Effective Field Theories of QCD

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Quantum Chromodynamics

Effective theory of Weak Force

HQET, Non-Relativistic QCD

Soft-Collinear Effective Theory

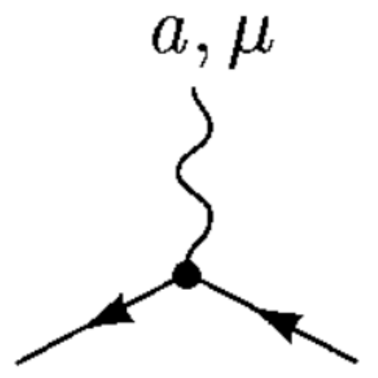
Applications: T^{+}_{cc}, X(3872) as hadronic molecules

Applications: Quarkonia Production in Jets

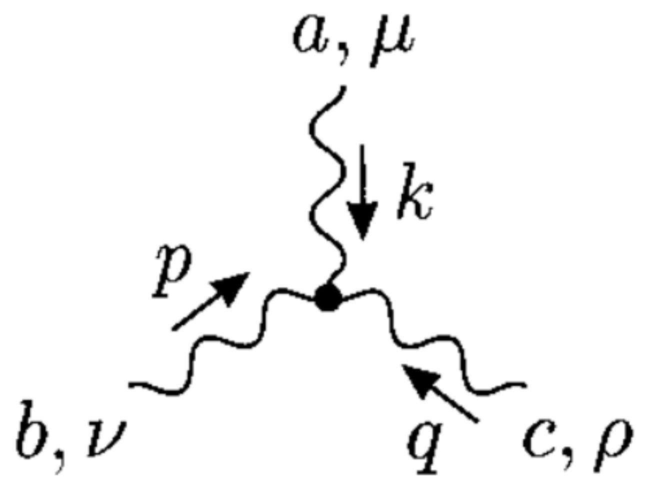
Quantum Chromodynamics

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_i \bar{\psi}_i (i\not{D} - m_i)\psi_i$$

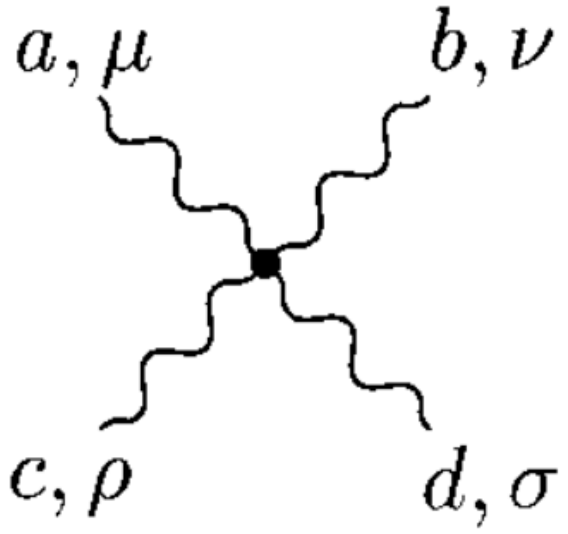
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu + igf^{abc} A^b_\mu A^c_\nu$$



$$= \quad ig\gamma^\mu t^a$$

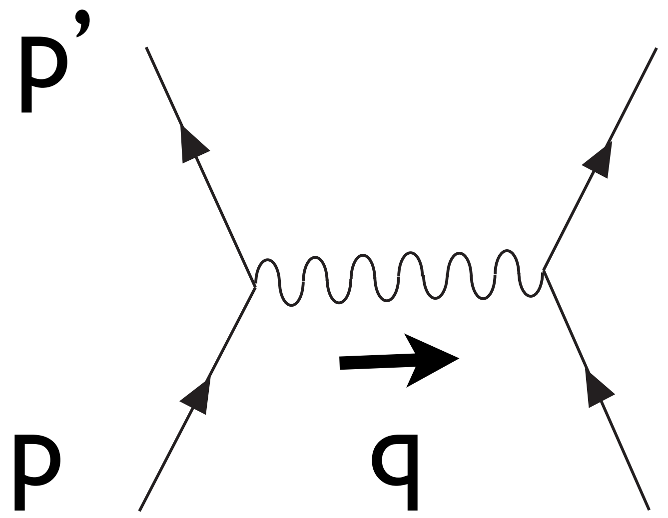


$$= \quad gf^{abc}\big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\big]$$



$$= \quad -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big]$$

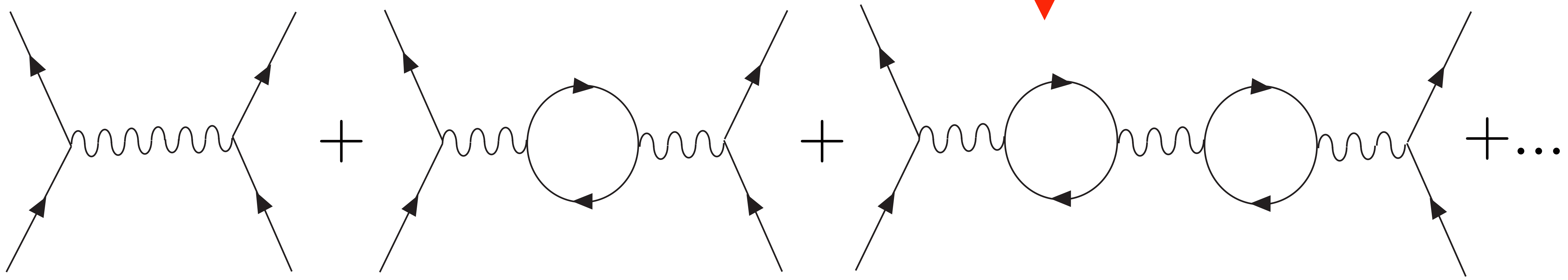
Running Coupling Constants



$\propto \frac{\alpha}{q^2}$

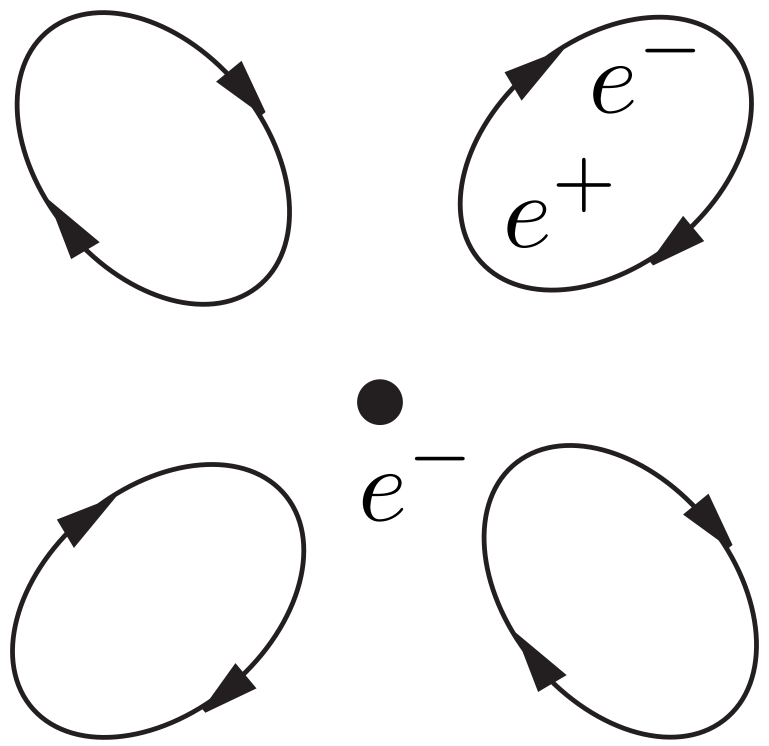
q = p - p' : momentum transfer

$\sim \alpha \log(|q^2|/m_e^2)$ $\qquad |q| \gg m_e$



$\propto \frac{\alpha_{\text{eff}}(q)}{q^2}$

Vacuum Polarization screens bare charge



$\alpha(m_e) = 1/137$

atomic physics

$\alpha(90\,\text{GeV}) = 1/128$

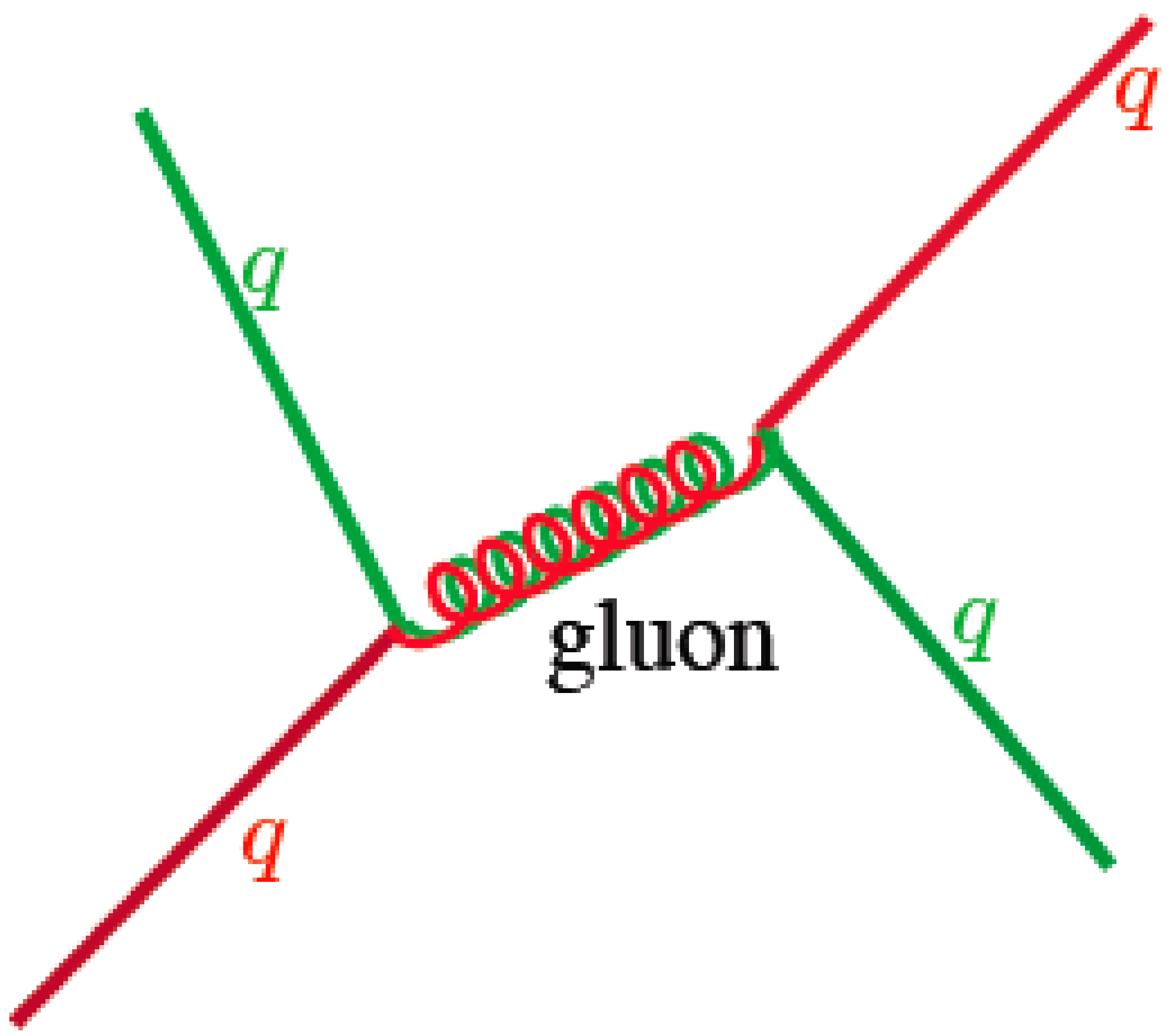
$\text{LEP} : e^+e^- \to Z_0 \to X$

Heisenberg: $q \leftrightarrow \hbar/r$

QED

electrons/positrons - (+,-) charges

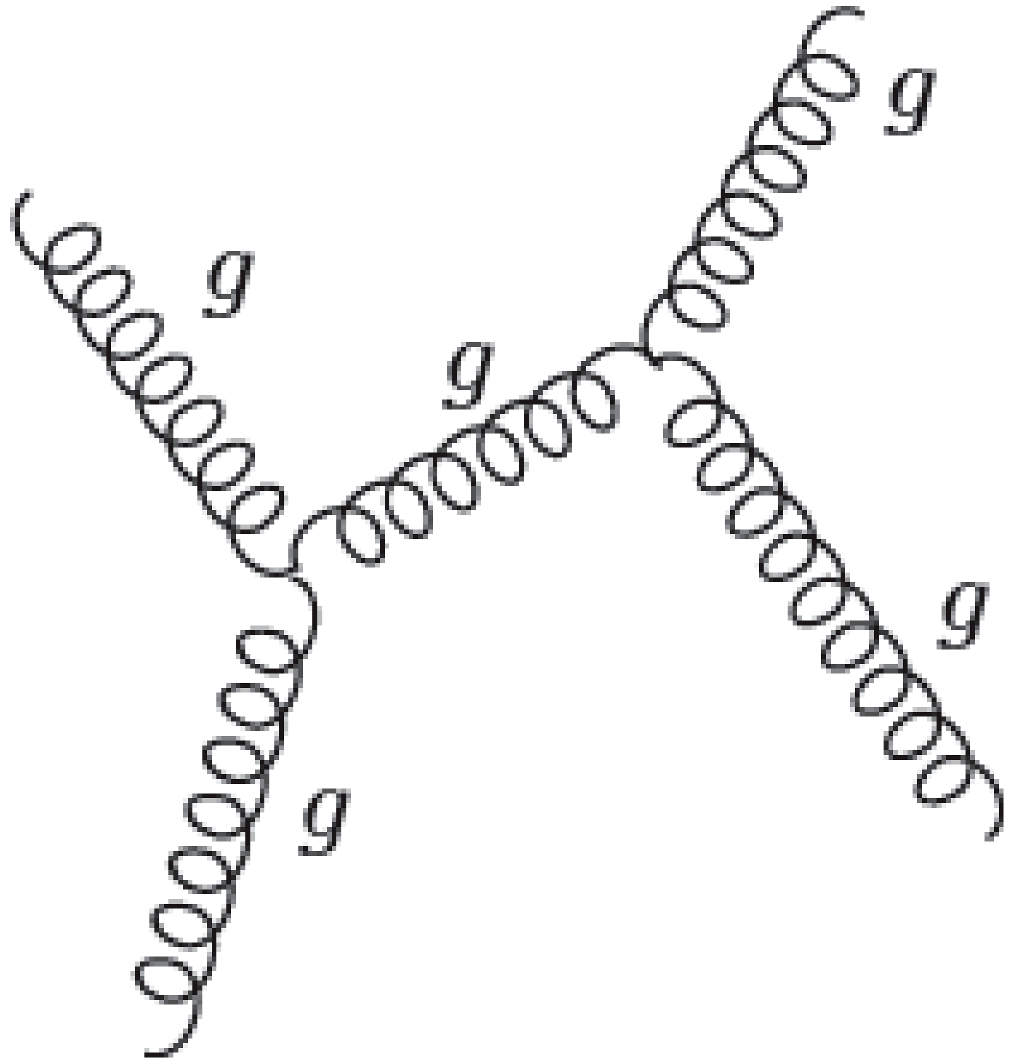
photon - neutral



QCD

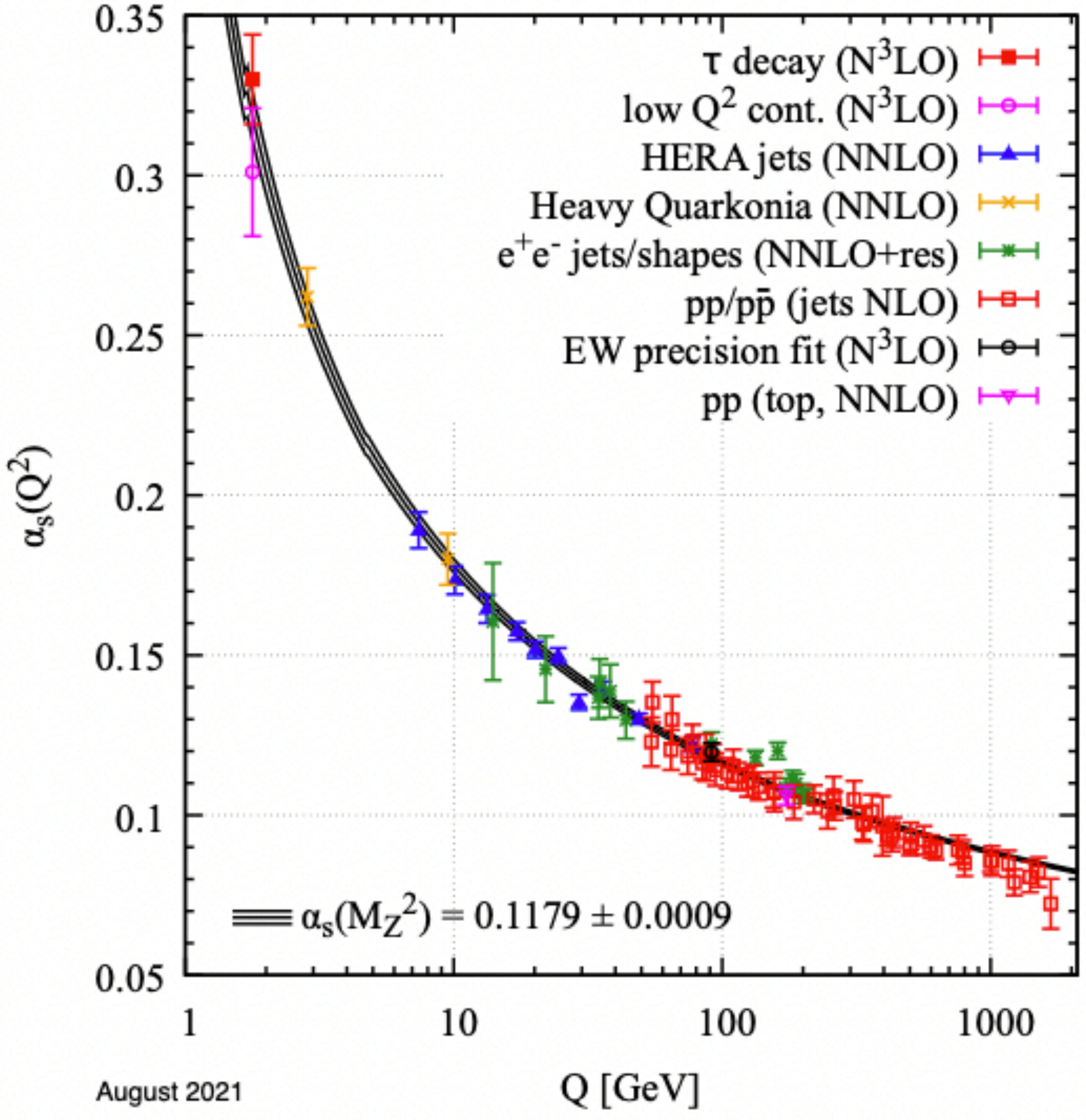
quarks - three colors (r,g,b)

gluons - 8 colors (e.g., r $\bar{g}$)

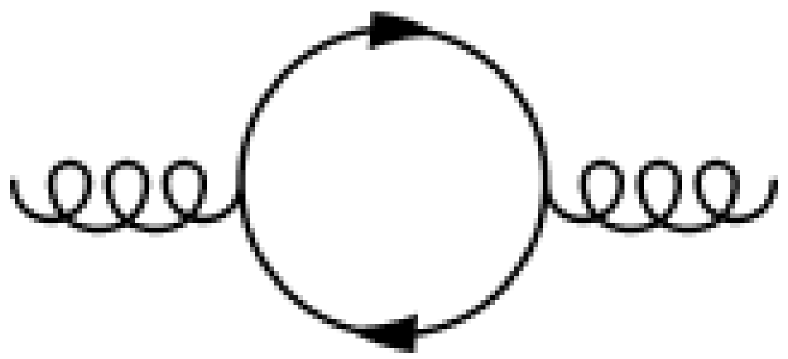


Key Difference: gluons colored and self-interact

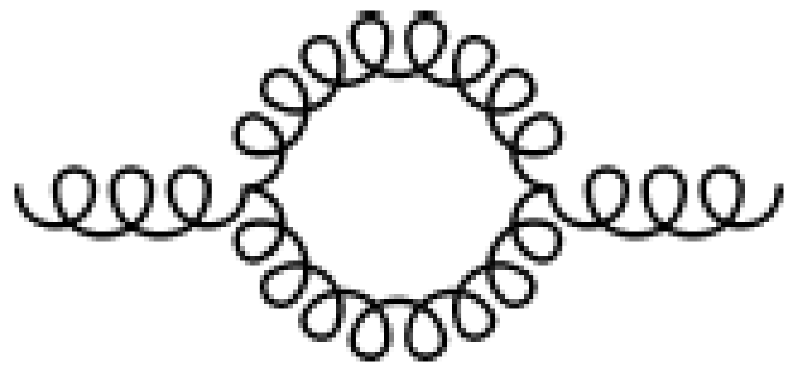
Asymptotic Freedom



(Gross, Politzer, Wilczek; Nobel 2004)



Screening



Anti-Screening

UV perturbation theory valid

IR - Strong Coupling

$$\left\{ \begin{array}{l} \text{Confinement} - \text{Colorless Hadrons} \\ \text{Chiral Symmetry Breaking} \\ \Lambda_{\text{QCD}} \sim 500\,\text{MeV} \end{array} \right.$$

Confinement

Physical Strongly Interacting Particles (hadrons) are Colorless

Mesons - q $\bar{q}$ Baryons - q q q

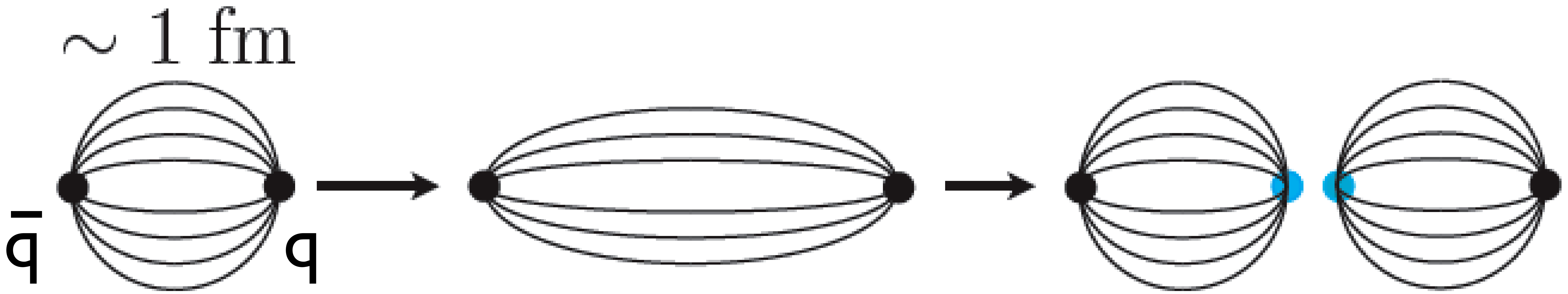
Potential Energy between quark and antiquark

$$V_{q\bar{q}}(r) \approx -\frac{4}{3}\frac{\alpha_s(1/r)}{r} + \sigma\, r$$

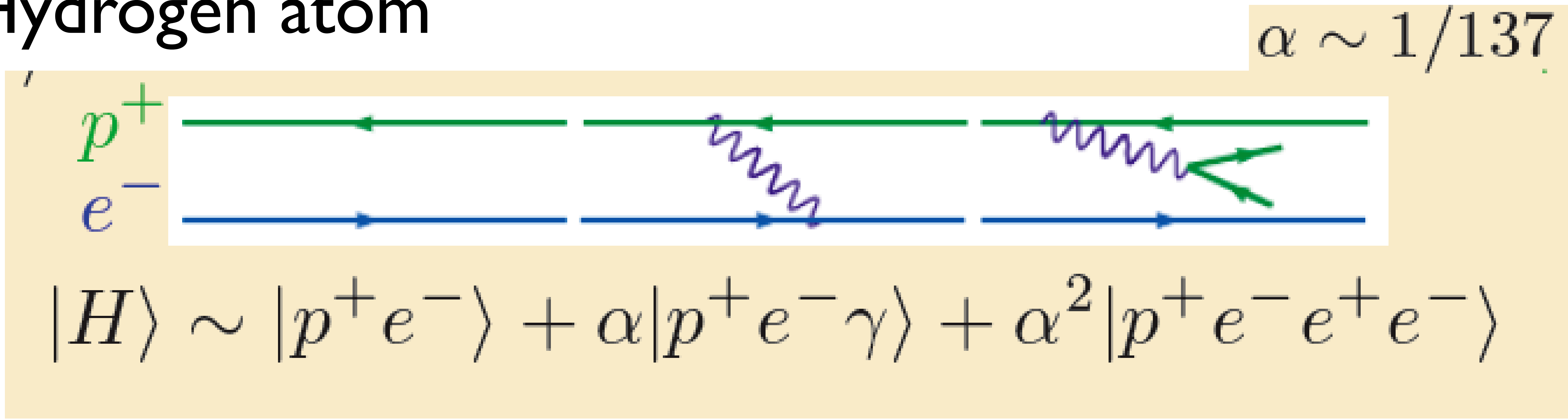
lattice QCD simulations

force between quark and antiquark separated by ~1 fm equivalent to 14 tons!

try to pull a quark-antiquark, create more mesons

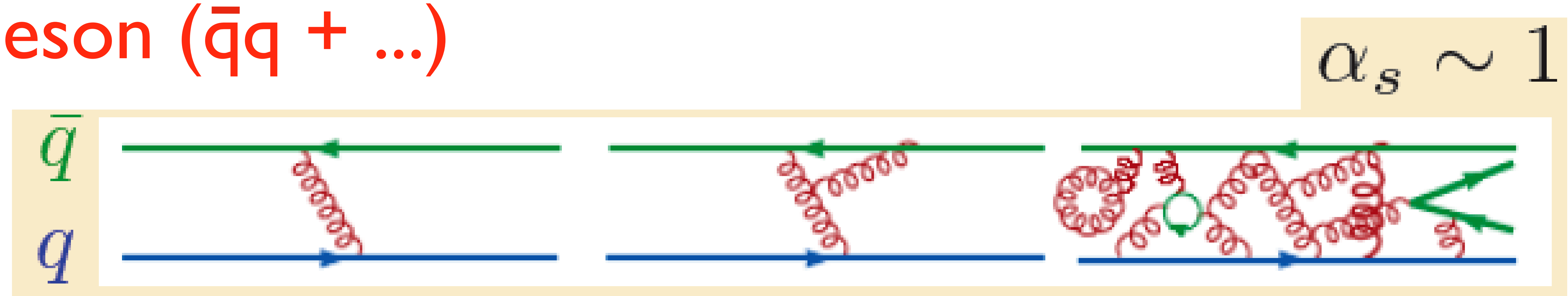


Hydrogen atom



Hydrogen a two-body system up to small corrections

Meson ($\bar{q}q$ + ...)

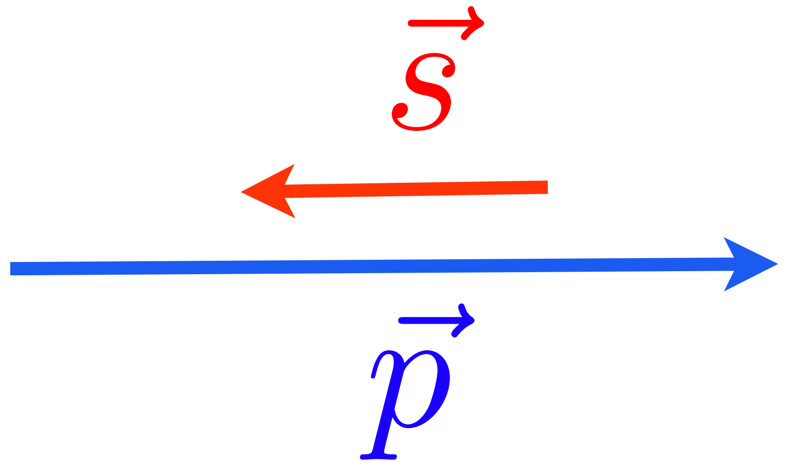


many-body system: many Fock states contribute

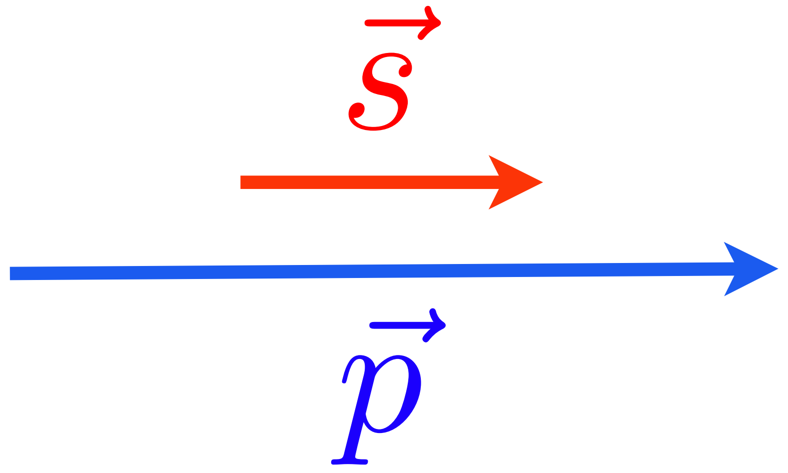
Chiral Symmetry

chirality = helicity (for a massless particle)

$\vec{s}\cdot\hat{p} = -1/2$



$\vec{s}\cdot\hat{p} = +1/2$

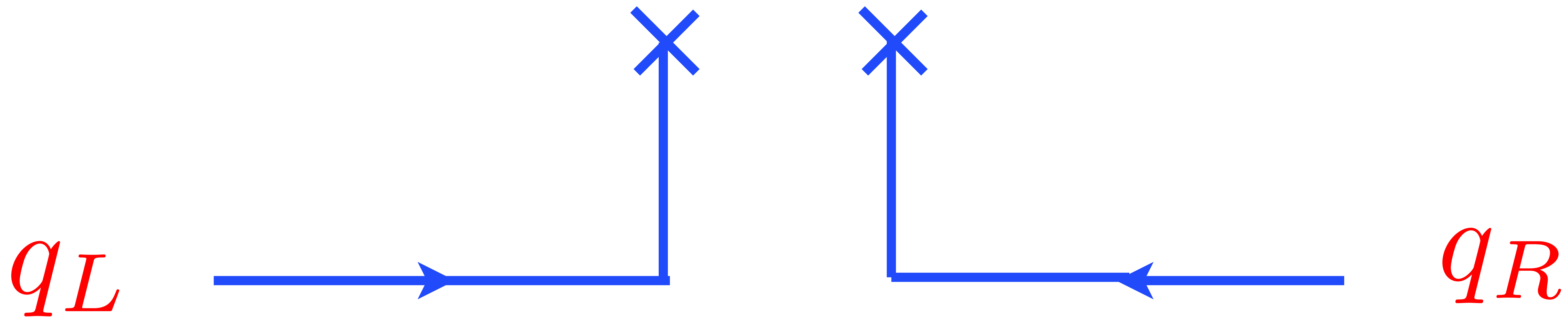


$q = q_L + q_R \quad SU_L(3) \times SU_R(3)$

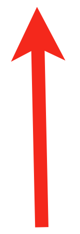
$q_L \rightarrow U_L q_L \qquad q_R \rightarrow U_R q_R$

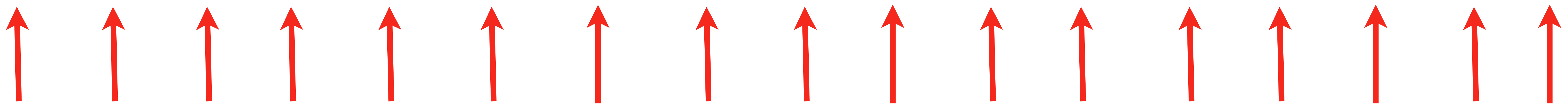
Symmetry of $\mathcal{L}_{\mathrm{QCD}}$, but not the vacuum

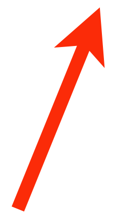
$\langle 0|\bar{q}_L q_R + \bar{q}_R q_L|0\rangle \neq 0$

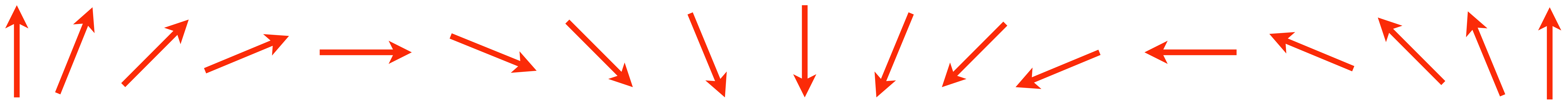


Spontaneous Chiral Symmetry Breaking

- Vacuum: $\langle \bar{q}_L q_R \rangle \leftrightarrow$ 



- Excited State: $\langle \bar{q}_L U_L^\dagger(x) U_R(x) q_R \rangle \leftrightarrow$ 



$\lambda = 2\pi/k$

- recover vacuum as $\lambda \to \infty \quad (k \to 0)$ $\qquad E(k) \propto k$

relativity: $E(k) = \sqrt{k^2 + m^2}$

- Spontaneous Symmetry Breaking $\longrightarrow$ Massless Particle

Goldstone Boson

- real world $m_u, m_d, m_s \neq 0$ pseudo-Goldstone Bosons

$$m_{PGB}^2 \propto m_q \langle 0|\bar{q}q|0\rangle$$

- Light hadrons of QCD: pions, kaons, and eta's

$m_\pi \sim 140\,\mathrm{MeV}$ $m_K \sim 496\,\mathrm{MeV}$ $m_\eta = 547\,\mathrm{MeV}$

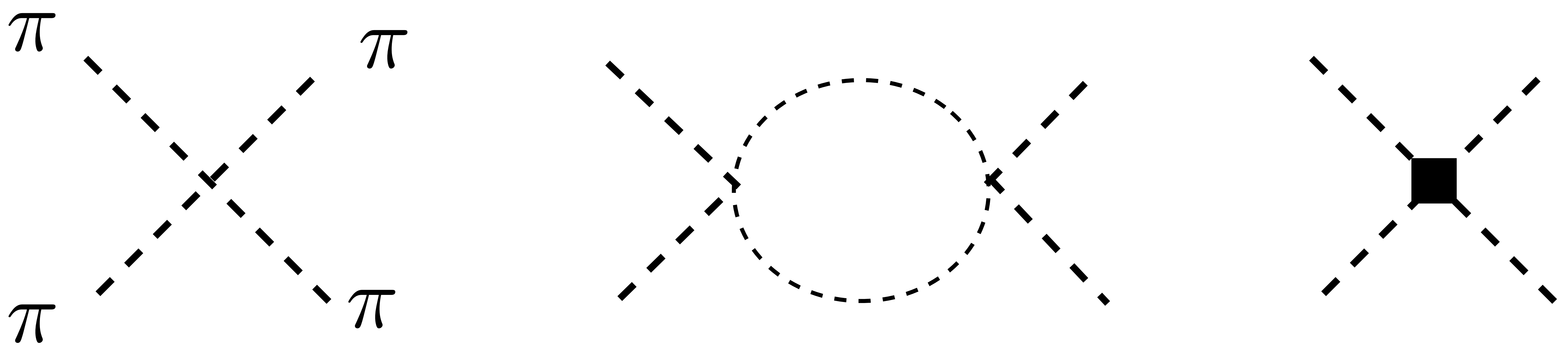
Chiral Symmetry Breaking determines the low lying degrees of freedom of QCD, constrains their self-interactions and interactions with other hadrons

Interactions vanish as $k \to 0$ derivatively coupled

weakly interacting at low energies:

Chiral Perturbation Theory (ChiPT)

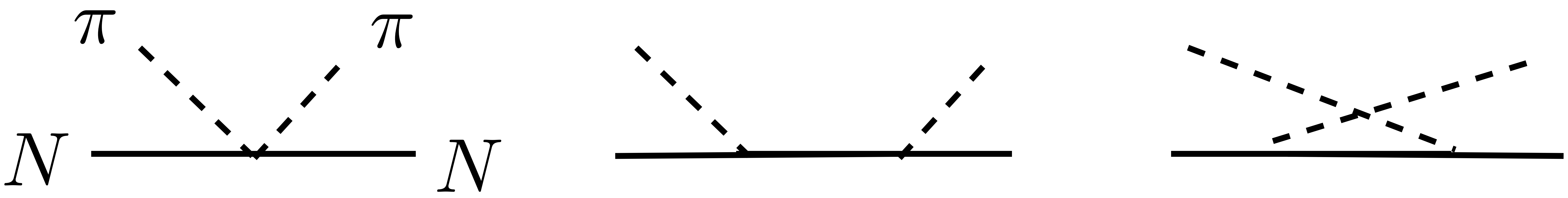
$$\mathcal{L} = \frac{f_\pi^2}{8}\mathrm{Tr}\,\partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \frac{f_\pi^2 B_0}{4}\mathrm{Tr}(m_q \Sigma + m_q \Sigma^\dagger) + \ldots \qquad \Sigma = e^{2i\pi/f_\pi}$$



$$\frac{Q^2}{\Lambda_\chi^2} \qquad \frac{Q^4}{\Lambda_\chi^4} \qquad Q \sim (p, m_\pi)$$

perturbative expansion in $\dfrac{Q^2}{\Lambda_\chi^2}$ $\quad \Lambda_\chi = 4\pi f_\pi \sim 1\,\mathrm{GeV}$

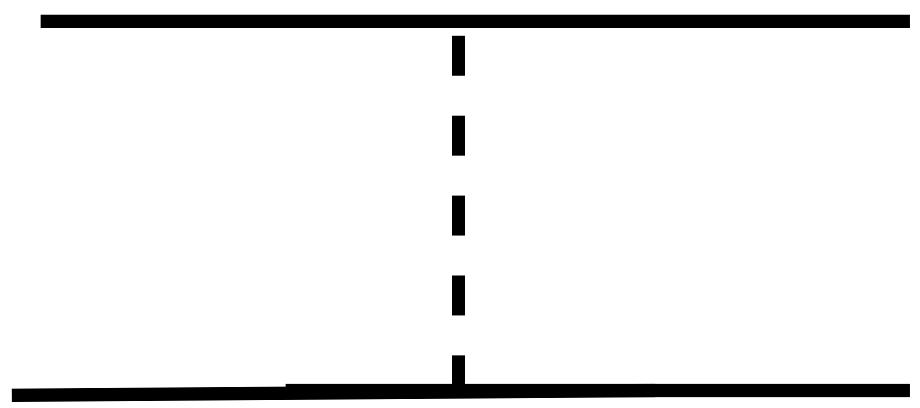
Coupling to nucleons



S-wave scattering length

$$a^I_{\pi N} = C_I \frac{m_\pi}{f_\pi^2}$$ (within 15%-25%)

Long range nuclear spin-tensor force



$$\frac{g_A^2}{2f_\pi^2} \frac{\vec{q}\cdot\vec{\sigma}\,\vec{q}\cdot\vec{\sigma}}{\vec{q}^2 + m_\pi^2}$$

Bottom-Up EFTs

isolate relevant degrees of freedom

small expansion parameter (ratio of scales):

Lagrangian is most general consistent w/
symmetries of underlying theory, i.e. QCD

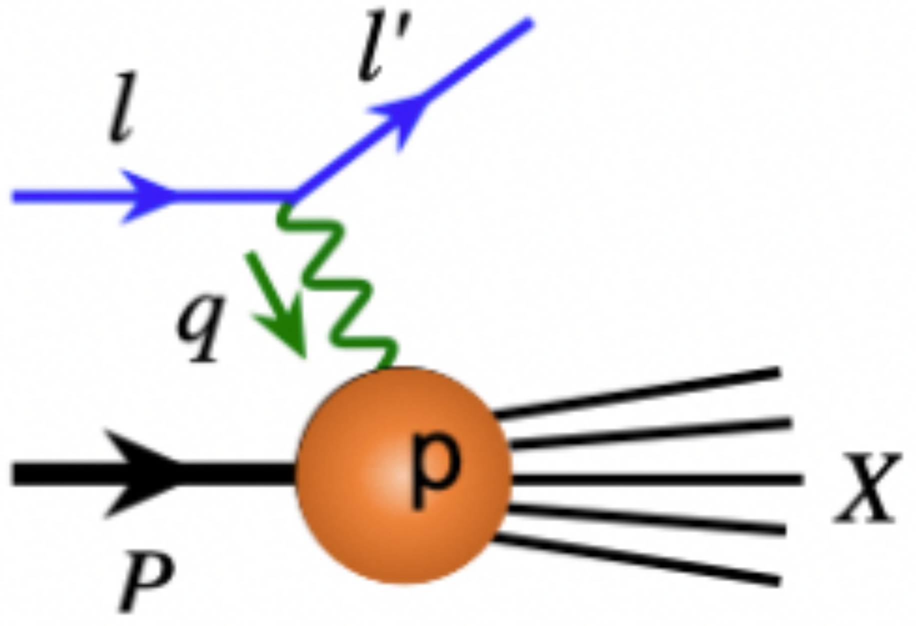
Power Counting - count powers of expansion
parameter in Lagrangian and in Feynman diagrams

coefficients in Lagrangian determined by experimental or lattice simulations

Examples: ChPT, effective theory of NN interactions, πN interactions,
Hadron molecules, e.g., X(3872), T_{cc}^{+}, ...

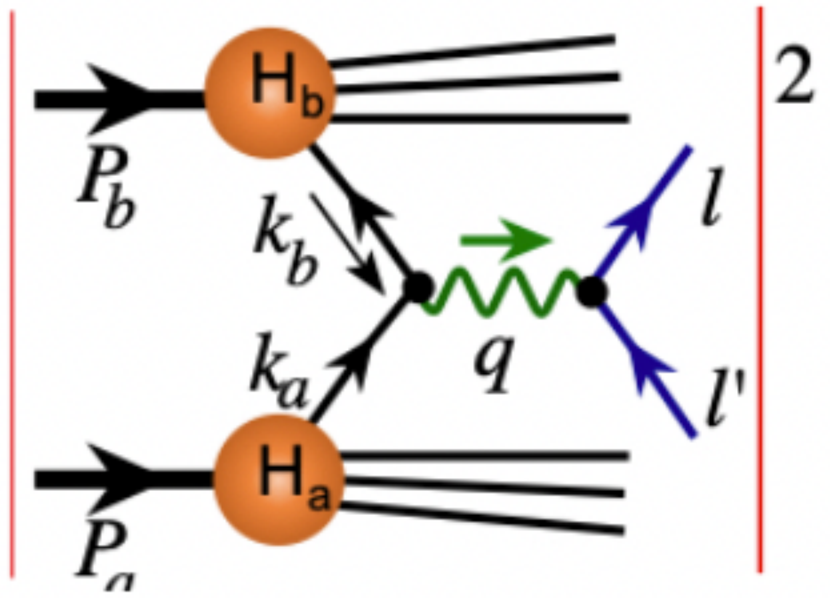
QCD Factorization

Deep Inelastic Scattering



$$d\sigma_{ep\to e'X} \sim \sum_i f_{i/p} \otimes d\hat{\sigma}(\gamma^* i \to X)$$

Drell-Yan



$$d\sigma_{pp\to \ell^+\ell^- X} \sim \sum_{ij} f_{i/p} \otimes f_{j/p} \otimes d\hat{\sigma}(ij \to \gamma^* X)$$

$Q^2 \gg \Lambda^2_{\rm QCD}$

$d\hat{\sigma}$ calculable in pQCD - $\alpha_s(Q)$

$f_{i/p}$ universal parton distribution functions

Deriving Factorization

DIS

$$d\sigma \propto \int d^4x e^{iq\cdot x} \langle p| T J^\mu(x) J^\nu(0) |p\rangle$$

apply Operator Product Expansion (OPE)

DY

All-orders diagrammatic proofs in pQCD

these can be challenging!

G. T. Bodwin, Phys. Rev. D 31, 2616 (1985) [Erratum-ibid. D 34, 3932 (1985)]

J. C. Collins, D. E. Soper and G. Sterman, Phys. Lett. B 134, 263 (1984)

G. T. Bodwin, Phys. Rev. D 31, 2616 (1985) [Erratum-ibid. D 34, 3932 (1985)]

J. C. Collins, D. E. Soper and G. Sterman, Nucl. Phys. B 308, 833 (1988)

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) \to \sum_n C_{12}{}^n(x)\mathcal{O}_n(0), \qquad \text{as} \quad x \to 0$$

Wilson coefficients obey RGE

$$\left[M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial g} + \gamma_1 + \gamma_2 - \gamma_n\right]C_{12}{}^n(x;M) = 0.$$

$$C_{12}{}^n(x) = \left(\frac{1}{|x|}\right)^{d_1+d_2-d_n} c(\bar{g}(1/x))\exp\left[\int_{1/x}^{M} d\log M'(\gamma_n - \gamma_1 - \gamma_2)\right],$$

Naive dimensional analysis

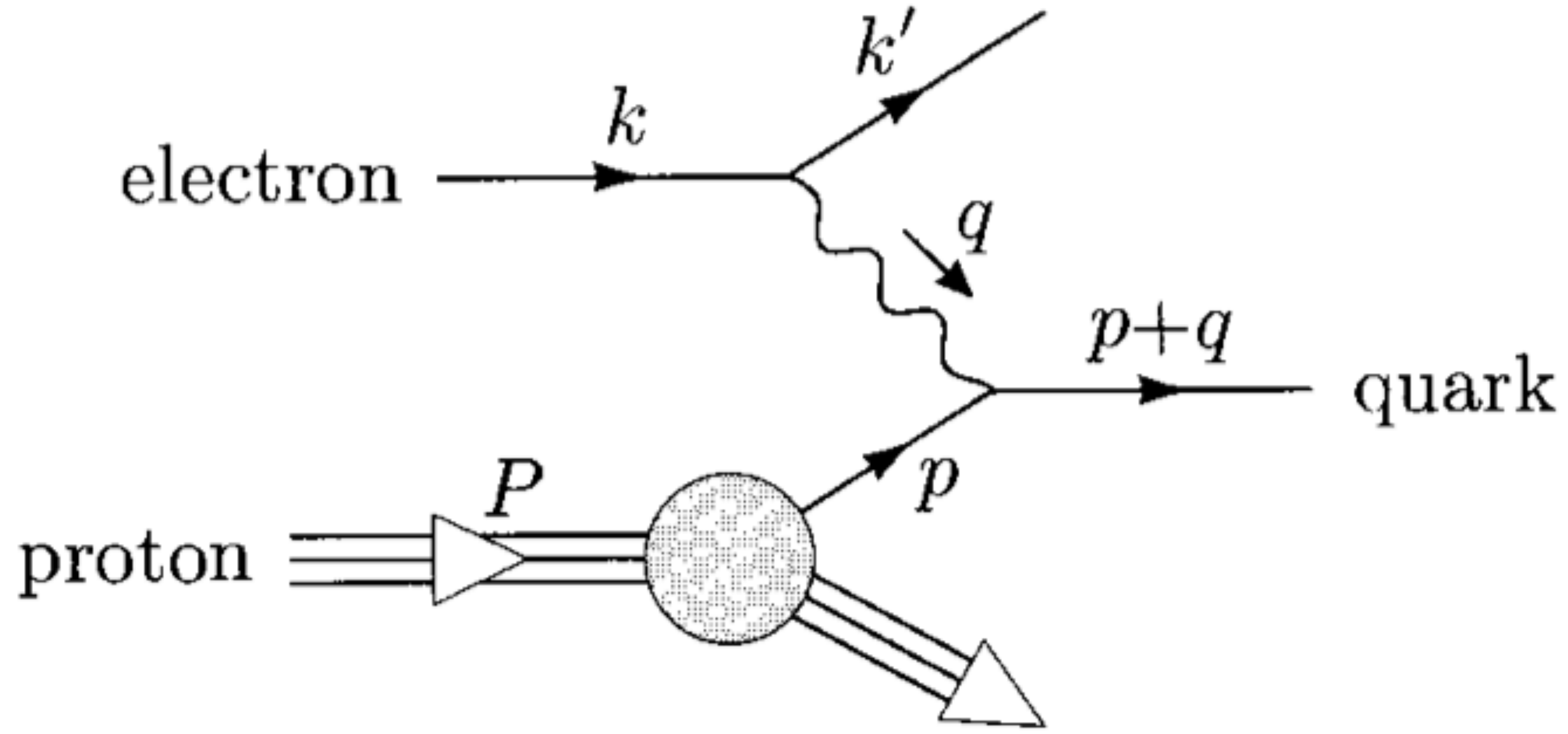
Anomalous dimension

General form of DIS cross section

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\left[(1-y)F_2(x,Q^2) + xy^2F_1(x,Q^2)\right]$$

$$x = \frac{Q^2}{2P\cdot q} \qquad\qquad Q^2 = -q^2 = xys$$

Parton Model



$$\frac{d^2\sigma}{dxdy}(e^-p \to e^-X) = \left(\sum_f xf_f(x)Q_f^2\right)\frac{2\pi\alpha^2 s}{Q^4}\left[1+(1-y)^2\right].$$

Bjorken scaling: $F_{1,2}(x,Q^2)$ independent of Q^2

Callan-Gross relation: $F_2(x) = 2xF_1(x)$ spin-1/2 quarks

OPE Analysis of DIS

Reproduce parton model results

Moments of pdf's are related to local twist-2 operators

Anomalous dimensions twist-2 operators imply scaling violation

DGLAP evolution equations for structure functions

QCD Evolution - DGLAP Equations

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

$$q \equiv f_{q/p} \qquad g \equiv f_{g/p}$$

$$P_{qq} = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

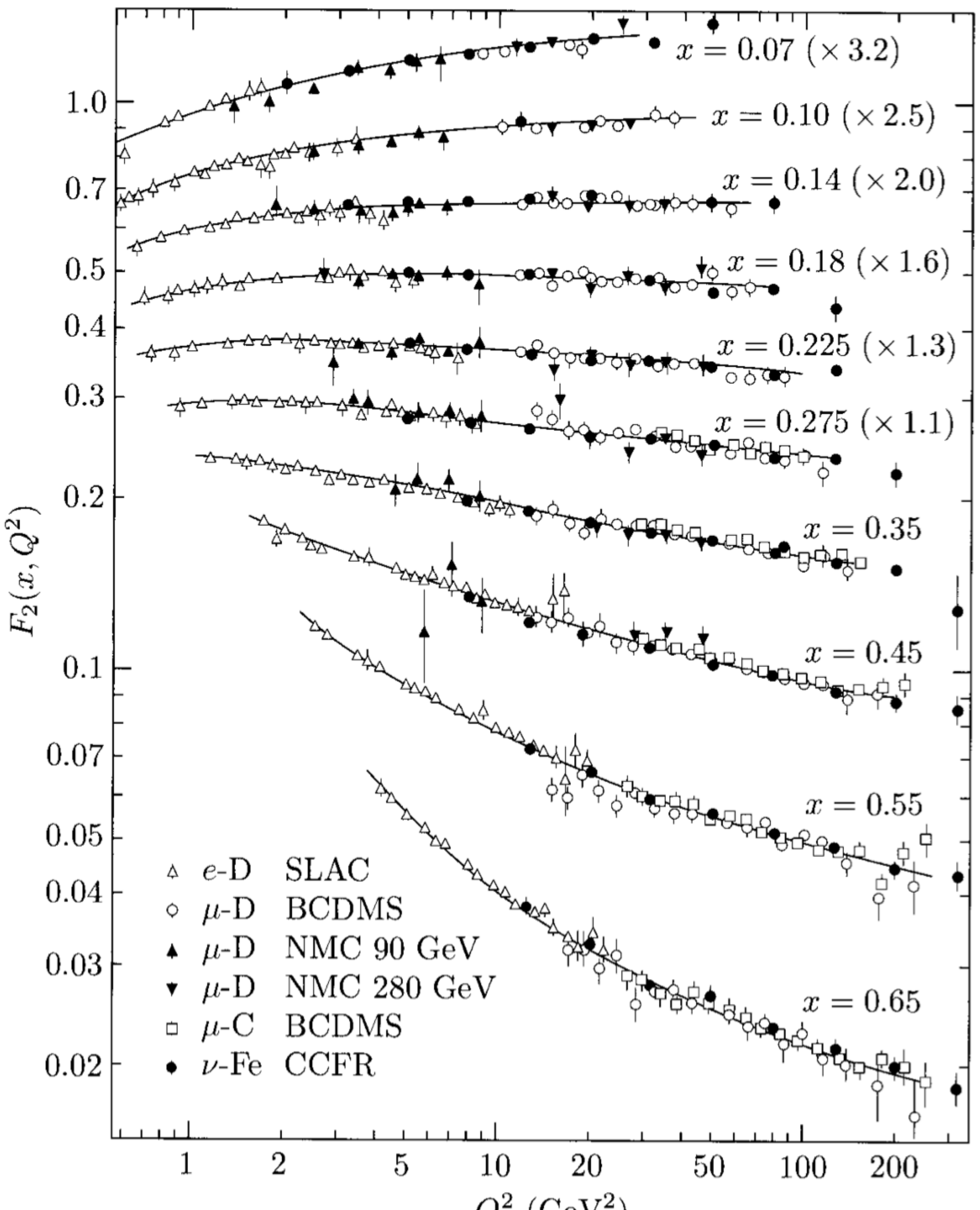
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}$$

Cross section is independent of μ

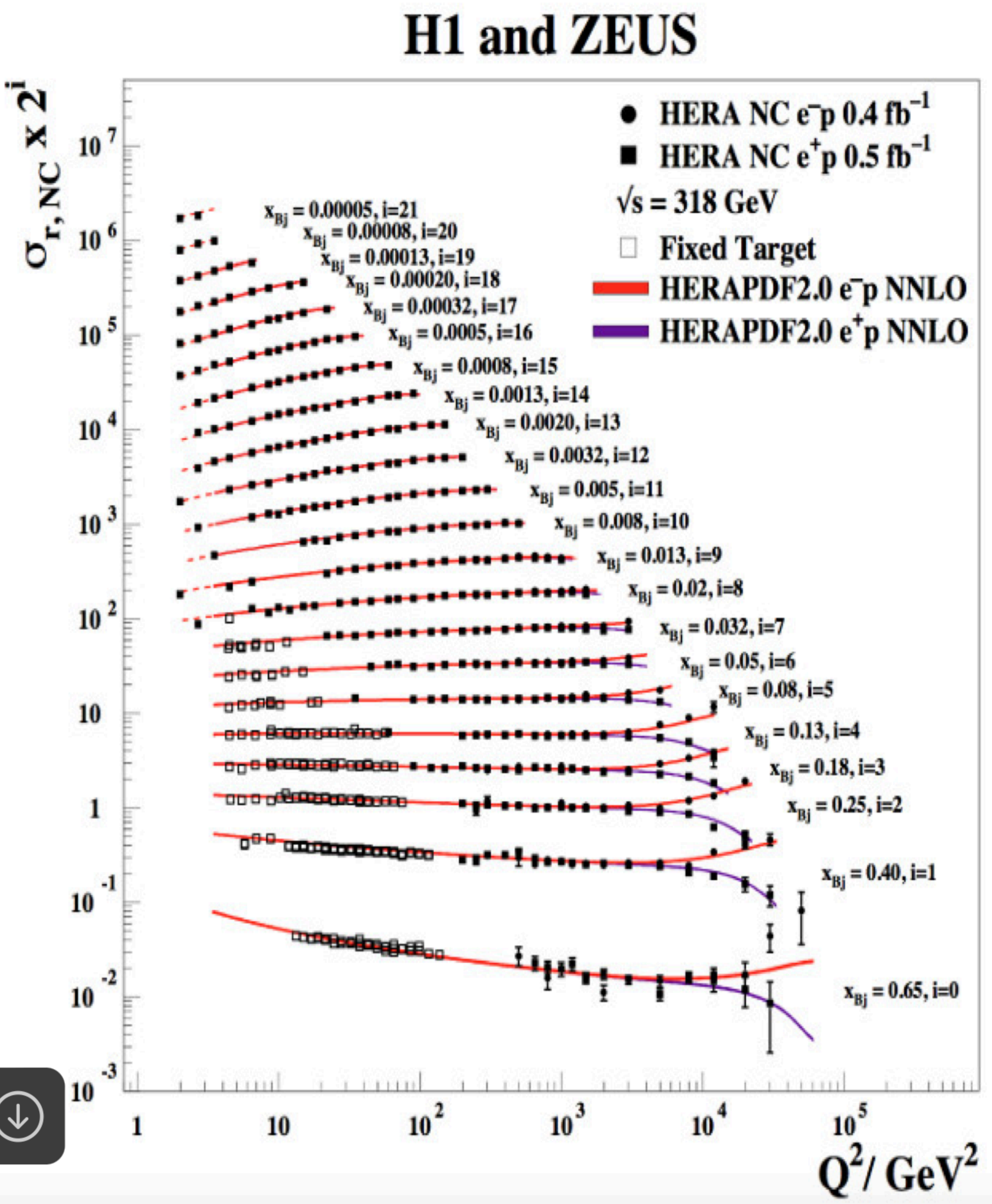
$$d\sigma(ep \to X) = \sum_i f_{i/p}(\mu) \otimes d\hat{\sigma}(i\gamma^* \to X)(\mu, \alpha_s(\mu))$$
$$= \sum_i f_{i/p}(Q) \otimes d\hat{\sigma}(i\gamma^* \to X)(Q, \alpha_s(Q))$$

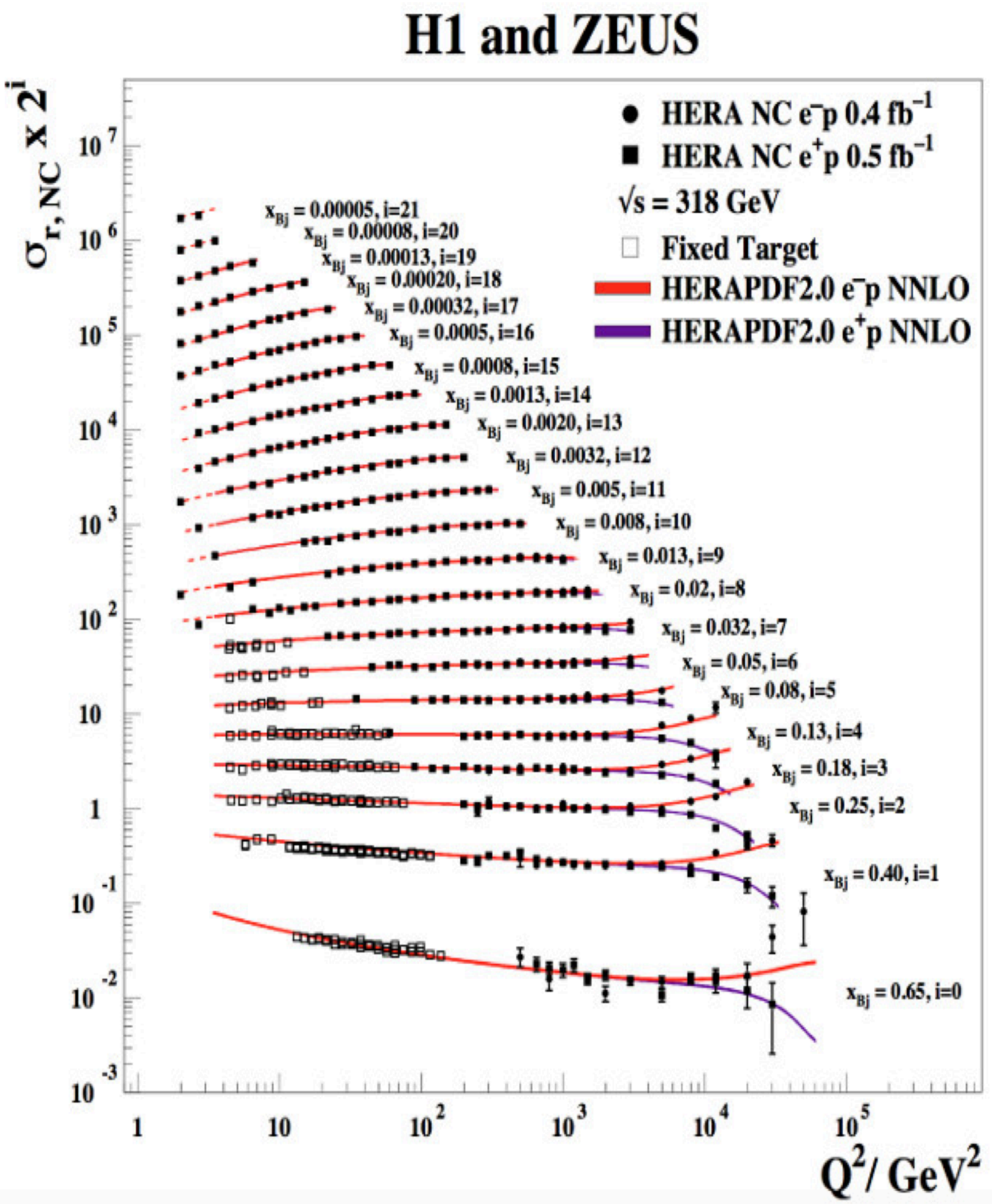
Minimize logs in short-distance cross section $\ln\left(\frac{Q^2}{\mu^2}\right)$



DGLAP tested over a large range of Q^2

Peskin, Schroeder, Intro to QFT, p.592, data from 1994


H1 and ZEUS
HERA NC e⁻p 0.4 fb⁻¹
HERA NC e⁺p 0.5 fb⁻¹
√s = 318 GeV
Fixed Target
HERAPDF2.0 e⁻p NNLO
HERAPDF2.0 e⁺p NNLO
$\sigma_{r,NC} \times 2^i$
Q^2/ GeV^2
x_Bj = 0.00005, i=21
x_Bj = 0.00008, i=20
x_Bj = 0.00013, i=19
x_Bj = 0.00020, i=18
x_Bj = 0.00032, i=17
x_Bj = 0.0005, i=16
x_Bj = 0.0008, i=15
x_Bj = 0.0013, i=14
x_Bj = 0.0020, i=13
x_Bj = 0.0032, i=12
x_Bj = 0.005, i=11
x_Bj = 0.008, i=10
x_Bj = 0.013, i=9
x_Bj = 0.02, i=8
x_Bj = 0.032, i=7
x_Bj = 0.05, i=6
x_Bj = 0.08, i=5
x_Bj = 0.13, i=4
x_Bj = 0.18, i=3
x_Bj = 0.25, i=2
x_Bj = 0.40, i=1
x_Bj = 0.65, i=0


H1 and ZEUS
HERA NC e⁻p 0.4 fb⁻¹
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Fixed Target
HERAPDF2.0 e⁻p NNLO
HERAPDF2.0 e⁺p NNLO
$\sigma_{r,NC} \times 2^i$
Q^2/ GeV^2

Summary of Important QCD Concepts

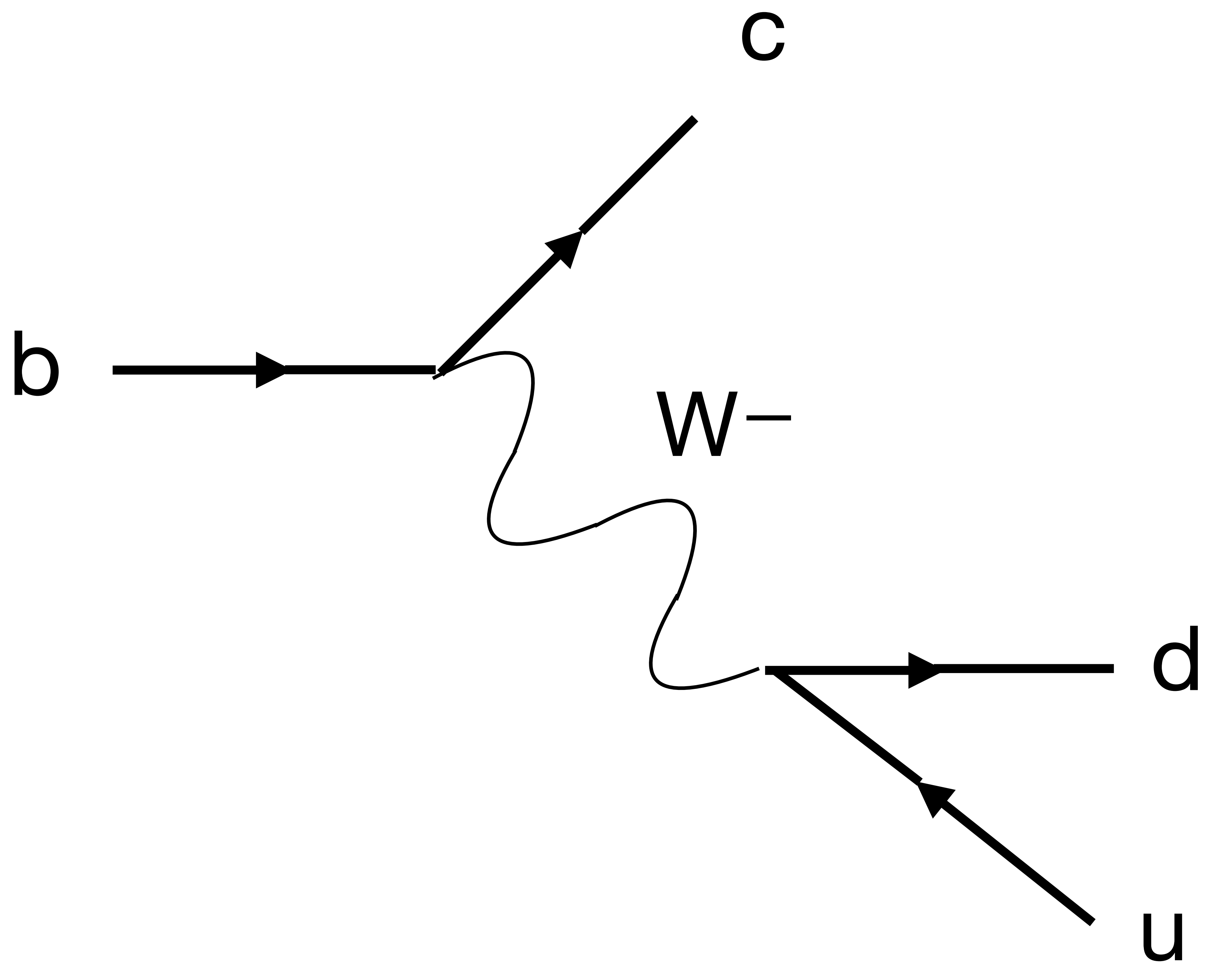
Asymptotic Freedom

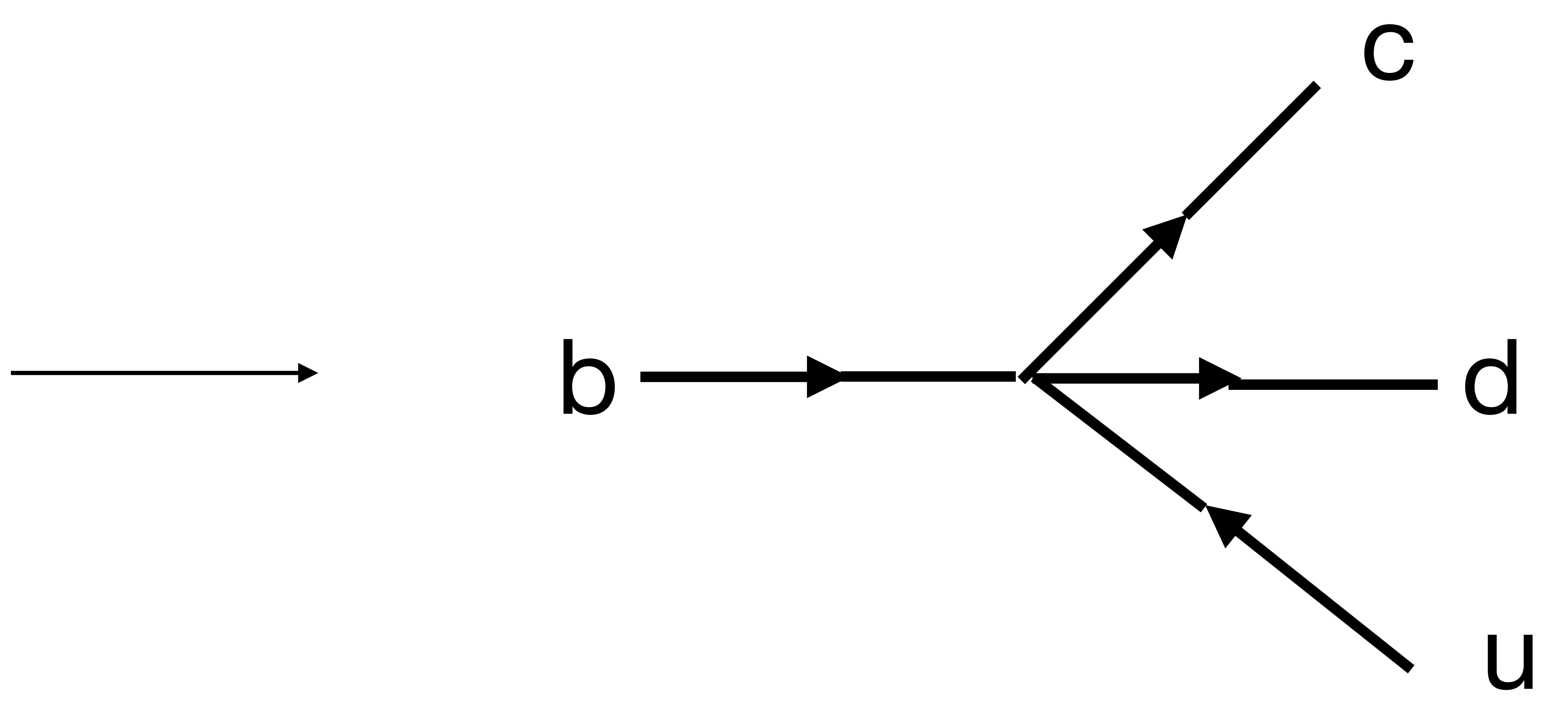
Confinement, Chiral Symmetry Breaking

High Energy QCD: Factorization, Evolution

Low Energy: Lattice QCD, Effective Theories

Effective Theory of Weak Interactions - b decay





$$\frac{1}{p^2 - m_W^2} = -\frac{1}{m_W^2} - \frac{p^2}{m_W^4} + \ldots$$

$$O_1(\mu) = [\bar{c}^\alpha \gamma_\mu P_L b_\alpha][\bar{d}^\beta \gamma^\mu P_L u_\beta],$$

$$O_2(\mu) = [\bar{c}^\beta \gamma_\mu P_L b_\alpha][\bar{d}^\alpha \gamma^\mu P_L u_\beta].$$

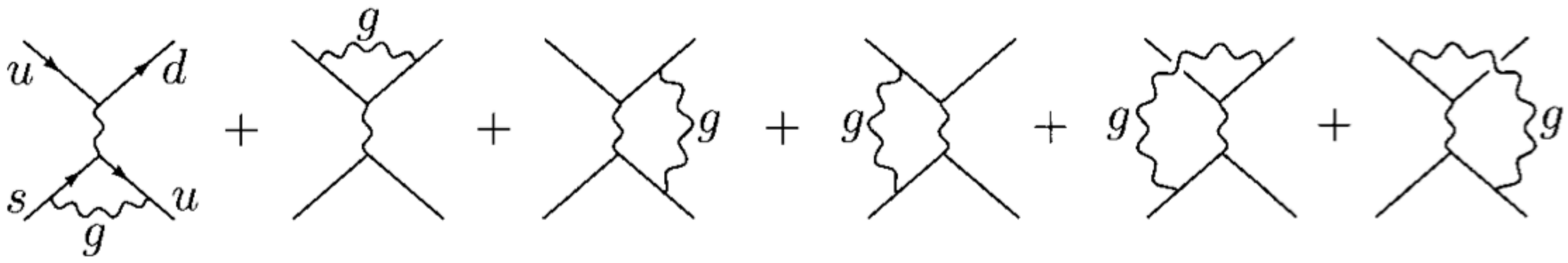
$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ C_1\left[\frac{M_W}{\mu}, \alpha_s(\mu)\right] O_1(\mu) + C_2\left[\frac{M_W}{\mu}, \alpha_s(\mu)\right] O_2(\mu) \right\},$$

$$C_1[1, \alpha_s(M_W)] = 1 + \mathcal{O}[\alpha_s(M_W)],$$

$$C_2[1, \alpha_s(M_W)] = 0 + \mathcal{O}[\alpha_s(M_W)].$$

Tree level - reproduces tree level Standard Model

One loop QCD corrections to effective operators



Evolution equations for Wilson Coefficients

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \gamma_{ji} C_j . \qquad \gamma(g) = \frac{g^2}{8\pi^2} \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} .$$

$$C_i \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] = P \exp \left[\int_{g(M_W)}^{g(\mu)} \frac{\gamma^T(g)}{\beta(g)} dg \right]_{ij} C_j [1, \alpha_s(M_W)].$$

Renormalization group improved Hamiltonian

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ C_+ \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_+(\mu) + C_- \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] O_-(\mu) \right\}$$

$$O_\pm = O_1 \pm O_2.$$

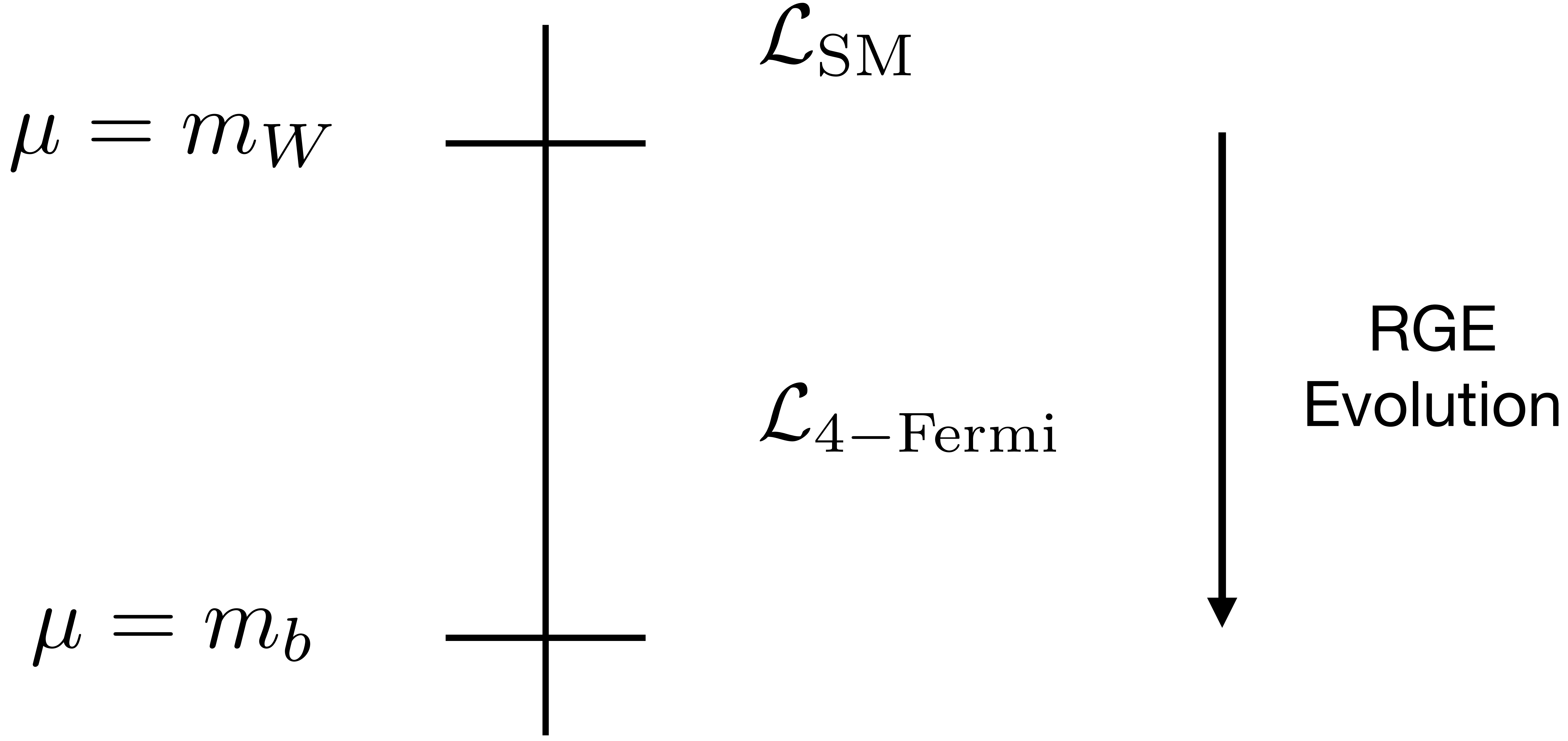
$$C_\pm \left[\frac{M_W}{\mu}, \alpha_s(\mu) \right] = \frac{1}{2} \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{a_\pm}$$

$$a_+ = \frac{6}{33 - 2N_q}, \qquad a_- = -\frac{12}{33 - 2N_q}.$$

Resum $\alpha_s^n(m_b) \ln^n \left(\frac{m_W^2}{m_b^2} \right)$ to all orders

$$\alpha_s(m_b) \sim 0.2 \qquad \log \left(\frac{m_W^2}{m_b^2} \right) \sim 6$$

Matching and Running



Effective Theory Principles for Top-Down construction of EFTs

Degrees of freedom - SM fermions

Power Counting - Dimensional analysis

Symmetries constrain Lagrangian - Lorentz Inv., color

Match effective Lagrangian at the high scale

RGE evolution to low scales

Effective Theories of this type for QCD:

Heavy Quark Effective Theory (HQET)

Non-Relativistic QCD (NRQCD)

Soft-Collinear Effective Theory (SCET)

Useful for:
- realizing emergent symmetries of QCD

  e.g. heavy quark spin-flavor symmetry $m_Q \to \infty$

- deriving new factorization theorems

- resummation of large logarithms

HQET

$$p^\mu = m_Q v^\mu + k^\mu$$

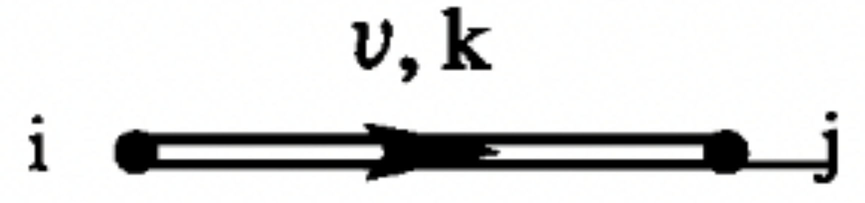
$v^\mu \sim (1, \vec{0})$ in heavy quark rest frame $\quad k^\mu \sim \Lambda_{\mathrm{QCD}}$

$$\frac{i}{\not{p} - m_Q} \approx \frac{1+\not{v}}{2}\frac{i}{v\cdot k}$$

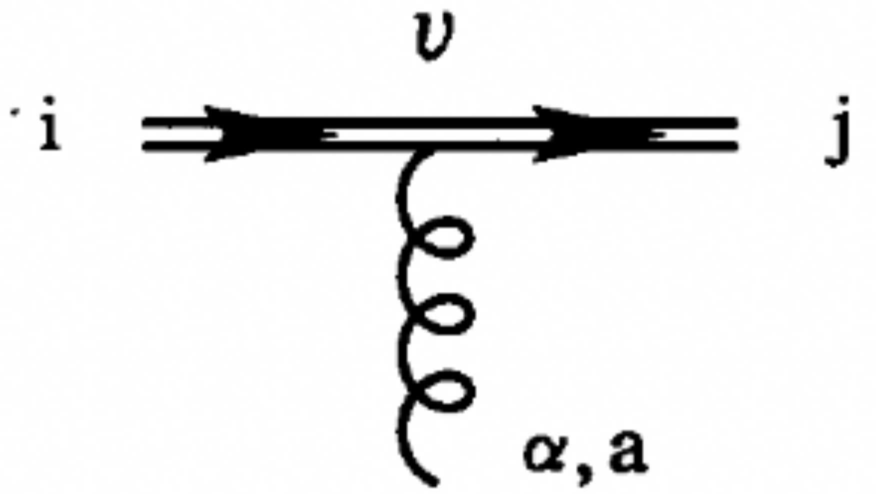
$$\bar{u}(p) i g T^a A^a_\mu \gamma^\mu u(p+k) \approx h_v^\dagger i g T^a A^a_\mu v^\mu h_v \qquad h_v = \frac{1+\not{v}}{2} h_v$$

HQET Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \bar{h}_v \mathrm{i} v \cdot D h_v + \frac{1}{2m_{\mathrm{Q}}} \bar{h}_v (\mathrm{i}D_\perp)^2 h_v + \frac{g}{4m_{\mathrm{Q}}} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v + \mathrm{O}(1/m_{\mathrm{Q}}^2).$$



$$= \frac{\mathrm{i}}{v\cdot \mathrm{k}} \frac{1+\not{v}}{2} \delta_{\mathrm{ji}}$$



$$= \mathrm{ig}\,(\mathrm{T_a})_{\mathrm{ji}}\, v^\alpha$$

Integrating out antiparticles

$$\psi(x) = e^{-im_Q v\cdot x}\left[h_v(x) + H_v(x)\right] \qquad \frac{1+\not{v}}{2}h_v = h_v \qquad \frac{1-\not{v}}{2}H_v = H_v$$

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \bar{h}_v i\not{D}_\perp H_v + \bar{H}_v i\not{D}_\perp h_v - \bar{H}_v(iv\cdot D + 2m_Q)H_v$$

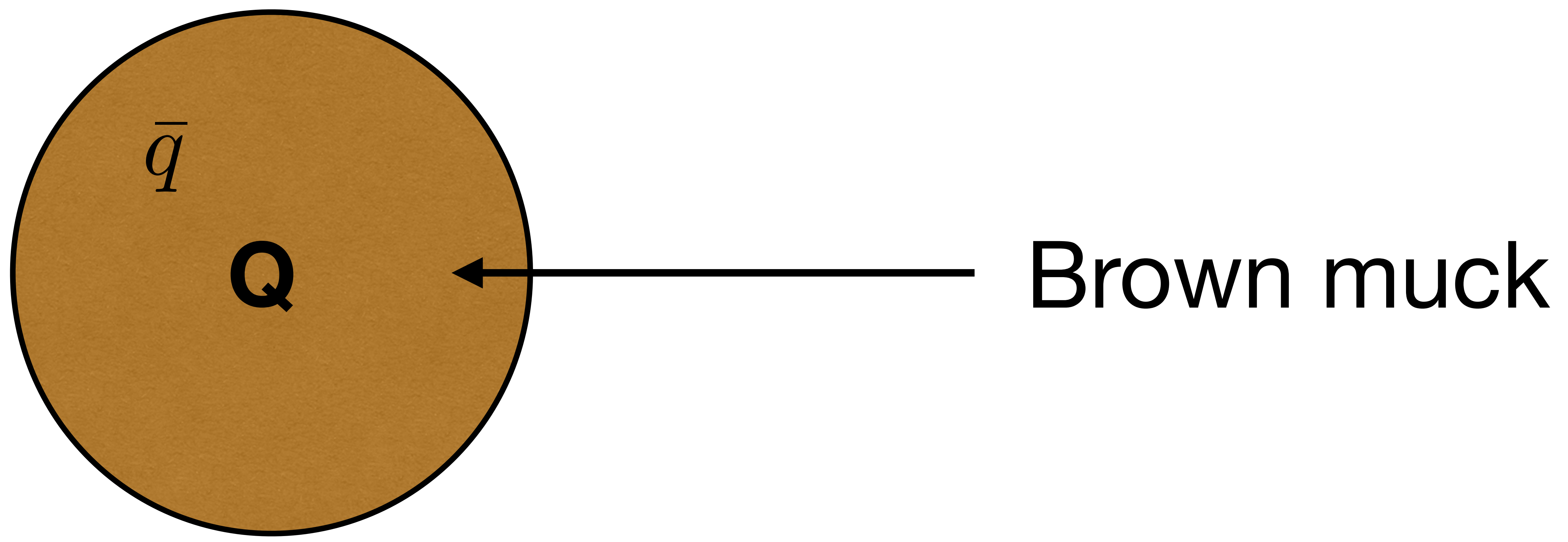
$$D_\perp^\mu = D^\mu - v^\mu v\cdot D$$

Integrating out H_v , eliminate using equation of motion

$$\mathcal{L}_{\text{eff}} = \bar{h}_v iv\cdot D h_v + \bar{h}_v\, i\not{D}_\perp \frac{1}{2m_Q + 2iv\cdot D} i\not{D}_\perp h_v$$

$$= \bar{h}_v iv\cdot D h_v + \frac{1}{2m_Q}\bar{h}_v (iD_\perp)^2 h_v + \frac{g}{4m_Q}\bar{h}_v \sigma_{\alpha\beta}G^{\alpha\beta}h_v + \ldots$$

Heavy quark is static source of color l.d.o.f. in $m_Q \to \infty$



$$m_M = m_Q + \bar{\Lambda} + \frac{\lambda}{2m_Q} + O\left(\frac{\Lambda^3_{\rm QCD}}{m_Q^2}\right)$$

$$m^2_{D^*} - m^2_D = 0.55\,\text{GeV}^2 \qquad m^2_{B^*} - m^2_B = 0.48\,\text{GeV}^2$$

Heavy meson decay form factors

$$\langle D(v')|V^\mu|B(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu,$$

$$\langle D^*(v',\epsilon)|V^\mu|B(v)\rangle = \mathrm{i}h_\mathrm{V}(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta,$$

$$\langle D^*(v',\epsilon)|A^\mu|B(v)\rangle = h_{\mathrm{A}_1}(w)(w+1)\epsilon^{*\mu} - [h_{\mathrm{A}_2}(w)v^\mu + h_{\mathrm{A}_3}(w)v'^\mu]\epsilon^*\cdot v.$$

Isgur-Wise function $\xi(w) = \xi(v'\cdot v),\ \xi(1) = 1$

$$h_+(w) = h_\mathrm{V}(w) = h_{\mathrm{A}_1}(w) = h_{\mathrm{A}_3}(w) = \xi(w)\ ,$$

$$h_-(w) = h_{\mathrm{A}_2}(w) = 0\,.$$

D^* and D are degenerate in heavy quark limit

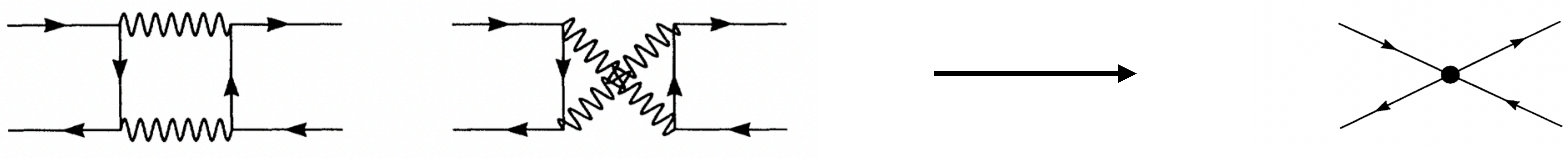
Heavy hadron chiral perturbation theory (HHChiPt)

$$H_{v,a} = \frac{1+\not{v}}{2}(\not{P}_a^* - \gamma_5 P_a) \qquad v^\mu \sim (1,\vec{0}) \longrightarrow H_a = \vec{P}_a^* \cdot \vec{\sigma} + P_a$$

$$\mathcal{L} = \mathrm{Tr}[H_a^\dagger (iD_0)_{ba} H_b] - g\mathrm{Tr}[H_a^\dagger H_b \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta_H}{4}\mathrm{Tr}[H_a^\dagger \sigma^i H_a \sigma^i]$$

NRQCD

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi$$



$$(\delta \mathcal{L}_{\text{4-fermion}})_{d=6} = \frac{f_1(^1S_0)}{M^2} \mathcal{O}_1(^1S_0) + \frac{f_1(^3S_1)}{M^2} \mathcal{O}_1(^3S_1) + \frac{f_8(^1S_0)}{M^2} \mathcal{O}_8(^1S_0) + \frac{f_8(^3S_1)}{M^2} \mathcal{O}_8(^3S_1)$$

$$\begin{aligned}
\mathcal{O}_1(^1S_0) &= \psi^\dagger \chi \, \chi^\dagger \psi, \\
\mathcal{O}_1(^3S_1) &= \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi, \\
\mathcal{O}_8(^1S_0) &= \psi^\dagger T^a \chi \, \chi^\dagger T^a \psi, \\
\mathcal{O}_8(^3S_1) &= \psi^\dagger \boldsymbol{\sigma} T^a \chi \cdot \chi^\dagger \boldsymbol{\sigma} T^a \psi.
\end{aligned}$$

Power counting: $E \sim m_Q v^2 \quad p \sim m_Q v^2$

Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

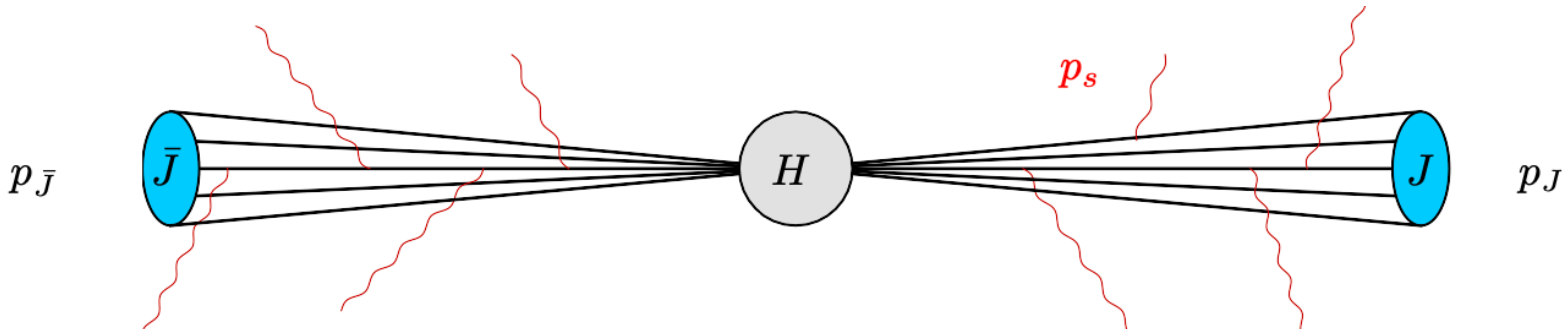
CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

Soft-Collinear Effective Theory

Two jet events in e+ e- collisions



Final state particles: energetic partons collinear to jets, soft radiation

Many scales: $Q, E_J, m_J,$ jet substructure

Soft Collinear Effective Theory (SCET)

Multipole expand QCD fields onto fields w/ definite momentum scaling

$$\phi^{\rm QCD} = \phi_n + \phi_{\bar{n}} + \phi_{us} + \ldots$$

Match QCD currents onto operators composed of these field

Decouple fields at level of SCET Lagrangian using BPS field redefinition

Factorization: cross section written in terms of matrix elements of different fields

Resummation: SCET RGE for each term in factorization theorem,
logs minimized at one scale

Glauber Modes: forward scattering of opposite collinear modes,
factorization violation, often neglected in SCET analyses

I. Z. Rothstein, I. W. Stewart, JHEP **1608** (2016) 025

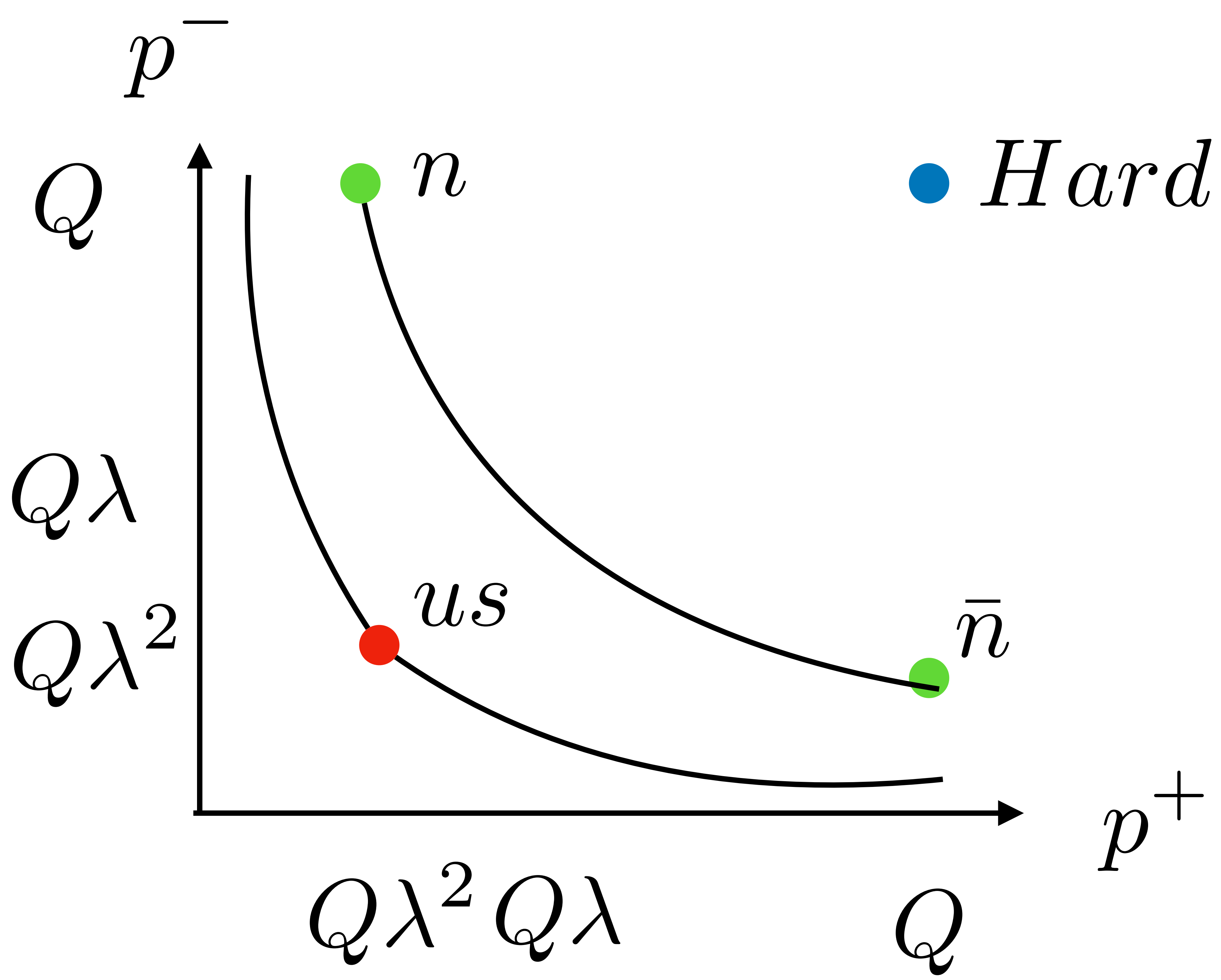
Soft Collinear Effective Theory (SCET$_\mathrm{I}$)

$$(p^+, p^-, p_\perp)$$

$$n \sim Q(1, \lambda^2, \lambda)$$

$$\bar{n} \sim Q(\lambda^2, 1, \lambda)$$

$$us \sim Q(\lambda^2, \lambda^2, \lambda^2)$$



$$d\sigma[e^+e^- \to 2\,\mathrm{jets}] = H \times J_n \otimes J_{\bar{n}} \otimes S$$

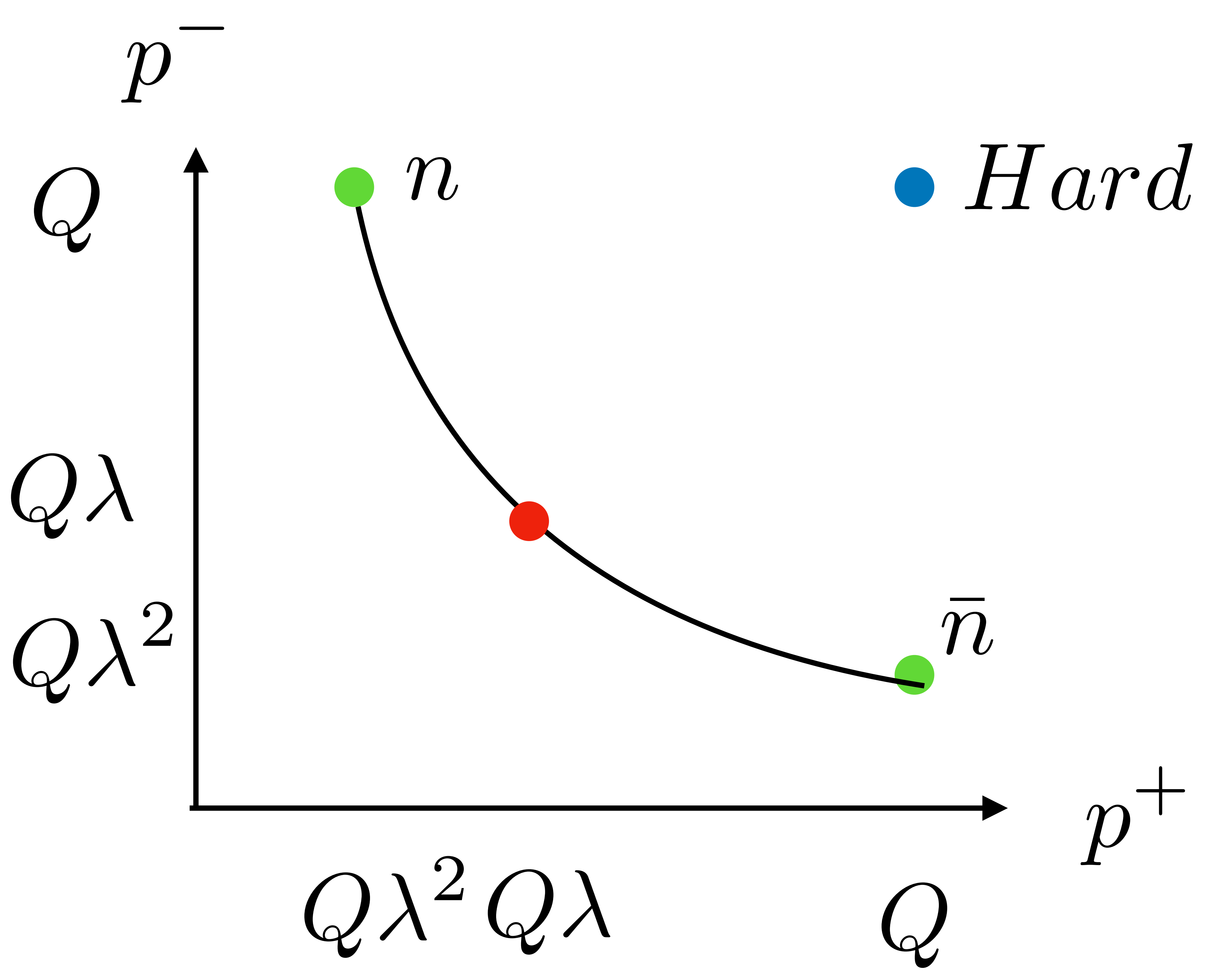
Soft Collinear Effective Theory (SCET$_{II}$)

$$(p^+, p^-, p_\perp)$$

$$n \sim Q(1, \lambda^2, \lambda)$$

$$\bar{n} \sim Q(\lambda^2, 1, \lambda)$$

$$s \sim Q(\lambda, \lambda, \lambda)$$

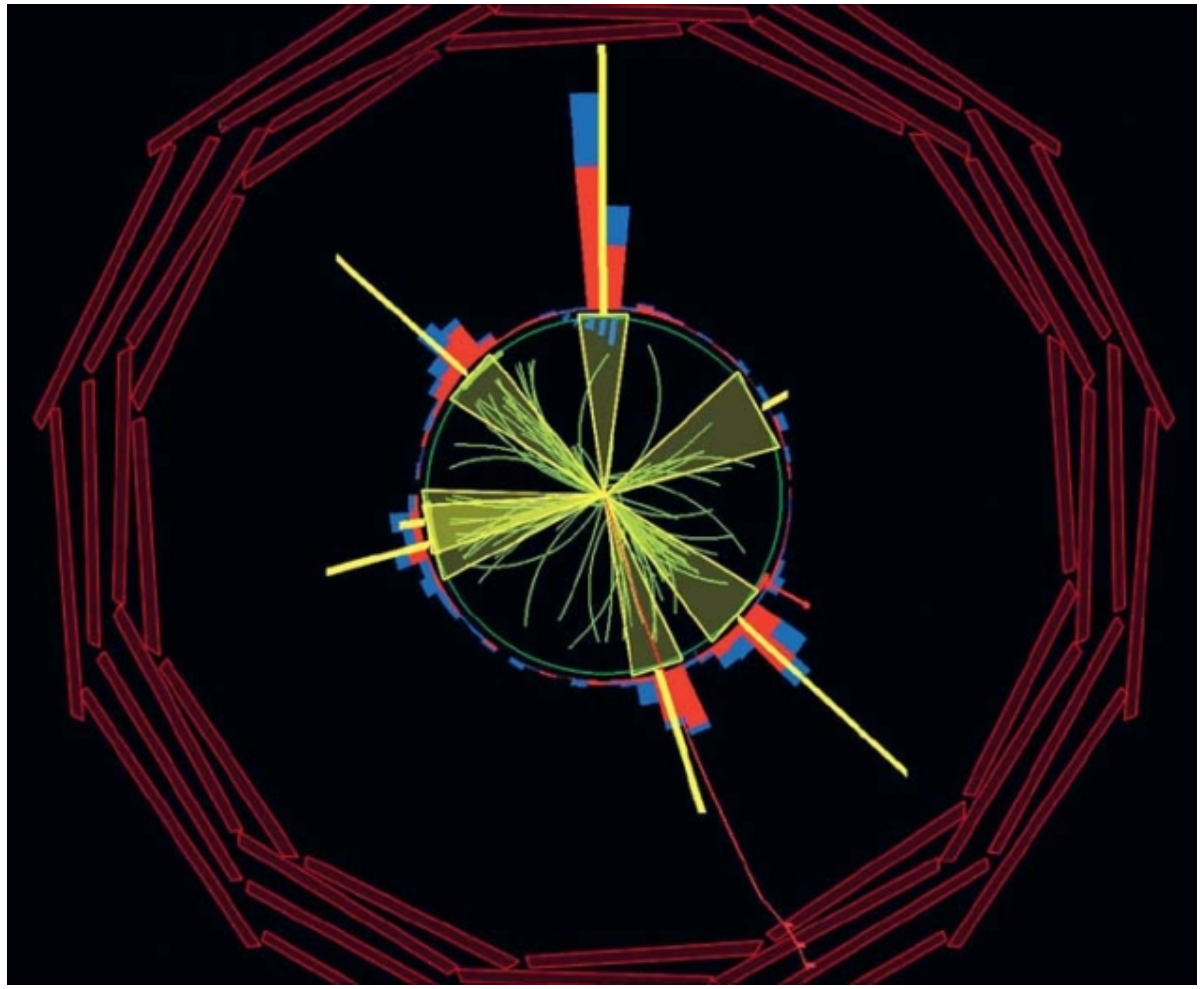


$$\frac{d\sigma[pp \to H + X]}{dp_T^2} = H \times \mathcal{B}^n_{g/p}(p_T) \otimes_T \mathcal{B}^{\bar{n}}_{g/p}(p_T) \otimes_T S(p_T)$$

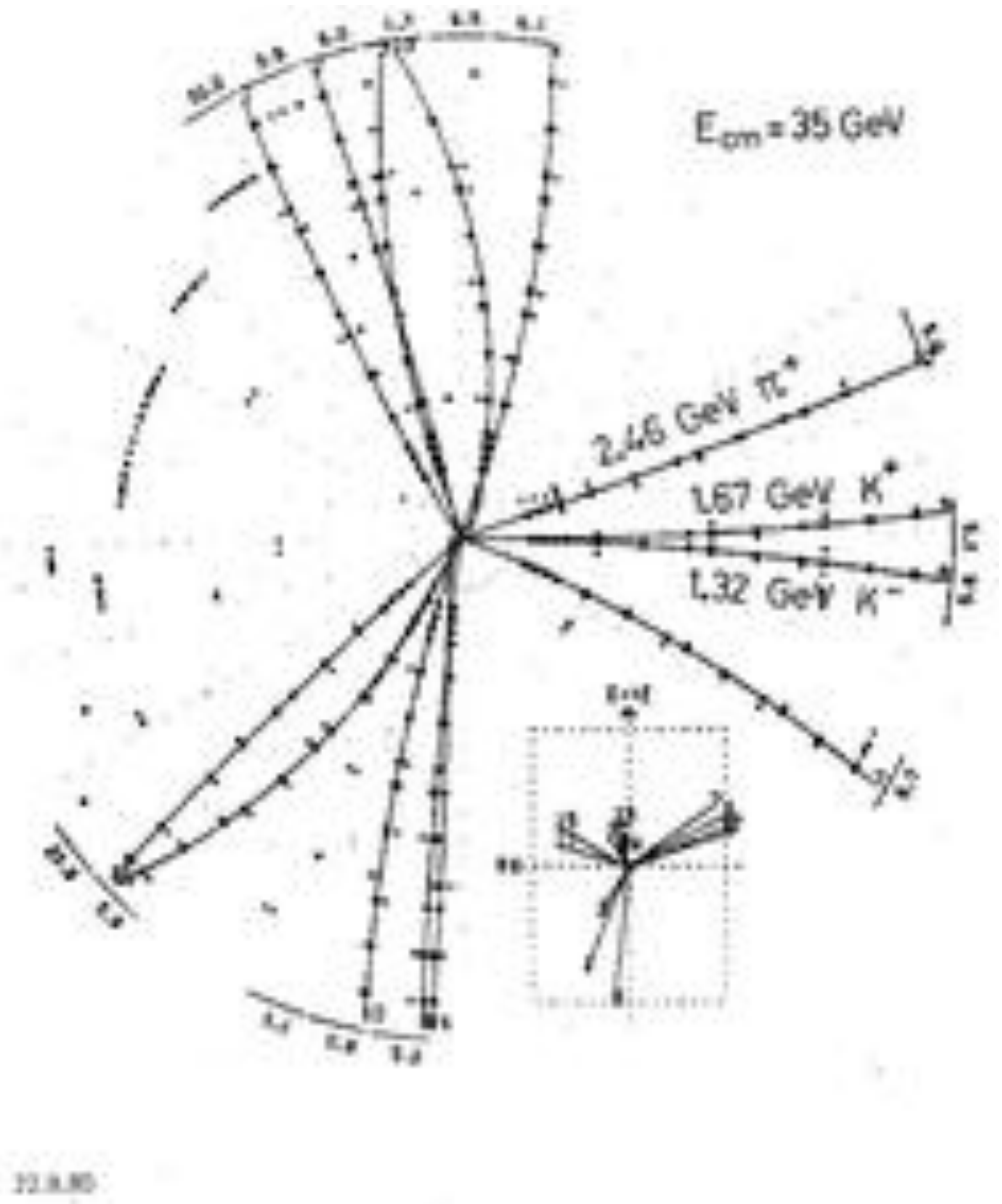
$$\mathcal{B}^n_{g/p} = \sum_i \mathcal{J}_{gi}(z, p_T) \otimes f_{i/p}(z)$$

What is a Jet?

Collimated shower of energetic final state particles



Jets at CMS



3 jet event at DESY
evidence for gluon

Jet Algorithms

sequential clustering algorithms

boost invariant distance measures (hadron collider)

$$d_{ij} = \min(p_{Ti}^{2p}, p_{Tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2},$$
$$d_{iB} = p_{Ti}^{2p},$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

p = 1: k_T algorithm

p = 0: Cambridge-Aachen

p = -1: anti-k_T

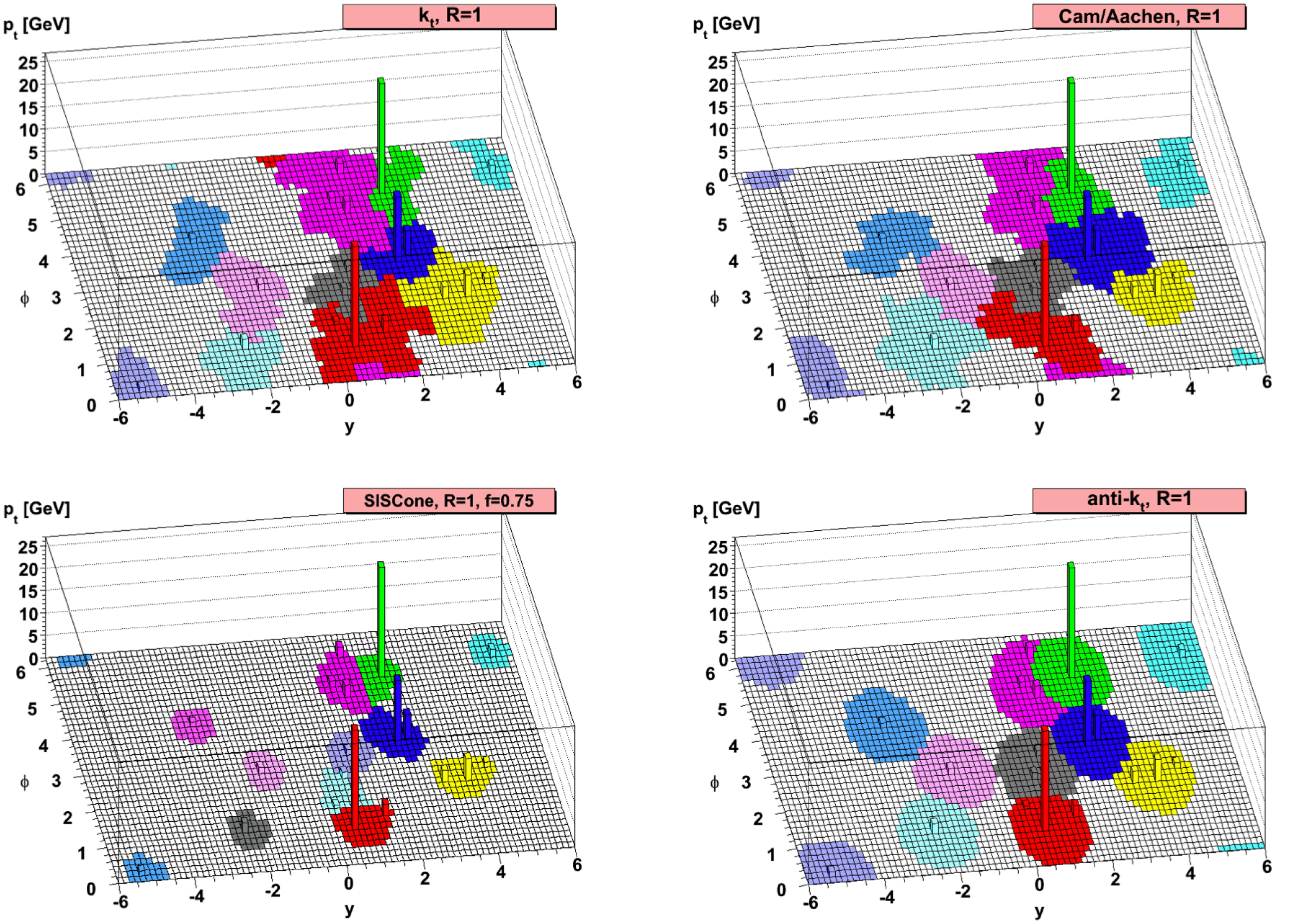
1) calculate d_{ij}, d_{iB} for all i,j

2) if d_{ij} smallest combine i and j, if d_{iB} smallest object is a jet and removed

3) repeat until every particle has been assigned to a jet

cone algorithms

more commonly used at $e^+ e^-$ colliders



M. Cacciari, G.P. Salam, G. Soyez, JHEP 0804:063,2008, arXiv:0802.1189

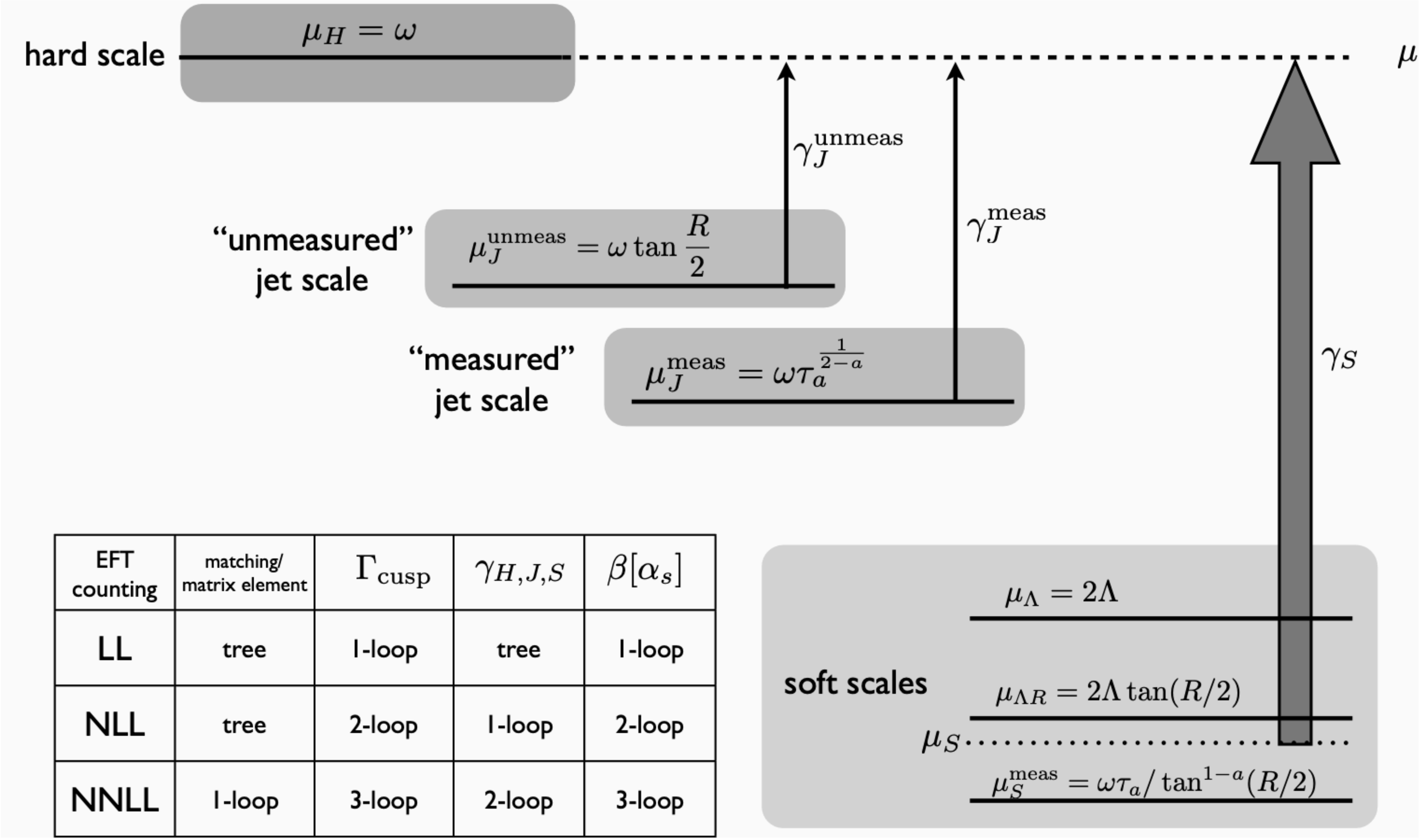
All jets characterized by: ω_J - light cone momentum R - jet radius

or P_T- transverse momentum

can also study jet substructure - functions of jet constituents

angularities $$\tau_a = \frac{1}{\omega}\sum_i (p_i^+)^{1-a/2}(p_i^-)^{a/2}$$ $$\omega = \sum_i p_i^-$$

identified hadrons within jet

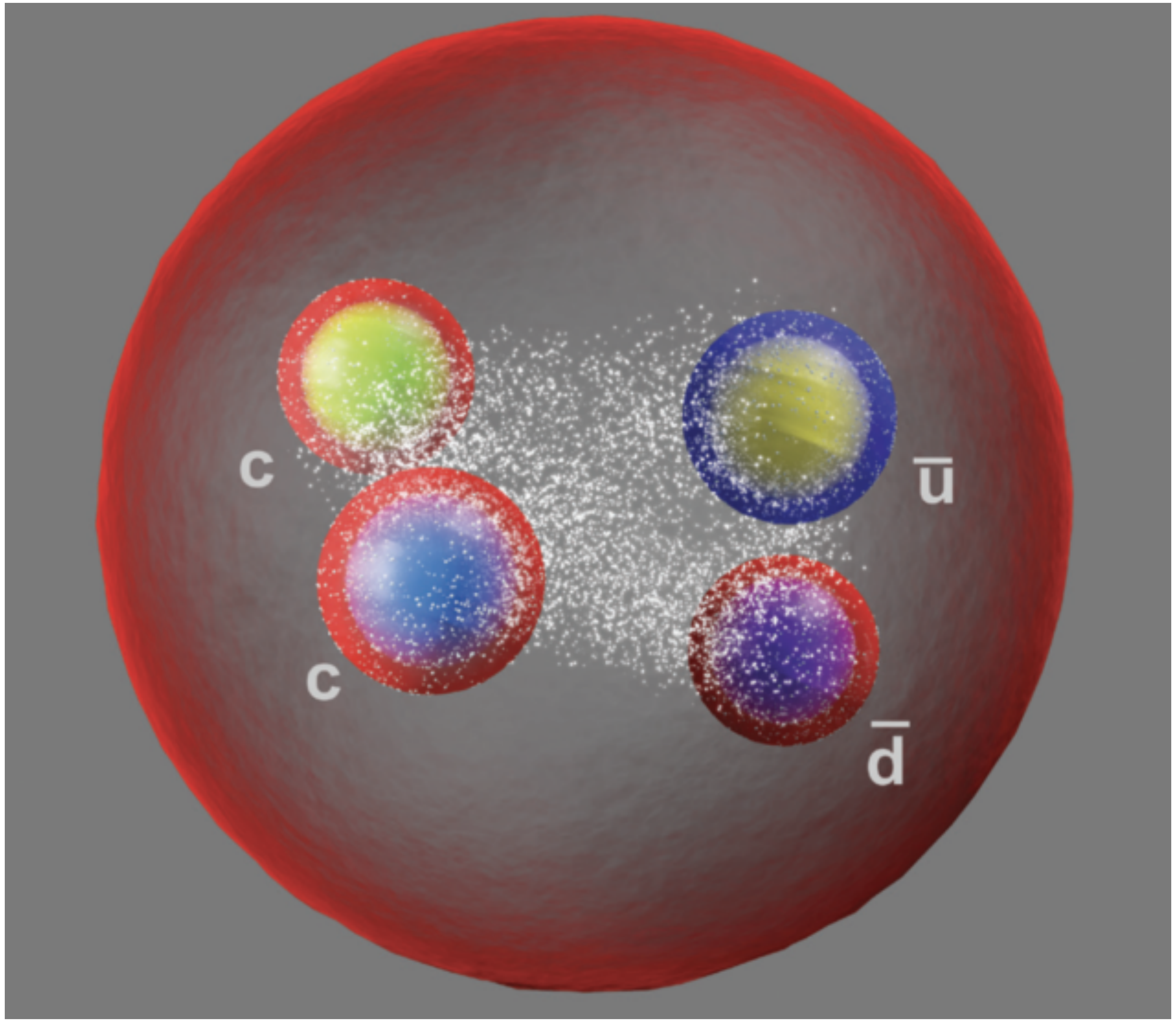


EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

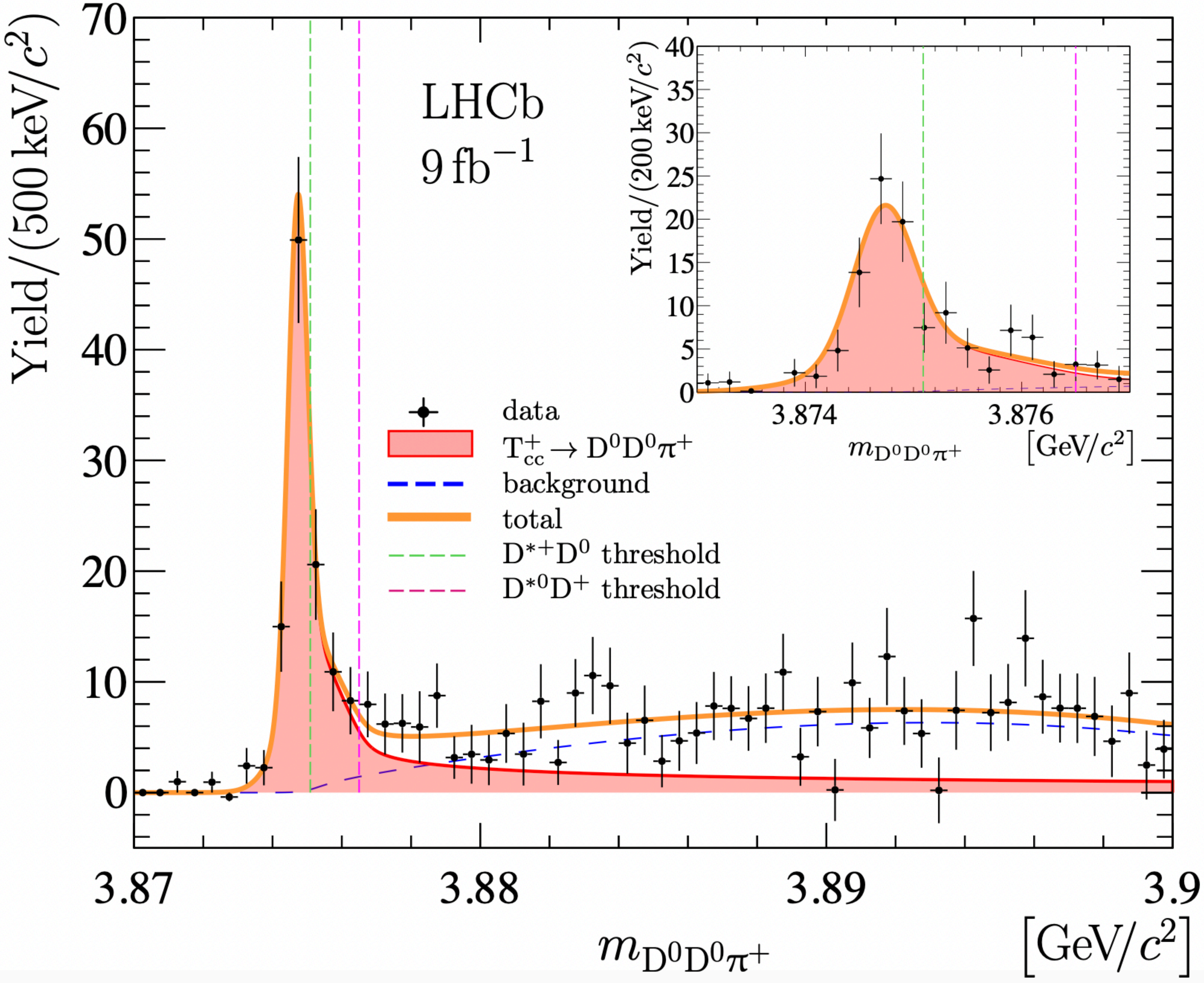
S.D. Ellis, et. al., *JHEP* 11 (2010) 101, arXiv:1001.0014

Applications: T_{cc}^{+}, X(3872) as hadronic molecules

Discovered 7/2021: T_{cc}^{+} doubly charm tetraquark



R. Aaij *et al.* (LHCb), (2021), arXiv:2109.01038 [hep-ex].
R. Aaij *et al.* (LHCb), (2021), arXiv:2109.01056 [hep-ex].



$$\delta m_{\text{pole}} = -360 \pm 40\,^{+4}_{-0}\ \text{keV}/c^2\,,$$
$$\Gamma_{\text{pole}} = 48 \pm 2\,^{+0}_{-14}\,\text{keV}\,,$$

LHCb
9 fb^{-1}
data
$T_{cc}^+ \to D^0D^0\pi^+$
background
total
Yield/(500 keV/c^2)
$m_{D^0D^0}$ [GeV/c^2]
LHCb
9 fb^{-1}
data
$T_{cc}^+ \to D^+D^0\pi^0$
$T_{cc}^+ \to D^+D^0\gamma$
background
total
Yield/(500 keV/c^2)
$m_{D^+D^0}$ [GeV/c^2]

HHChiPt plus contact terms for S-wave D^*D scattering

$$
\begin{aligned}
\mathcal{L} \quad = \quad & H^{*i\dagger}\left(i\partial^0 + \frac{\nabla^2}{2m_{H^*}} - \delta^*\right)H^{*i} + H^\dagger\left(i\partial^0 + \frac{\nabla^2}{2m_H} - \delta\right)H \\
& + \frac{g}{f_\pi} H^\dagger \partial^i \pi H^{*i} + \text{h.c.} + \frac{1}{2} H^\dagger \mu_D \vec{B}^i H^{*i} + \text{h.c.} \\
& - C_0 (H^{*T}\tau_2 H)^\dagger (H^{*T}\tau_2 H) - C_1 (H^{*T}\tau_2\tau_a H)^\dagger (H^{*T}\tau_2\tau_a H)\,.
\end{aligned}
$$

$$
H = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}, \quad H^{*i} = \begin{pmatrix} D^{*0i} \\ D^{*+i} \end{pmatrix}
$$

S. Fleming, R. Hodges, TM, Phys. Rev. D. 104 (2021) 11, 116010, arXiv:2109.02188

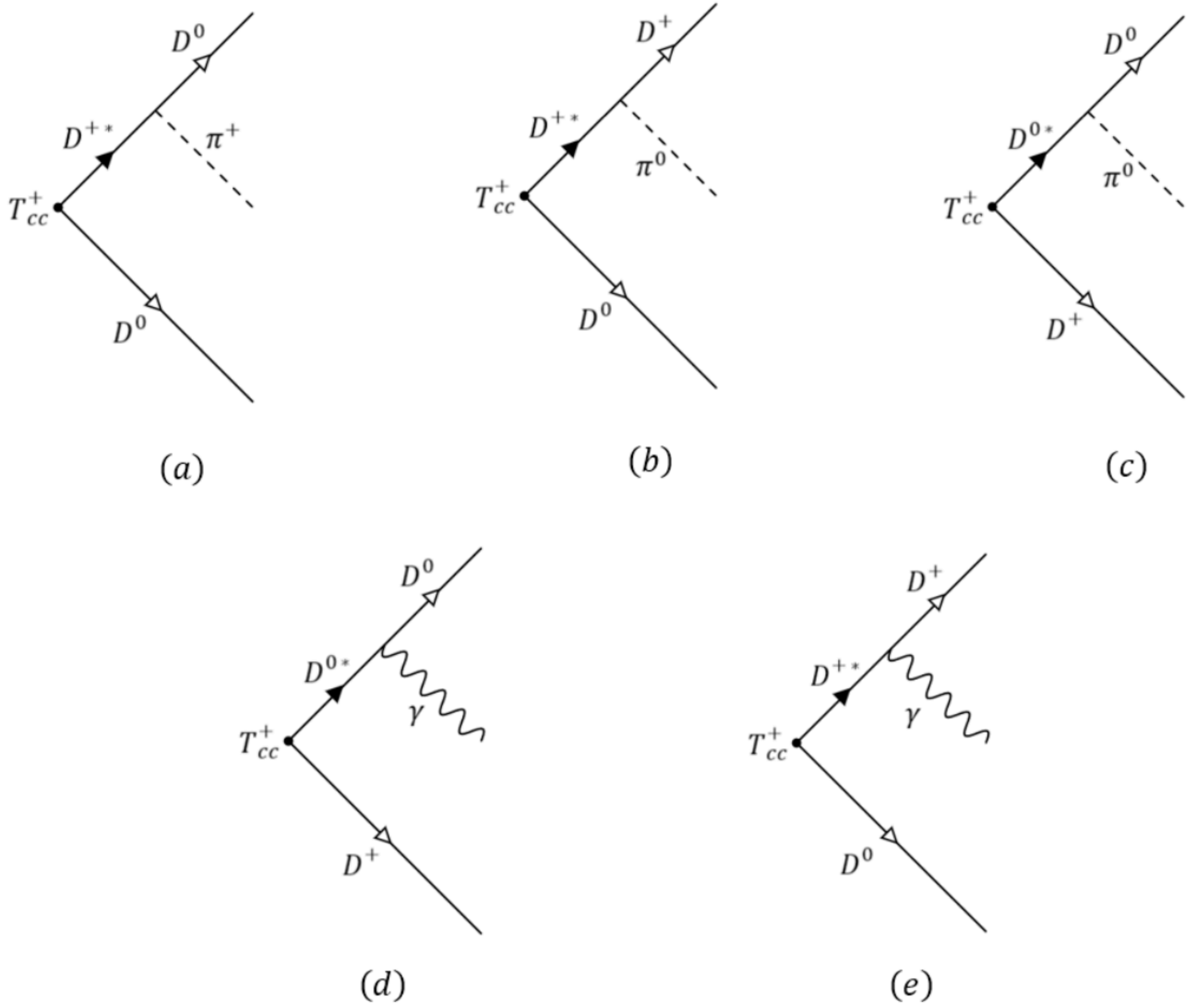
T-Matrix for D^*D scattering

$$T = \frac{1}{E + E_T} \begin{pmatrix} g_0^2 & g_0 g_1 \\ g_0 g_1 & g_1^2 \end{pmatrix}$$

$$g_0^2 \Sigma_0'(-E_T) + g_+^2 \Sigma_+'(-E_T) = 1 ,$$

Tune interactions to produce pole at T_{cc}^+

Decay Diagrams



Decay rate formulae

$$\frac{d\Gamma[T_{cc}^{+} \to D^0 D^0 \pi^+]}{dp^2_{D^0_1} dp^2_{D^0_2}} = c_\theta^2 \frac{g^2}{(4\pi f_\pi)^2} \frac{2\gamma_0 p_\pi^2}{3} \left[\frac{1}{p^2_{D^0_1} + \gamma_0^2} + \frac{1}{p^2_{D^0_2} + \gamma_0^2} \right]^2 ,$$

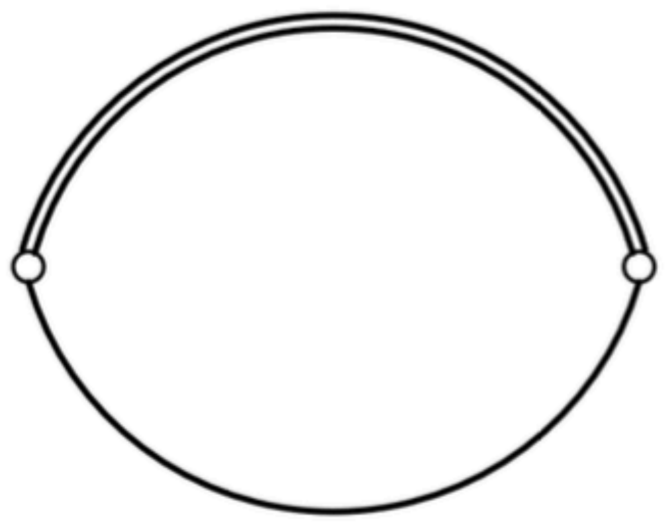
$$\frac{d\Gamma[T_{cc}^{+} \to D^+ D^0 \pi^0]}{dp^2_{D^+} dp^2_{D^0}} = \frac{g^2}{(4\pi f_\pi)^2} \frac{2p_\pi^2}{3} \left[\frac{\sqrt{\gamma_0}\, c_\theta}{p^2_{D^+} + \gamma_0^2} - \frac{\sqrt{\gamma_+}\, s_\theta}{p^2_{D^0} + \gamma_+^2} \right]^2 ,$$

$$\frac{d\Gamma[T_{cc}^{+} \to D^+ D^0 \gamma]}{dp^2_{D^+} dp^2_{D^0}} = \frac{E_\gamma^2}{6\pi^2} \left[\frac{\sqrt{\gamma_0}\, c_\theta \mu_{D^0}}{p^2_{D^+} + \gamma_0^2} - \frac{\sqrt{\gamma_+}\, s_\theta \mu_{D^+}}{p^2_{D^0} + \gamma_+^2} \right]^2 .$$

$$\gamma_0^2 = 2\mu_0(m_{D_0} + m_{D^{*+}} - m_T)$$
$$\gamma_+^2 = 2\mu_0(m_{D_+} + m_{D^{*0}} - m_T) .$$

$$g_0^2 = \frac{\cos^2\theta}{\Sigma_0'(-E_T)} \qquad g_+^2 = \frac{\sin^2\theta}{\Sigma_+'(-E_T)} ;$$

$$i\Sigma_i(E) =$$



Predictions for Decay Rate

	I=0	I=1	Γ_{max}
θ	$-32.4°$	$32.4°$	$-8.34°$
$\Gamma[T_{cc}^{+} \to D^0 D^0 \pi^+]$	32	32	44
$\Gamma[T_{cc}^{+} \to D^+ D^0 \pi^0]$	15	3.8	13
$\Gamma[T_{cc}^{+} \to D^+ D^0 \gamma]$	6.1	2.8	1.9
$\Gamma[T_{cc}^{+}]$	52	38	58

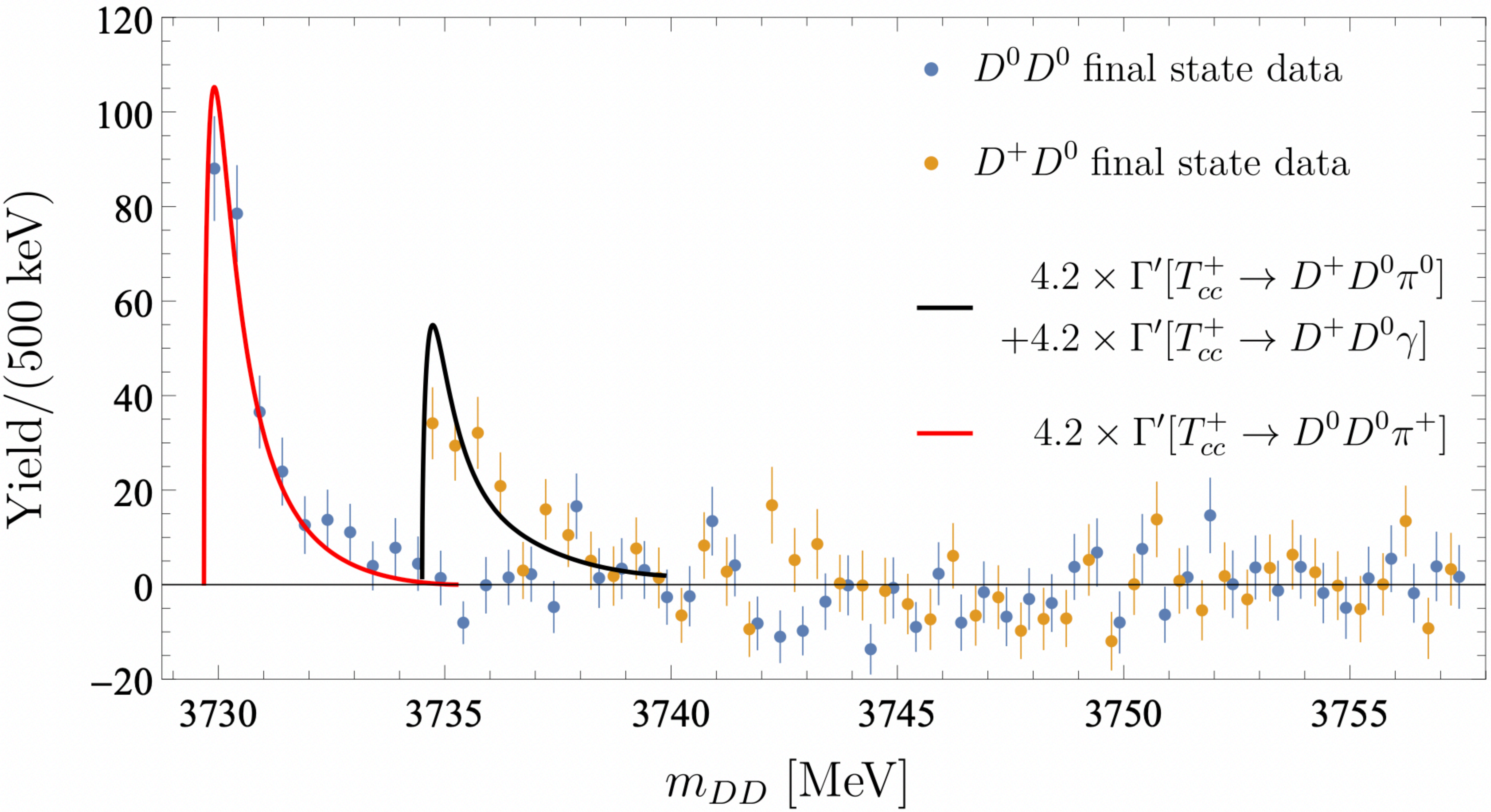
$$I = 0 : g_0 = -g_1$$

$$I = 1 : g_0 = g_1$$

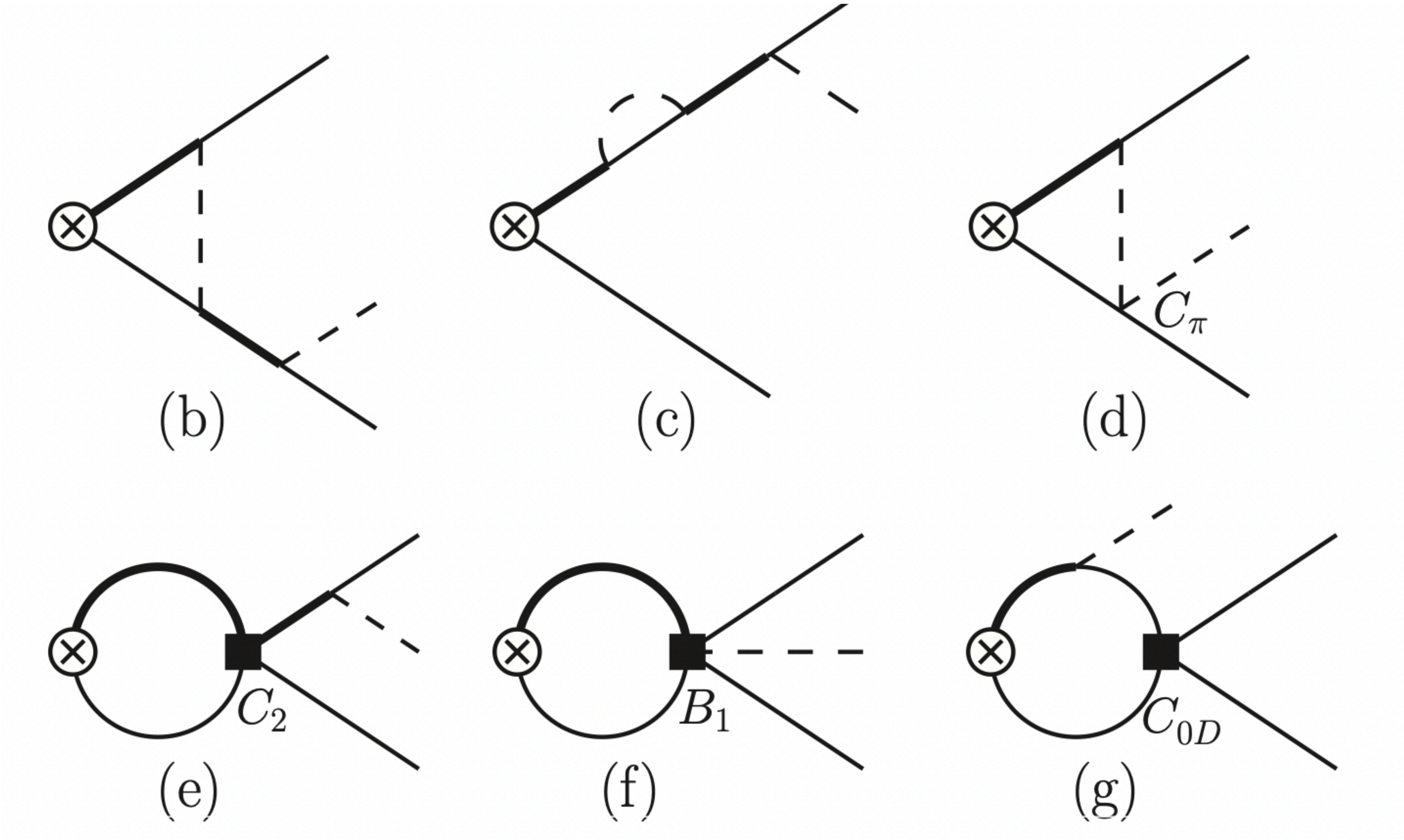
Other Predictions

	$\Gamma(T_{cc}^{+})$ (keV)
Fleming et al.	**52**
Meng et al.	$46.7^{+2.7}_{-2.9}$
Ling et al.	53
Feijoo et al.	43
Yan & Valderrama	49 ± 16
Albaladejo	77

$d\Gamma/dm_{DD}$ vs. data

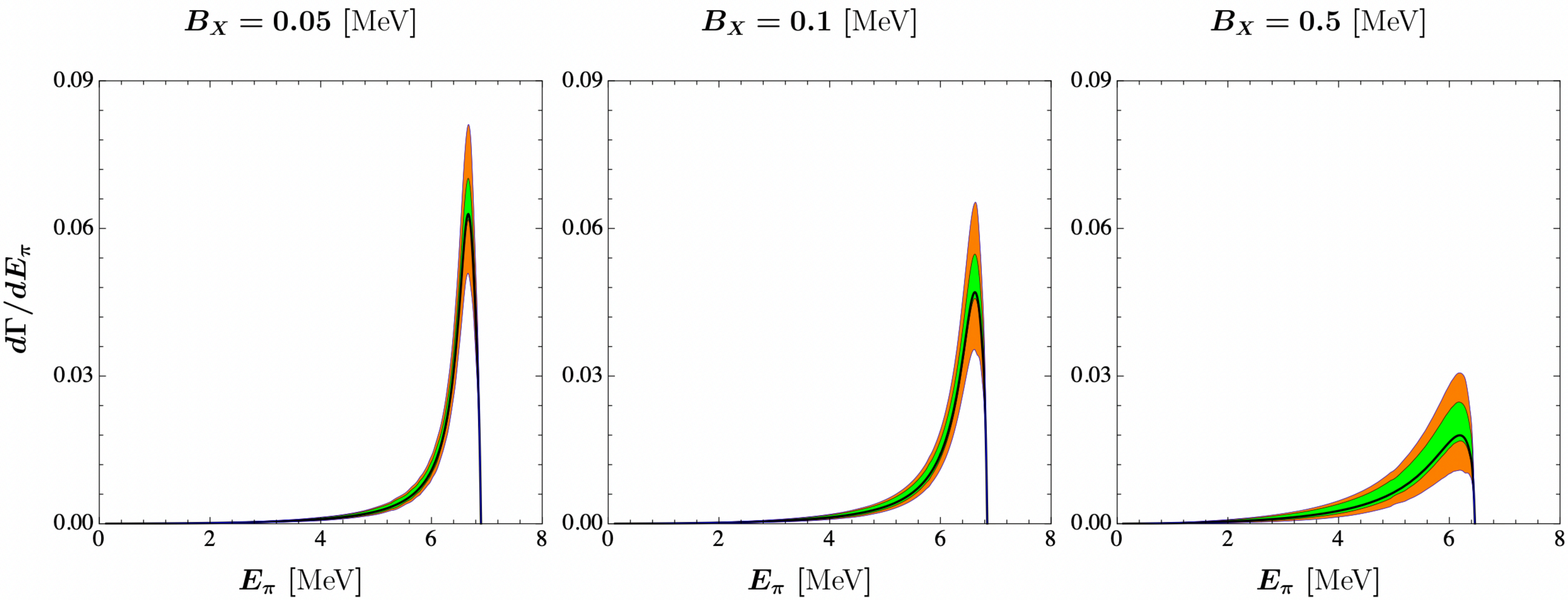


NLO Calculation for $X(3872) \to D^0 \bar{D}^0 \pi^0$



$\pi^0 D^0, \bar{D}^0 D^0$ rescattering effects

L. Dai, F.-K. Guo, TM, Phys. Rev. D. 101 (2020) 5, 054024, arXiv:1912.04317



$B_X = 0.05$ [MeV]
$B_X = 0.1$ [MeV]
$B_X = 0.5$ [MeV]
$d\Gamma/dE_\pi$
0.09
0.06
0.03
0.00
0
2
4
6
8
E_π [MeV]

Applications: Quarkonia Production in Jets

Color-Singlet Model (pre-1995)

$$\sigma(pp \to J/\psi + X) = f_{g/p} \otimes f_{g/p} \otimes \sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]\, |\psi_{c\bar{c}}(0)|^2$$

$\bar{c}c$ pair produced with same quantum numbers as J/ψ

Predictive Formalism

$\sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]$ calculable in QCD perturbation theory

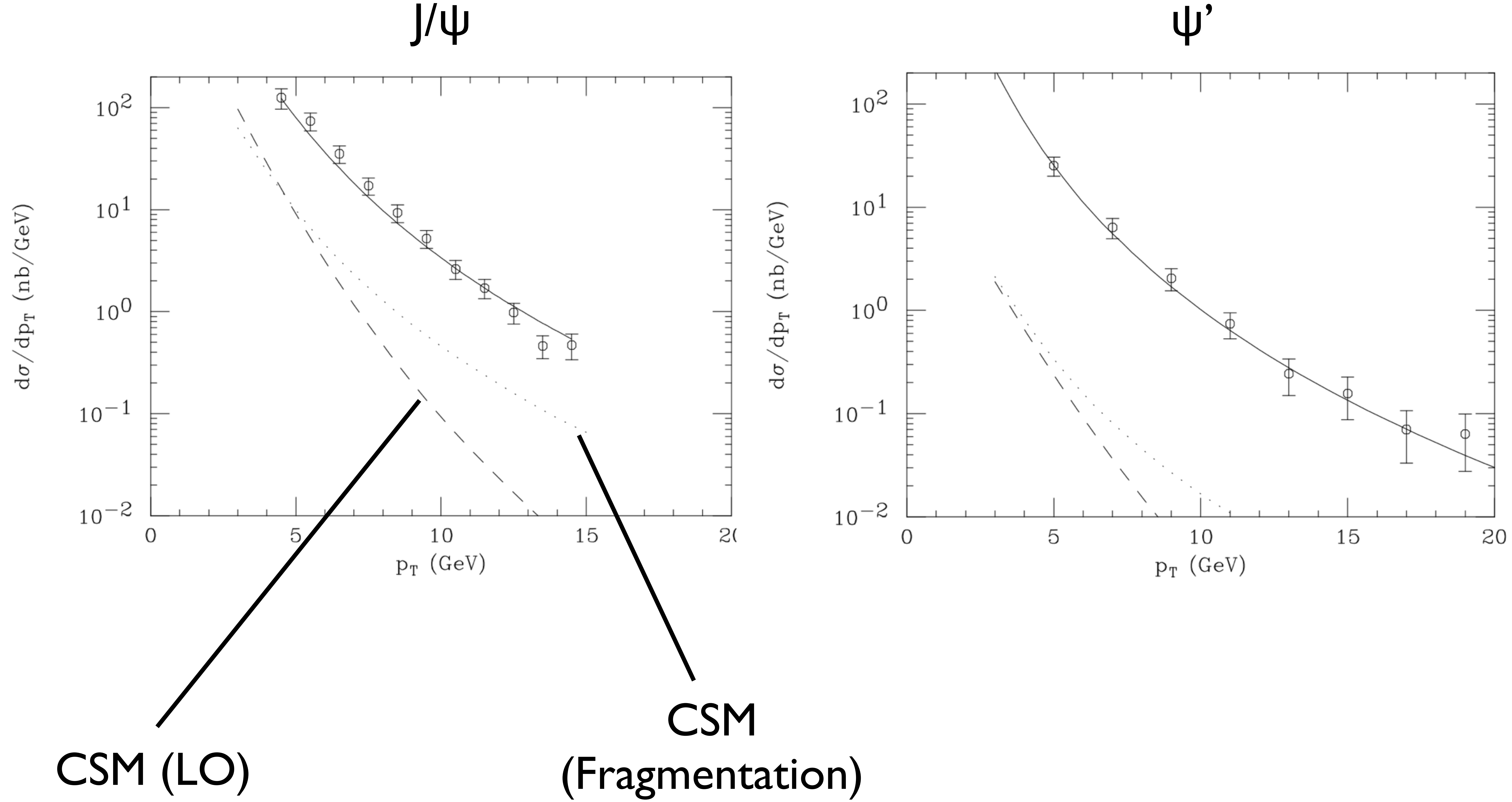
$|\psi_{c\bar{c}}(0)|^2$ fixed by $\Gamma[J/\psi \to \ell^+\ell^-]$

Suffers from theoretical inconsistencies when applied to χ_{cJ}

$\Gamma[\chi_{cJ} \to \text{hadrons}] = |\psi'_{c\bar{c}}(0)|^2 \sigma(c\bar{c}(^3P_J^{(1)}) \to gg)$ ⟵ **Not IR Safe**

J/ψ production at Tevatron (1996)

CSM badly underpredicts J/ψ and ψ’ production at large p_T



Non-Relativistic QCD (NRQCD) Factorization Formalism

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

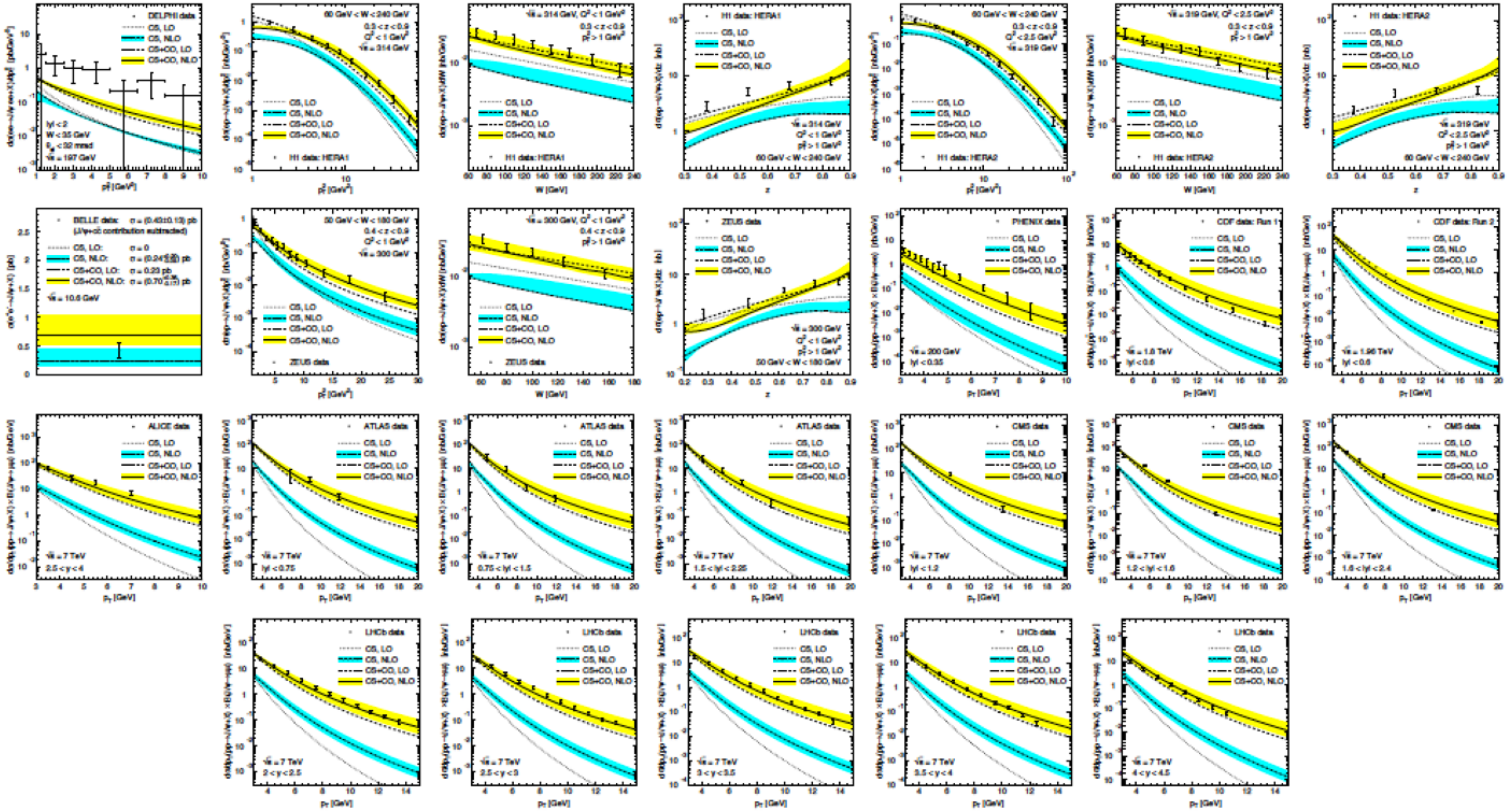
CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

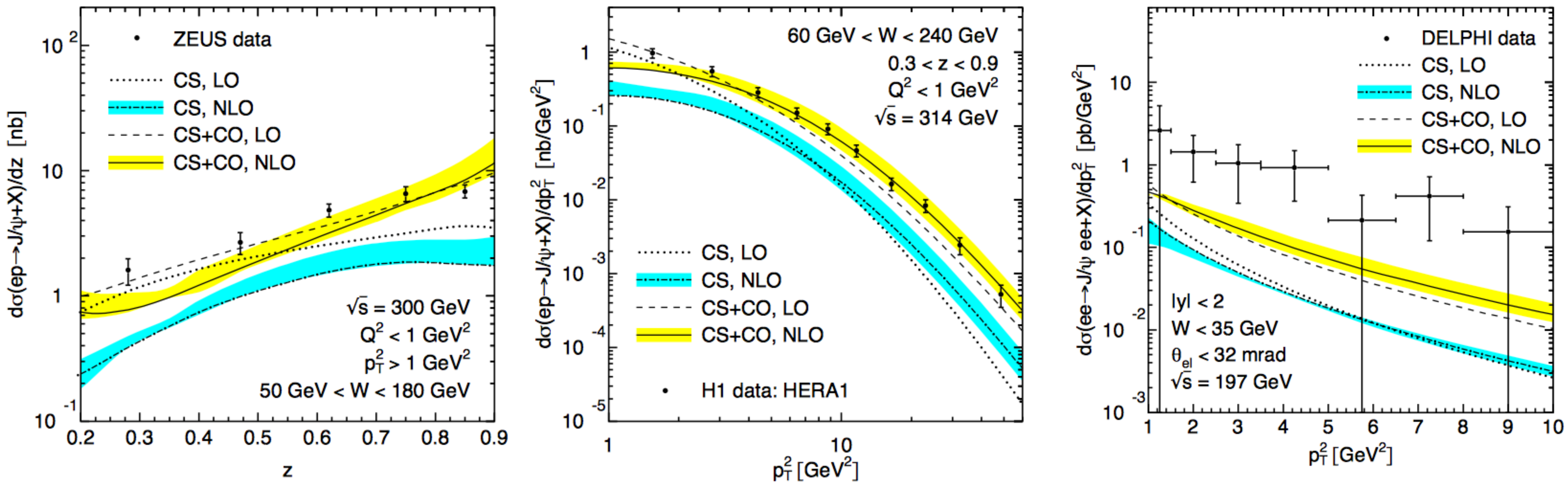
Global Fits with NLO CSM + COM

Butenschoen and Kniehl, PRD 84 (2011) 051501



$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$ fit to 194 data points, 26 data sets

NLO: CSM + COM Required to Fit Data



$$ep \to J/\psi + X \qquad\qquad \gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

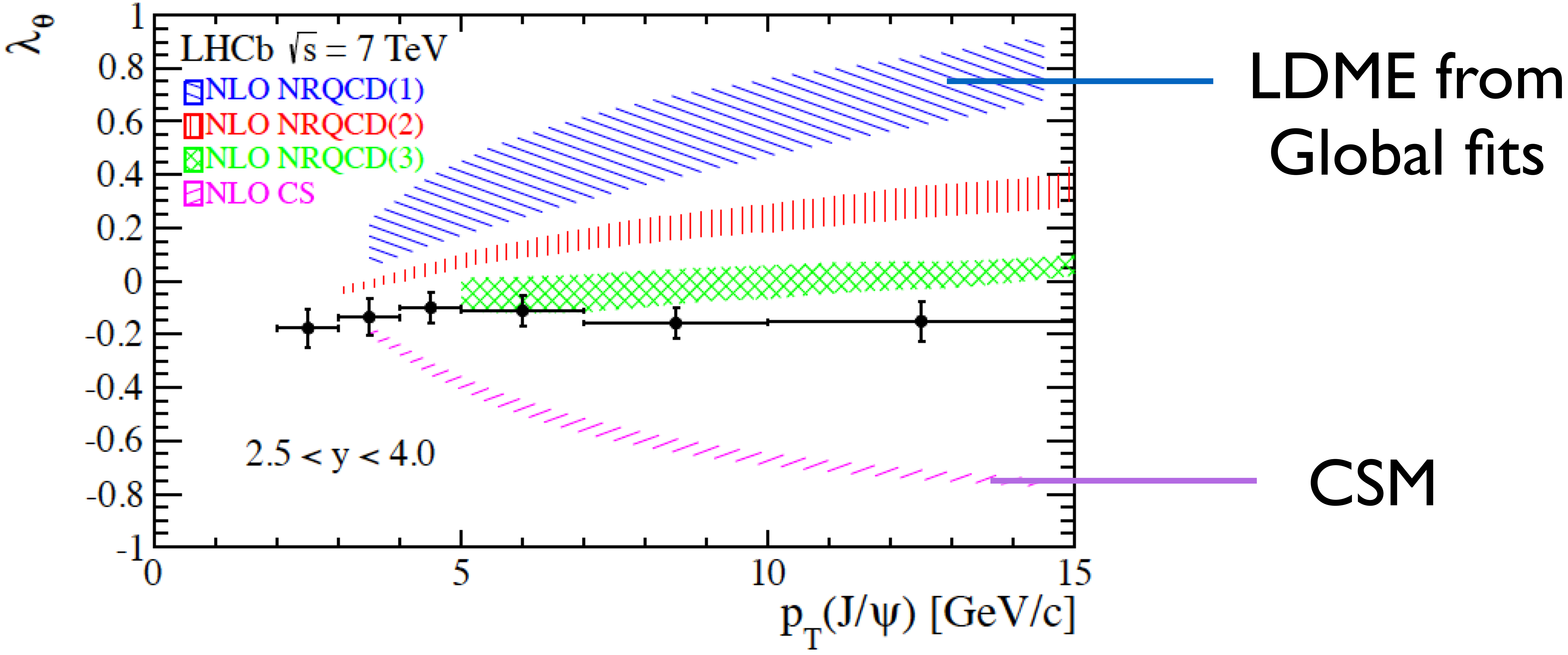
extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle = 1.32 \text{ GeV}^3$$

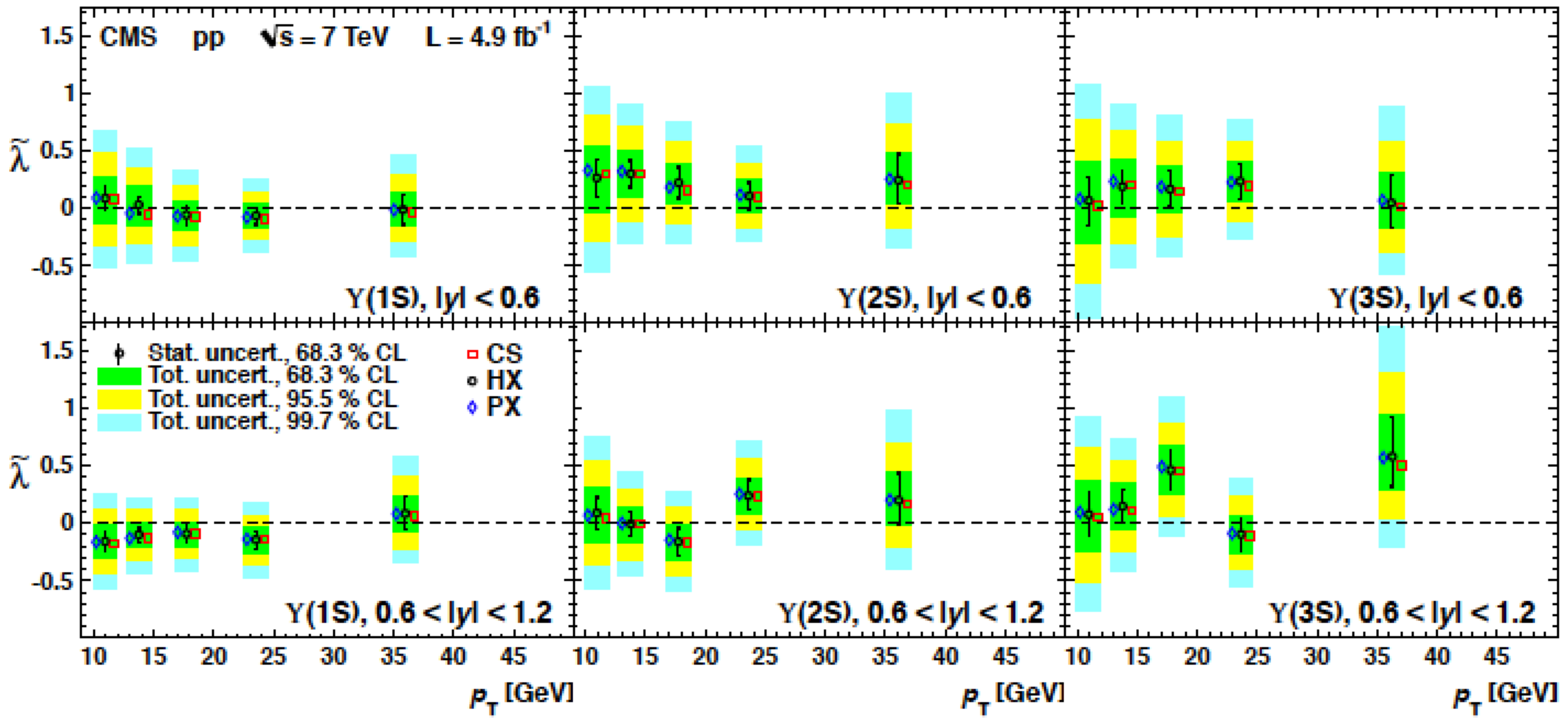
$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV^3
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV^5

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

Polarization of J/ψ at LHCb



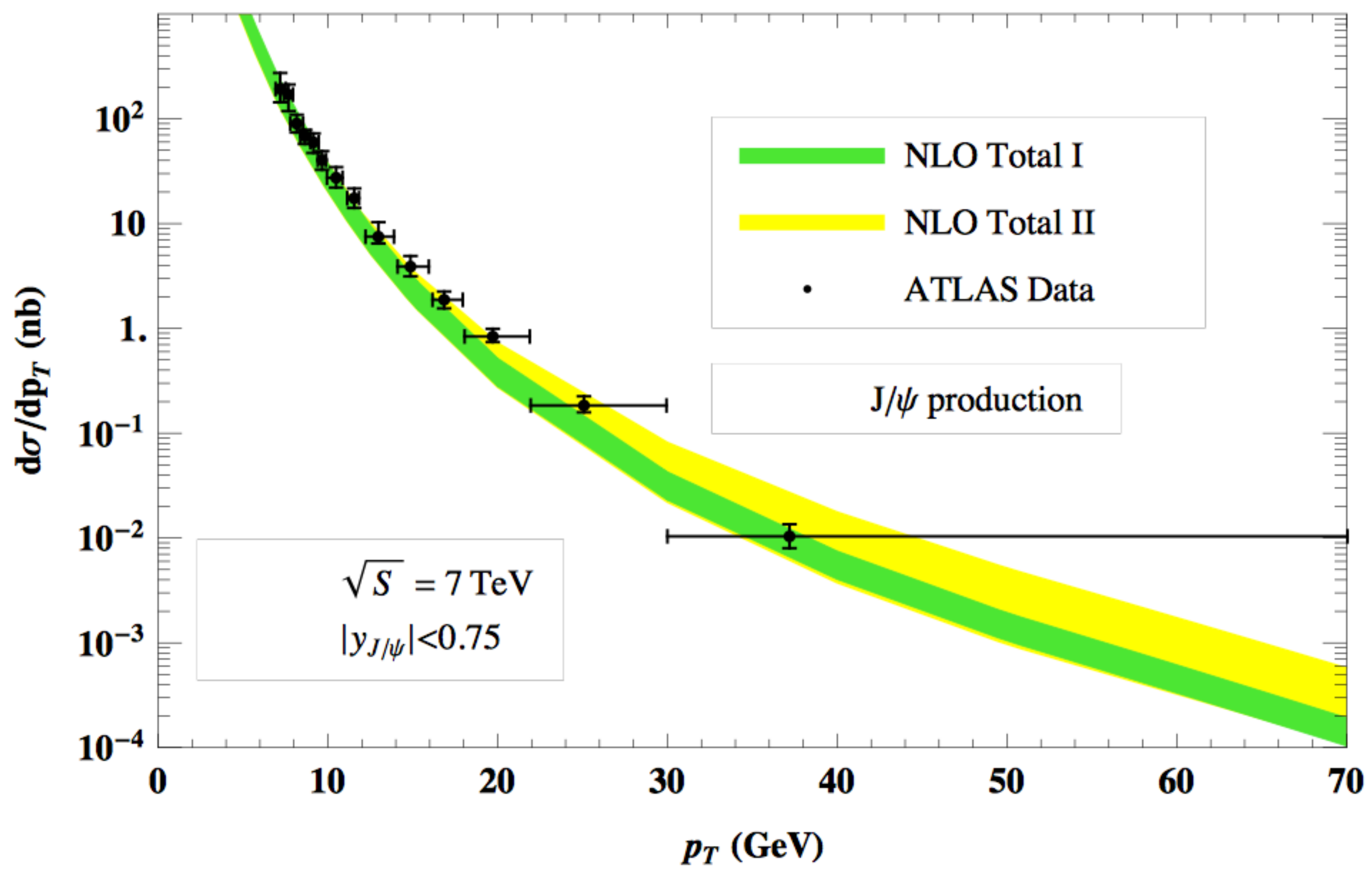
Polarization of Υ(nS) at CMS

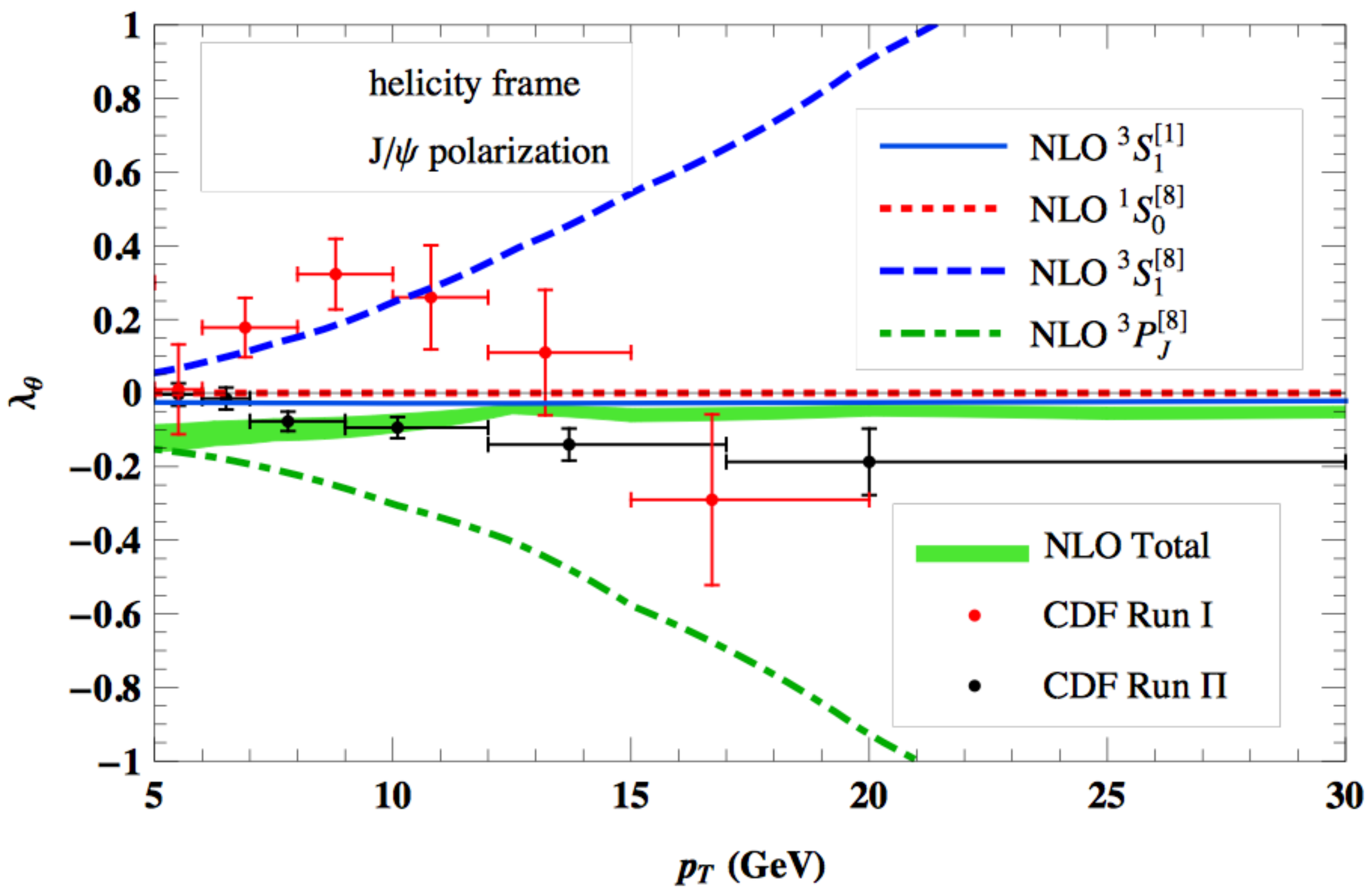


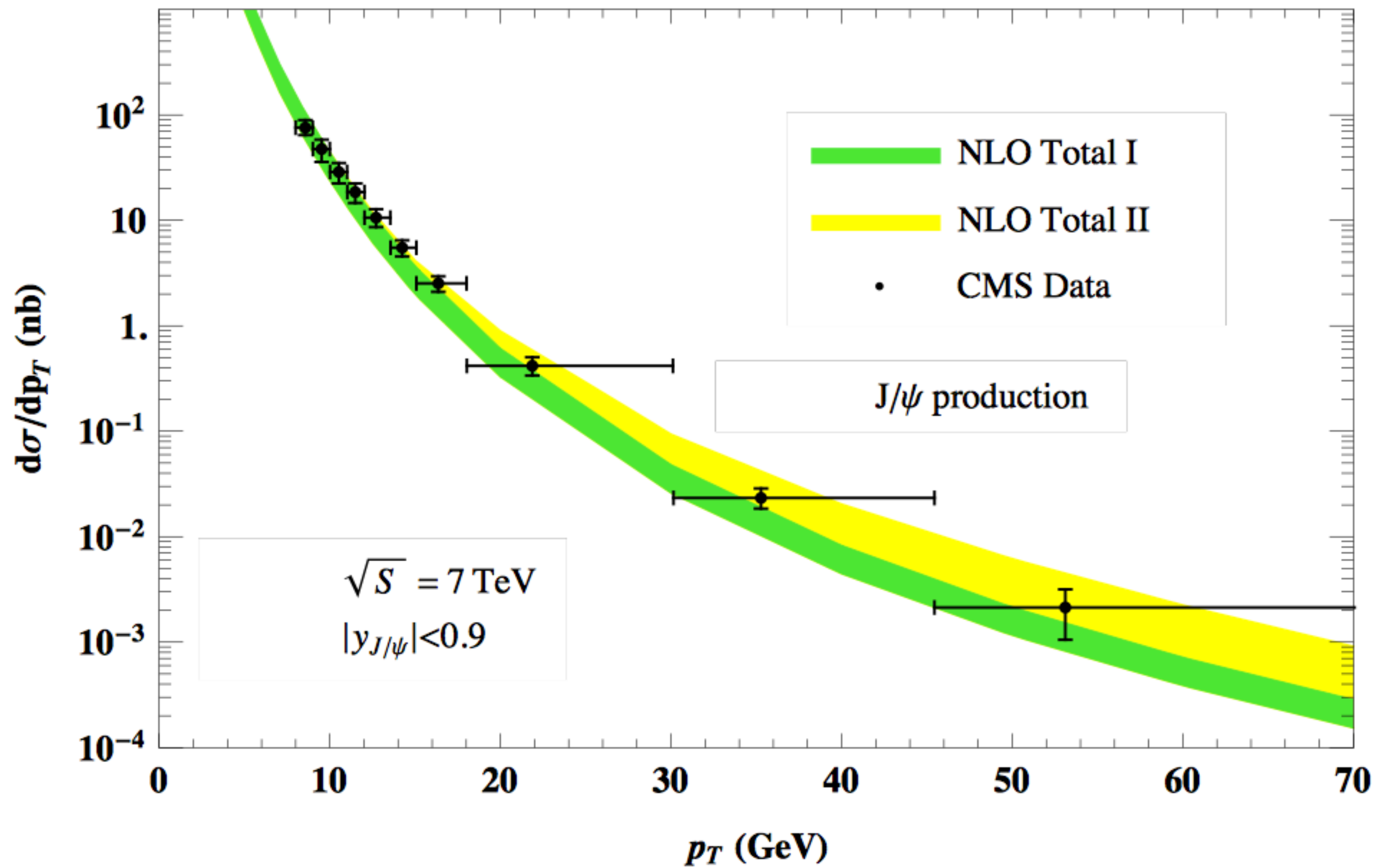
Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_t production, polarization

Chao, et. al. PRL 108, 242004 (2012)







$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle/m_c^2$ 10^{-2}GeV^3
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

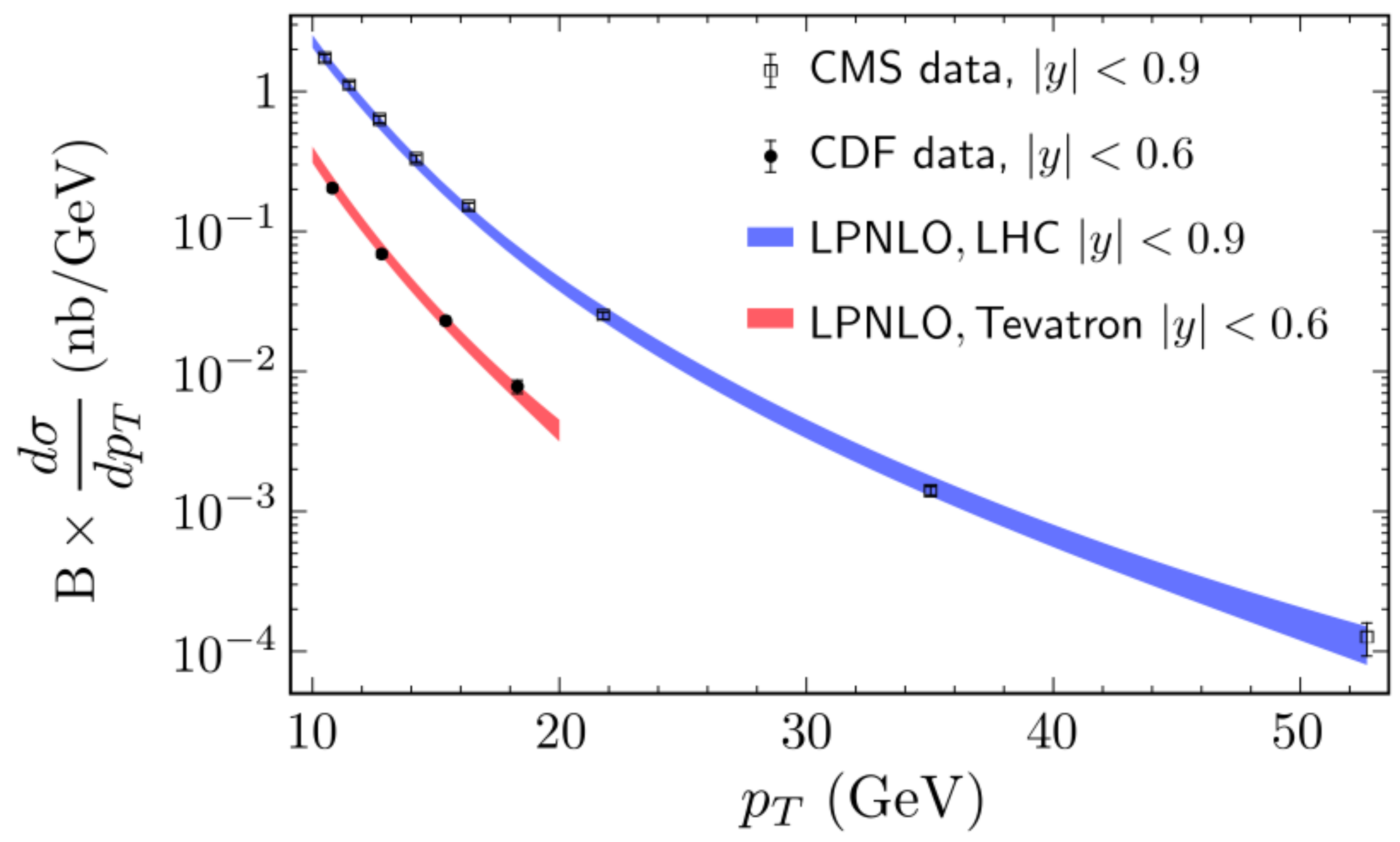
Recent Attempts to Resolve J/ψ Polarization Puzzle

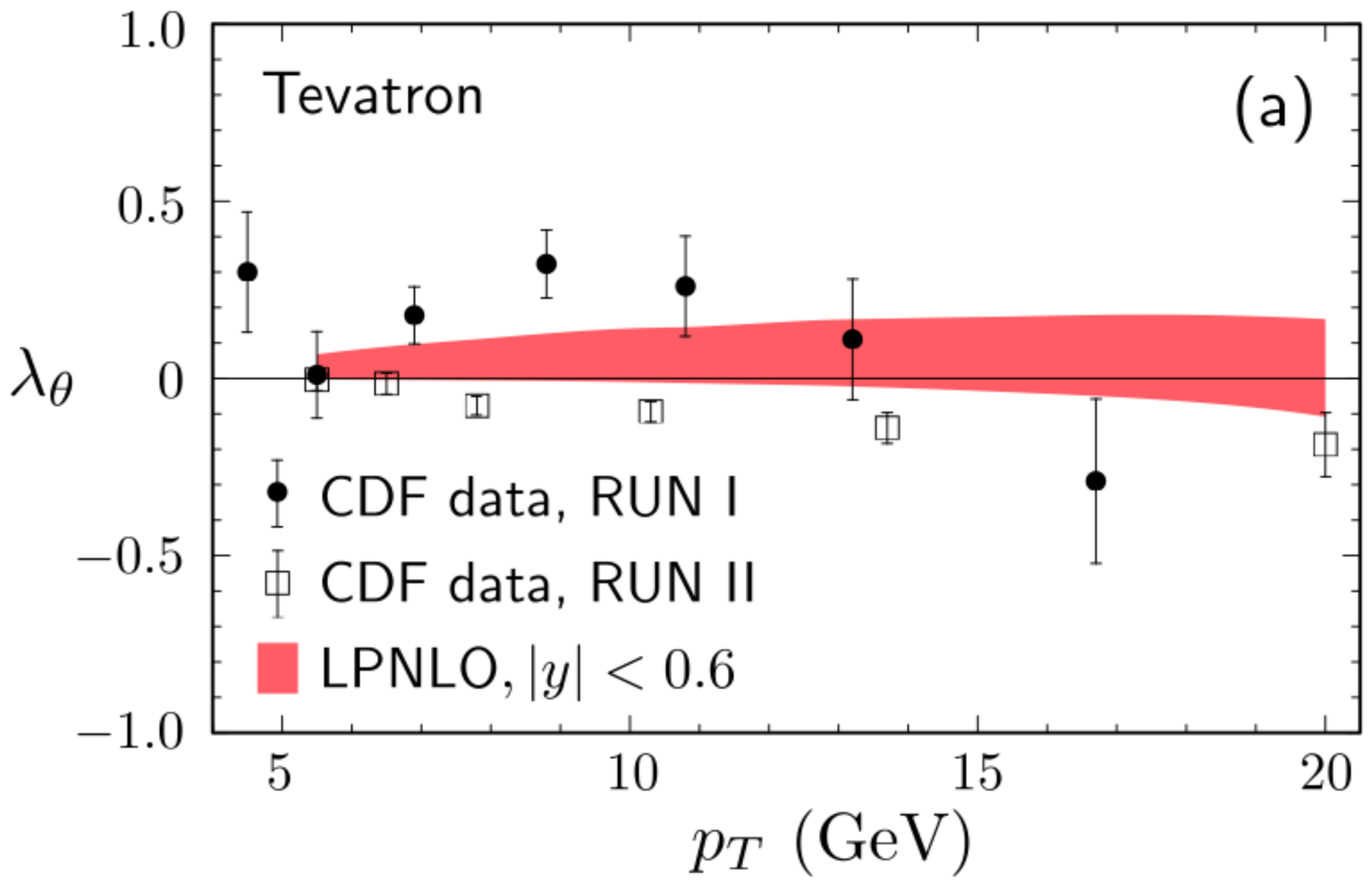
i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001(2014)

ii) resum logs of p_t/m_c using DGLAP evolution

iii) fit COME to p_t spectrum, predict basically no polarization





Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)})\rangle = 0.099 \pm 0.022\,\mathrm{GeV}^3$$

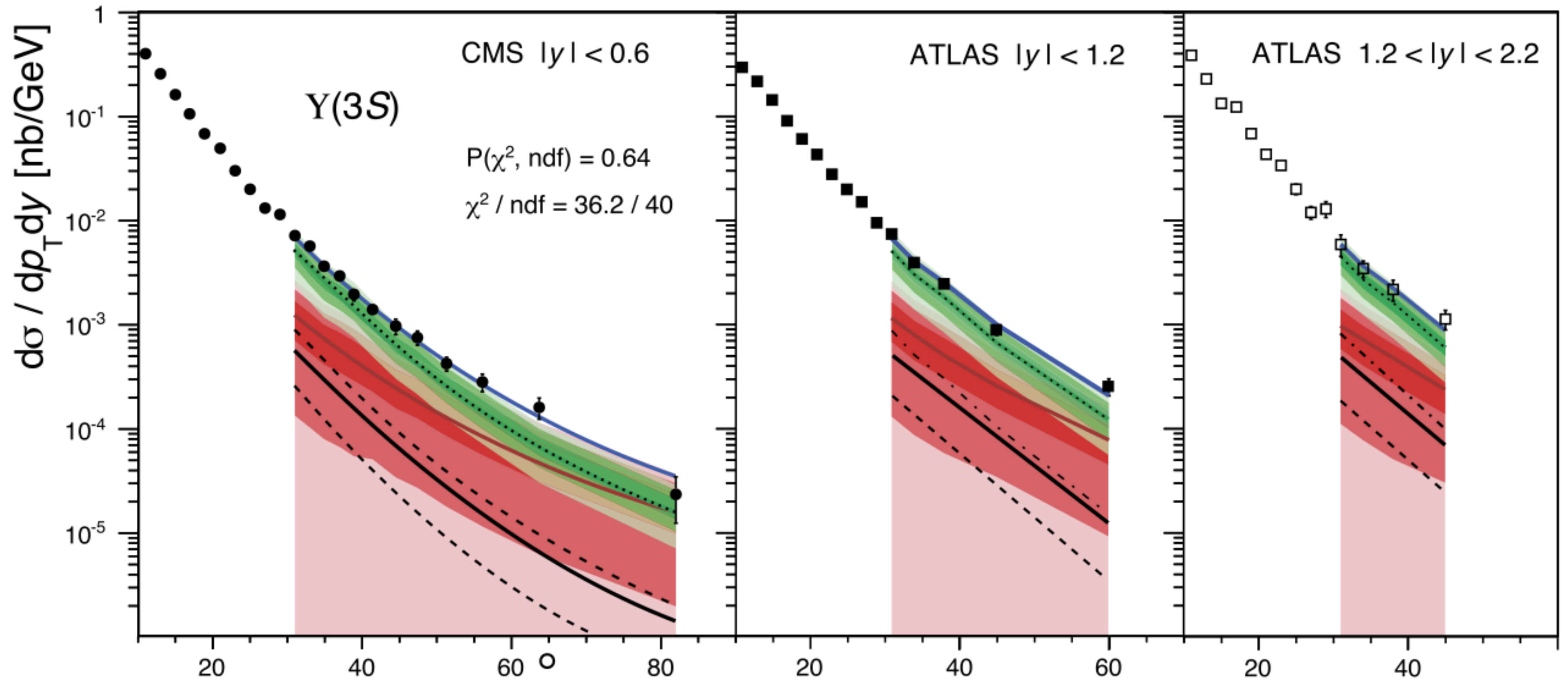
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(8)})\rangle = 0.011 \pm 0.010\,\mathrm{GeV}^3$$

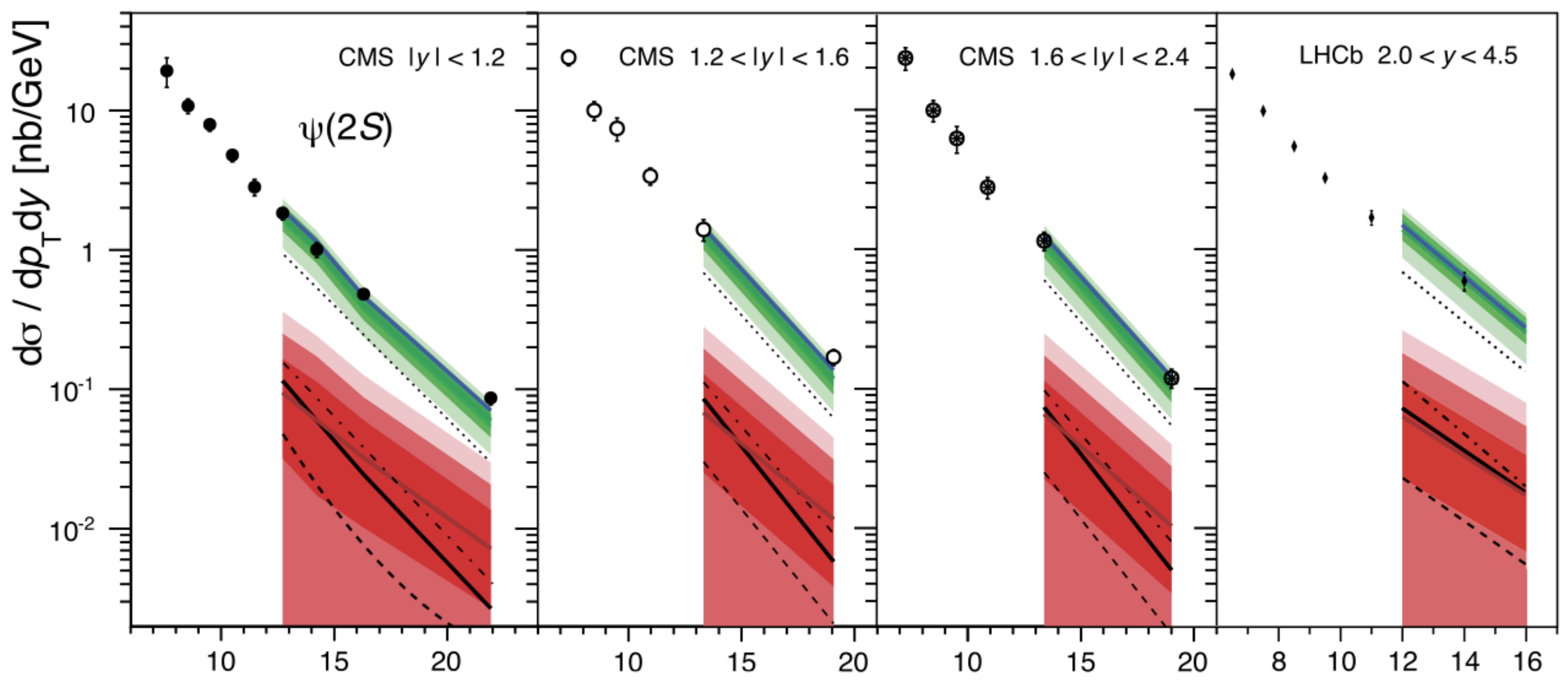
$$\langle \mathcal{O}^{J/\psi}(^3P_0^{(8)})\rangle = 0.011 \pm 0.010\,\mathrm{GeV}^5$$

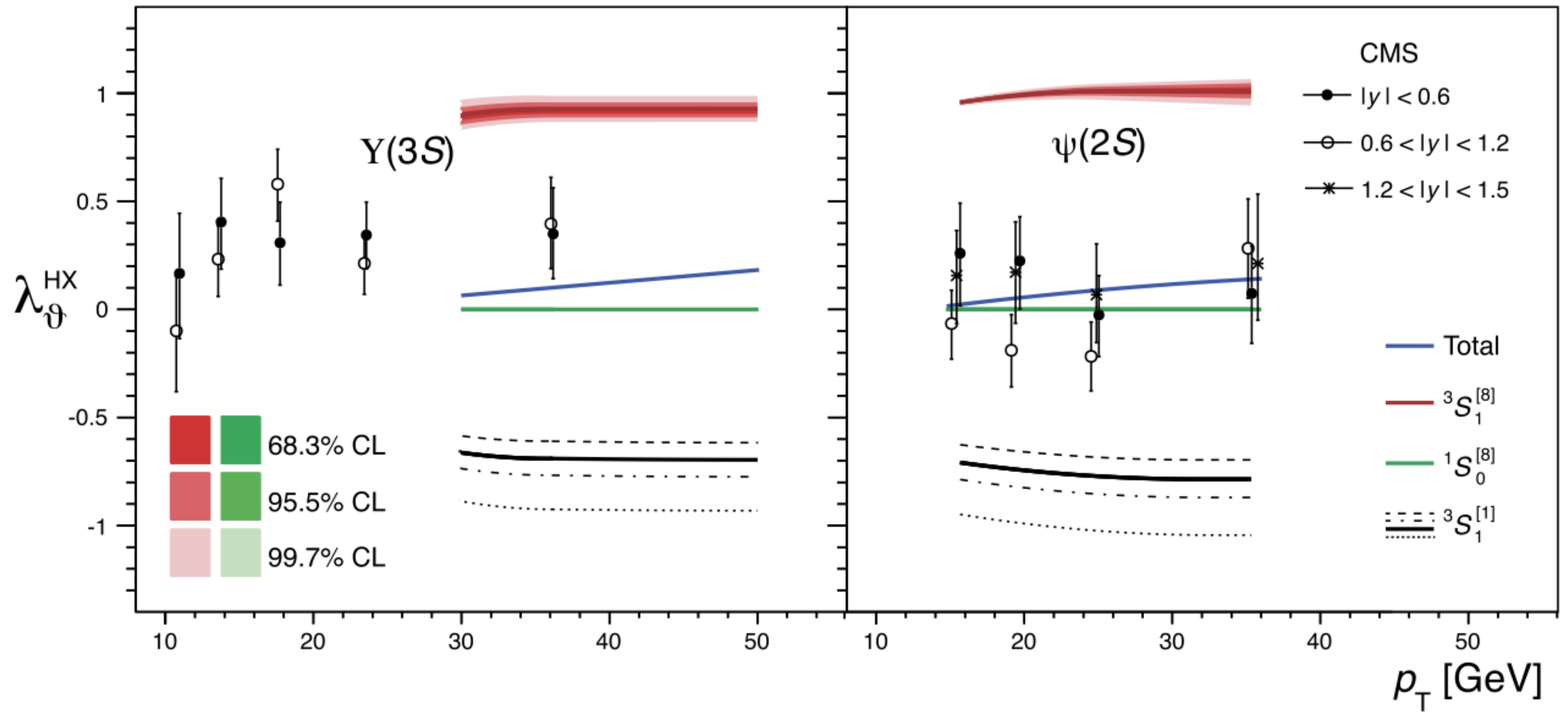
Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press





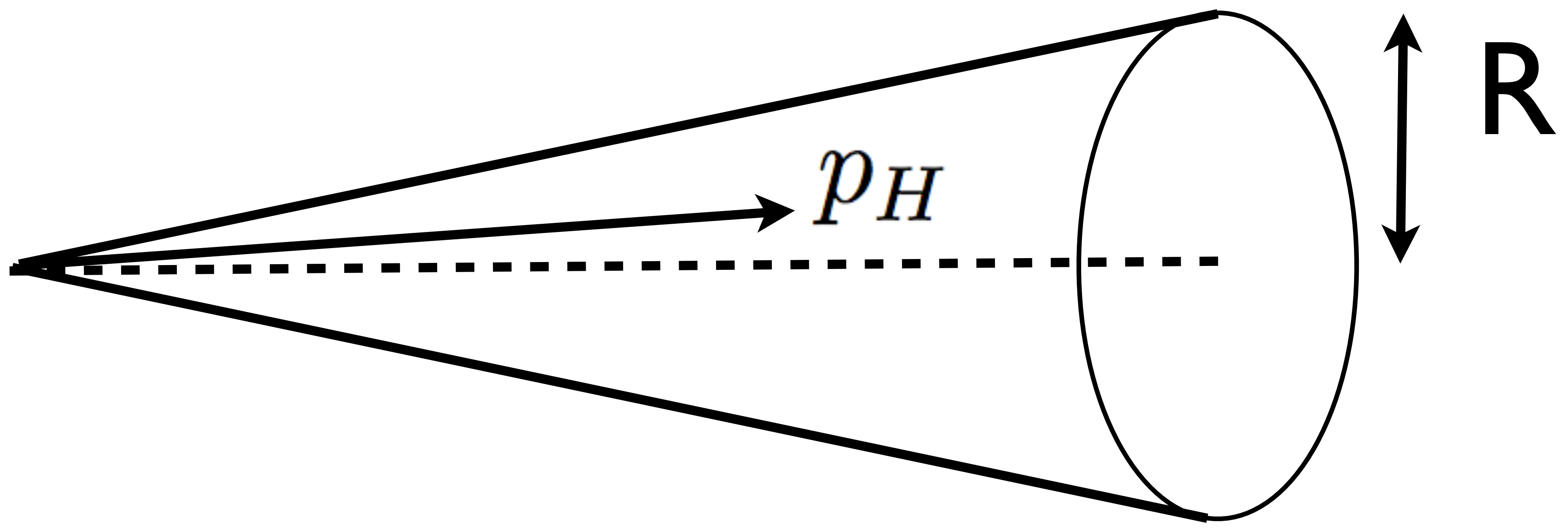


argue for $^1S_0^{(8)}$ dominance in both ψ(2S) & Υ(3S) production

Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009
A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035
A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\rm jet}^+$$

cross sections determined by **fragmenting jet function (FJF):**

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: **fragmentation functions**

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_i \int_z^1 \frac{dx}{x}\, C_i(E_{\rm cm}, x, \mu)\; D_i^h(z/x, \mu)$$

jet cross sections: **jet functions**

$$d\sigma(E,R) = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\boxed{\mathcal{G}_g^h(E,R,\mu,z) \longrightarrow D_i^h(z/x,\mu)\ ,\ J_\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 \mathrm{d}z\, z\, D_j^h(z,\mu) = 1$$

$$J_i(E,R,z,\mu) = \frac{1}{2}\sum_h \int \frac{dz}{(2\pi)^3}\, z\, \mathcal{G}_i^h(E,R,z,\mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell J_i(E,R,\mu)$$

$$\frac{d\sigma}{dE dz} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell\, \mathcal{G}_i^h(E,R,z,\mu)$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E,R,z,\mu) = \sum_i \int_z^1 \frac{dz'}{z'} \mathcal{J}_{ij}(E,R,z',\mu) D_j^h\left(\frac{z}{z'},\mu\right)\left[1+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{4E^2\tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E,R,z,\mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi}\left[\left(L^2-\frac{\pi^2}{24}\right)\delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z)\right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z)\ln z & z \le 1/2 \\ \frac{2(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ & z \ge 1/2. \end{cases}$$

$$L = \ln[2E\tan(R/2)/\mu].$$

scale for $\mathcal{J}_{ij}(E,R,z,\mu)$

sum rule for matching coefficients

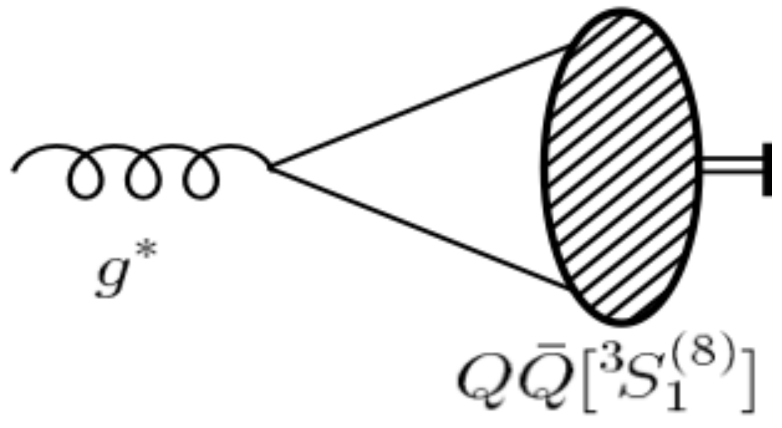
$$\sum_j \int_0^1 dz\, z\, \mathcal{J}_{ij}(R,z,\mu) = 2(2\pi)^3 J_i(R,\mu)$$

NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230
Braaten, Chen, PRD 54 (1996) 3216
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale 2m$_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3}\langle O^\psi({}^3S_1^{(8)})\rangle\delta(1-z)$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2}\frac{\langle O^\psi({}^3S_1^{(1)})\rangle}{M_\psi^3}\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2} dy\frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}$$
$$\sum_{i=0}^{2} z^i\left(f_i(r,y) + g_i(r,y)\frac{1+r-2y}{2(y-r)\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right),$$

DGLAP evolution: 2m$_c$ to 2E tan(R/2)

FJF in terms of fragmentation function

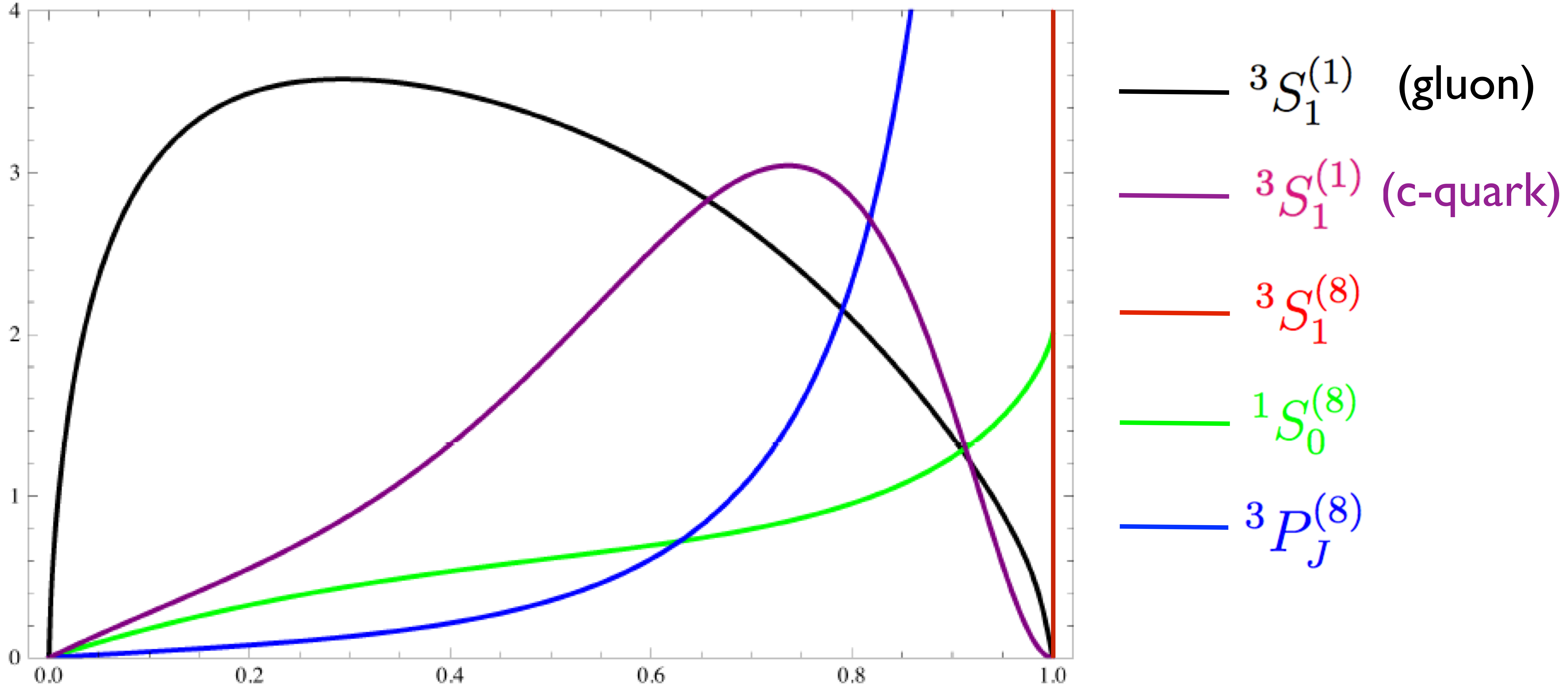
$$\mathcal{G}_g^{\psi}(E,R,z,\mu) = D_{g\to\psi}(z,\mu)\left(1+\frac{C_A\alpha_s}{\pi}\left(L_{1-z}^2-\frac{\pi^2}{24}\right)\right)$$
$$+\frac{C_A\alpha_s}{\pi}\left[\int_z^1\frac{dy}{y}\tilde{P}_{gg}(y)L_{1-y}D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right.$$
$$+2\int_z^1 dy\frac{D_{g\to\psi}(z/y,\mu)-D_{g\to\psi}(z,\mu)}{1-y}L_{1-y}$$
$$\left.+\theta\left(\frac{1}{2}-z\right)\int_z^{1/2}\frac{dy}{y}\hat{P}_{gg}(y)\ln\left(\frac{y}{1-y}\right)D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right]$$

$$L_{1-z}=\ln\left(\frac{2E\tan(R/2)(1-z)}{\mu}\right)$$

For large E, FJF ~ NRQCD frag. function (at scale 2E tan(R/2))

$$\mathcal{G}_g^h(E,R,\mu=2E\tan(R/2),z)\to D_g^{\psi}(z,2E\tan(R/2))+O(\alpha_s)$$

NRQCD FF's (at scale 2m$_c$)

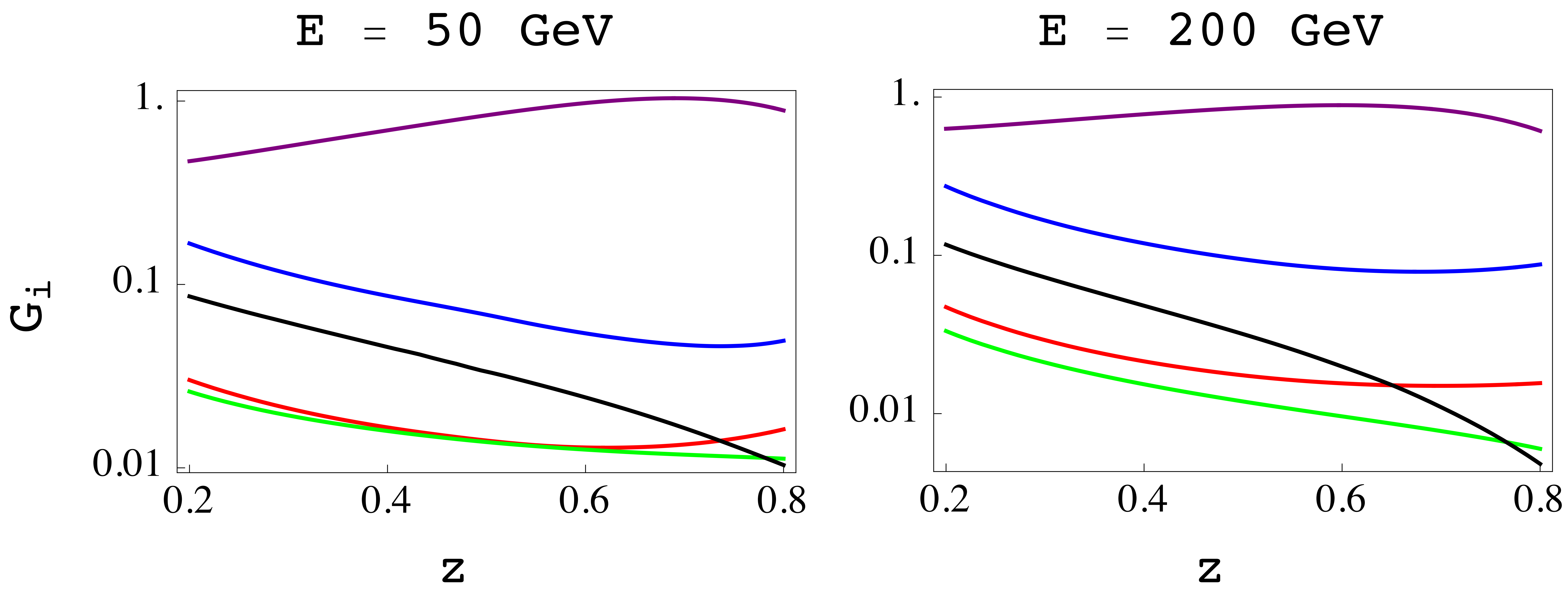


(normalization arbitrary)

Evolution to 2E tan(R/2) will soften discrepancies

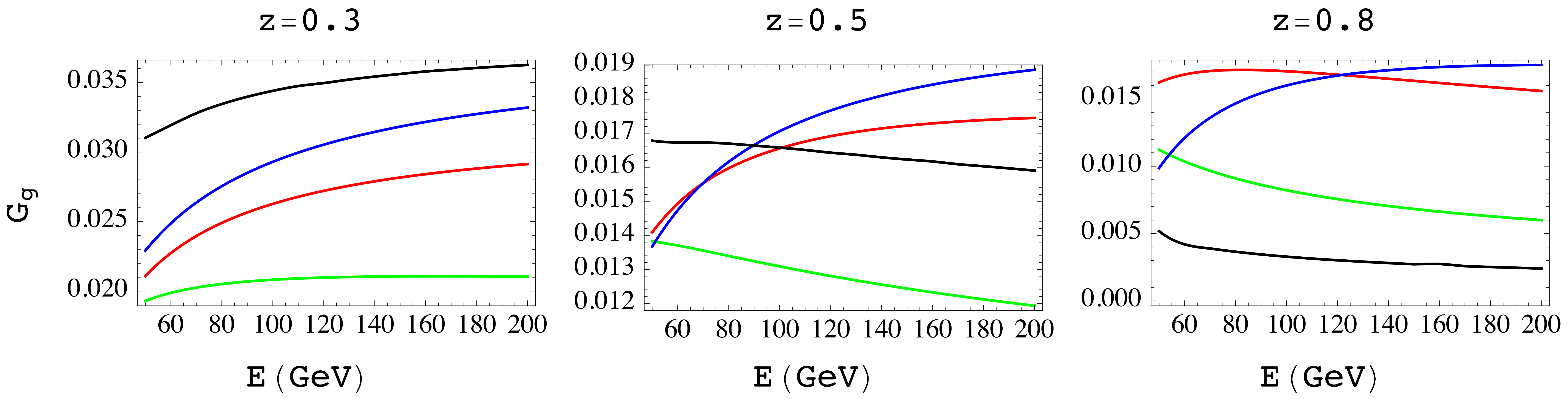
FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



FJF's at Fixed z vs. Energy

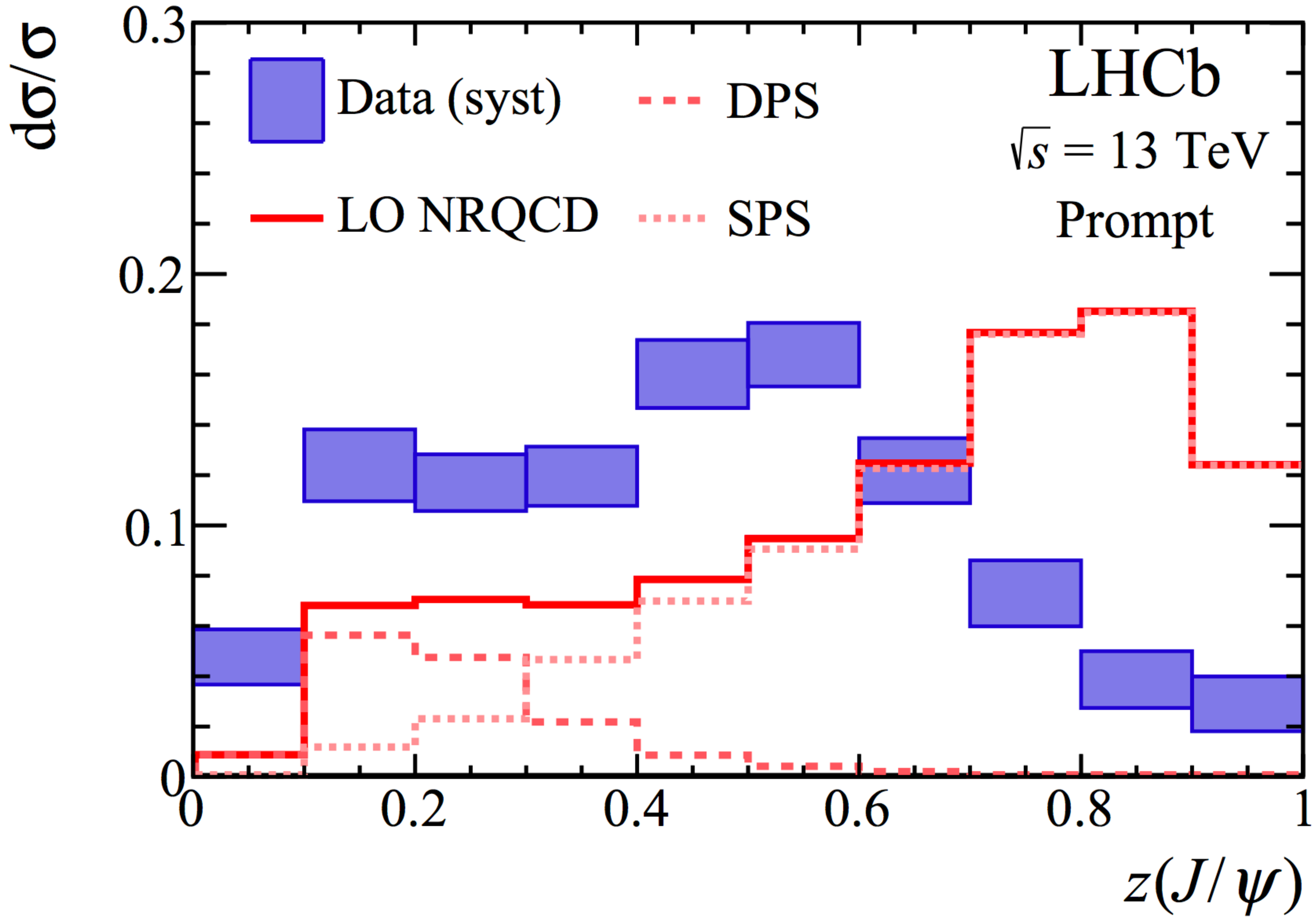
M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if z > 0.5

Recent Observations of Quarkonia within Jets

LHCb collaboration, Phys. Rev. Lett. 118 (2017) no.19, 192001



cuts: $2.5 < \eta_{\rm jet} < 4.0 \quad p_{T,jet} > 20\,{\rm GeV} \quad p(\mu) > 5\,{\rm GeV}$

This result was anticipated in:

Jets w/ Heavy Mesons: NLL' vs. Monte Carlo

(w/ R. Bain, L. Dai, A. Hornig, A.Leibovich, Y. Makris)

JHEP 1606 (2016) 121 (arXiv:1601.05815)

$$e^+e^- \to b\bar{b}$$
$$\hookrightarrow \text{B jet}$$

$$e^+e^- \to q\bar{q}g$$
$$\hookrightarrow J/\psi \text{ jet}$$

$e^+e^- \longrightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010) 101

$$d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes J_g \otimes S$$

$$\longrightarrow \quad d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes \mathcal{G}_g^{J/\psi} \otimes S$$

unmeasured jets:

E, R

measured jets:

angularity: $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

$$\omega = \sum_i p_i^- \qquad s = \omega^2 \tau_0$$

$e^+e^- \longrightarrow$ Jets Formula (NLL')

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(i)}}{dz d\tau_a} = \sum_j \int_z^1 \frac{dx}{x} D_j(x;\mu_J) H_2(\mu_H)\left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu,\mu_H)} S^{\text{unmeas}}(\mu_\Lambda) J_\omega(\mu_R)\left(\frac{\mu_R}{\omega\tan\frac{R}{2}}\right)^{\omega_R(\mu,\mu_R)}$$
$$\times\left\{\left[\delta_{ij}\delta(1-z/x)(1+f_S(\tau_a,\mu_S)) + f_{\mathcal{J}}^{ij}(\tau_z, z/x;\mu_J)\right]\left(\frac{\mu_S\tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)}\right.$$
$$\left.\times\left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)}\frac{1}{\Gamma[-\omega_J(\mu,\mu_J)-\omega_S(\mu,\mu_S)]}\frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}}\right\}_+$$
$$\times\exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda)+\gamma_E\Omega(\mu;\mu_J,\mu_S)\right].$$

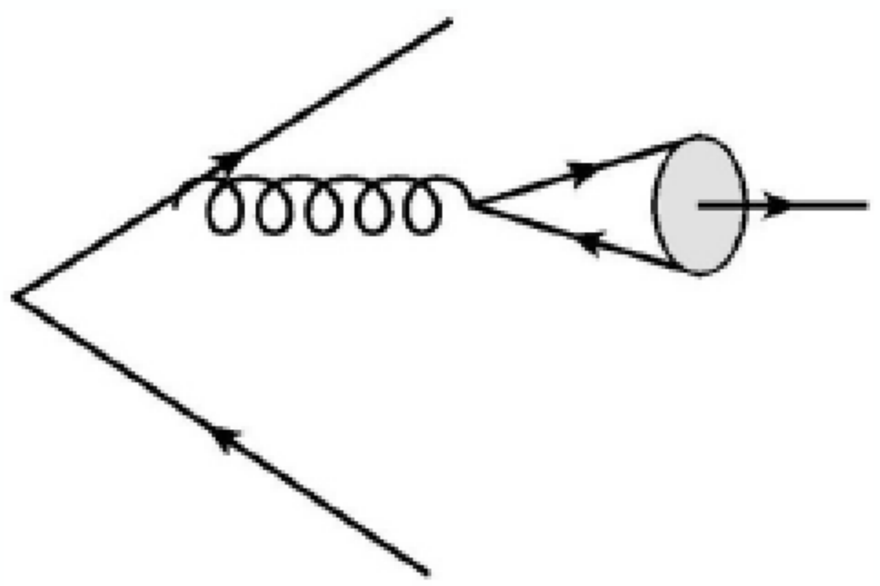
$e^+e^- \longrightarrow$ Jets Formula (NLL')

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{(i)}}{dz d\tau_a} = \sum_j \int_z^1 \frac{dx}{x} D_j(x;\mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu,\mu_H)} S^{\text{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan\frac{R}{2}}\right)^{\omega_R(\mu,\mu_R)}$$
$$\times \left\{ \left[\delta_{ij}\delta(1-z/x)(1+f_S(\tau_a,\mu_S)) + f_{\mathcal{J}}^{ij}(\tau_z, z/x;\mu_J)\right] \left(\frac{\mu_S \tan^{1-a}\frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu,\mu_S)}\right.$$
$$\left.\times \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu,\mu_J)} \frac{1}{\Gamma[-\omega_J(\mu,\mu_J) - \omega_S(\mu,\mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu,\mu_J)+\omega_S(\mu,\mu_S)}}\right\}_+$$
$$\times \exp\left[\mathcal{K}(\mu;\mu_H,\mu_R,\mu_J,\mu_S,\mu_\Lambda) + \gamma_E \Omega(\mu;\mu_J,\mu_S)\right].$$

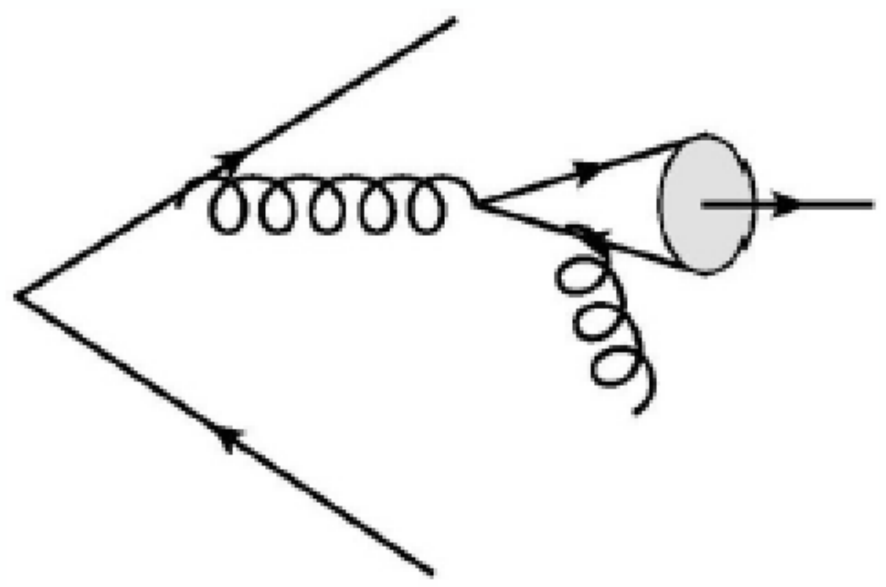
RGE evolution

Madgraph + PYTHIA

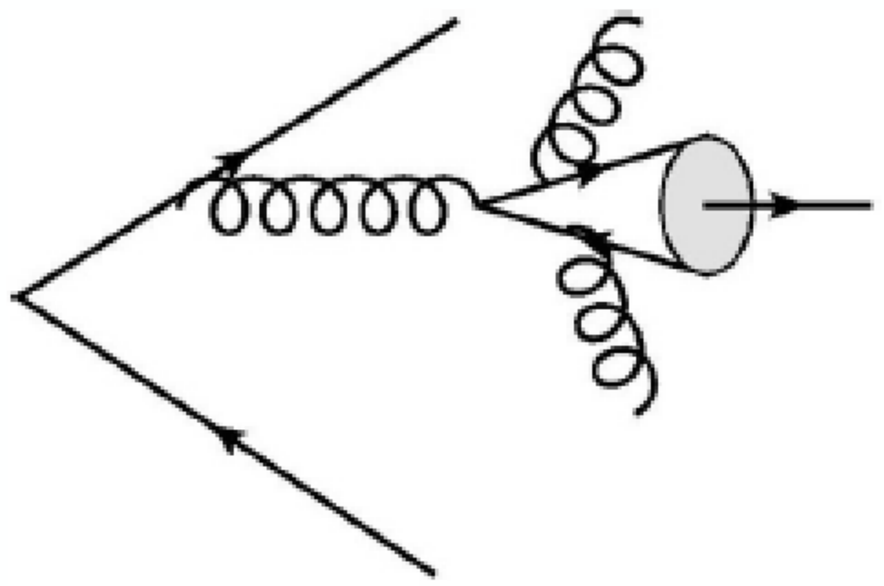
$e^+e^- \to b\,\bar{b}\,c\,\bar{c}\left[{}^3S_1^{(8)}\right]$



$e^+e^- \to b\,\bar{b}\,g\,c\,\bar{c}\left[{}^1S_0^{(8)}\right]$



$e^+e^- \to b\,\bar{b}\,g\,g\,c\,\bar{c}\left[{}^3S_1^{(1)}\right]$

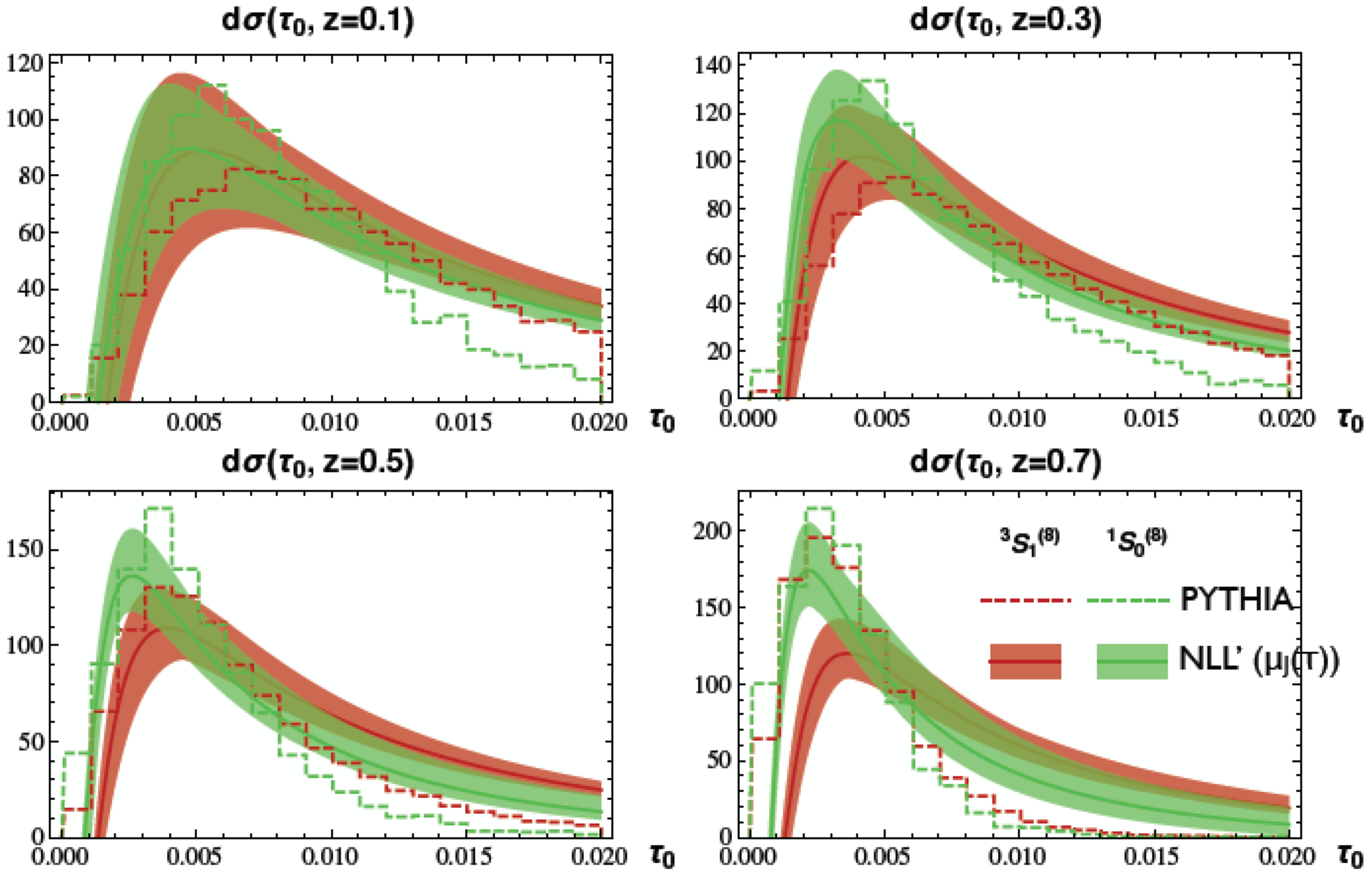


Force Madgraph to create J/ψ from gluon initiated jet

PYTHIA: parton shower, hadronization

NLL' vs. Monte Carlo

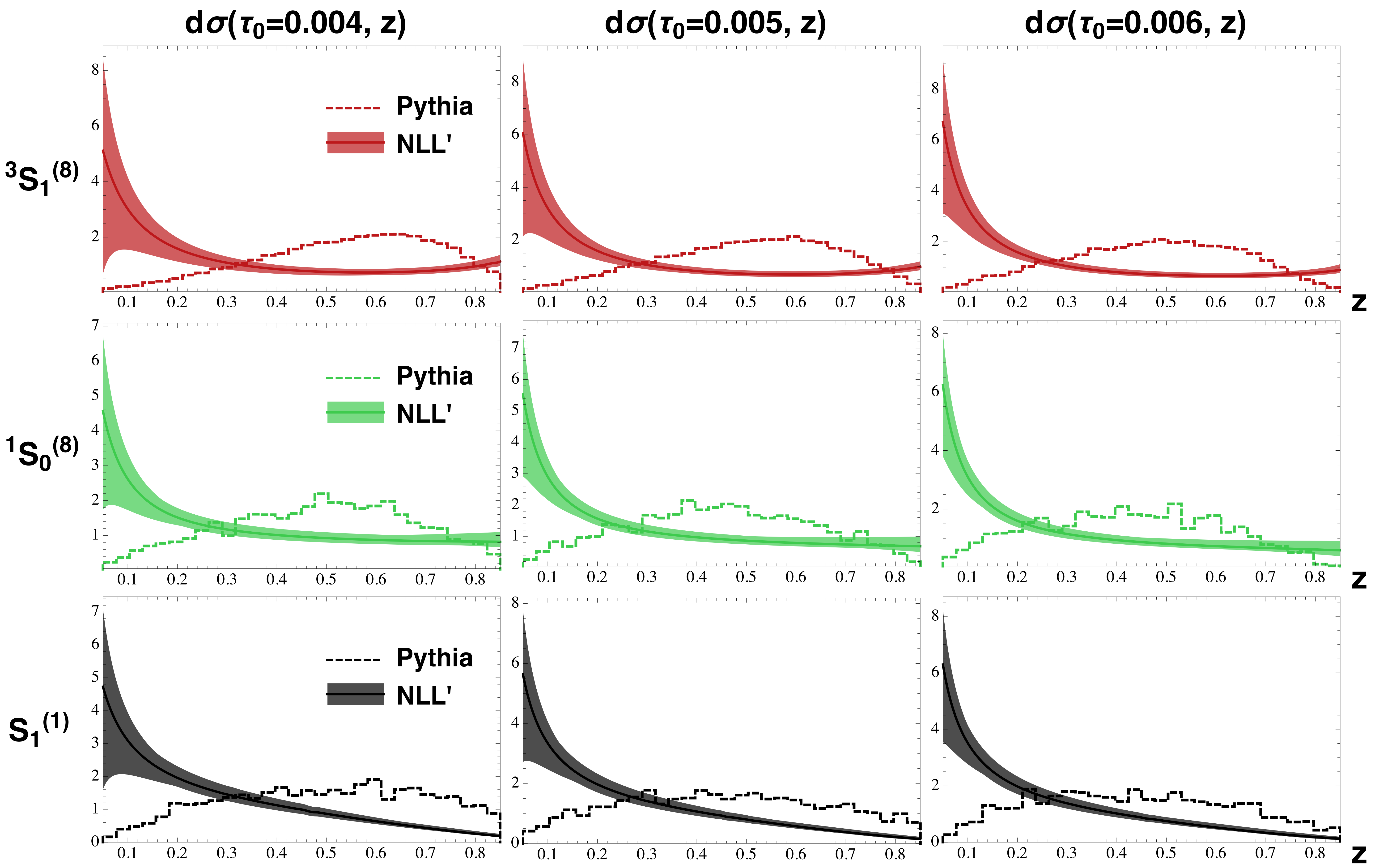
fixed z, variable τ_0



good agreement, some discrimination for large z

NLL' FJF vs. Pythia

R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T. Mehen JHEP 1606 (2016) 121

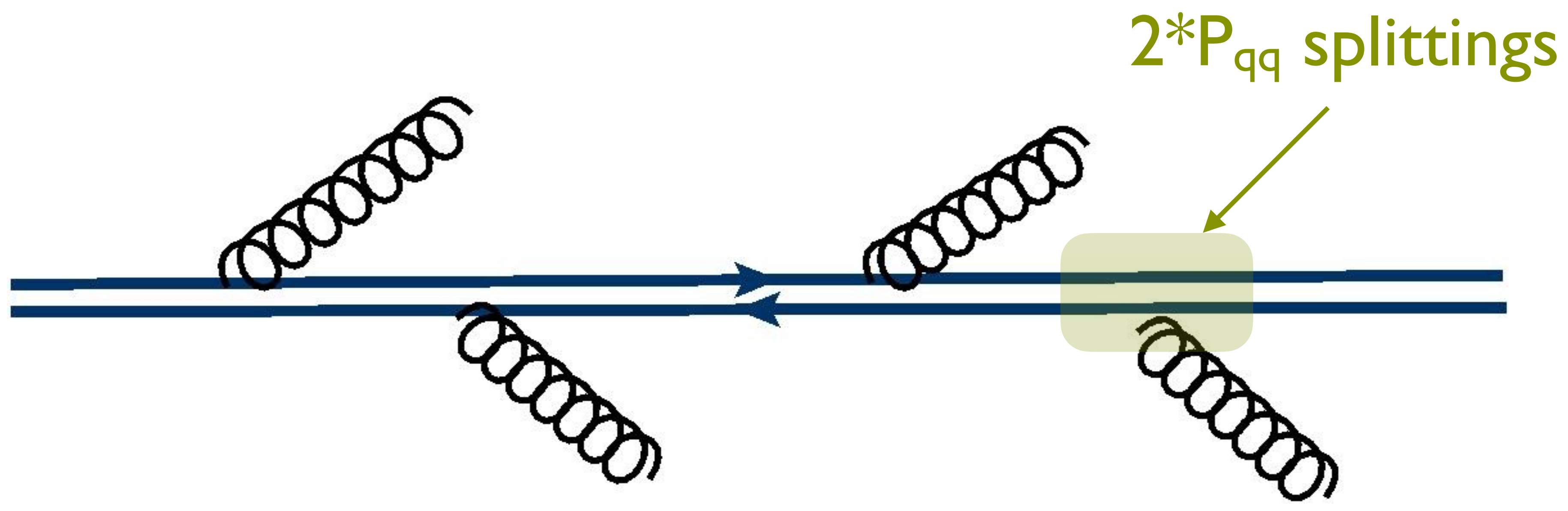


$e^+e^- \rightarrow \bar{q}qg$ $\qquad E_{CM} = 250\,\mathrm{GeV} \qquad \tau_0 = s/\omega^2$

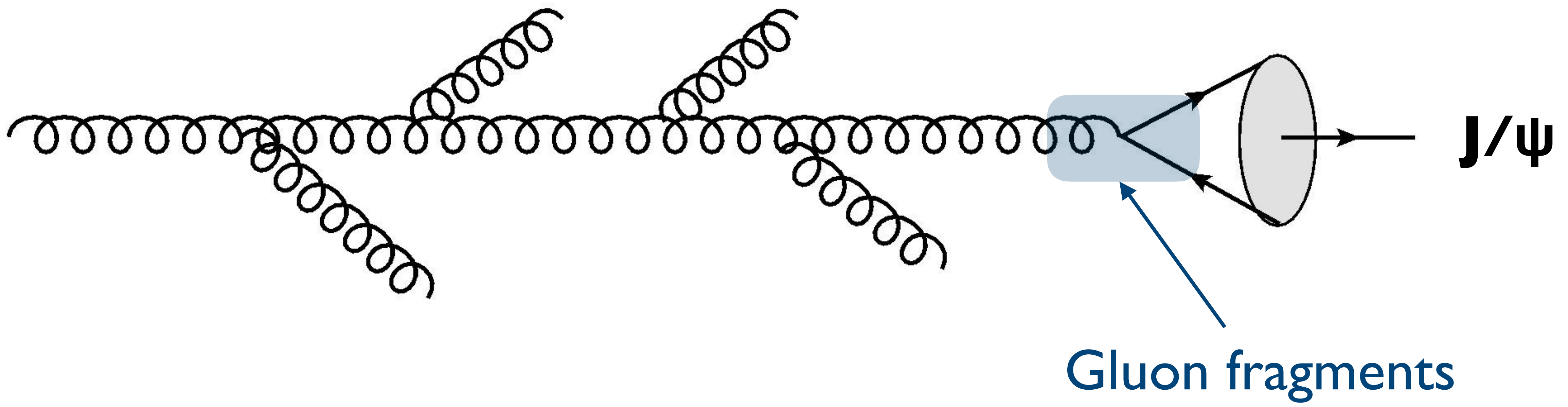
└→ jet w/ J/ψ

Explaining difference between NLL' vs Pythia

PYTHIA's model for showering color-octet $c\bar{c}$ pairs:



Physical picture of analytical calculation

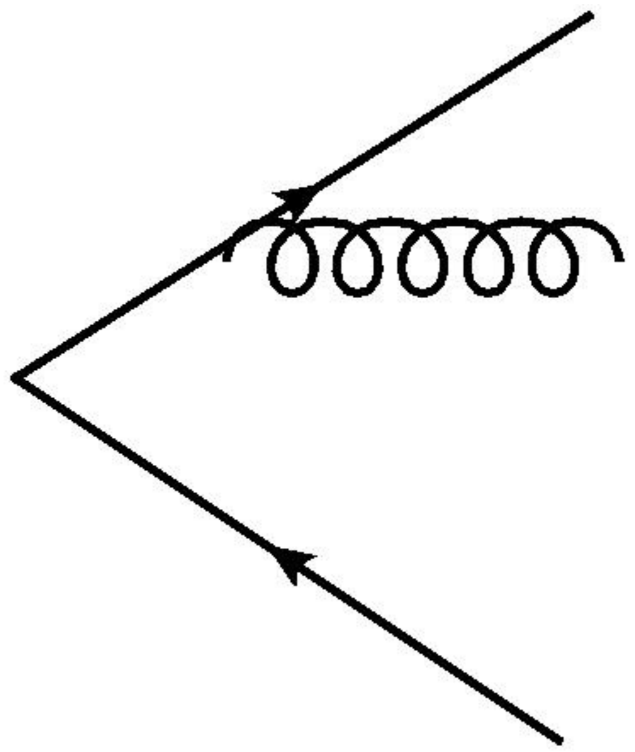


Pythia z distributions much harder than NLL' calculations

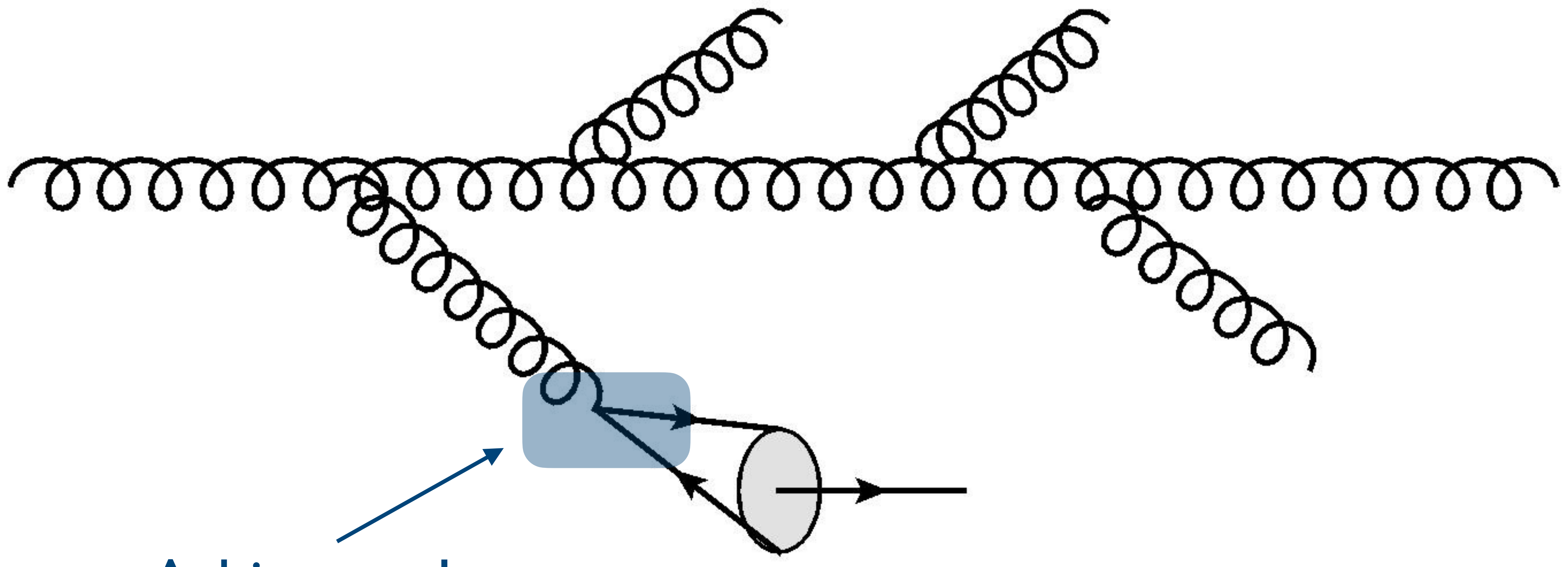
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+e^- \to b\,\bar{b}\,g$$



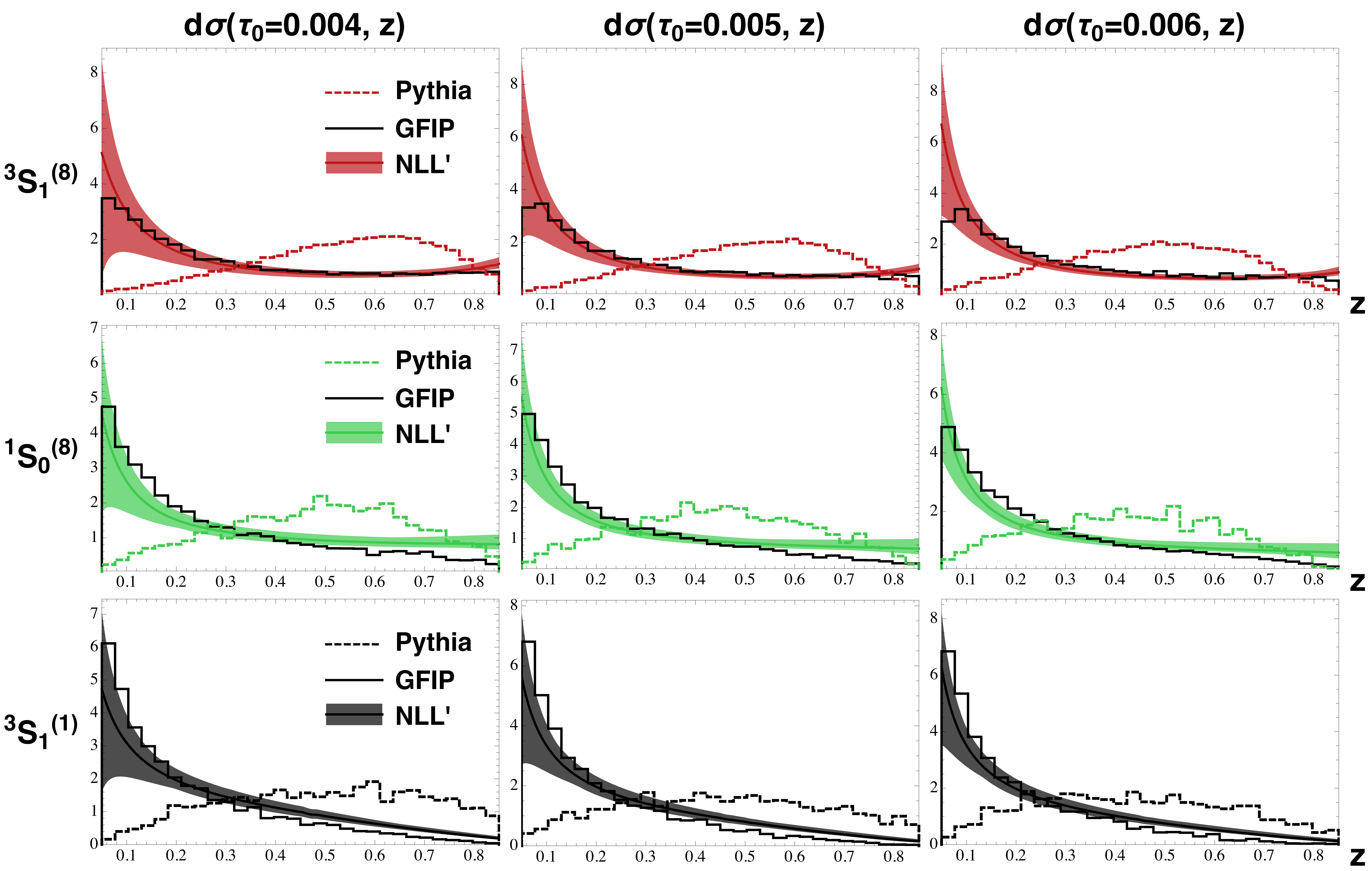
PYTHIA + Convolution



shower gluon with PYTHIA down to scale ~$2m_c$, no hadronization

convolve final state gluon distribution w/ NRQCD FFs

NLL', PYTHIA, and GFIP



GFIP and Recent LHCb Observations

R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. Mehen, PRL 119 (2017) 3, 032002, arXiv:1702.5525

generate events with hard c-quark , gluons

LHCb: pp collisions $\sqrt{s} = 13\,\mathrm{TeV}$ cuts:

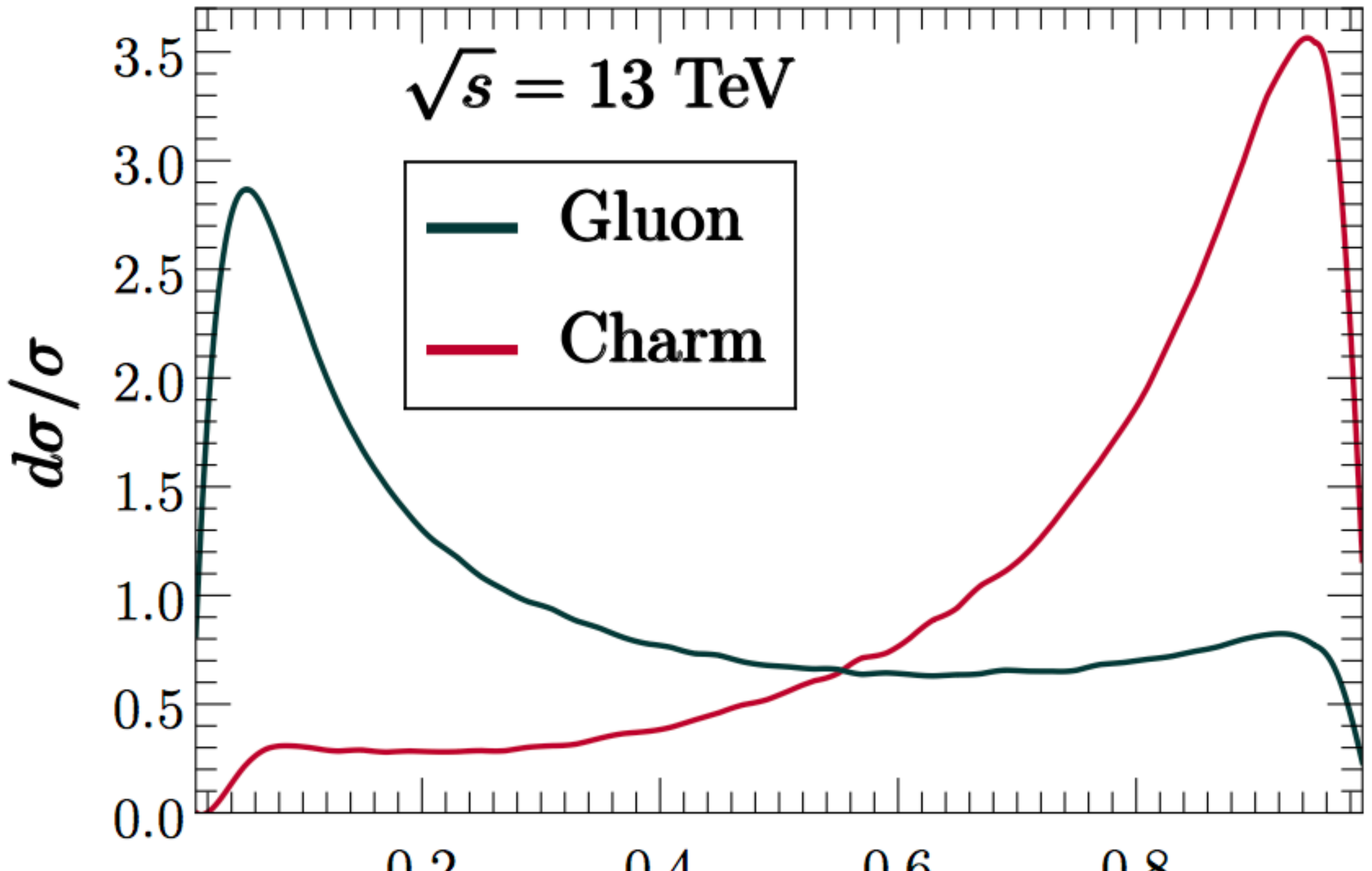
$2<\eta<4.5$

R=0.5

$p_{T,JET} < 20$ GeV

$p_\mu < 5$ GeV

evolve shower to scale ~$2m_c$



convolve w/ NRQCD FF for c quarks, gluons ~$2m_c$

LHCb data is normalized so $\sum_i \Delta z \left(\frac{d\sigma}{\sigma}\right)_i = \Delta z$

compare 0.1 < z < 0.9

Use following three sets of LDMEs

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ $\times 10^{-2}$GeV^3
B & K [5, 6]	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao, et al. [12]	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al. [13]	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

Butenschoen and Kniehl, PRD 84 (2011) 051501 — **global fits to world's data**

Chao, et. al. PRL 108, 242004 (2012)
Bodwin, et. al., PRL 113, 022001(2014) — **fits to high p_T hadron collider data**

FJF and Recent LHCb Observations

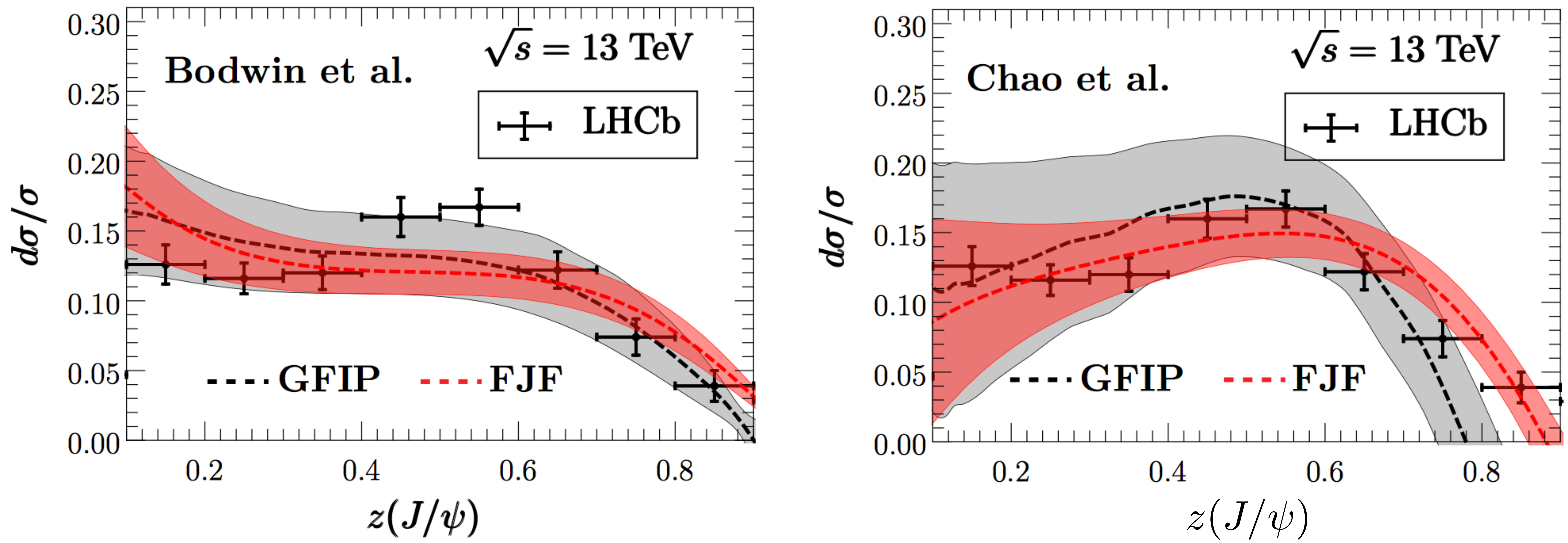
combine FJFs with hard events generated by Madgraph

NRQCD FFs evolved from $2m_c$ to jet energy scale using DGLAP

factorization theorem with tree level hard function,
trivial soft function, no NLL' resummation

FJF is only term in factorization dependent on $z(J/\psi)$

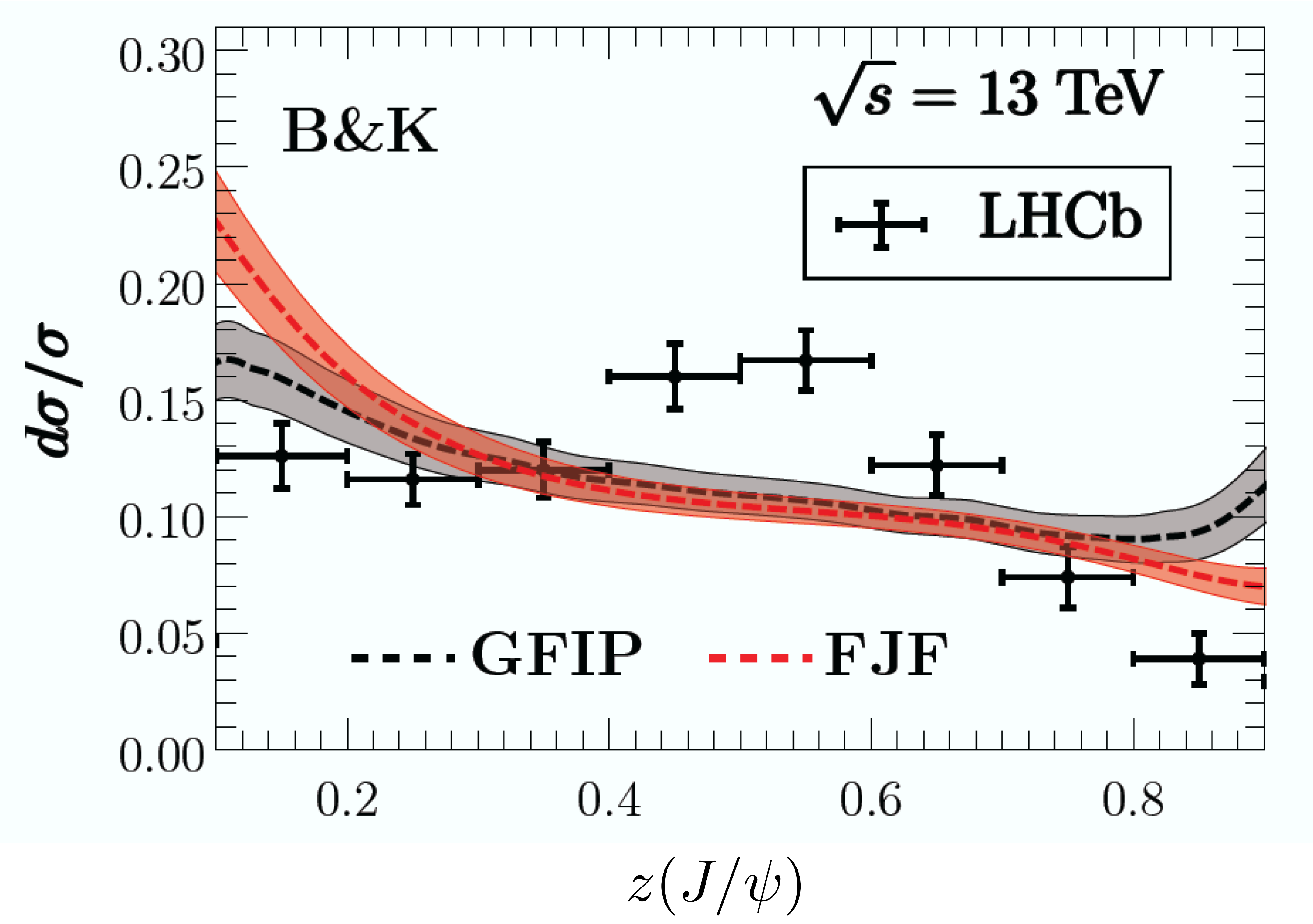
Results



FJFs, GFIP consistent

LDME from fits high p_T agree with LHCb

Results



LDME from global fits:
poorer agreement with LHCb, better than PYTHIA

Future Measurements

polarization of J/ψ in jets

Z.-B. Kang, .J.-W. Qiu, F. Ringer, H. Xing, H. Zhang, PRL 119 (2017) 032001

absolute cross sections

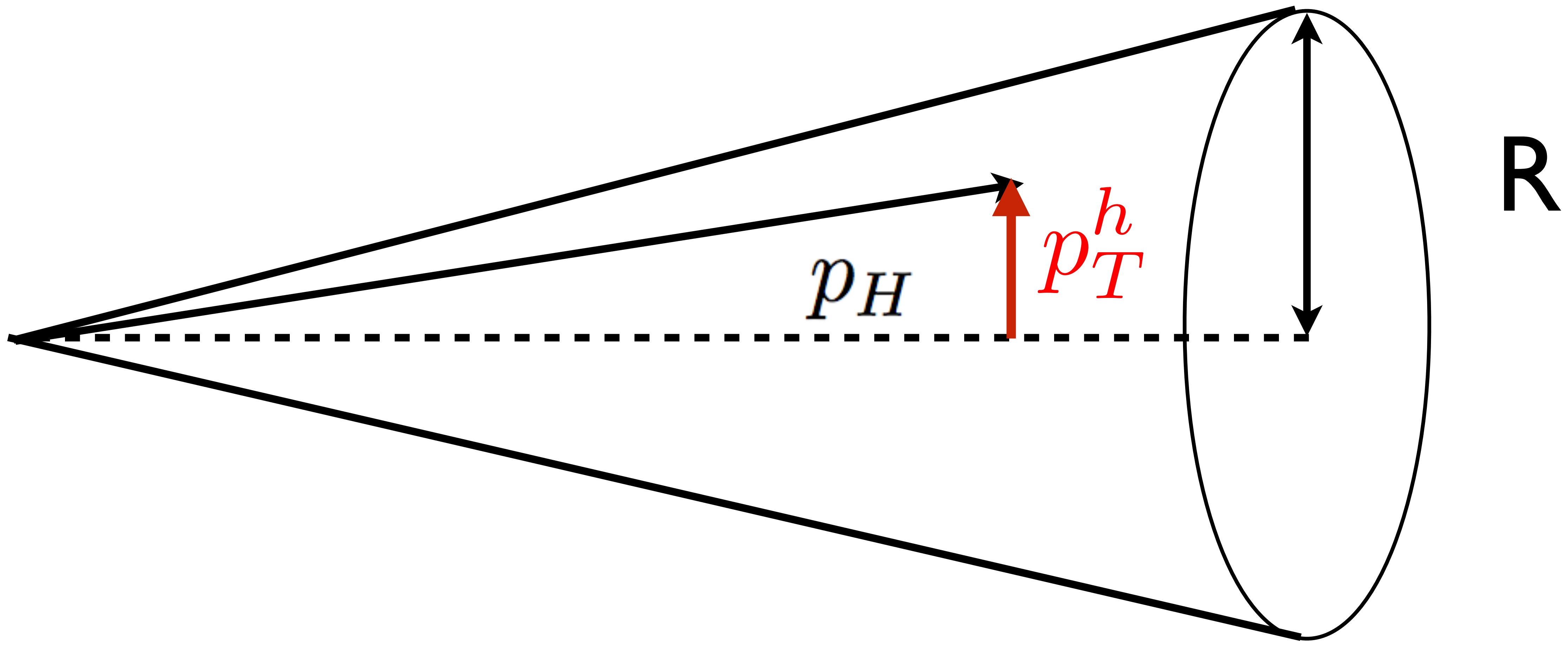
alternative jet definitions, e.g., soft drop

p_T dependent FJFs

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z, p_T are both measured



Jet Energy: E

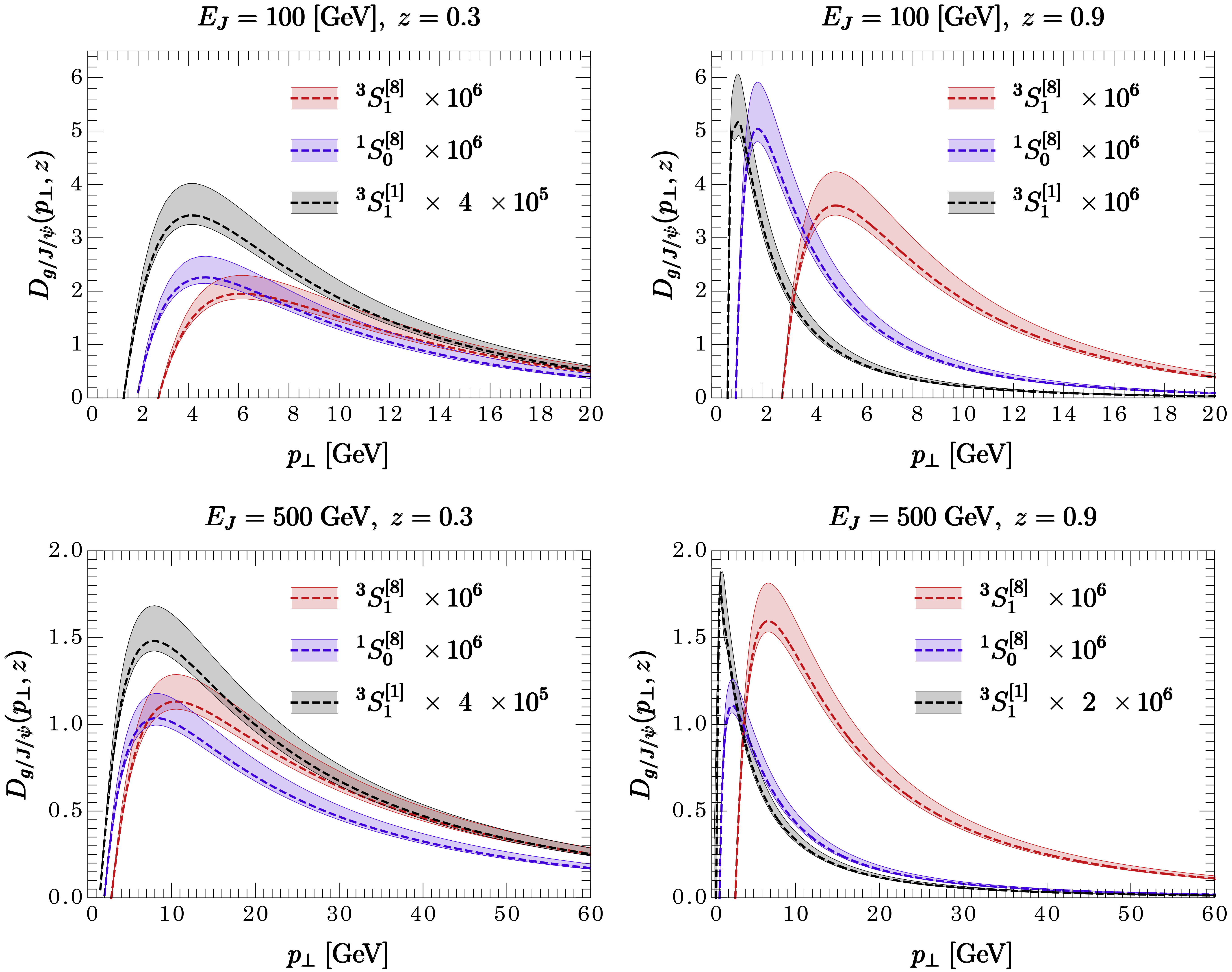
$$p_H^+ = z p_{\rm jet}^+$$

transverse momentum measured w/ rspt. to jet axis

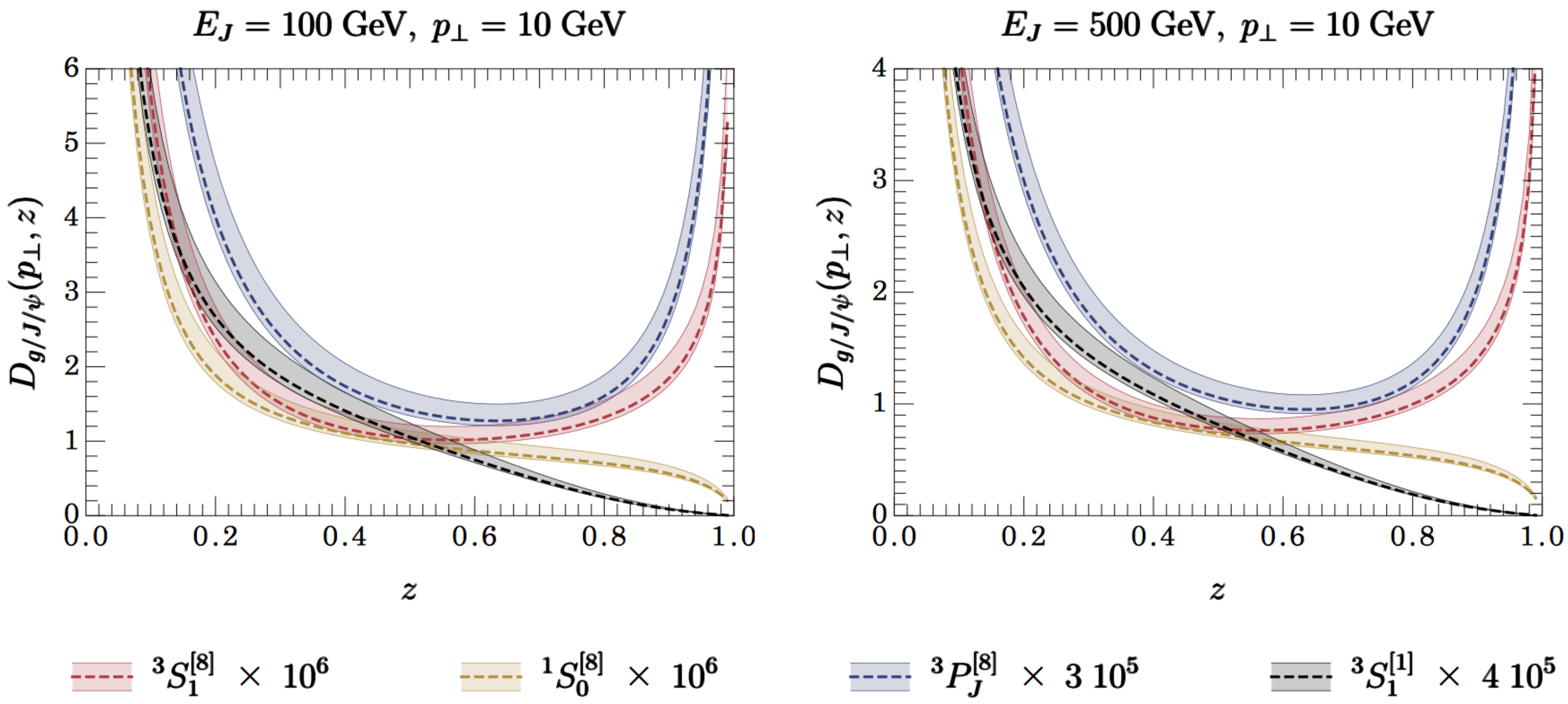
jet axis ~ parton initiating jet if out of jet radiation is ultrasoft

$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\rm QCD}$$

Application to Quarkonium Production

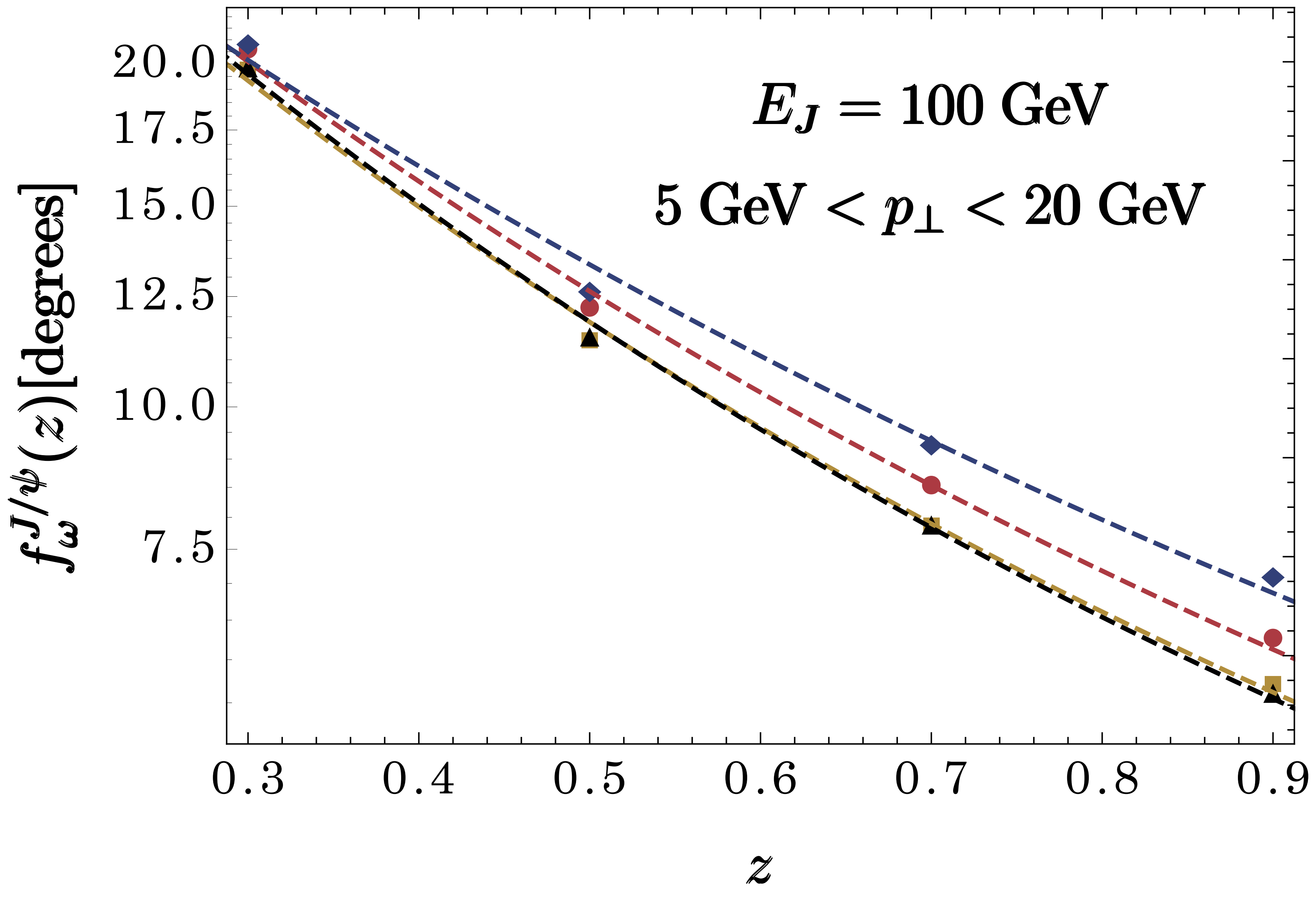


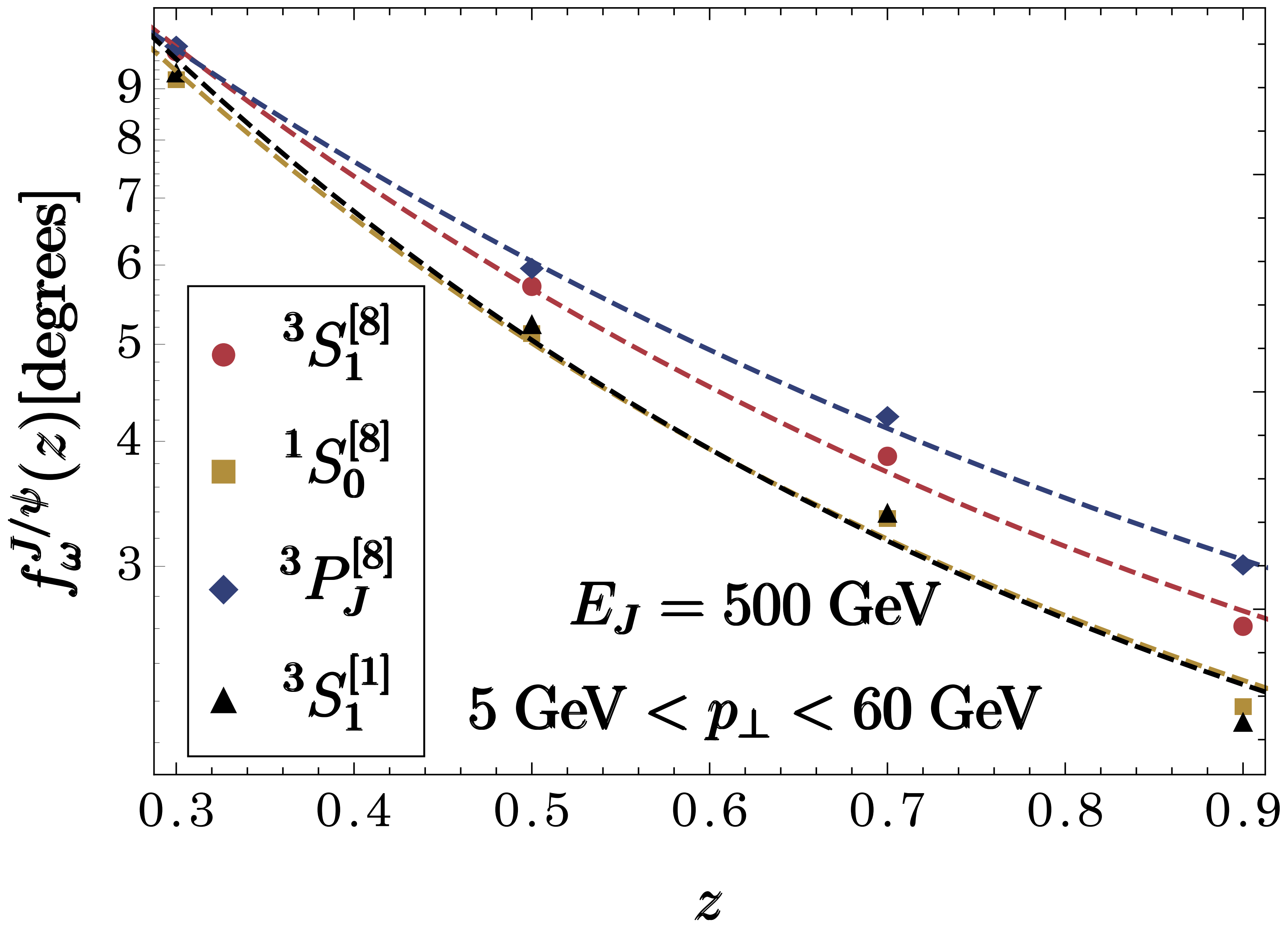
Application to Quarkonium Production



Application to Quarkonium Production

$$\langle\theta\rangle(z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f^h_\omega(z)$$





$E_J = 100$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.92	0.92
$^3S_1^{[8]}$	3.86	0.84
$^1S_0^{[8]}$	3.88	0.90
$^3P_J^{[8]}$	3.75	0.74

$E_J = 500$ GeV		
$^{2S+1}L_J^{[1,8]}$	C_0	C_1
$^3S_1^{[1]}$	3.75	1.68
$^3S_1^{[8]}$	3.48	1.39
$^1S_0^{[8]}$	3.66	1.64
$^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

Conclusions

measuring quarkonia within jets and using jet observables
should provide insights into quarkonia production

If $^1S_0^{(8)}$ mechanism dominates high p_T production
FJF should have negative slope for z(E), for z > 0.5

LHCb data on $z(J/\psi)$ well-described by FJF, GFIP
improvement over default PYTHIA, consistent w/ NLL' calculations
LDME extracted from high p_T slightly preferred

TMD FJFs: p_T^h, θ discriminate between NRQCD mechanisms

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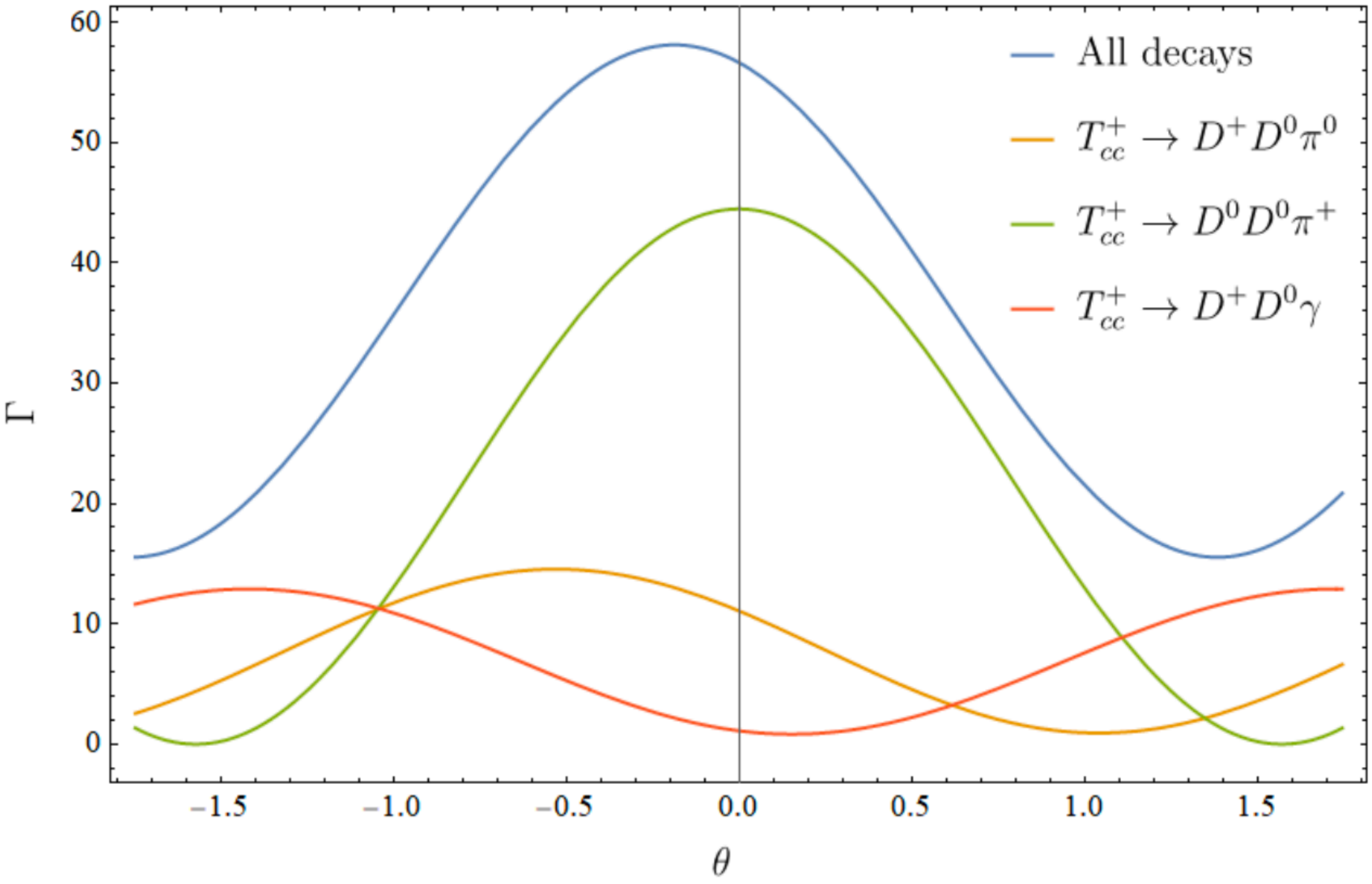

this agrees well with the same plot by Meng et al. (2107.14784)