Brookhaven National Laboratory

From Heavy Ion Physics to the Electron Ion Collider

The 2022 CFNS Summer School on the Physics of the Electron-Ion Collider

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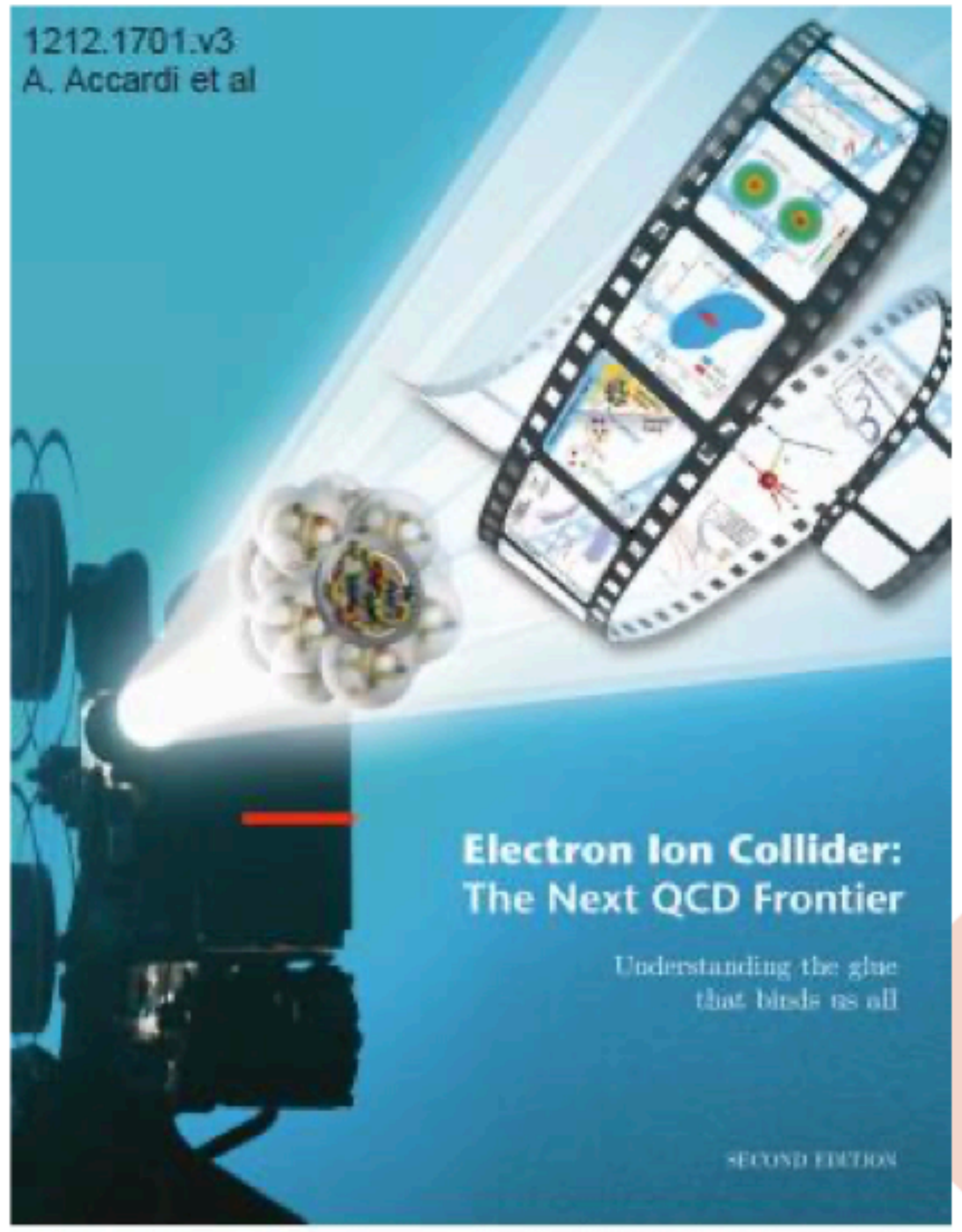
07/15/2022

U.S. DEPARTMENT OF ENERGY | Office of Science

Three lectures

- Lecture 1: Introduction to saturation - theory basics, experimental observables
  based heavily on “*Mining for Gluon Saturation at Colliders*”
  by Astrid Morreale and Farid Salazar, Universe 7 (2021) 8, 312
- Lecture 2: The Color Glass Condensate and applications to heavy ion collisions
- Lecture 3: Applications to physics in ultra-peripheral HICs and at the Electron Ion Collider

Science goals of the Electron Ion Collider



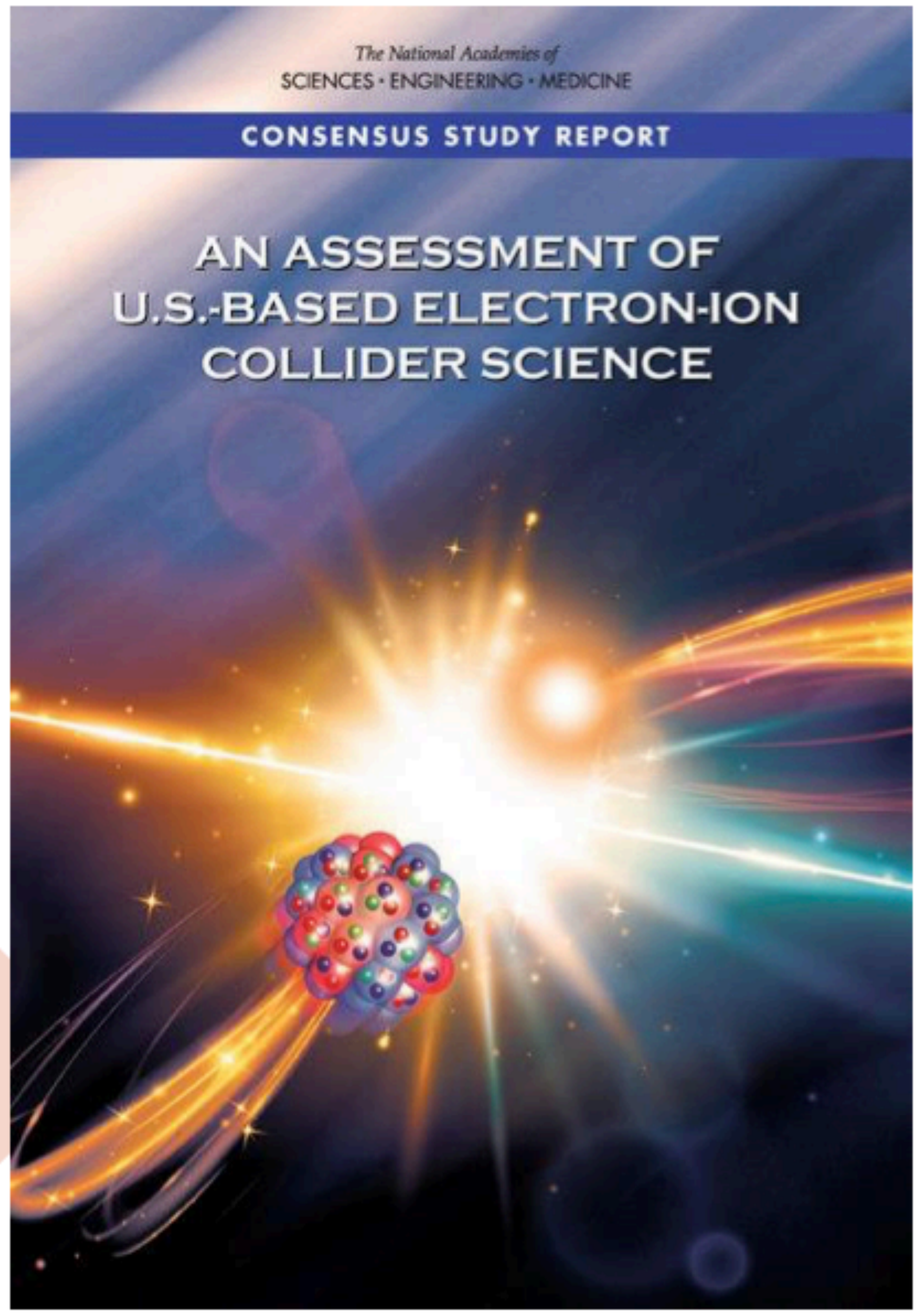
White paper
arXiv:1212.1701

Origin of nucleon mass

Origin of nucleon spin

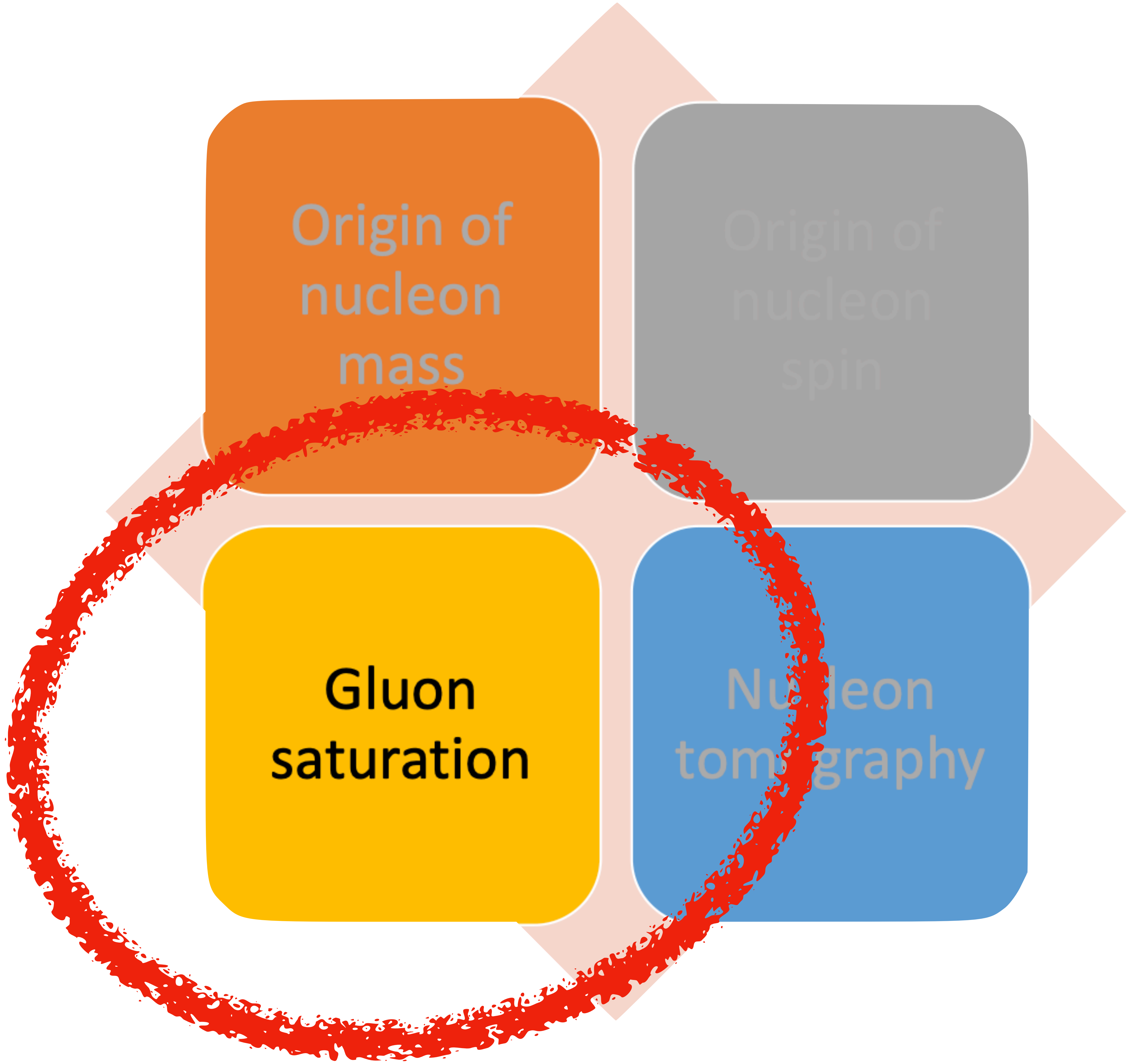
Gluon saturation

Nucleon tomography

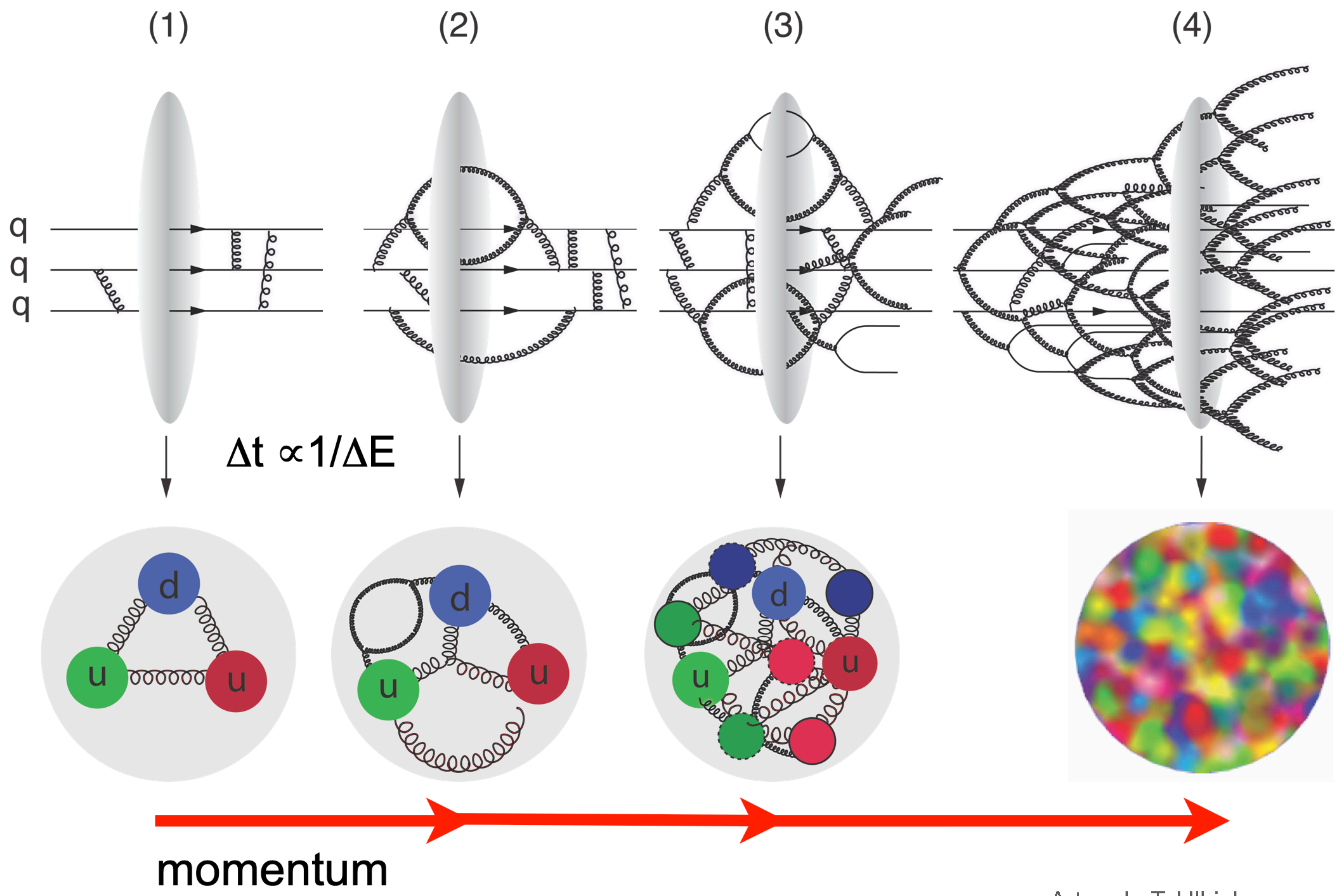


NAS report
July 2018

Science goals of the Electron Ion Collider



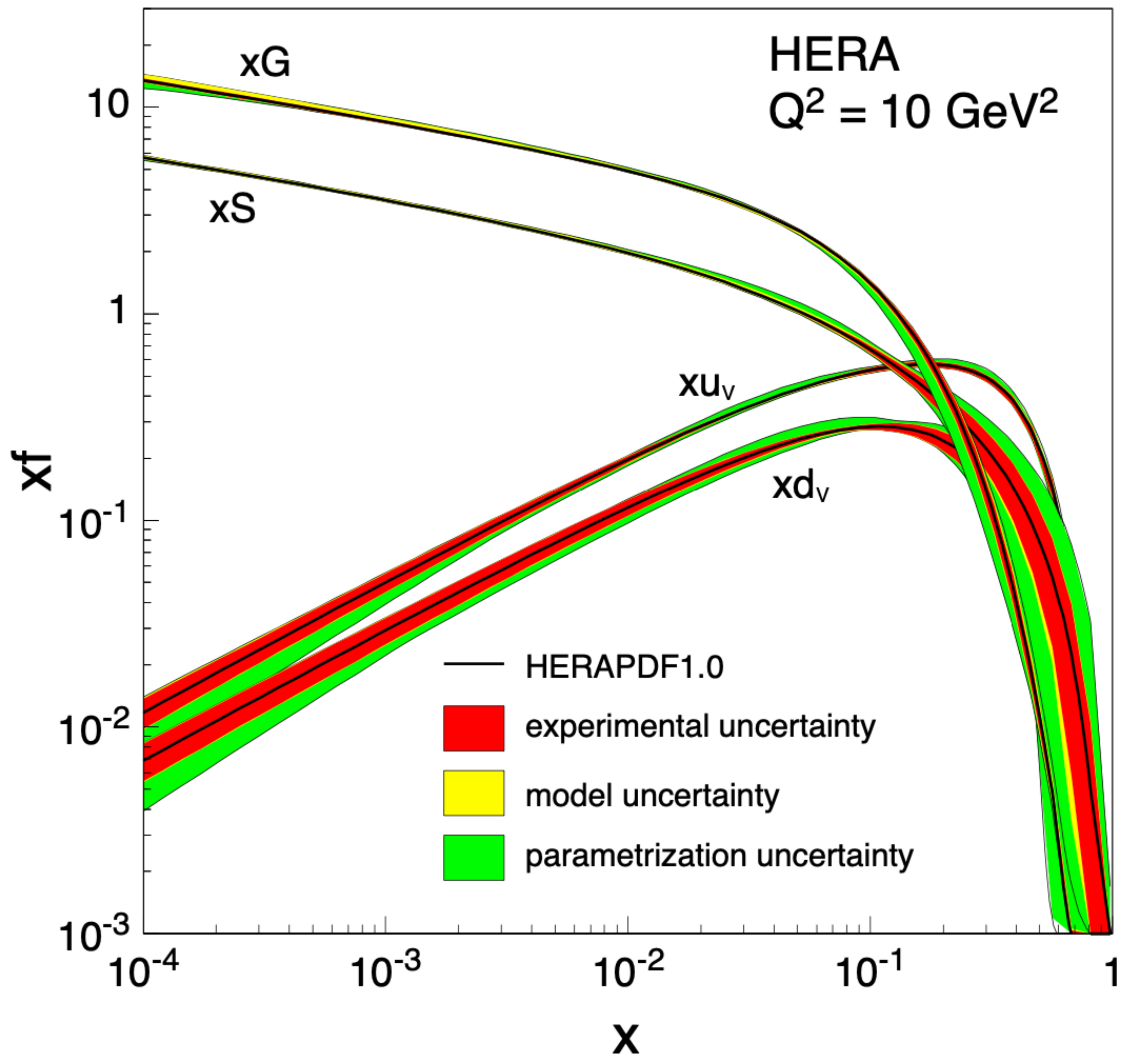
The boosted proton

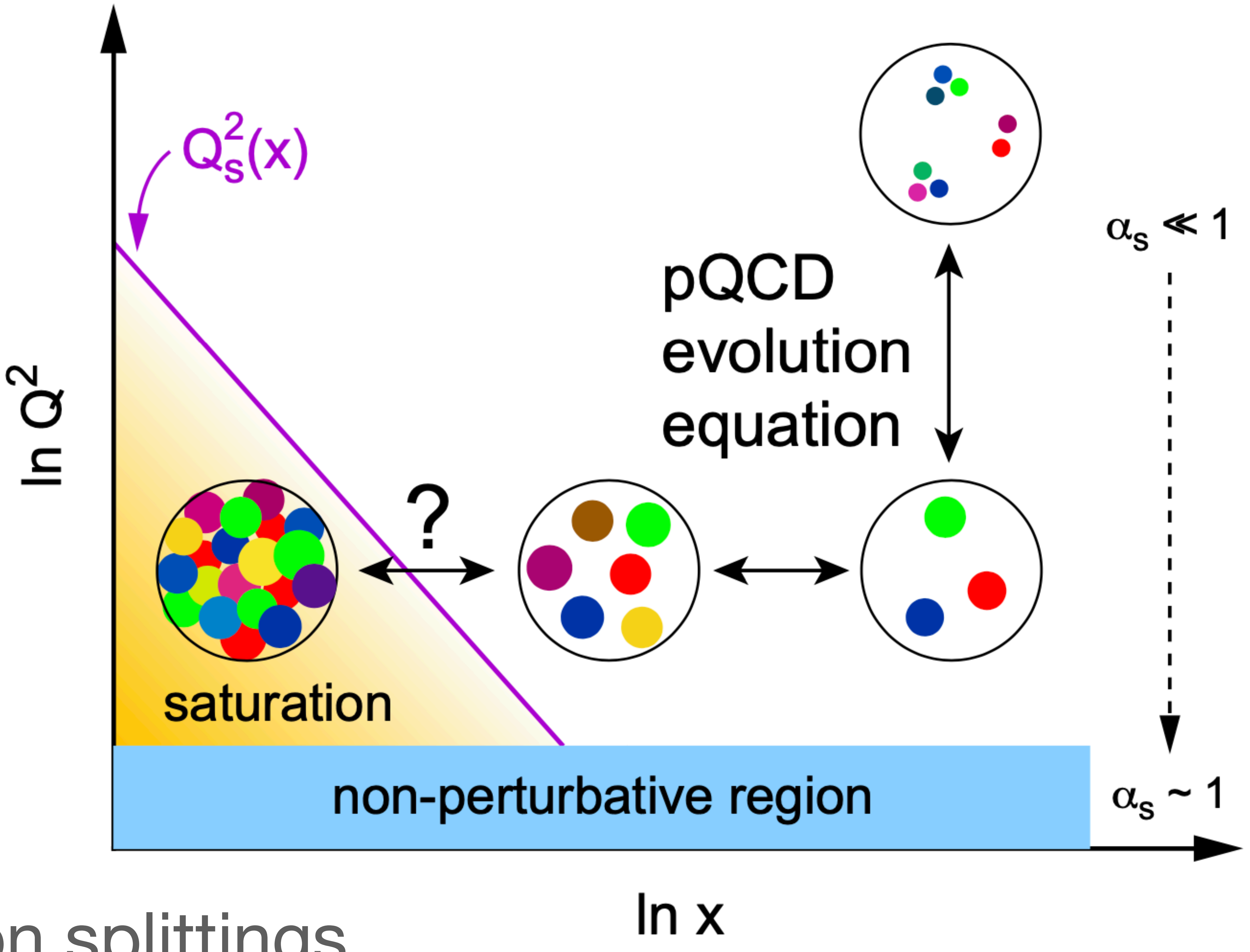


Artwork: T. Ullrich

Saturation

Explosive growth of gluon density violates unitarity



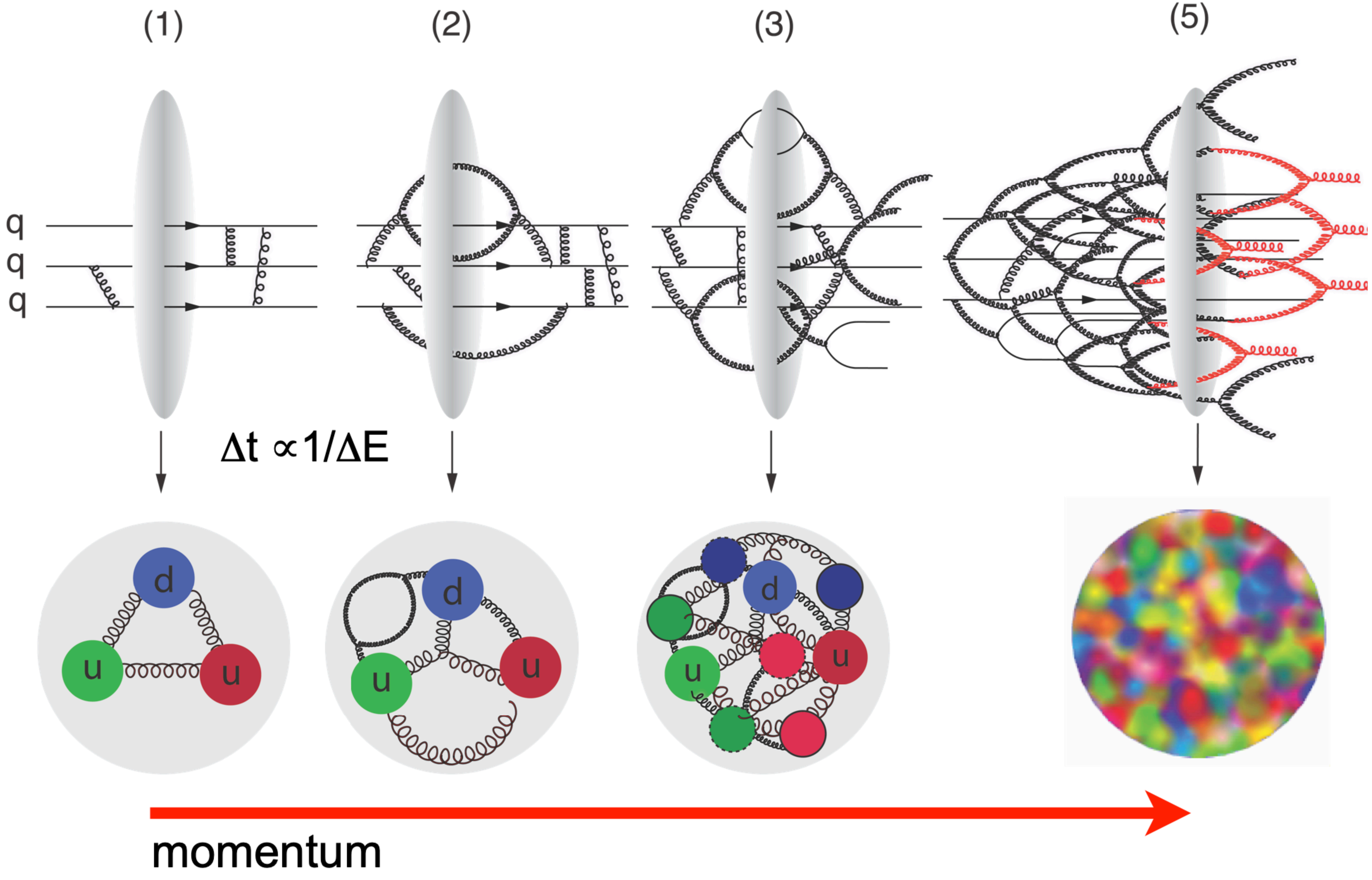


BUT: Recombination will balance gluon splittings

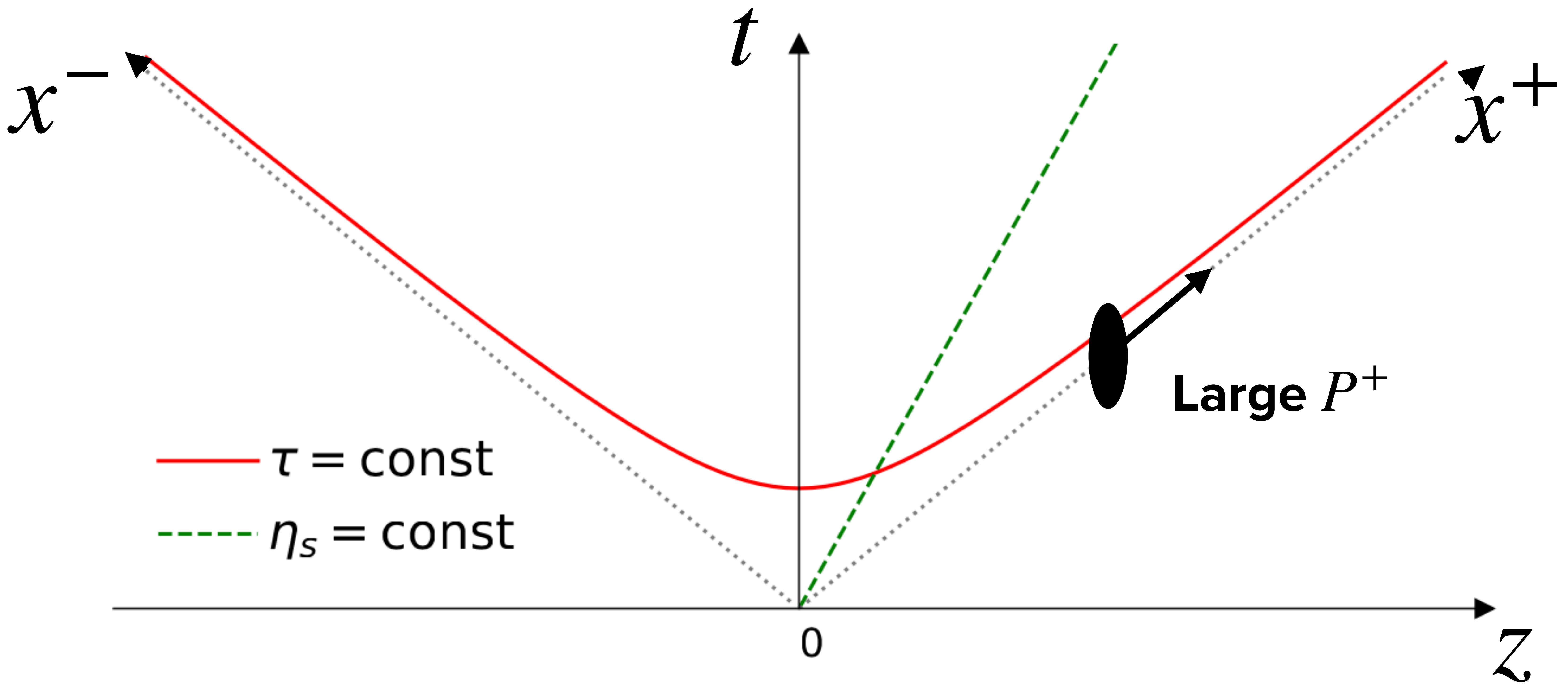
Need non-linear evolution equations at low x and low to moderate Q^2

Saturation of gluon densities is characterized by the scale $Q_s(x)$

The boosted proton



Light cone

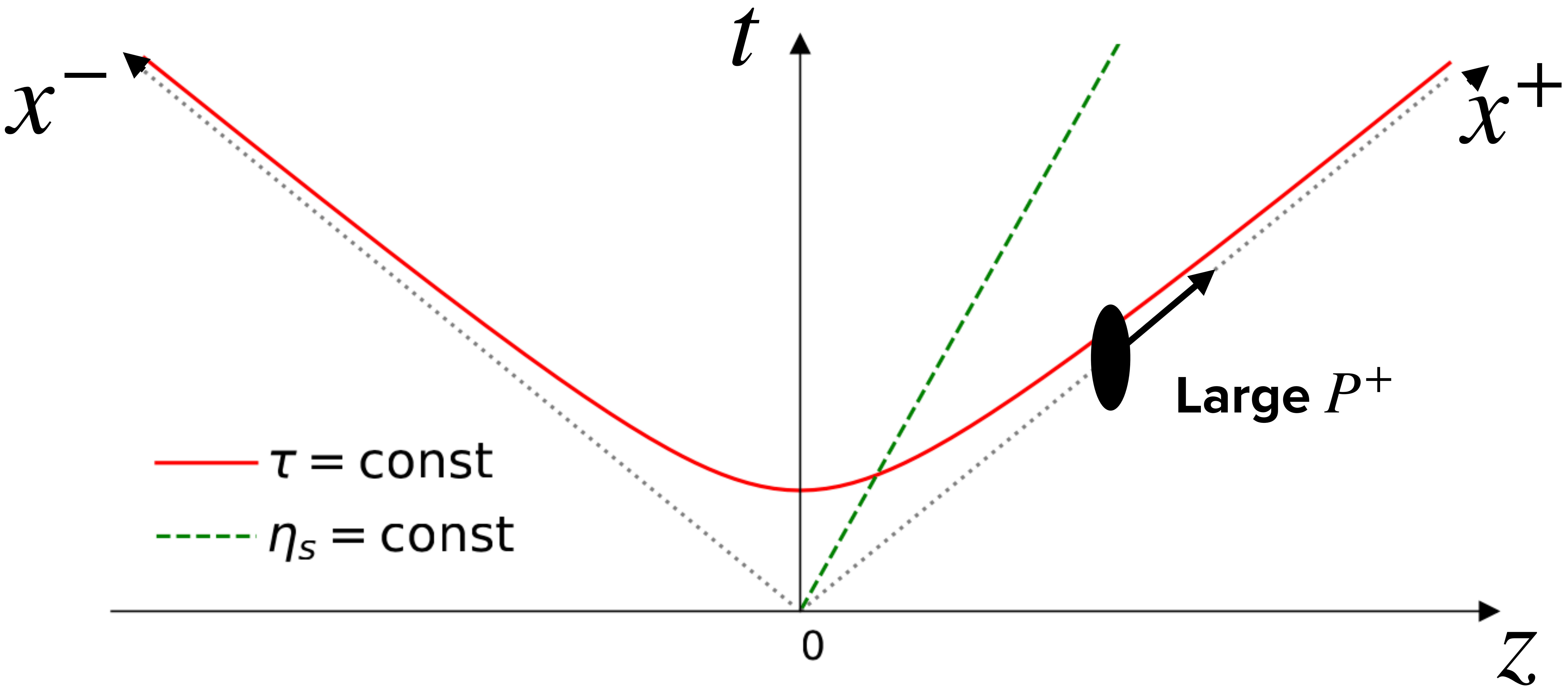


Light cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$

In the future light cone define $x^+ = \dfrac{\tau}{\sqrt{2}} e^{+\eta}$, and $x^- = \dfrac{\tau}{\sqrt{2}} e^{-\eta}$

or inverted $\tau = \sqrt{2x^+x^-}$, and $\eta = \dfrac{1}{2} \ln\left(\dfrac{x^+}{x^-}\right)$

Light cone

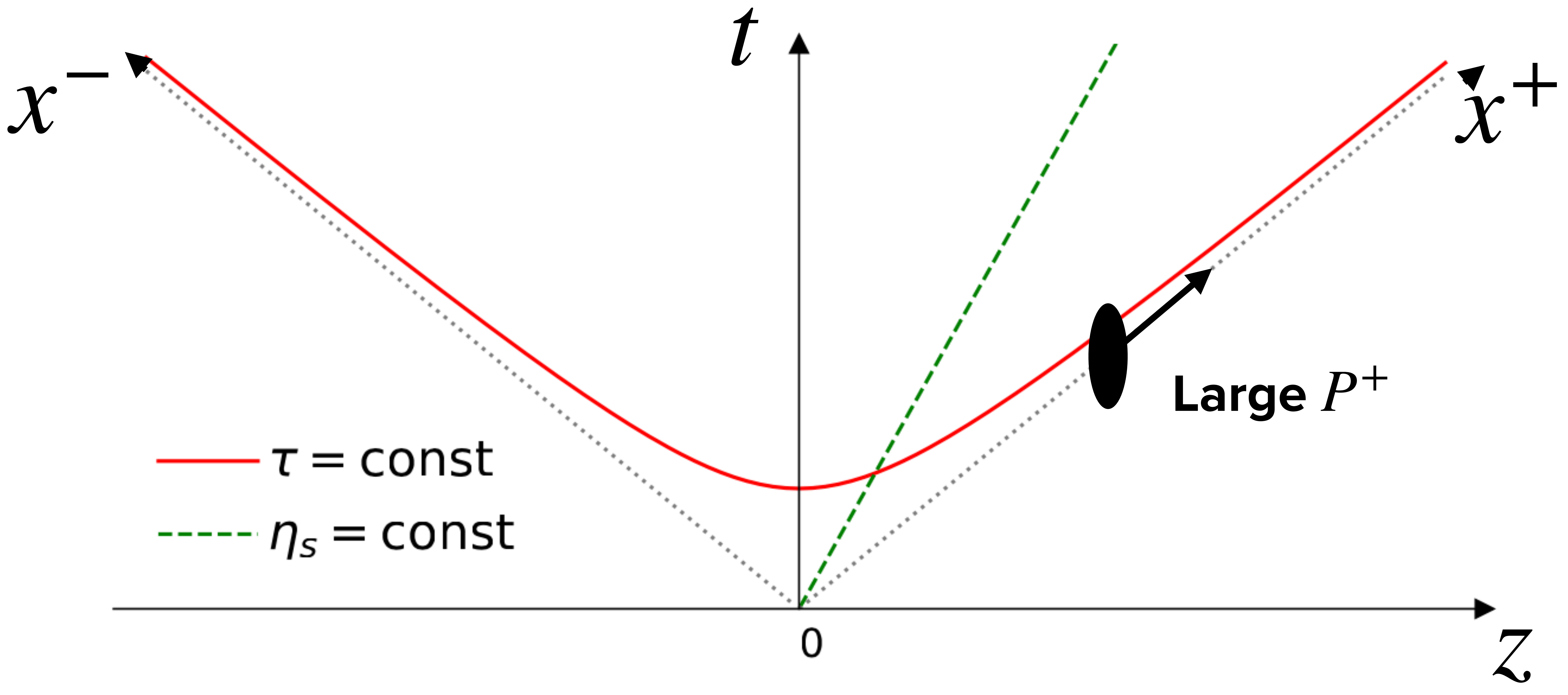


Probe hadron (or nucleus) moving with large P^+ at scale $x_0 P^+$ with $x_0 \ll 1$

Separate partonic content based on longitudinal momentum $k^+ = xP^+$

Large $x > x_0$: Static and localized color sources ρ

Color sources

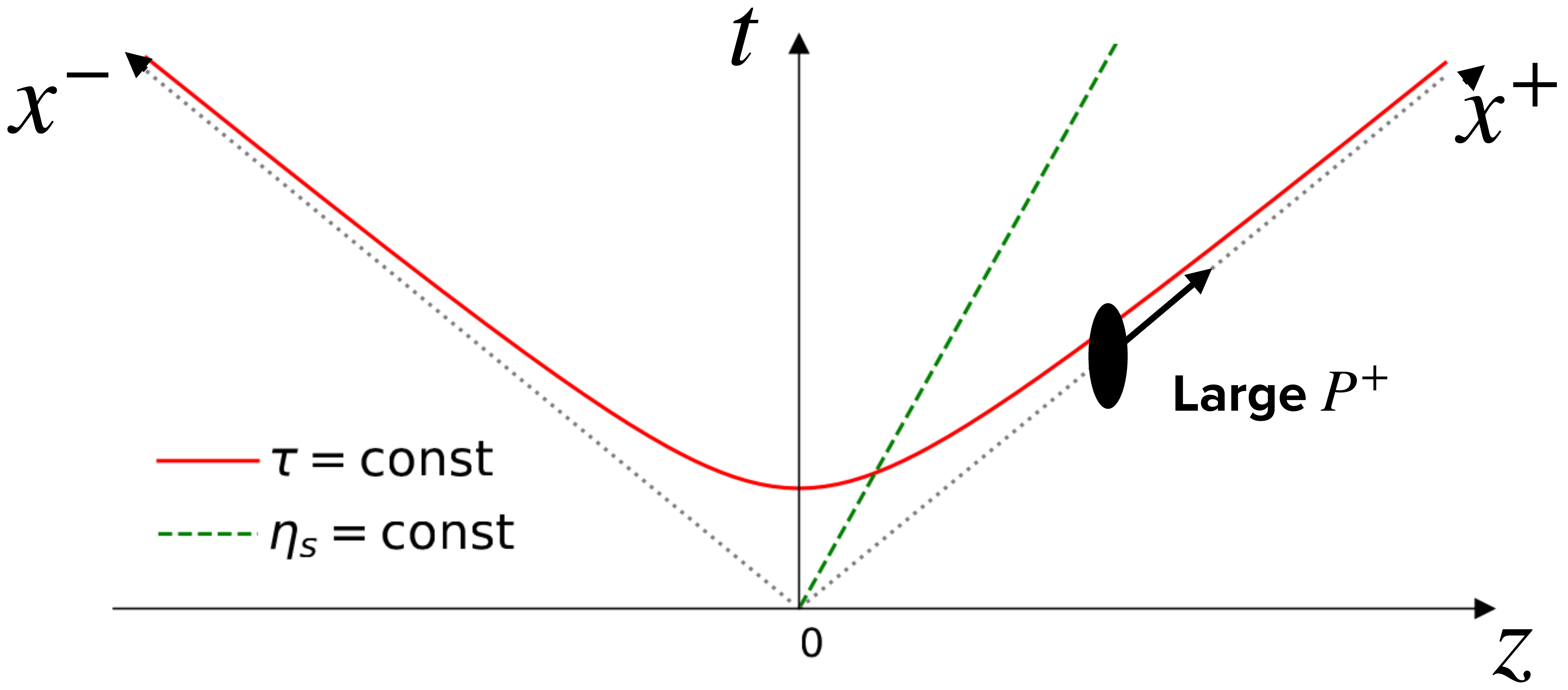


How localized are these sources? $\Delta z^- \sim \dfrac{1}{k^+} = \dfrac{1}{xP^+}$

What is the resolution scale of the probe? $\dfrac{1}{x_0 P^+} > \dfrac{1}{xP^+}$ **for** $x > x_0$

→**Look fully localized in** z^- **to the probe**

Color sources



How fast do they evolve? $\Delta z^+ \sim \dfrac{1}{k^-} = \dfrac{2k^+}{k_T^2} = \dfrac{2xP^+}{k_T^2}$ **(because** $a_\mu b^\mu = a^+b^- + a^-b^+ - \overrightarrow{a}_T \cdot \overrightarrow{b}_T$**)**

What is the time scale of the probe? $\tau \approx \dfrac{2x_0P^+}{k_T^2} < \dfrac{2xP^+}{k_T^2}$

→Look static in light cone time z^+ **to the probe**

Dynamic color fields

The moving color sources generate a current, independent of light cone time z^+:

$$J^{\mu,a}(z) = \delta^{\mu+}\rho^a(z^-, z_T)$$

a is the color index of the gluon
the name "color" comes from this

This current generates delocalized dynamical fields $A^{\mu,a}(z)$ described by the Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

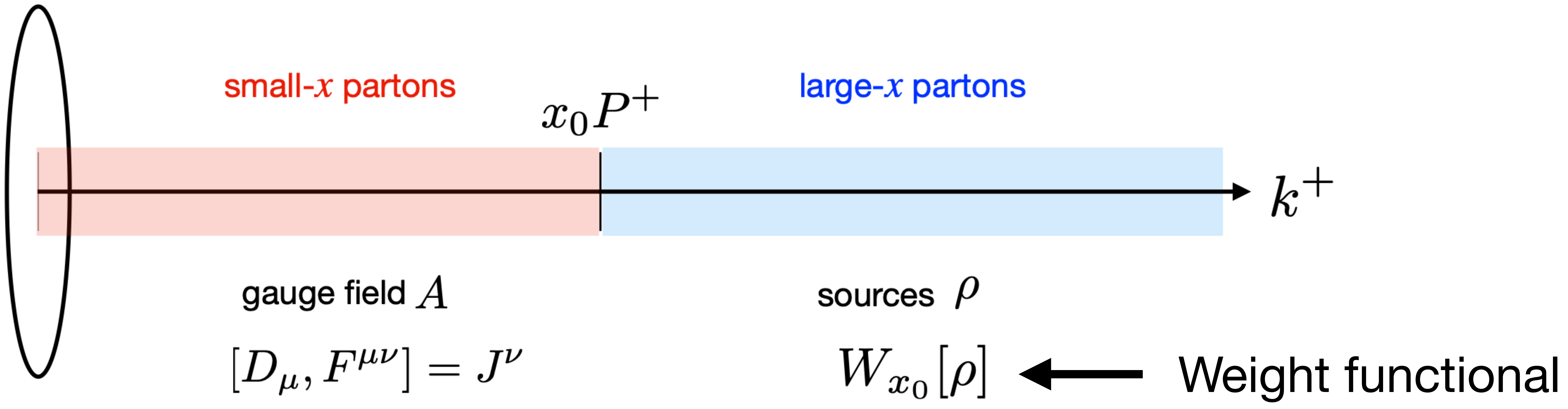
with $D_\mu = \partial_\mu + igA_\mu$ and $F_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

These fields are the small $x < x_0$ degrees of freedom

They can be treated classically, because their occupation number is large $\langle AA \rangle \sim 1/\alpha_s$

the name "condensate" comes from this scaling

Color Glass Condensate: Sources and fields



Two steps to compute expectation value of an observable $\mathcal{O}$:

1) Compute quantum expectation value $\mathcal{O}[\rho] = \langle \mathcal{O} \rangle_\rho$ for sources drawn from a given $W_{x_o}[\rho]$

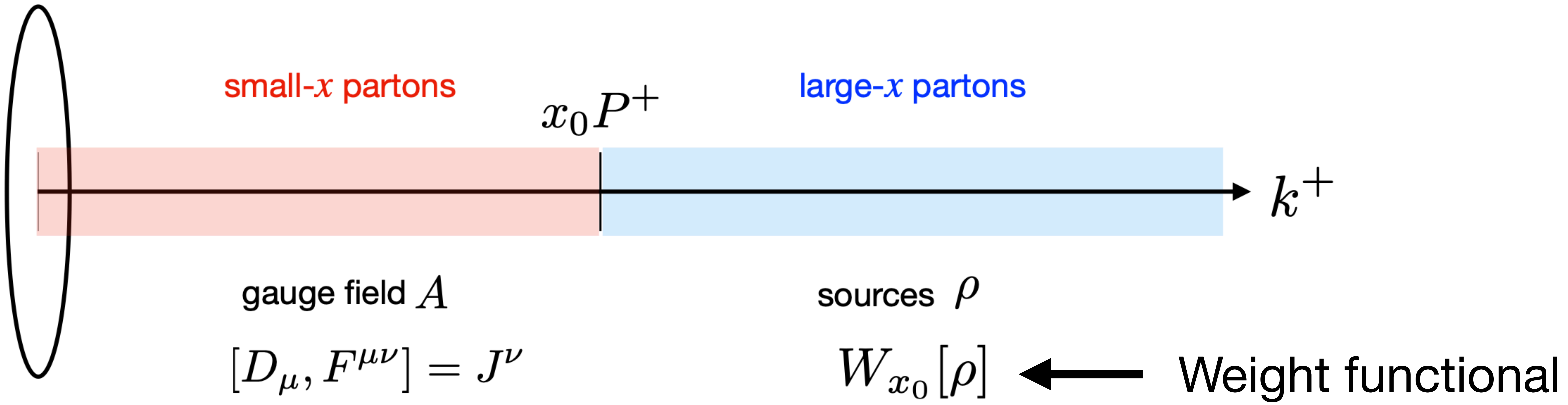
2) Average over all possible configurations given the appropriate gauge invariant weight functional $W_{x_o}[\rho]$ this situation is similar to spin glasses - the name "glass" comes from this

When $x \lesssim x_0$ the path integral $\langle \mathcal{O} \rangle_\rho$ is dominated by classical solution and we are done

For smaller x we need to do quantum evolution

Weight functional

https://arxiv.org/pdf/hep-ph/0406169.pdf



What is the weight functional?

Need to model. E.g. the McLerran-Venugopalan model:

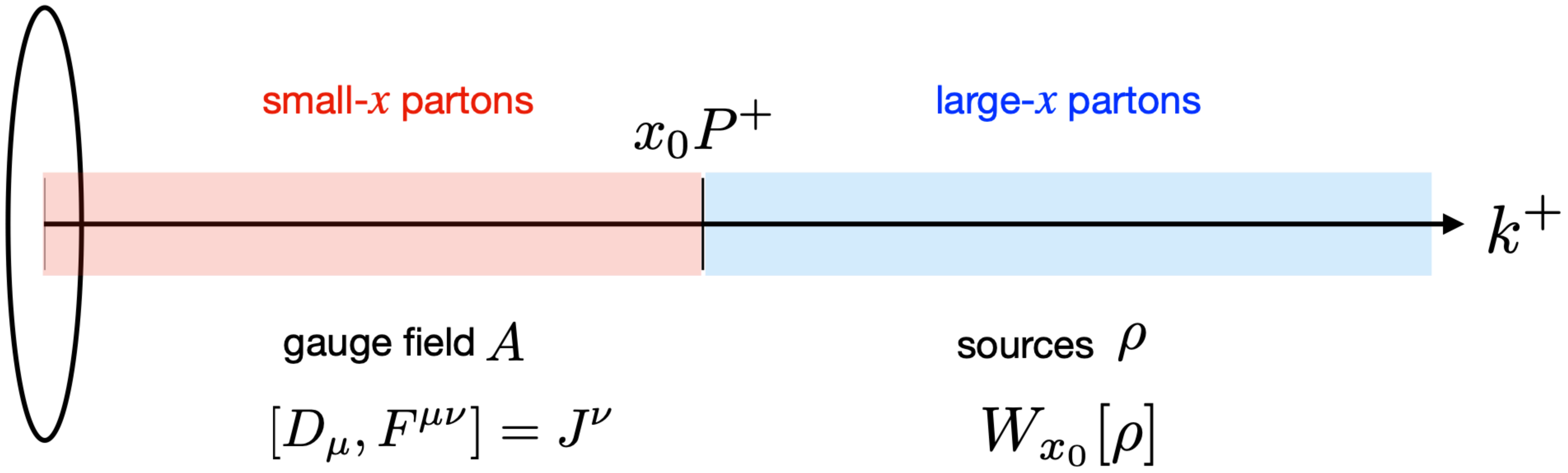
Assume a large nucleus, invoke central limit theorem. All correlations of ρ^a are Gaussian

$$W_{x_0}[\rho] = \mathcal{N} \exp\left(-\frac{1}{2} \int dx^- d^2 x_T \frac{\rho^a(x^-, x_T)\rho^a(x^-, x_T)}{\lambda_{x_0}(x^-)} \right)$$

where $\lambda_{x_0}(x^-)$ is related to the transverse color charge density distribution of the nucleus

Weight functional

https://arxiv.org/pdf/hep-ph/0406169.pdf



…where $\lambda_{x_0}(x^-)$ is related to the transverse color charge density distribution of the nucleus

$$\mu^2 = \int dx^- \lambda_{x_0}(x^-) = \frac{(g^2 C_F)(AN_c)}{\pi R_A^2} \frac{1}{N_c^2 - 1} = \frac{g^2 A}{2\pi R_A^2} \sim A^{1/3}$$

That color charge density is related to Q_s, the saturation scale.

normalized per color degree of freedom

Wilson lines

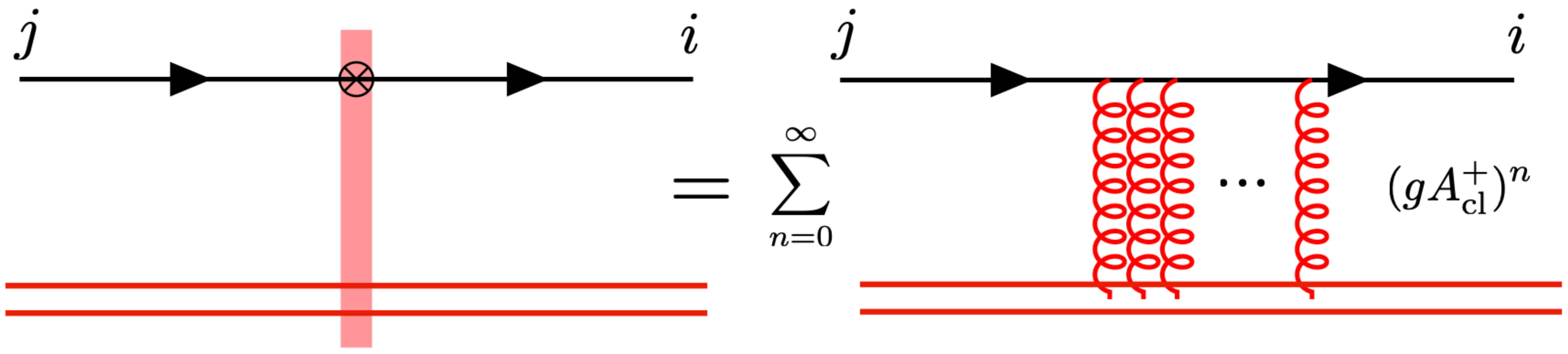
Interaction of high energy color-charged particle with large k^- momentum (and small $k^+ = \dfrac{k_T^2}{2k^-}$)

with a classical field of a nucleus can be described in the **eikonal approximation:**

The scattering rotates the color, but keeps k^-, transverse position $\vec{x}_T$, and any other quantum numbers the same.

The color rotation is encoded in a light-like Wilson line, which for a quark reads

$$V_{ij}(\vec{x}_T) = \mathscr{P}\left(ig \int_{-\infty}^{\infty} A^{+,c}(z^-, \vec{x}_T)\, t^c_{ij}\, dz^- \right)$$



MULTIPLE INTERACTIONS NEEDED TO BE RESUMMED, BECAUSE
$A^+ \sim 1/g$

Wilson lines

Interaction of high energy color-charged particle with large k^- momentum (and small $k^+ = \frac{k_T^2}{2k^-}$)

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path ordering
(for the lattice formulation we actually use anti-path ordering)

SU(3) generator (fundamental rep.)

Wilson lines

For a gluon interacting with the target, we have

$$U_{ab}(\overrightarrow{x}_T) = \mathscr{P}\left(ig\int_{-\infty}^{\infty} A^{+,c}(z^-, \overrightarrow{x}_T)\, T^c_{ab}\, dz^- \right)$$

SU(3) generator (adjoint rep.)

Wilson lines and correlators

$$U_{ab}(\overrightarrow{x}_T) = \mathscr{P}\left(ig \int_{-\infty}^{\infty} A^{+,c}(z^-, \overrightarrow{x}_T)\, T^c_{ab}\, dz^- \right)$$ gluon scattering

$$V_{ij}(\overrightarrow{x}_T) = \mathscr{P}\left(ig \int_{-\infty}^{\infty} A^{+,c}(z^-, \overrightarrow{x}_T)\, t^c_{ij}\, dz^- \right)$$ quark scattering

These Wilson lines are the building blocks of the CGC. At the EIC for example, cross sections will be calculated as convolutions of Wilson line correlators with perturbatively calculable and process-dependent impact factors

In heavy ion collisions, one can compute particle production by determining Wilson lines after the collision from the Wilson lines of the colliding nuclei. We will get to that next time.

Everything is Wilson lines + some perturbative, process dependent stuff

$$V$$

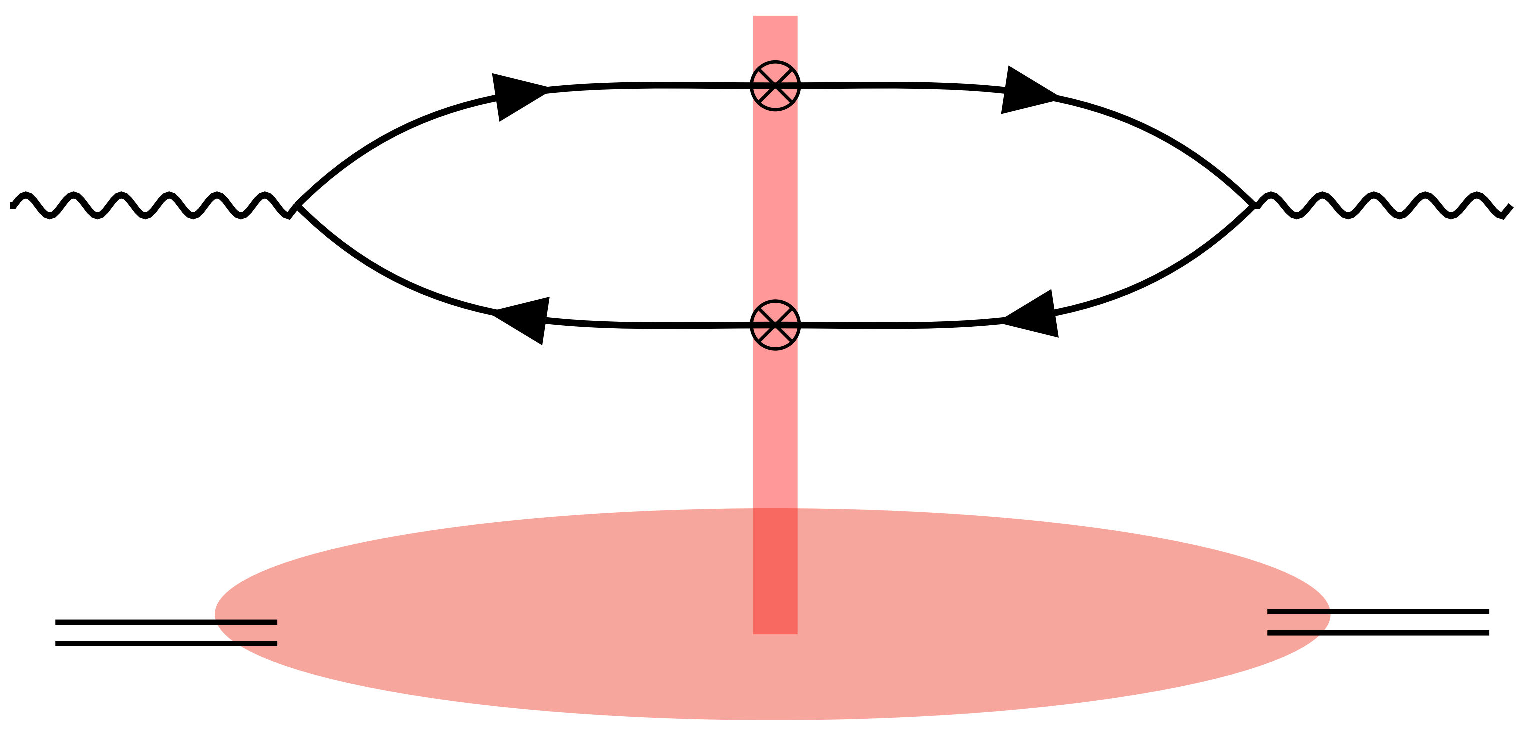
Dipole correlator

$$S^{(2)}_{x_0}(\vec{x}_T, \vec{y}_T) = \frac{1}{N_c} \left\langle \mathrm{Tr} \left[V(\vec{x}_T) V^\dagger(\vec{y}_T) \right] \right\rangle_{x_0}$$

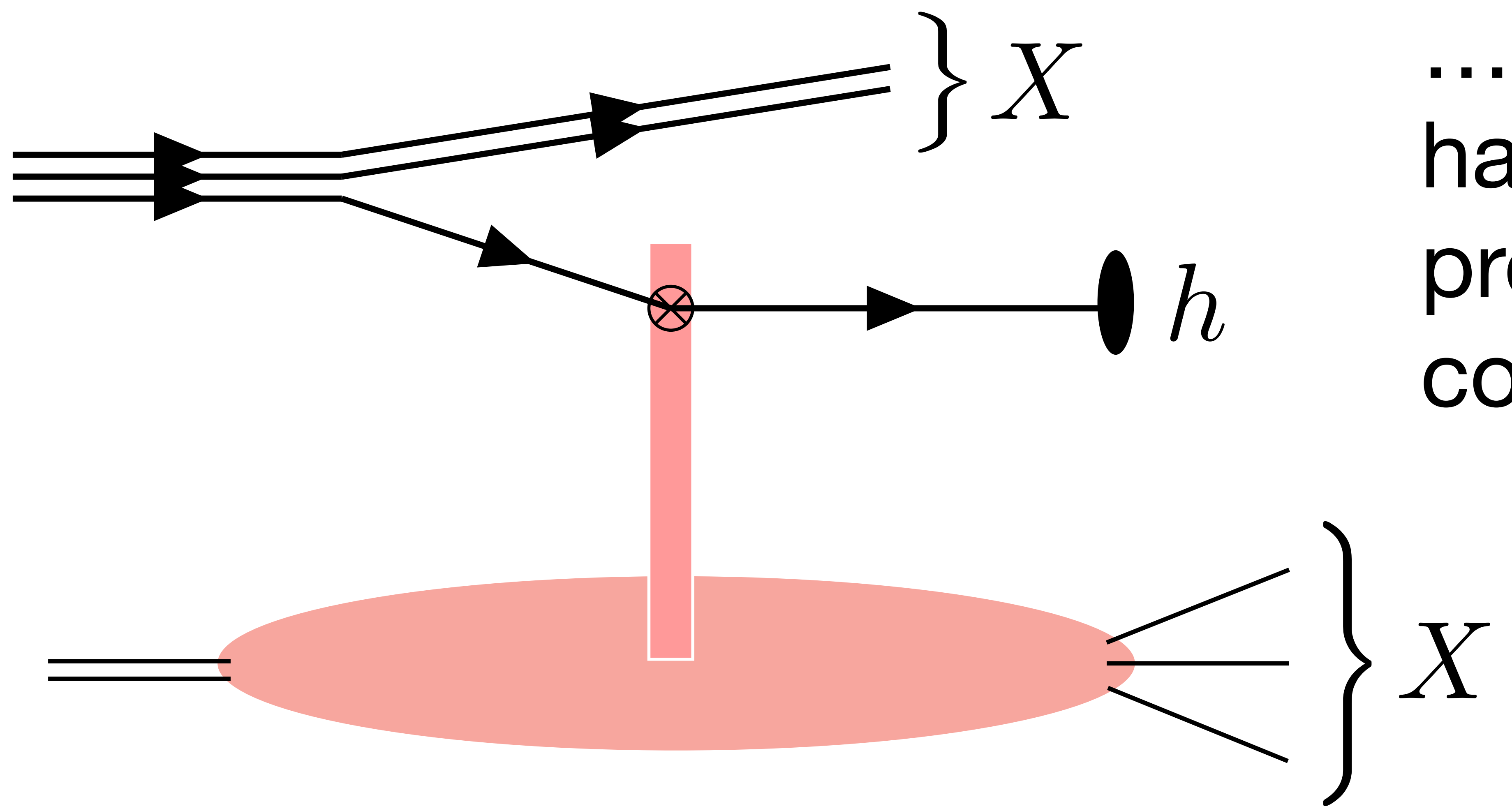
Dipole correlator is the simplest and most important

It appears for example in the deep inelastic scattering (DIS) cross section at small x …

$eA \to e + X$



$pA \to h + X$



… and also in hadron production in p+A collisions

Dipole correlator in the MV model

$$S_{x_0}^{(2)}(r_T) = \exp\left[-\frac{1}{4}\alpha_s C_F \mu^2 r_T^2 \ln\left(\frac{1}{\Lambda r_T} + e\right)\right]$$

depends only on the dipole size $r_T = |\overrightarrow{x}_T - \overrightarrow{y}_T|$

Λ is an infrared cutoff

Small r_T: scattering matrix is close to unity, i.e. there is no scattering

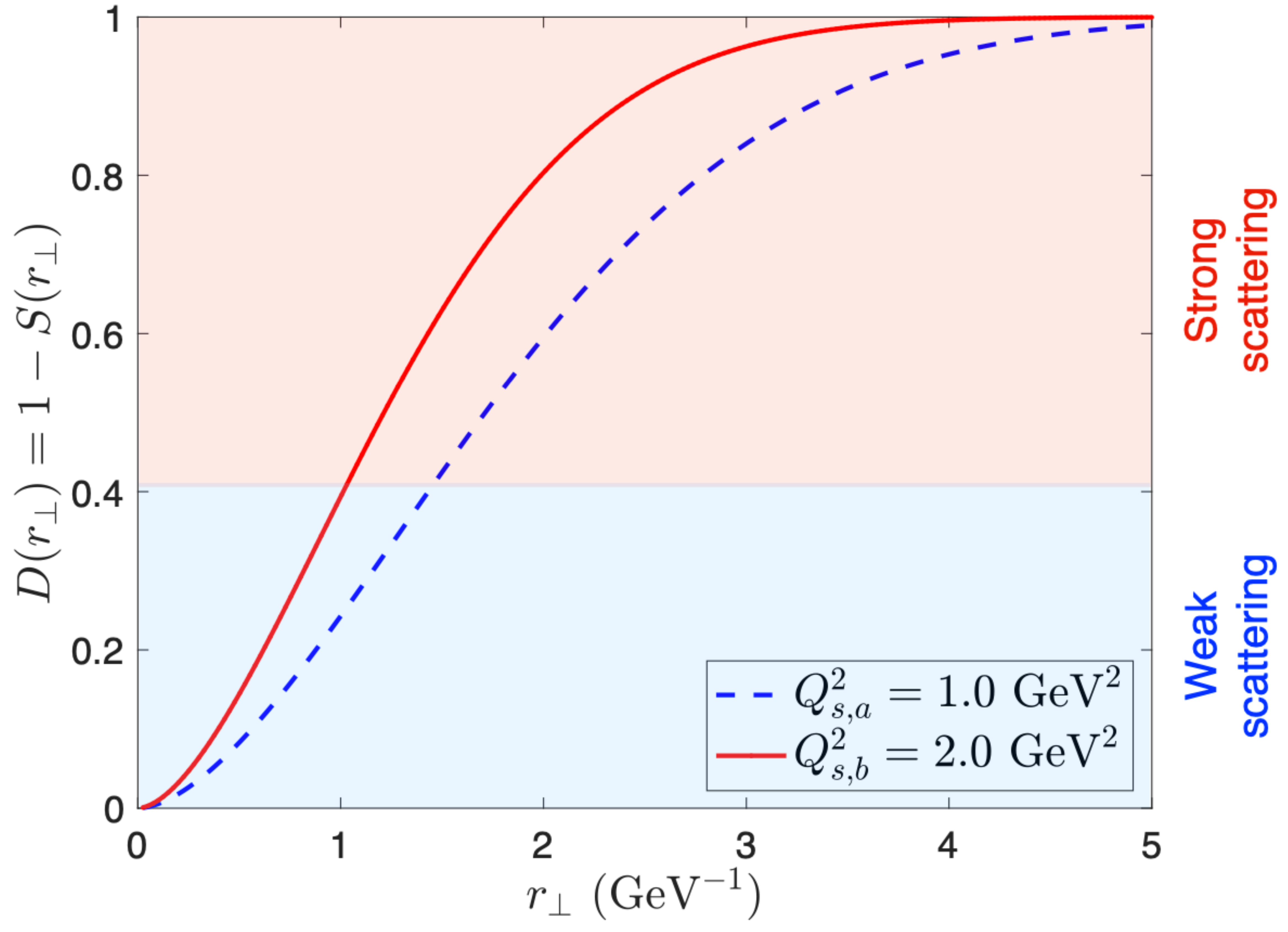
Makes sense: an infinitesimally small dipole is a color neutral object

Mathematically this follows from the unitarity of the Wilson lines $V(\overrightarrow{x})V^\dagger(\overrightarrow{x}) = 1$

Large r_T: Wilson lines decorrelate, correlator goes to zero as it should in the black disk limit

Dipole amplitude and saturation scale

$$D_{x_0}(r_T) = 1 - S^{(2)}_{x_0}(r_T) = 1 - \exp\left[-\frac{1}{4}\alpha_s C_F \mu^2 r_T^2 \ln\left(\frac{1}{\Lambda r_T} + e\right)\right]$$



The transition between the weak and strong scattering regimes defines the saturation scale

$$Q_s^2 = \frac{2}{r_{T,s}^2}$$

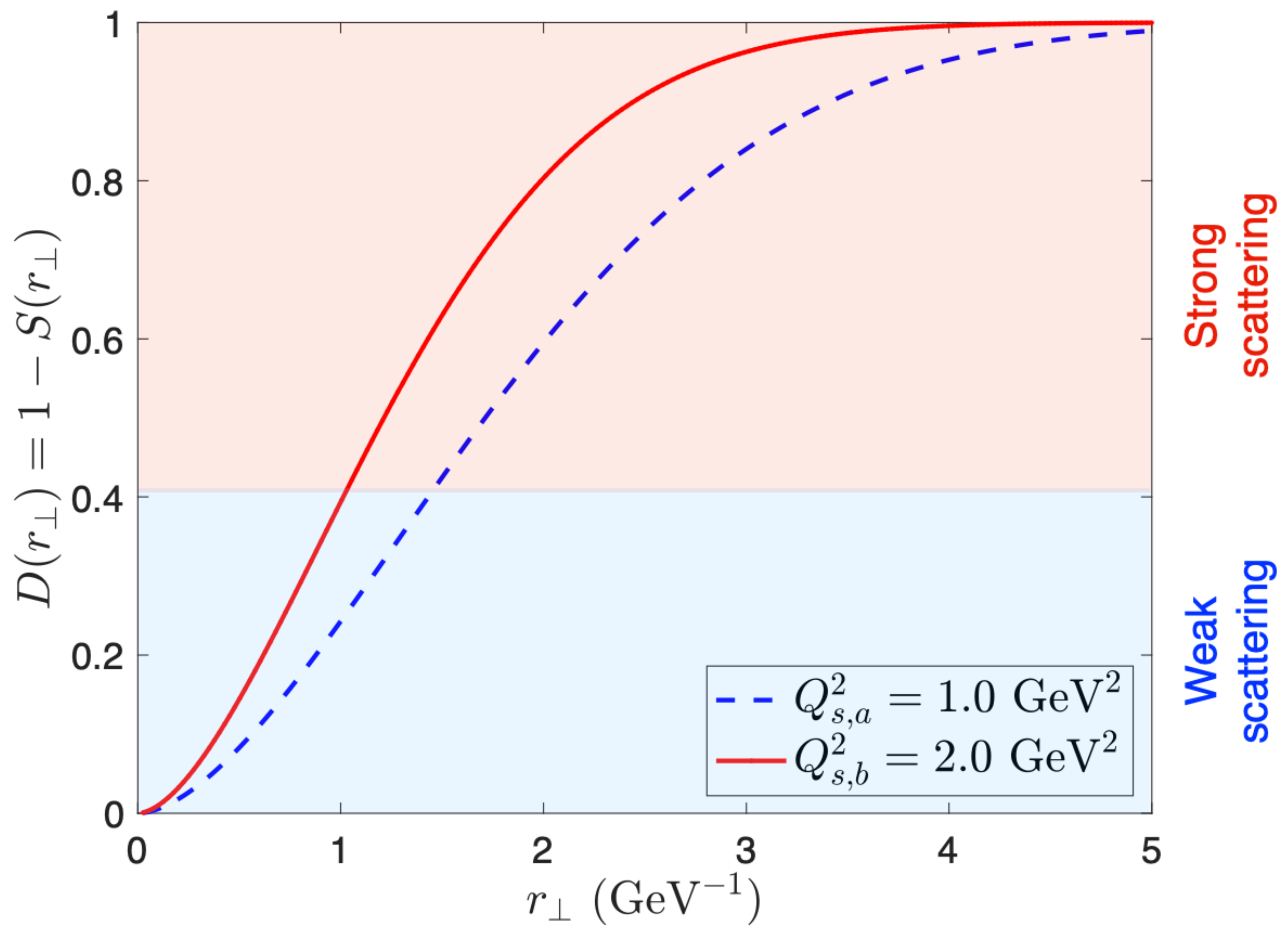
where

$$S^{(2)}_{x_0}(r_{T,s}) = \exp(-c)$$

with c a constant usually chosen to be $1/2$

Dipole amplitude and saturation scale

$$D_{x_0}(r_T) = 1 - S^{(2)}_{x_0}(r_T) = 1 - \exp\left[-\frac{1}{4}\alpha_s C_F \mu^2 r_T^2 \ln\left(\frac{1}{\Lambda r_T} + e\right)\right]$$

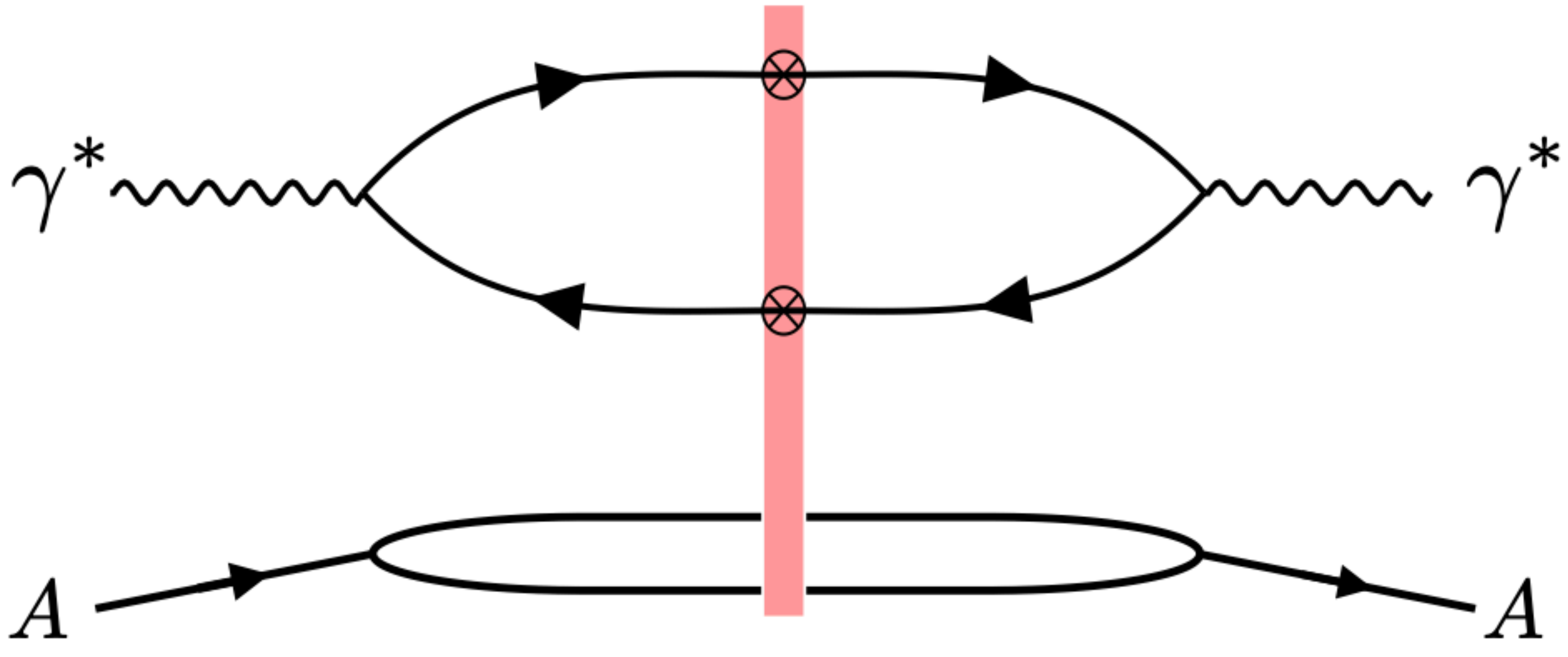


The form above shows $Q_s^2 \sim \mu^2 \sim A^{1/3}$

So Q_s gains a nuclear *oomph* factor when scattering off large nuclei compared to protons

Total DIS cross section

The dipole amplitude appears e.g. in the total DIS cross section



$\mathcal{M}^{\gamma^* A \to \gamma^* A}$

$$\sigma_\lambda^{\gamma^* A} = 2\,\mathrm{Im}(\mathscr{M}_\lambda^{\gamma^* A \to \gamma^* A})$$

$$= 2 \int d^2\vec{r}_T\, d^2\overrightarrow{b}_T \int_0^1 dz\, \left|\psi_\lambda^{\gamma^*}(\vec{r}_T, Q^2, z)\right|^2 \left[1 - S_x^{(2)}\left(\overrightarrow{b}_T + \frac{\vec{r}_T}{2}, \overrightarrow{b}_T - \frac{\vec{r}_T}{2}\right)\right]$$

Q^2 is the virtuality of the photon, λ its polarization
$\psi_\lambda^{\gamma^*}(\vec{r}_T, Q^2, z)$ is the light-cone wave-function that describes the splitting of the virtual photon into the q-$\bar{q}$ pair. Quark has longitudinal momentum fraction z, anti-quark $1 - z$

$\overrightarrow{b}_T$ dependence needs to be modeled…

Total DIS cross section - sensitive to saturation?

x is the longitudinal momentum fraction at which the nucleus is probed

$x = Q^2/W^2$ with W the center of mass energy per nucleon of the photon-nucleus system

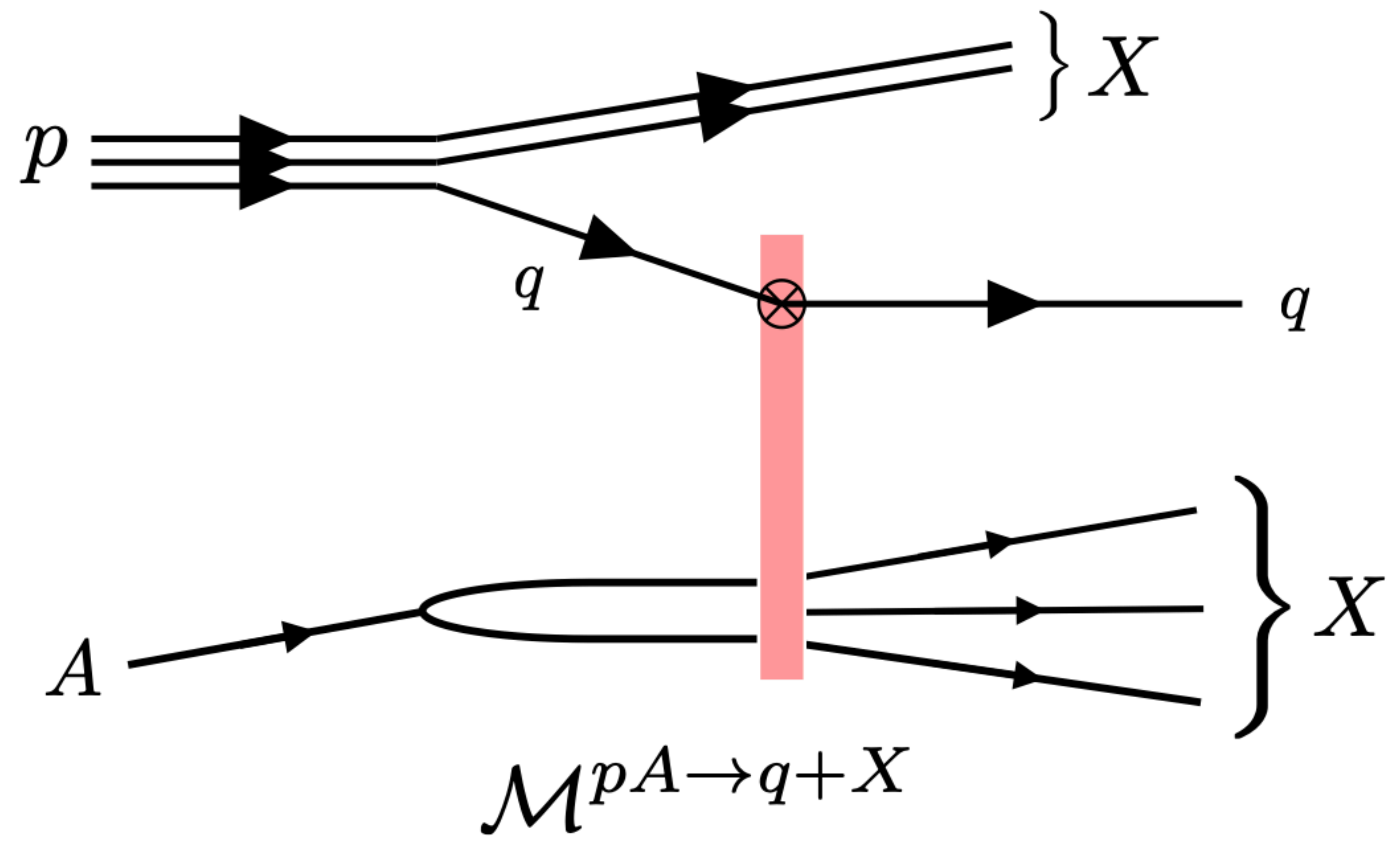
$$\sigma_\lambda^{\gamma^*A} = 2\,\mathrm{Im}(\mathscr{M}_\lambda^{\gamma^*A\to\gamma^*A})$$
$$= 2\int d^2\vec{r}_T\, d^2\overrightarrow{b}_T \int_0^1 dz\, \left|\psi_\lambda^{\gamma^*}(\vec{r}_T, Q^2, z)\right|^2 \left[1 - S_x^{(2)}\left(\overrightarrow{b}_T + \frac{\vec{r}_T}{2}, \overrightarrow{b}_T - \frac{\vec{r}_T}{2}\right)\right]$$

Accessing the saturated regime requires dipole sizes $r_T \sim 1/Q_s$

Now, the lightcone wave function $\psi_\lambda^{\gamma^*}$ suppresses dipole sizes with $r_T^2 \gtrsim 1/Q^2$

So we are limited to virtualities in the range $\Lambda_{QCD}^2 \ll Q^2 \lesssim Q_s^2$ to access saturation in DIS

Forward quark production in p+A collisions



The dipole correlator also appears in the cross section for forward quark production: $V(\vec{x}_T)$ from the amplitude, $V^\dagger(\vec{y}_T)$ from the conjugate amplitude.

The cross section reads $$\frac{d\sigma^{pA\to qX}}{dy d^2k_T} = \frac{1}{(2\pi)^2} x_p\, q(x_p)\, C_{x_A}(\vec{k}_T)$$

where $\vec{k}_T$ and y are the transverse momentum and rapidity of produced quark

$x_p\, q(x_p)$ is the quark distribution in the proton for a collinear quark with momentum fraction x_p

x_A is the longitudinal momentum fraction of gluons probed in the nucleus

Forward quark production

The cross section reads $$\frac{d\sigma^{pA\to qX}}{dy d^2k_T} = \frac{1}{(2\pi)^2} x_p\, q(x_p)\, C_{x_A}(\vec{k}_T)$$

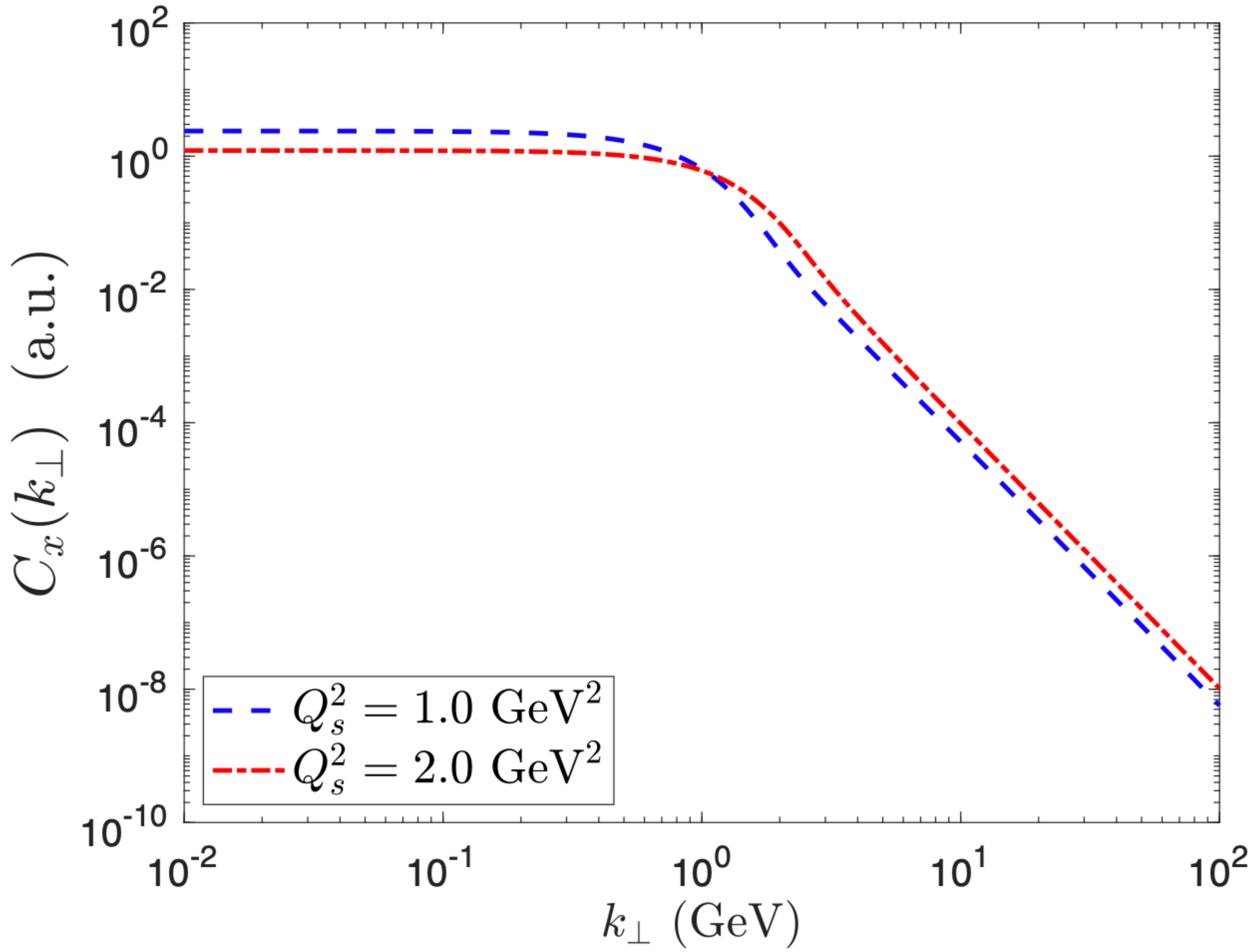
$$C_{x_A}(\vec{k}_T) = \int d^2x_T d^2y_T e^{-i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} S^{(2)}_{x_A}(\vec{x}_T, \vec{y}_T)$$

determining the transverse momentum kick $\vec{k}_T$ acquired by the quark as it traverses the nucleus

The function has two limits:

Perturbative regime: $C_x(\vec{k}_T) \sim \frac{Q_s^2(x)}{k_T^4}$

Saturation regime: $C_x(\vec{k}_T) \sim \frac{1}{Q_s^2(x)}$



Multi-gluon correlators

Besides the dipole correlator, more complicated correlators of Wilson lines appear for example in cross sections of less exclusive processes or in the small-x evolution equations (which we will get to)

For example:

$$S^{(2,2)}_{x_0} = \frac{1}{N_c^2} \left\langle \mathrm{Tr}[V(\vec{x}_T)V^\dagger(\vec{y}_T)]\mathrm{Tr}[V(\vec{y}'_T)V^\dagger(\vec{x}'_T)] \right\rangle_{x_0}$$

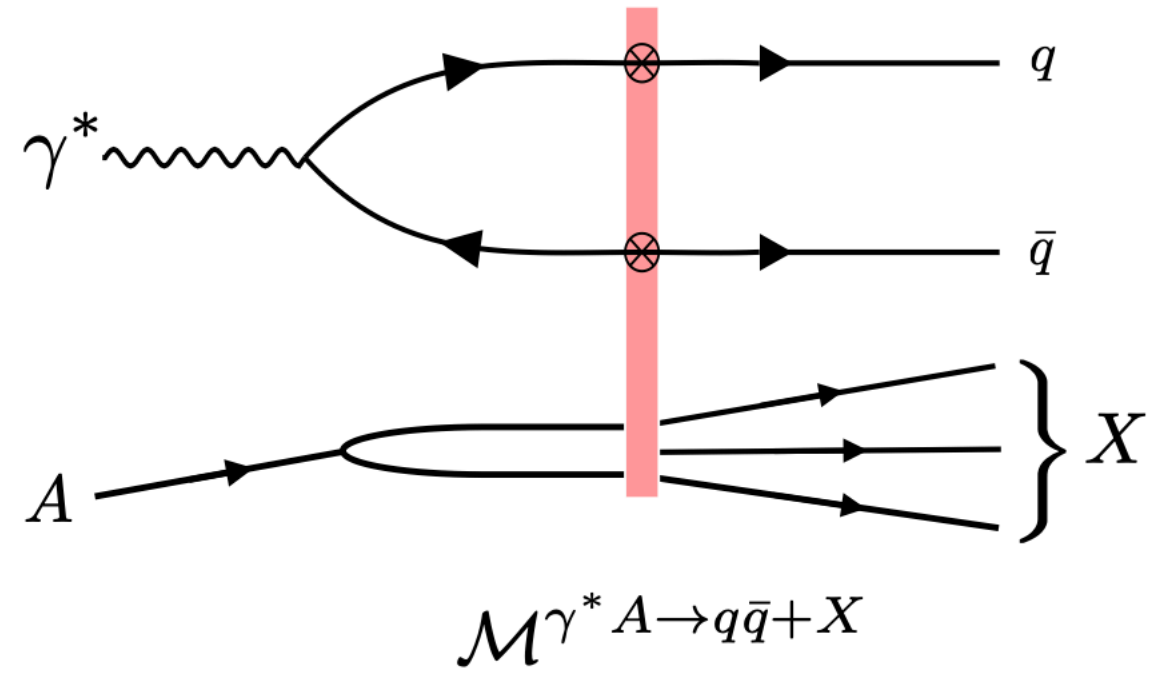
double dipole

$$S^{(4)}_{x_0} = \frac{1}{N_c} \left\langle \mathrm{Tr}[V(\vec{x}_T)V^\dagger(\vec{y}_T)V(\vec{y}'_T)V^\dagger(\vec{x}'_T)] \right\rangle_{x_0}$$

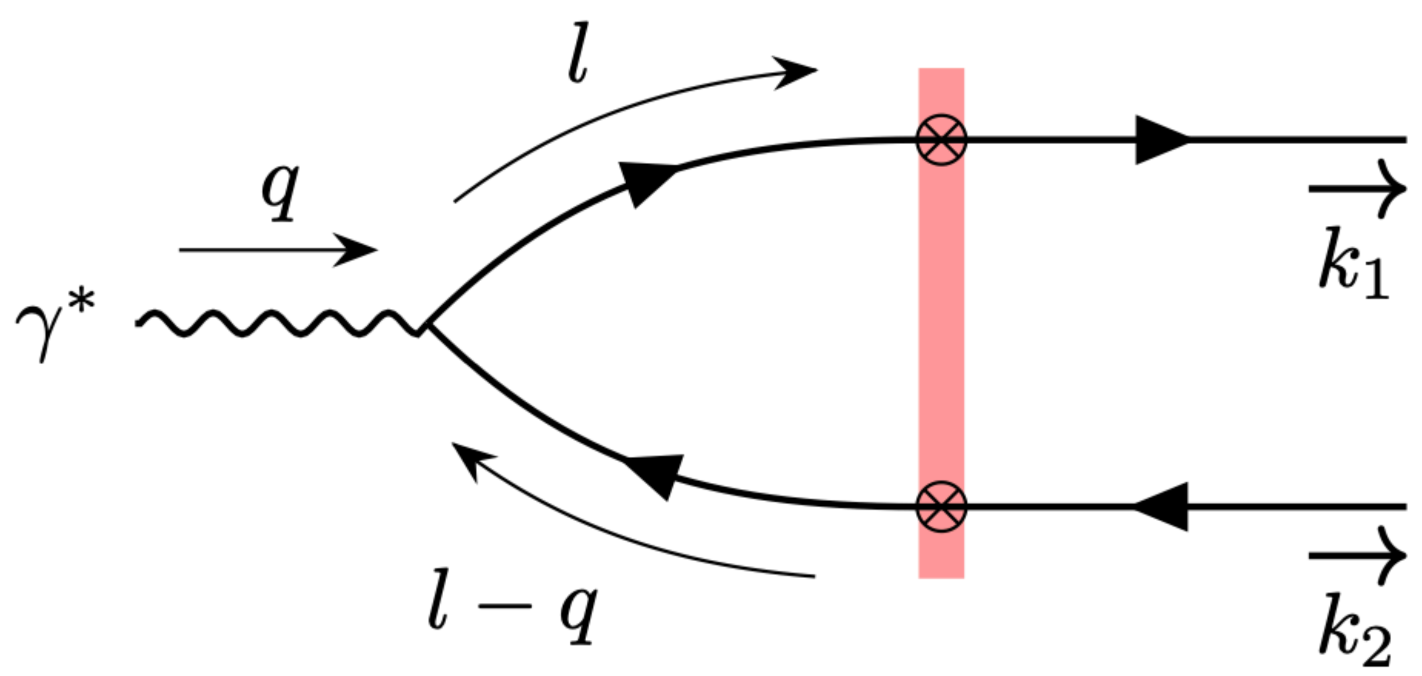
quadrupole

quadrupole appears for example in inclusive dijet production in DIS

double dipole appears in diffractive dijet production



TMD factorization



https://arxiv.org/abs/1101.0715

Define $\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}\,,$

$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}\,,$

In the back-to-back limit for the dijet (produced q-$\bar{q}$ pair), where $k_T \ll P_T$ one can establish TMD factorization (TMD: Transverse momentum dependent parton distribution function)

That is assuming that the back to back limit is equivalent to the limit $r_T \ll b_T$ *

Then, one expands $\left[\mathbb{1} - V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp)\right] = -\boldsymbol{r}_\perp^j \left[V(\boldsymbol{b}_\perp)\,\partial^j V^\dagger(\boldsymbol{b}_\perp)\right] + \mathcal{O}\left(\boldsymbol{r}_\perp^2\right)$ in the amplitude

What appears is the gluon field $A^i(\vec{b}_T) = \frac{i}{g} V(\vec{b}_T)\partial^i V^\dagger(\vec{b}_T)$

* : that is not quite true and there are corrections to this:
R. Boussarie, H. Mäntysaari, F. Salazar, B. Schenke, JHEP 09 (2021) 178

TMD factorization

https://arxiv.org/abs/1101.0715

Differential cross section:

$$\frac{d\sigma_\lambda^{\gamma^*+A\to q\bar{q}+X}}{dz_1 dz_2 d^2k_T d^2P_T} = \delta(1-z_1-z_2)\, H^{ij,\lambda}_{\gamma^* g\to q\bar{q}}(Q^2, \overrightarrow{P}_T, z)\, xG^{ij}_{\mathrm{WW}}(x, \overrightarrow{k}_T)$$

where $H^{ij,\lambda}_{\gamma^* g\to q\bar{q}}(Q^2, \overrightarrow{P}_T, z)$ is the hard factor, calculable perturbatively,

with $\overrightarrow{P}_T$ the mean transverse momentum of the jets

and $xG^{ij}_{\mathrm{WW}}(x, \overrightarrow{k}_T)$ is the Weizsäcker-Williams gluon TMD with $\overrightarrow{k}_T$ the momentum imbalance

$$xG^{ij}_{\mathrm{WW}}(x, \overrightarrow{k}_T) = \frac{4}{(2\pi)^3}\int d^2b_T d^2b'_T\, e^{-i\overrightarrow{k}_T\cdot(\overrightarrow{b}_T-\overrightarrow{b}'_T)} \left\langle \mathrm{Tr}[A^i(\overrightarrow{b}_T)A^j(\overrightarrow{b}'_T)]\right\rangle_x$$

WW gluon TMD

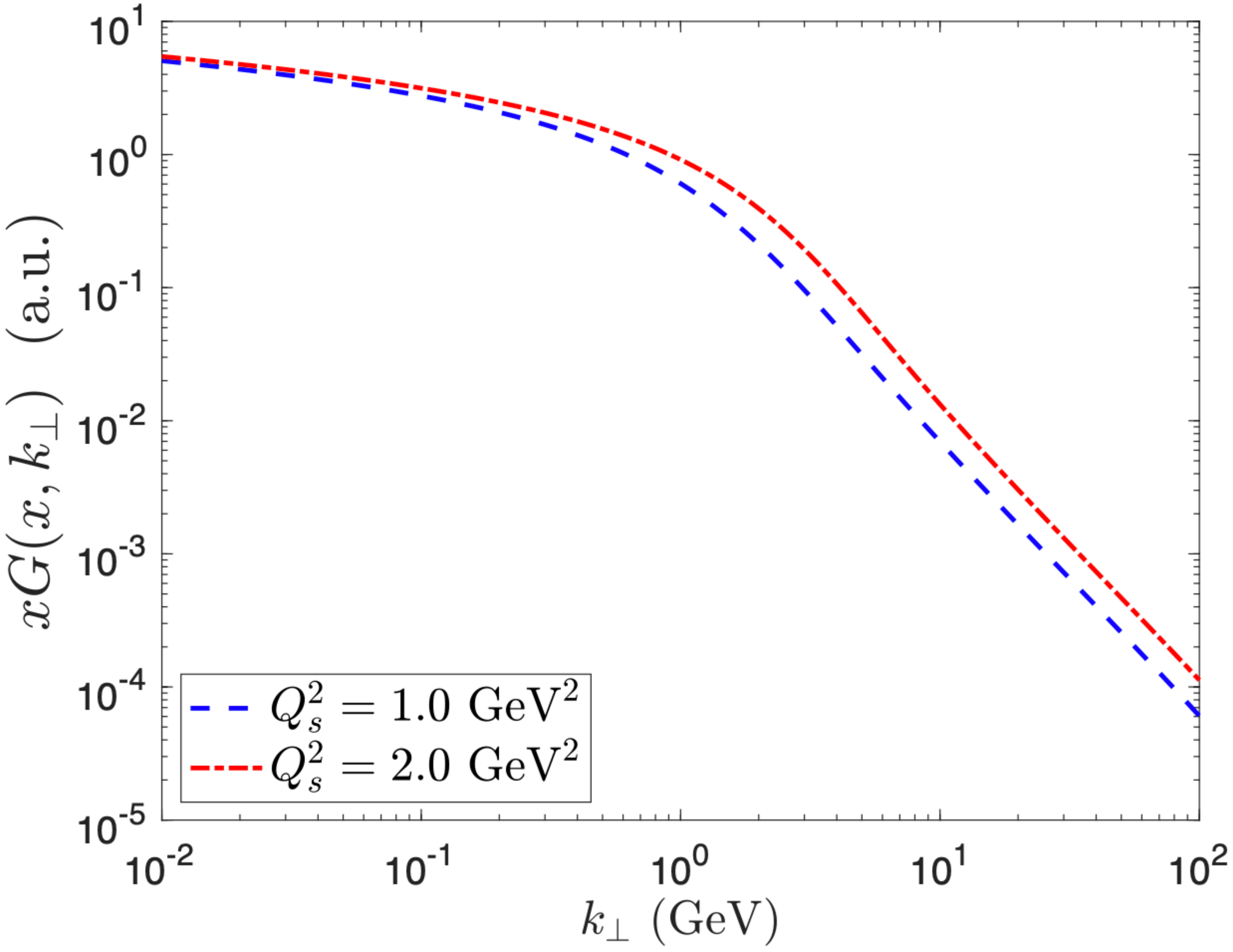
https://arxiv.org/abs/1101.0715

This distribution

$$xG^{ij}_{\rm WW}(x, \vec{k}_T) = \frac{4}{(2\pi)^3} \int d^2b_T d^2b'_T \, e^{-i\vec{k}_T\cdot(\vec{b}_T - \vec{b}'_T)} \left\langle {\rm Tr}[A^i(\vec{b}_T)A^j(\vec{b}'_T)] \right\rangle_x$$

has a probability density interpretation

(unlike the FT of the dipole correlator)



Perturbative regime: $xG^{ii}(x, \vec{k}_T) \sim \frac{Q_s^2(x)}{\vec{k}_T^2}$

Saturation regime: $xG^{ii}(x, \vec{k}_T) \sim \ln\left(\frac{Q_s^2(x)}{\vec{k}_T^2}\right)$

WW gluon TMD

https://arxiv.org/abs/1101.0715

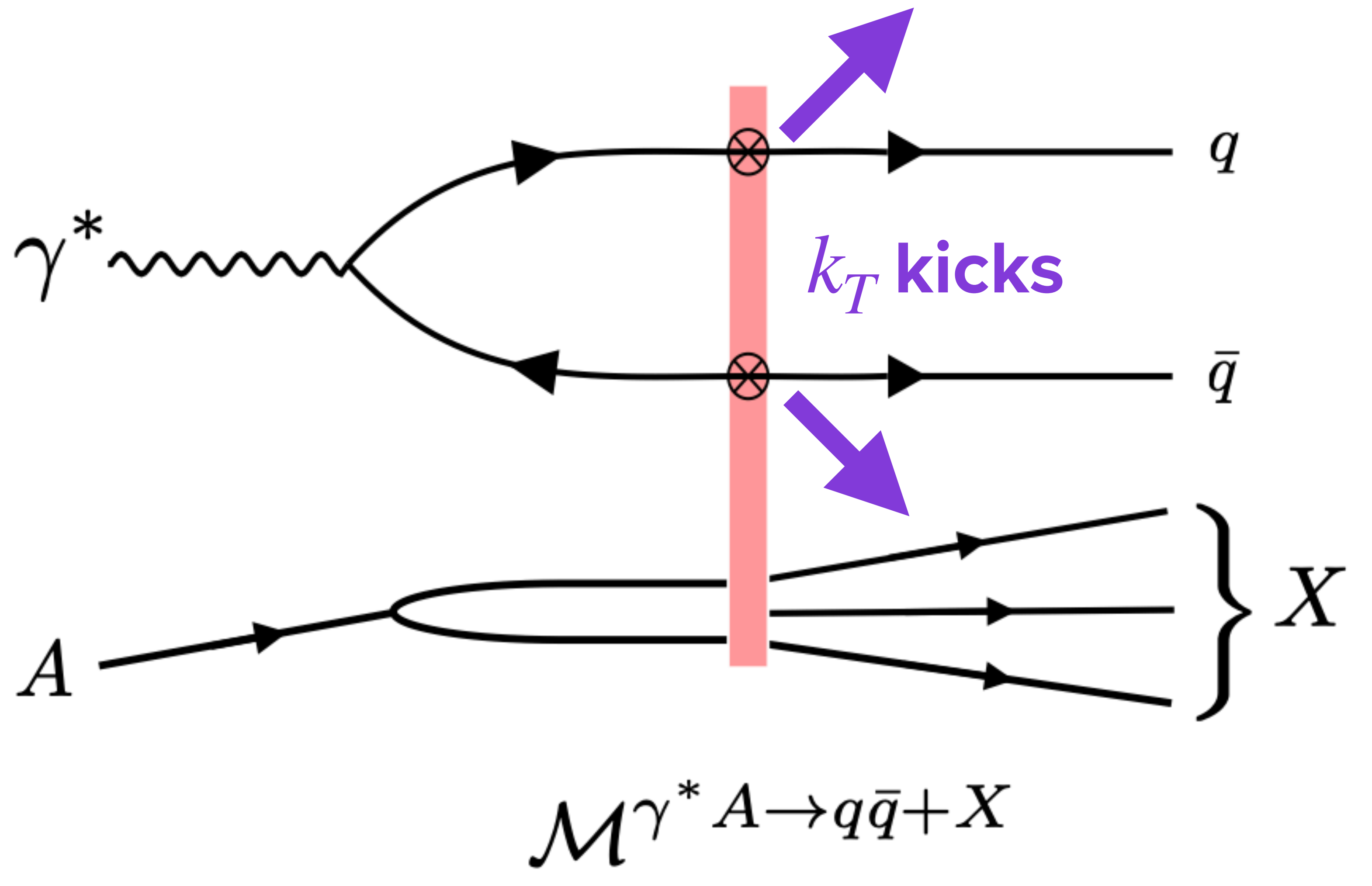
The WW gluon TMD contains a correlator of light-like Wilson lines and their derivatives:

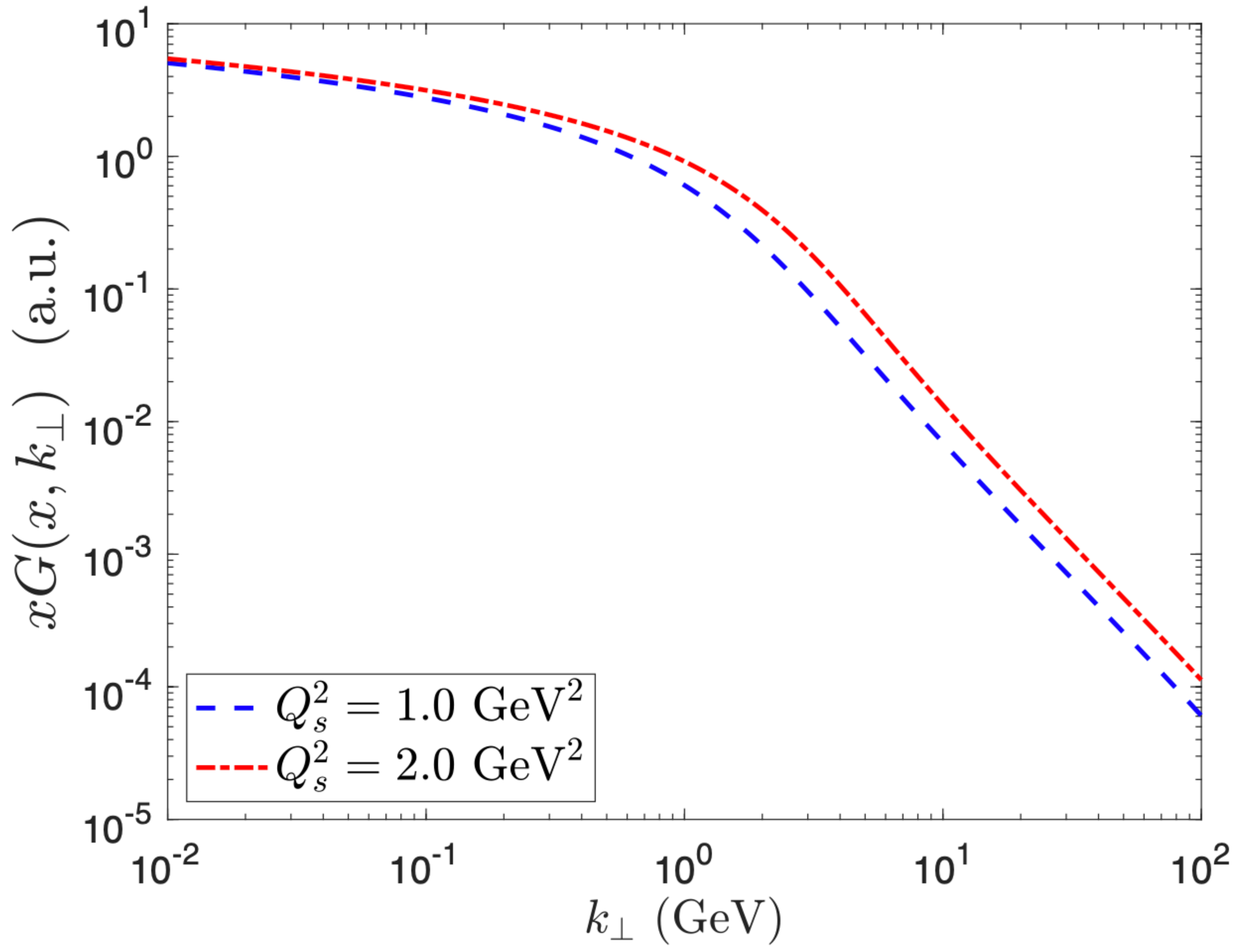
$$xG^{ij}_{\rm WW}(x, \vec{k}_T) = \frac{4}{(2\pi)^3} \int d^2b_T d^2b'_T\, e^{-i\vec{k}_T\cdot(\vec{b}_T - \vec{b}'_T)} \left\langle \mathrm{Tr}[A^i(\vec{b}_T)A^j(\vec{b}'_T)] \right\rangle_x$$

Transverse momentum imbalance of a dijet

is dictated by WW gluon transverse momentum distribution!

Azimuthal distribution of dijets/dihadrons near back-to-back limit provides access to WW gluon TMD and saturation effects

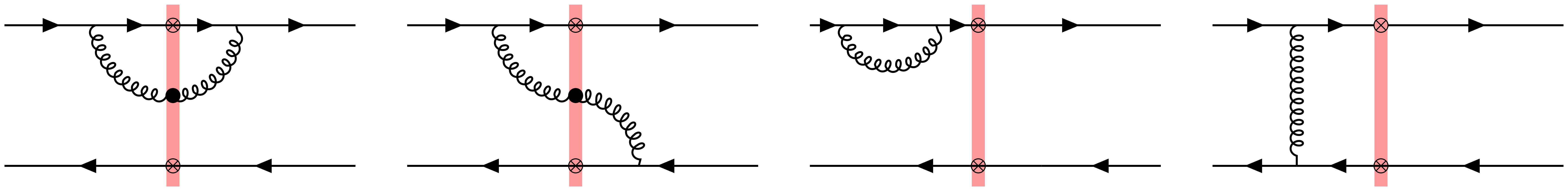




Quantum evolution

Quantum fluctuations around the classical solution are enhanced by terms $\sim \alpha_s \ln(x_0/x)$

Can be understood as gluon emissions in the interval $[x, x_0]$



At large N_c, resumming terms enhanced by $\sim \alpha_s \ln(x_0/x)$ results in the Balitsky-Kovchegov (BK) equations Balitsky, Nucl. Phys. B 1996, 463, 99–160, [hep-ph/9509348] Kovchegov, Phys. Rev. D 1999, 60, 034008, [hep-ph/9901281]

$$\frac{dS^{(2)}_{x_0}(\vec{r}_T)}{d\ln(1/x)} = \frac{\alpha_s N_c}{2\pi^2}\int d^2 r'_T \frac{\vec{r}_T^{\,2}}{\vec{r}_T^{\,\prime 2}(\vec{r}_T - \vec{r}'_T)^2}\left[S^{(2)}_x(r'_T)S^{(2)}_x(|\vec{r}_T - \vec{r}'_T|) - S^{(2)}_x(r_T)\right]$$

Terms non-linear in $S^{(2)}_x$ arise from real diagrams above where gluon crosses the shockwave

The linear term comes from the virtual contributions

Weak scattering limit, $D_x(r_T) = 1 - S^{(2)}_x(r_T) \ll 1$: BK equations reduce to BFKL equations

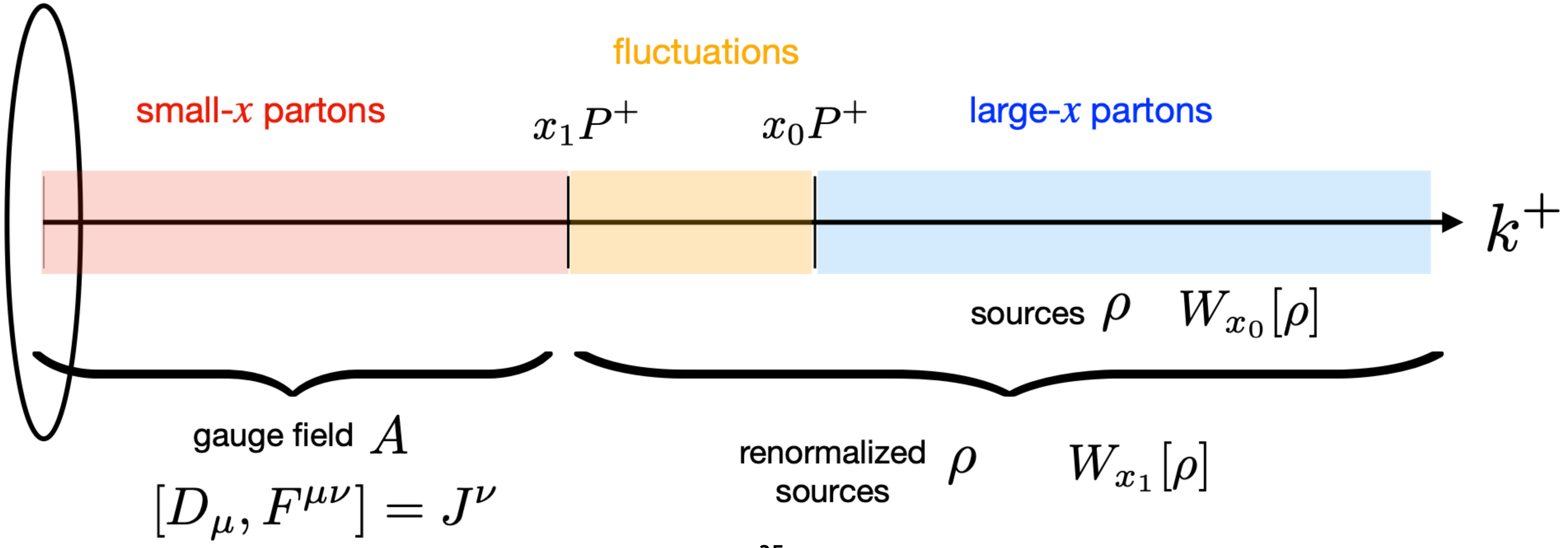
JIMWLK equation

Jalilian-Marian, J.; Kovner, A.; McLerran, L.D.; Weigert, H., Phys. Rev. D 1997, 55, 5414–5428, [hep-ph/9606337]
Jalilian-Marian, J.; Kovner, A.; Weigert, H., Phys. Rev. D 1998, 59, 014015, [hep-ph/9709432]
Kovner, A.; Milhano, J.G.; Weigert, H., Phys. Rev. D 2000, 62, 114005, [hep-ph/0004014]
Iancu, E.; Leonidov, A.; McLerran, L.D., Nucl. Phys. A 2001, 692, 583–645,[hep-ph/0011241]
Iancu, E.; Leonidov, A.; McLerran, L.D., Phys. Lett. B 2001, 510, 133–144, [hep-ph/0102009]
Ferreiro, E.; Iancu, E.; Leonidov, A.; McLerran, L., Nucl. Phys. A 2002, 703, 489–538, [hep-ph/0109115]

Alternatively, resum the large logarithmic corrections by evolving the weight functional:

$$\frac{dW_x[\rho]}{d\ln(1/x)} = -\mathscr{H}_{\mathrm{JIMWLK}}\, W_x[\rho]$$

Physically, one absorbs the quantum fluctuations in the interval $[x_0 - dx, x_0]$ into stochastic fluctuations of the color sources by redefining the color sources ρ:



JIMWLK equation

Jalilian-Marian, J.; Kovner, A.; McLerran, L.D.; Weigert, H., Phys. Rev. D 1997, 55, 5414–5428, [hep-ph/9606337]
Jalilian-Marian, J.; Kovner, A.; Weigert, H., Phys. Rev. D 1998, 59, 014015, [hep-ph/9709432]
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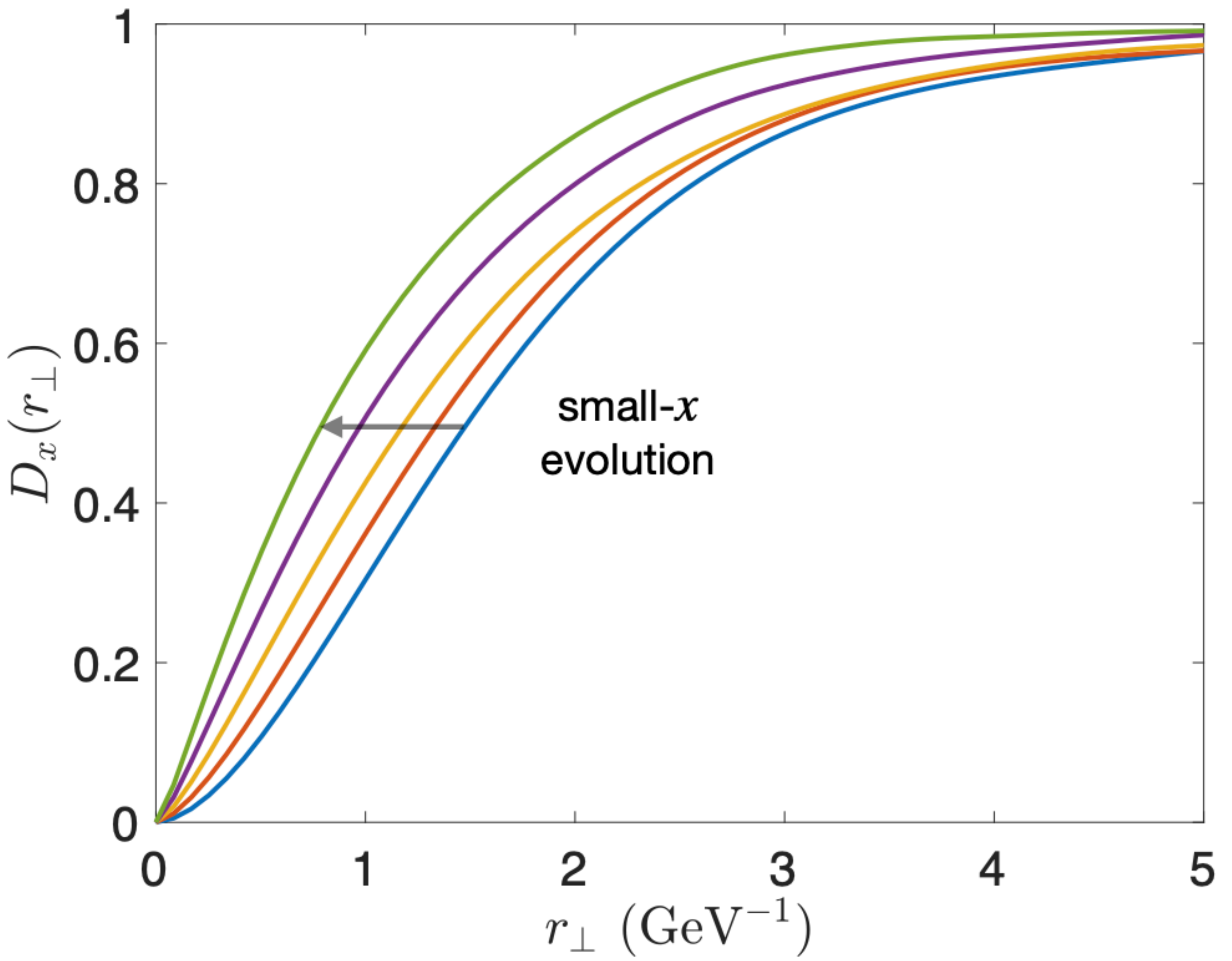
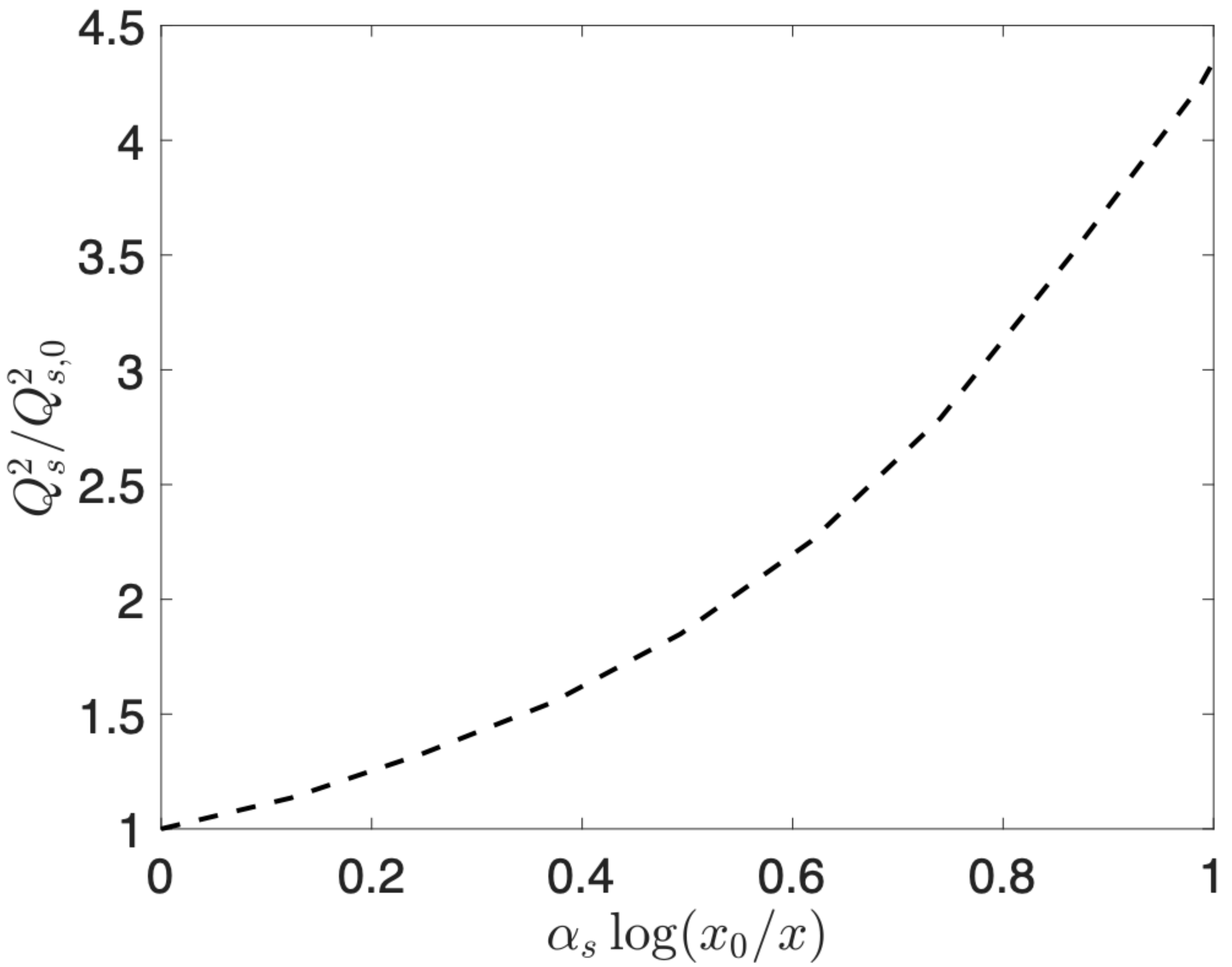
The quantum evolution effectively increases the color charge density, and hence Q_s

Effect on the dipole amplitude

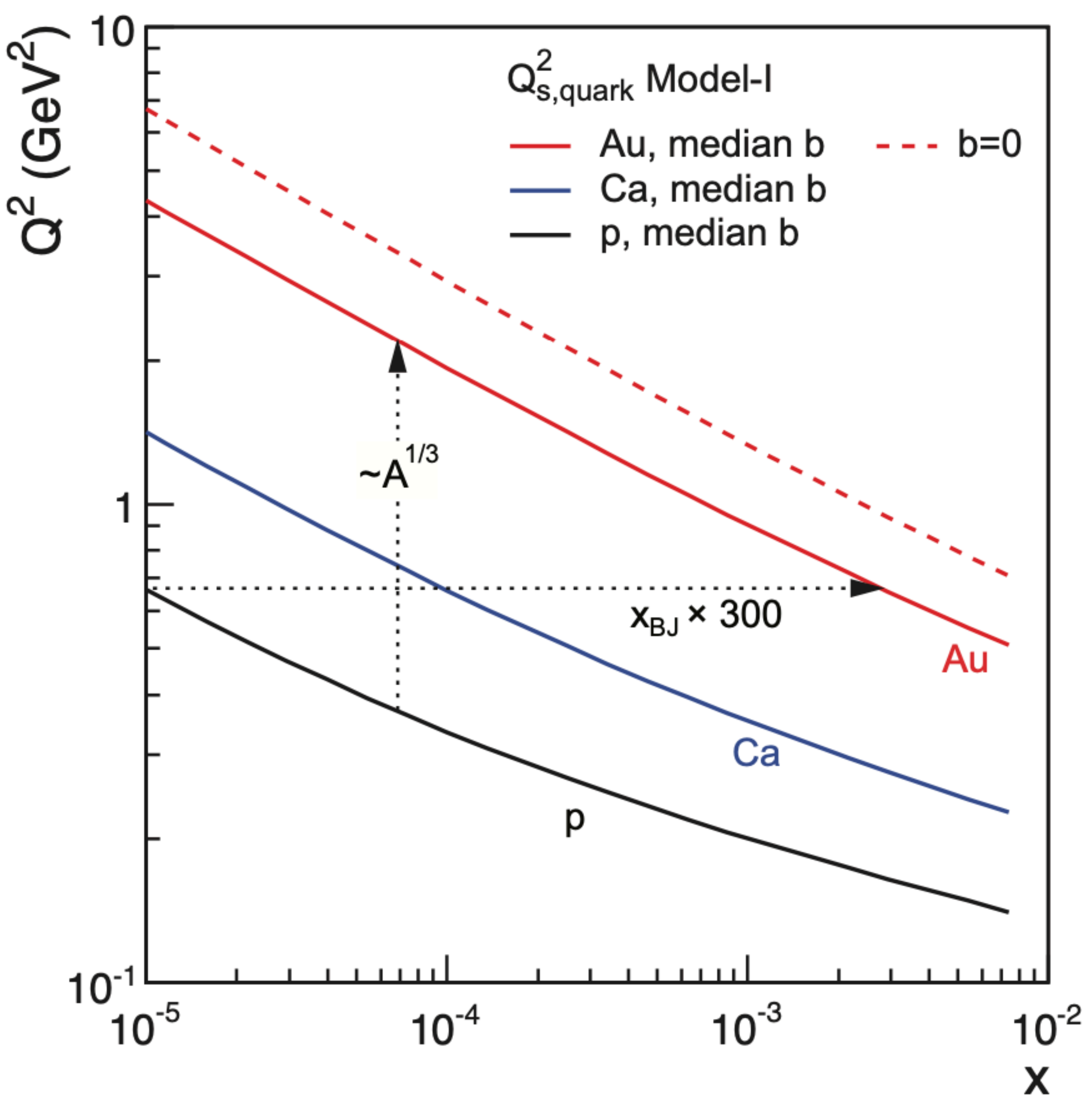


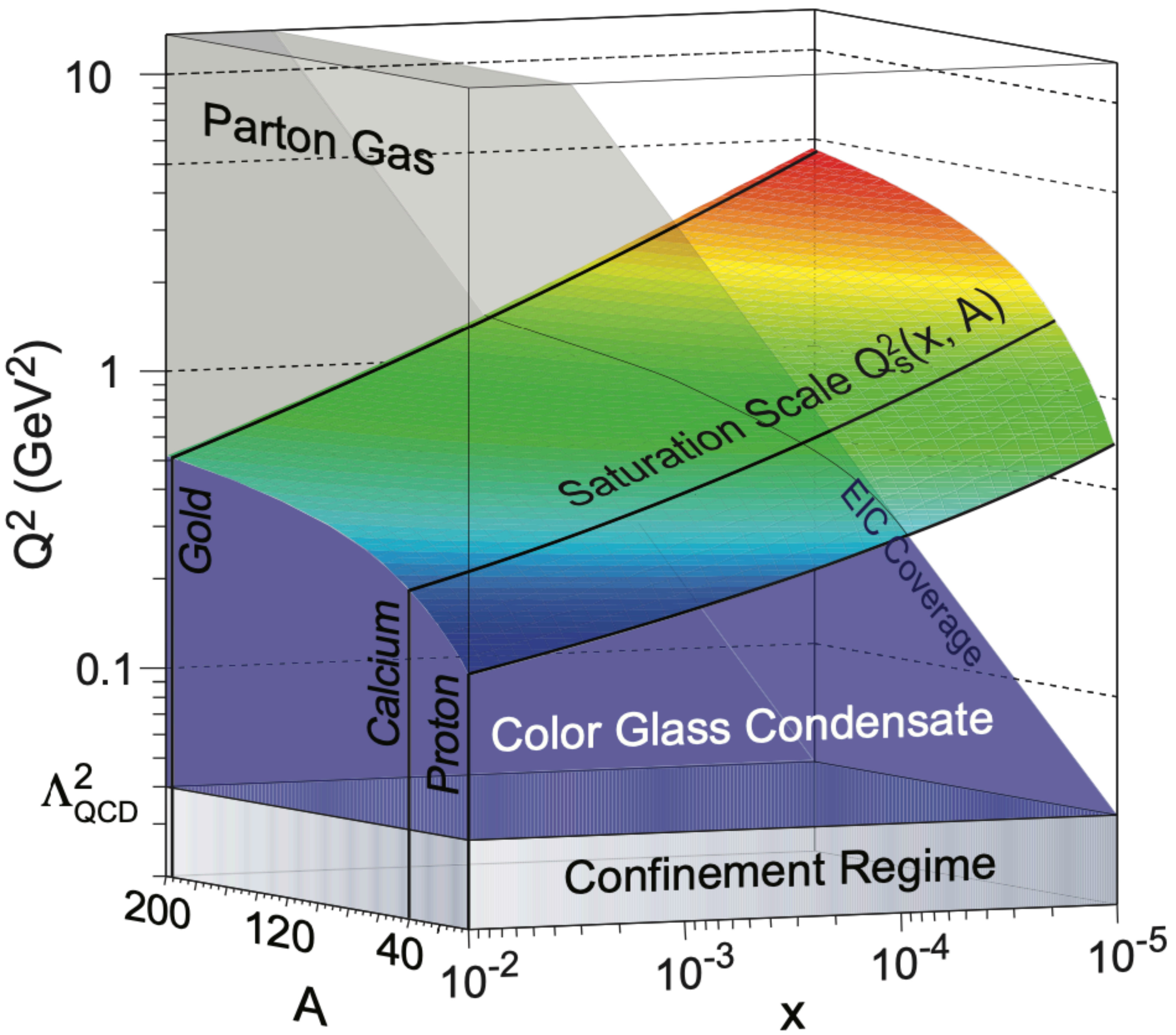
Increasing Q_s with decreasing x

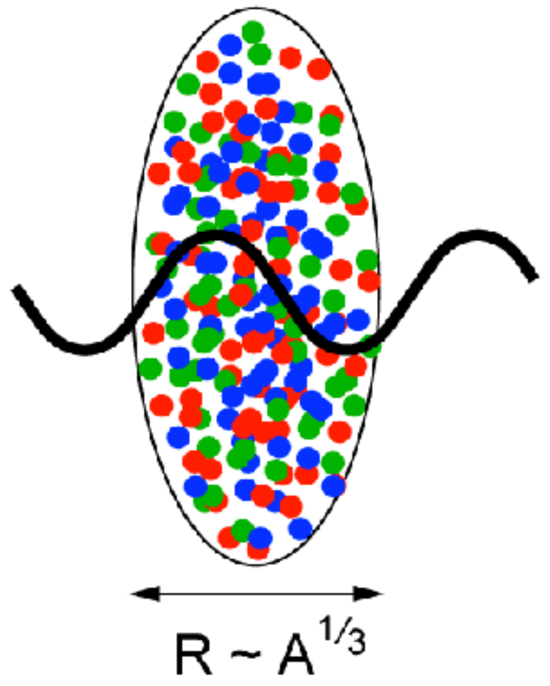


We will return to the effect of the small-x evolution on observables in HICs and at the EIC later

Recap: Saturation scale's A and x dependence







$$(Q_s^A)^2 \approx c Q_0^2 \left(\frac{A}{x}\right)^{1/3}$$

A. Accardi et al., EIC White Paper, Eur.Phys.J.A 52 (2016) 9, 268