Machine Learning for Collinear QCD

Lecture 3

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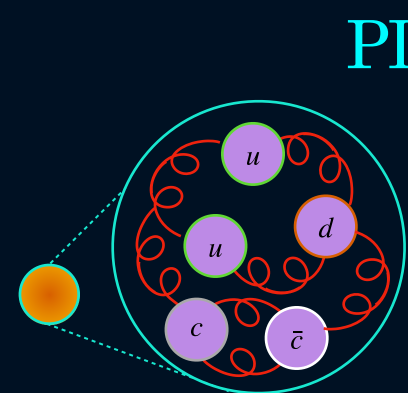
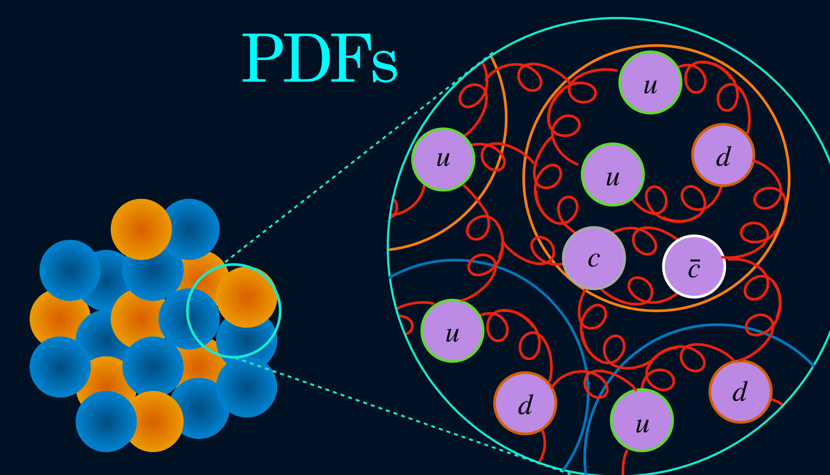
Outline

Substructure of nucleons and nuclei

To a certain approximation, the probability of finding quarks and gluons in a hadron carrying a momentum-fraction x of the hadron's momentum is encoded in non-perturbative parton distribution functions (PDFs).



Free proton



Proton bound within a nucleus

Substructure of nucleons and nuclei

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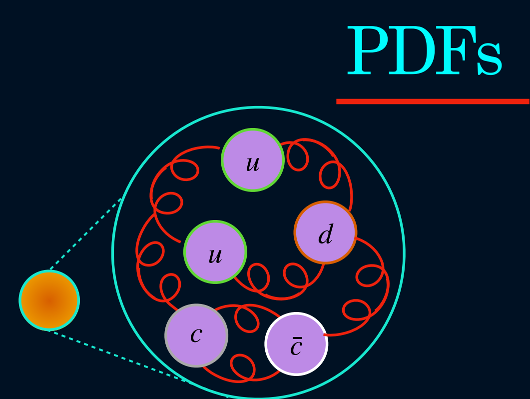
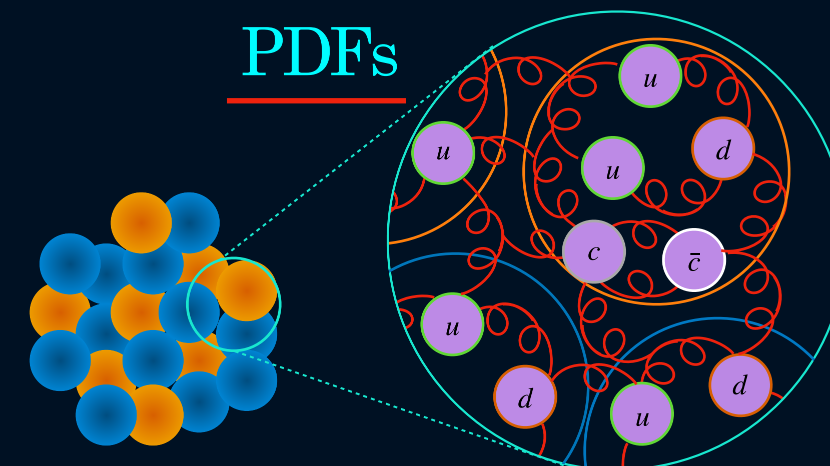
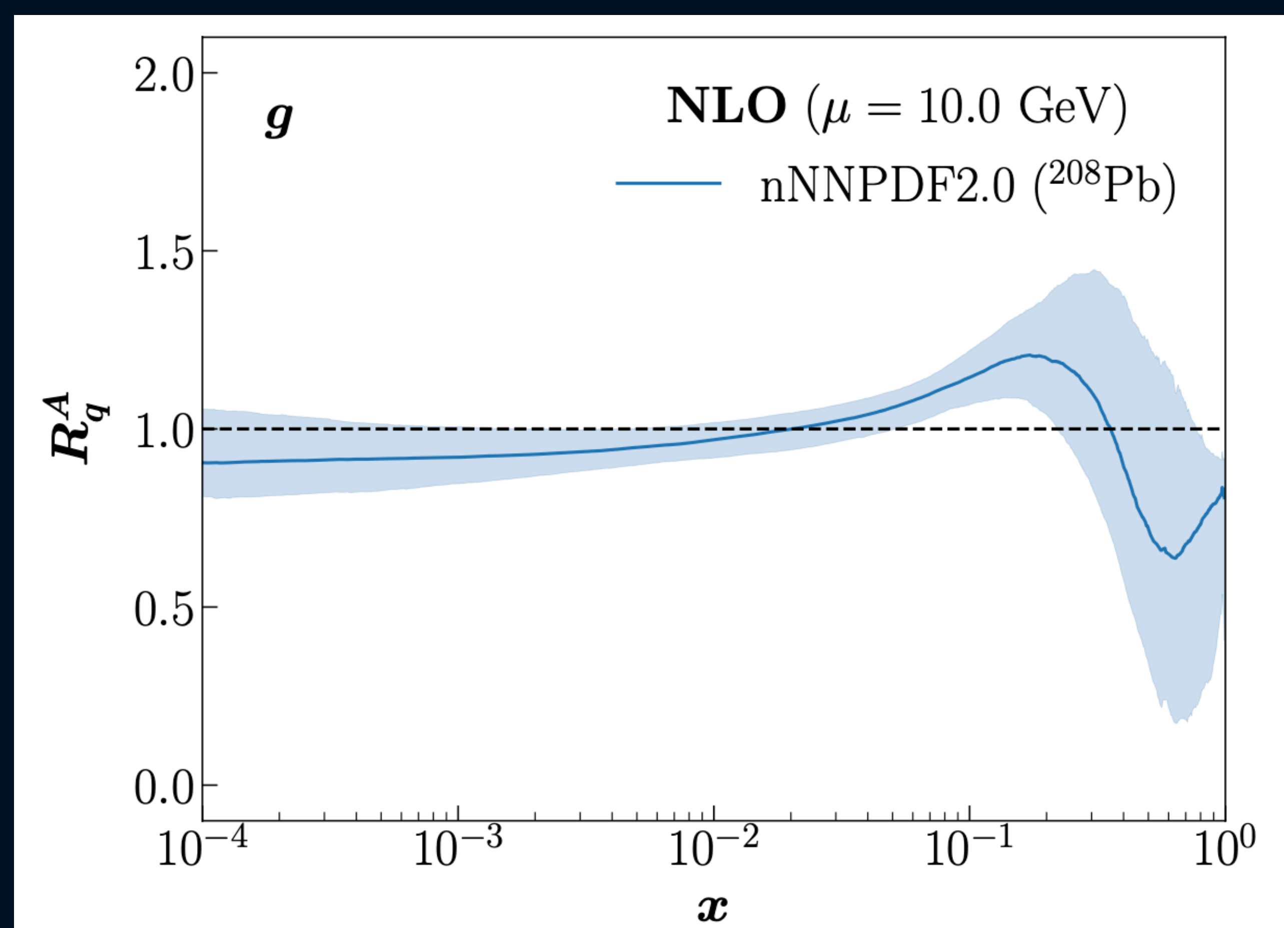


Free proton

Turns out they are quite different!!

Proton bound within a nucleus





Substructure of nucleons and nuclei

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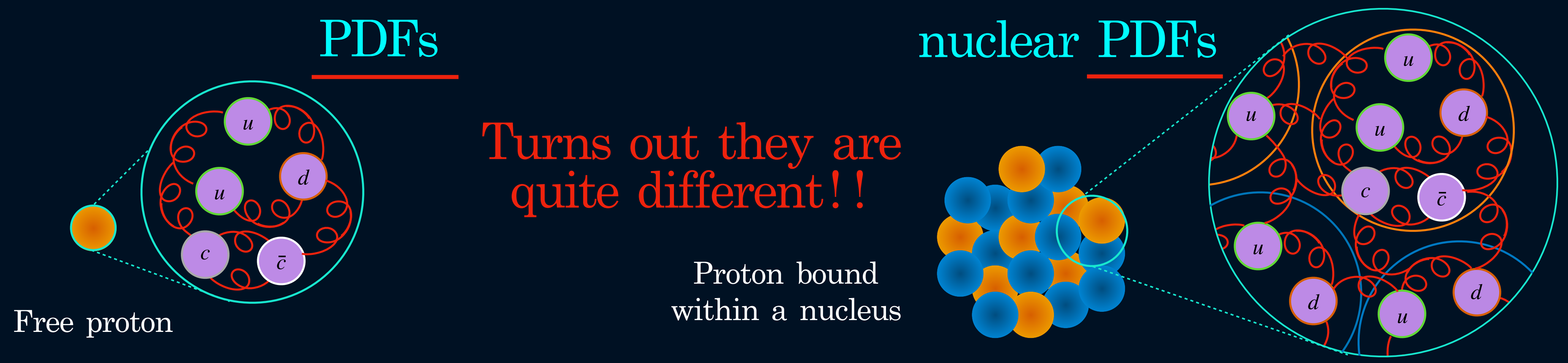

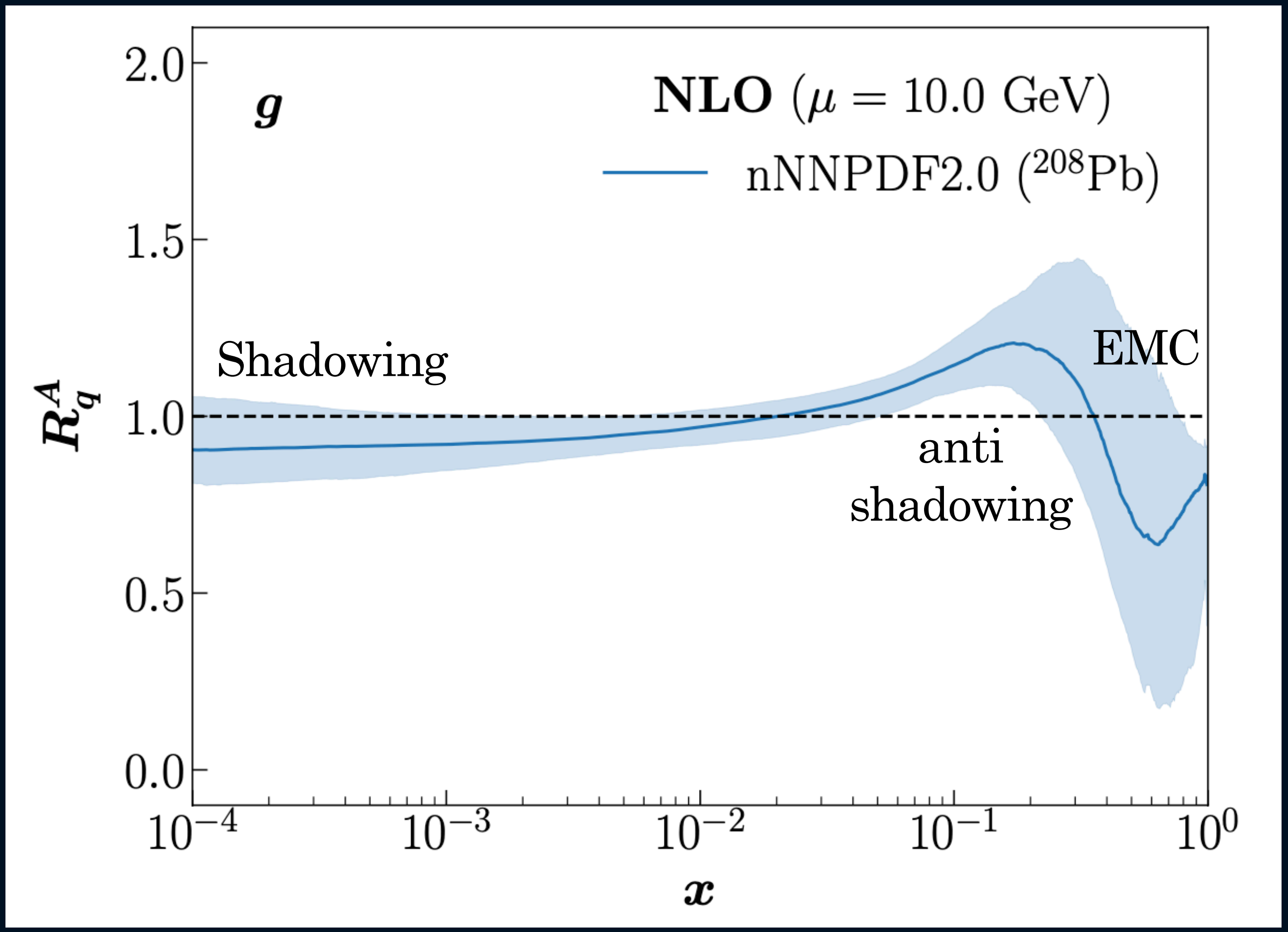

Substructure of nucleons and nuclei

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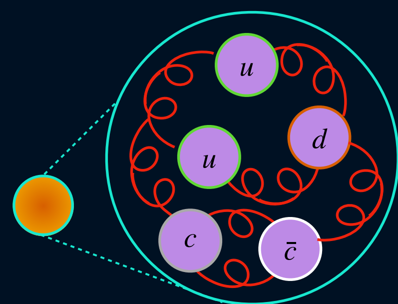
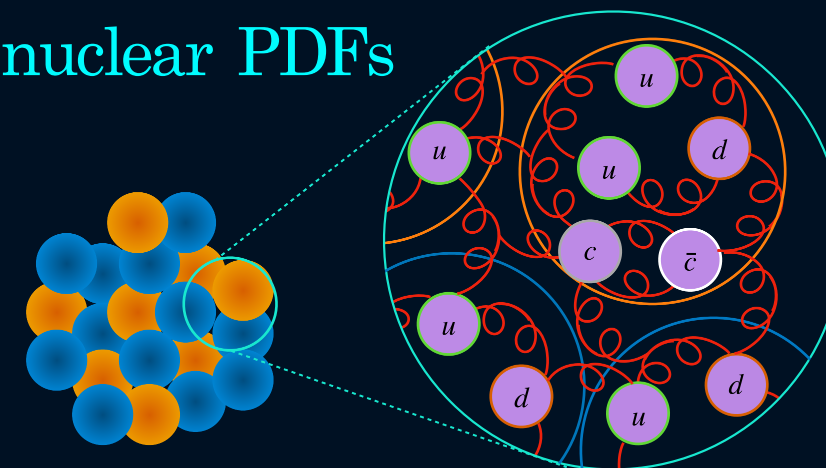
PDFs



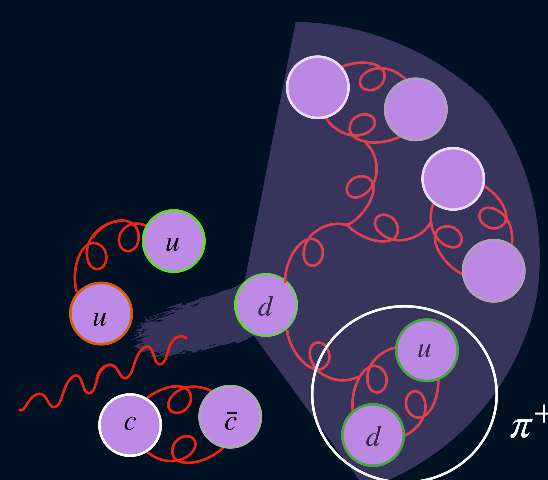
Free proton

nuclear PDFs



Proton bound within a nucleus

In addition, fragmentation functions (FFs) encodes the probability of producing a hadron from a quark fragmentation.

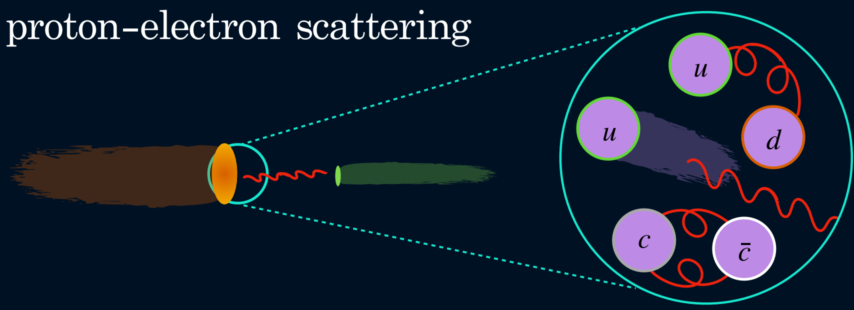


FFs

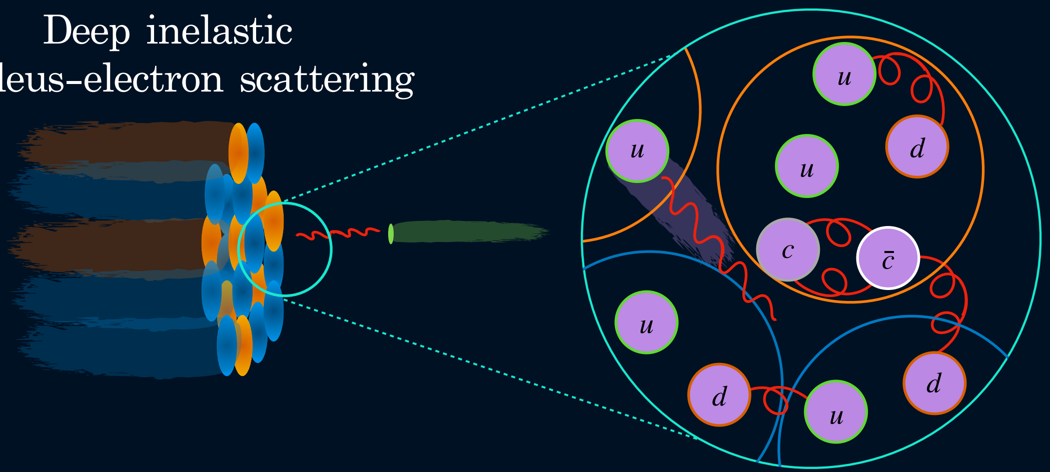
Substructure of nucleons and nuclei

We extract these objects from the scattering-probabilities off protons and nuclei that we measure in collider-experiments.

Deep inelastic
proton-electron scattering



Deep inelastic
Nucleus-electron scattering

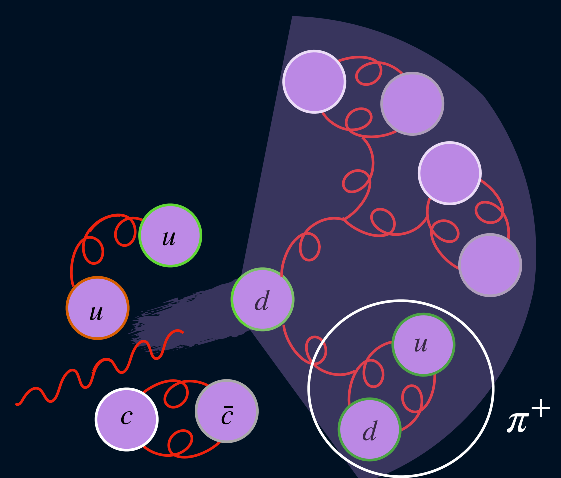


$$\text{Scattering probability} = \hat{\sigma} \otimes f^{(p)}$$

Perturbative — Non-perturbative PDFs

$$\text{Scattering probability} = \hat{\sigma} \otimes f^{(A)}$$

Perturbative — Non-perturbative nuclear PDFs



$$\text{Scattering probability} = \hat{\sigma} \otimes f^{(p)} \otimes D^{(\pi^{\pm})}$$

Perturbative — Non-perturbative

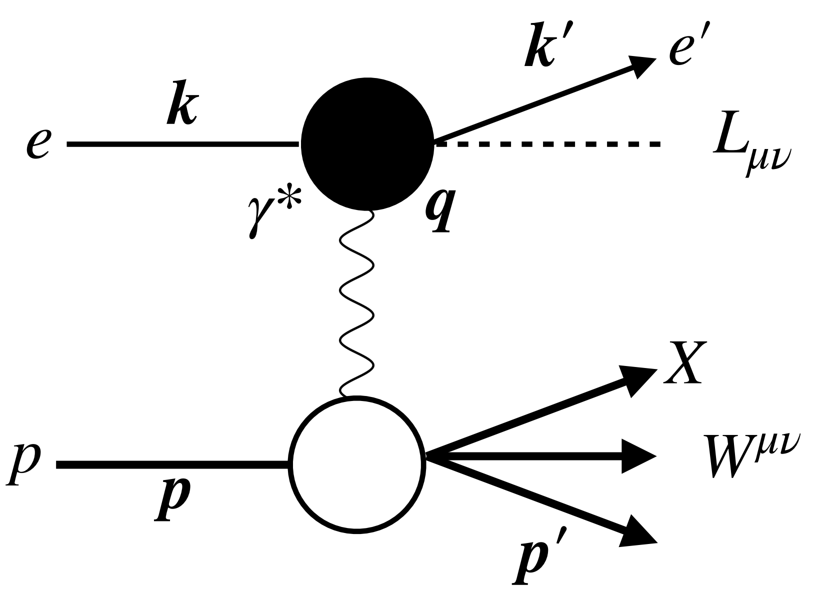
Proton PDFs

To calculate a cross section involving a hadron in renormalizable perturbation series:

$$\mu \sim Q \gg 0 \implies \alpha_s(\mu^2) \ll 0 \implies O(Q, m, \mu) = O^{(0)} + O^{(1)} + O^{(2)} + \dots$$

$$\mu \text{ appears in } O(Q,m,\mu) \text{ as } \begin{cases} \frac{Q}{\mu} \sim 1 \implies & \text{short-distance} \\ \frac{\mu}{m} \gg 1 \implies & \text{long-distance} \end{cases}$$

Deep inelastic scattering



Collinear factorization theorems allows us to separate them systematically.

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4y e^{iq\cdot y} \sum_X \langle p \,|\, j^\mu(y) \,|\, X \rangle \langle X \,|\, j^\nu(0) \,|\, p \rangle$$

$$\overset{\text{Bjorken}}{\underset{\text{limit}}{=}} \sum_a \int_x^1 \frac{d\xi}{\xi} \, f_{a/p}(\xi, \mu) \, H_a^{\mu\nu}(q^\mu, \xi p^\mu, \mu, \alpha_s(\mu)) + \text{H.T.}$$

PDF — Hard scattering coefficient

$$\frac{d^2\sigma^{lp\to lX}}{dxdQ^2} \propto L_{\mu\nu} W^{\mu\nu} \propto \sum_a \int_x^1 d\xi \, f_{a/H}(\xi, Q^2) \, \frac{d^2\hat{\sigma}}{dxdQ^2}(x/\xi, Q^2) = \hat{\sigma} \otimes f_{a/H}$$

Nuclear PDFs

EMC Effect: $$\frac{F_2^A}{F_2^D} \neq \frac{2}{A} \cdot C \otimes \frac{Zf^p + (A-Z)f^n}{f^p + f^n}$$

Nucleus is not an ensemble of Z free protons and (A-Z) free neutrons.

Four equally possible scenarios:

a) the fundamental interactions are the same but PDFs in nucleus are different.
b) The fundamental interactions are different in the medium but PDFs are the same.
c) Both (a) and (b).
d) The factorization picture is no longer valid.

Replace proton with nuclear states

Working with the assertion (a): $$\tilde{W}^{\mu\nu} = \frac{1}{4\pi} \int d^4y e^{iq\cdot y} \sum_X \langle A | j^\mu(y) | X \rangle \langle X | j^\nu(0) | A \rangle$$

$$\overset{\text{Bjorken}}{\underset{\text{limit}}{=}} \sum_a \int_x^1 \frac{d\xi}{\xi} f_{a/A}(\xi, \mu)\, H_a^{\mu\nu}(q^\mu, \xi p^\mu, \mu, \alpha_s(\mu)) + \textbf{H.T.}$$

Nuclear PDF — Hard scattering coefficient

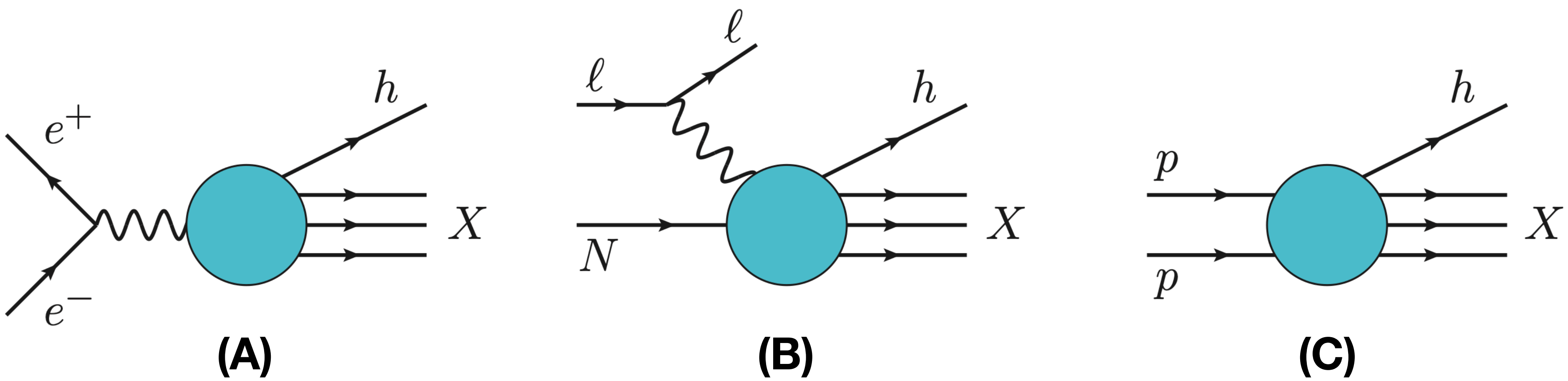
Physical quantity to be fitted

$$f_{a/A} \doteq Z \cdot \tilde{f}_{a/p} + (A-Z) \cdot \tilde{f}_{a/n} \implies \frac{d^2\sigma^{lA\to lX}}{dxdQ^2} \propto L_{\mu\nu}\tilde{W}^{\mu\nu} \propto Z \cdot \tilde{F}_2^{\,p/A} + (A-Z) \cdot \tilde{F}_2^{\,n/A} = \hat{\sigma} \otimes f_{a/A}$$

Effective quantities

Fragmentation Functions

In measurements where a hadron is identified…



A. SIA: single-inclusive hadron production in electron-positron annihilation, $e^+ + e^- \to h + X$.

$$\sigma^{e^+ + e^- \to h+X} = \hat{\sigma} \otimes D^h$$

B. SIDIS: semi-inclusive deep-inelastic lepton-nucleon scattering, $\ell + N \to \ell + h + X$

$$\sigma^{\ell + p \to \ell + h + X} = \hat{\sigma} \otimes f^p \otimes D^h$$

C. Single-inclusive hadron production in proton-proton collisions, $p + p(\bar{p}) \to h + X$

$$\sigma^{p+p(\bar{p}) \to h+X} = \hat{\sigma} \otimes f^p \otimes f^p \otimes D^h$$

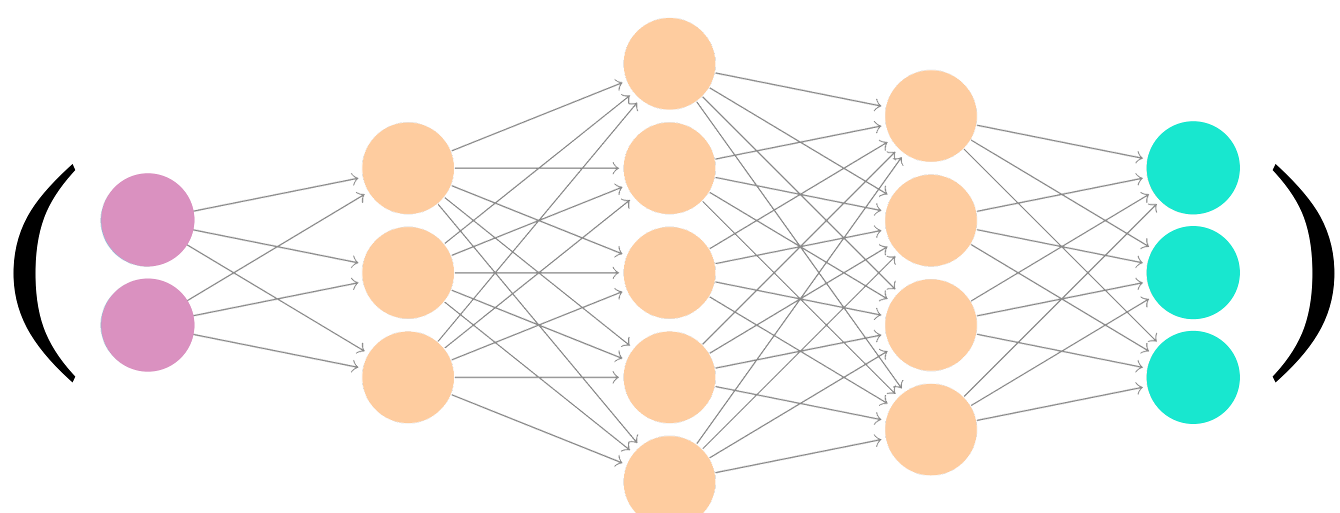
Neural Networks parameterisation

Due to DGLAP being perturbative, we only need to extract the renormalised PDFs or FFs at one arbitrary scale μ

$$t(\theta) = \boxed{H^{\text{hard}}(Q) \otimes \Gamma^{\text{DGLAP}}(Q,\mu)} \otimes \left[xf(\mu,\theta) \,||\, zD(\mu,\theta)\right]$$

$$\left[xf(\mu,\theta) \,||\, zD(\mu,\theta)\right] \to \text{What kind of Parameterisation?}$$

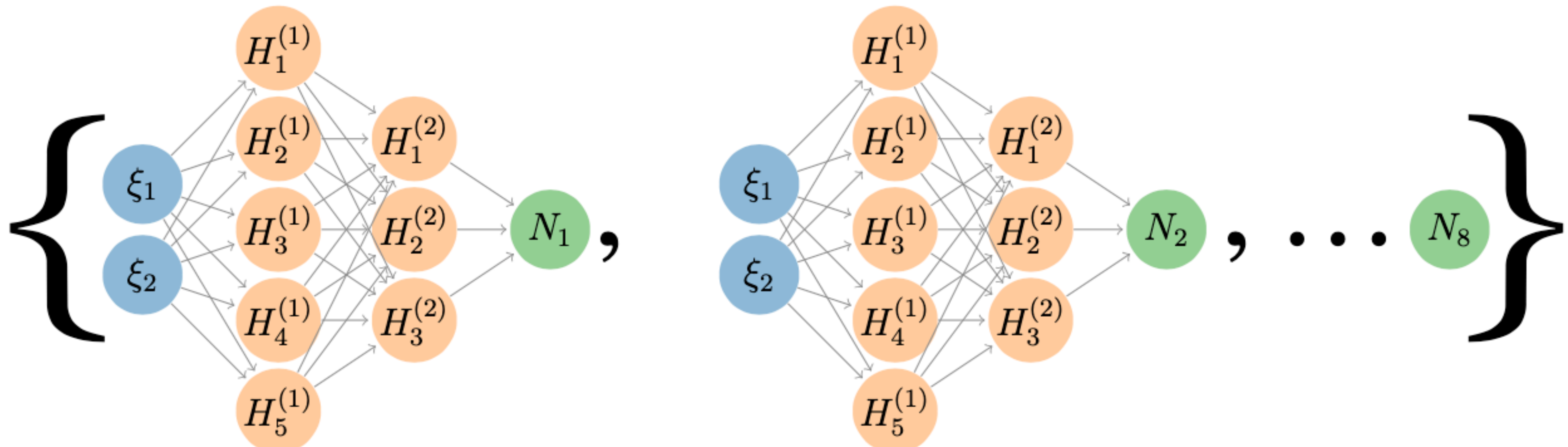
Since PDFs and FFs functional form is not known from first principles, we parameterise them using feed-forward neural networks (NNs) since NNs can approximate any continuous function within the data range

Scattering probability = $\hat{\sigma}$ $\otimes$ 

NNPDF3.1 — proton PDFs

Parameterisation $\{\boldsymbol{g}\ \boldsymbol{\Sigma}\ \boldsymbol{T_3}\ \boldsymbol{T_8}\ \boldsymbol{V}\ \boldsymbol{V_3}\ \boldsymbol{V_8}\ (\boldsymbol{c^+})\}$

NNPDF3.1 — arXiv:1706.00428
NNPDF4.0 — arXiv:2109.02653



$$f_i(x,\mu_0) = A_i \hat{f}_i(x,\mu_0), \quad \hat{f}_i(x,\mu_0) = x^{\alpha_i}(1-x)^{\beta_i} N_i(x)$$

$$A_g = \frac{1-\int_0^1 dx x\Sigma(x,\mu_0)}{\int_0^1 dx x \hat{g}(x,\mu_0)}, \quad A_\Sigma = A_{T_3} = A_{T_8} = 1$$

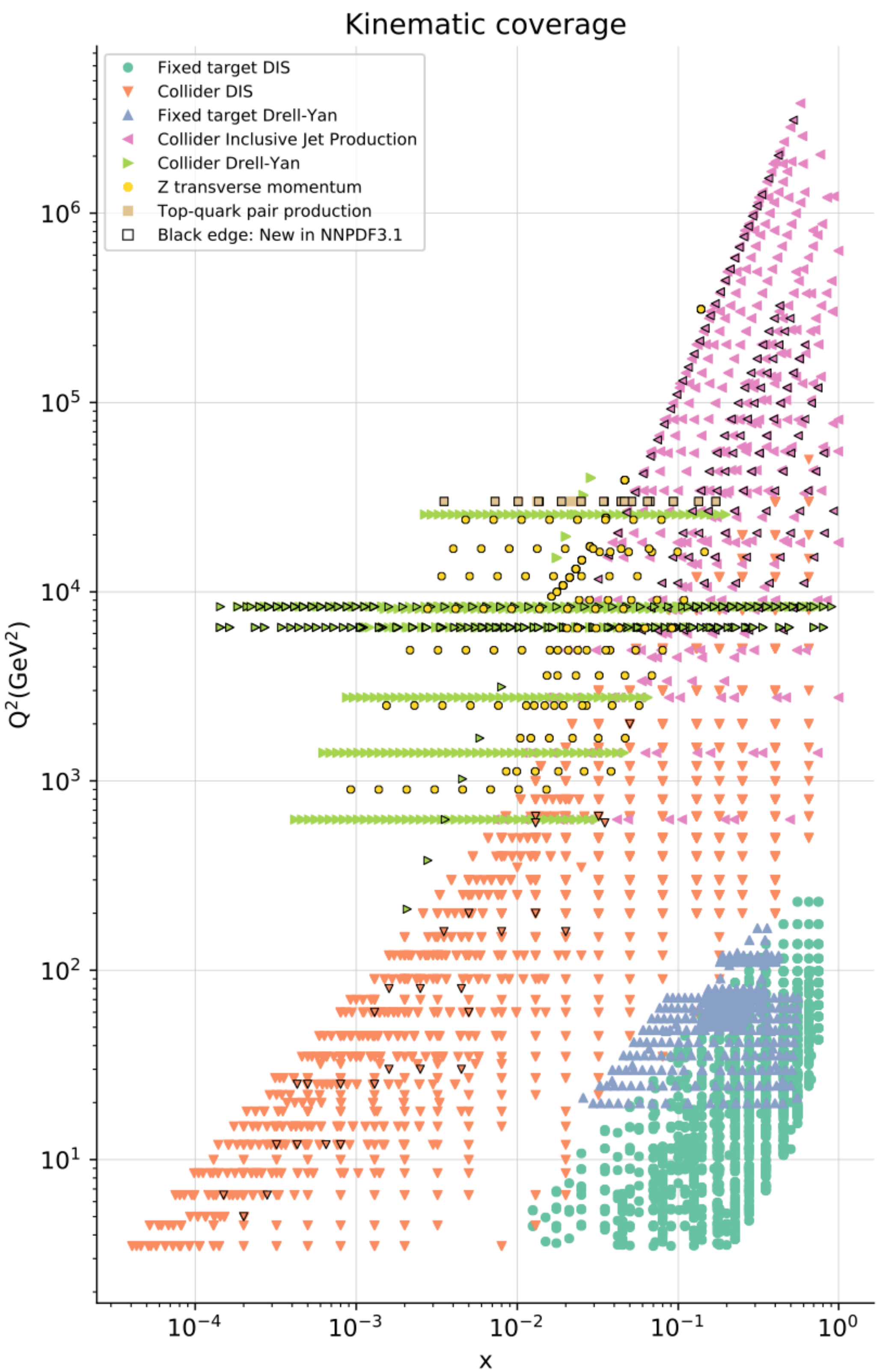
$$A_V = \frac{3}{\int_0^1 dx \hat{V}(x,\mu_0)}, \quad A_{V_3} = \frac{1}{\int_0^1 dx \hat{V}_3(x,\mu_0)}, \quad A_{V_8} = \frac{3}{\int_0^1 dx \hat{V}_8}$$

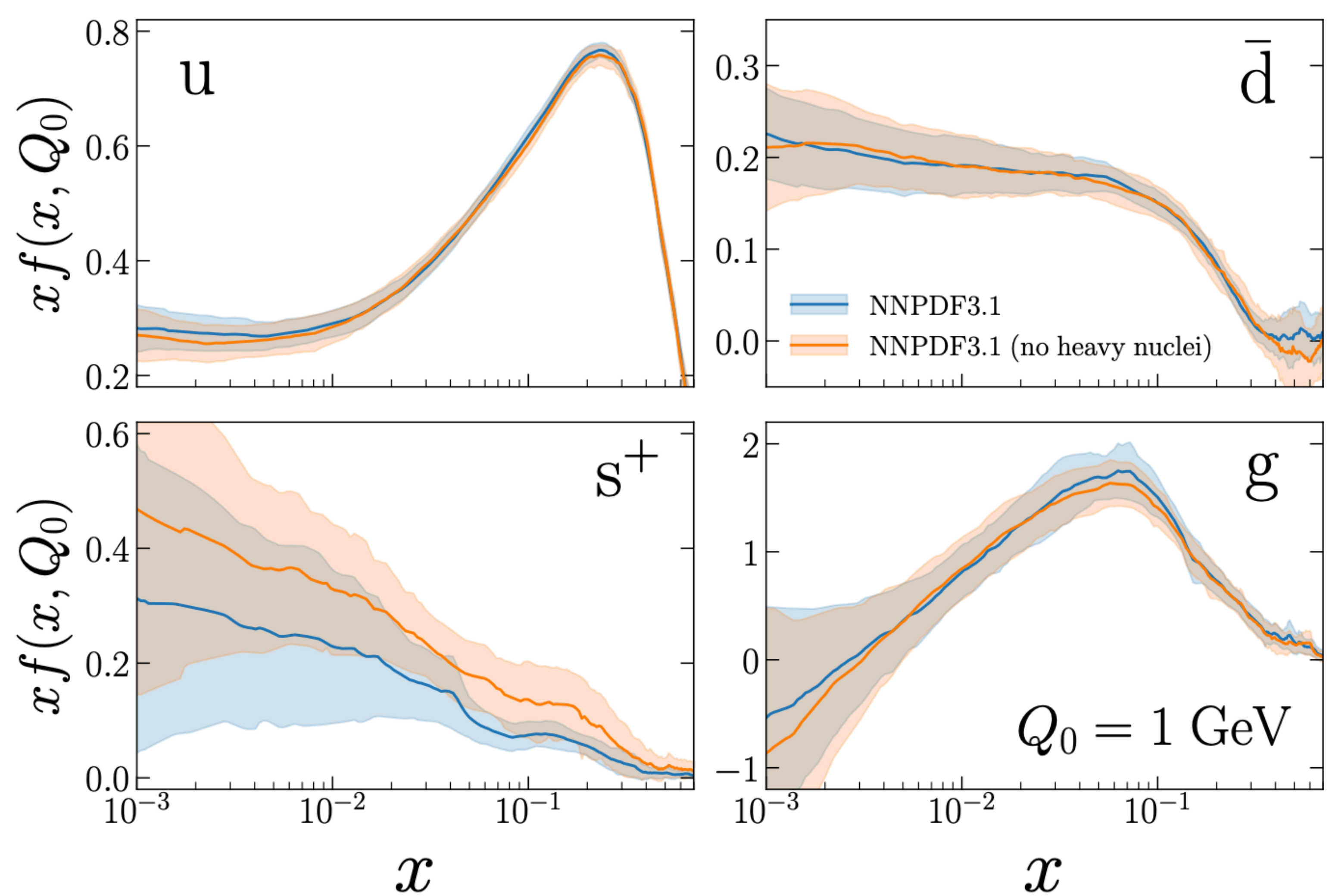
$$t(x,Q;\boldsymbol{\theta}) = \sum_i H_i^{\text{hard}}(Q) \otimes \sum_j \Gamma_{ij}^{\text{DGLAP}}(Q,\mu) \otimes f_j(\mu;\boldsymbol{\theta})$$

Loss function

$$\chi^2 = \chi^2_{\text{exp}} + \lambda_{\text{pos}} \sum_{l=1}^{N_{\text{pos}}} \sum_{j=1}^{N_A} \sum_{i_l=1}^{N_{\text{dat}}^{(l)}} \max\left(0, -\mathcal{F}_{i_l}^{(l)}(A_j)\right)$$

NNPDF3.1 — proton PDFs

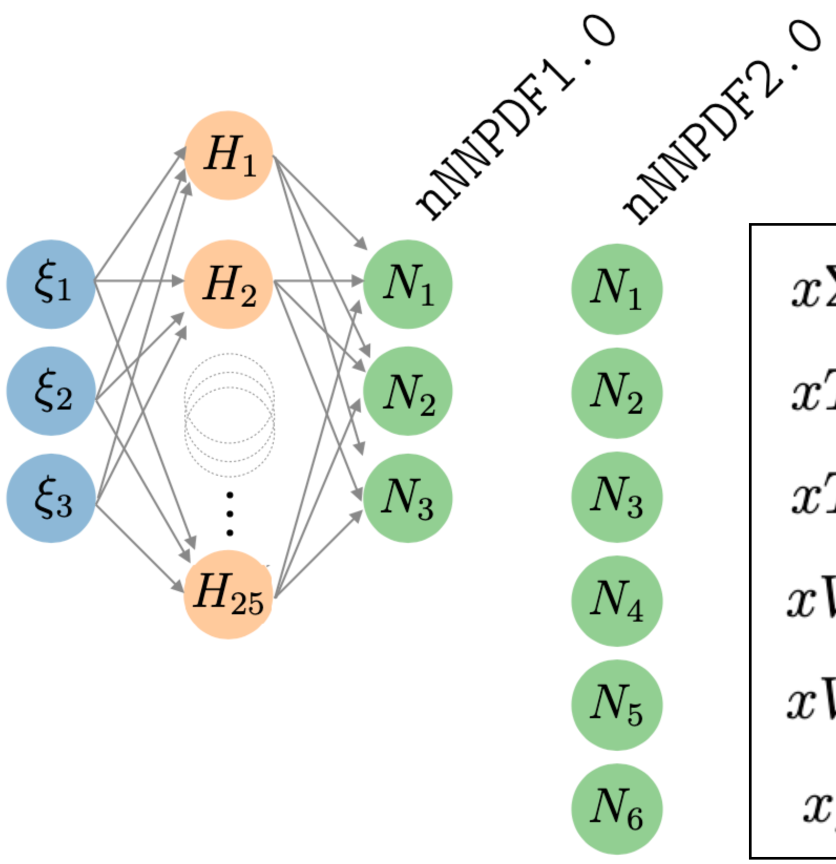




nNNPDF2.0 — nuclear PDFs

Parameterisation $\{g \quad \Sigma \quad T_3 \quad T_8 \quad V \quad V_3 \quad V_8\}$

nNNPDF1.0 — arXiv:1904.00018
nNNPDF2.0 — arXiv:2006.14629
nNNPDF3.0 — arXiv:2201.12363



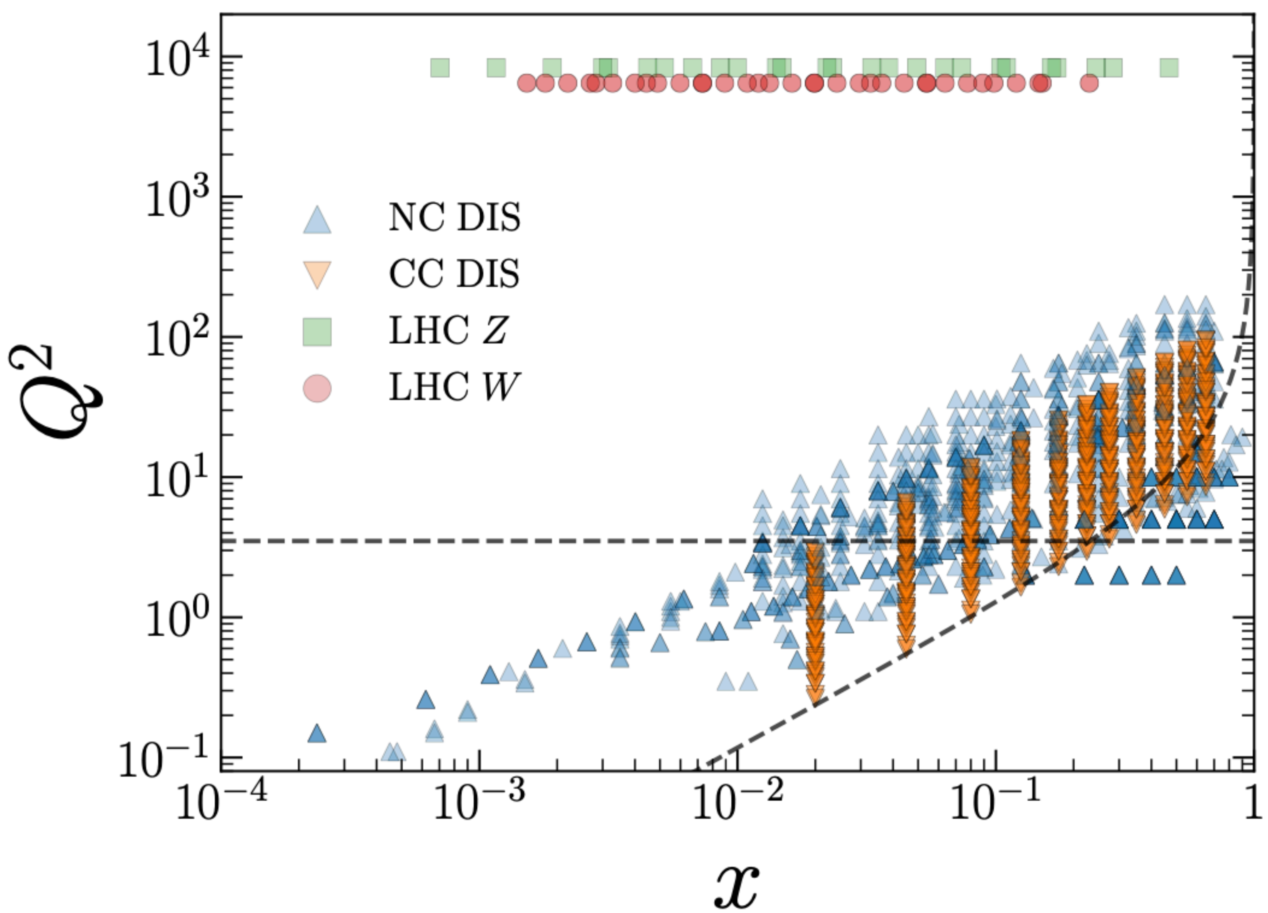
$$
\begin{aligned}
x\Sigma^{(p/A)}(x,Q_0) &= x^{\alpha_\Sigma}(1-x)^{\beta_\Sigma} N_1(x,A)\,,\\
xT_3^{(p/A)}(x,Q_0) &= x^{\alpha_{T_3}}(1-x)^{\beta_{T_3}} N_2(x,A)\,,\\
xT_8^{(p/A)}(x,Q_0) &= x^{\alpha_{T_8}}(1-x)^{\beta_{T_8}} N_3(x,A)\,,\\
xV^{(p/A)}(x,Q_0) &= B_V x^{\alpha_V}(1-x)^{\beta_V} N_4(x,A)\,,\\
xV_3^{(p/A)}(x,Q_0) &= B_{V_3} x^{\alpha_{V_3}}(1-x)^{\beta_{V_3}} N_5(x,A)\,,\\
xg^{(p/A)}(x,Q_0) &= B_g x^{\alpha_g}(1-x)^{\beta_g} N_6(x,A)\,.
\end{aligned}
$$

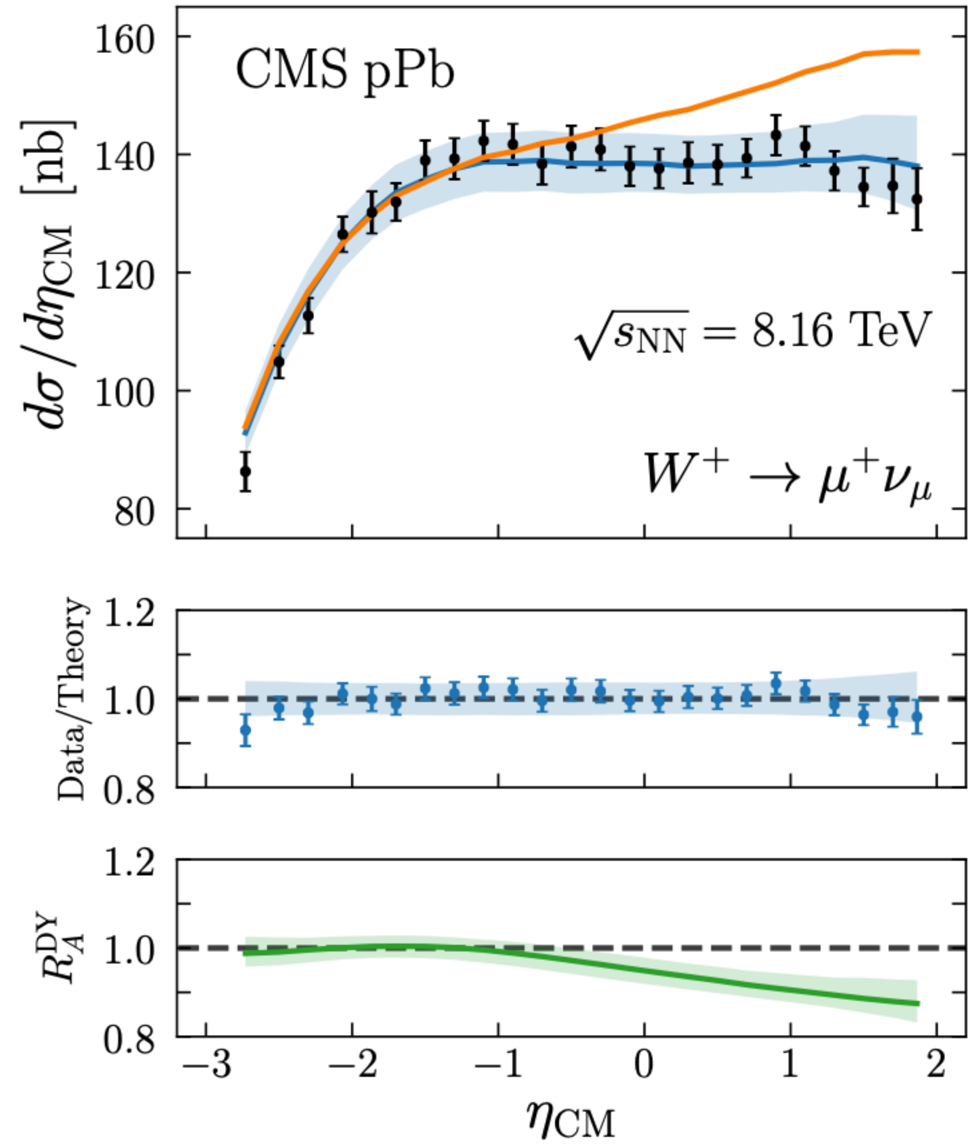
$$
t(x,Q;\boldsymbol{\theta}) = \sum_i H_i^{\text{hard}}(Q) \otimes \sum_j \Gamma_{ij}^{\text{DGLAP}}(Q,\mu) \otimes f_j^{A}(\mu;\boldsymbol{\theta})
$$

Loss function

$$
\chi^2 = \chi^2_{\text{exp}} + \lambda_{\text{BC}} \sum_f \sum_{i=1}^{N_x} \left(q_f^{(p/A)}(x_i,Q_0,A=1) - q_f^{(p)}(x_i,Q_0) \right)^2 + \lambda_{\text{pos}} \sum_{l=1}^{N_{\text{pos}}} \sum_{j=1}^{N_A} \sum_{i_l=1}^{N_{\text{dat}}^{(l)}} \max\left(0, -\mathcal{F}_{i_l}^{(l)}(A_j)\right)\,,
$$

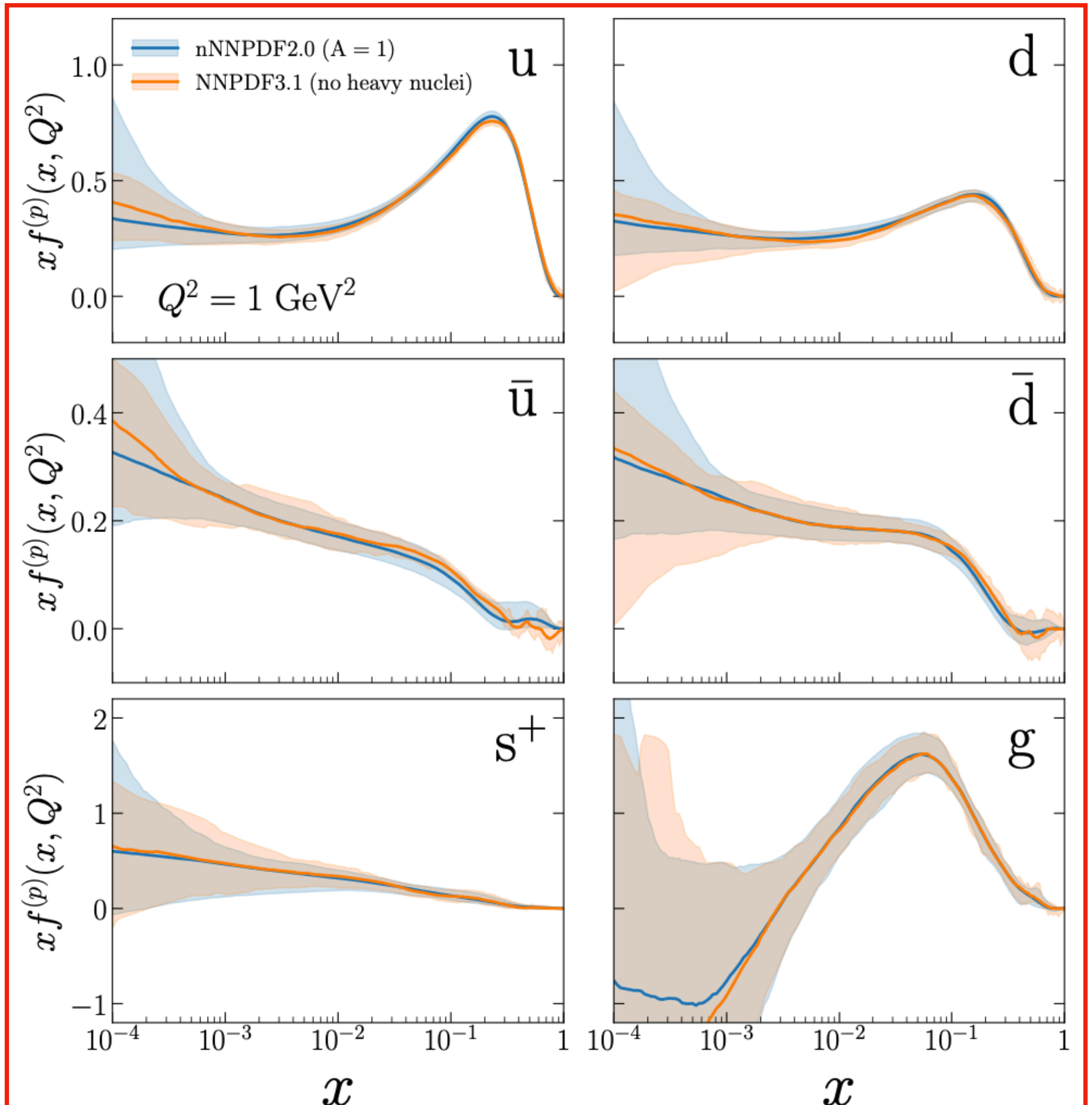
nNNPDF2.0 — nuclear PDFs

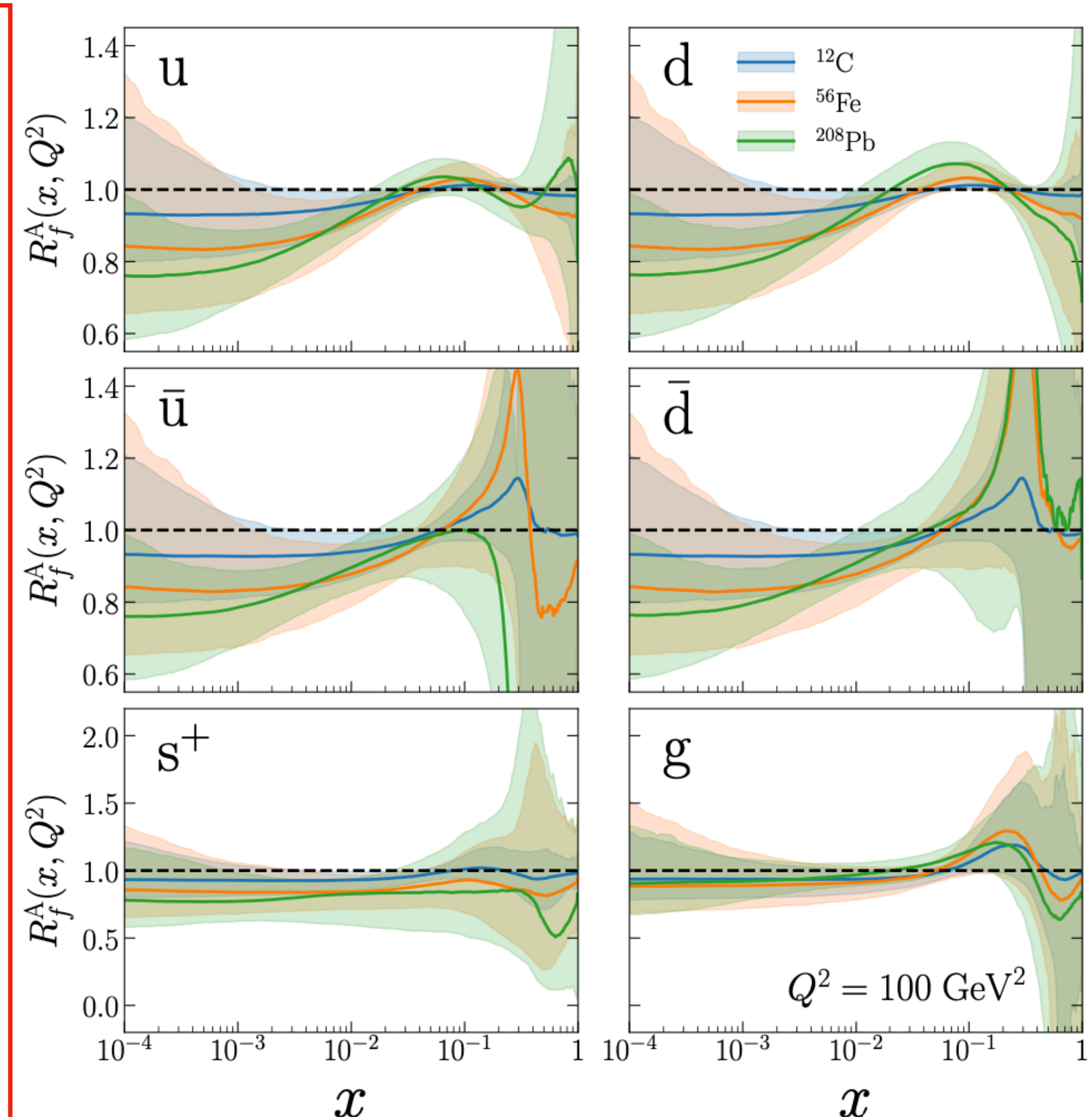




nNNPDF2.0 — nuclear PDFs

$$\chi^2 = \chi^2_{\rm exp} + \lambda_{\rm BC}\sum_f\sum_{i=1}^{N_x}\left(q_f^{(p/A)}(x_i,Q_0,A=1) - q_f^{(p)}(x_i,Q_0)\right)^2$$
$$+\, \lambda_{\rm pos}\sum_{l=1}^{N_{\rm pos}}\sum_{j=1}^{N_A}\sum_{i_l=1}^{N_{\rm dat}^{(l)}}\max\left(0,-\mathcal{F}_{i_l}^{(l)}(A_j)\right)\,,$$





MAPFF1.0 — Fragmentation Functions

Parameterisation

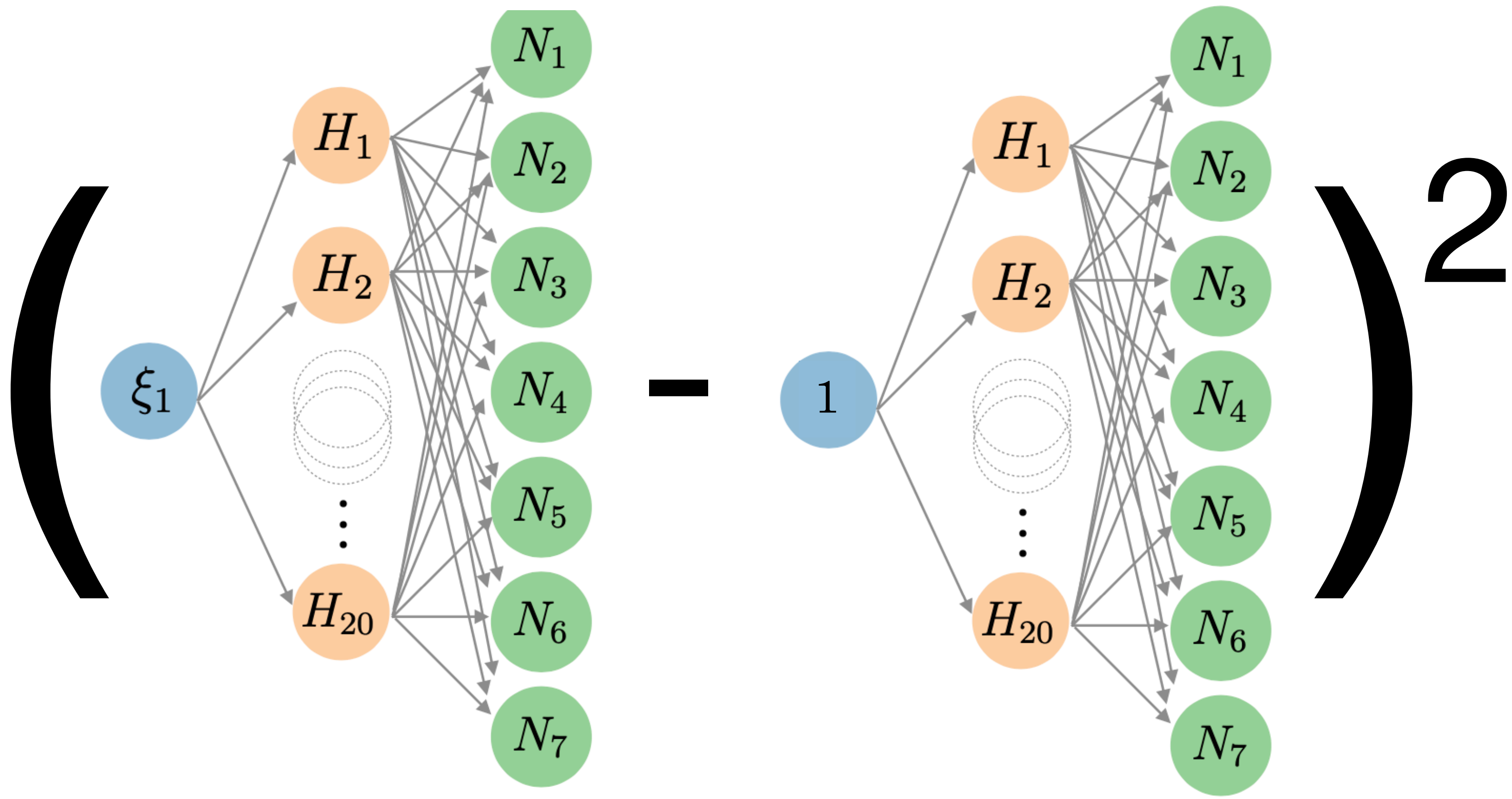
MAPFF1.0 — arXiv:2105.08725

7 flavours assuming a partially symmetric sea

$$\pi^+: \{u, \quad \bar{d}, \quad d = \bar{u}, \quad s^+, \quad c^+, \quad b^+, \quad g\}$$

$$K^+: \{u, \quad \bar{s}, \quad s = \bar{u}, \quad d^+, \quad c^+, \quad b^+, \quad g\}$$

$$\boxed{zD_i^{\pi^+}(z, \mu_0 = 5\,\text{GeV}) = \Big(N_i(z;\boldsymbol{\theta}) - N_i(1;\boldsymbol{\theta})\Big)^2}$$

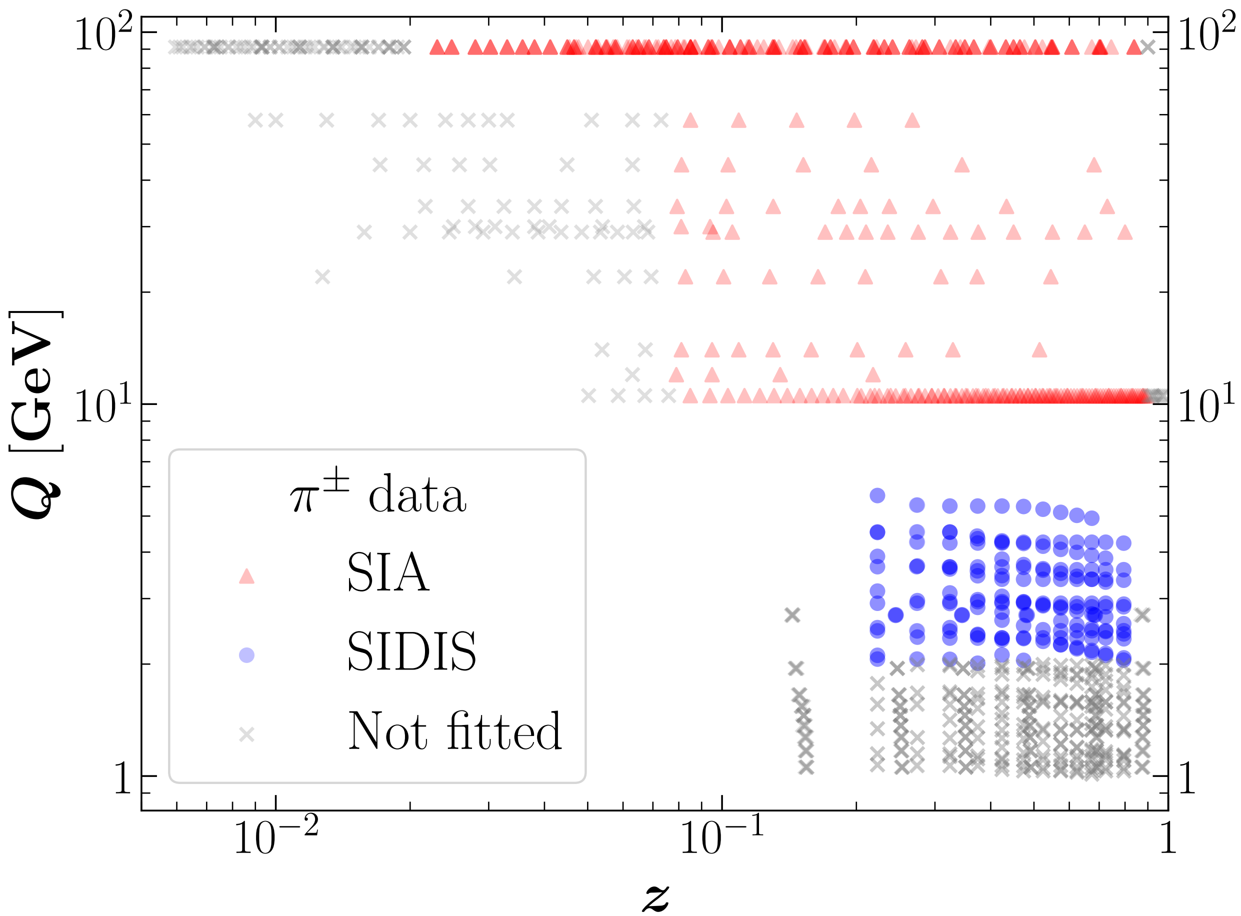


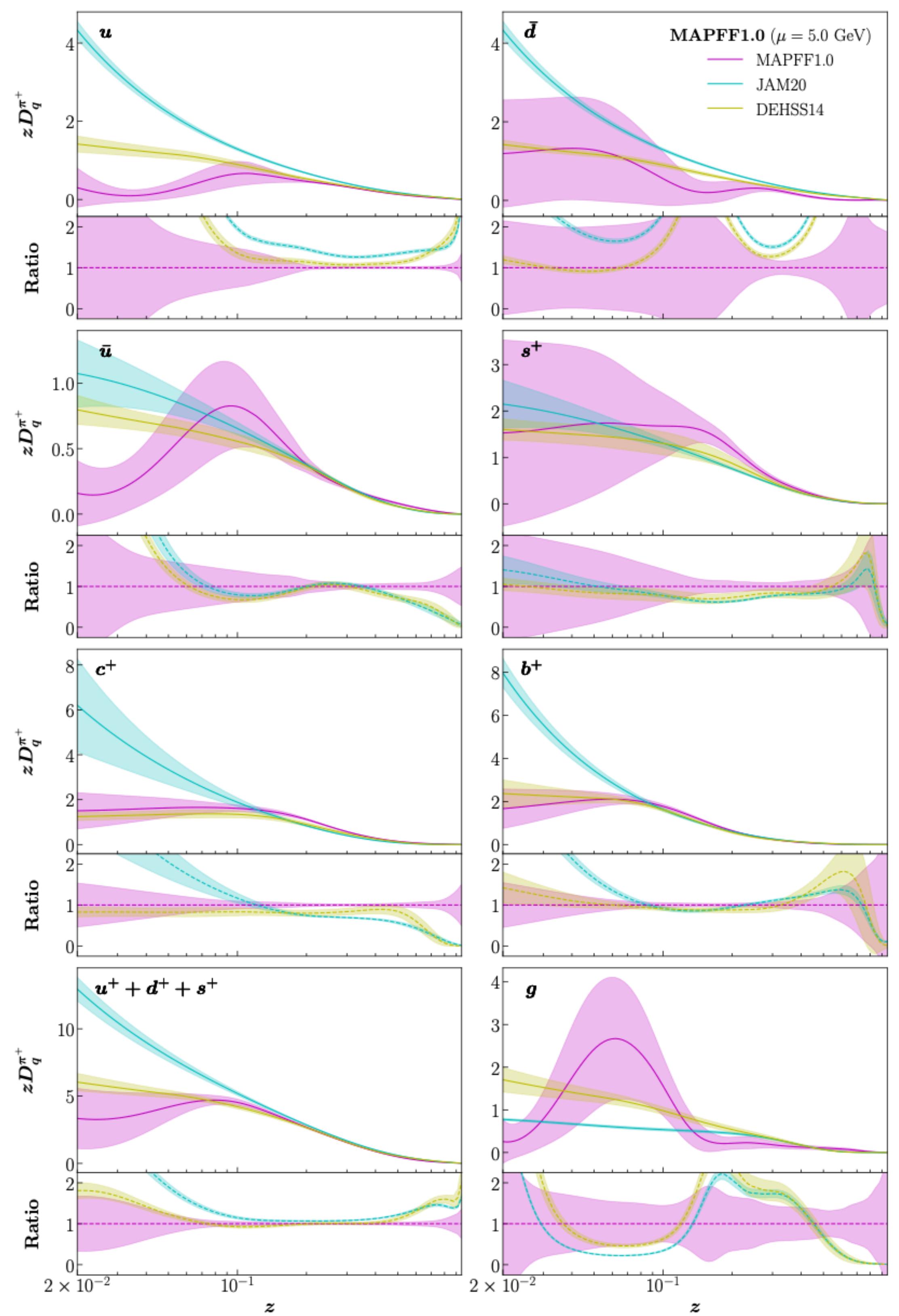
$$F_i(x,z,Q) = x\sum_{q\bar{q}} e_q^2 \Big\{ [C_{i,qq}(x,z,Q)\otimes f_q(x,Q) + C_{i,qg}(x,z,Q)\otimes f_g(x,Q)]\otimes D_q^{\pi^\pm}(z,Q)$$
$$+ [C_{i,gq}(x,z,Q)\otimes f_q(x,Q)]\otimes D_g^{\pi^\pm}(z,Q)\Big\}, \qquad i = 2, L\,.$$

Loss function

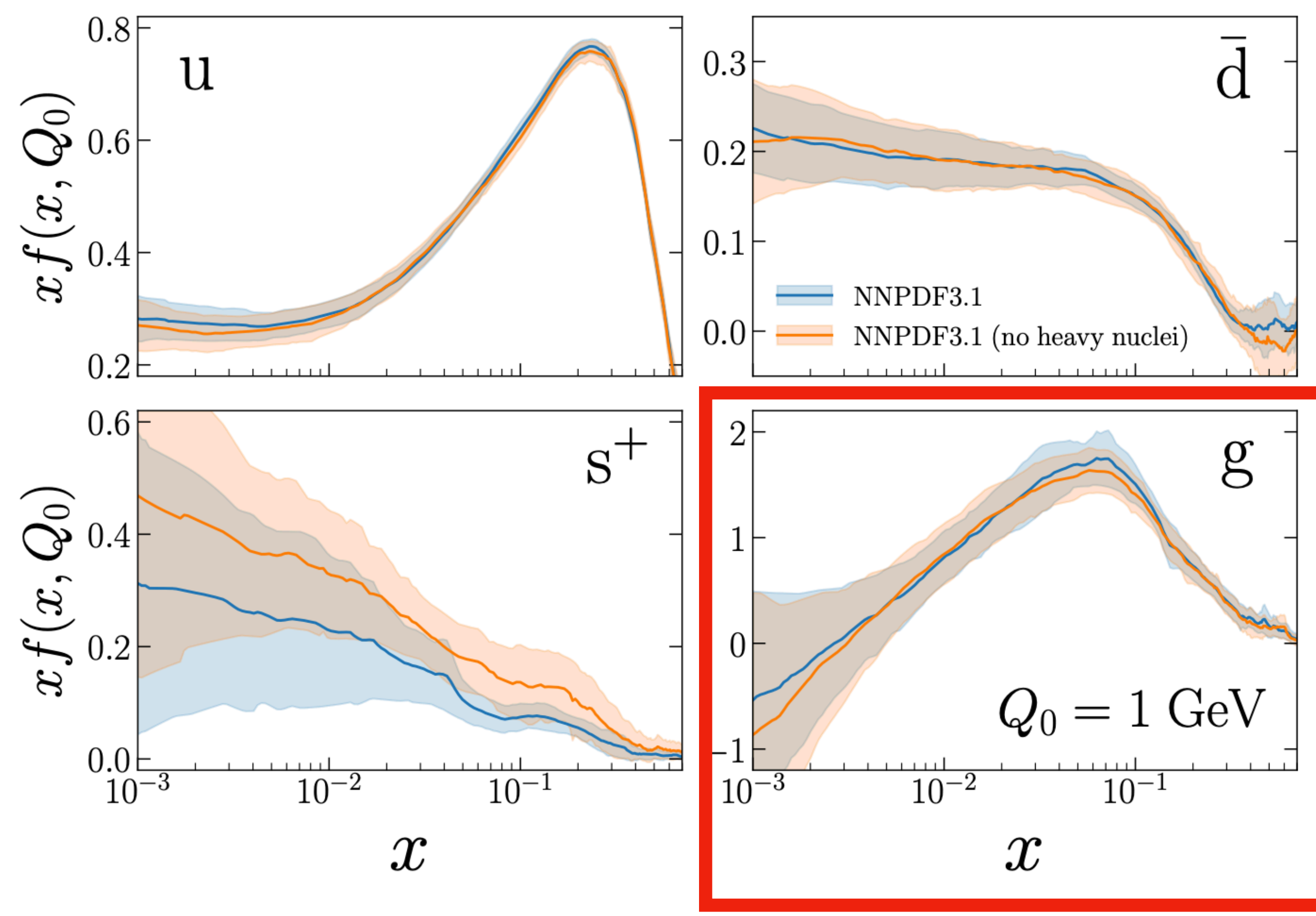
$$\boxed{\chi^2 = \chi^2_{\text{exp}}}$$

MAPFF1.0 — Fragmentation Functions





Tutorial 2 — Fitting the gluon PDF



Time for Tutorial 2!

Tutorial 2: Non-linear regression with neural networks

Author and references

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https://github.com/rabah-khalek/TF_tutorials

Learning Goals

This notebook will serve as an introduction to non-linear regression using neural networks. We will learn how to build and train neural networks with TensorFlow, a powerful machine learning library.