Effective Field Theories of QCD

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Quantum Chromodynamics

Effective theory of Weak Force

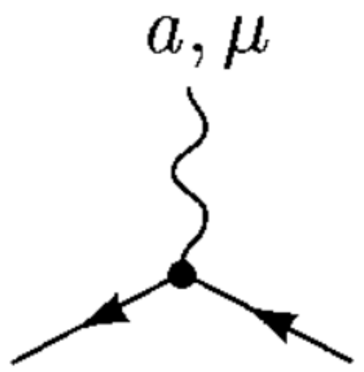
HQET, Non-Relativistic QCD

Soft-Collinear Effective Theory

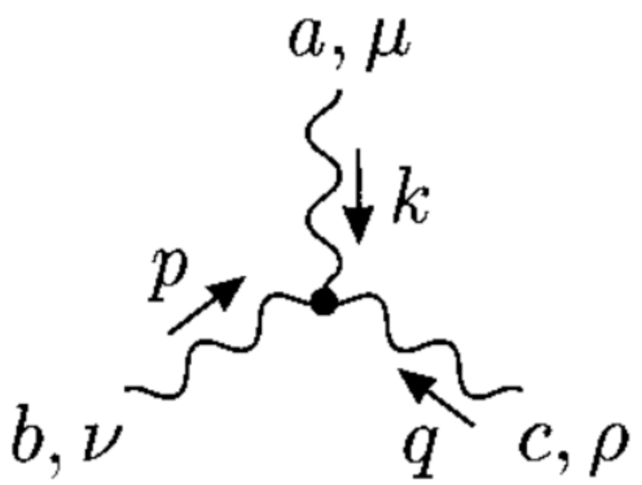
Quantum Chromodynamics

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_i \bar{\psi}_i (i\not{D} - m_i)\psi_i$$

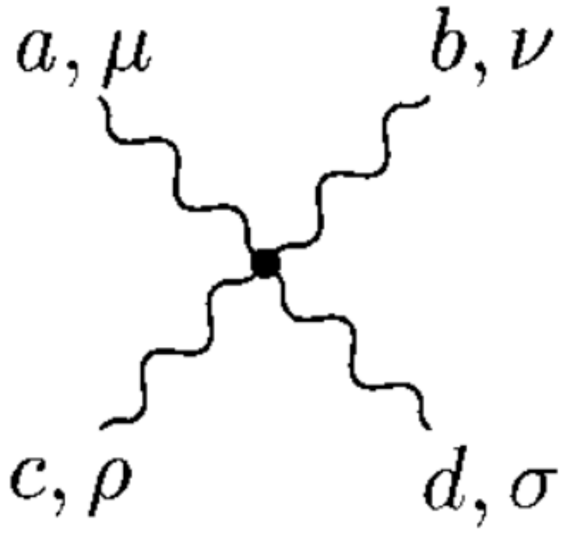
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + igf^{abc} A^b_\mu A^c_\nu$$



$$= \quad ig\gamma^\mu t^a$$

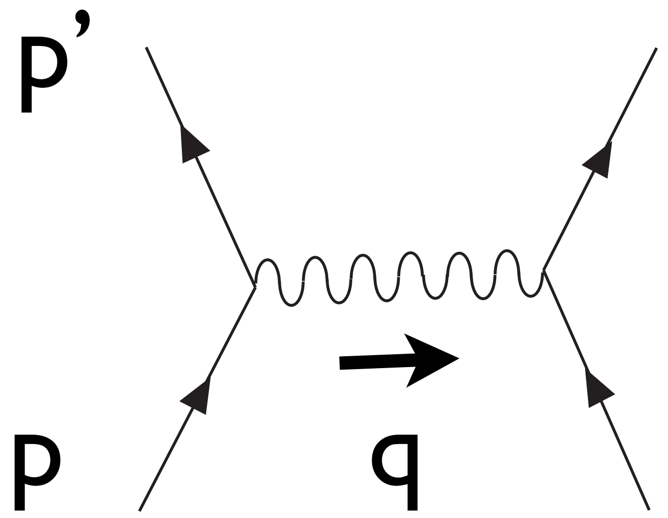


$$= \quad gf^{abc}\big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\big]$$



$$= \quad -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big]$$

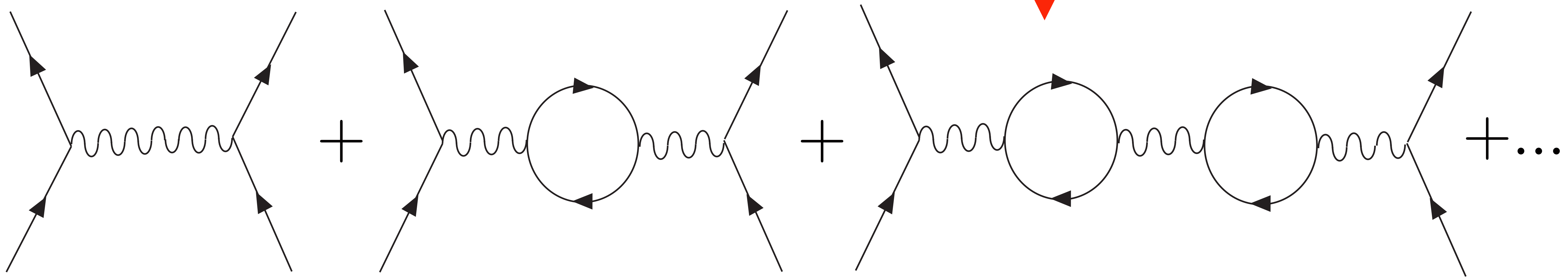
Running Coupling Constants



$\propto \dfrac{\alpha}{q^2}$

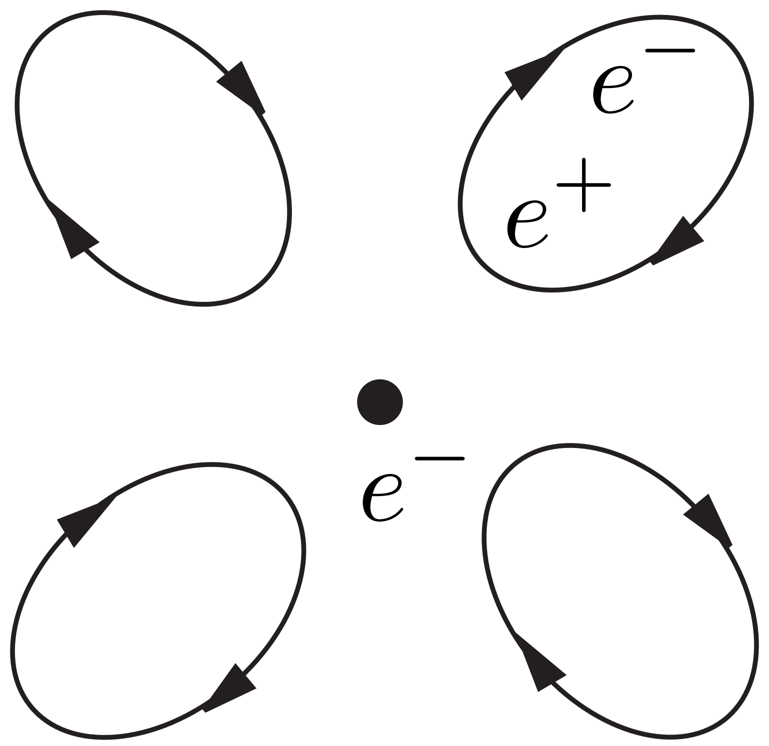
q = p - p' : momentum transfer

$\sim \alpha \log(|q^2|/m_e^2)$ $\qquad |q| \gg m_e$



$\propto \dfrac{\alpha_{\text{eff}}(q)}{q^2}$

Vacuum Polarization screens bare charge



$\alpha(m_e) = 1/137$

atomic physics

$\alpha(90\,\text{GeV}) = 1/128$

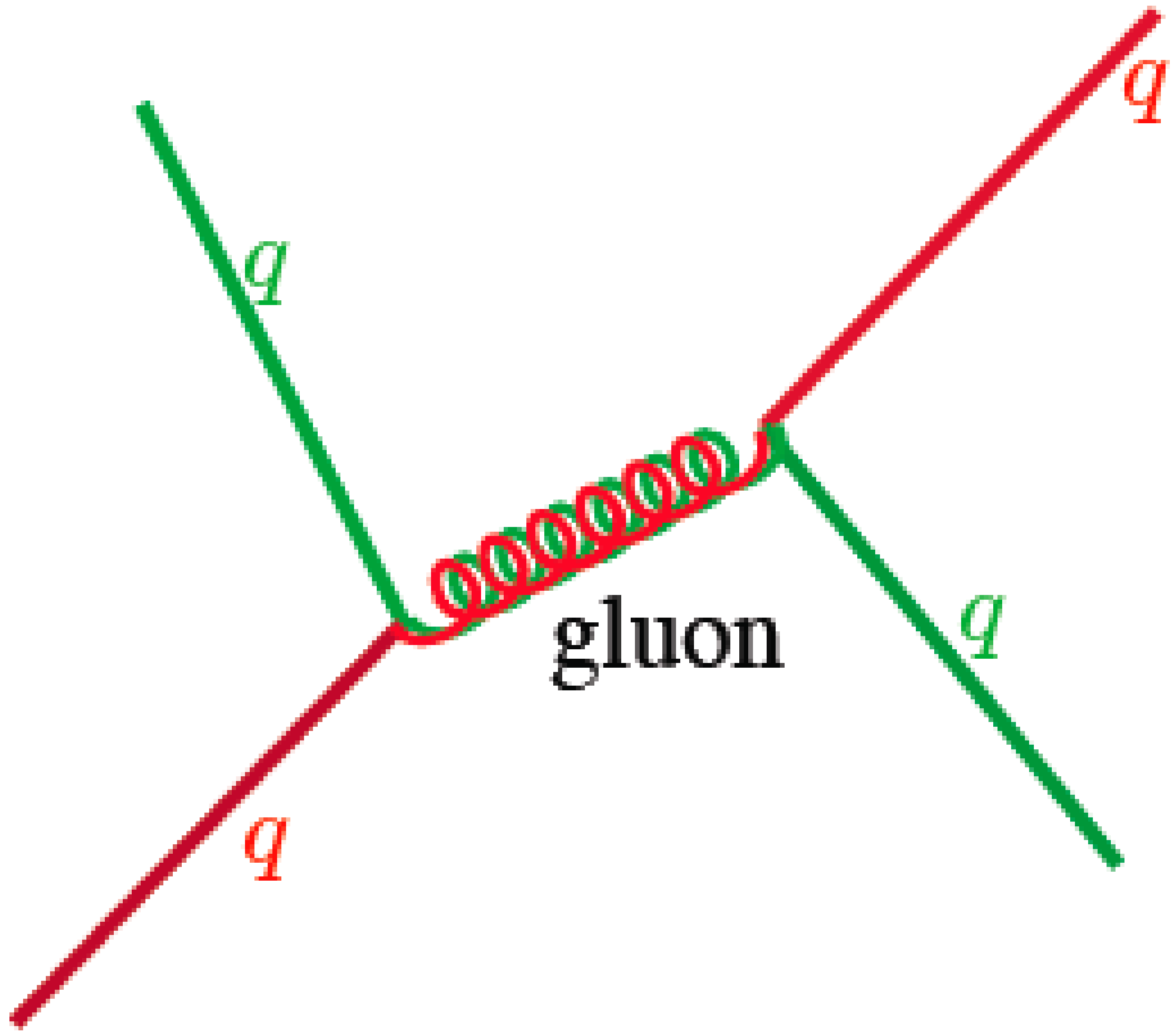
$\text{LEP} : e^+e^- \to Z_0 \to X$

Heisenberg: $q \leftrightarrow \hbar/r$

QED

electrons/positrons - (+,-) charges

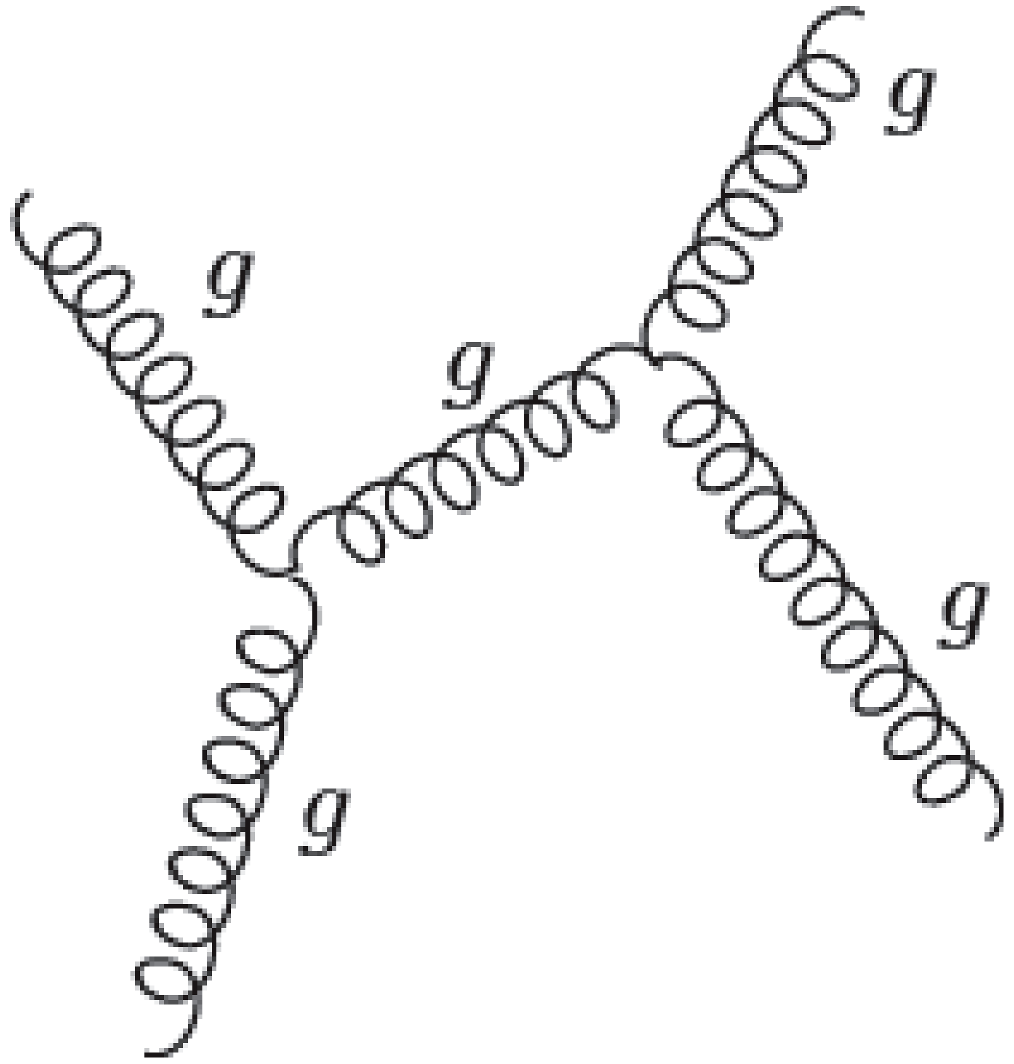
photon - neutral



QCD

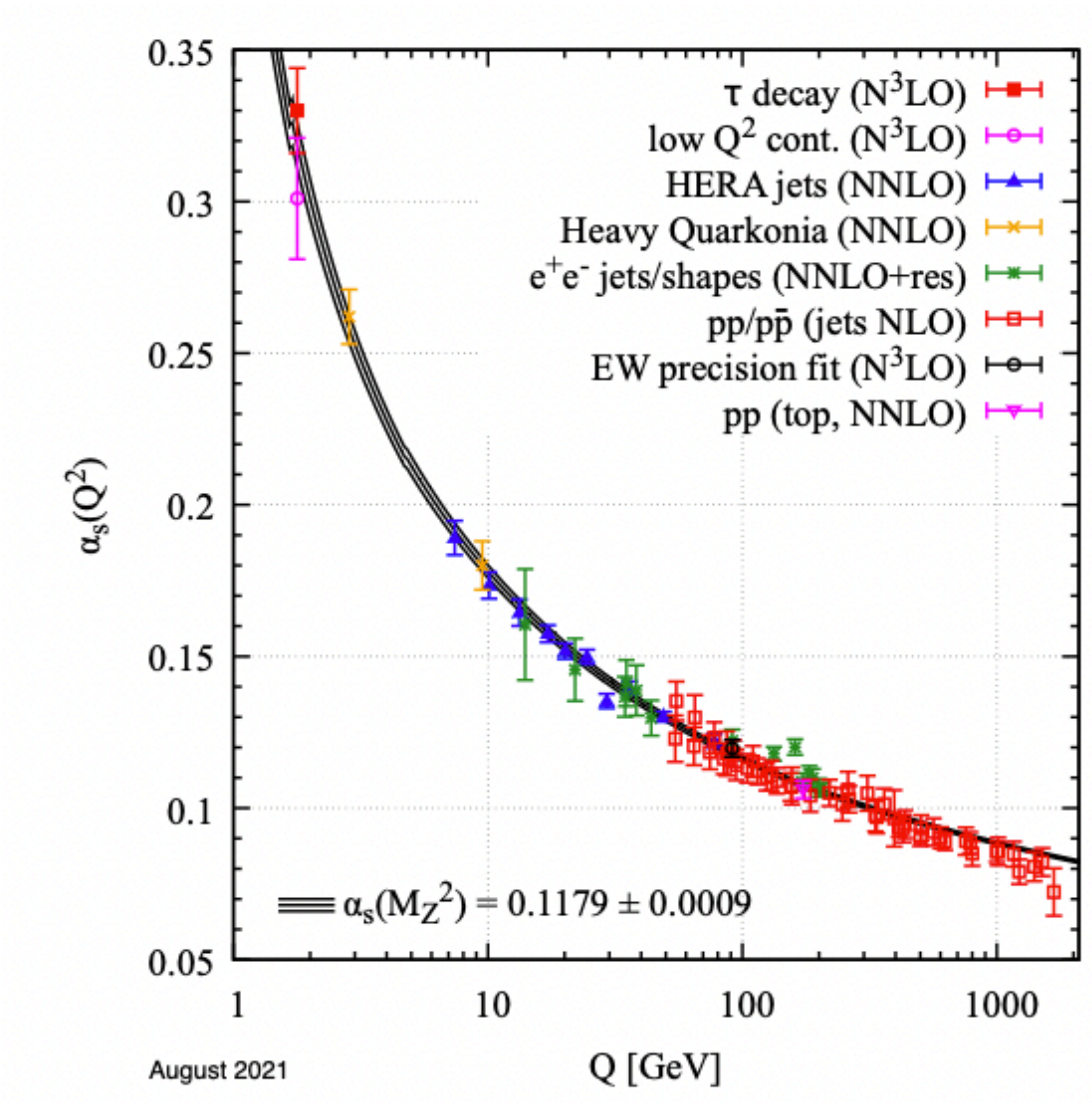
quarks - three colors (r,g,b)

gluons - 8 colors (e.g., r $\bar{g}$)

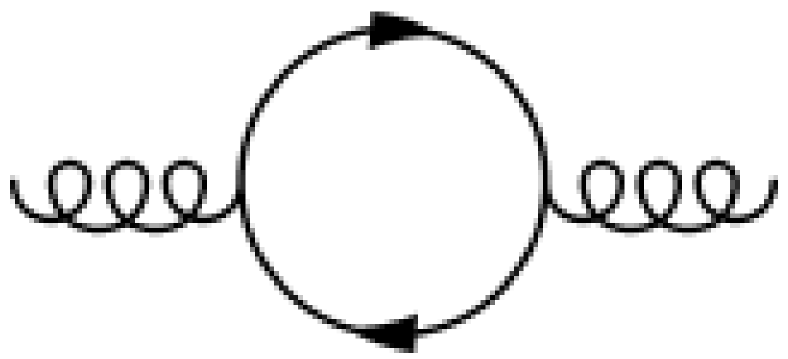


Key Difference: gluons colored and self-interact

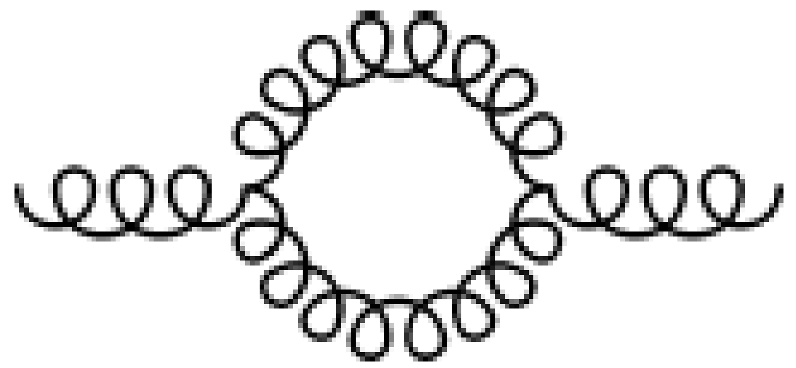
Asymptotic Freedom



(Gross, Politzer, Wilczek; Nobel 2004)



Screening



Anti-Screening

UV perturbation theory valid

IR - Strong Coupling
$$\left\{ \begin{array}{l} \text{Confinement} - \text{Colorless Hadrons} \\ \text{Chiral Symmetry Breaking} \\ \Lambda_{\text{QCD}} \sim 500\,\text{MeV} \end{array} \right.$$

Confinement

Physical Strongly Interacting Particles (hadrons) are Colorless

Mesons - q $\bar{q}$

Baryons - q q q

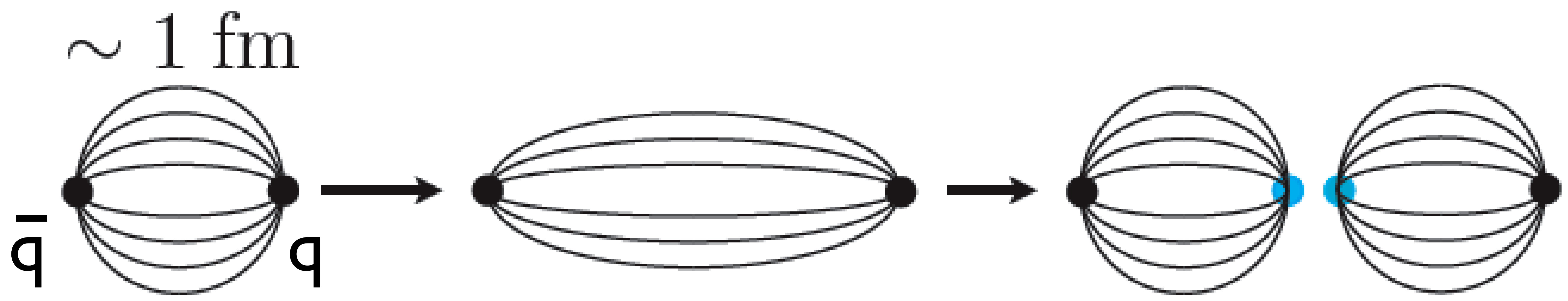
Potential Energy between quark and antiquark

$$V_{q\bar{q}}(r) \approx -\frac{4}{3}\frac{\alpha_s(1/r)}{r} + \sigma\, r$$

lattice QCD simulations

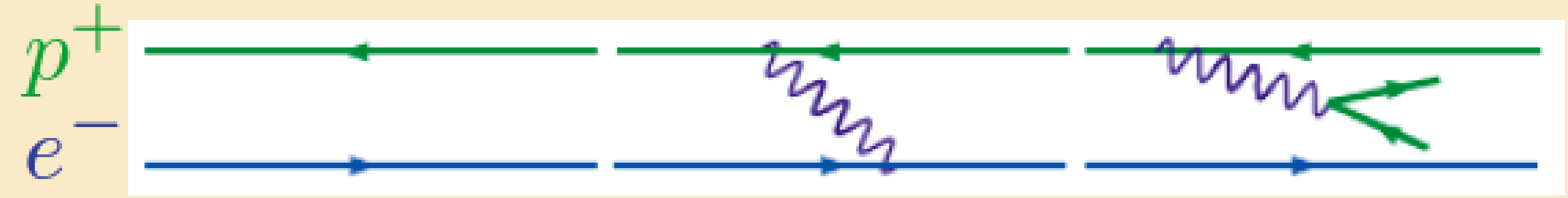
force between quark and antiquark separated by ~1 fm equivalent to 14 tons!

try to pull a quark-antiquark, create more mesons



Hydrogen atom

$\alpha \sim 1/137$

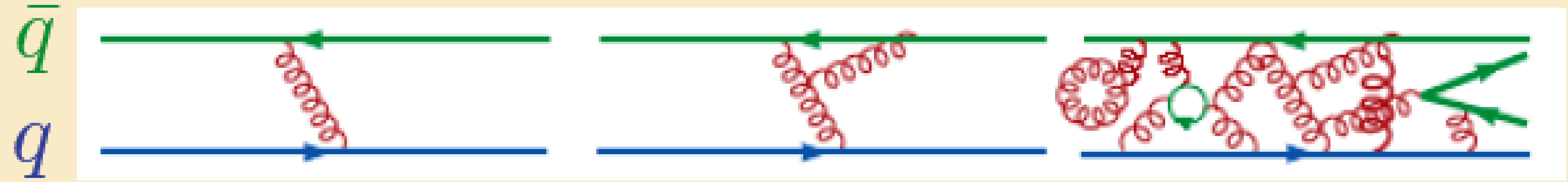


$$|H\rangle \sim |p^+e^-\rangle + \alpha|p^+e^-\gamma\rangle + \alpha^2|p^+e^-e^+e^-\rangle$$

Hydrogen a two-body system up to small corrections

Meson ($\bar{q}q$ + ...)

$\alpha_s \sim 1$

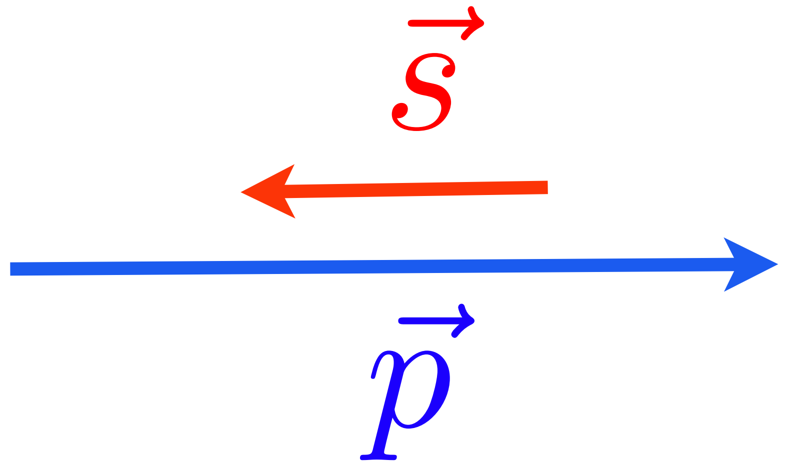


many-body system: many Fock states contribute

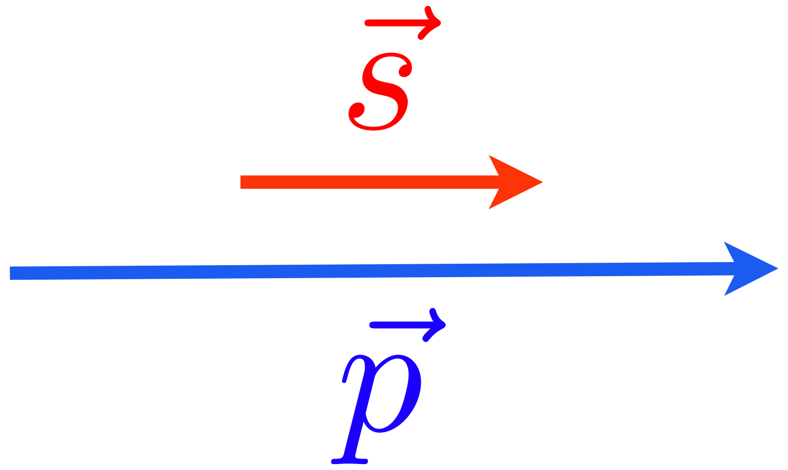
Chiral Symmetry

chirality = helicity (for a massless particle)

$\vec{s}\cdot\hat{p} = -1/2$



$\vec{s}\cdot\hat{p} = +1/2$

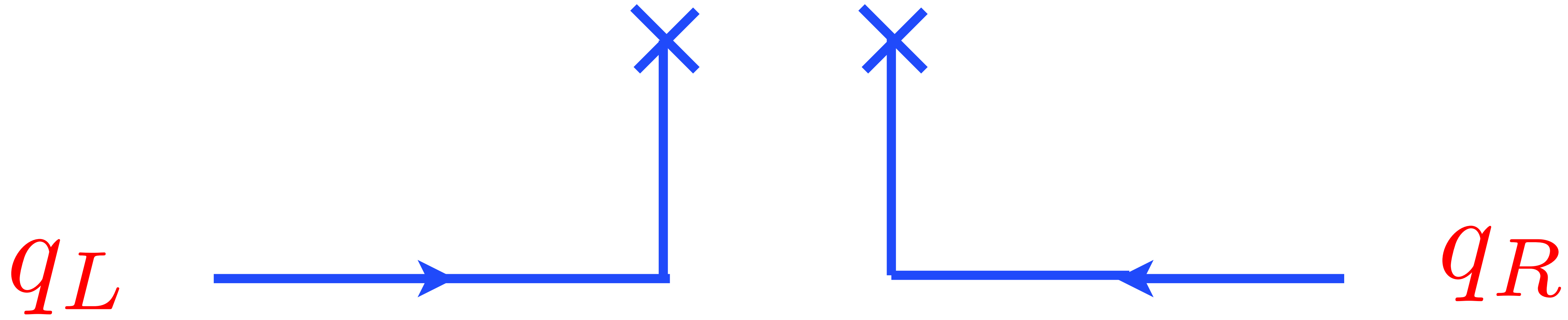


$q = q_L + q_R \qquad SU_L(3)\times SU_R(3)$

$$q_L \to U_L q_L \qquad q_R \to U_R q_R$$

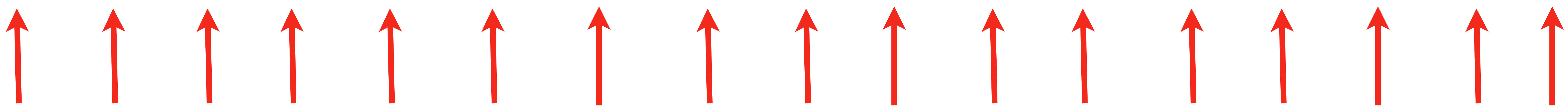
Symmetry of $\mathcal{L}_{\rm QCD}$, but not the vacuum

$$\langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle \neq 0$$

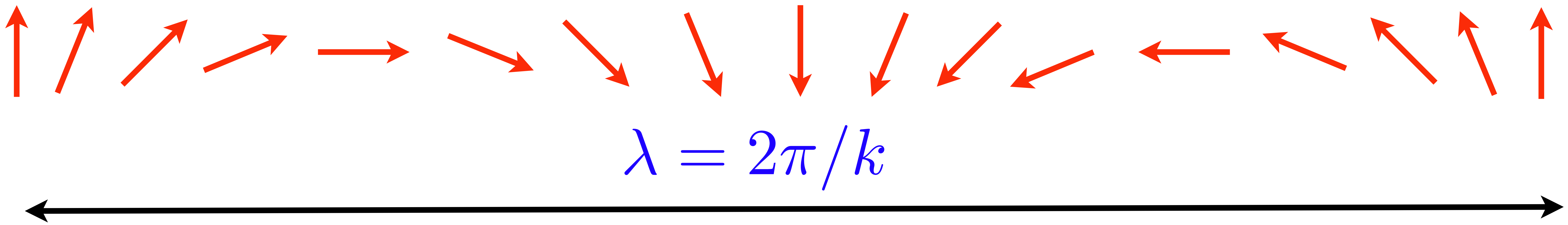


Spontaneous Chiral Symmetry Breaking

- Vacuum: $\langle \bar{q}_L q_R \rangle \leftrightarrow \uparrow$



- Excited State: $\langle \bar{q}_L U_L^\dagger(x) U_R(x) q_R \rangle \leftrightarrow \nearrow$



- recover vacuum as $\lambda \to \infty \quad (k \to 0)$ $\qquad E(k) \propto k$

relativity: $E(k) = \sqrt{k^2 + m^2}$

- Spontaneous Symmetry Breaking $\longrightarrow$ Massless Particle

Goldstone Boson

- real world $m_u, m_d, m_s \neq 0$ pseudo-Goldstone Bosons

$$m^2_{PGB} \propto m_q \langle 0|\bar{q}q|0\rangle$$

- Light hadrons of QCD: pions, kaons, and eta's

$m_\pi \sim 140\,\mathrm{MeV}$ $m_K \sim 496\,\mathrm{MeV}$ $m_\eta = 547\,\mathrm{MeV}$

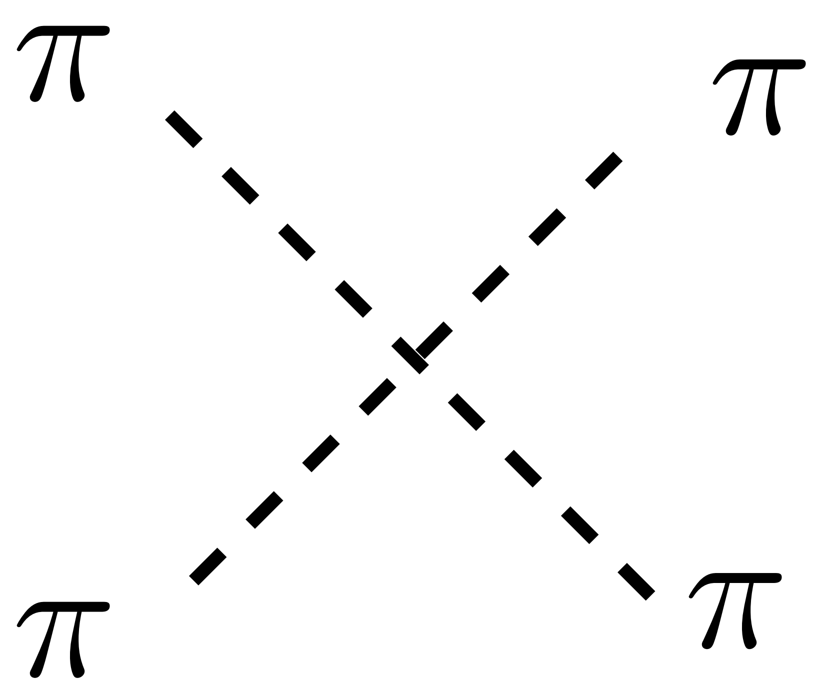
Chiral Symmetry Breaking determines the low lying degrees of freedom of QCD, constrains their self-interactions and interactions with other hadrons

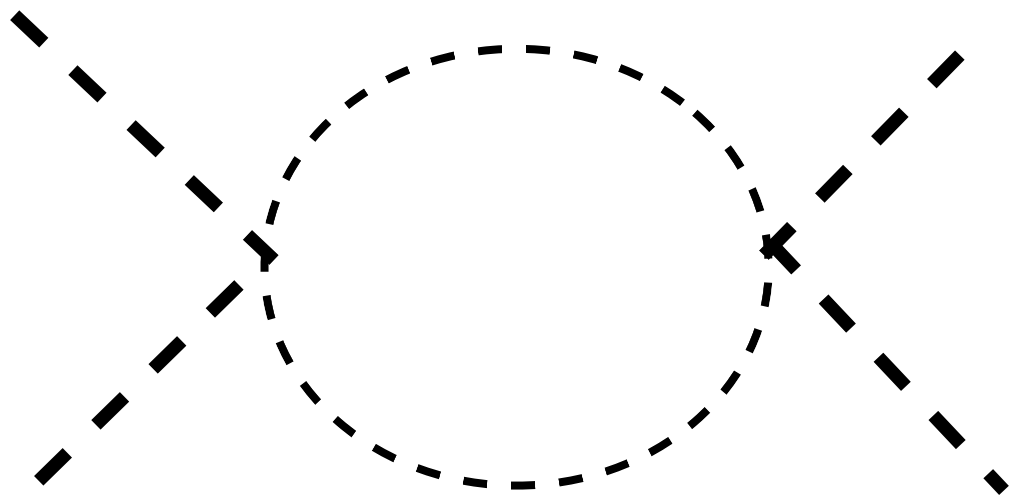
Interactions vanish as $k \to 0$ derivatively coupled

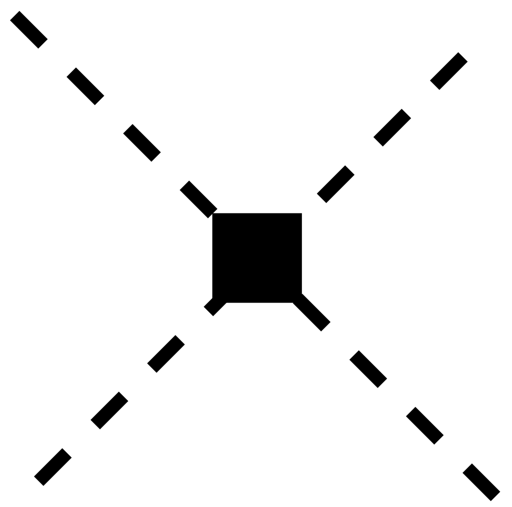
weakly interacting at low energies:

Chiral Perturbation Theory (ChiPT)

$$\mathcal{L} = \frac{f_\pi^2}{8} \mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \frac{f_\pi^2 B_0}{4} \mathrm{Tr}(m_q \Sigma + m_q \Sigma^\dagger) + \ldots \qquad \Sigma = e^{2i\pi/f_\pi}$$



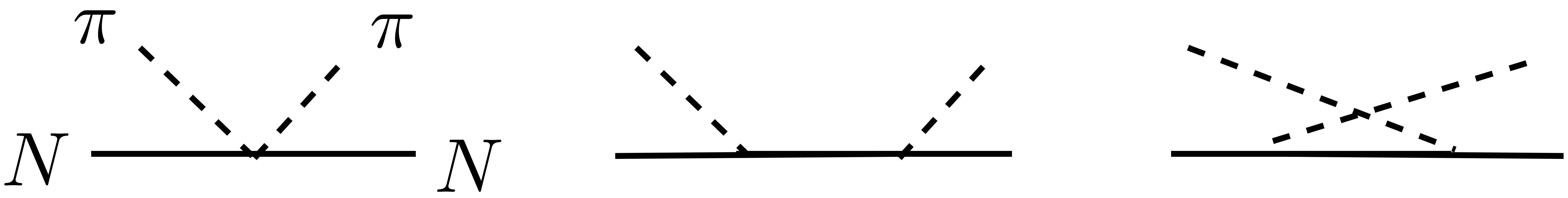




$$\frac{Q^2}{\Lambda_\chi^2} \qquad \frac{Q^4}{\Lambda_\chi^4} \qquad Q \sim (p, m_\pi)$$

perturbative expansion in $\dfrac{Q^2}{\Lambda_\chi^2}$ $\quad \Lambda_\chi = 4\pi f_\pi \sim 1\,\mathrm{GeV}$

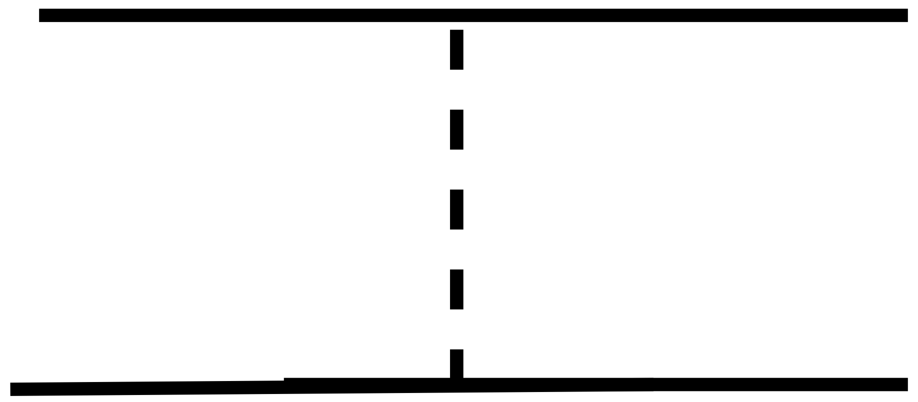
Coupling to nucleons



S-wave scattering length

$$a^I_{\pi N} = C_I \, \frac{m_\pi}{f_\pi^2}$$ (within 15%-25%)

Long range nuclear spin-tensor force



$$\frac{g_A^2}{2f_\pi^2} \frac{\vec{q}\cdot\vec{\sigma}\,\vec{q}\cdot\vec{\sigma}}{\vec{q}^2 + m_\pi^2}$$

Bottom-Up EFTs

isolate relevant degrees of freedom

small expansion parameter (ratio of scales):

Lagrangian is most general consistent w/
symmetries of underlying theory, i.e. QCD

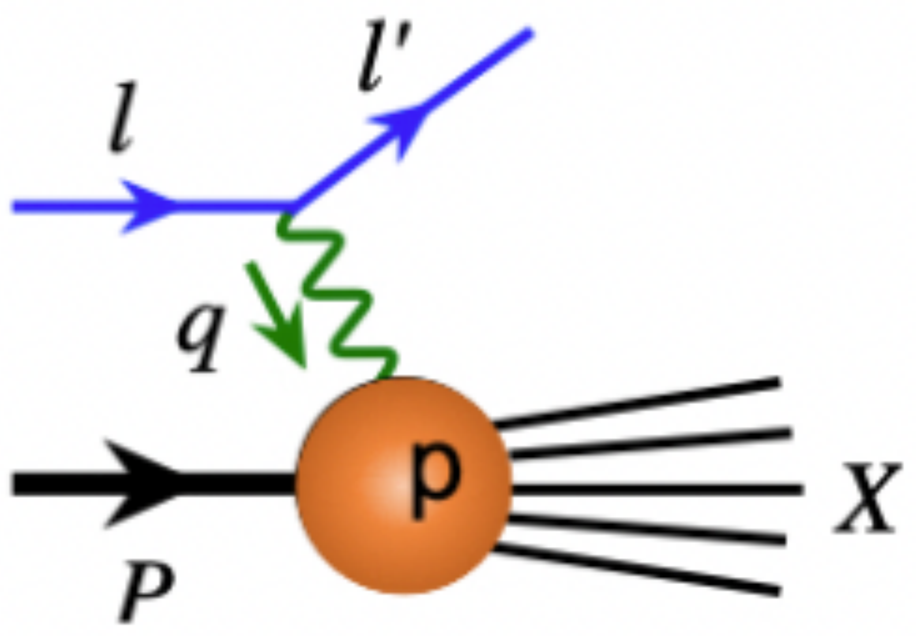
Power Counting - count powers of expansion
parameter in Lagrangian and in Feynman diagrams

coefficients in Lagrangian determined by experimental or lattice simulations

Examples: ChPT, effective theory of NN interactions, πN interactions,
Hadron molecules, e.g., X(3872), T_{cc}^{+}, ...

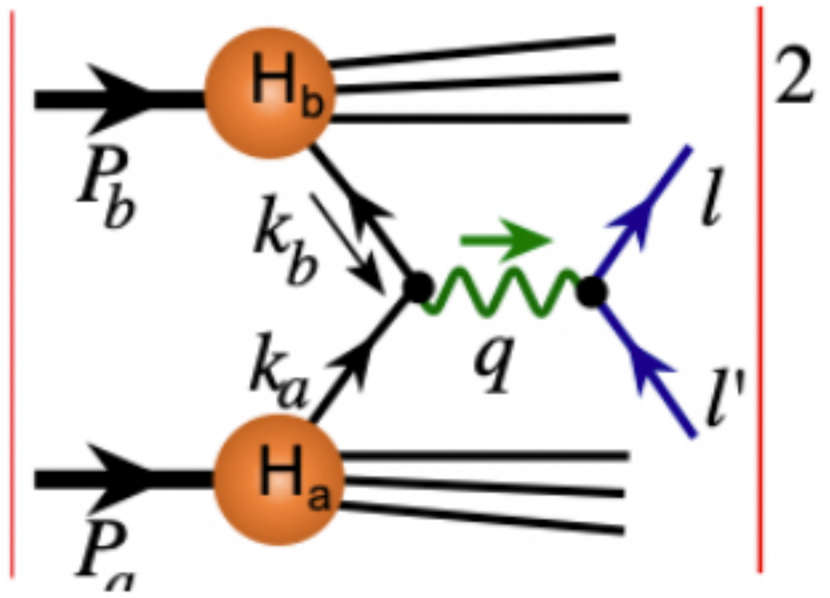
QCD Factorization

Deep Inelastic Scattering



$$d\sigma_{ep\to e'X} \sim \sum_i f_{i/p} \otimes d\hat{\sigma}(\gamma^* i \to X)$$

Drell-Yan



$$d\sigma_{pp\to \ell^+\ell^- X} \sim \sum_{ij} f_{i/p} \otimes f_{j/p} \otimes d\hat{\sigma}(ij \to \gamma^* X)$$

$Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

$d\hat{\sigma}$ calculable in pQCD - $\alpha_s(Q)$

$f_{i/p}$ universal parton distribution functions

Deriving Factorization

DIS $$d\sigma \propto \int d^4x e^{iq\cdot x} \langle p | T J^\mu(x) J^\nu(0) | p \rangle$$ apply Operator Product Expansion (OPE)

DY All-orders diagrammatic proofs in pQCD

these can be challenging!

G. T. Bodwin, Phys. Rev. D 31, 2616 (1985) [Erratum-ibid. D 34, 3932 (1985)]

J. C. Collins, D. E. Soper and G. Sterman, Phys. Lett. B 134, 263 (1984)

G. T. Bodwin, Phys. Rev. D 31, 2616 (1985) [Erratum-ibid. D 34, 3932 (1985)]

J. C. Collins, D. E. Soper and G. Sterman, Nucl. Phys. B 308, 833 (1988)

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) \to \sum_n C_{12}{}^n(x)\mathcal{O}_n(0), \qquad \text{as} \quad x \to 0$$

Wilson coefficients obey RGE

$$\left[M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial g} + \gamma_1 + \gamma_2 - \gamma_n\right] C_{12}{}^n(x; M) = 0.$$

$$C_{12}{}^n(x) = \left(\frac{1}{|x|}\right)^{d_1+d_2-d_n} c(\bar{g}(1/x)) \exp\left[\int_{1/x}^{M} d\log M'(\gamma_n - \gamma_1 - \gamma_2)\right],$$

Naive dimensional analysis

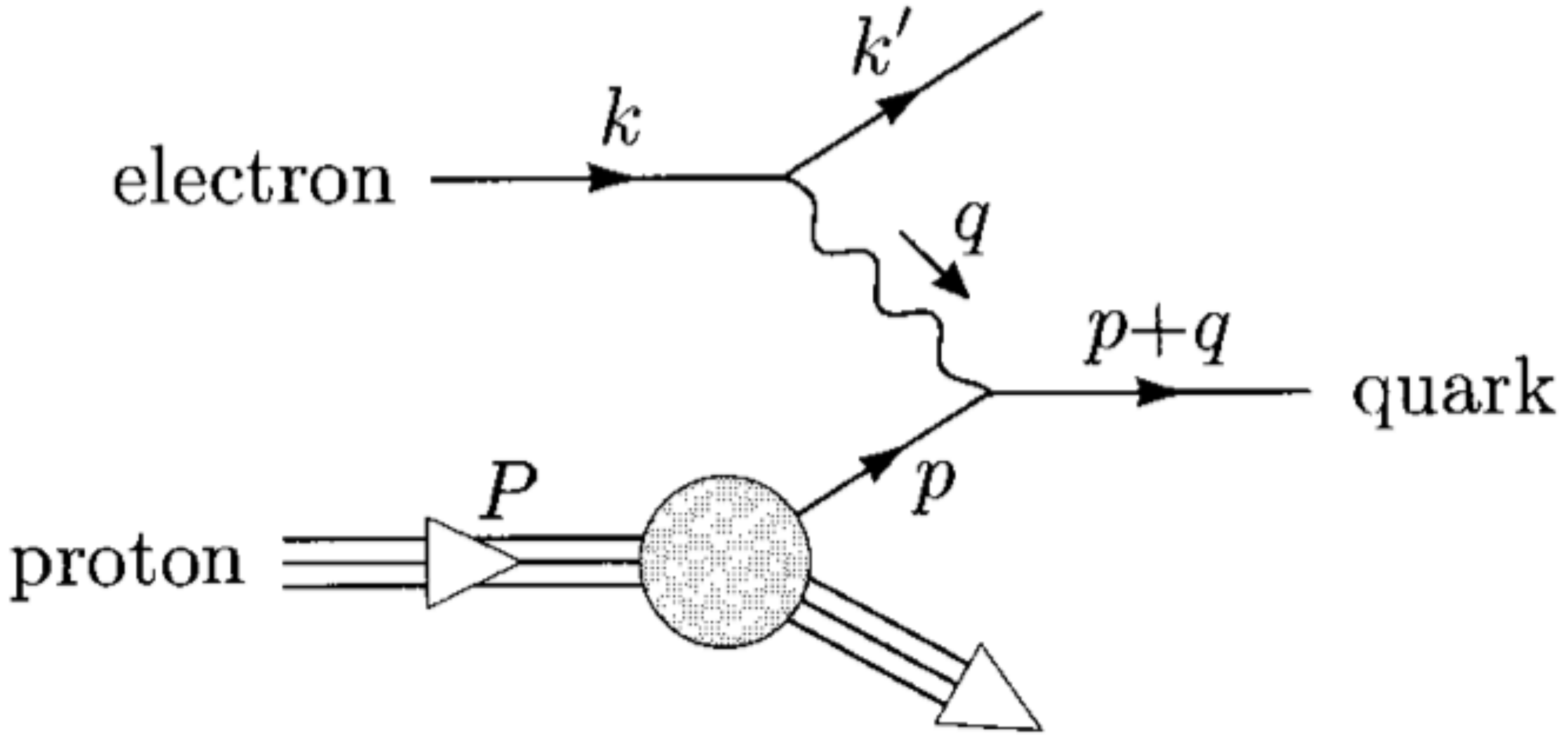
Anomalous dimension

General form of DIS cross section

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\left[(1-y)F_2(x,Q^2) + xy^2F_1(x,Q^2)\right]$$

$$x = \frac{Q^2}{2P\cdot q} \qquad\qquad Q^2 = -q^2 = xys$$

Parton Model



$$\frac{d^2\sigma}{dxdy}(e^-p\to e^-X) = \left(\sum_f x f_f(x) Q_f^2\right)\frac{2\pi\alpha^2 s}{Q^4}\left[1+(1-y)^2\right].$$

Bjorken scaling: $F_{1,2}(x,Q^2)$ independent of Q^2

Callan-Gross relation: $F_2(x) = 2xF_1(x)$ spin-1/2 quarks

OPE Analysis of DIS

Reproduce parton model results

Moments of pdf's are related to local twist-2 operators

Anomalous dimensions twist-2 operators imply scaling violation

DGLAP evolution equations for structure functions

QCD Evolution - DGLAP Equations

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

$$q \equiv f_{q/p} \qquad g \equiv f_{g/p}$$

$$P_{qq} = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

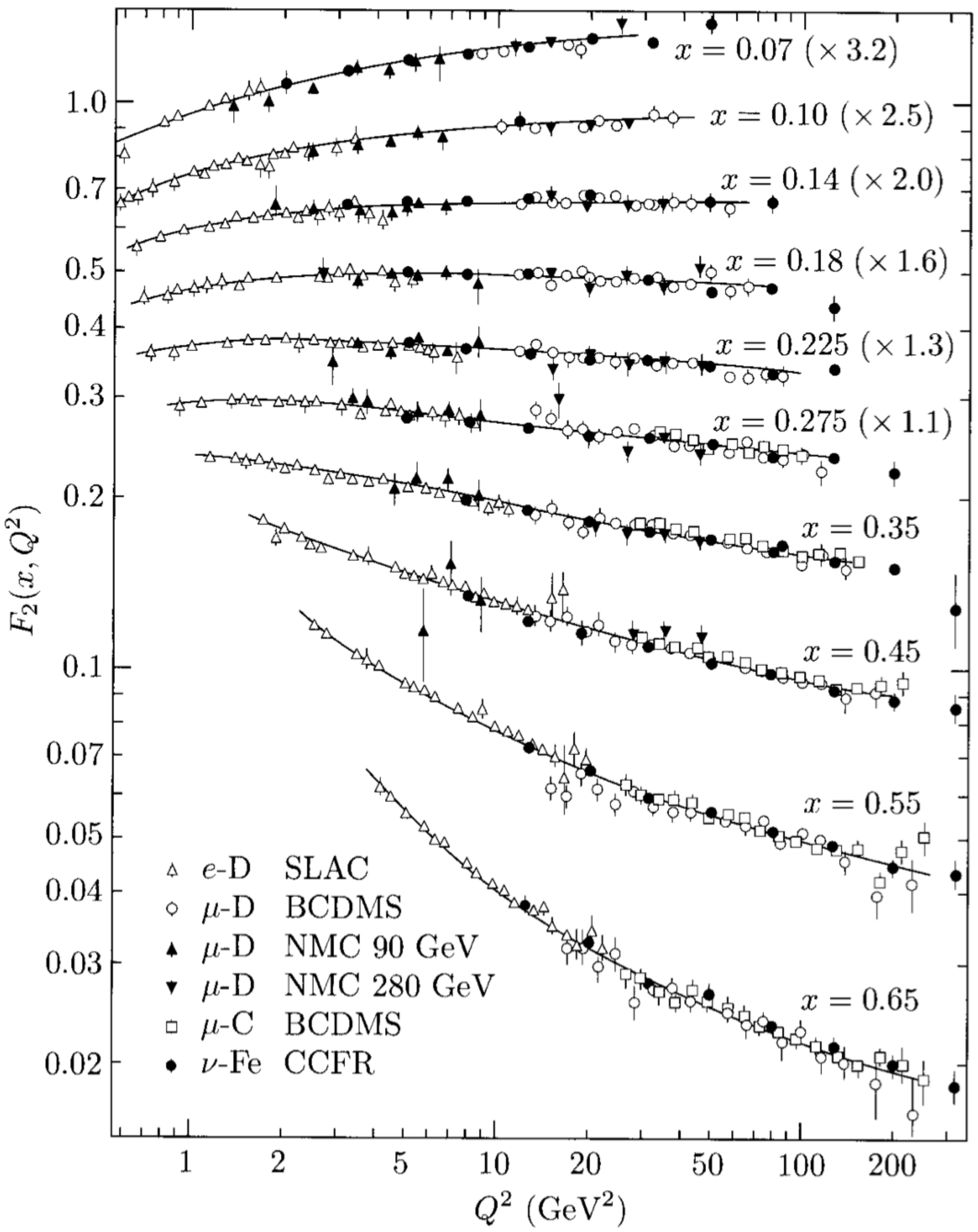
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}$$

Cross section is independent of μ

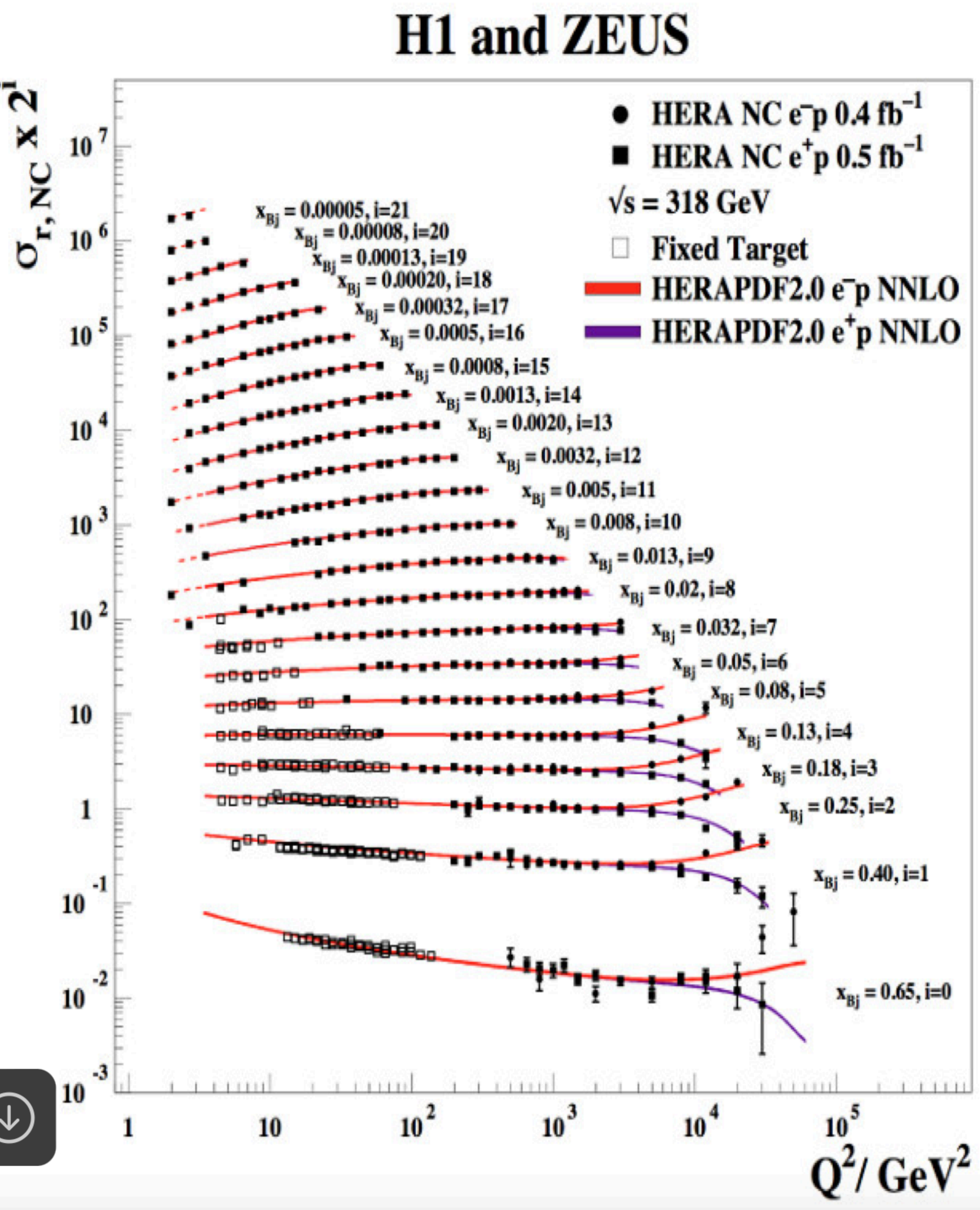
$$
\begin{aligned}
d\sigma(ep \to X) &= \sum_i f_{i/p}(\mu) \otimes d\hat{\sigma}(i\gamma^* \to X)(\mu, \alpha_s(\mu)) \\
&= \sum_i f_{i/p}(Q) \otimes d\hat{\sigma}(i\gamma^* \to X)(Q, \alpha_s(Q))
\end{aligned}
$$

Minimize logs in short-distance cross section $\ln\left(\frac{Q^2}{\mu^2}\right)$

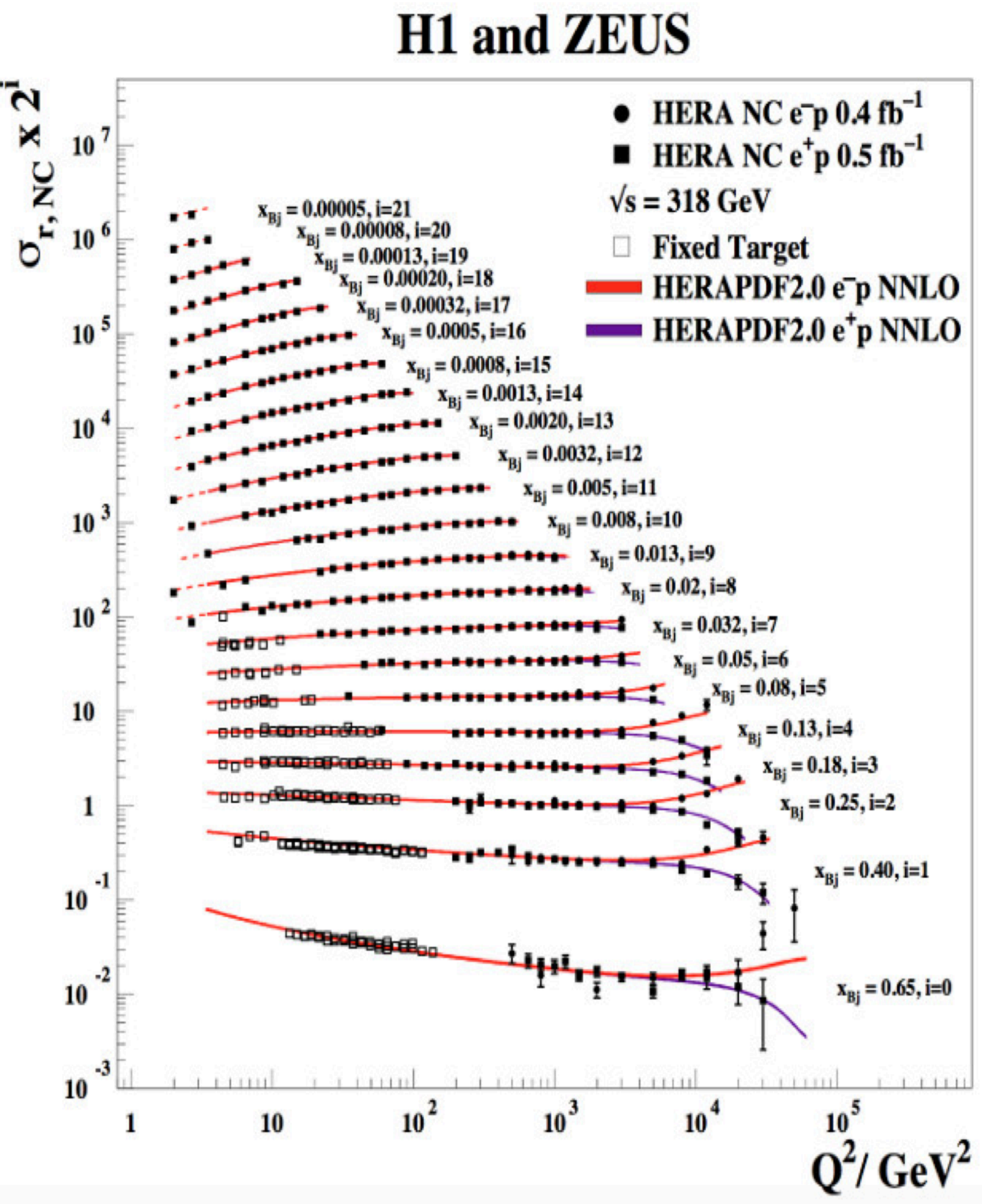


DGLAP tested over a large range of Q^2

Peskin, Schroeder, Intro to QFT, p.592, data from 1994



H1 and ZEUS
HERA NC e⁻p 0.4 fb⁻¹
HERA NC e⁺p 0.5 fb⁻¹
√s = 318 GeV
Fixed Target
HERAPDF2.0 e⁻p NNLO
HERAPDF2.0 e⁺p NNLO
σr, NC x 2^i
x_Bj = 0.00005, i=21
x_Bj = 0.00008, i=20
x_Bj = 0.00013, i=19
x_Bj = 0.00020, i=18
x_Bj = 0.00032, i=17
x_Bj = 0.0005, i=16
x_Bj = 0.0008, i=15
x_Bj = 0.0013, i=14
x_Bj = 0.0020, i=13
x_Bj = 0.0032, i=12
x_Bj = 0.005, i=11
x_Bj = 0.008, i=10
x_Bj = 0.013, i=9
x_Bj = 0.02, i=8
x_Bj = 0.032, i=7
x_Bj = 0.05, i=6
x_Bj = 0.08, i=5
x_Bj = 0.13, i=4
x_Bj = 0.18, i=3
x_Bj = 0.25, i=2
x_Bj = 0.40, i=1
x_Bj = 0.65, i=0
Q²/ GeV²



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x_Bj = 0.65, i=0
Q²/ GeV²

Summary of Important QCD Concepts

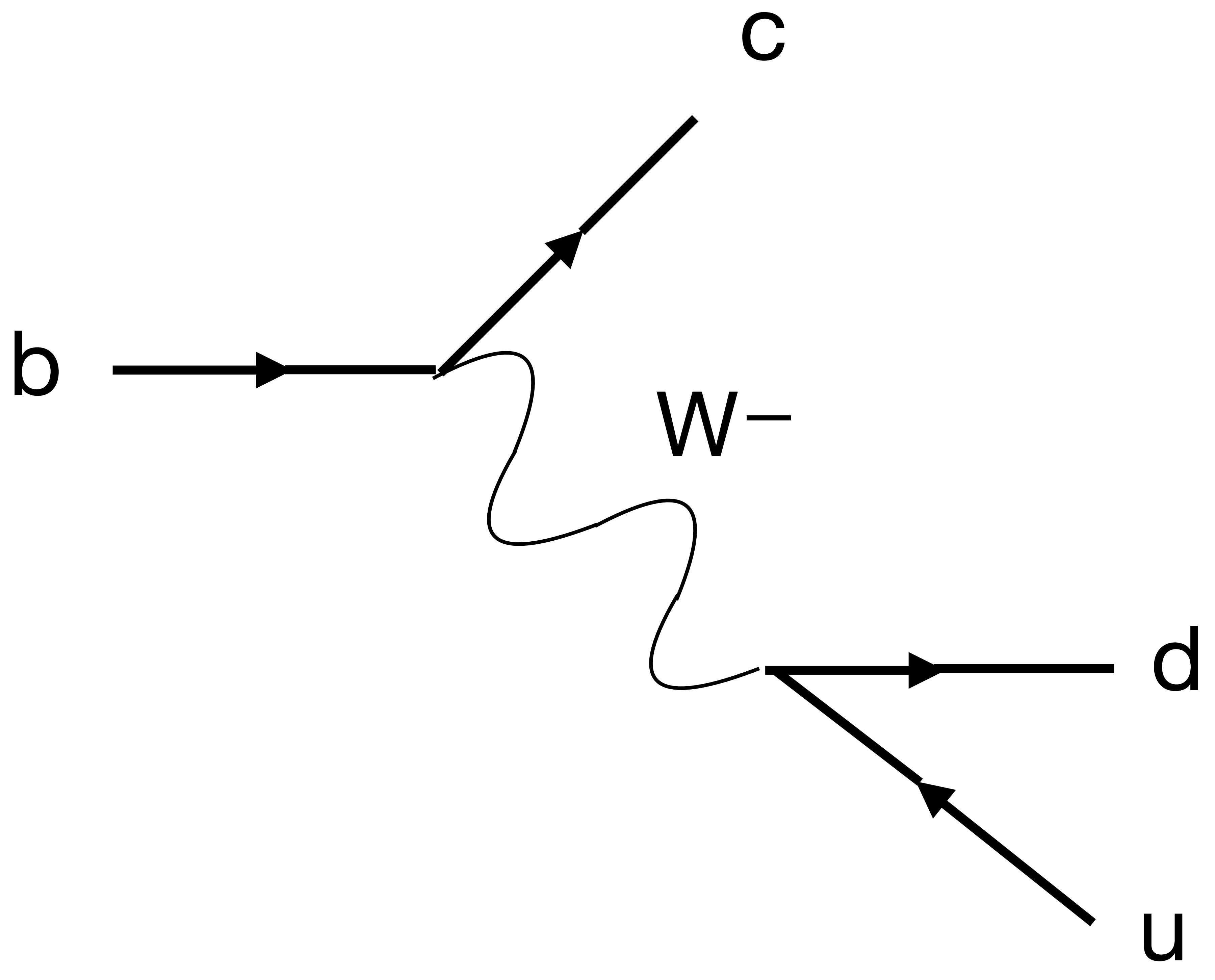
Asymptotic Freedom

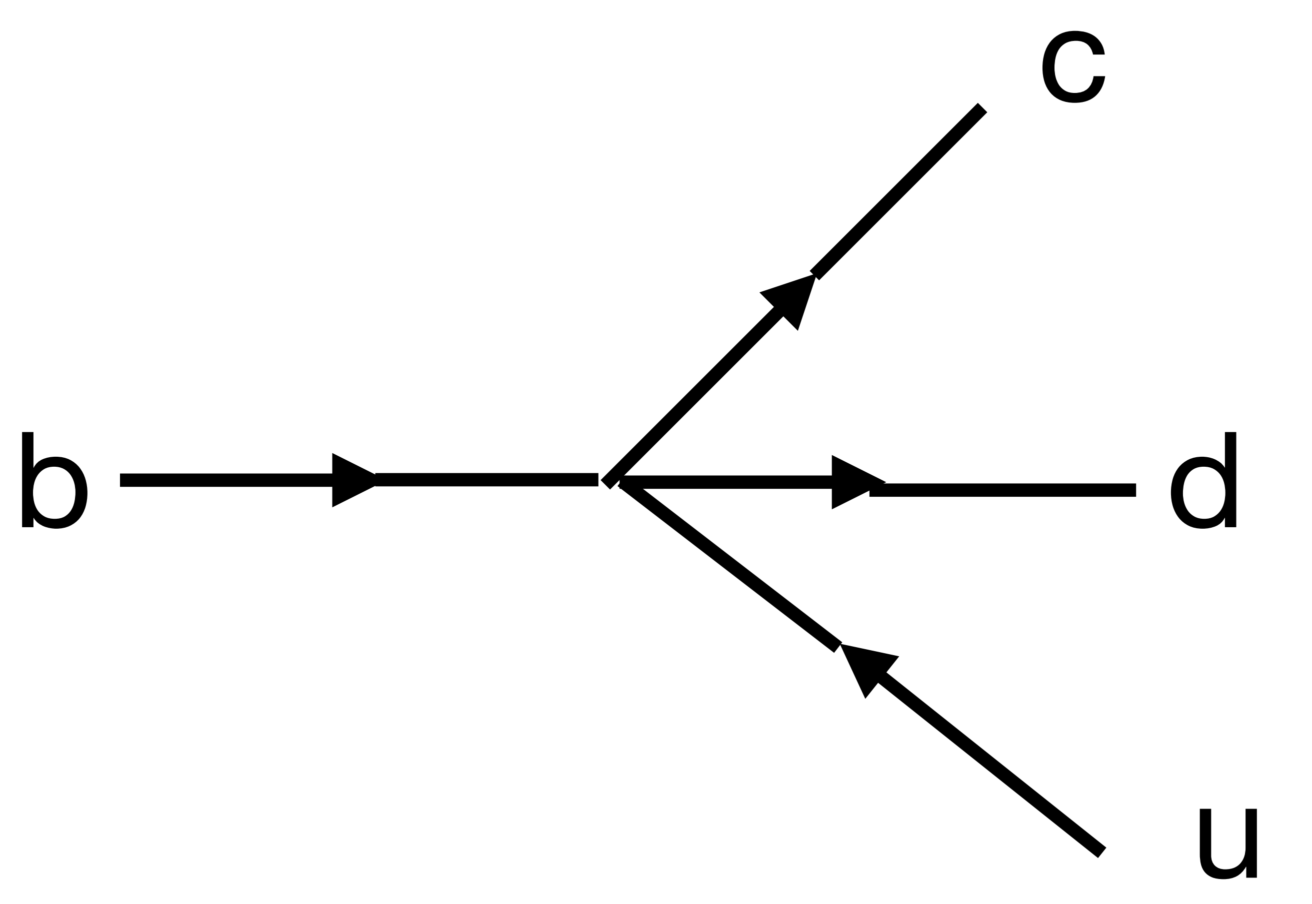
Confinement, Chiral Symmetry Breaking

High Energy QCD: Factorization, Evolution

Low Energy: Lattice QCD, Effective Theories

Effective Theory of Weak Interactions - b decay





$$\frac{1}{p^2 - m_W^2} = -\frac{1}{m_W^2} - \frac{p^2}{m_W^4} + \ldots$$

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ C_1 \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_1(\mu) + C_2 \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_2(\mu) \right\},$$

$$O_1(\mu) = [\bar{c}^\alpha \gamma_\mu P_L b_\alpha][\bar{d}^\beta \gamma^\mu P_L u_\beta],$$

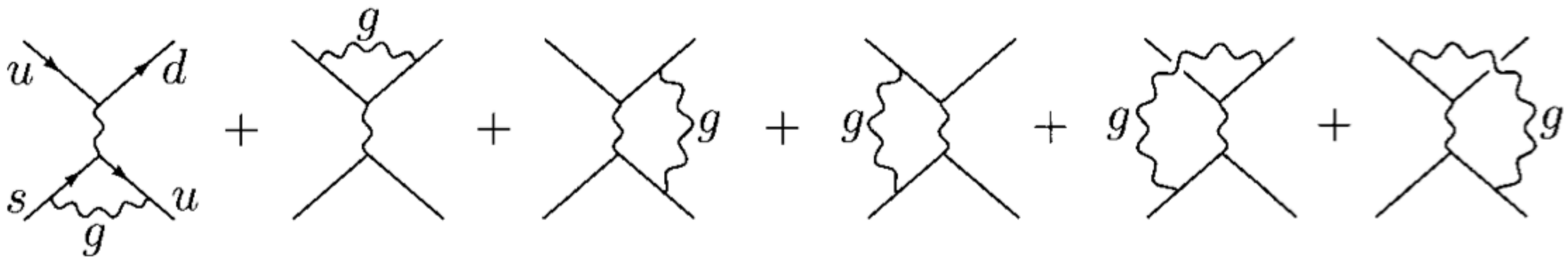
$$O_2(\mu) = [\bar{c}^\beta \gamma_\mu P_L b_\alpha][\bar{d}^\alpha \gamma^\mu P_L u_\beta].$$

$$C_1[1, \alpha_s(M_W)] = 1 + \mathcal{O}[\alpha_s(M_W)],$$

$$C_2[1, \alpha_s(M_W)] = 0 + \mathcal{O}[\alpha_s(M_W)].$$

Tree level - reproduces tree level Standard Model

One loop QCD corrections to effective operators



Evolution equations for Wilson Coefficients

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \gamma_{ji} C_j . \qquad \gamma(g) = \frac{g^2}{8\pi^2} \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} .$$

$$C_i \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] = P \exp \left[\int_{g(M_W)}^{g(\mu)} \frac{\gamma^T(g)}{\beta(g)} dg \right]_{ij} C_j [1, \alpha_s(M_W)].$$

Renormalization group improved Hamiltonian

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ C_+ \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_+(\mu) + C_- \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_-(\mu) \right\}$$

$$O_\pm = O_1 \pm O_2.$$

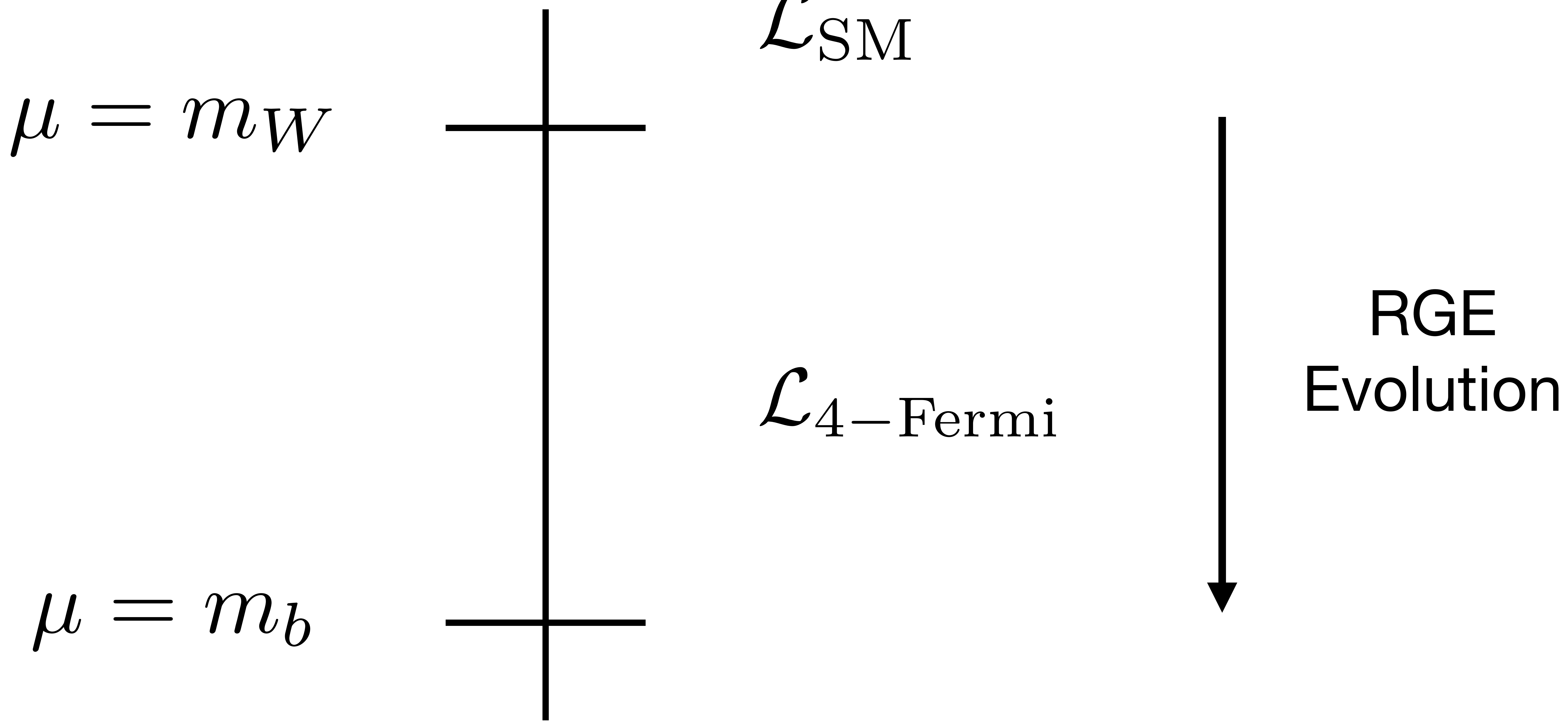
$$C_\pm \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] = \frac{1}{2} \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{a_\pm}$$

$$a_+ = \frac{6}{33 - 2N_q}, \qquad a_- = -\frac{12}{33 - 2N_q}.$$

Resum $\alpha_s^n(m_b) \ln^n \left(\frac{m_W^2}{m_b^2} \right)$ to all orders

$$\alpha_s(m_b) \sim 0.2 \qquad \log \left(\frac{m_W^2}{m_b^2} \right) \sim 6$$

Matching and Running



Effective Theory Principles for Top-Down construction of EFTs

Degrees of freedom - SM fermions

Power Counting - Dimensional analysis

Symmetries constrain Lagrangian - Lorentz Inv., color

Match effective Lagrangian at the high scale

RGE evolution to low scales

Effective Theories of this type for QCD:

Heavy Quark Effective Theory (HQET)

Non-Relativistic QCD (NRQCD)

Soft-Collinear Effective Theory (SCET)

Useful for: - realizing emergent symmetries of QCD

e.g. heavy quark spin-flavor symmetry $m_Q \to \infty$

- deriving new factorization theorems

- resummation of large logarithms

HQET

$$p^\mu = m_Q v^\mu + k^\mu$$

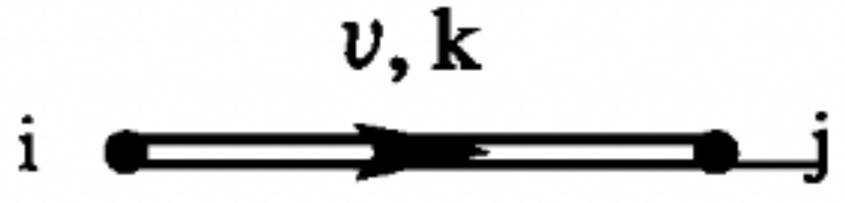
$v^\mu \sim (1, \vec{0})$ in heavy quark rest frame $\quad k^\mu \sim \Lambda_{\rm QCD}$

$$\frac{i}{\not p - m_Q} \approx \frac{1+\not v}{2}\frac{i}{v\cdot k}$$

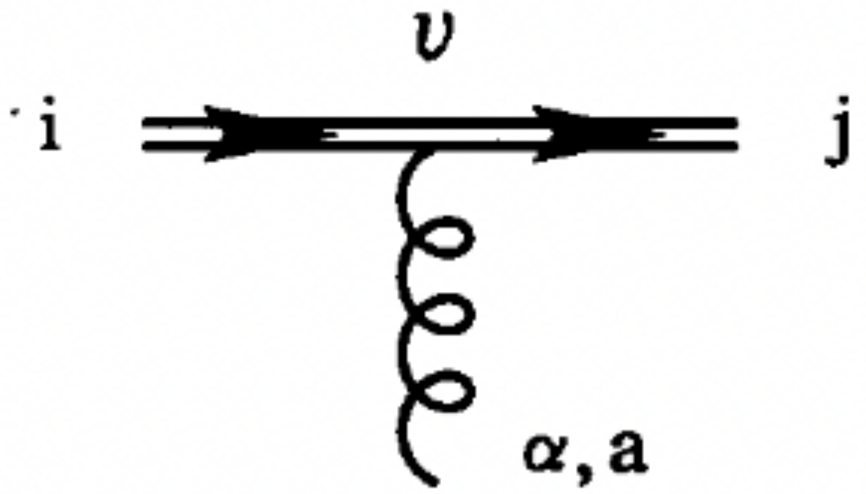
$$\bar u(p) i g T^a A^a_\mu \gamma^\mu u(p+k) \approx h_v^\dagger i g T^a A^a_\mu \gamma^\mu v^\mu h_v \qquad h_v = \frac{1+\not v}{2} h_v$$

HQET Lagrangian

$$\mathcal{L}_{\rm eff} = \bar h_v \mathrm{i} v\cdot D h_v + \frac{1}{2m_Q}\bar h_v(\mathrm{i}D_\perp)^2 h_v + \frac{g}{4m_Q}\bar h_v \sigma_{\alpha\beta}G^{\alpha\beta}h_v + O(1/m_Q^2).$$

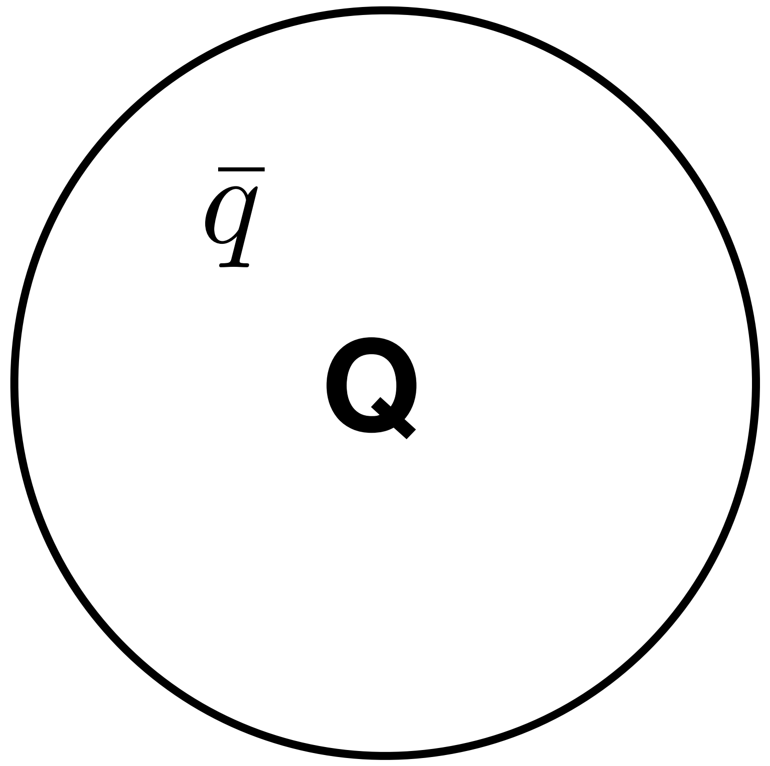


$= \dfrac{\mathrm{i}}{v\cdot \mathrm{k}}\,\dfrac{1+\not v}{2}\,\delta_{\rm ji}$



$= \mathrm{ig}\,(\mathrm{T_a})_{\rm ji}\, v^\alpha$

Heavy quark is static source of color l.d.o.f. same in $m_Q \to \infty$



$$m_M = m_Q + \bar{\Lambda} + \frac{\lambda}{2m_Q} + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_Q^2}\right)$$

$$m^2_{D^*} - m^2_D = m^2_{B^*} - m^2_B$$

Heavy meson decay form factors

$$\langle D(v')|V^\mu|B(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu,$$

$$\langle D^*(v',\epsilon)|V^\mu|B(v)\rangle = \mathrm{i}h_{\mathrm{V}}(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta,$$

$$\langle D^*(v',\epsilon)|A^\mu|B(v)\rangle = h_{\mathrm{A}_1}(w)(w+1)\epsilon^{*\mu} - [h_{\mathrm{A}_2}(w)v^\mu + h_{\mathrm{A}_3}(w)v'^\mu]\epsilon^*\cdot v.$$

Isgur-Wise function $\xi(w) = \xi(v'\cdot v),\ \xi(1) = 1$

$$h_+(w) = h_{\mathrm{V}}(w) = h_{\mathrm{A}_1}(w) = h_{\mathrm{A}_3}(w) = \xi(w)\,,$$

$$h_-(w) = h_{\mathrm{A}_2}(w) = 0\,.$$

D^* and D are degenerate in heavy quark limit

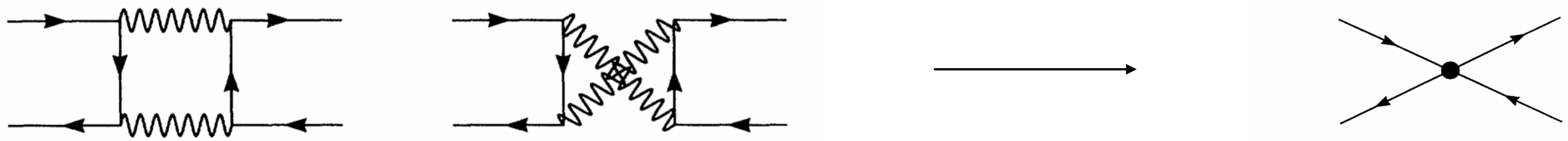
Heavy hadron chiral perturbation theory

$$H_v = \frac{1+\not{v}}{2}(\not{D}^* + D) \qquad v^\mu \sim (1,\vec{0}) \longrightarrow H_a = \vec{P}_a^* \cdot \vec{\sigma} + P_a$$

$$\mathcal{L} = \mathrm{Tr}[H_a^\dagger (iD_0)_{ba} H_b] - g\mathrm{Tr}[H_a^\dagger H_b \, \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta_H}{4}\mathrm{Tr}[H_a^\dagger \, \sigma^i \, H_a \, \sigma^i]$$

NRQCD

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi$$



$$(\delta\mathcal{L}_{\text{4-fermion}})_{d=6} = \frac{f_1(^1S_0)}{M^2}\mathcal{O}_1(^1S_0) + \frac{f_1(^3S_1)}{M^2}\mathcal{O}_1(^3S_1) + \frac{f_8(^1S_0)}{M^2}\mathcal{O}_8(^1S_0) + \frac{f_8(^3S_1)}{M^2}\mathcal{O}_8(^3S_1)$$

$$\begin{aligned}
\mathcal{O}_1(^1S_0) &= \psi^\dagger\chi\,\chi^\dagger\psi, \\
\mathcal{O}_1(^3S_1) &= \psi^\dagger\boldsymbol{\sigma}\chi\cdot\chi^\dagger\boldsymbol{\sigma}\psi, \\
\mathcal{O}_8(^1S_0) &= \psi^\dagger T^a\chi\,\chi^\dagger T^a\psi, \\
\mathcal{O}_8(^3S_1) &= \psi^\dagger\boldsymbol{\sigma}T^a\chi\cdot\chi^\dagger\boldsymbol{\sigma}T^a\psi.
\end{aligned}$$

Power counting: $E \sim m_Q v^2 \quad p \sim m_Q v^2$

Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle\mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle\mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

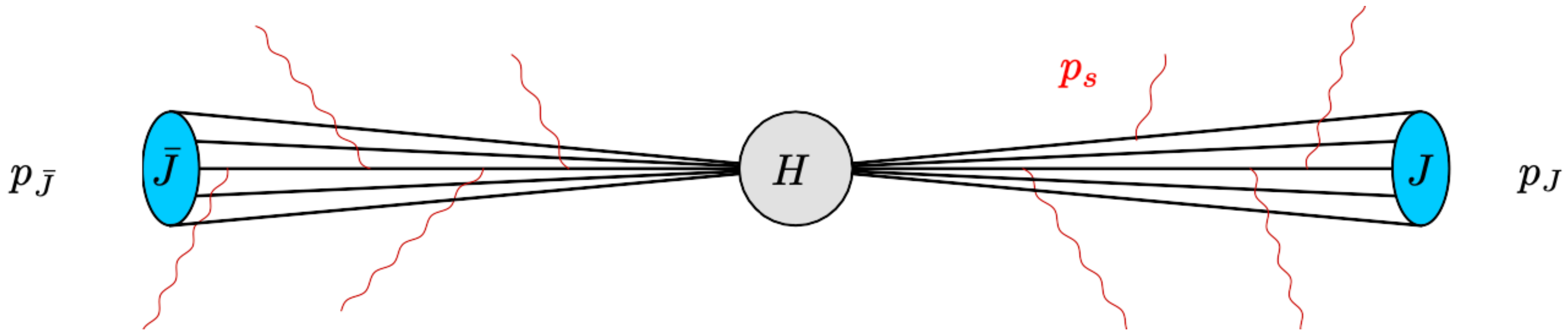
CSM - lowest order in v

$$\langle\mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle\mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle\mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

Soft-Collinear Effective Theory

Two jet events in e+ e- collisions



Final state particles: energetic partons collinear to jets, soft radiation

Many scales: $Q, E_J, m_J,$ jet substructure

Soft Collinear Effective Theory (SCET)

Multipole expand QCD fields onto fields w/ definite momentum scaling

$$\phi^{\mathrm{QCD}} = \phi_n + \phi_{\bar{n}} + \phi_{us} + \ldots$$

Match QCD currents onto operators composed of these field

Decouple fields at level of SCET Lagrangian using BPS field redefinition

Factorization: cross section written in terms of matrix elements of different fields

Resummation: SCET RGE for each term in factorization theorem,
logs minimized at one scale

Glauber Modes: forward scattering of opposite collinear modes,
factorization violation, often neglected in SCET analyses

I. Z. Rothstein, I. W. Stewart, JHEP **1608** (2016) 025

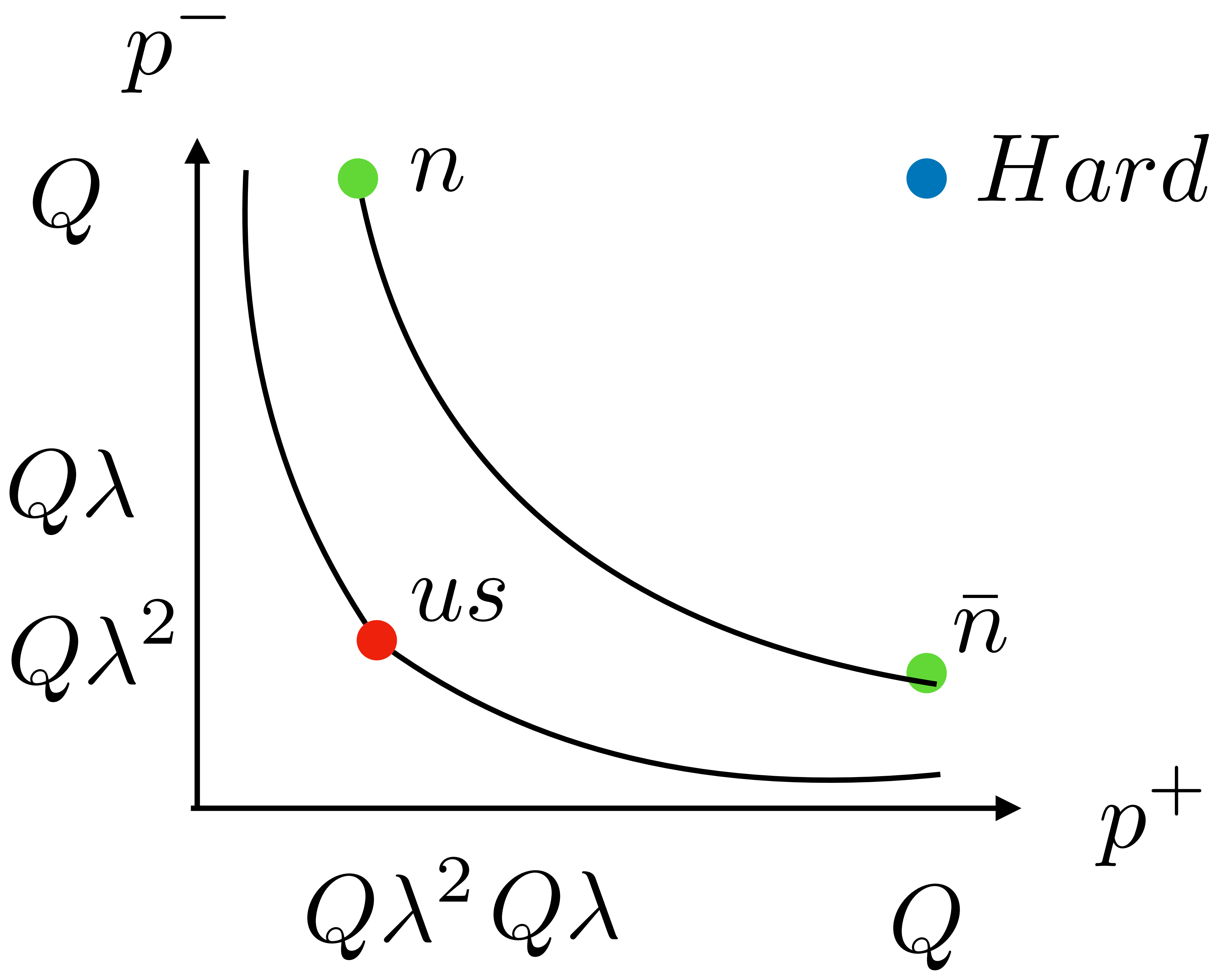
Soft Collinear Effective Theory (SCET$_\mathrm{I}$)

$$(p^+, p^-, p_\perp)$$

$$n \sim Q(1, \lambda^2, \lambda)$$

$$\bar{n} \sim Q(\lambda^2, 1, \lambda)$$

$$us \sim Q(\lambda^2, \lambda^2, \lambda^2)$$



$$d\sigma[e^+e^- \to 2\,\mathrm{jets}] = H \times J_n \otimes J_{\bar{n}} \otimes S$$

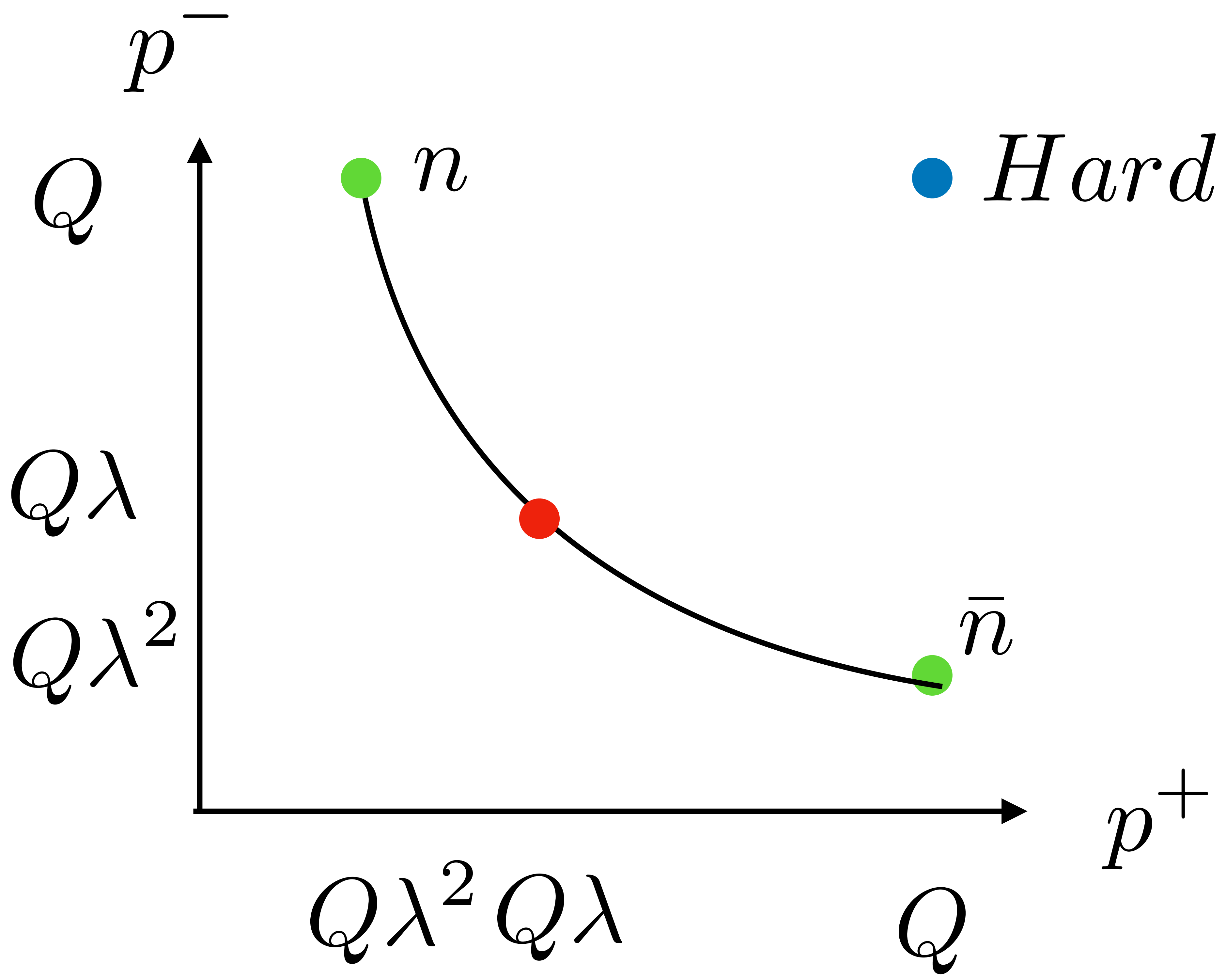
Soft Collinear Effective Theory (SCET$_{II}$)

$$(p^+, p^-, p_\perp)$$

$$n \sim Q(1, \lambda^2, \lambda)$$

$$\bar{n} \sim Q(\lambda^2, 1, \lambda)$$

$$s \sim Q(\lambda, \lambda, \lambda)$$

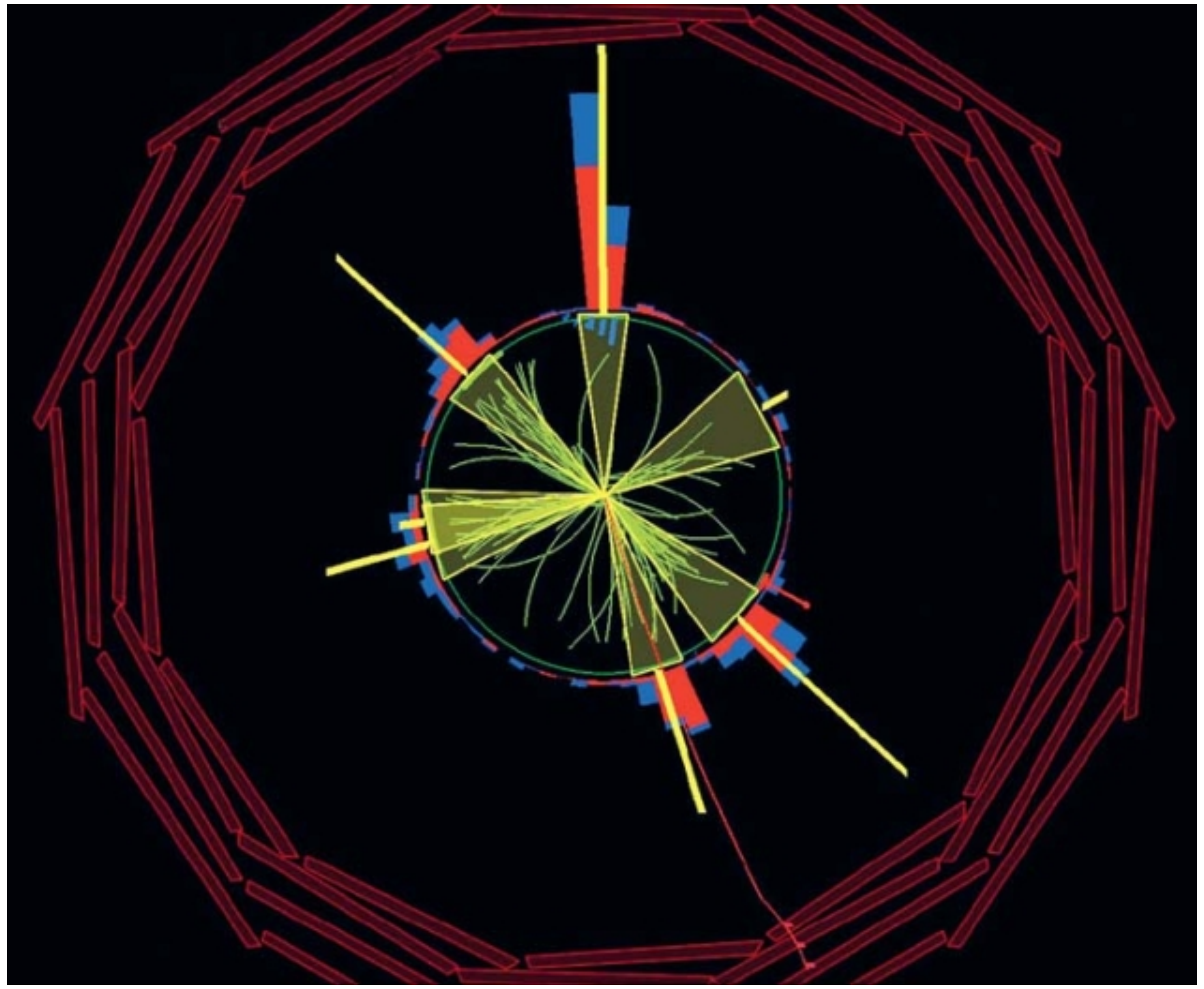


$$\frac{d\sigma[pp \to H + X]}{dp_T^2} = H \times \mathcal{B}^n_{g/p}(p_T) \otimes_T \mathcal{B}^{\bar{n}}_{g/p}(p_T) \otimes_T S(p_T)$$

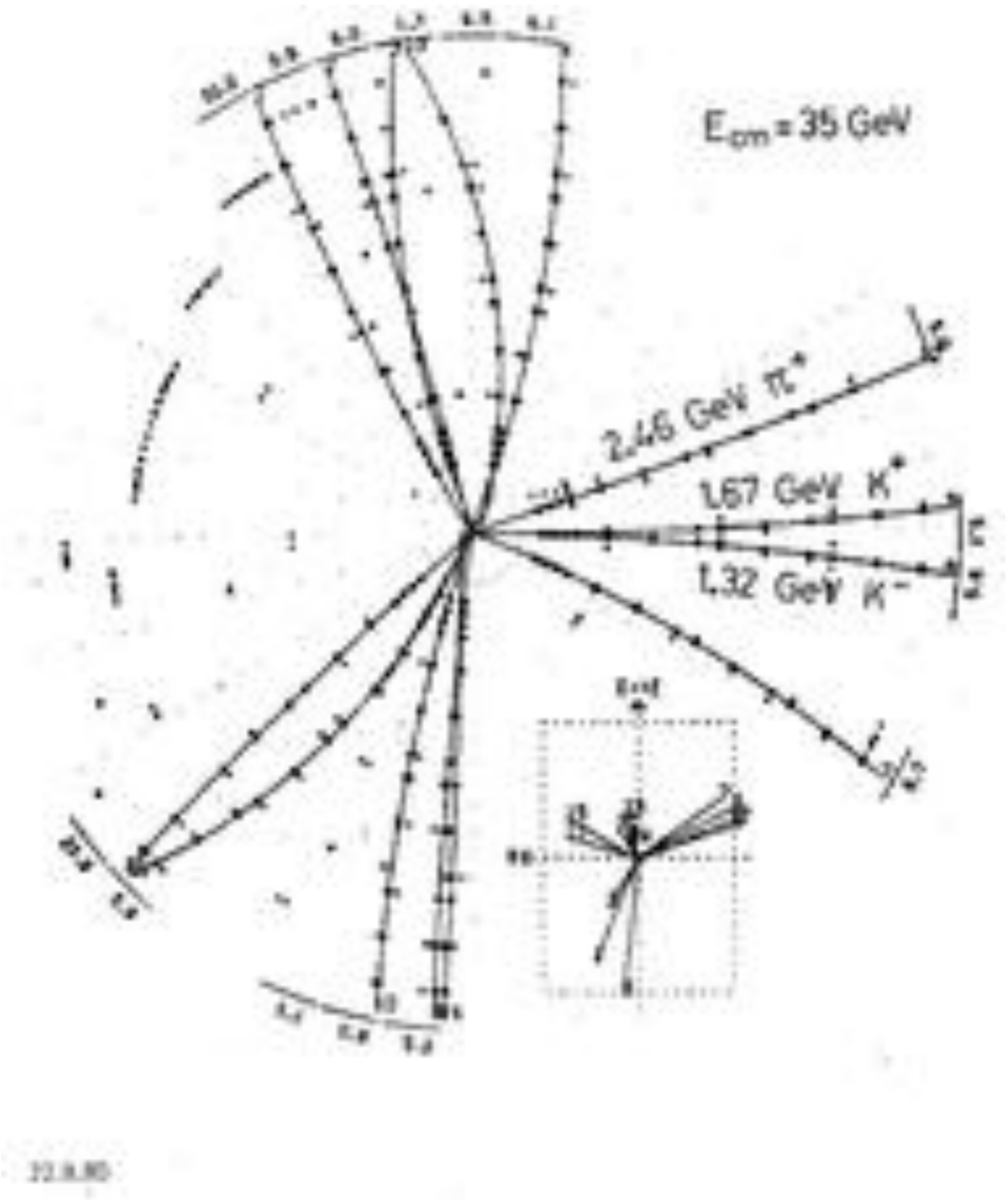
$$\mathcal{B}^n_{g/p} = \sum_i \mathcal{J}_{gi}(z, p_T) \otimes f_{i/p}(z)$$

What is a Jet?

Collimated shower of energetic final state particles



Jets at CMS



3 jet event at DESY
evidence for gluon

Jet Algorithms

sequential clustering algorithms

boost invariant distance measures (hadron collider)

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p})\frac{\Delta R_{ij}^2}{R^2},$$
$$d_{iB} = p_{Ti}^{2p},$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

p = 1: k_T algorithm

p = 0: Cambridge-Aachen

p = -1: anti-k_T

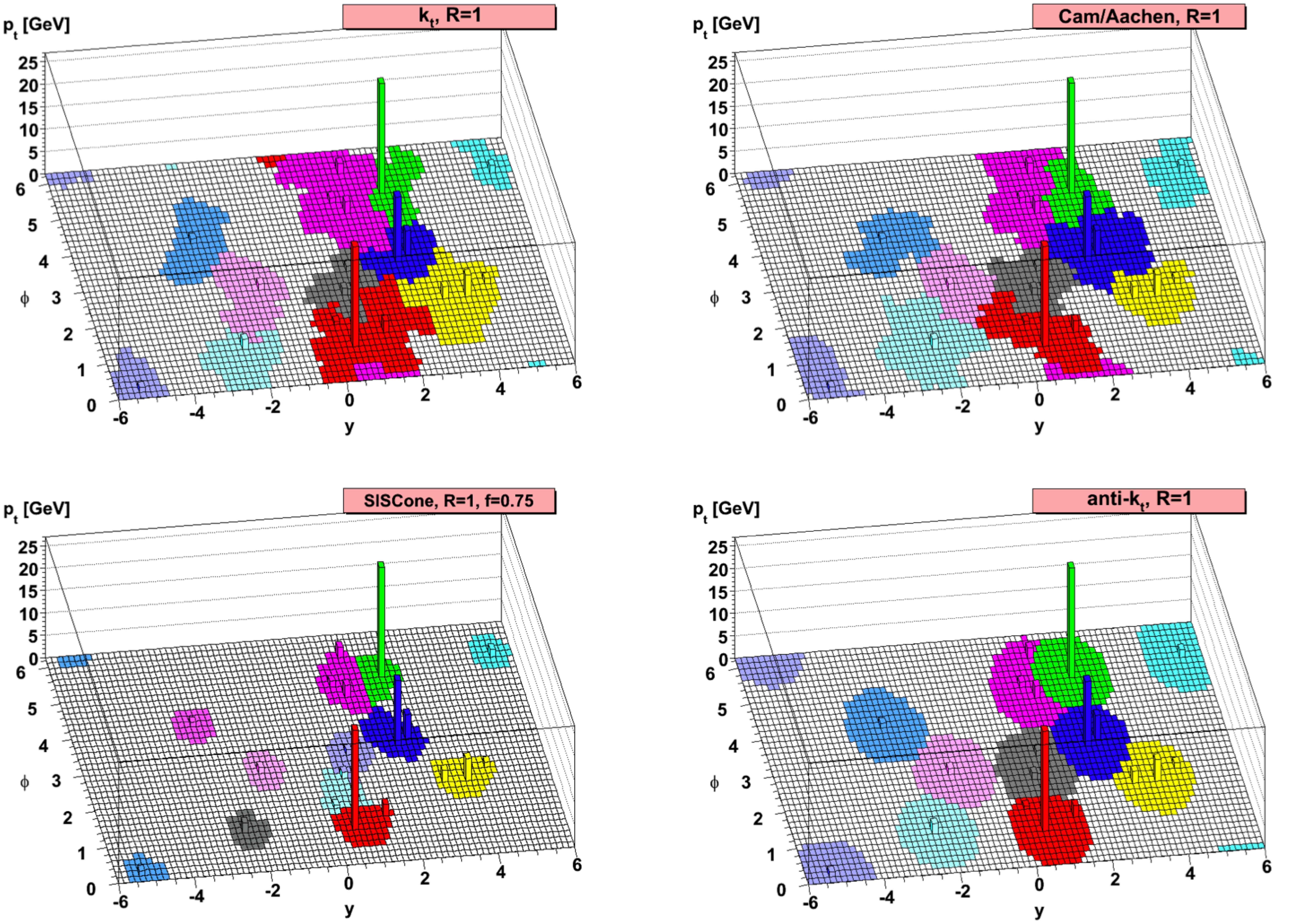
1) calculate d_{ij}, d_{iB} for all i,j

2) if d_{ij} smallest combine i and j, if d_{iB} smallest object is a jet and removed

3) repeat until every particle has been assigned to a jet

cone algorithms

more commonly used at $e^+ e^-$ colliders



M. Cacciari, G.P. Salam, G. Soyez, JHEP 0804:063,2008, arXiv:0802.1189

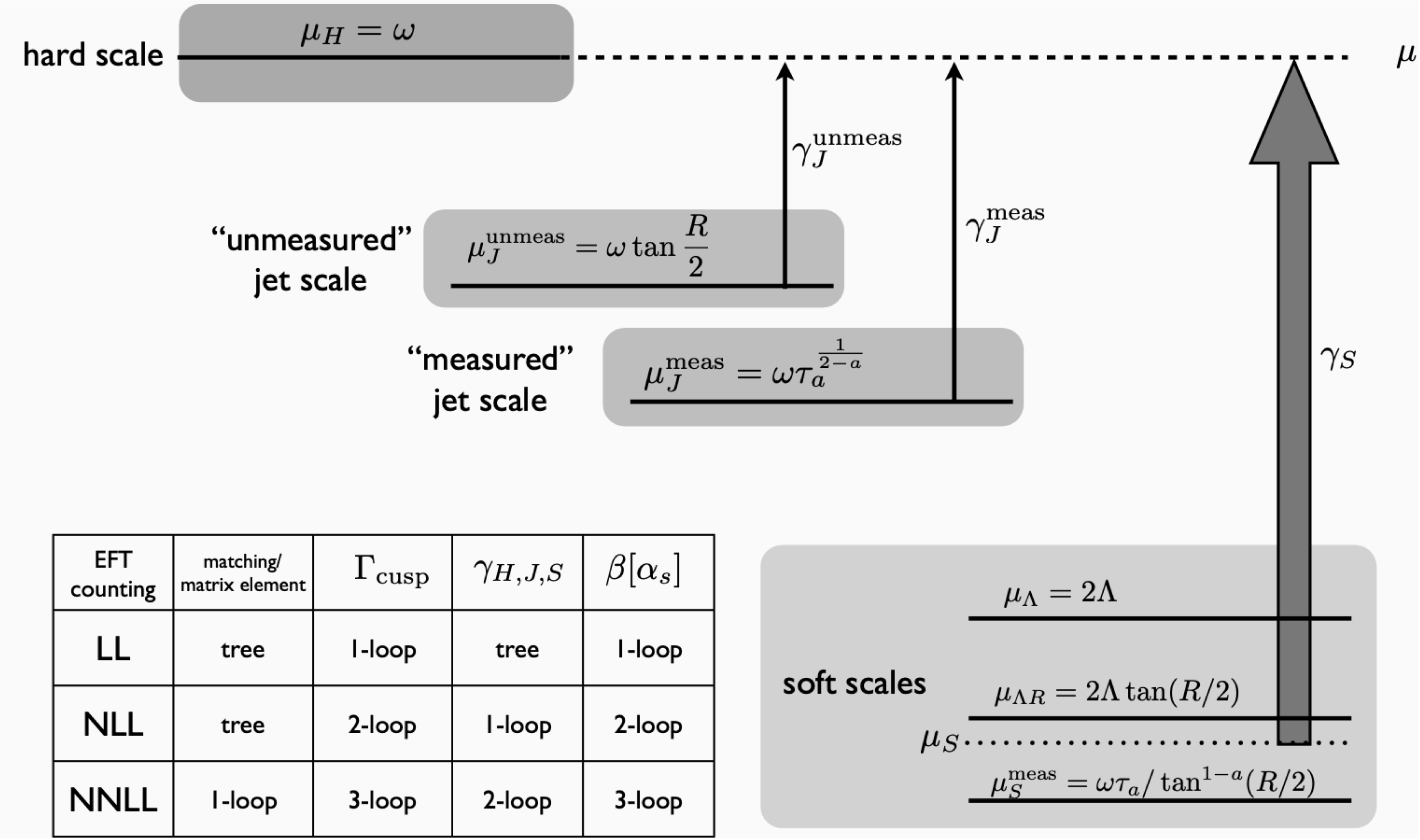
All jets characterized by: ω_J - light cone momentum R - jet radius

or P_T- transverse momentum

can also study jet substructure - functions of jet constituents

angularities $$\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$$ $$\omega = \sum_i p_i^-$$

identified hadrons within jet



EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop