Transverse Momentum PDFs (TMDs)

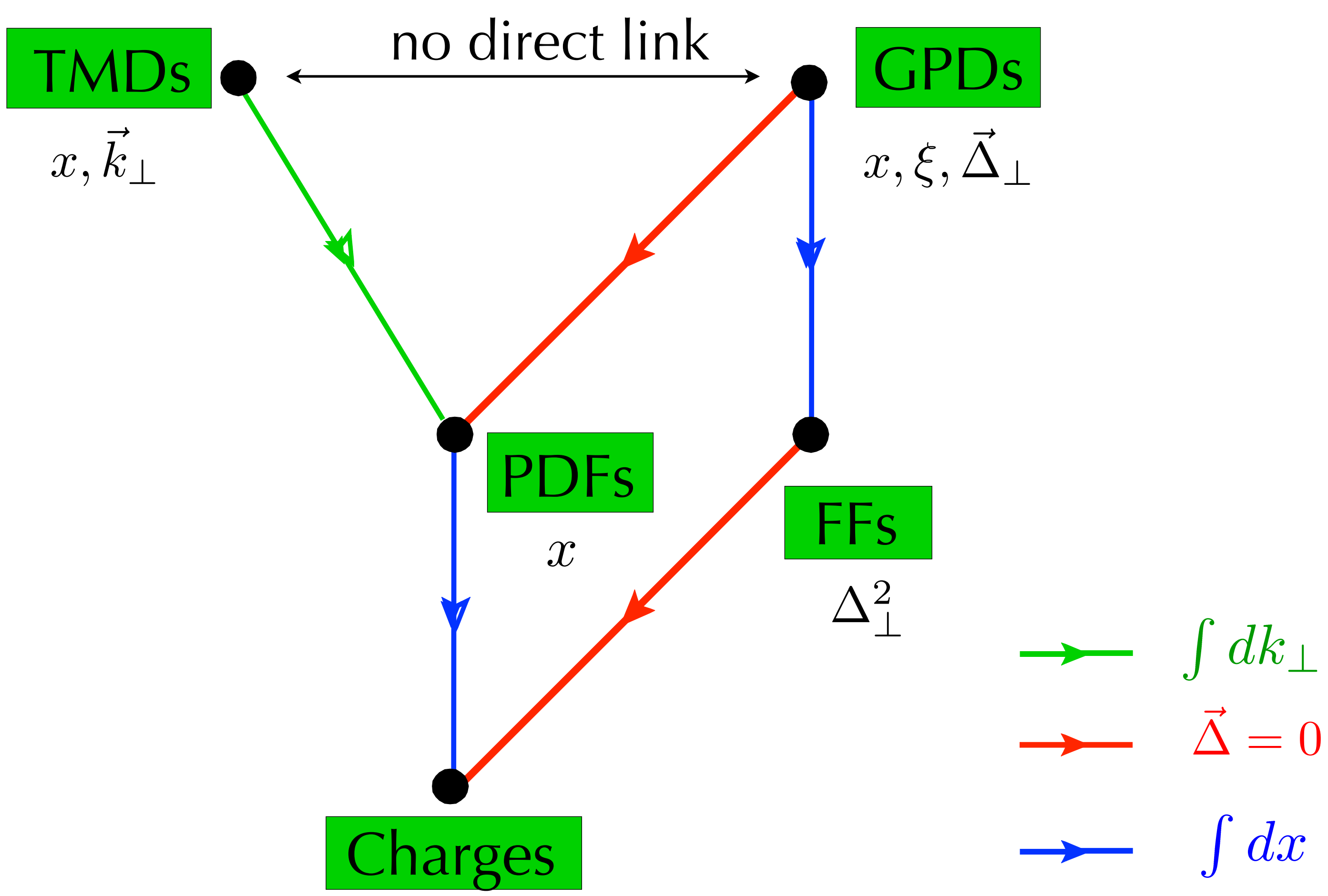
$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}e^{ik\cdot z}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0}}$$

Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction

$k_\perp$: parton transverse momentum



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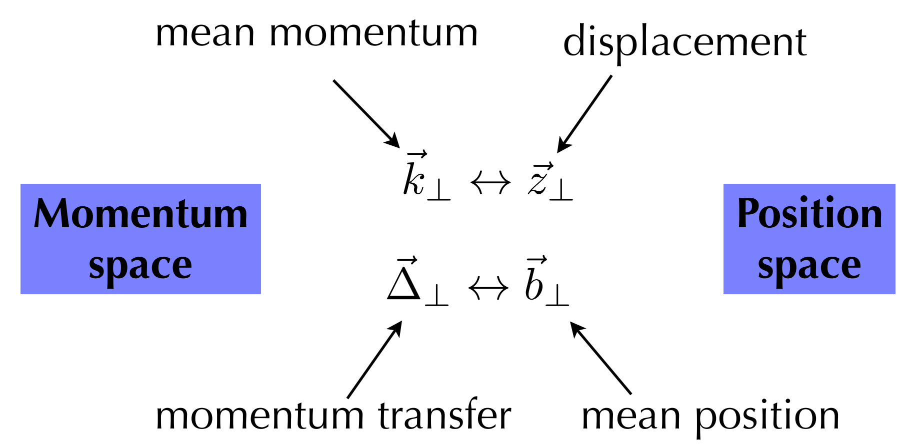

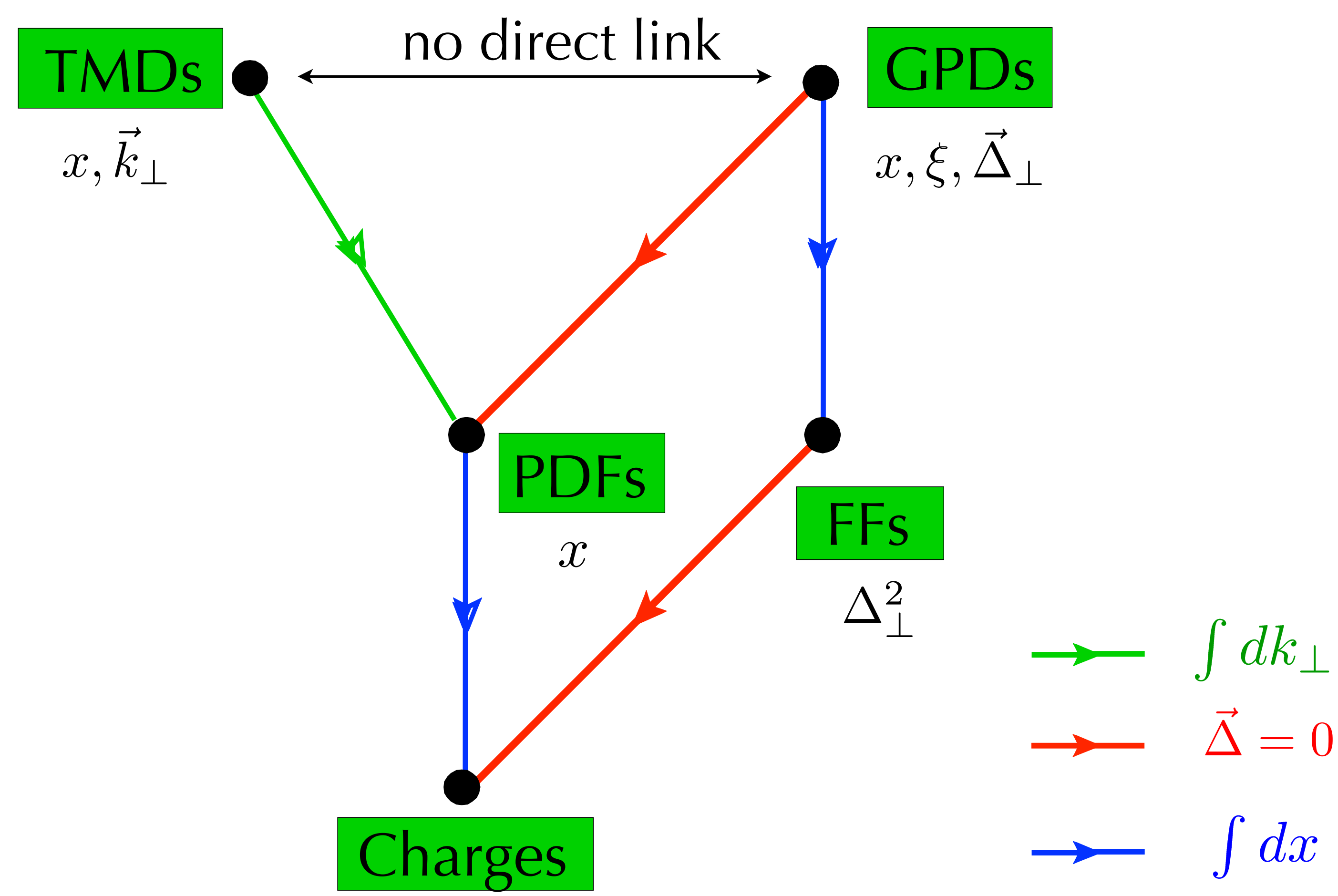

Transverse Momentum PDFs (TMDs)

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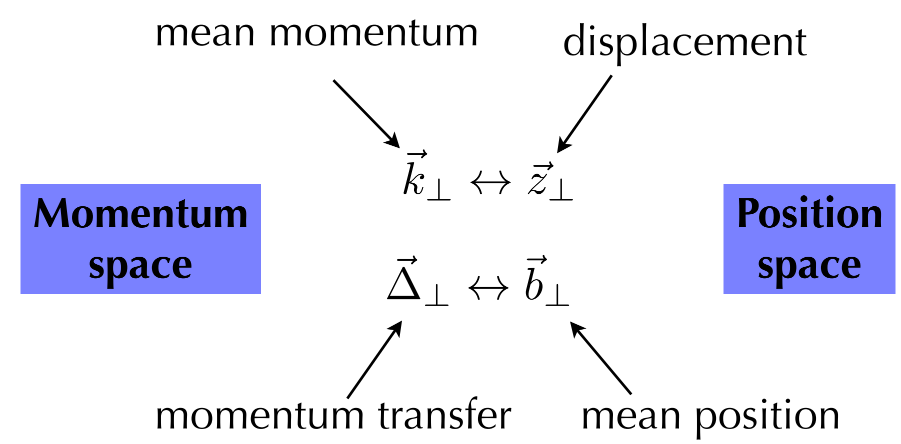
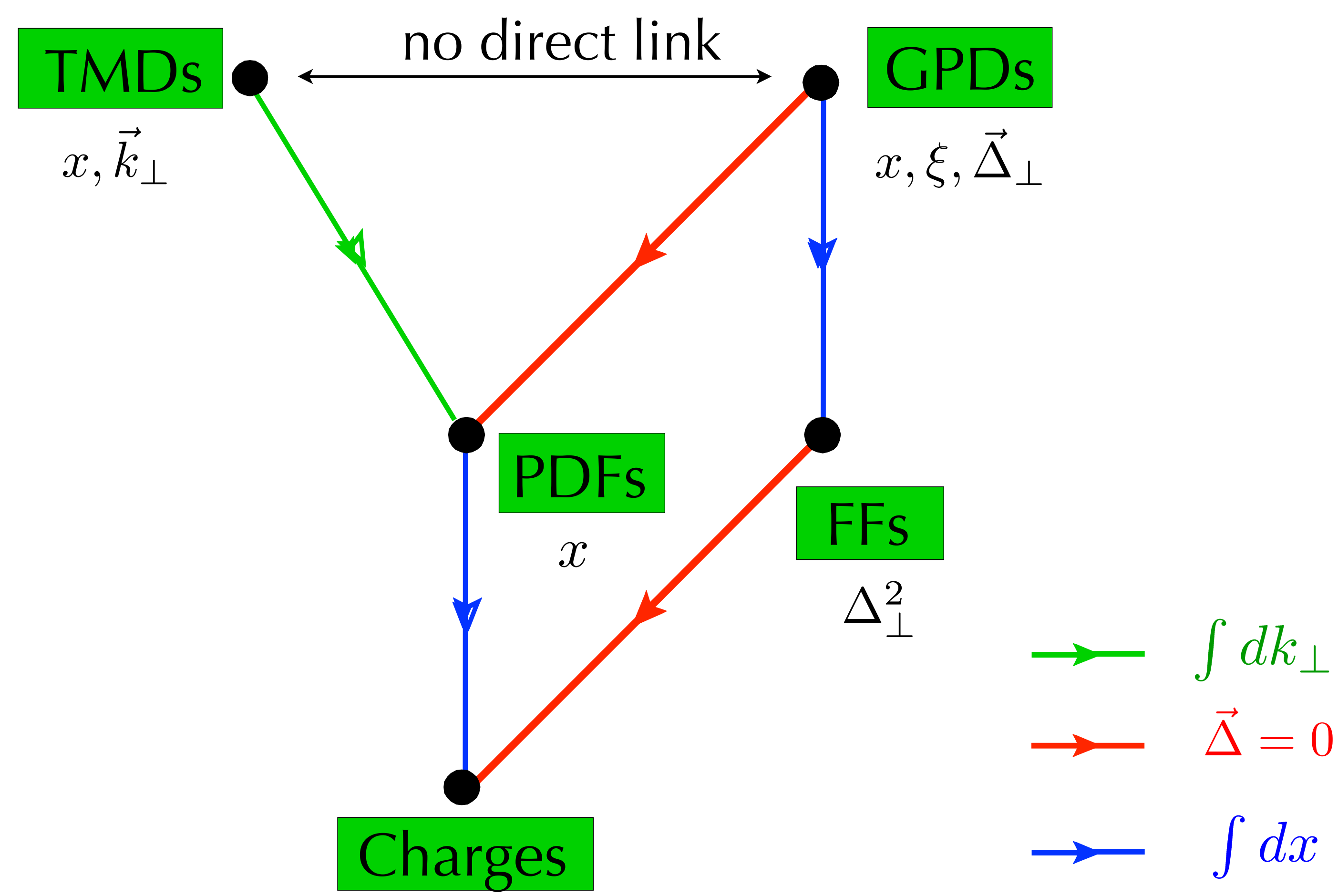
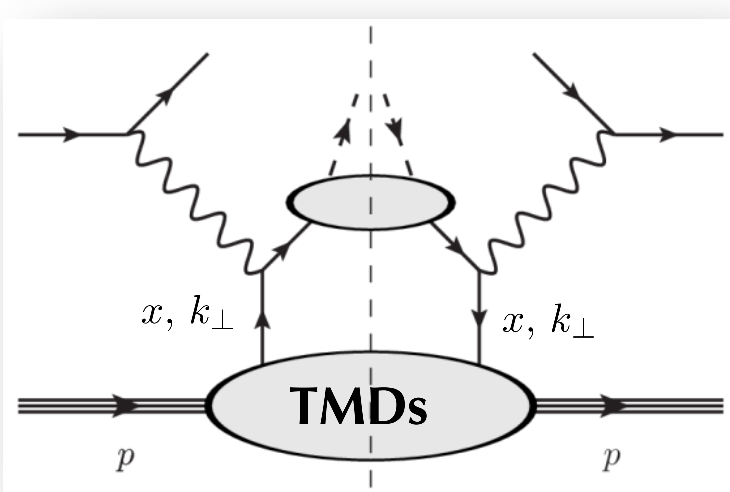
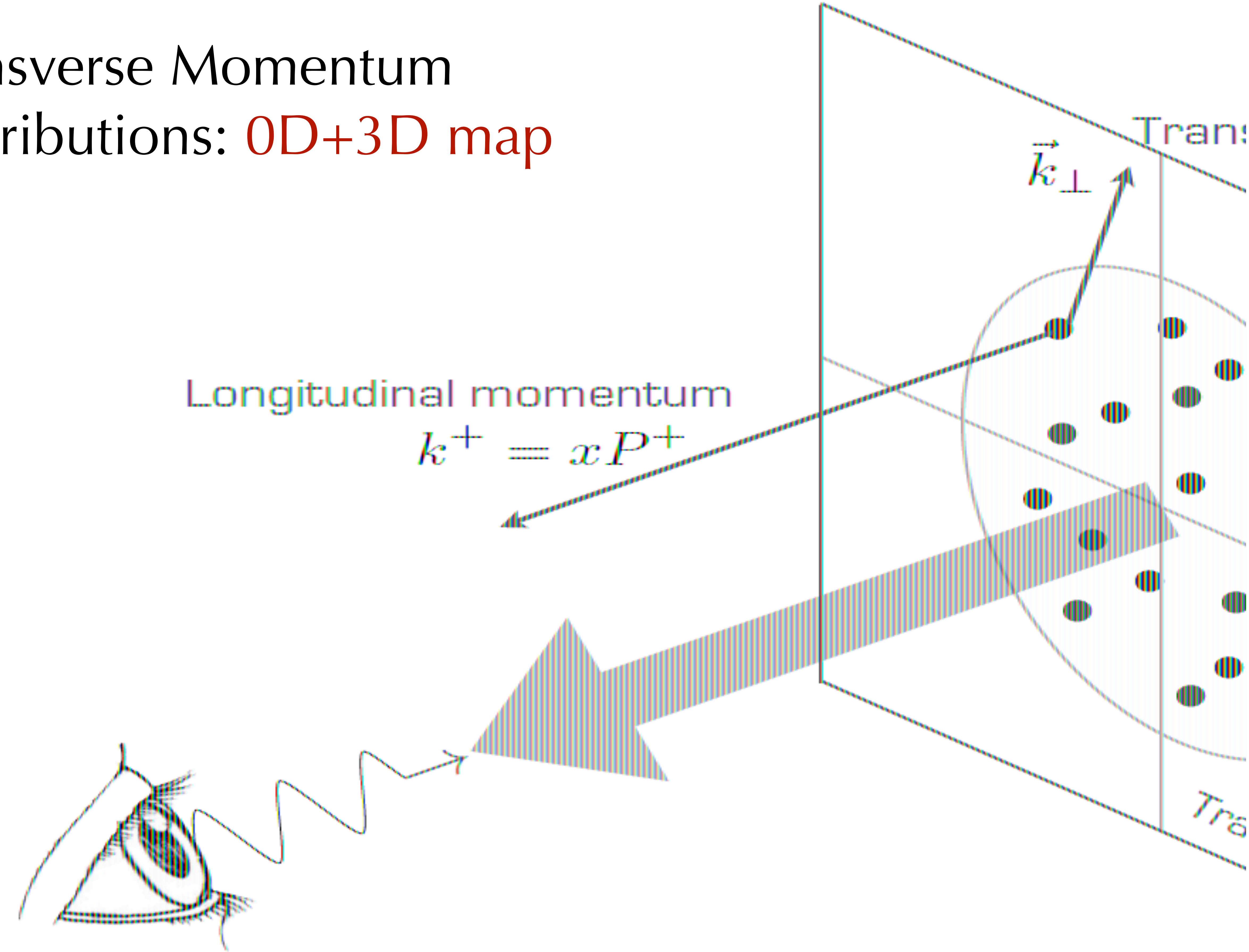




Semi-Inclusive
Deep Inelastic Scattering



Transverse Momentum Distributions: 0D+3D map



Key information from TMDs

- Complete momentum spectrum of single particle

- Transverse momentum size as function of x (3D map)

- Spin-Spin and Spin-Orbit Correlations of partons

- Information on parton orbital angular momentum
  (no direct model-independent relation)

- Study interesting new non-trivial aspects of pQCD: role of re-scattering of active partons, factorization, universality, evolution,....

- Non-perturbative structure we cannot calculate with QCD

A few references on TMDs

- V. Barone, A. Drago, P. Ratcliffe, Phys. Rept. 359 (2002) 1
- U. D'Alesio, F. Murgia, Prog. Part. Nucl. Phys. 61 (2008) 394
- A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. Mulders, M. Schlegel, JHEP 0702 (2007) 93
- M. Anselmino, et al., Eur. Phys. J. A47 (2011) 35
- C. Aidala, S. Bass, D. Hasch, G. Mallot, Rev. Mod. Phys. 85 (2013) 655
- Collins, *Foundations of Perturbative QCD*, Cambridge U. Press, 2011
- A. Metz, A. Vossen, Prog. Part. Nucl. Phys. 91 (2016) 136
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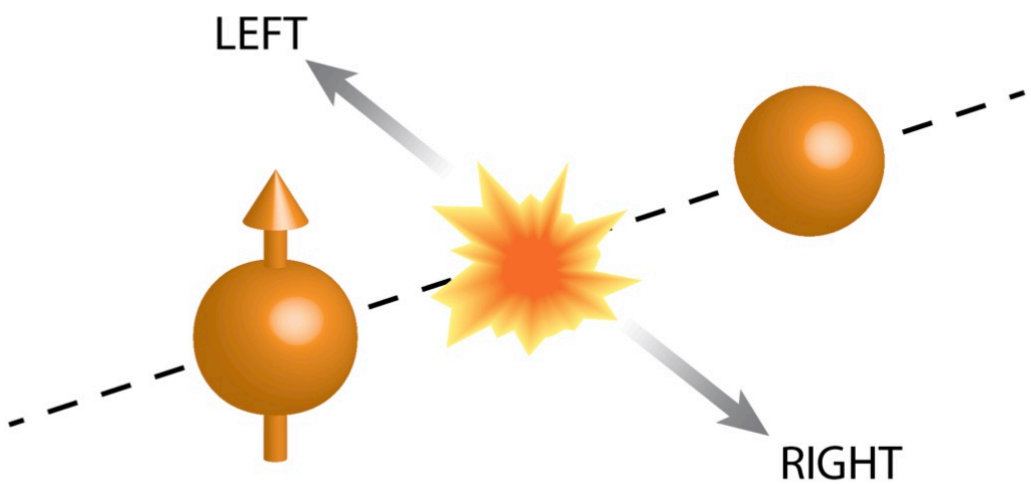
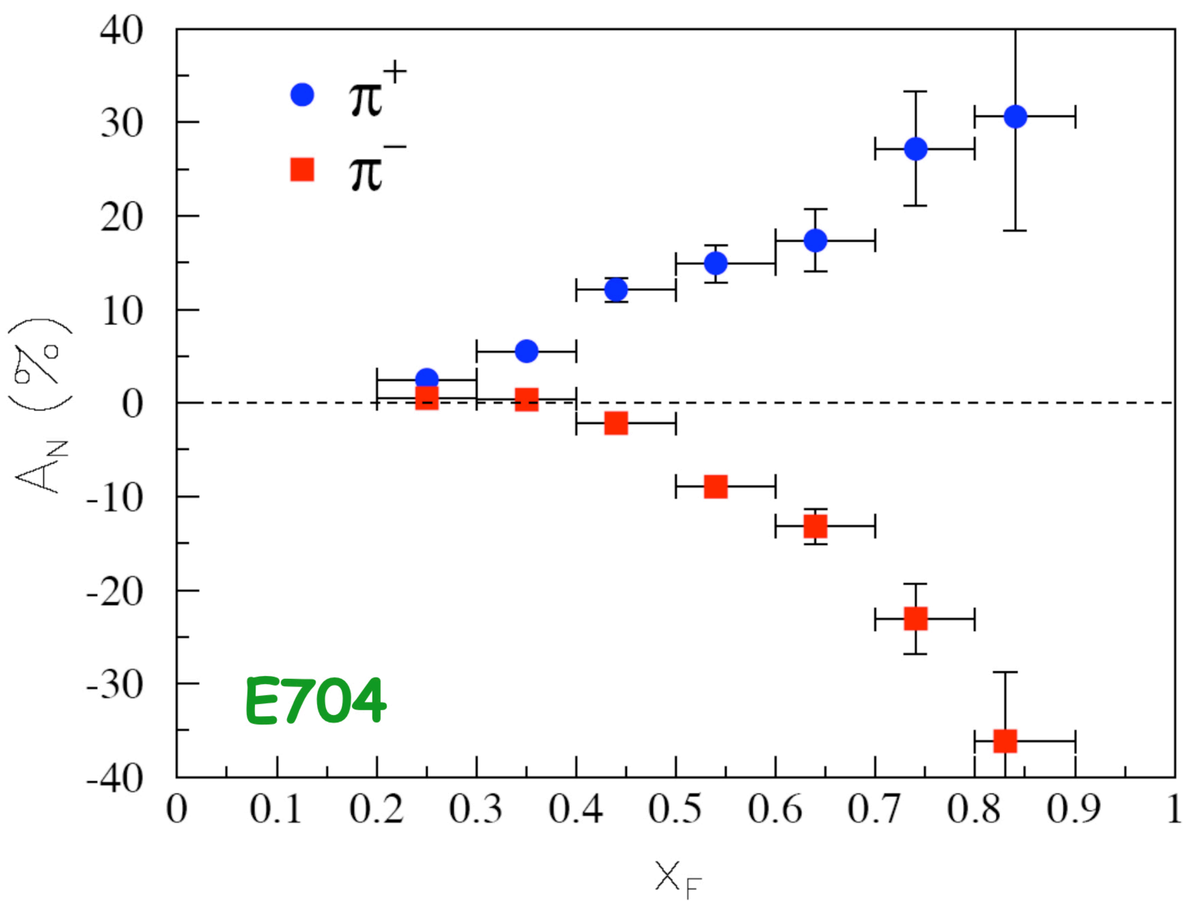
Where TMDs started from ….. 1991

Consider the scattering of a transversely polarized proton off an unpolarized proton

$$p^{\uparrow}p \to \pi X$$



$$A_N \equiv \frac{L-R}{L+R} = \frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$$



$\sqrt{s} = 20\,\mathrm{GeV}$

FNAL E704
PLB 264 (1991) 462
PLB 261 (1991) 201

[ANL: ZGS ('76) ; Fermilab: E704 ('91); BNL: AGS ('99); STAR ('02); BRAHMS ('05); PHENIX ('13)]

Early theory prediction:

If partons have only longitudinal momentum, SSA would vanish

G.L. Kane, J. Pumplin, W. Repko, Phys. Rev. Lett. 41 (1978) 1689

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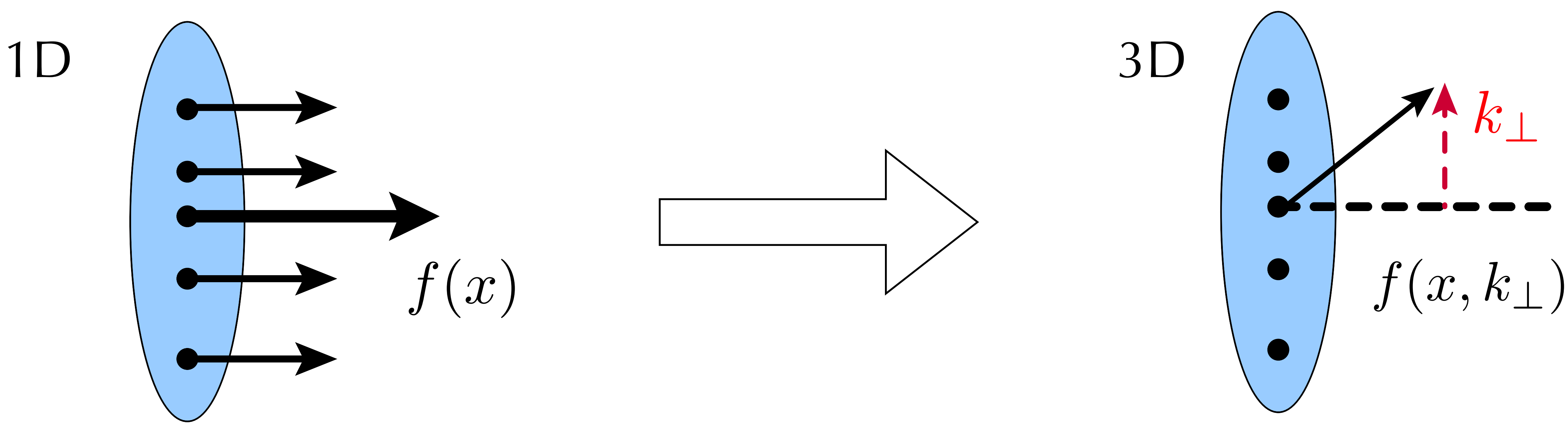
PHYSICAL REVIEW D VOLUME 41, NUMBER 1 1 JANUARY 1990

Single-spin production asymmetries from the hard scattering of pointlike constituents

Dennis Sivers
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

When one takes into account the transverse momenta of the constituents in a polarized proton, there exists a kinematic, "trigger-bias," effect in the formulation of the QCD-based hard-scattering model which can lead to single-spin production asymmetries.



TMDs: rich quantum correlations between spin and momentum

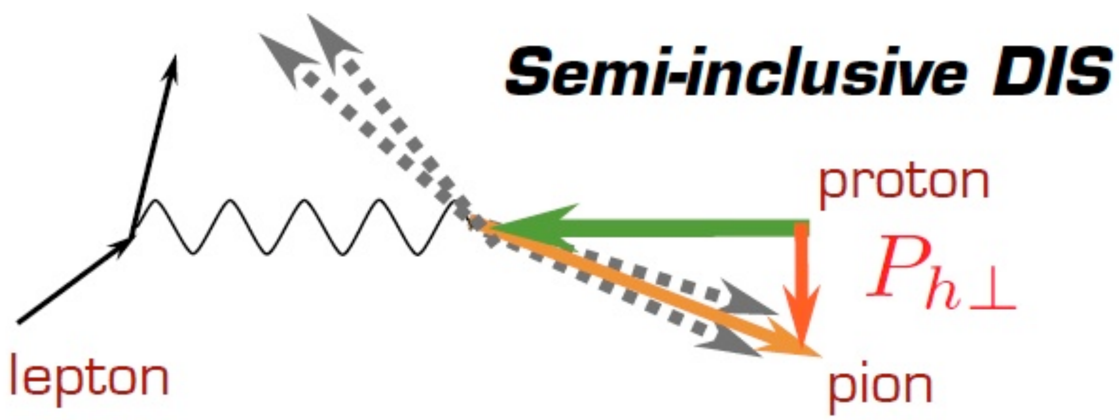
nucleon pol. \ quark pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

(similar classification for gluons)

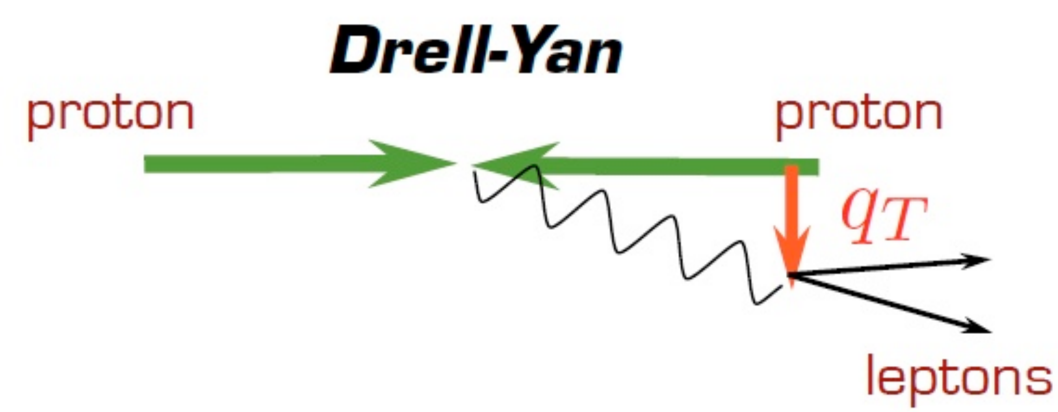
- TMDs in **black** survive transverse-momentum integration
- TMDs in **red** are T-odd (change sign in SIDIS and DY processes)
- TMDs in blue require OAM transfer
- No effects for U/L and L/U polarizations due to parity invariance

Where can we access TMDs?

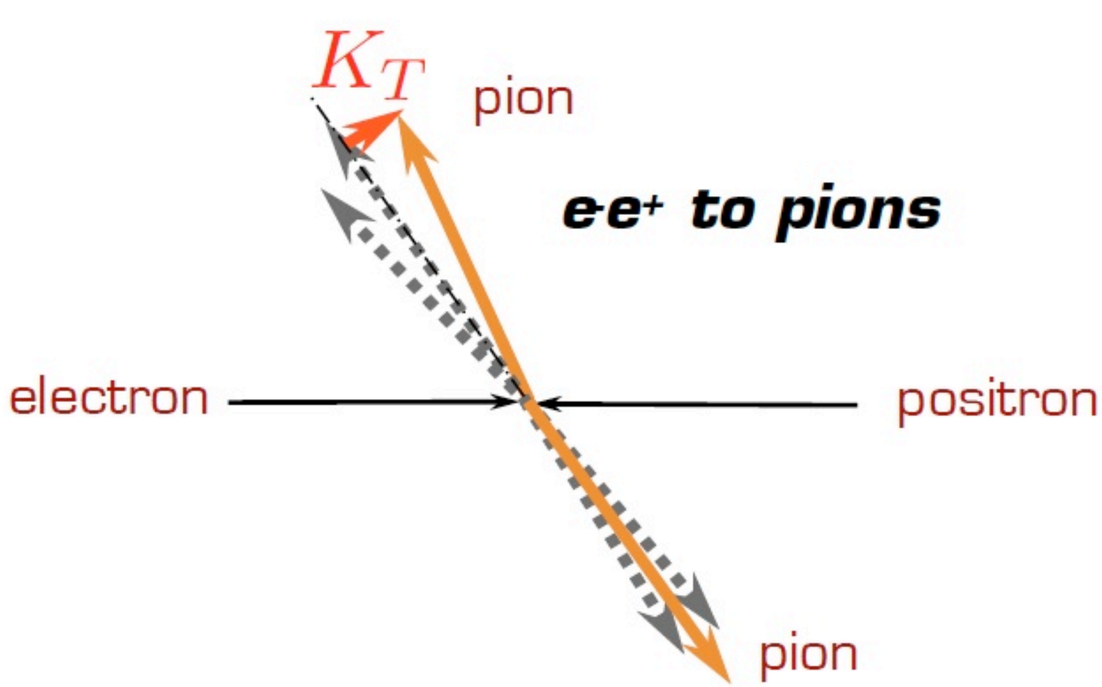
$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$



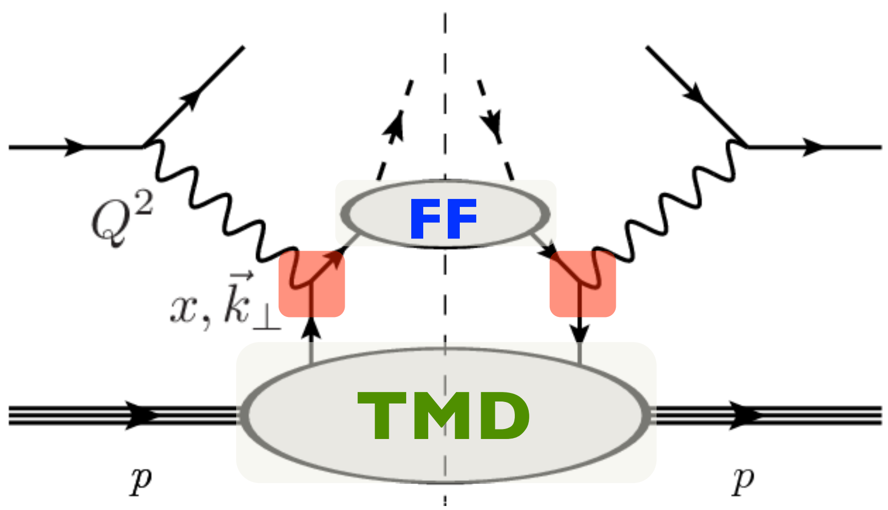
$$e^+e^- \to hh'X$$



we are interested
in the region of small transverse-momenta
sensitive to non-perturbative QCD effects

Where can we access TMDs?

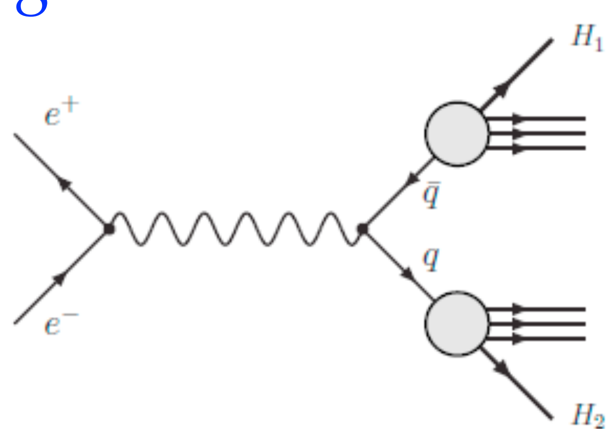
$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



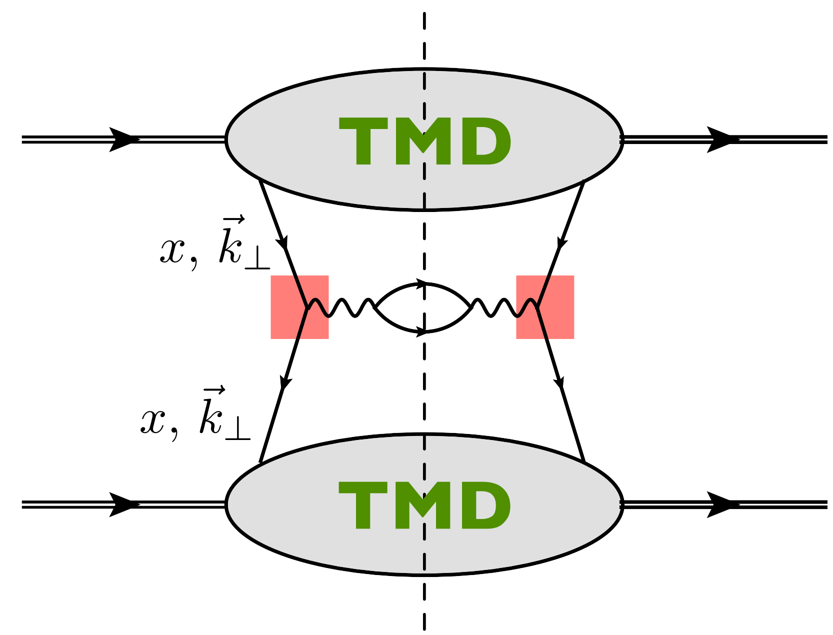
$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Fragmentation Functions

$$e^+e^- \to hh'X$$



$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$



$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

✓Factorization

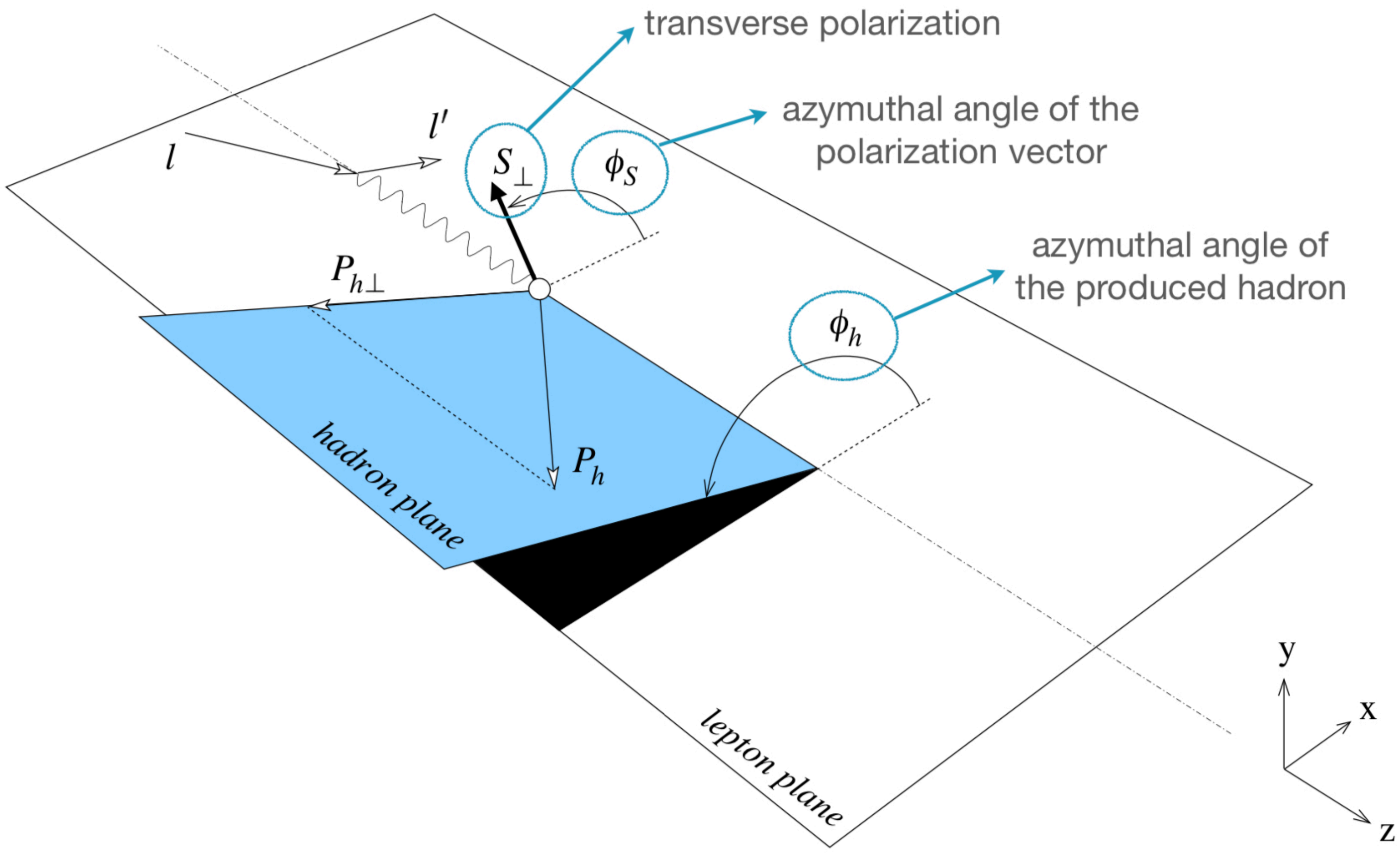
✓Universality

SIDIS

$$\ell(l, \lambda_\ell) + N(P, S) \to \ell(l', \lambda'_\ell) + h(P_h, S_h) + X$$

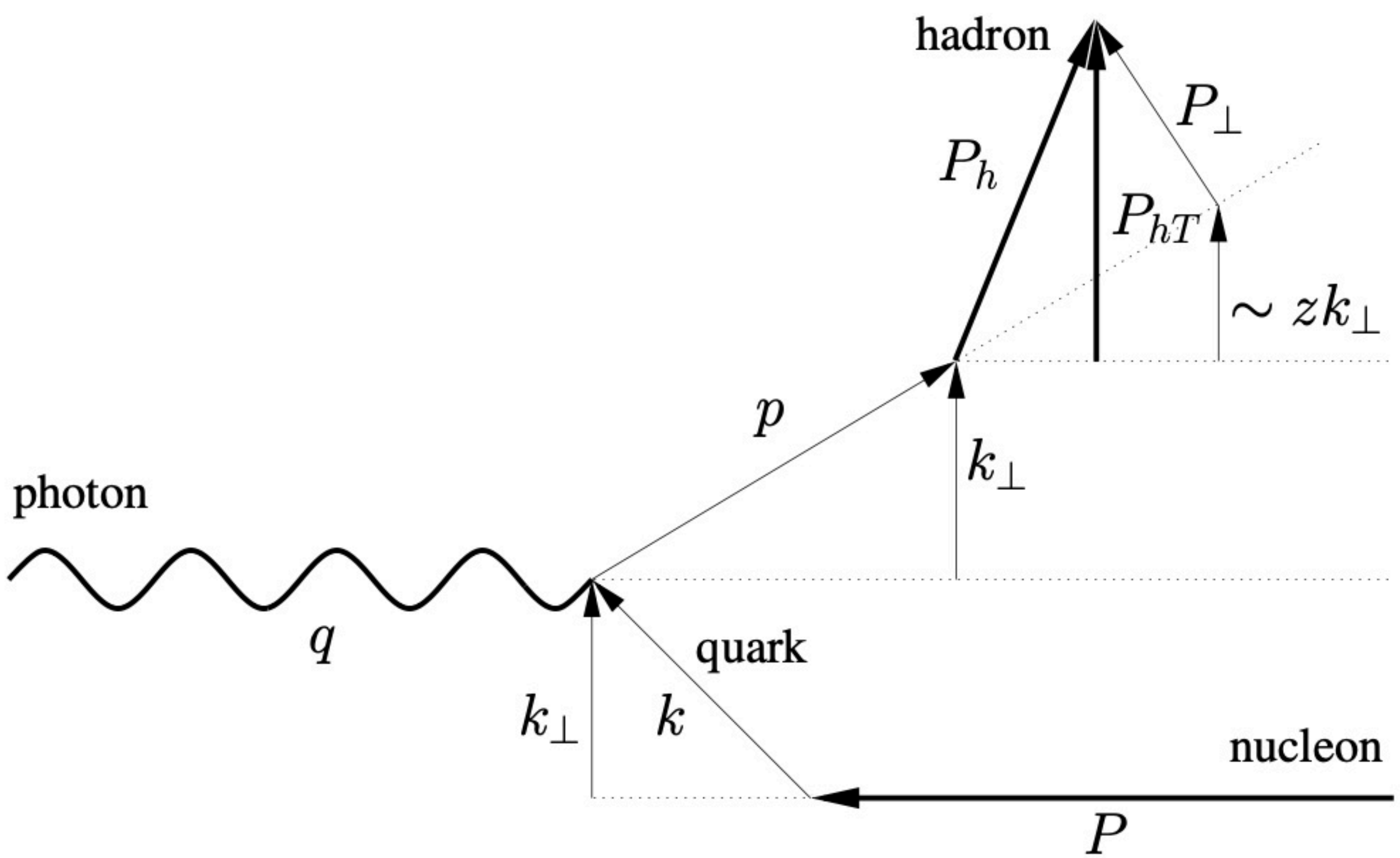
- 6 independent kinematical variables

$$x_B = \frac{Q^2}{2P \cdot q} \qquad Q^2 \qquad \phi_S \qquad z = \frac{P \cdot P_h}{P \cdot q} \qquad P_{h\perp} = |\vec{P}_{h\perp}| \qquad \phi_h$$



$$d\sigma \sim L^{\mu\nu} W_{\mu\nu}$$

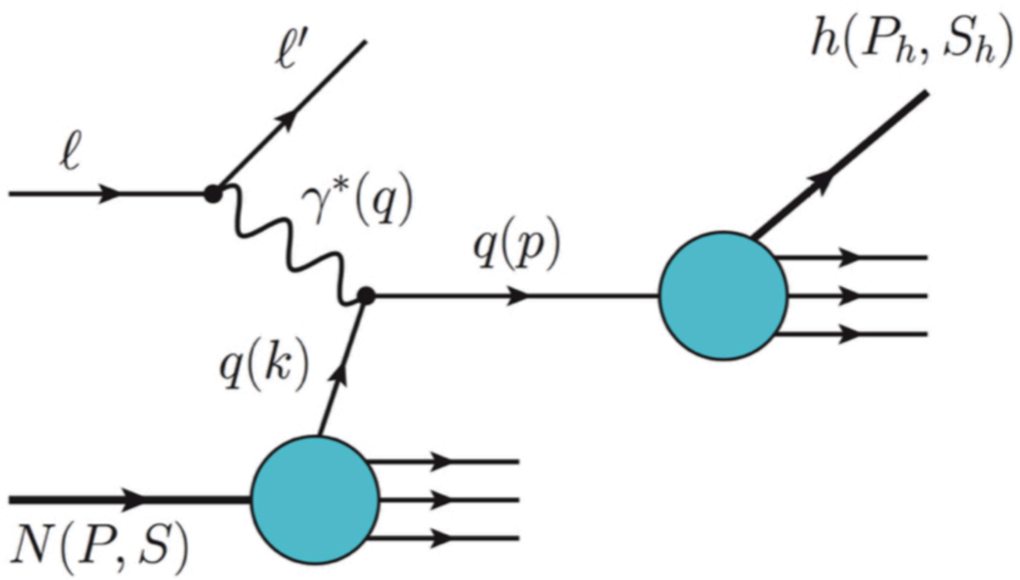
SIDIS kinematics

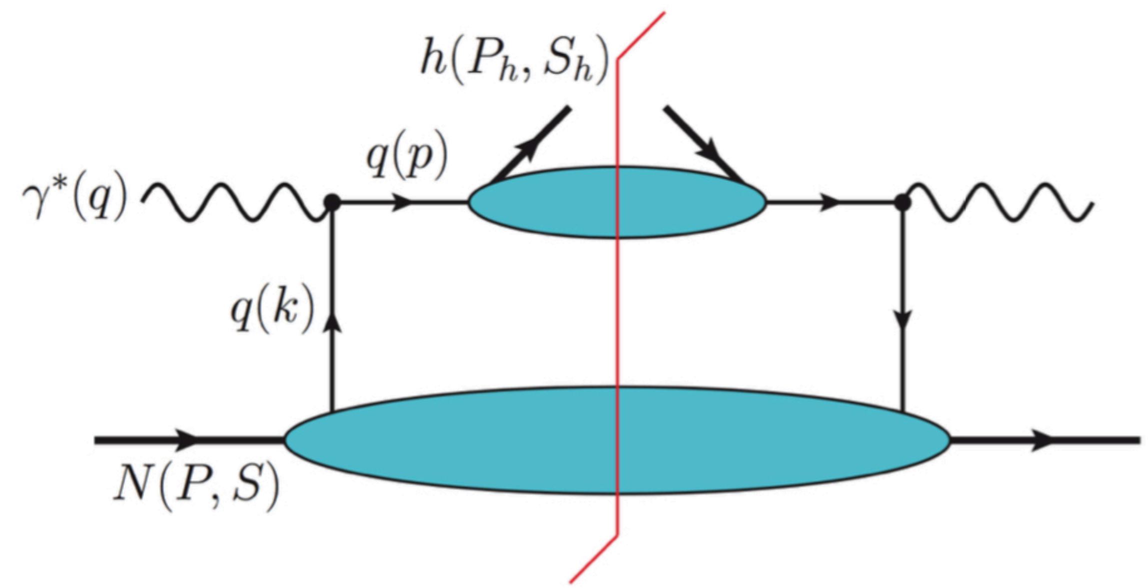


$$\vec{P}_{hT} \approx z\vec{k}_\perp + \vec{P}_\perp$$

Figure from Bacchetta et al., arXiv: 2206.07598

Hadronic tensor at tree level



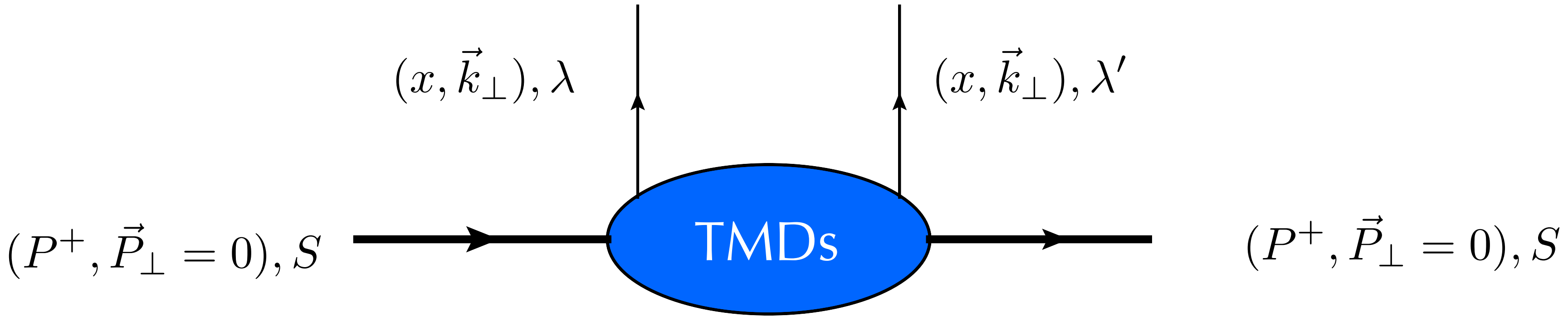


$$W^{\mu\nu} \sim \frac{2x_B z_h}{Q^2} \sum_q e_q^2 \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)$$

$$\times \mathrm{Tr}\left[\int dk^- \, \Phi^q(k, P, S) \, \gamma^\mu \int dp^+ \, \Delta(p, P_h) \, \gamma^\nu \right] \Big|_{k^+ = x_B P^+}$$

TMD Correlators

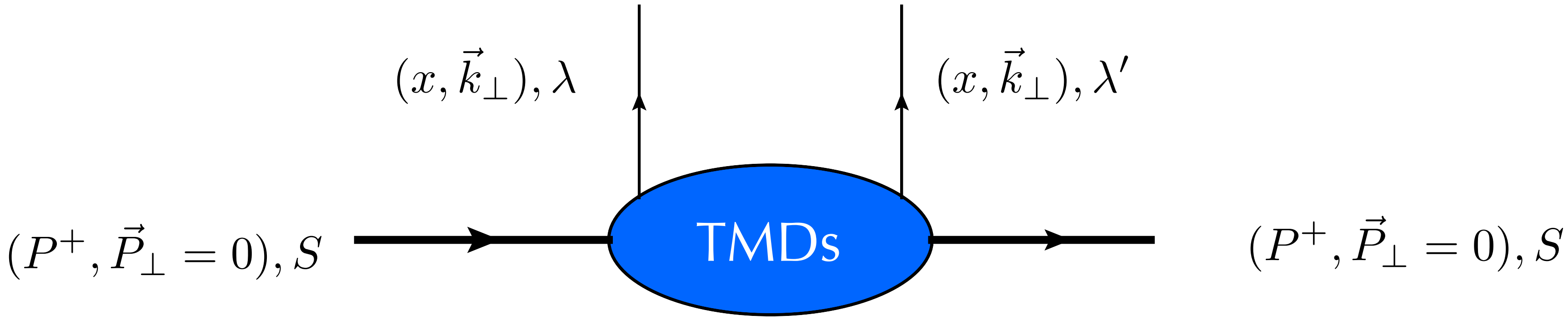
$$\Phi^q(x,\vec{k}_\perp,P,S) = \int dk^- \, \Phi^q(k,P,S) = \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \langle P,S|\bar{\psi}(-\frac{z}{2})\psi(\frac{z}{2})|P,S\rangle\Big|_{z^+=0}$$



TMDs parametrize quark-quark correlator

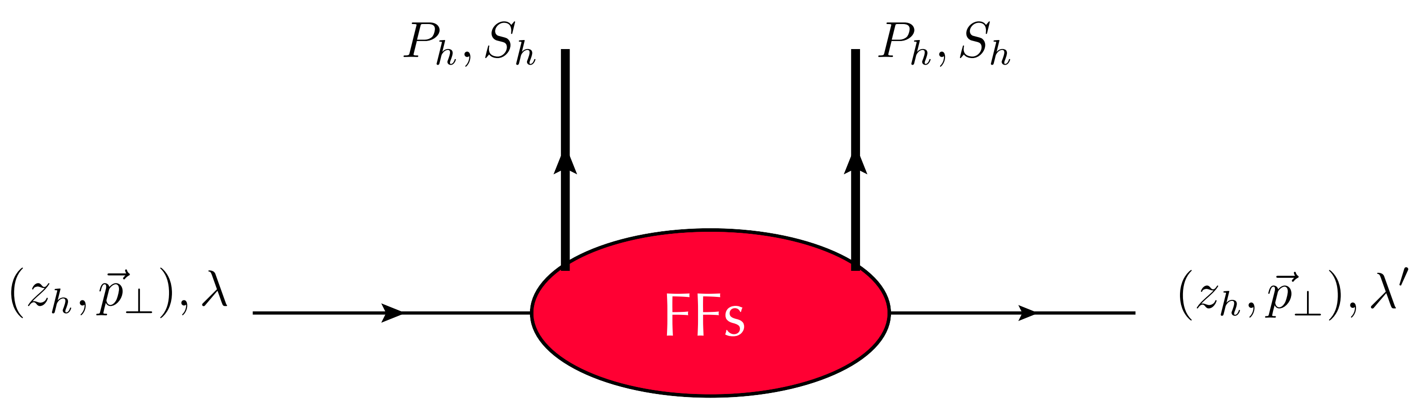
TMD Correlators

$$\Phi^q(x,\vec{k}_\perp,P,S) = \int dk^- \, \Phi^q(k,P,S) = \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \langle P,S|\bar{\psi}(-\frac{z}{2})\psi(\frac{z}{2})|P,S\rangle\Big|_{z^+=0}$$



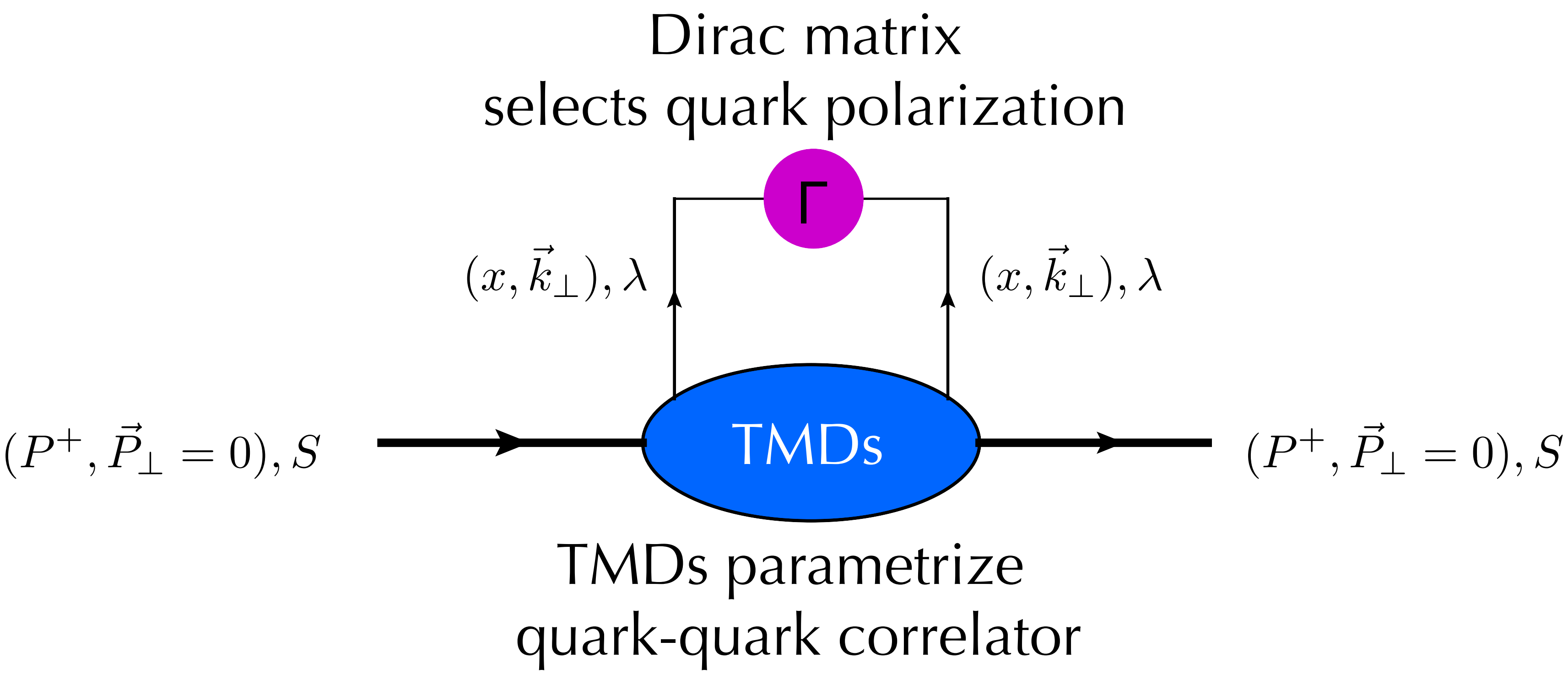
TMDs parametrize quark-quark correlator

$$\Delta(z_h,\vec{p}_\perp,P_h,S_h) = \int dp^+ \, \Delta(p,P_h,S_h) = \sum_X \int \frac{dz^+ d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ip\cdot z} \langle 0|\psi(\frac{z}{2})|P_h,S_h;X\rangle\langle P_h,S_h;X|\bar{\psi}(-\frac{z}{2})|0\rangle\Big|_{z^-=0}$$



FFs parametrize fragmentation correlator

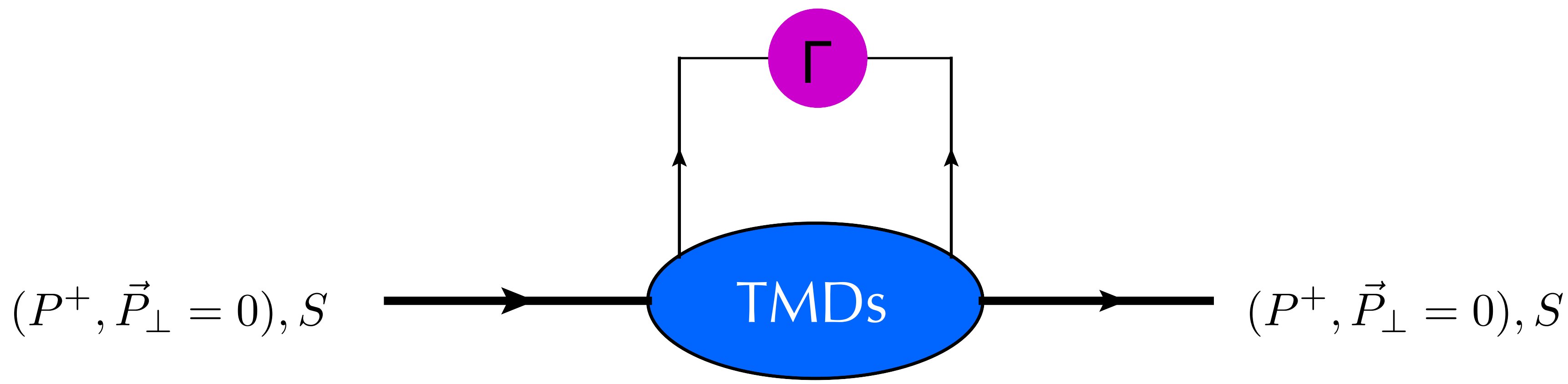
TMDs - PDFs of Quarks



$$\Phi_{ij}(x, k_\perp, S) = \int \frac{dz^- d^2 z_\perp}{2(2\pi)^3} e^{ik\cdot z} \langle p, S | \bar{\psi}_j(-\frac{z}{2})\, \mathcal{W}[-\frac{z}{2}, \frac{z}{2}] \psi_i(\frac{z}{2}) | p, S \rangle \Big|_{z^+=0}$$

$$\Phi^{[\Gamma]} = \frac{1}{2} \mathrm{Tr}[\Phi \Gamma]$$

TMD-PDFs of quarks at leading twist



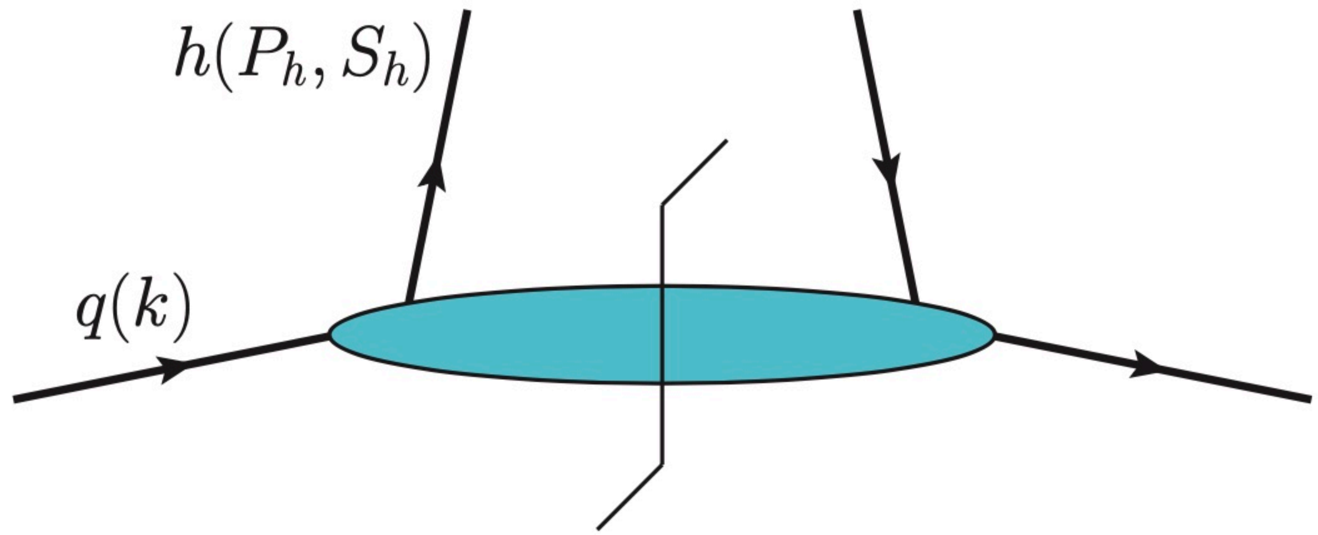
- leading-twist: $\Gamma = (\gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{i+}\gamma_5)$

- spin four-vector: $S = \left(\dfrac{\Lambda\, P^+}{M}, -\dfrac{\Lambda\, P^-}{M}, \vec{S}_\perp \right) \qquad S^2 = \Lambda^2 - \vec{S}_\perp^{\,2} = -1 \qquad P \cdot S = 0$

$$\begin{aligned}
\Phi^{q[\gamma^+]}(x, \vec{k}_\perp) &= f_1^q - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{M}\, f_{1T}^{\perp\, q} \\
\Phi^{q[\gamma^+\gamma_5]}(x, \vec{k}_\perp) &= \lambda\, \Lambda\, g_1^q + \frac{\lambda\, \vec{k}_\perp \cdot \vec{S}_\perp}{M}\, g_{1T}^{\perp\, q} \\
s_\perp^i\, \Phi^{q[i\sigma^{i+}\gamma_5]}(x, \vec{k}_\perp) &= \vec{s}_\perp \cdot \vec{S}_\perp\, h_1^q + \frac{\Lambda\, \vec{k}_\perp \cdot \vec{s}_\perp}{M}\, h_{1L}^{\perp\, q} - \frac{\epsilon_\perp^{ij} k_\perp^i s_\perp^j}{M}\, h_1^{\perp\, q} \\
&\quad + \frac{1}{2M^2} \left(2\vec{k}_\perp \cdot \vec{s}_\perp\, \vec{k}_\perp \cdot \vec{S}_\perp - \vec{k}^{\,2}\, \vec{s}_\perp \cdot \vec{S}_\perp \right) h_{1T}^{\perp\, q}
\end{aligned}$$

- TMDs depend on x and $\vec{k}_\perp^{\,2}$

Quark fragmentation functions



quark polarization

final hadron pol.

	U	L	T
U	D_1		$H_1^\perp$
L		G_1	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

- Same interpretation as for TMDs, but with the role of quark and hadron interchanged
- FFs in red are T-odd

SIDIS cross section

$$\frac{d\sigma}{dx_B dy d\phi_S dz_h d\phi_h dP_{h\perp}^2} \sim \Big\{ (1-y+\frac{1}{2}y^2)\, F_{UU,T} + (1-y)\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h}$$

$$+\Lambda(1-y)\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} + \lambda_l \Lambda y(1-\frac{1}{2}y)\, F_{LL}$$

$$+|\vec{S}_\perp|(1-y+\frac{1}{2}y^2)\sin(\phi_h-\phi_S)\, F_{UT,T}^{\sin(\phi_h-\phi_S)}$$

$$+|\vec{S}_\perp|(1-y)\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+|\vec{S}_\perp|(1-y)\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\lambda_\ell|\vec{S}_\perp|y(1-\frac{1}{2}y)\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \text{10 additional terms} \Big\}$$

- Structure functions depend on 4 variables: $F_i = F_i(x_B, z_h, P_{h\perp}^2, Q^2)$

$F_{XY,L(T)}^{\text{weight}}$

X beam polarization

Y target polarization

weight angular distribution of produced hadron

$L(T)$ virtual-photon polarization

Bacchetta, et al., JHEP 0702 (2007) 93

SIDIS structure functions at tree level

$$F_{UU,T} = x_B \sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1(x, \vec{k}_\perp^{\,2}) \, D_1(z_h, \vec{p}_\perp^{\,2})$$

$$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$

$$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

$$F_{LL} \sim g_1 \otimes D_1$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^\perp \otimes D_1$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} \sim h_1 \otimes H_1^\perp$$

$$F_{UT,T}^{\sin(3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$$

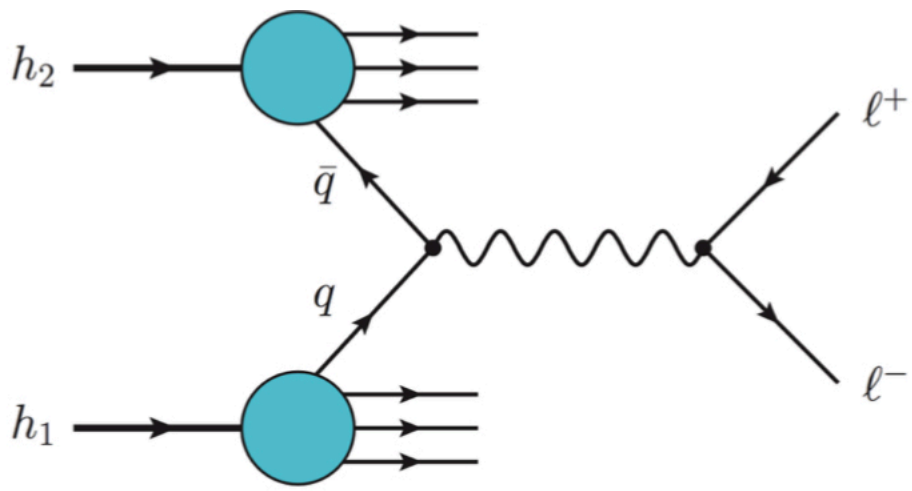
$$F_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T} \otimes D_1$$

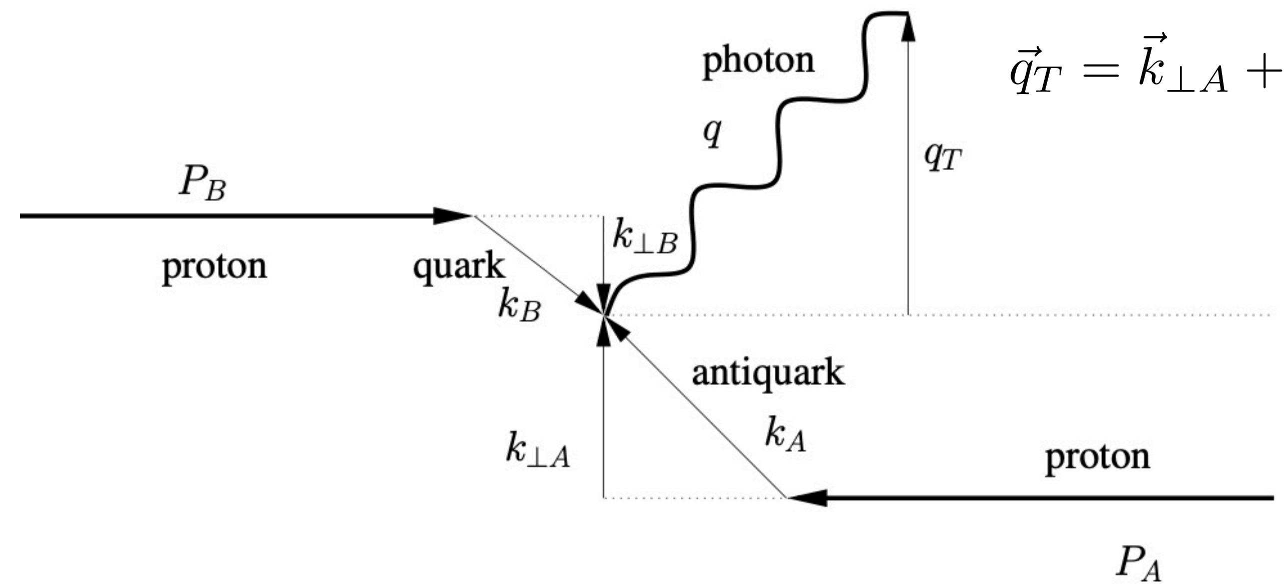
- transverse parton momenta of TMDs and FFs are convoluted
  (convolutions may contain additional powers of transverse parton momenta)

Bacchetta et. al, JHEP 0702 (2007) 93

Drell-Yan process at tree level

$$h_1 + h_2 \to l^+ + l^- + X$$





$$\vec{q}_T = \vec{k}_{\perp A} + \vec{k}_{\perp B}$$

- Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{k}_{a\,\perp}\, d^2\vec{k}_{b\,\perp}\, \delta^{(2)}(\vec{k}_{a\,\perp} + \vec{k}_{b\,\perp} - \vec{q}_\perp)$$
$$\times \mathrm{Tr}\left[\Phi^q(x_a, \vec{k}_{a\,\perp}, P_1, S_1)\,\gamma^\mu\,\Phi^{\bar{q}}(x_b, \vec{k}_{b\,\perp}, P_2, S_2)\,\gamma^\nu\right]\Big|_{\substack{k_a^+ = x_B P_1^+ \\ k_b^+ = x_b P_2^+}}$$

- It involves two TMD-PDFs
- transverse parton momenta are convoluted
- longitudinal momentum fractions fixed by the kinematics
- cross section parametrised by 48 structure functions
  [Arnold, Metz, Schlegel, PRD 79 (2009) 489]

Light-front wave function representation

Proton state

Probability Amplitude for the N,β Fock state

$$|(P^+,\vec{P}_\perp),\Lambda\rangle \quad = \quad \sum_{N,\beta}[dx]_N[d\vec{k}_\perp]_N\Psi^\Lambda_{N,\beta}(x_i,\vec{k}_{\perp\, i})|N,\beta;(x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp\, i}),\lambda_i\rangle$$

Light-front wave functions

Internal variables: $x_i = \frac{p_i^+}{P^+}$ $\quad \sum_{i=1}^{N} x_i = 1$ $\quad \sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz}\Psi^\Lambda_{\lambda_1\ldots\lambda_N} = \lambda_i\Psi^\Lambda_{\lambda_1\ldots\lambda_N}$$

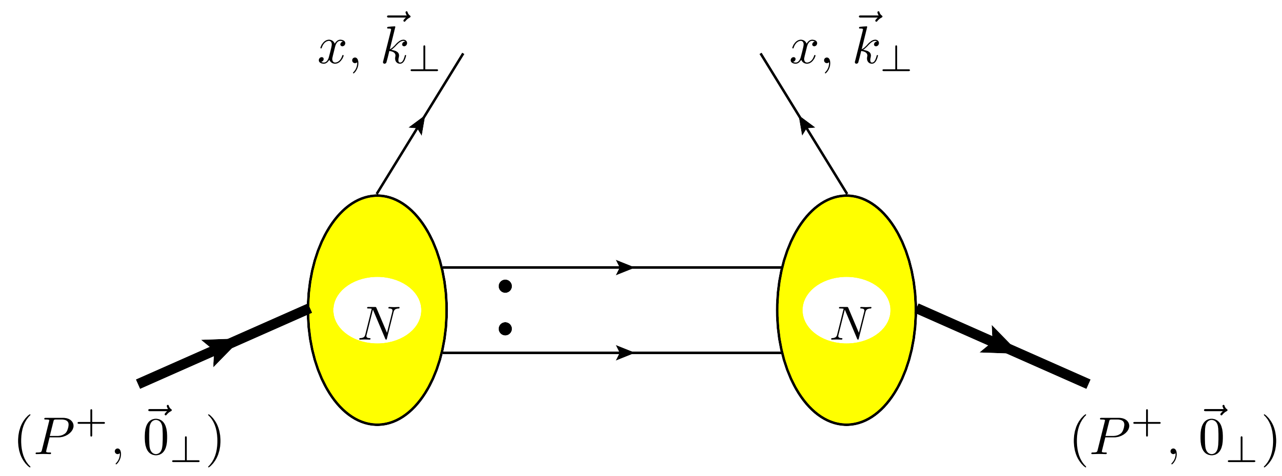
$$\boxed{\Lambda = \sum_{i=1}^N \lambda_i + \ell_z}$$

Eigenstates of total OAM

$$\hat{L}_z\Psi^\Lambda_{\lambda_1\ldots\lambda_N} = \ell_z\Psi^\Lambda_{\lambda_1\ldots\lambda_N}$$

$A^+ = 0$ **gauge**

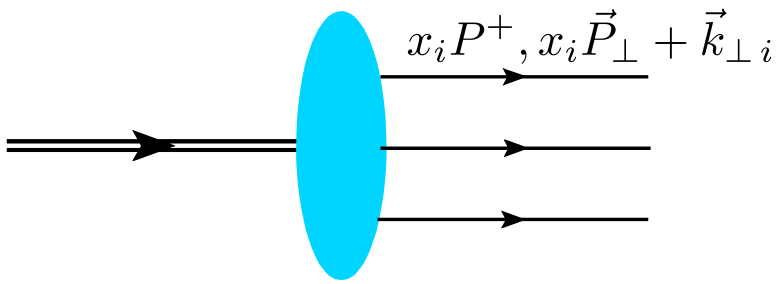
Light-Front Wave Function Overlap Representation



$$\text{TMDs} \sim \sum_N \int [dx]_N \, |\Psi_N(k_N)|^2 \delta(\dots)$$ probability density in 3D momentum space

$$\text{PDFs} \sim \sum_N \int [d^3k]_N \, |\Psi_N(k_N)|^2 \delta(\dots)$$ probability density in 1D momentum space

Quark-OAM: partial wave decomposition of LFWF



$$| P, \Lambda\rangle = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \; \underbrace{\Psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})}_{\text{LCWF: eigenstate of OAM}} \; \frac{\varepsilon^{ijk}}{\sqrt{6}} \, u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3) | \, 0\rangle$$

J^q_z	$\rightarrow$	$(\uparrow\uparrow\uparrow)_{LC} = \frac{3}{2}$	$(\uparrow\uparrow\downarrow)_{LC} = \frac{1}{2}$	$(\uparrow\downarrow\downarrow)_{LC} = -\frac{1}{2}$	$(\downarrow\downarrow\downarrow)_{LC} = -\frac{3}{2}$
$L^q_z = \frac{1}{2} - J^q_z$	$\rightarrow$	$L^q_z = -1$	$L^q_z = 0$	$L^q_z = 1$	$L^q_z = 2$

${}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$:probability to find the proton in a state with eigenvalue of OAM L_z

$\Downarrow$

$$\mathcal{L}_z = \sum_{L_z} L_z \; {}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$$

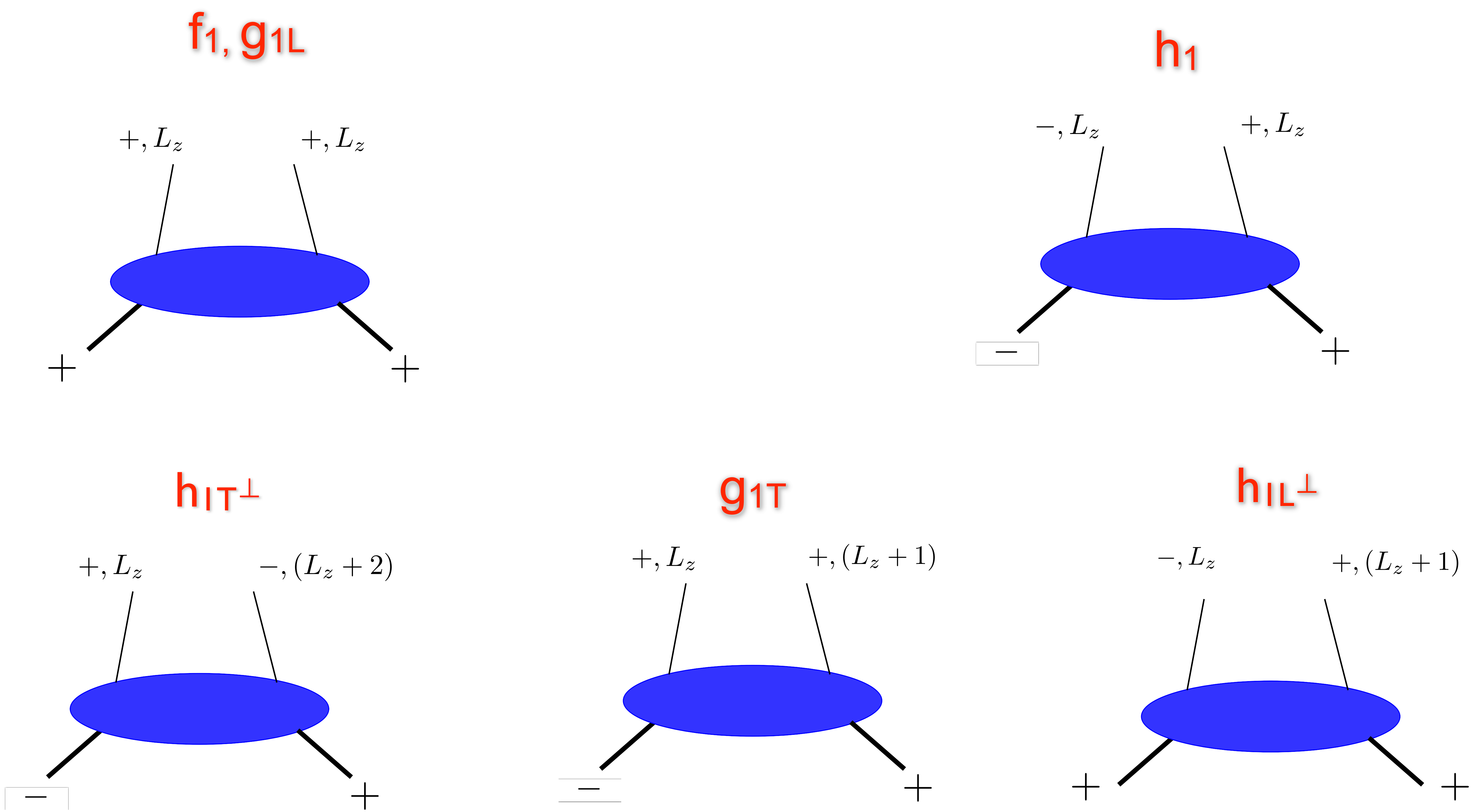
$\downarrow$

squared of LFWFs

$A^+ = 0$ **gauge**

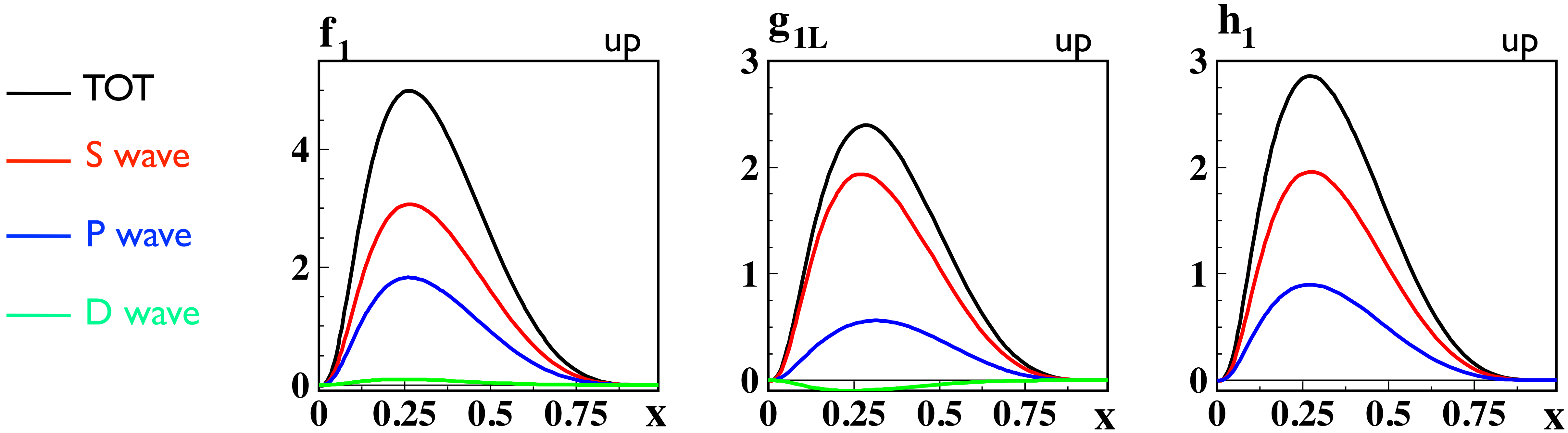
OAM decomposition of T-even TMDs

$\Delta J = \Delta J^q + \Delta L_z^q$ total angular momentum conservation

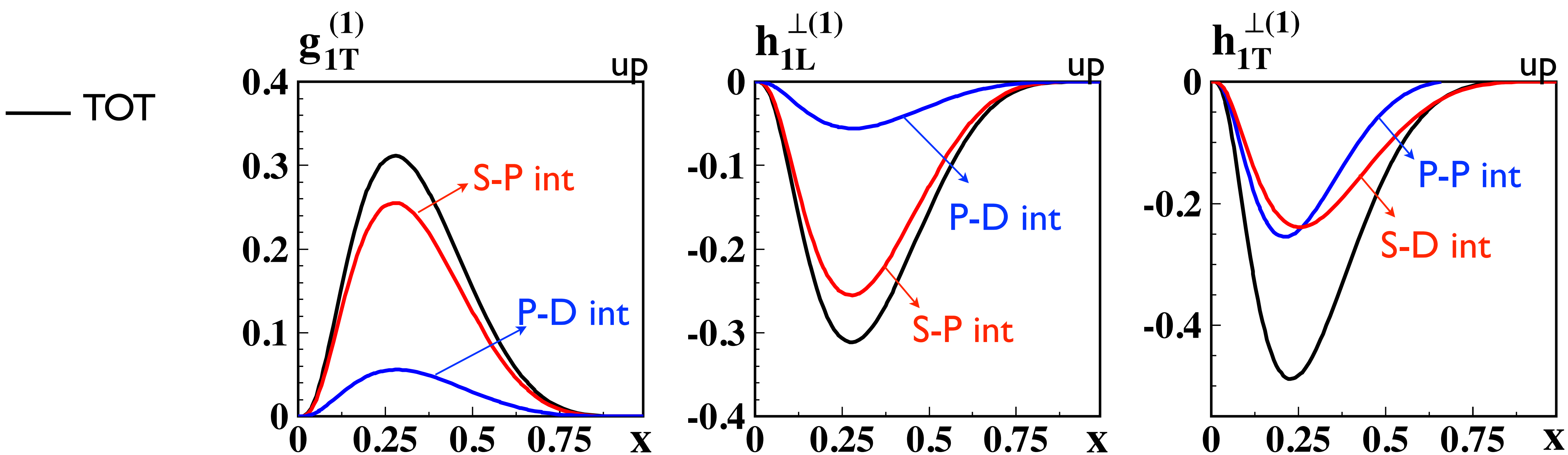


OAM content of TMDs

Model results with light-front wave functions fitted to nucleon electromagnetic form factors

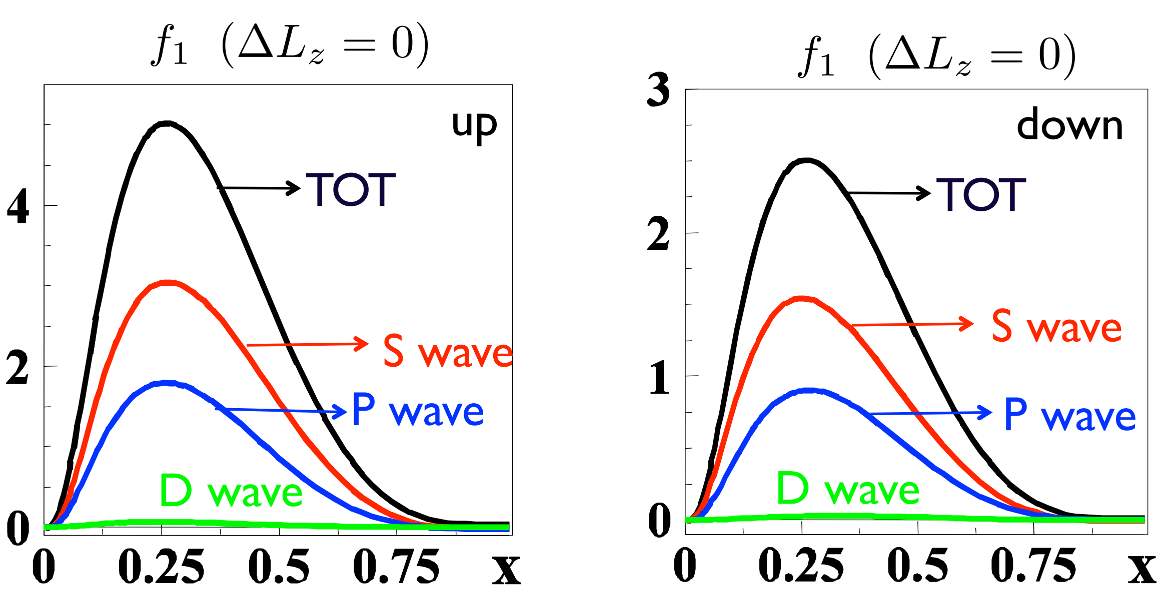


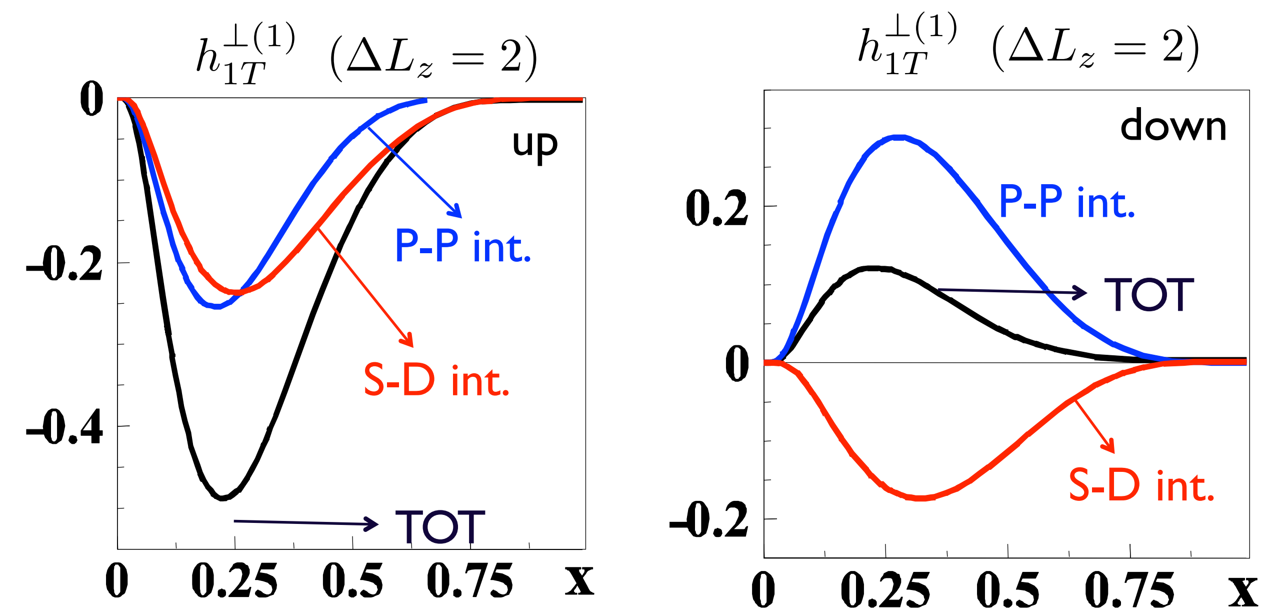
$$j^{(1)}(x) = \int \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, j(x, k_\perp^2)$$



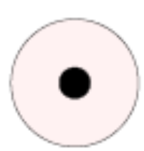
BP, Cazzaniga, Boffi, PRD78 (2008)

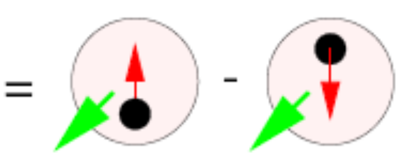
OAM content of TMDs in observables



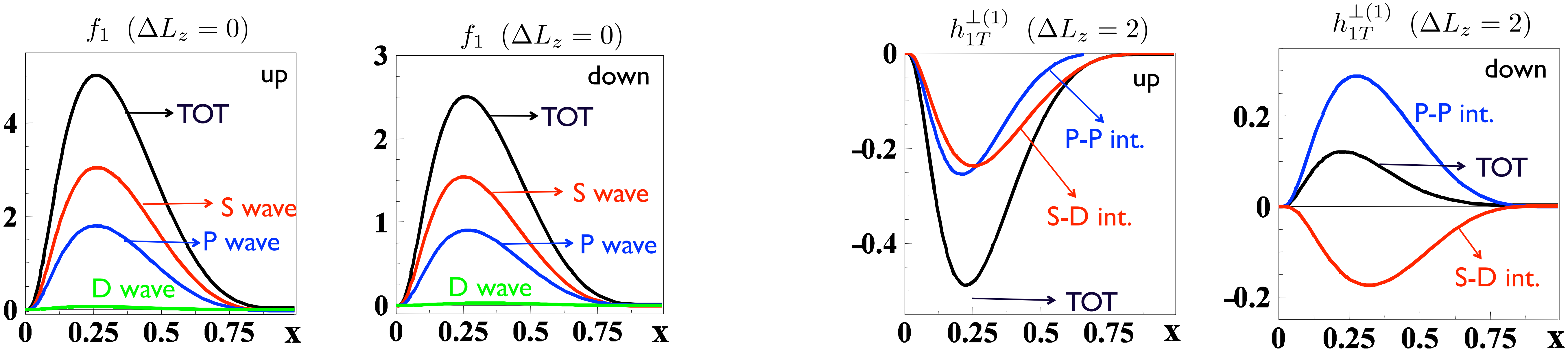


"pretzelosity"

$f_1 =$ 

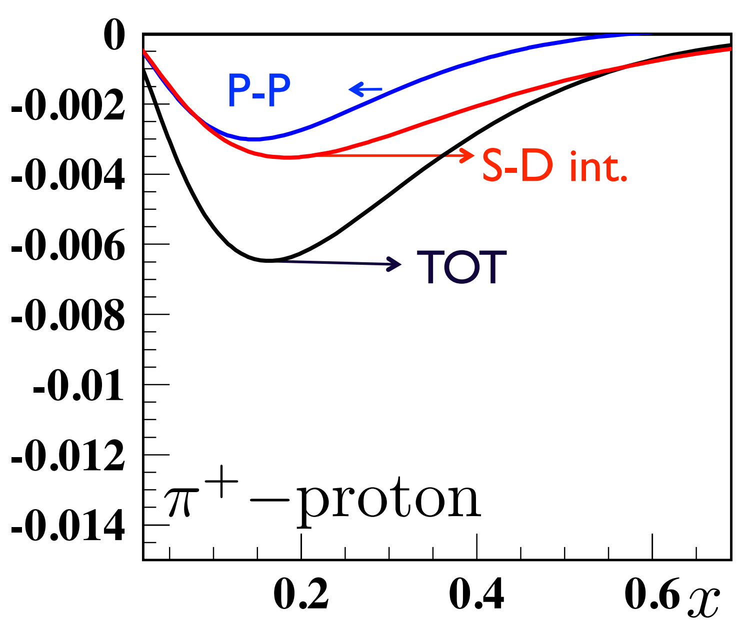
$h_{1T}^{\perp} =$ 

OAM content of TMDs in observables

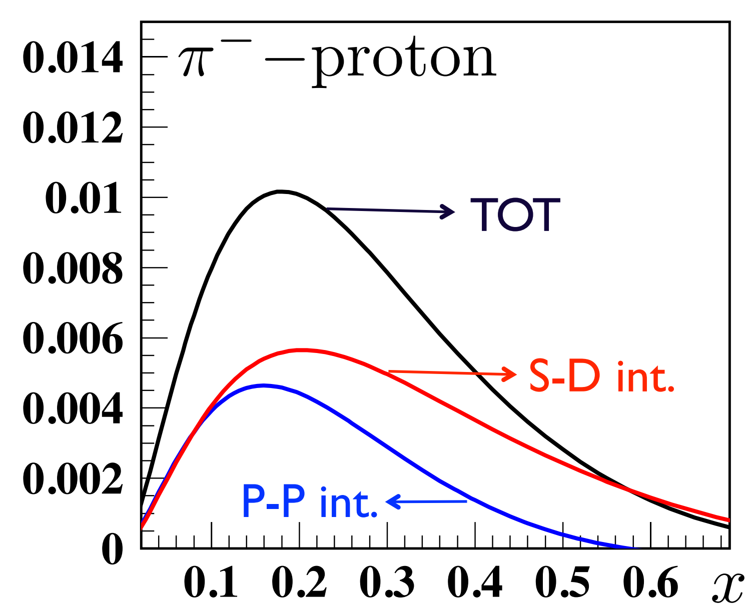


✦ Effects on SIDIS observables

$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



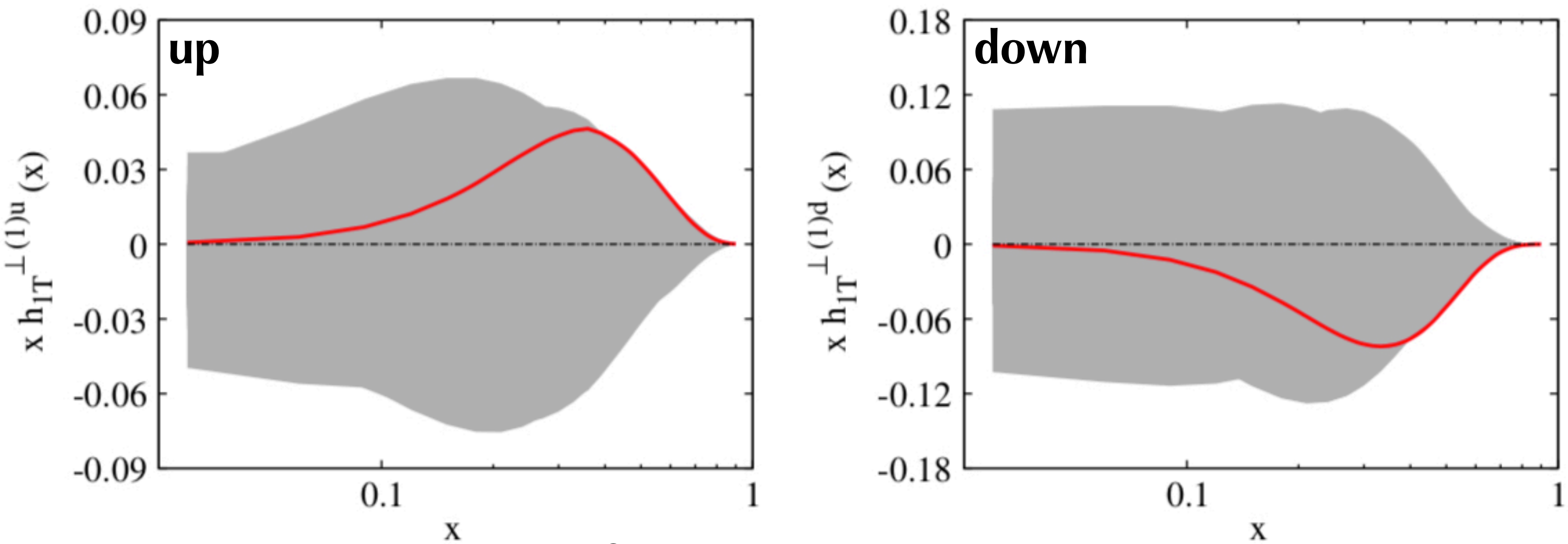
$\langle Q^2 \rangle = 2.5\ \text{GeV}^2$



Boffi, Efremov, BP, Schweitzer, PRD79(2009)

First attempt to extract pretzelosity from data

Lefky, Prokudin, PRD91(2015) 034010

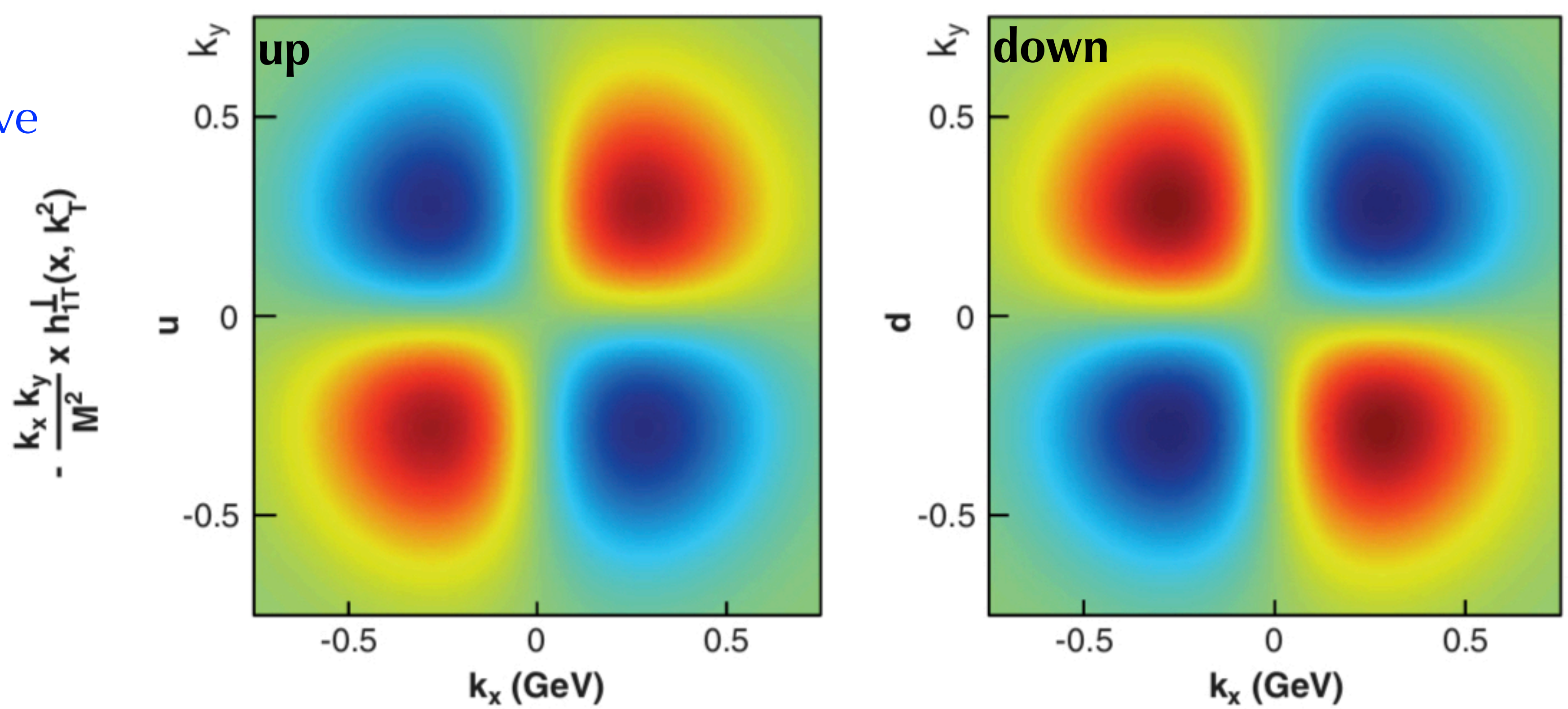


$Q^2 = 2.4\ \mathrm{GeV}^2$

convolution in SIDIS with $P^3_{h,\perp}$ factor $\longrightarrow$ suppressed for $\langle P_{h,\perp}\rangle < 1\,\mathrm{GeV}$

red: positive

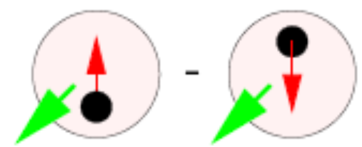
blue: negative



$x = 0.1$

future measurements will be very important to clarify the sign and size of the pretzelosity

Quark OAM from pretzelosity

$h_{1T}^{\perp} =$  "pretzelosity"

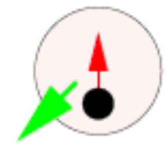
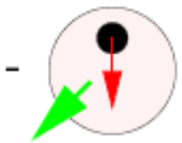
model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

Quark OAM from pretzelosity

$h_{1T}^{\perp}$ =  -  "pretzelosity"

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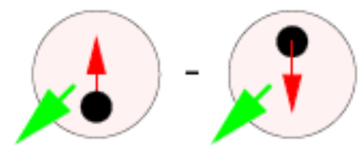
[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert\Delta L_z\rvert = 2$

no operator identity

relation at level of matrix elements of operators

Quark OAM from pretzelosity

$h_{1T}^{\perp} =$  "pretzelosity"

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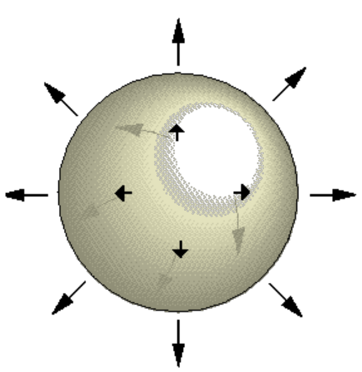
relation at level of matrix elements of operators

$\Downarrow$



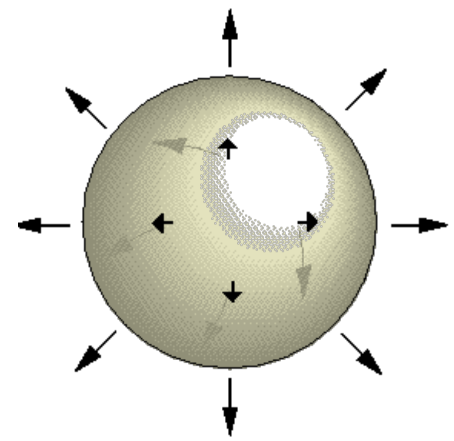
valid in all quark models with spherical symmetry in the rest frame

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame
- SU(6) symmetry

spherical symmetry
in the rest frame



rest frame

$$|\vec{0},\sigma\rangle$$

zero OAM

the quark distribution does not depend
on the direction of polarization

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
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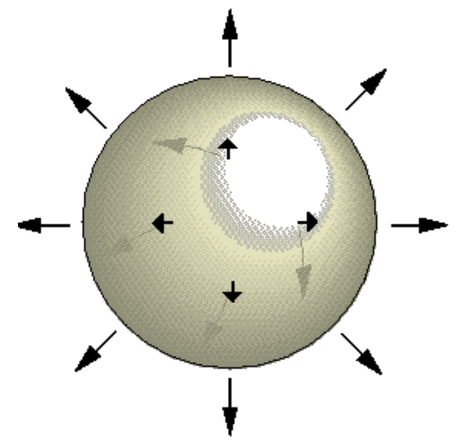
spherical symmetry
in the rest frame



rest frame

$|\vec{0},\sigma\rangle$

zero OAM

the quark distribution does not depend
on the direction of polarization

Light-cone boost →

infinite-momentum frame

$|\vec{k},\lambda\rangle_{LF}$

NON-zero OAM

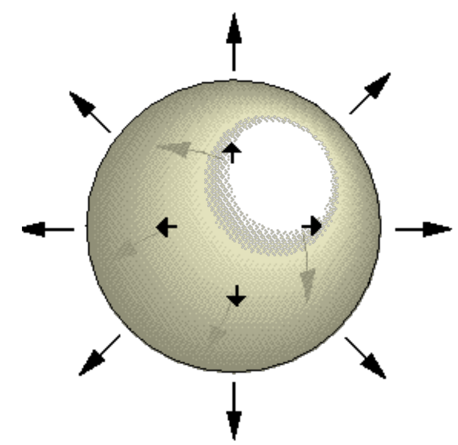
LC polarizations of quark and nucleon
are NOT all independent

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame
- SU(6) symmetry

spherical symmetry
in the rest frame



rest frame

$|\vec{0}, \sigma\rangle$

infinite-momentum frame

$|\vec{k}, \lambda\rangle_{LF}$

Light-cone boost

zero OAM

the quark distribution does not depend
on the direction of polarization

NON-zero OAM

LC polarizations of quark and nucleon
are NOT all independent

relations
among polarized TMDs

[Lorcé, BP, PLB 710 (2012) 486]

Relations among T-even TMDs

[Avakian, Efremov, Schweitzer, Yuan, 2008]
[Lorcé, Pasquini, 2011]

	Linear Relations	Quadratic Relations
*=SU(6) Flavor dependent $D^u = \frac{2}{3},\ D^d = -\frac{1}{3}$	$D^q\, f_1^q + g_{1L}^q = 2h_1^q$	
Flavor independent	$g_{1T}^q = -h_{1L}^{\perp q}$ $g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}$	$2\, h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2$

Bag [Jaffe, Ji 1991); Signal (1997); Barone & al. (2002); Avakian & al., (2008-2010)]

χQSM [Lorcé, Pasquini, Vanderhaeghen (2011)]

LFQM [Pasquini & al. (2008)]

S Diquark [Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]

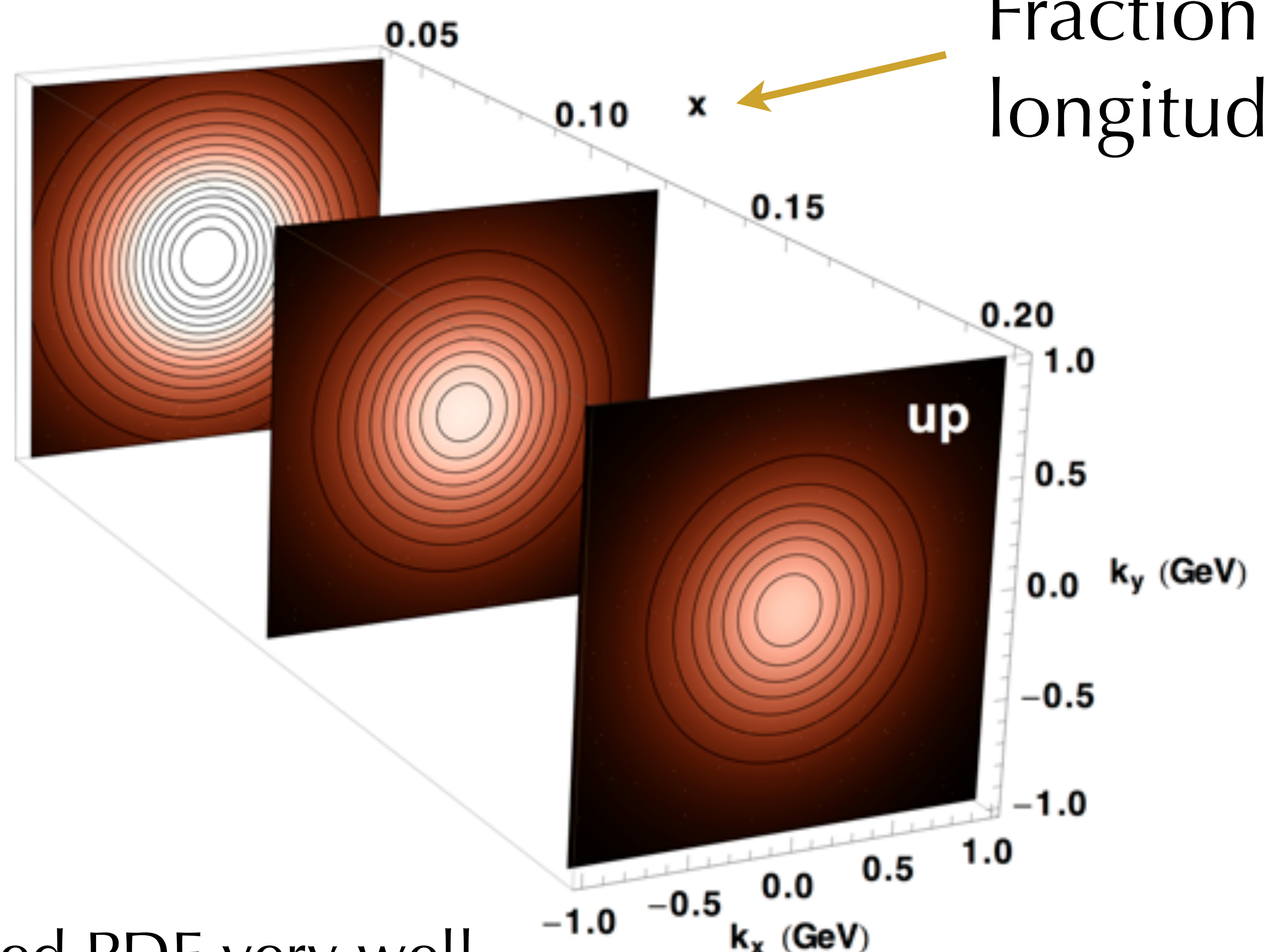
AV Diquark [Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]

Cov. Parton [Efremov & al. (2009)]

Quark Target [Meissner & al. (2007)]

Common assumptions:
- No gluons
- Independent quarks

The unpolarized TMD f_1



Fraction of longitudinal momentum

Transverse momentum

We know the integrated PDF very well.

We aim to the the TMD still poorly.

How wide is the distribution
Is there a difference between flavours
How does it change with x

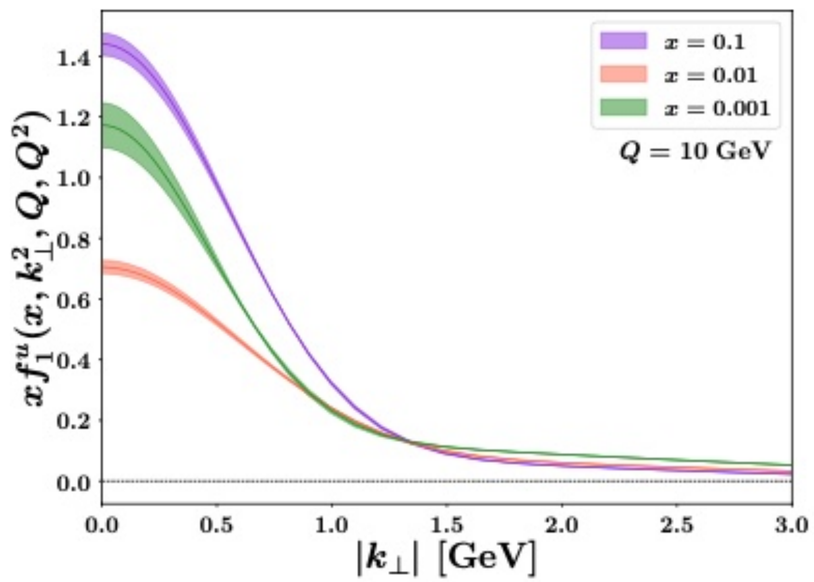
?

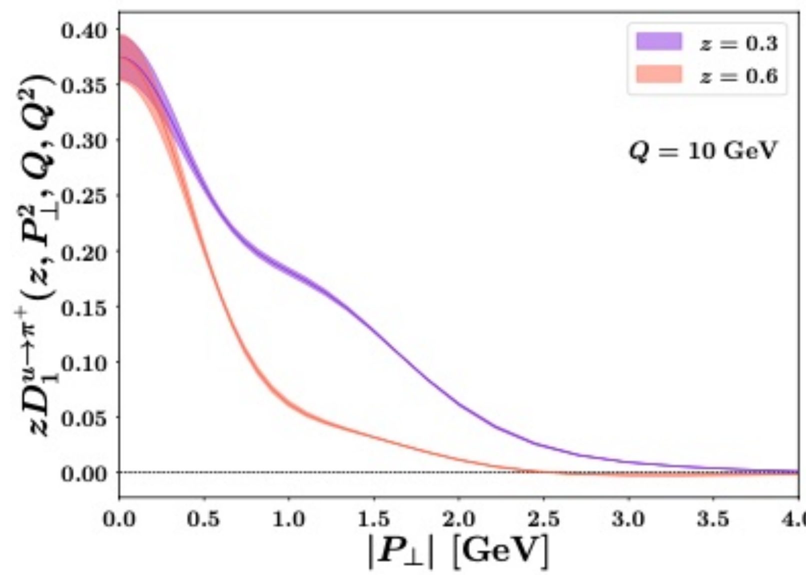
Quark unpolarized TMD extractions

	Framework	HERMES	COMPASS	DY	Z Production	N of points
Pavia 2016 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL	✗	✗	✓	✓	457
Pavia 2019 arXiv:1912.07550	NNNLL	✗	✗	✓	✓	353
SV 2020 arXiv:1912.06532	NNNLL	✓	✓	✓	✓	1039
MAP 2022 arXiv:2206.07598	NNNLL	✓	✓	✓	✓	2031

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

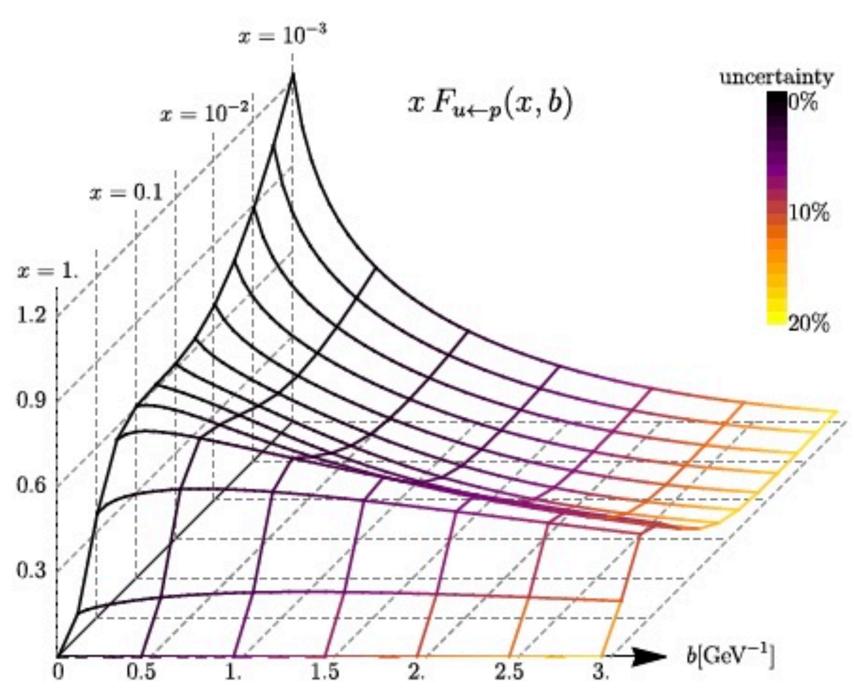
DY data at NNNLL

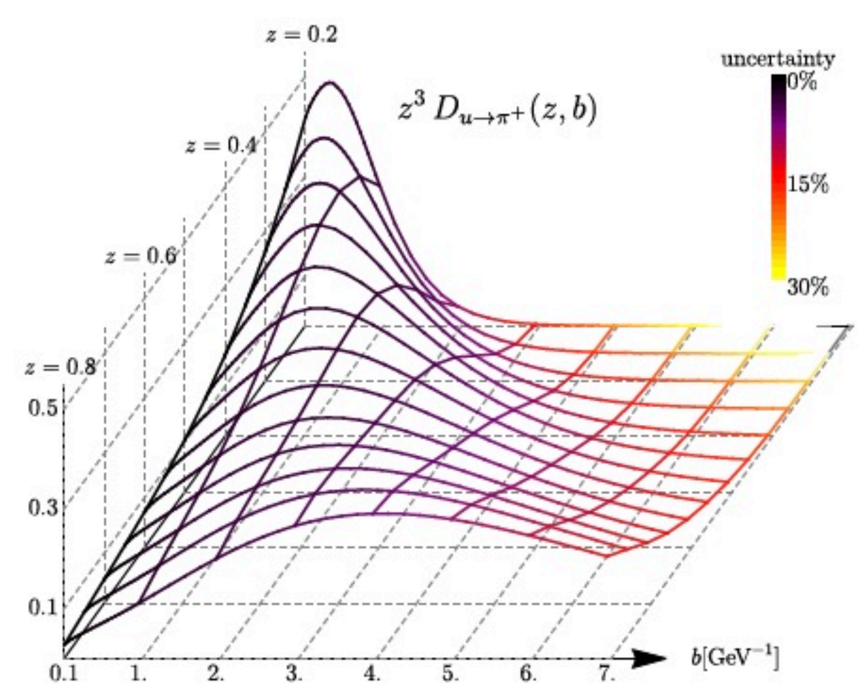




Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

DY+ SIDIS data at NNNLL

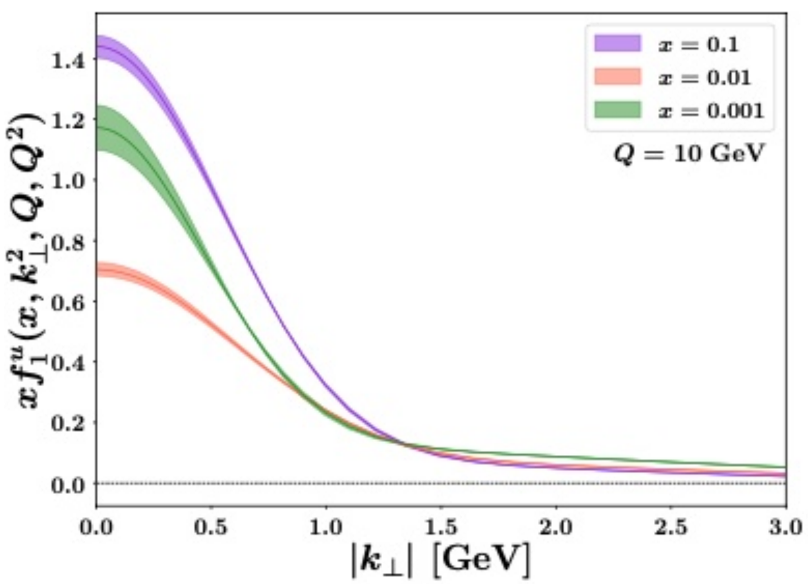


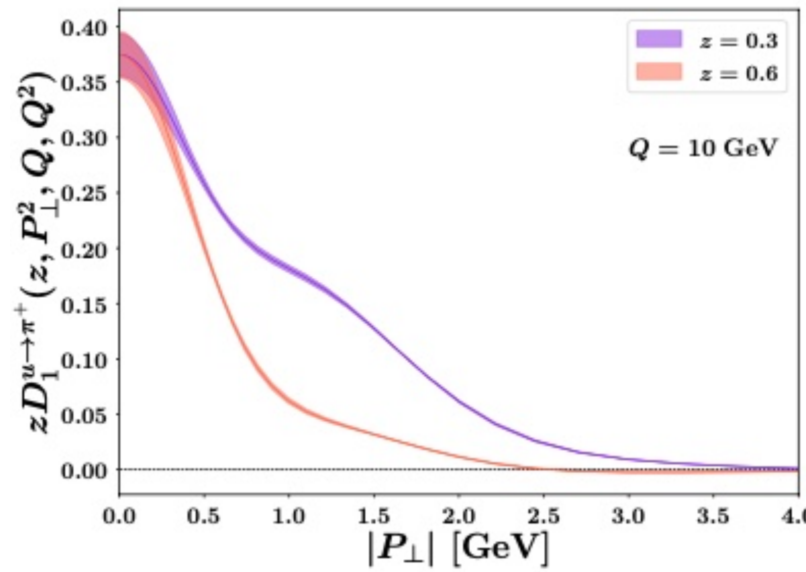


Scimemi, Vladimirov, JHEP 06 (2020) 137

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

DY data at NNNLL

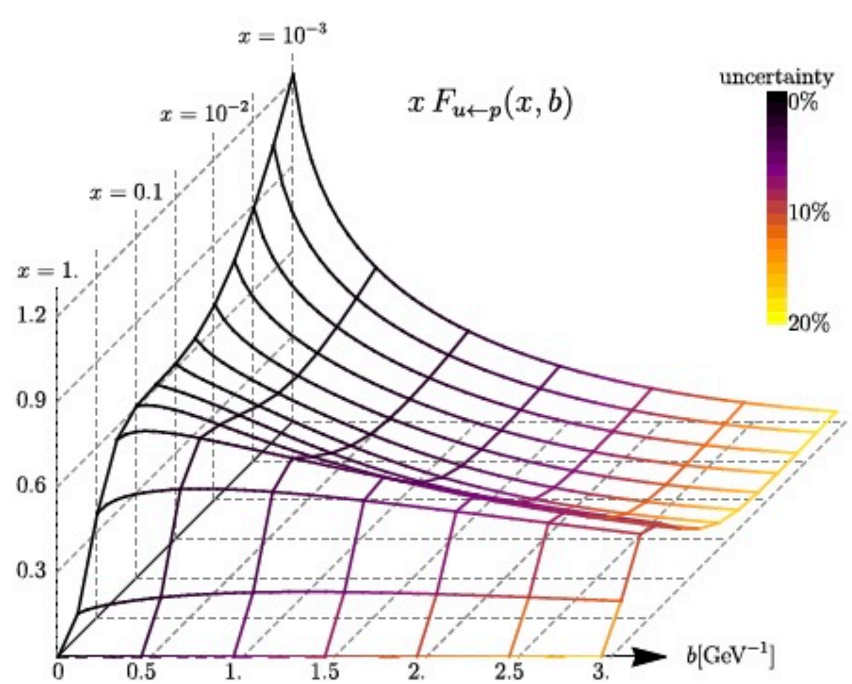


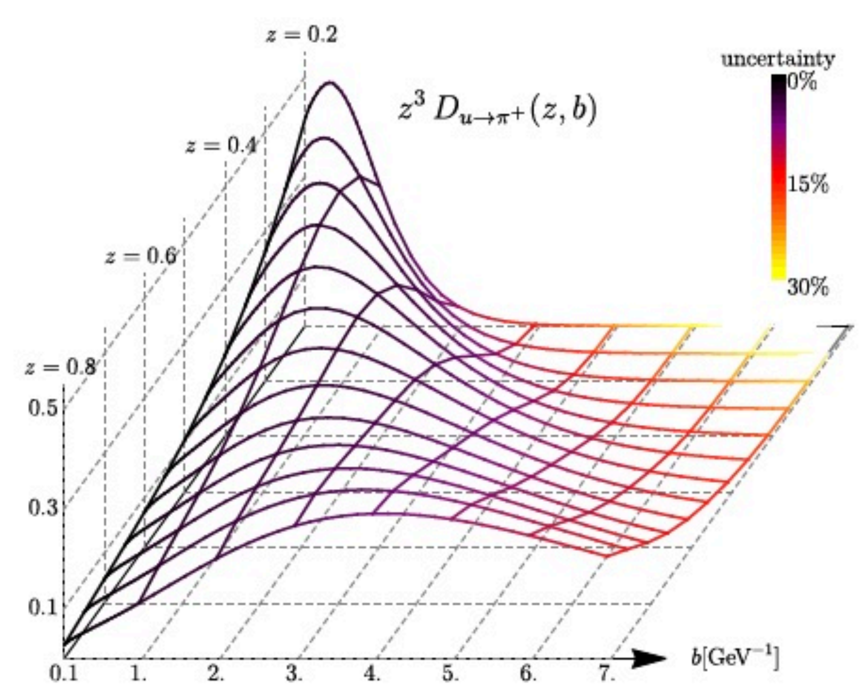


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

- The shape changes in a non trivial way as function of x
- It deviates from a simple gaussian
- Flavor dependence remains an open issue
- Larger uncertainties at small x -> needed EIC data with better accuracy

DY+ SIDIS data at NNNLL





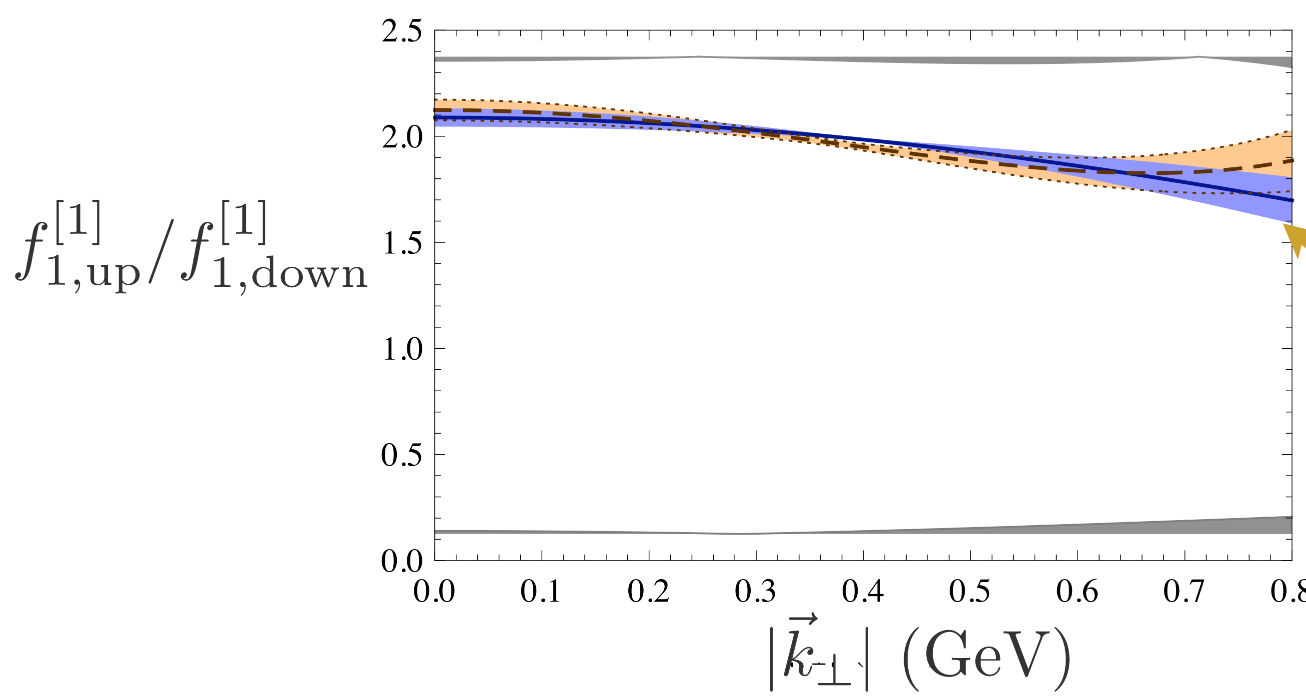
Scimemi, Vladimirov, JHEP 06 (2020)137

Flavor structure of TMDs: indications from lattice QCD

$$f_{1,\,q}^{[1]}(\vec{k}_\perp^2) = \int_0^1 \mathrm{d}x\,(f_{1,\,q}(x,\vec{k}_\perp^2) - f_{1,\,\bar{q}}(x,\vec{k}_\perp^2))$$

number of quarks as function of transverse momentum

$$\frac{\int d^2\vec{k}_\perp^{\,2}\, f_{1,\mathrm{up}}^{[1]}(\vec{k}_\perp^{\,2})}{\int d^2\vec{k}_\perp^{\,2}\, f_{1,\mathrm{down}}^{[1]}(\vec{k}_\perp^{\,2})} = \frac{n_{\mathrm{up}}}{n_{\mathrm{down}}} = 2$$

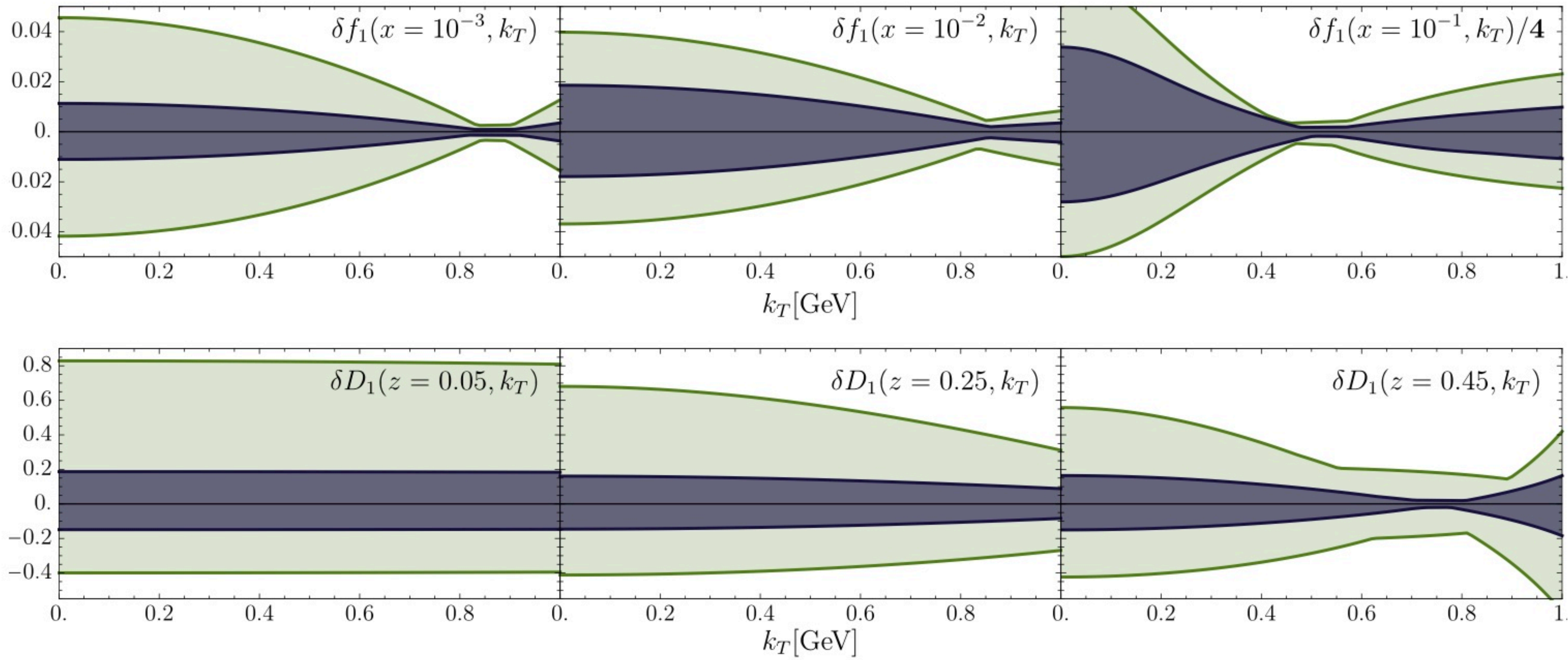


"less" up quarks

Pioneering lattice-QCD studies hint at a down distribution being wider than up

*Musch, Hagler, Negele, Schaefer, PRD**83** (2011) 094507*

Unpolarized TMD at EIC

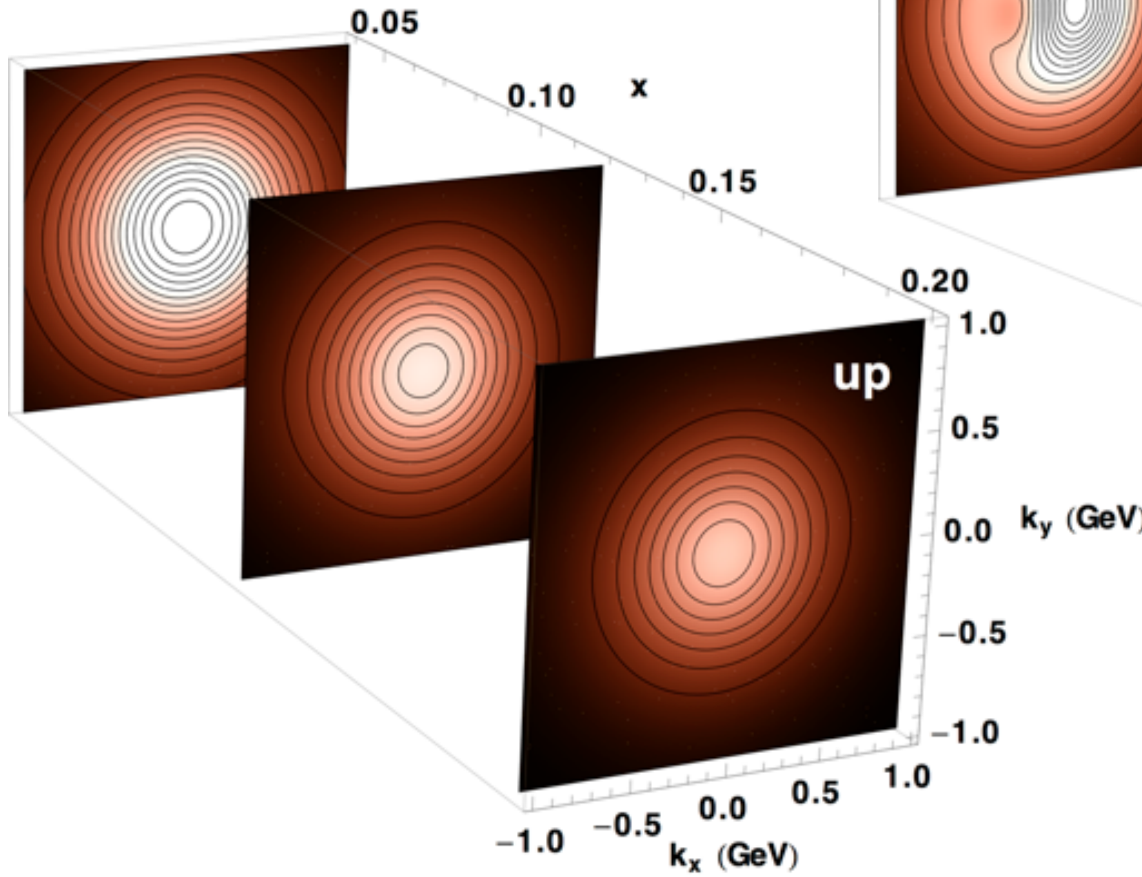


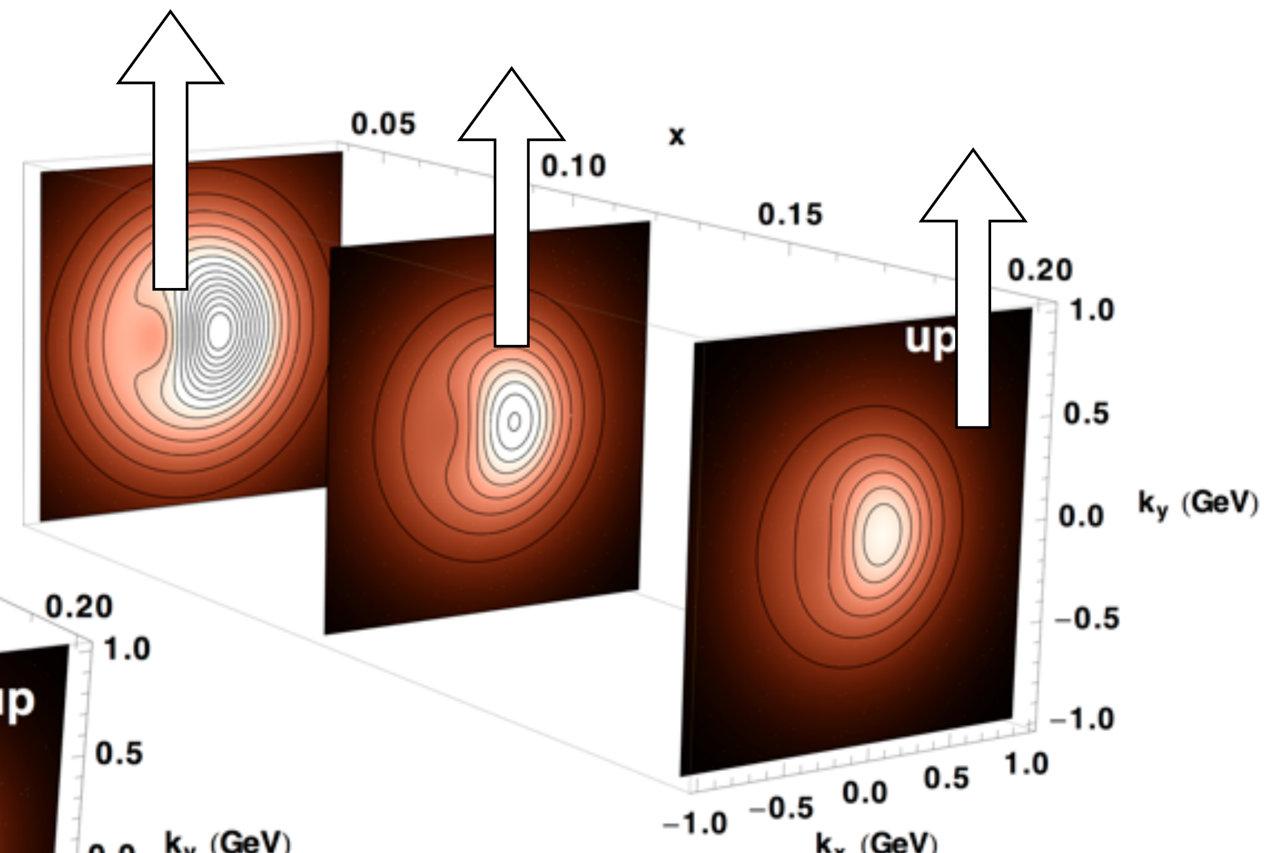
uncertainty band from SV19: Scimemi, Vladimirov, JHEP06 (2020) 137

uncertainty band from SV17+ EIC pseudodata

EIC Yellow Report, arXiv: 2103.05019

Adding the spin





correlation between x, $k_\perp$ and spin

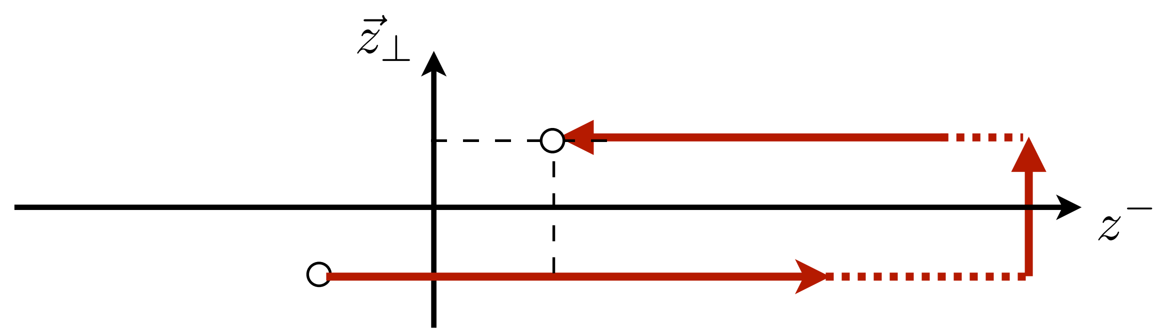
information on parton OAM

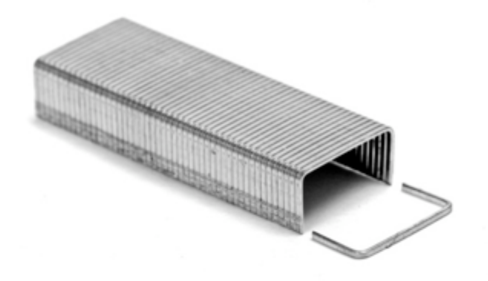
correlation between x and $k_\perp$

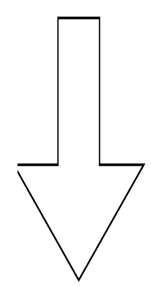
Need of the Gauge-Link

$$\Phi^{[\Gamma]}(x,\vec{k}_\perp,P,S) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3}\, e^{ik\cdot z}\langle P,S|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\boxed{\text{Gauge Link}}\,\psi(\frac{z}{2})|P,S\rangle\Big|_{z^+=0}$$

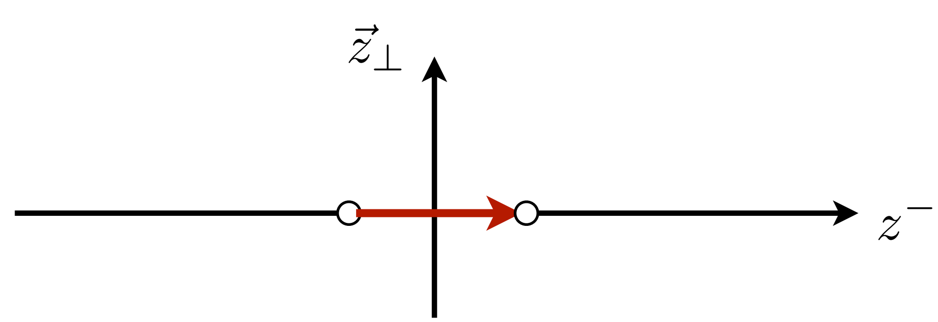
The staple gauge-link







$k_\perp$ integration



Wilson lines have physical consequences….

$$f_{q/p\uparrow}(x,\vec{k}_\perp) = f_{1T}^{\perp}(x,k_\perp^2) + \frac{(\vec{S}\times\vec{k}_\perp)\cdot\vec{p}_\perp}{M|\vec{p}_\perp|} f_{1T}^{\perp}(x,k_\perp^2)$$

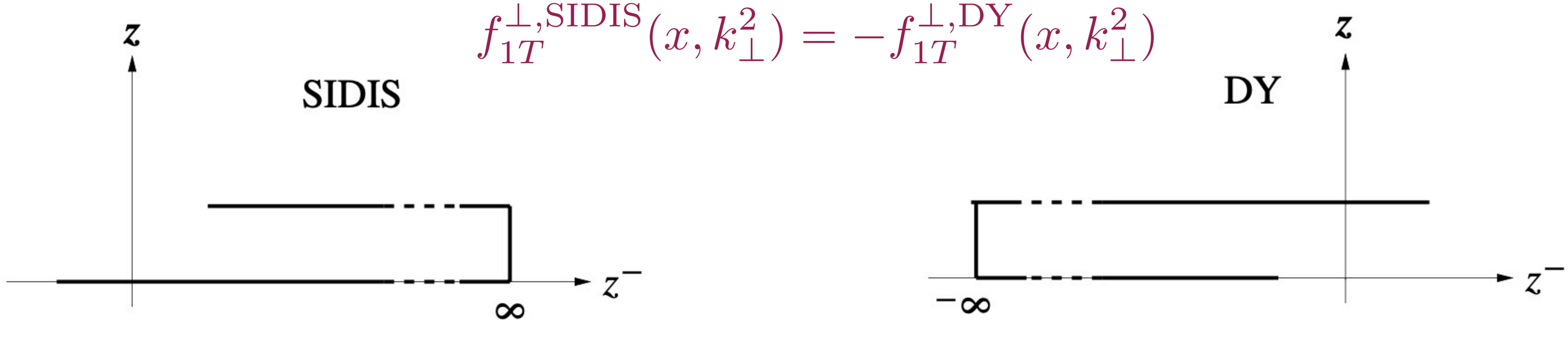
- 1990: introduced by D. Sivers to describe the large SSA measured in inclusive hadron production in p+p collisions at Fermilab.

- 1993 Collins: time reversal changes sign of $(\vec{S}\times\vec{k}_\perp)\cdot\vec{p}_\perp$ $\longrightarrow$ Sivers function = 0 ???

- 2002, Brodsky, Hwang, Schmidt, Collins:

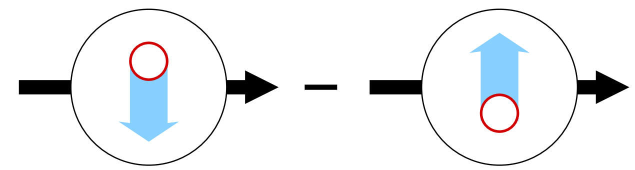
NO!
Time reversal interchanges Wilson line
for SIDIS (future pointing) and Drell-Yan (past pointing)

$$f_{1T}^{\perp,\mathrm{SIDIS}}(x,k_\perp^2) = -f_{1T}^{\perp,\mathrm{DY}}(x,k_\perp^2)$$

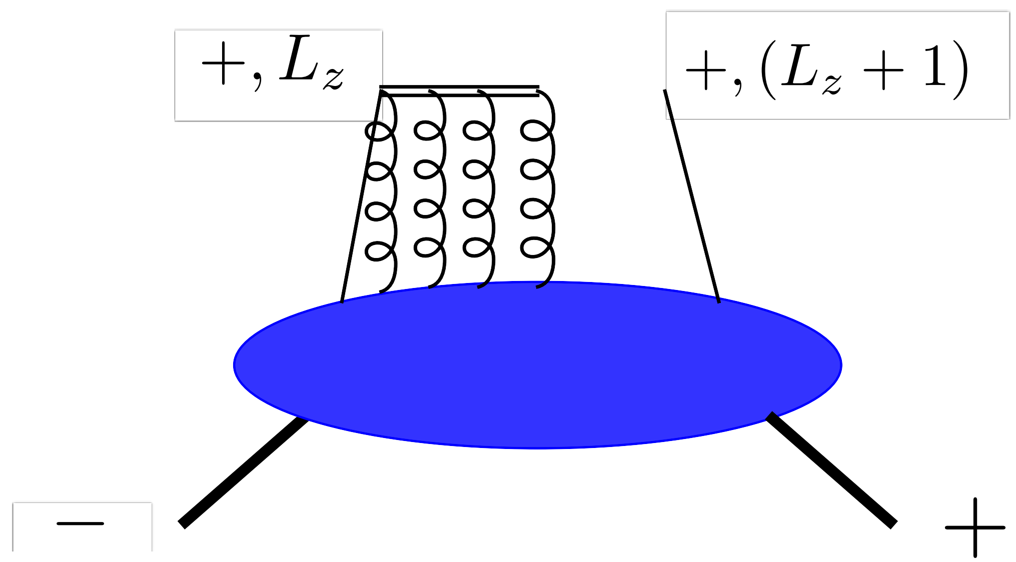


- 2016/17: sign change has been observed/hinted

Sivers function

$f_{1T}^{\perp} =$ 

unpolarized quarks in $\perp$ pol. nucleon



non-zero ONLY with final-state interaction

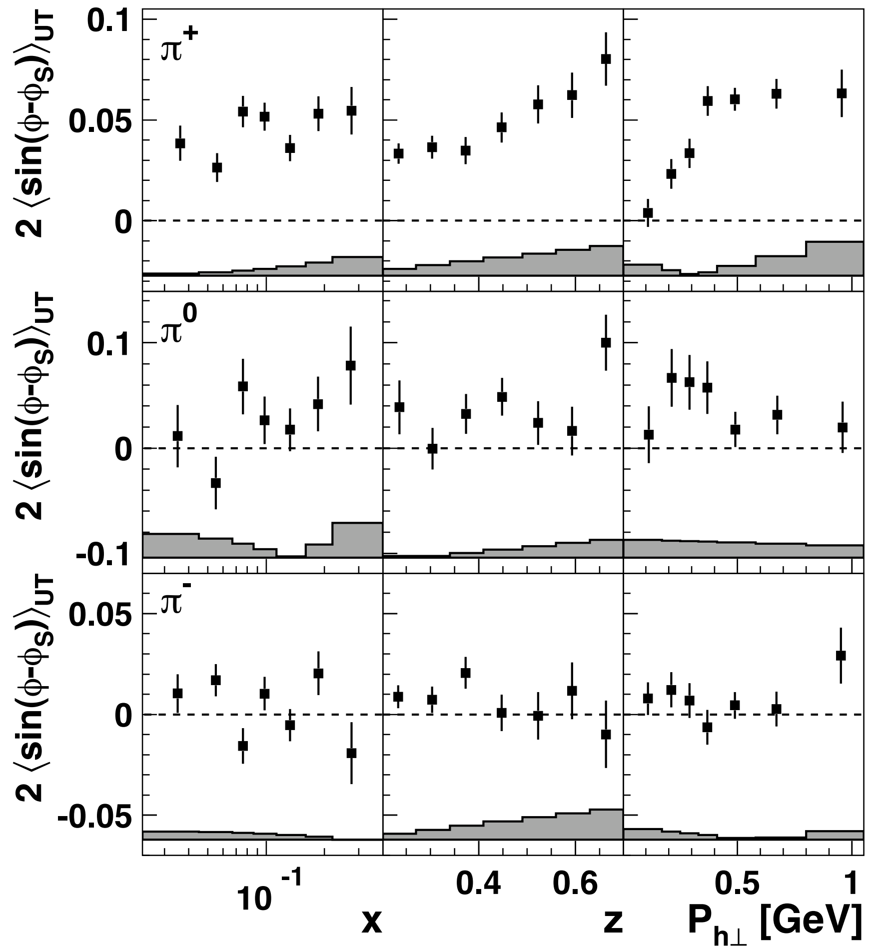
non trivial correlation between quark OAM and nucleon transverse spin

the helicity mismatch requires orbital angular momentum (OAM)

Sivers effect has been measured in SIDIS

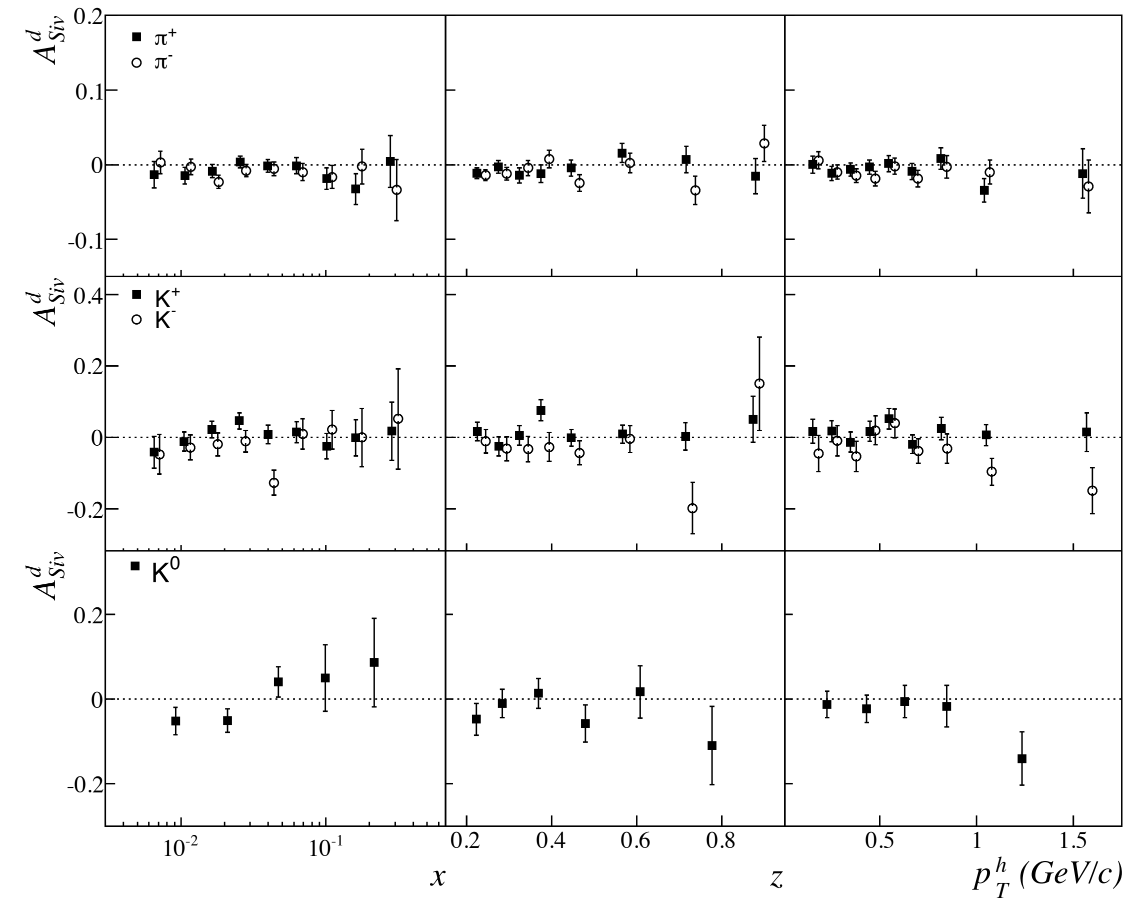
hermes

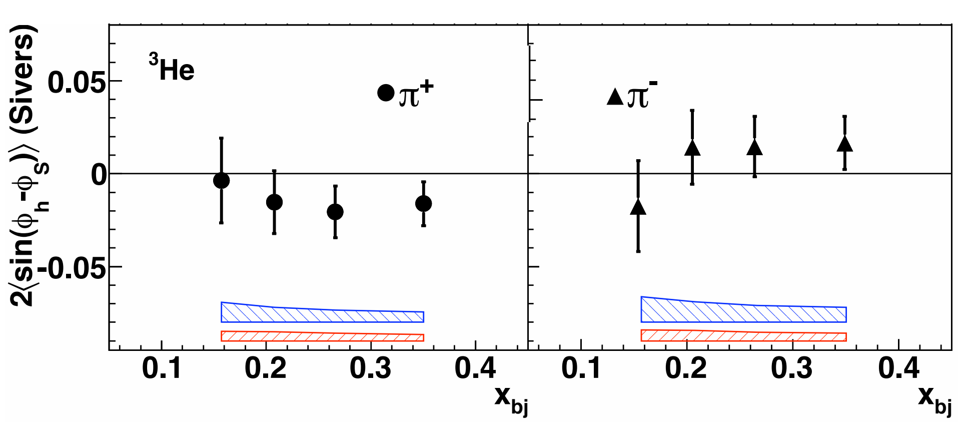
*PRL**103** (09) 152002*



COMPASS

*PL B**673** (09) 127*



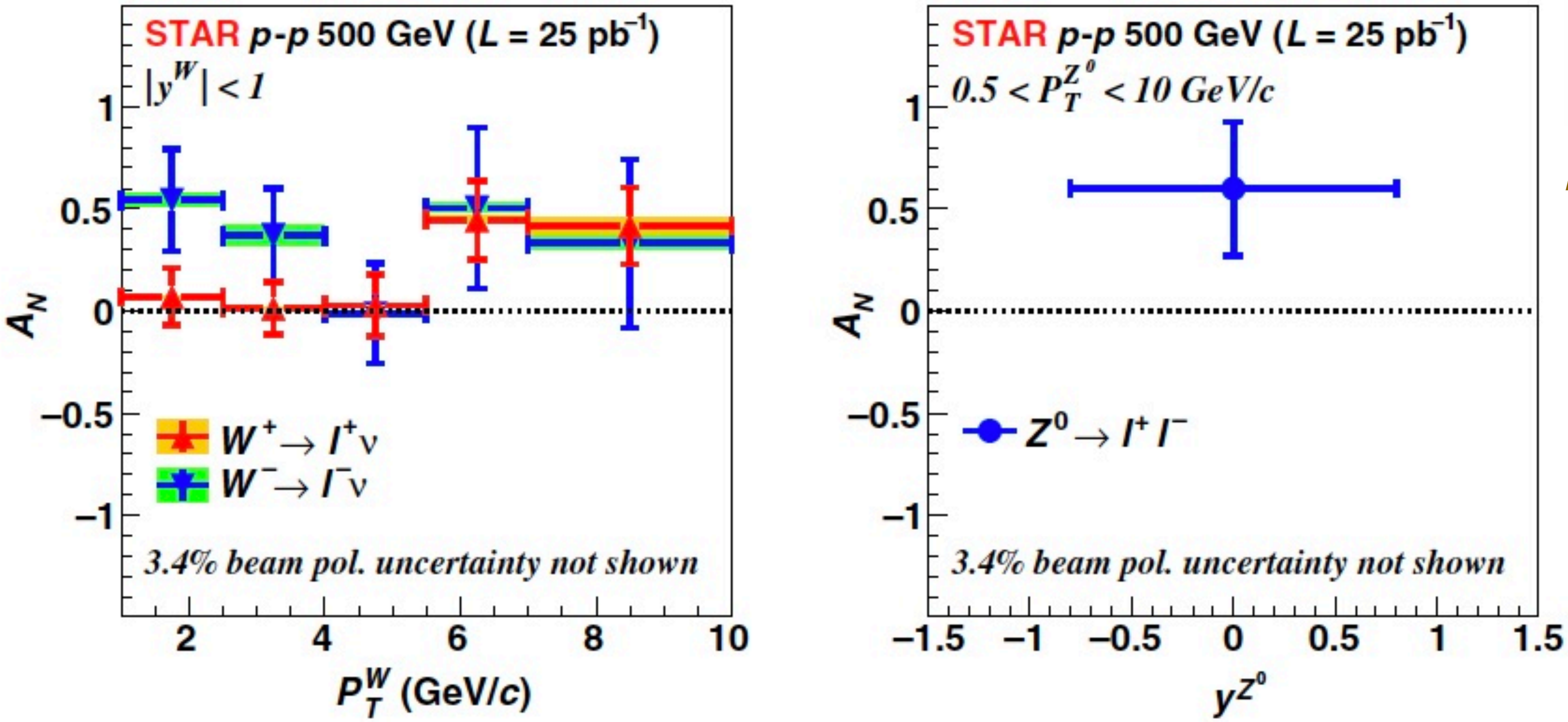


Jefferson Lab

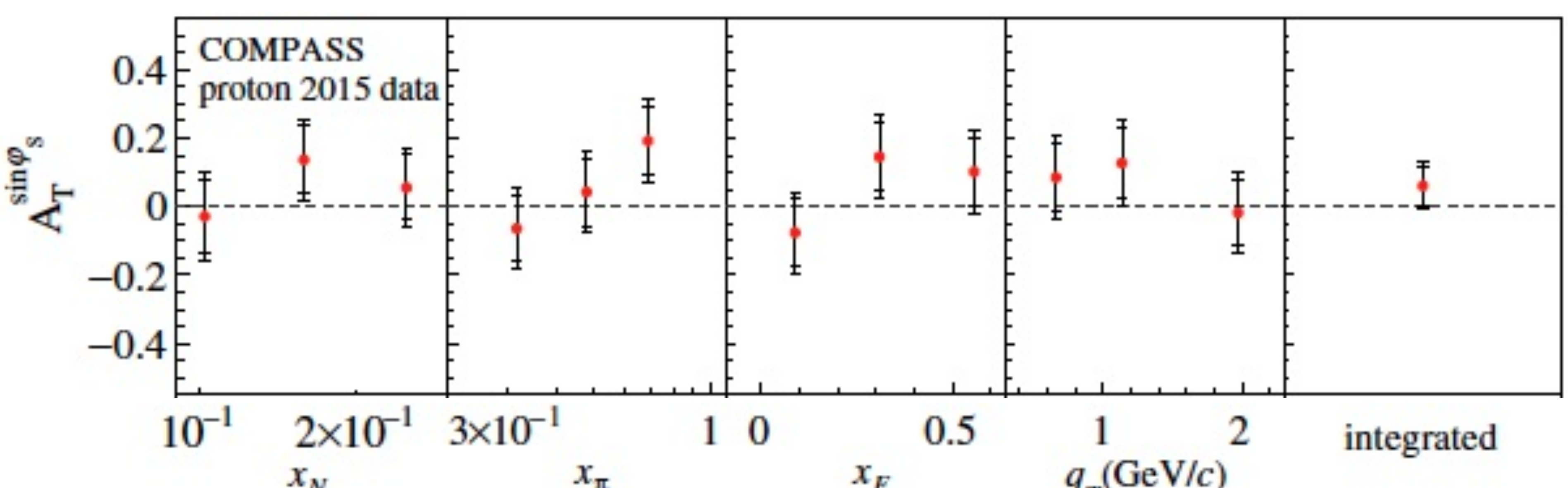
Hall A

*PRL**107** (2011) 072003*

Sivers effect has been measured in DY and $W^{\pm}/Z^0$ production

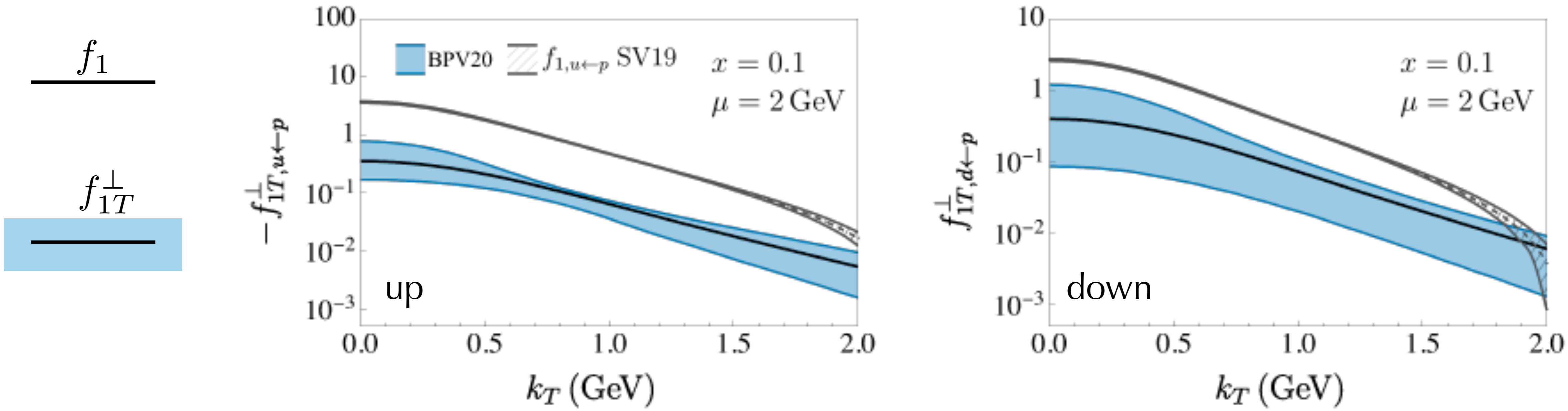


PRL 116 (2016) 132301

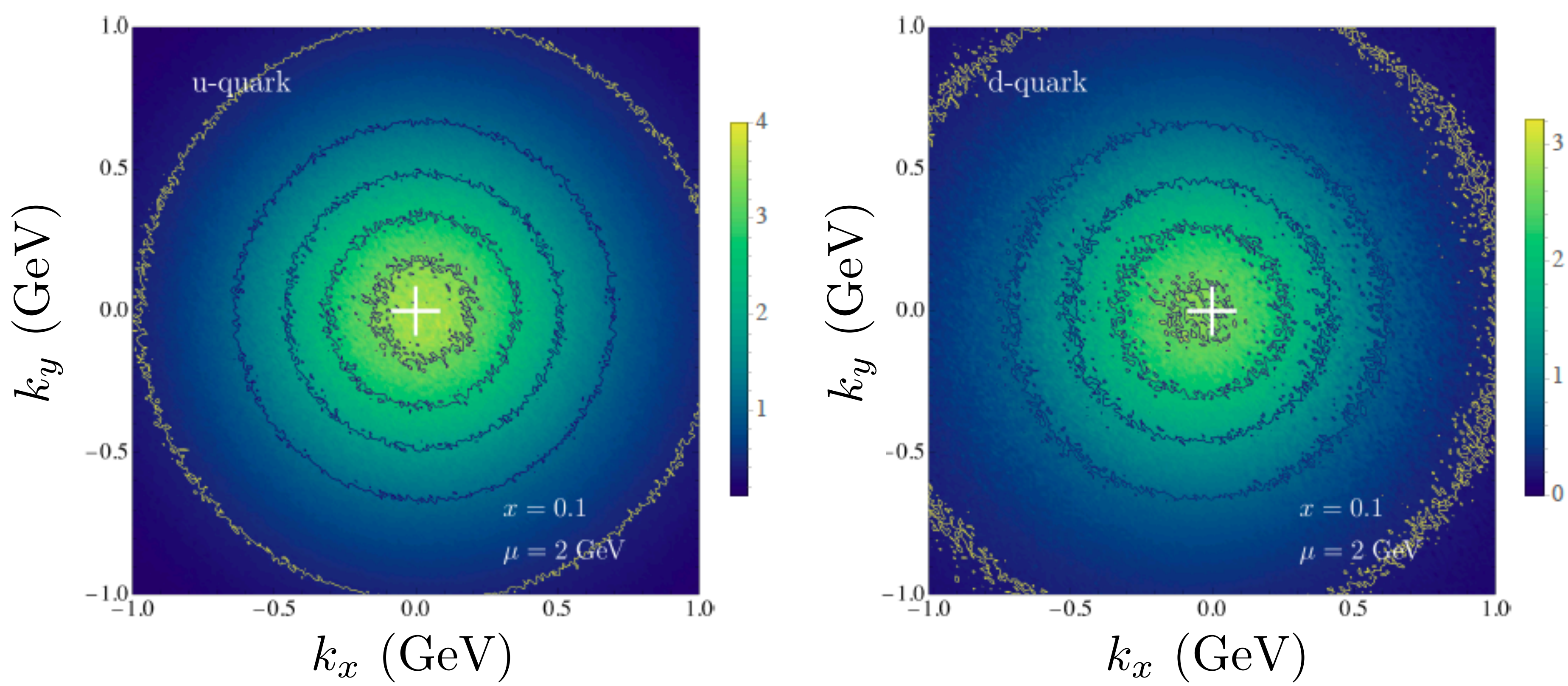


PRL 119(2017)112002

Global fit to SIDIS, DY, $W^{\pm}/Z$ boson production



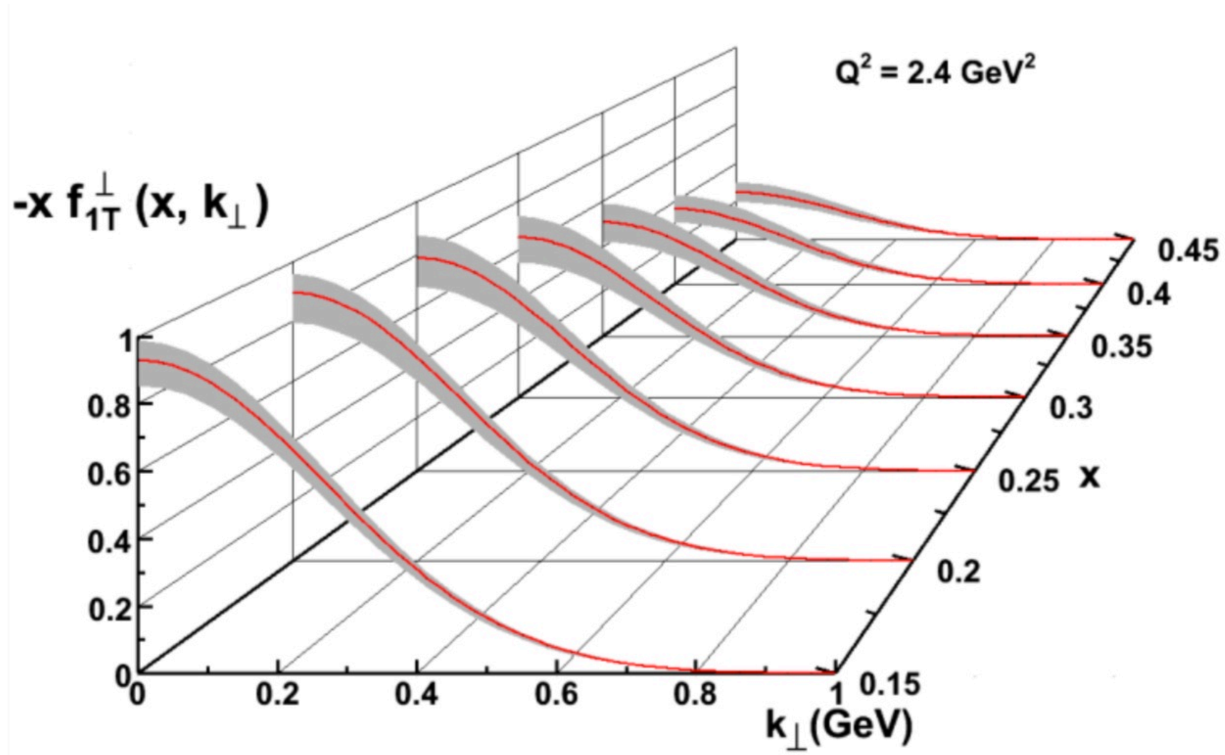
$$\rho_{UT_y}(x, \vec{k}_\perp, S_y) = f_1(x, k_\perp) - \frac{k_x}{M} f_{1T}^{\perp}(x, k_\perp)$$



M. Bury, A. Prokudin, A. Vladimirov, JHEP 05 (2021) 151

See also JAM20: PRD102 (2020) 054002; PV22: PLB827 (2022) 136961; EKT20: JHEP 01 (2021) 126

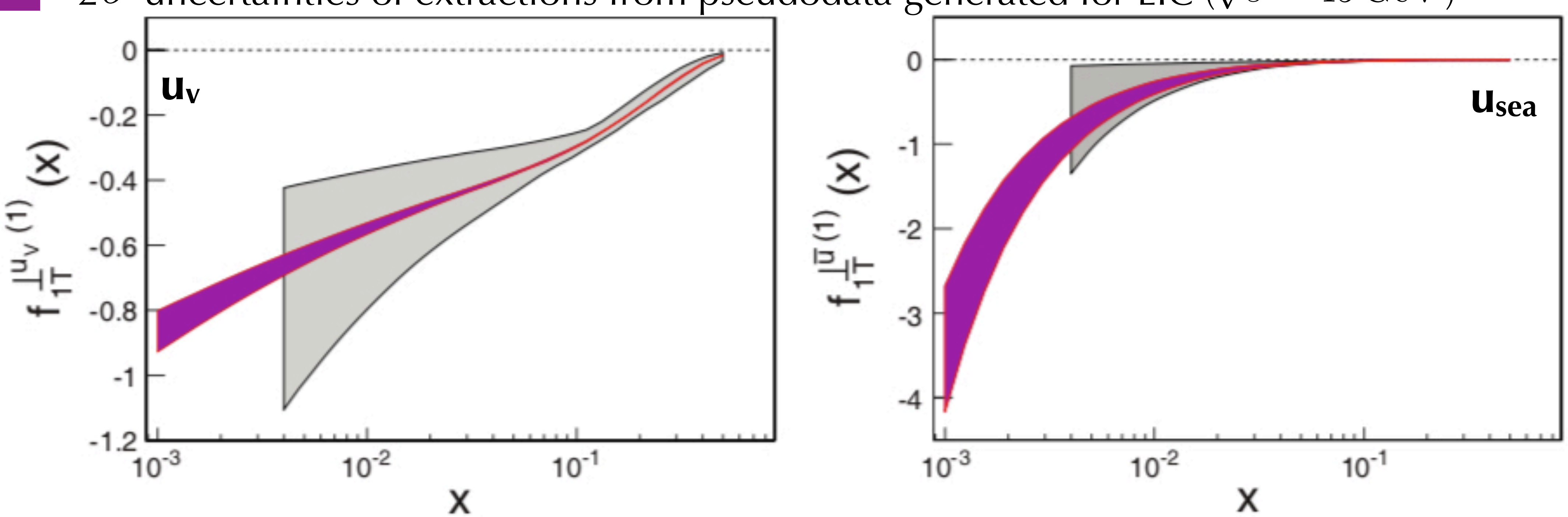
Sivers function at JLab12 and EIC



Dudek et al., Physics opportunities with the 12 GeV JLab upgrade at JLab, EPJA48(2012) 187

2σ uncertainties of extractions from currently available data

2σ uncertainties of extractions from pseudodata generated for EIC ($\sqrt{s} = 45\,\mathrm{GeV}$)

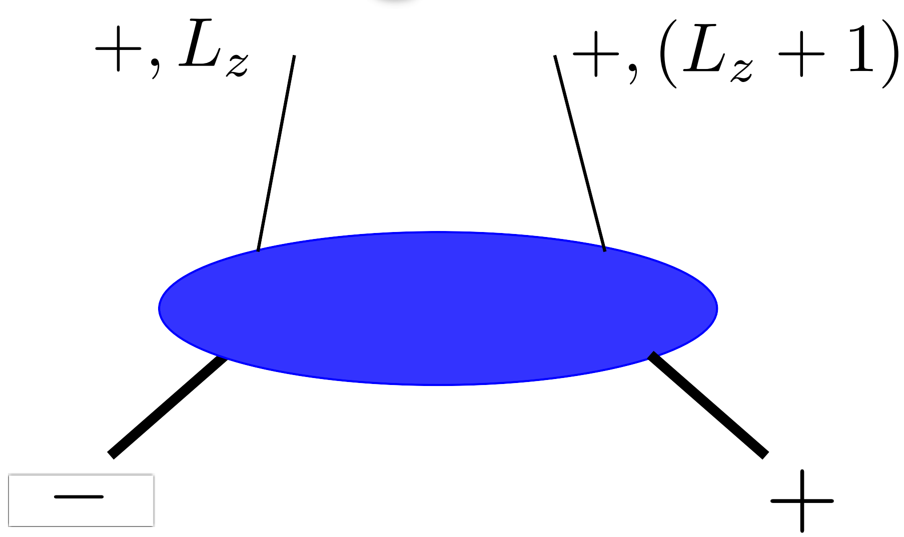


Accardi et al., The Electron Ion Collider: the next QCD Frontier, EPJA52 (2016) 268

The Worm-Gear functions

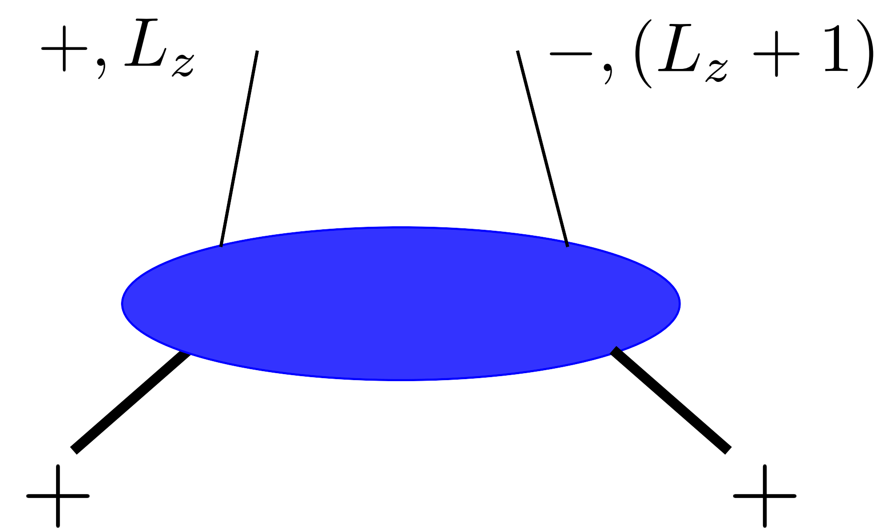
Another way to access orbital angular momentum
information without final state interactions:
Worm gear functions

g_{1T}



longitudinally pol. quarks
in transversely pol. nucleon

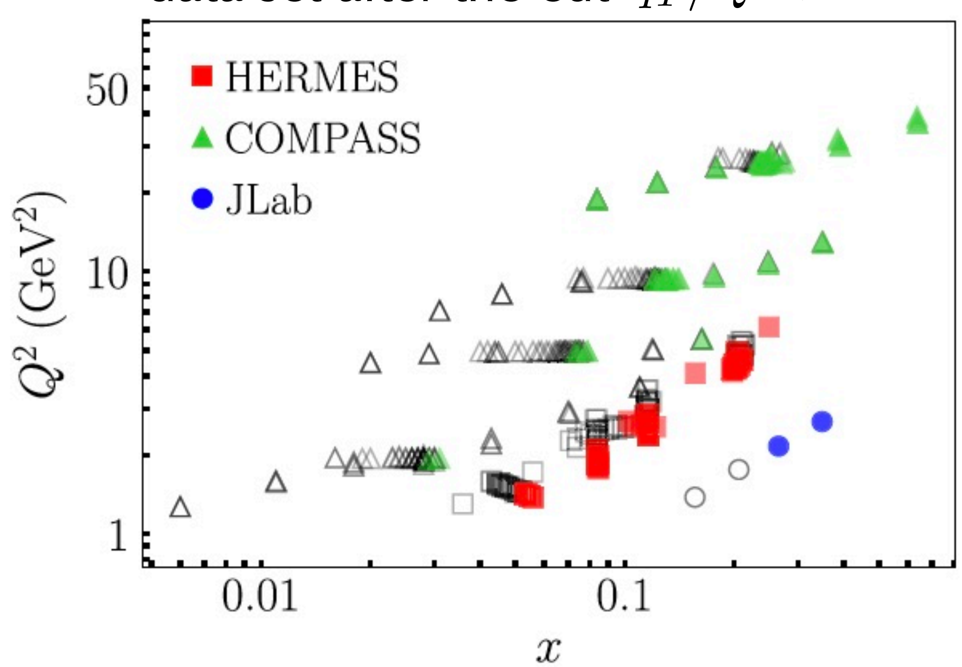
$h_{1L}^{\perp}$



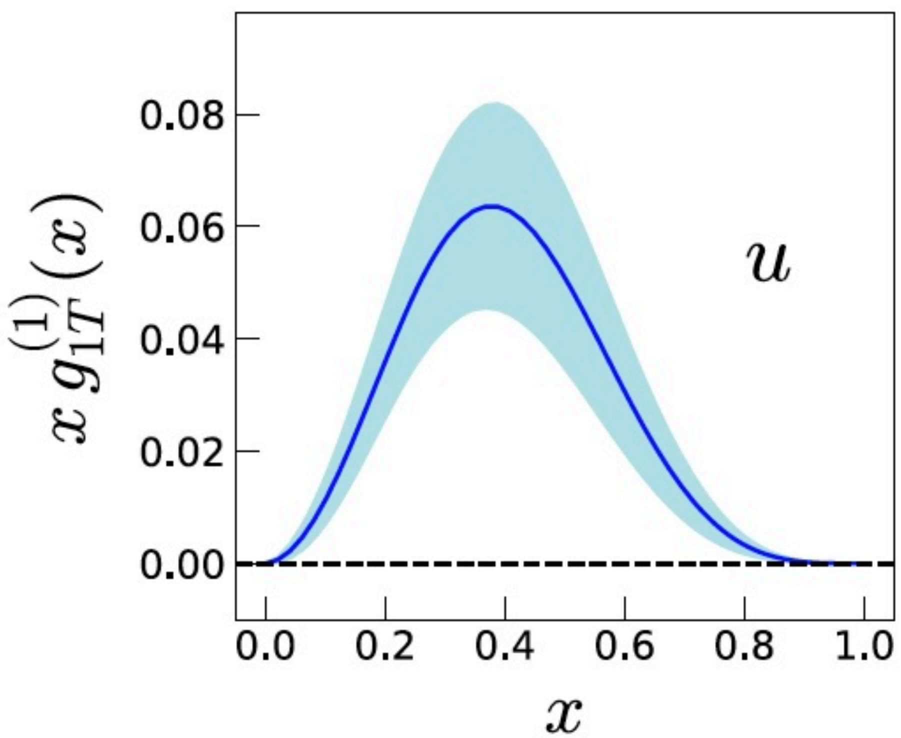
transversely pol. quarks
in longitudinal pol. nucleon

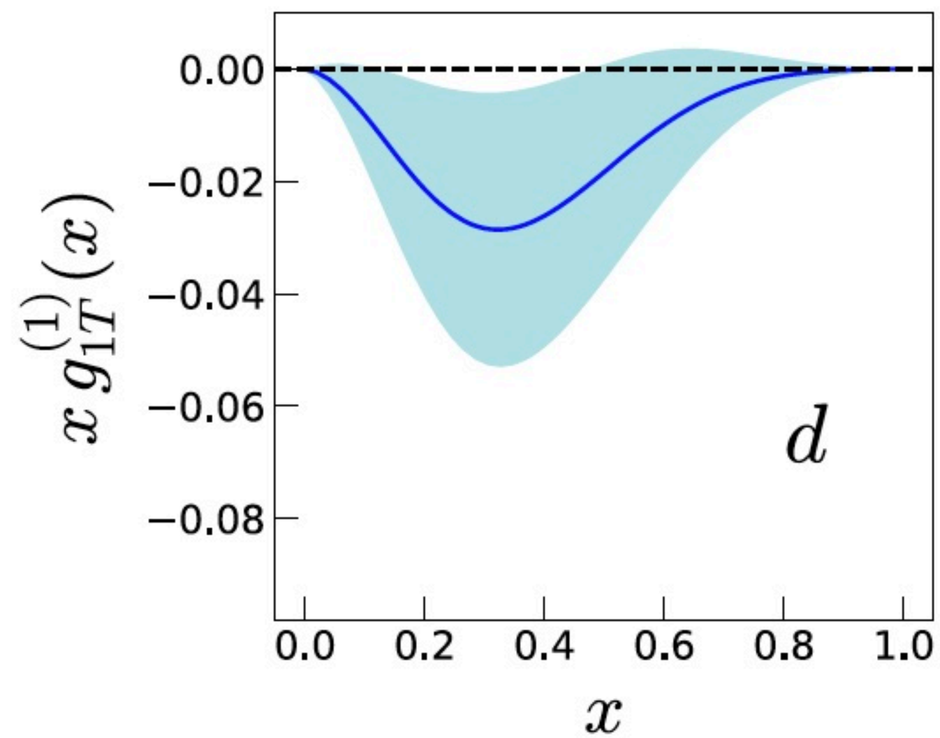
First extraction of g_{1T}

Bhattacharya et al, PRD105, 034007 (2022)



$$g_{1T}^{(1)} = \int d^2\vec{k}_\perp \left(\frac{k_\perp^2}{2M^2}\right) g_{1T}(x, \vec{k}_\perp^2)$$



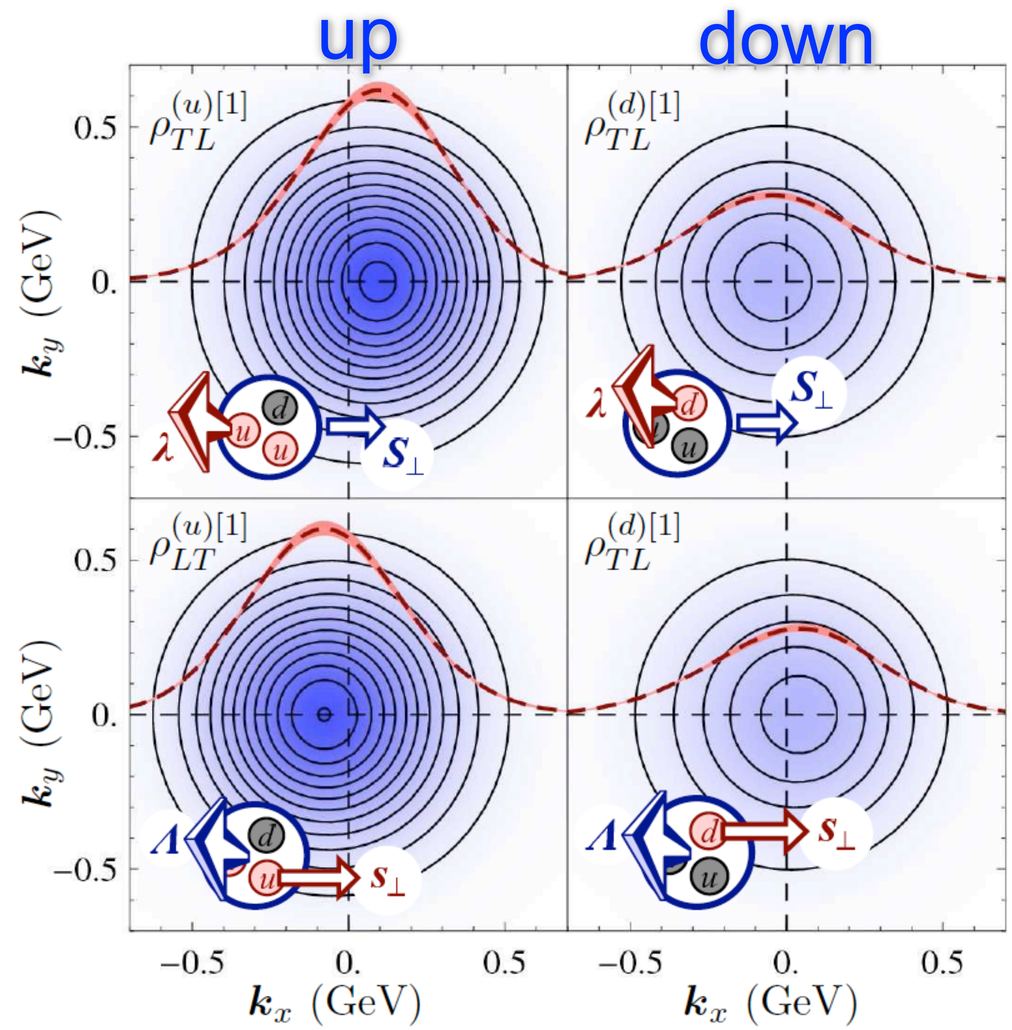


Worm-gear shift $\langle k_x \rangle_{TL}$ compatible with lattice results

Pioneering lattice QCD studies

Musch, Hagler, Negele, Schaefer, Europhysics Lett. ***88*** *(2009) 61001*

$\langle k_x^u \rangle = 67(5)\ \mathrm{MeV}$ g_{1T}



$\langle k_x^d \rangle = -30(5)\ \mathrm{MeV}$

$\langle k_x^u \rangle = -60(5)\ \mathrm{MeV}$ $h_{1L}^{\perp}$

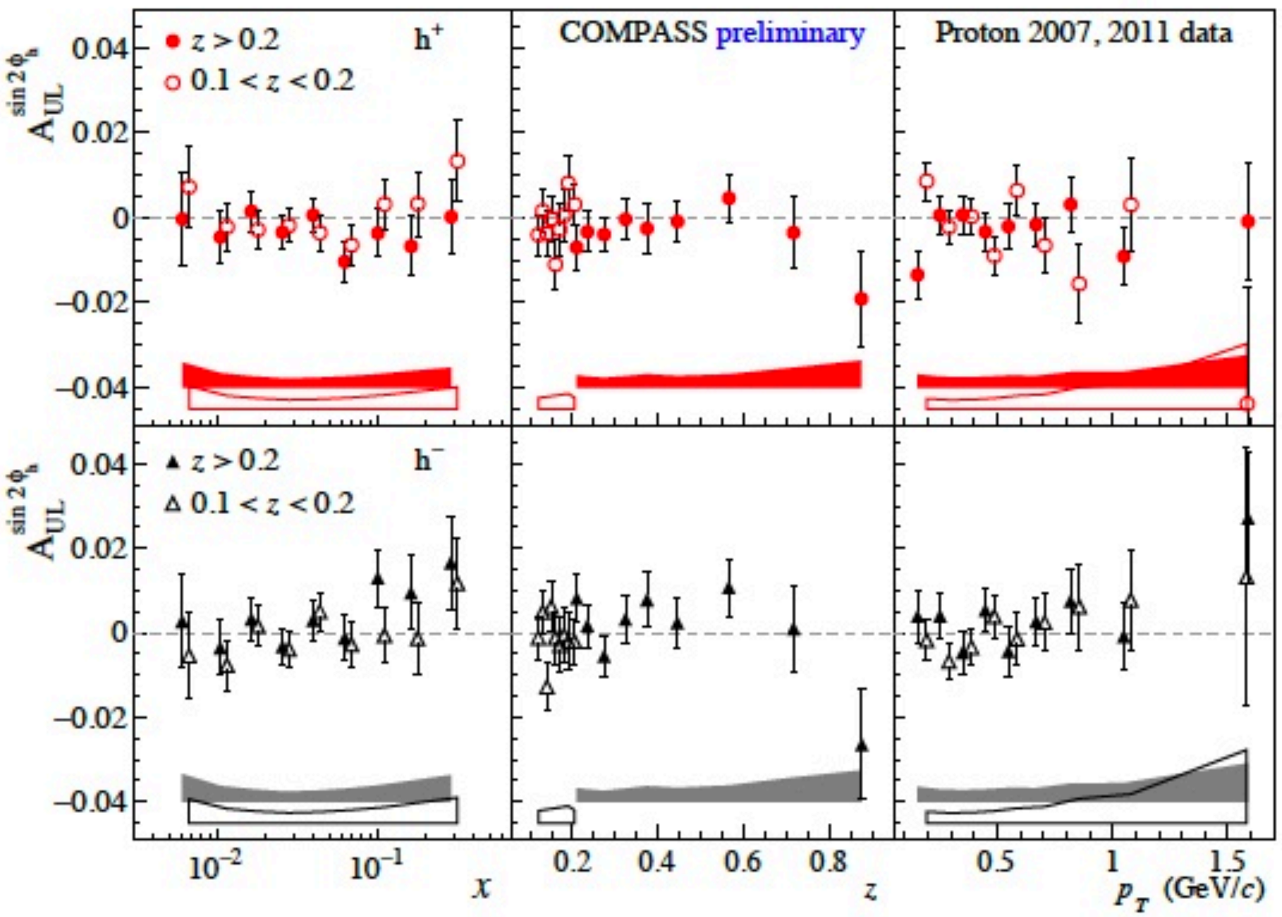
$\langle k_x^d \rangle = 16(5)\ \mathrm{MeV}$

$$\langle k_x^q \rangle_{g_{1T}} \approx -\langle k_x^q \rangle_{h_{1L}^{\perp}}$$

~~g_{1T}, $h_{1L}^{\perp}$ $\longleftrightarrow$ IPDs~~

genuine effect of intrinsic transverse momentum of quarks!
not counterpart in impact parameter space distributions

Limitations of existing data/facilities



- sample data for $A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$

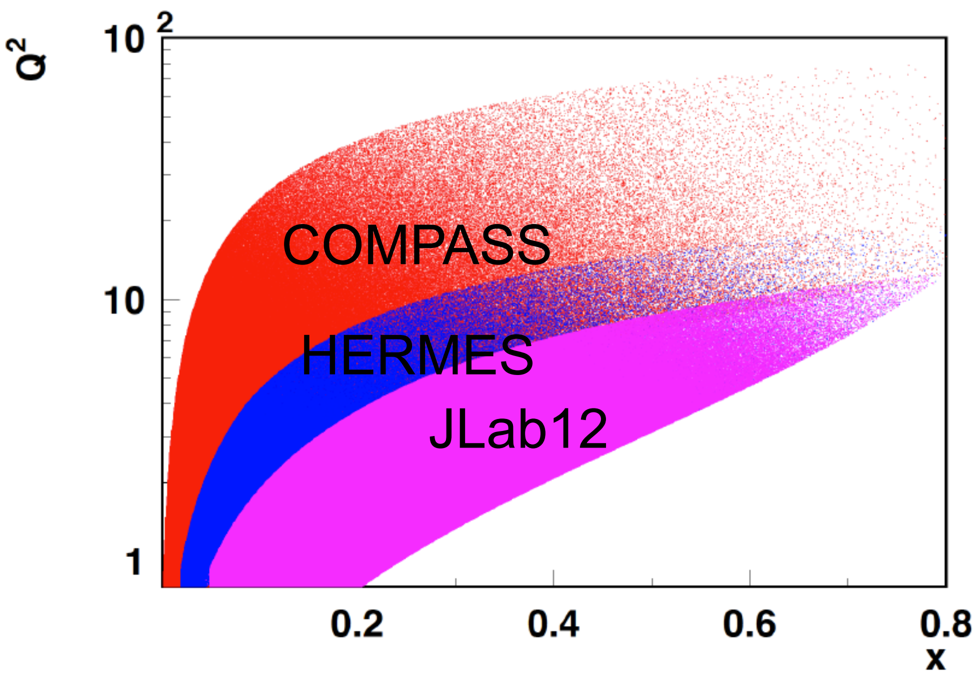
- models predict small effects

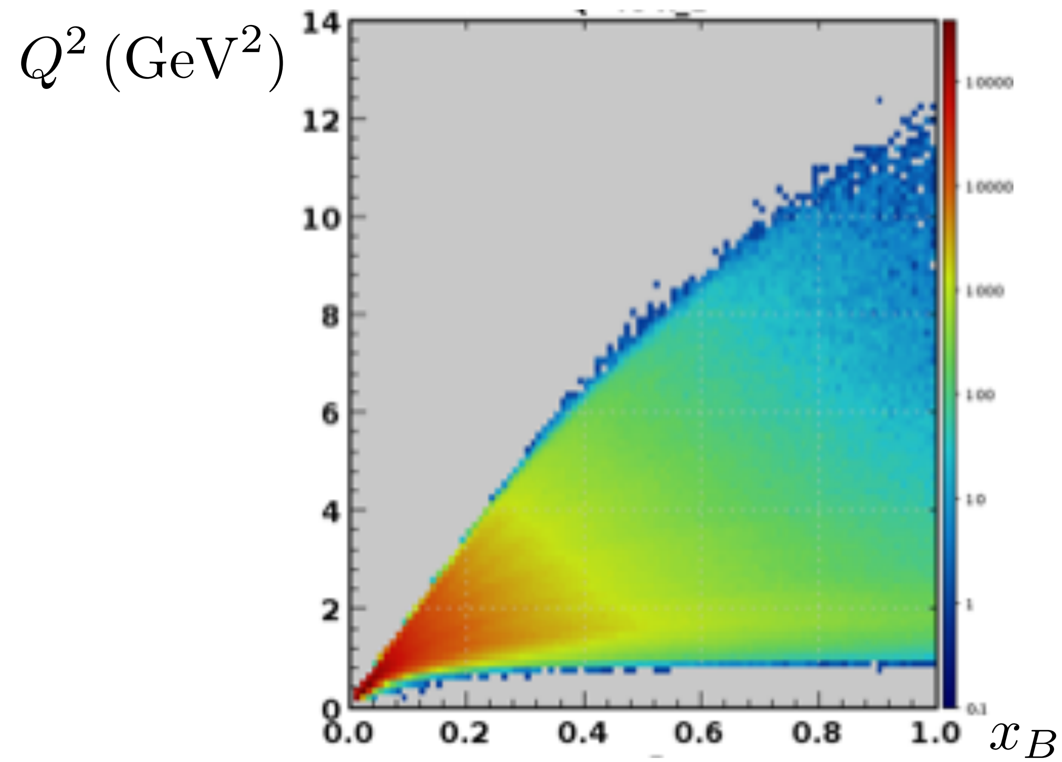
- data basically only allow conclusion that effect is compatible with zero

Existing data/facilities often suffer from one or more of the following:

- lack of data precision (due to lack of machine luminosity)
- lack of kinematical coverage
- lack of polarization
- limited detector capabilities

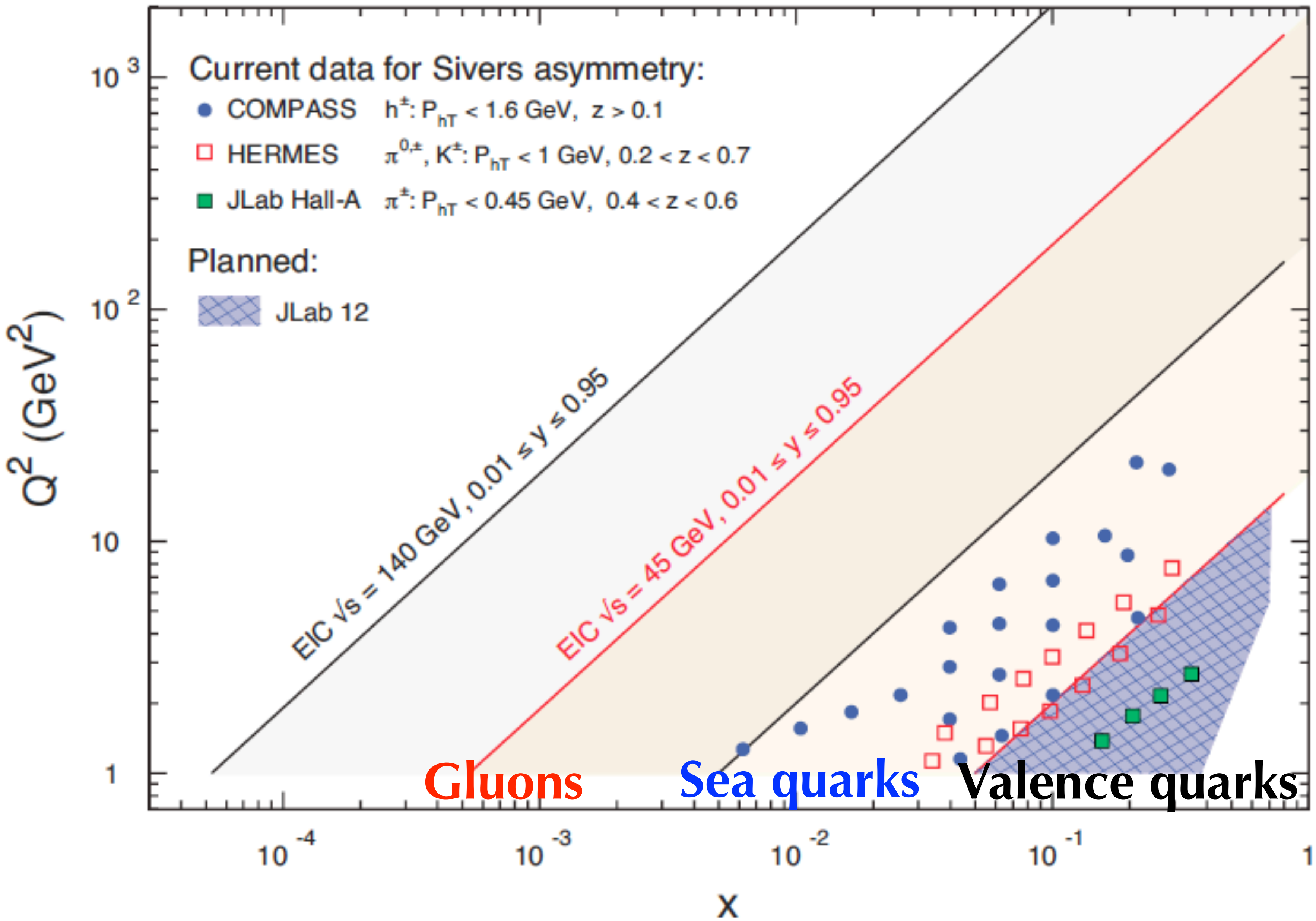
JLab12 SIDIS program





- JLab12 program very important to constrain TMD distributions at large x_B
- complementary measurements with different targets
- Hall B: large acceptance (CLAS), unpolarized and polarized H e D targets; cross sections, single and double-spin asymmetries; start kaon SIDIS program with RICH detector
- Hall C: SHMS + HMS, precision magnetic spectrometer setup, unpolarized target; L/T separation in SIDIS, precision cross section of π^+ and π^-, and K^+ and K^-
- Hall A: forward large acceptance (SOLID), longitudinal and transversely polarized ^3He target; pion and kaon run; access to neutron structure at high x_B and Q^2

Paste, present and future TMD measurements



hermes

Jefferson Lab

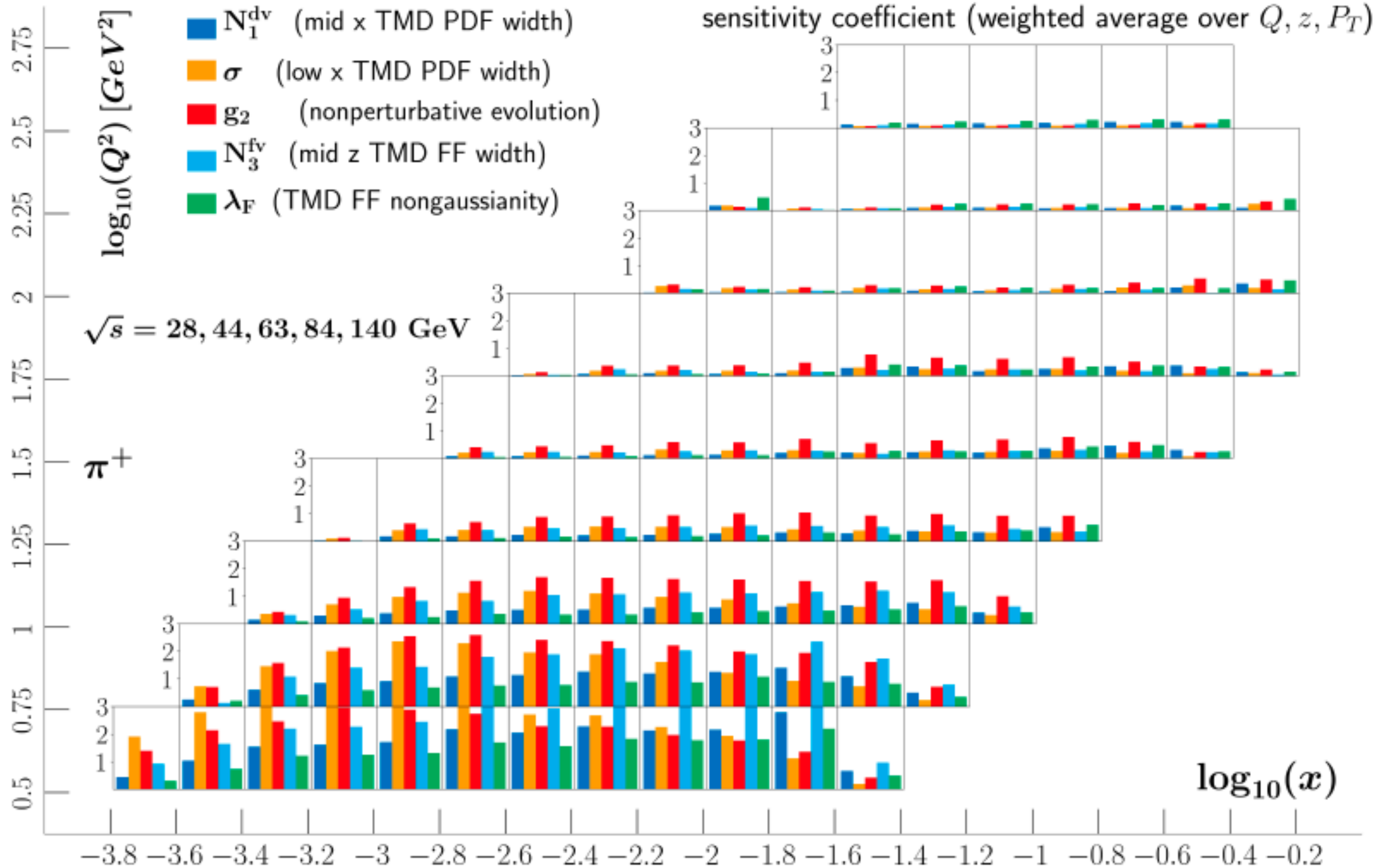
COMPASS

EIC

- multidimensional binning
- high Q^2 reach
- large range in transverse momentum

Accardi et al., The Electron Ion Collider: the next QCD Frontier, EPJA52 (2016) 268

Foreseen EIC impact on unp. TMDs: SIDIS measurements



Sensitivity coefficients: measure of the correlation between fit parameters and measurable quantities at EIC

EIC Yellow Report: arXiv: 2103.05419
based on Bacchetta, et al., JHEP 07 (2020) 117

Library and Plotting tools for collinear parton distributions

LHAPDF
lhapdf.hepforge.org

xFitter
www.xfitter.org

APFEL++
github.com/vbertone/apfelxx
apfel.mi.infn.it

Dedicated Softwares to study GPDs

PARTONS
partons.cea.fr

PARtonic
Tomography
Of
Nucleon
Software

GeParD

not yet public

Dedicated software to study and fit TMDs

arTeMiDe
teorica.fis.ucm.es/artemide

TMD lib and TMD Plotter
tmdlib.hepforge.org

NangaParbat
MapCollaboration/NangaParbat

Efforts to combine different inputs to understand PDFs, TMDs and GPDs in an unified framework

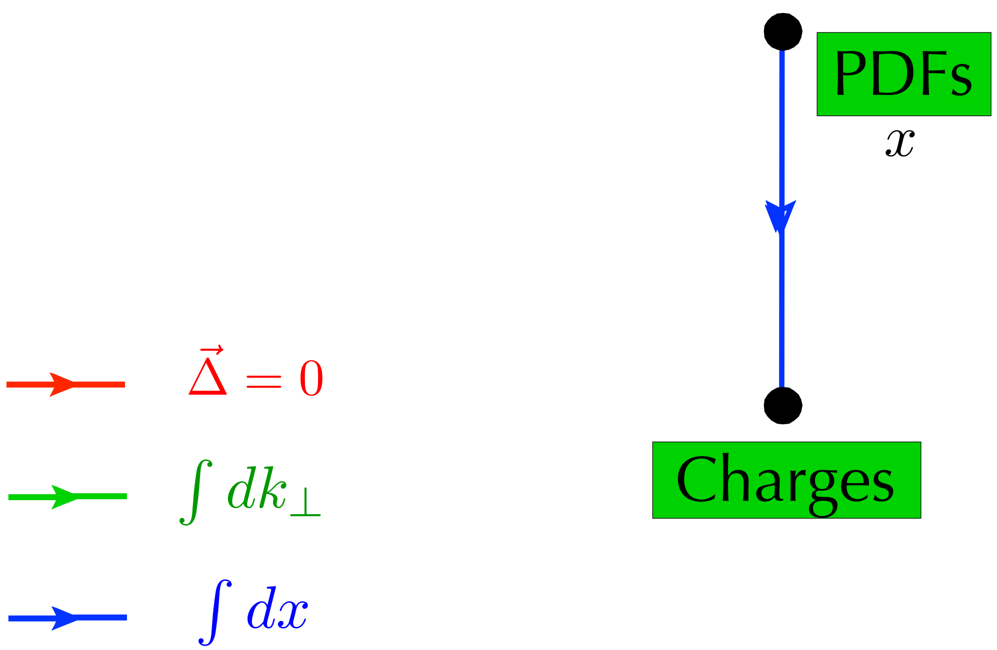
$\longrightarrow$ $\vec{\Delta} = 0$

$\longrightarrow$ $\int dk_\perp$

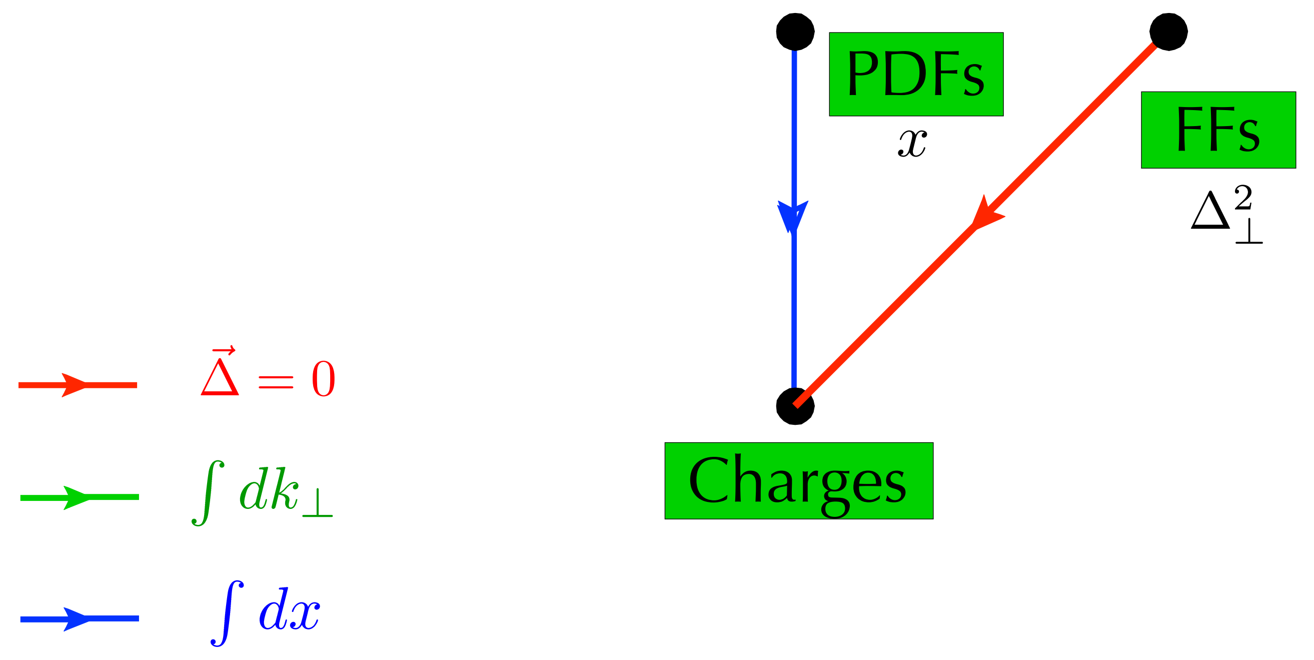
$\longrightarrow$ $\int dx$

Charges

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

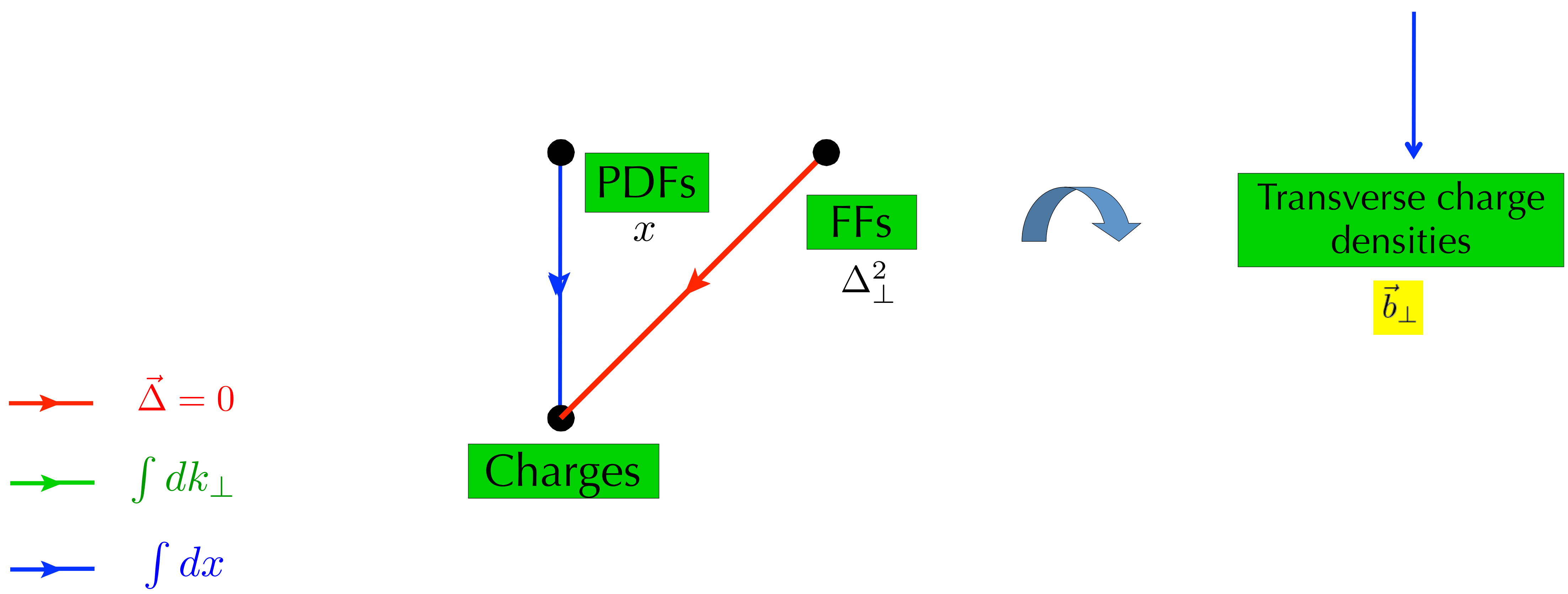


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



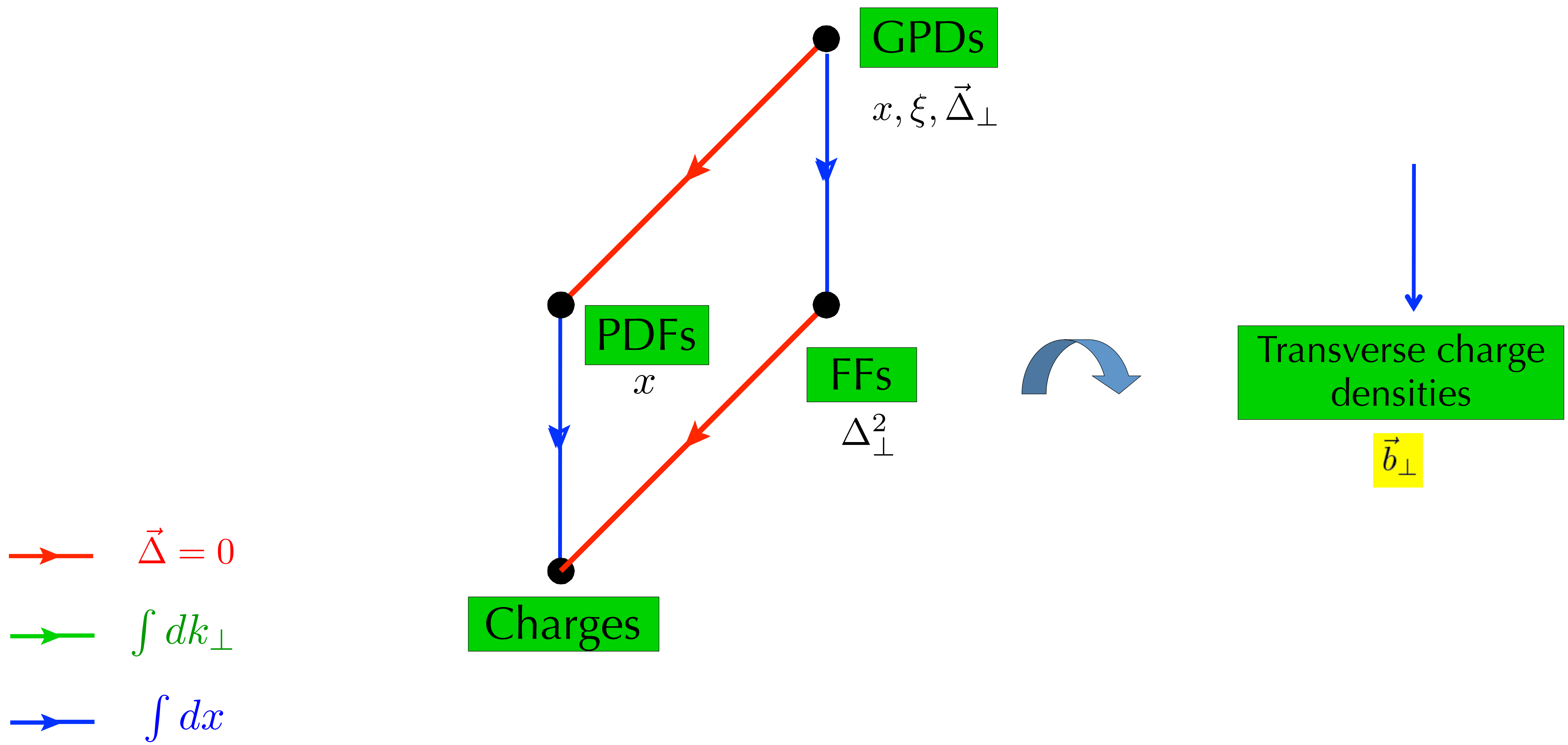
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier transform $\Delta_\perp \leftrightarrow b_\perp$

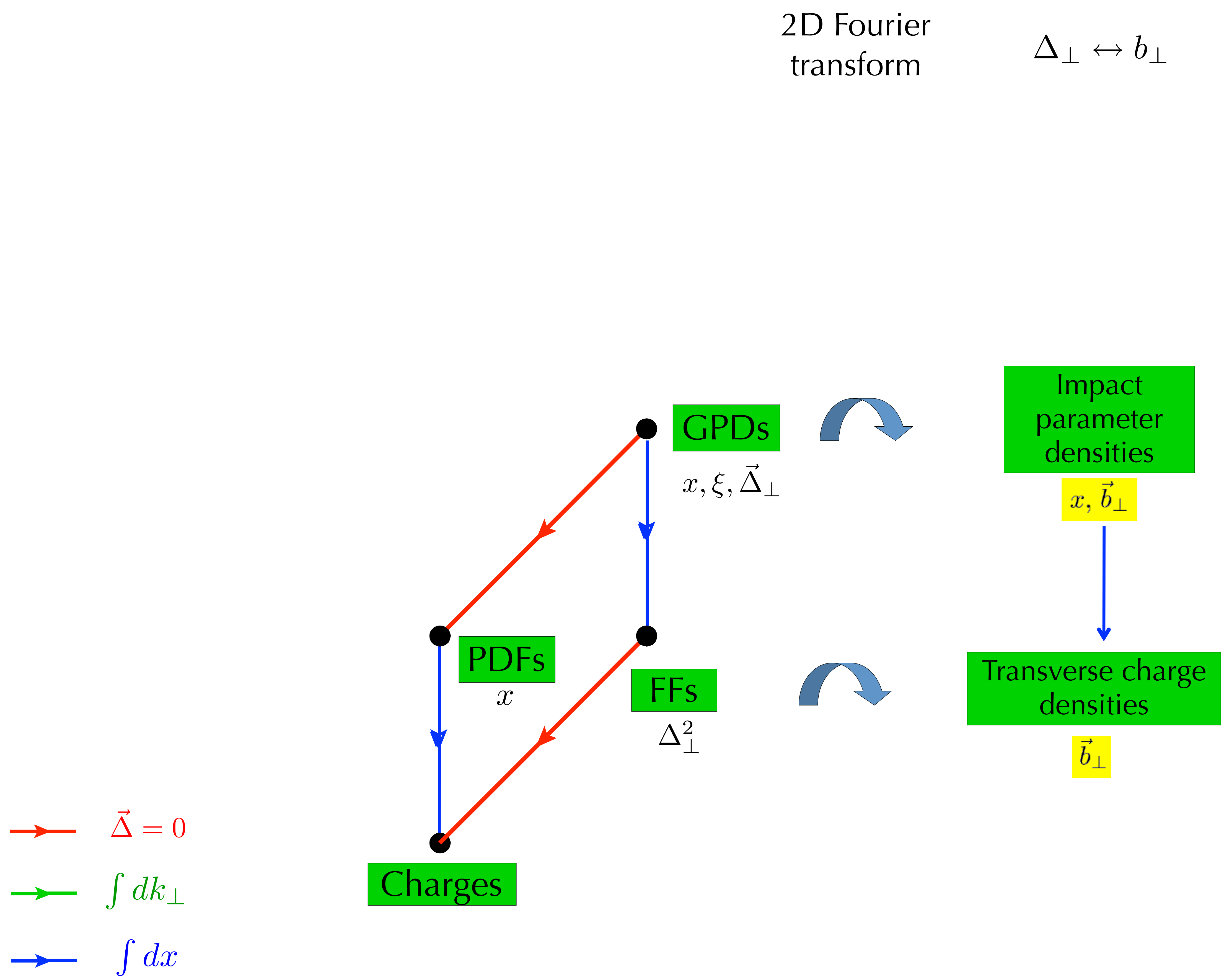


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

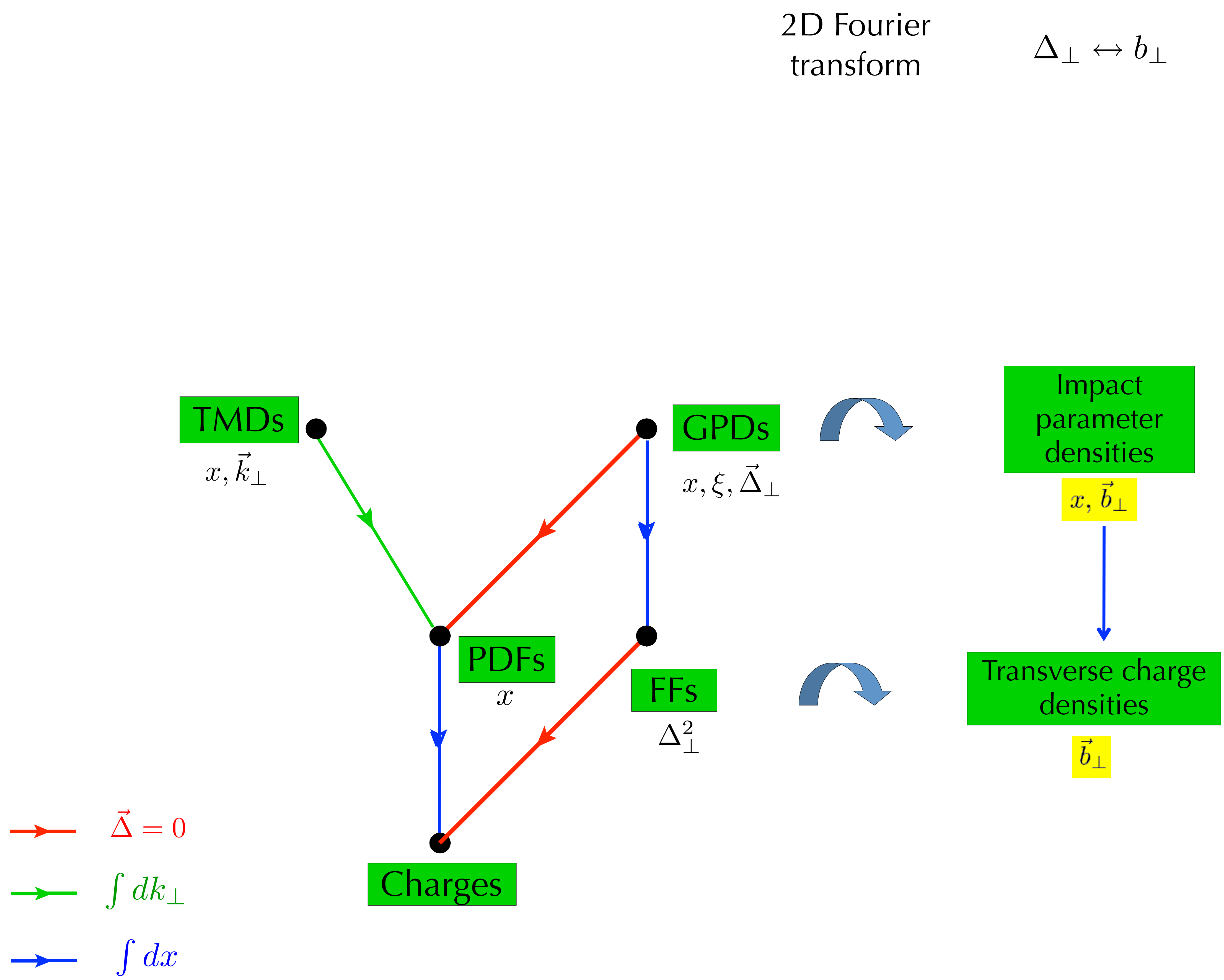
2D Fourier transform $\Delta_\perp \leftrightarrow b_\perp$



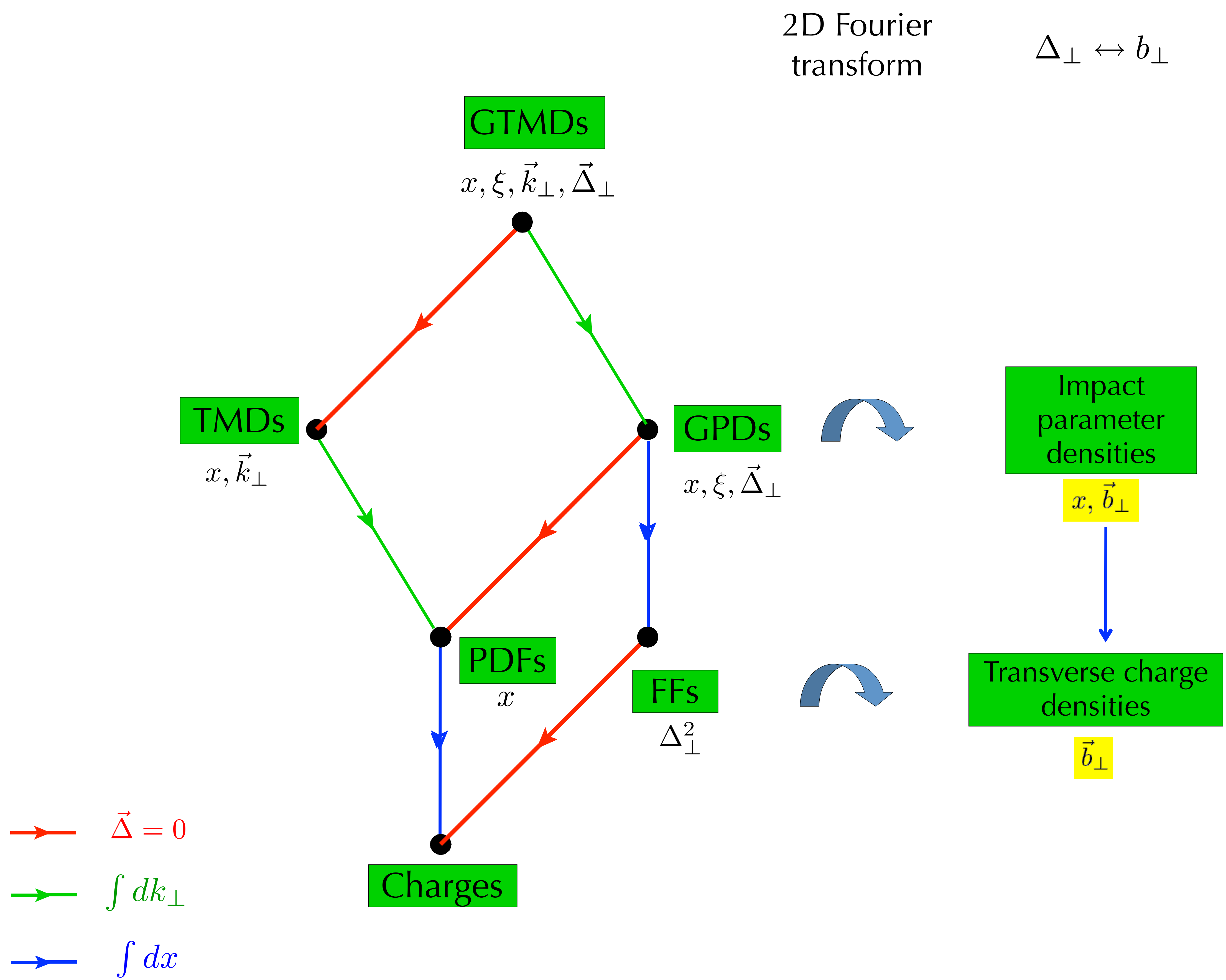
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



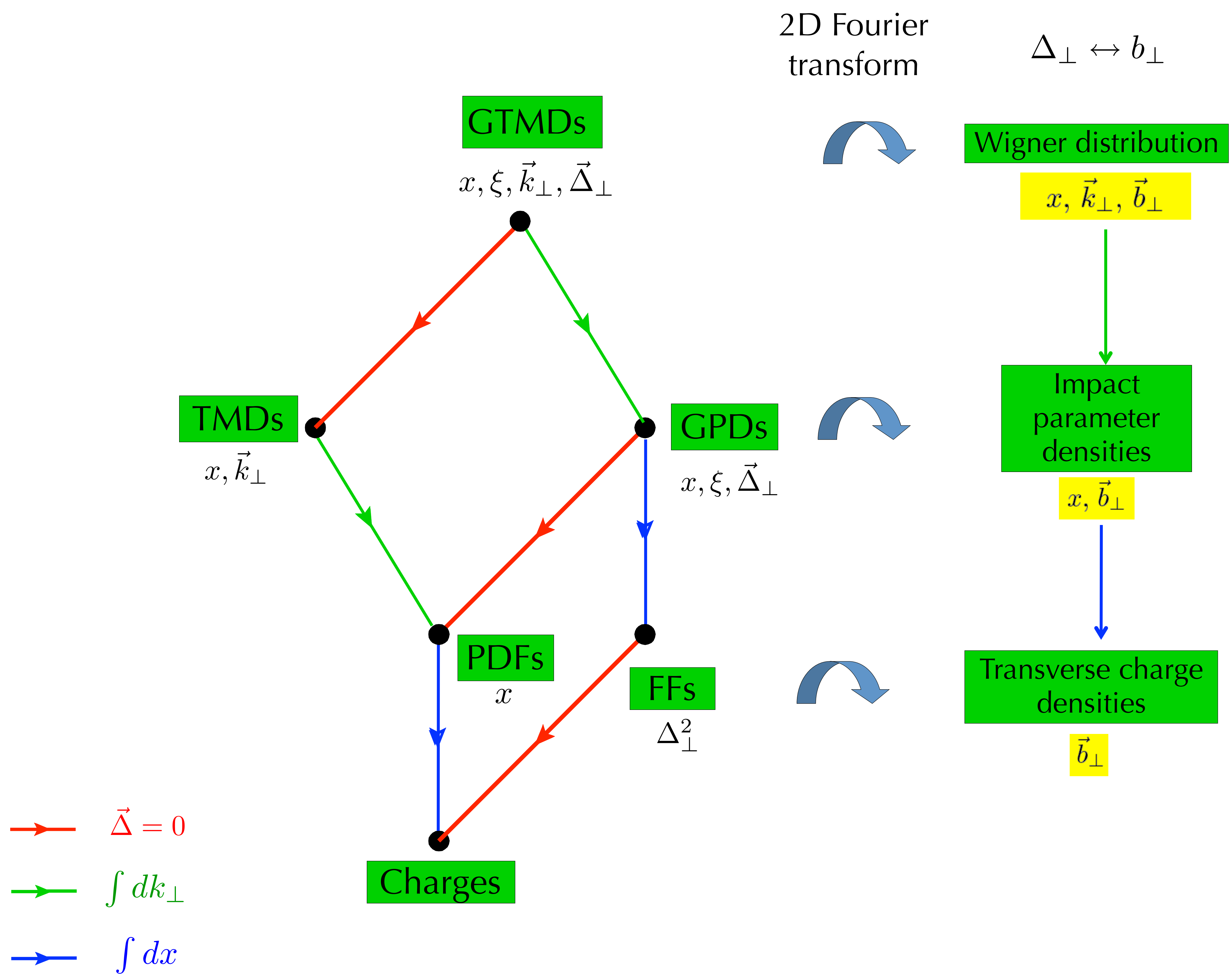
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



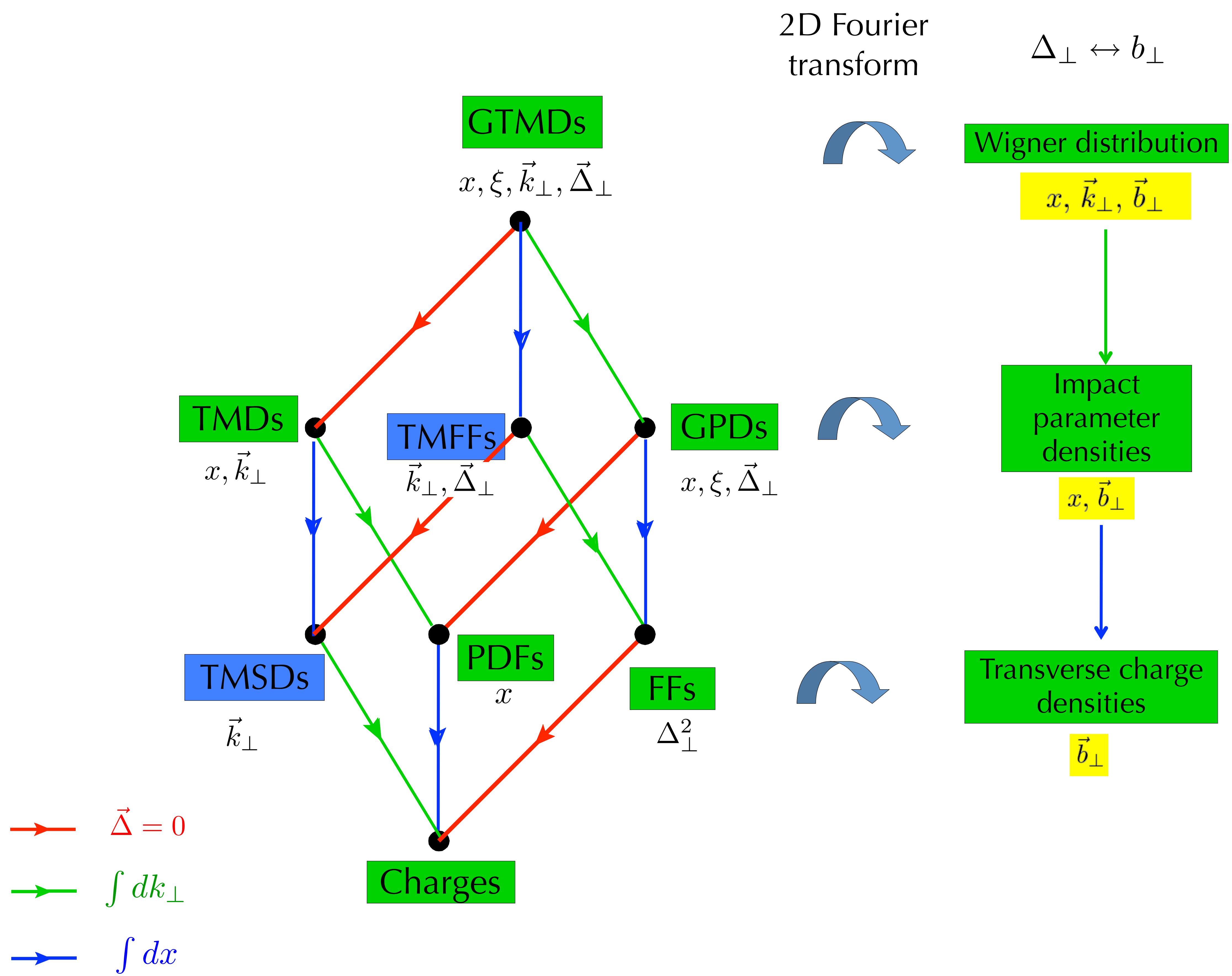
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

Angular Correlations

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1\rangle$	$\langle S_L^q \ell_L^q\rangle$	$\langle S_x^q \ell_x^q\rangle$	$\langle S_y^q \ell_y^q\rangle$
L	$\langle S_L \ell_L^q\rangle$	$\langle S_L S_L^q\rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q\rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q\rangle$
T_x	$\langle S_x \ell_x^q\rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q\rangle$	$\langle S_x S_x^q\rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q\rangle$
T_y	$\langle S_y \ell_y^q\rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q\rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q\rangle$	$\langle S_y S_y^q\rangle$

$\boxed{\xi = 0}$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T,\ \tilde{H}_T$

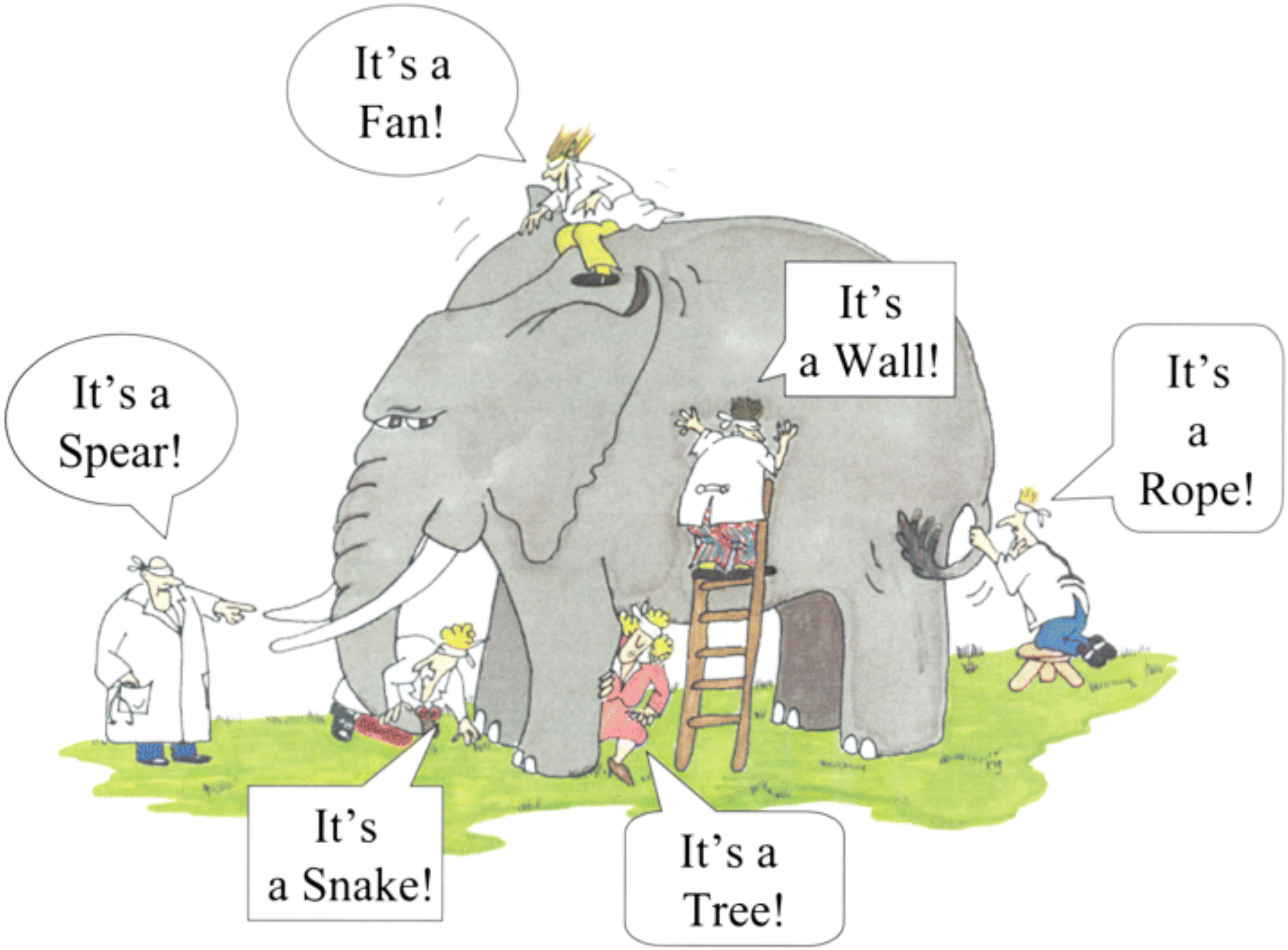
TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

The blind men and the elephant



Different observables in different kinematical regimes
need to talk to each other
to reconstruct the full picture of the nucleon