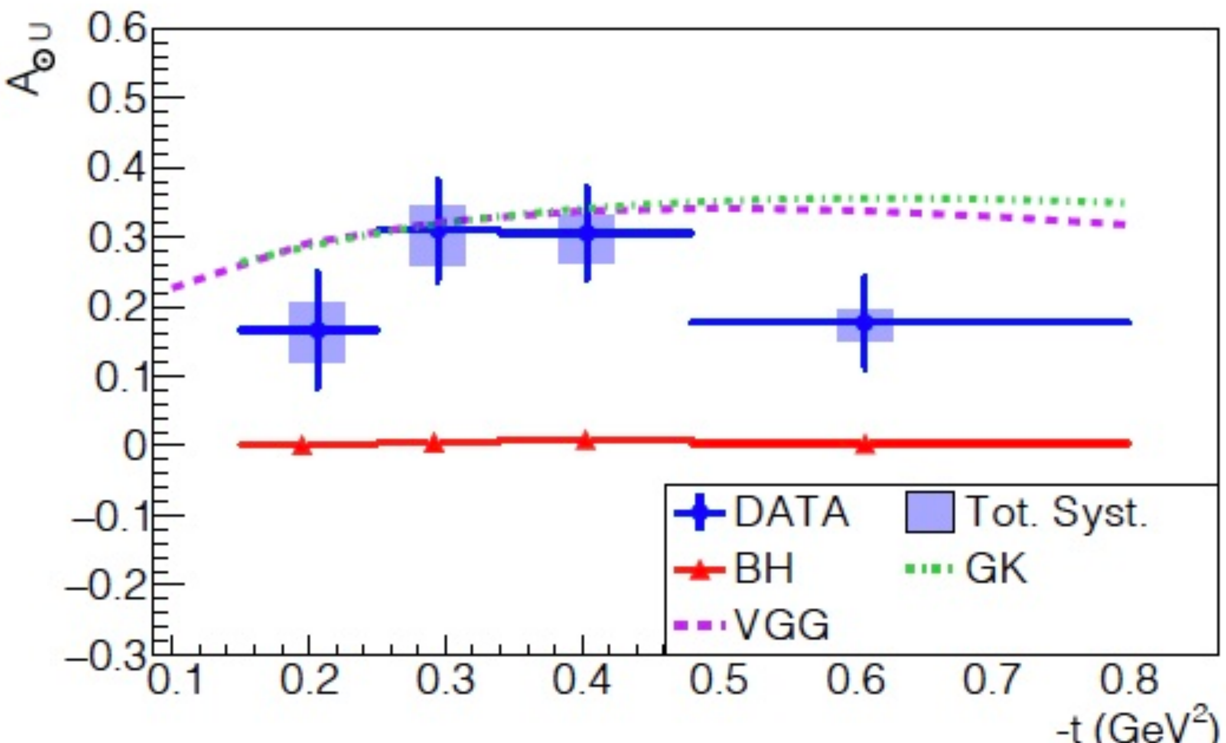


photon polarization asymmetry

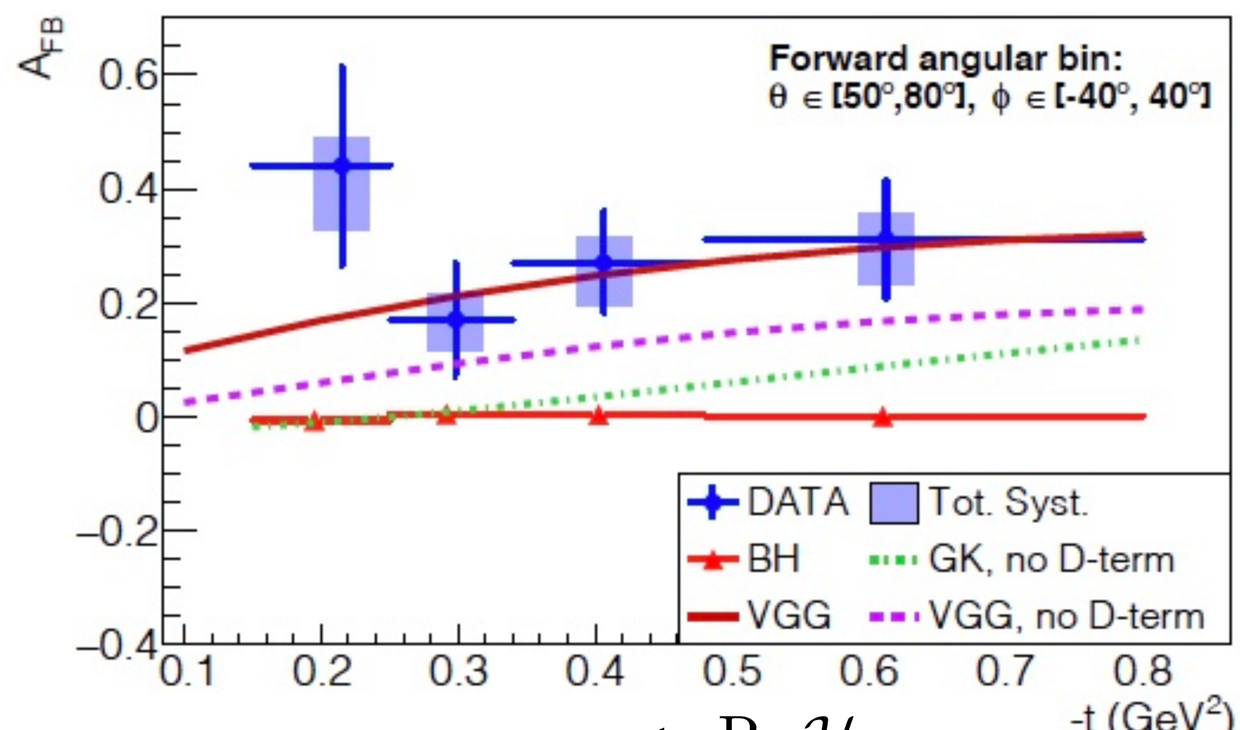
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

forward-backward asymmetry

$$A_{FB} = \frac{d\sigma(\theta,\phi) - d\sigma(180^\circ - \theta, 180^\circ + \phi)}{d\sigma(\theta,\phi) + d\sigma(180^\circ - \theta, 180^\circ + \phi)}$$



access to Im $\mathcal{H}$



access to Re $\mathcal{H}$

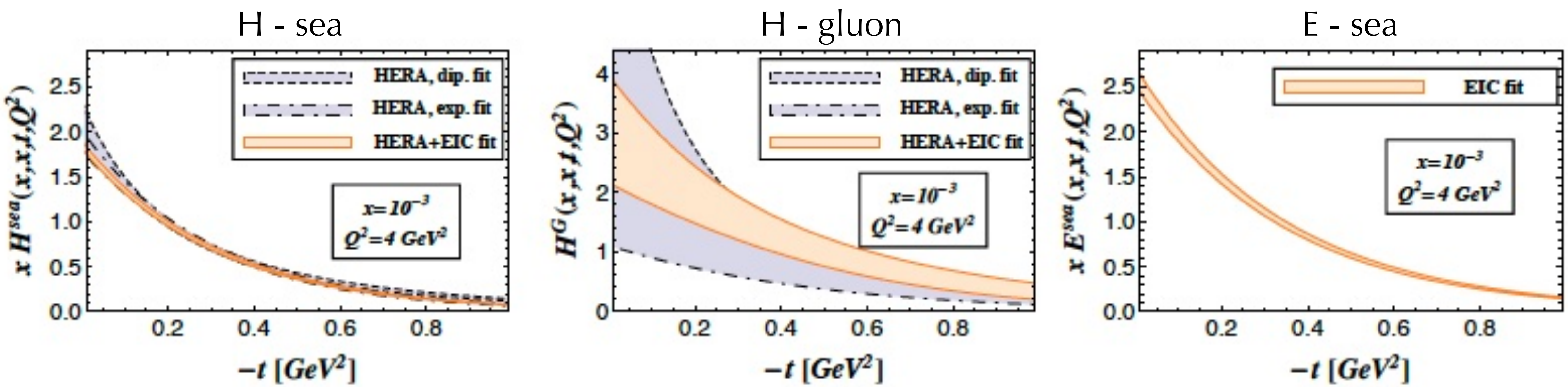
✓ Test of the universality of GPDs

✓ Further data from JLab12 and future EIC

✓ New promising path towards the extraction of Re $\mathcal{H}$ and then the D-term

Impact of EIC on GPD measurements

EIC Yellow Report: arXiv: 2103.05419



$x = 10^{-3}, Q^2 = 4\,\mathrm{GeV}^2$