CGC baseline for double inclusive gluon production in ultra-peripheral collisions

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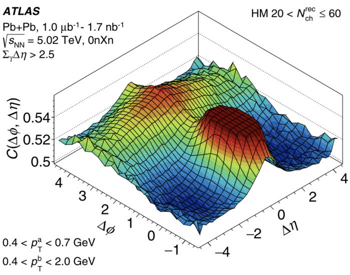
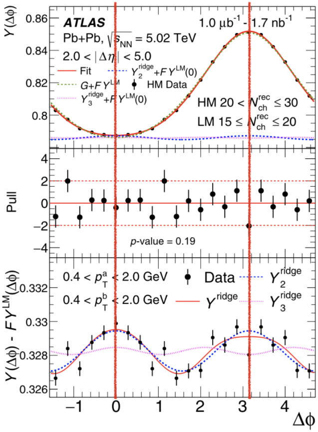
What's new?

Two particle angular correlation observed in UPC measurement at LHC



(a) PHYSICAL REVIEW C 104, 014903 (2021), ATLAS



(b) Backgrounds & signals

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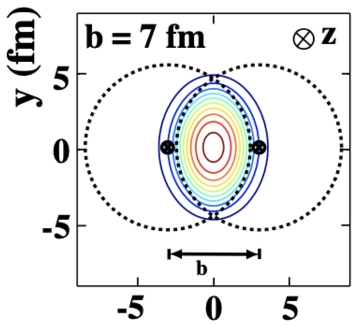
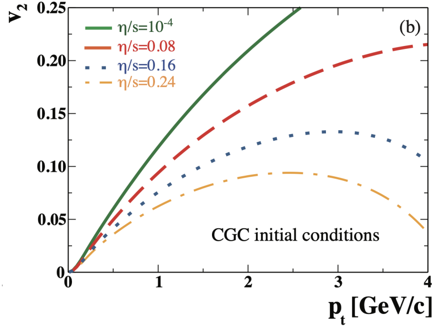
Elliptic flow



(a) Peripheral collision for AA



(b) $v_2 \to$ viscosity

Small viscosity η/s leads to higher v_2. Signal of QGP as nearly perfect fluid.

$$\frac{dN}{dq_1^2 dq_2^2} \propto 1 + \sum_n 2v_n^2 \cos(n\Delta\theta)$$

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Flow signal in small systems

- If flow signal indicates Quark-gluon plasma, what is the smallest collision system to create QGP?
  - High multiplicity p+p (2010), p+Pb (2012) at LHC
  - p+Au, d+Au, ^{3}He+Au at RHIC (2013-2020)

- Is there alternative origin of the flow signal without creating QGP?

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Flow signal in small systems

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- Is there alternative origin of the angular correlation without creating QGP?

- The smallest projectile is photon! But we don't have EIC yet.

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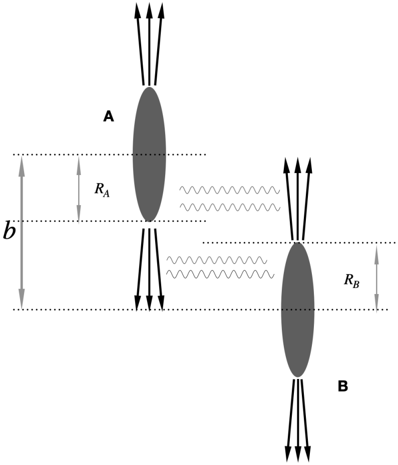
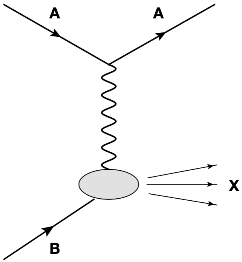
Ultra-peripheral collisions



- $b > R_A + R_B$
- equivalent photon approximation
- photon-nuclear interaction



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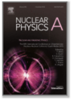
Origins of the angular correlation

- Highly non-equilibrium right after the scattering
- Hydrodynamics assumed instant local equilibrium which leads to superluminal modes.

ELSEVIER

Nuclear Physics A
Volume 956, December 2016, Pages 890-893



The unreasonable effectiveness of hydrodynamics in heavy ion collisions

Jacquelyn Noronha-Hostler [a, b], Jorge Noronha [c], Miklos Gyulassy [a]

- Quantum correlations
  - Bose-Einstein statistics
  - HBT(Hanbury Brown and Twiss) effect
  - Both are presented

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Bose enhancement

- When produced two gluons are identical, there is a quantum correction to two particle distribution.

$$\langle a_m^\dagger(\boldsymbol{k}) a_m(\boldsymbol{k}) a_n^\dagger(\boldsymbol{p}) a_n(\boldsymbol{p})\rangle$$
$$= n(\boldsymbol{k})n(\boldsymbol{p}) + \frac{(2\pi)^2\delta^{(2)}(\boldsymbol{k}-\boldsymbol{p})}{(N_c^2-1)} n(\boldsymbol{k})n(\boldsymbol{p})$$

- Due to the symmetry of 2-d kinematics, at $\boldsymbol{k} = -\boldsymbol{p}$, we have

$$\frac{(2\pi)^2\delta^{(2)}(\boldsymbol{k}+\boldsymbol{p})}{(N_c^2-1)} n(\boldsymbol{k})n(\boldsymbol{p})$$

- In practice, δ function smeared and leads to peaks at $\theta = 0, \pi, 2\pi$

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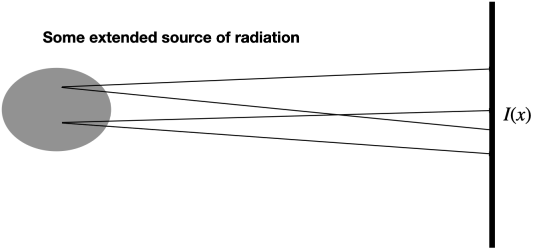
HBT



- Collecting data for the intensity of the signal $I(\boldsymbol{x})$
- $I(\boldsymbol{x}) \Rightarrow I(\boldsymbol{k}) = \bar{I}(\boldsymbol{k}) + \Delta_{\boldsymbol{k}}$
- If $\Delta_{\boldsymbol{k}}$ satisfy Gaussian distribution

$$\langle \Delta_{\boldsymbol{k}} \Delta_{\boldsymbol{p}} \rangle \propto \delta^{(2)}(\boldsymbol{k} - \boldsymbol{p})$$

- 2-d symmetry leads to another peak at $\theta = \pi$

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Non-perturbative photon

- Photon emitted by the nucleus coherently

- Resolution bounded by nucleus size

$$\frac{1}{Q} \gtrsim R_A$$

- For $A > 16$

$$Q^2 \lesssim (60Mev)^2$$

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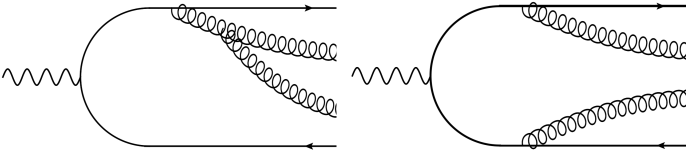
Dipole model ($|Q| < \Lambda_{QCD}$)



- Dipole model to approximate the photon
  Small Q^2 suppresses the longitudinal polarization

$$\Psi_\lambda^T(z, \boldsymbol{r}, s_1) = -i\frac{2ee_f}{2\pi}\delta_{s_1,-s_2}(2z-1+2\lambda s_1)\sqrt{z(1-z)}\frac{\boldsymbol{r}\cdot\boldsymbol{\epsilon_\lambda}}{|r|}\varepsilon_f K_1(\varepsilon_f|r|)$$

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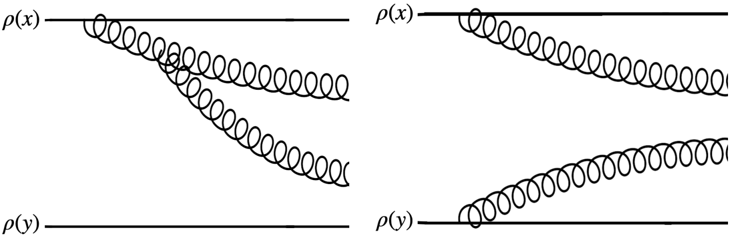
MV model



- Inspired by Vector Meson Dominance Model
- Due to the existence of the high energy fixed point, ρ-meson at asymptotically high energy $\equiv$ nucleus
- Valence degrees of freedom $\rho_a(\boldsymbol{x})$ follow the distribution defined by McLerran-Venugopalan (MV) model

$$W(\rho_a) = \exp\left\{-\int_{\boldsymbol{x}} \frac{\rho_a(\boldsymbol{x})\rho_a(\boldsymbol{x})}{2\mu^2}\right\}$$

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Angular correlation from the cross section

From the cross section of the two gluon production

$$\Sigma = \frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2}$$

one can extract the angular correlation function

$$C(q,\theta) = \frac{\Sigma(q,\theta)}{\frac{1}{2\pi}\int_0^{2\pi}\Sigma(q,\theta)d\theta}$$

set $|q_1| = |q_2| = q$, and θ is the angle between the two particles

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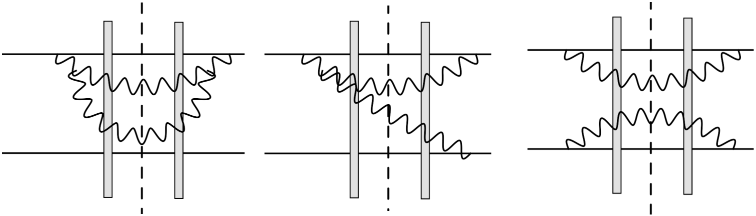
Organize the cross section

Organize the cross section Σ according to the order of ρ

$$\Sigma = \Sigma_2 + \Sigma_3 + \Sigma_4$$



(a) $\Sigma_2(\rho^2)$ (b) $\Sigma_3(\rho^3)$ (c) $\Sigma_4(\rho^4)$

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Isolating the signal

- Symmetrization of $\hat{\rho}s$ (MV model)

$$2\hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y}) = \{\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})\} + [\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})]$$
$$= \rho_a(\boldsymbol{x})\rho_b(\boldsymbol{y}) - \delta^{(2)}(x-y)T^c_{ab}\rho_c(\boldsymbol{x})$$

- Symmetrization of color factors (Dipole model)

$$2t^a t^b = \left\{t^a, t^b\right\} + i f^c_{ab} t^c$$

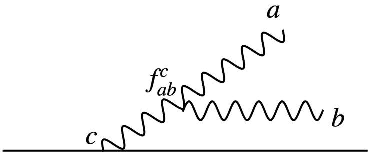

Figure: The diagram for the correction term

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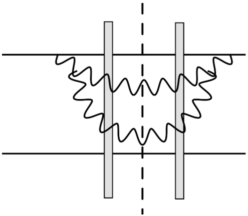
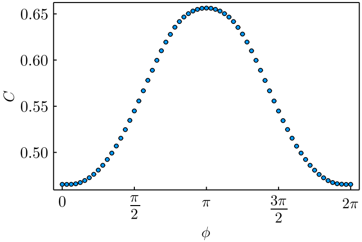
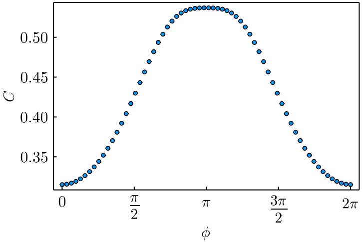
$\Sigma_2,\ q = Q_s$





(a) Dipole



(b) MV

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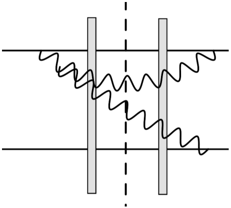
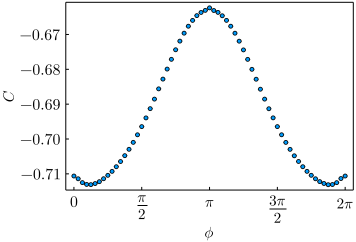
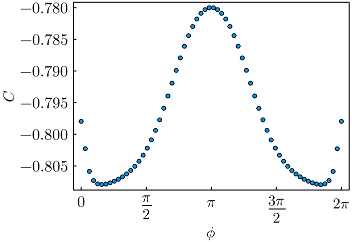
As expected, a strong back-to-back correlation.

$\Sigma_3,\ q = Q_s$





(a) Dipole



(b) MV

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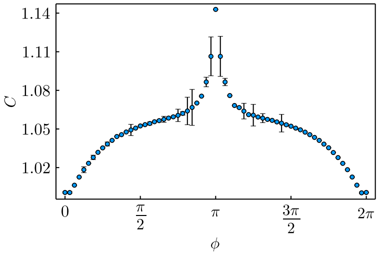
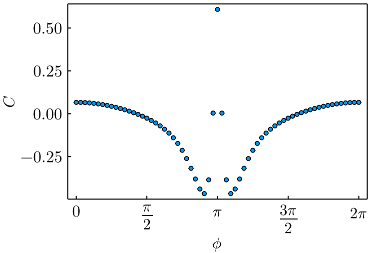
Back-to-back correlation

Σ_4^{nsym}, non-symmetric part, $q = Q_s$



(a) Dipole



(b) MV

Also gives us back-to-back correlation.

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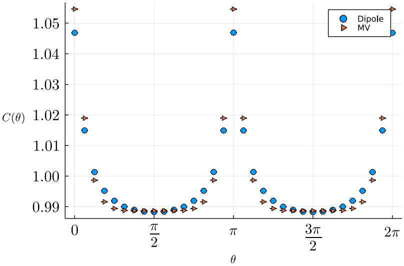
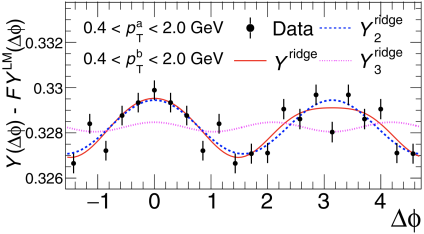
Σ_4^{sym}, symmetric part, $q = Q_s$





As what was done in experimental analysis, we subtract backgrounds and normalize the signal. The preliminary results shows similar correlations in CGC calculation.

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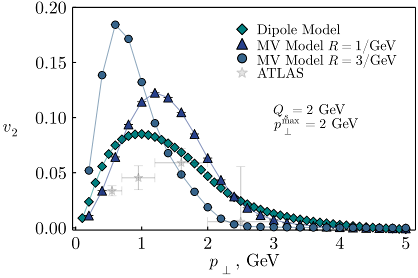
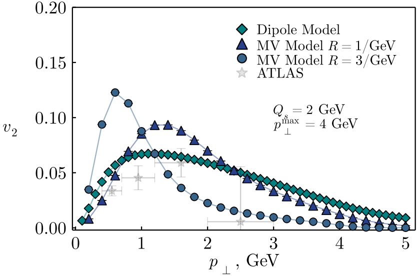
Preliminary v_2 results





Phenomenology to appear in follow up work. The key takeaway is that similar correlation shows up even the projectile is dipole.

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Summary and outlook

- We analytically derived inclusive two gluon production in UPC at mid-rapidity.
- To estimate systematic uncertainty originated from the poor knowledge of the real photon wave function, we studied two limiting cases.
- Both models result in qualitatively similar correlation. Quantitatively, the amplitude of azimuthal anisotropy for MV model is about two times the dipole model.
- Our results show similar correlation as experimental data.
- Further developments
  - To compare with experimental data
  - To incorporate rapidity difference
  - To extend to EIC physics (large Q^2)
  - To add rapidity gap between dipole and gluons.

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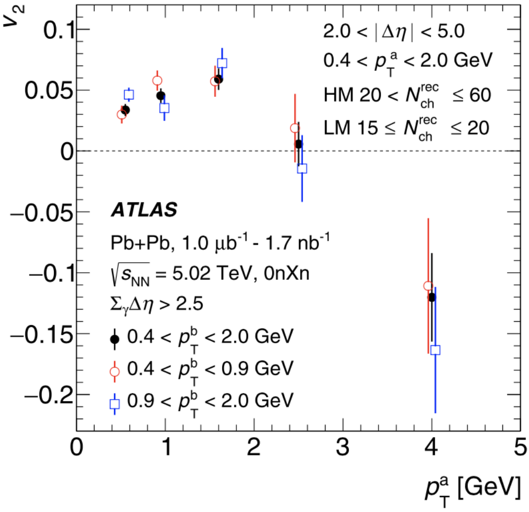
v_2 data



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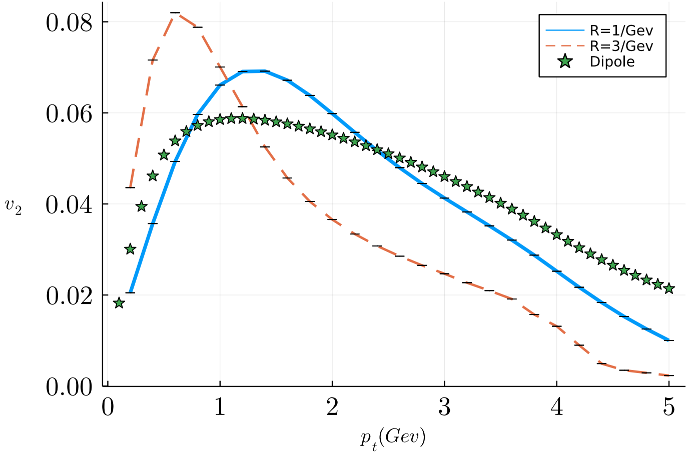
v_2 preliminary numerical results



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Gluon production

Create gluons within initial states

One account for the emission of the gluons using coherent operators

$$C = \mathcal{P} e^{i\sqrt{2}\int d^2x d\xi\, \hat{b}^i_a(\xi,\boldsymbol{x})\left[a^\dagger_{i,a}(\xi,\boldsymbol{x})+a_{i,a}(\xi,\boldsymbol{x})\right]}$$

with the background field

$$\hat{b}^i_a(\xi,\boldsymbol{x}) = \frac{g}{2\pi}\int d^2y \frac{(\boldsymbol{x}-\boldsymbol{y})^i}{|\boldsymbol{x}-\boldsymbol{y}|^2}\hat{\rho}^a_{\mathrm{P}}(\xi,\boldsymbol{y})$$

- MV model source ρ_a
- $\hat{\rho}^a_D(\boldsymbol{x}) = b^\dagger_{\alpha\sigma}(\boldsymbol{x_1})t^a_{\alpha\beta}b_{\beta\sigma}(\boldsymbol{x_1})\delta^{(2)}(\boldsymbol{x}-\boldsymbol{x_1}) - d^\dagger_{\alpha\sigma}(\boldsymbol{x_2})t^a_{\beta\alpha}d_{\beta\sigma}(\boldsymbol{x_2})\delta^{(2)}(\boldsymbol{x}-\boldsymbol{x_2})$
- $\hat{\rho}^a_g(\zeta,\boldsymbol{x}) = a^{i\dagger}_b(\eta,\boldsymbol{x})T^a_{bc}a_c(\eta,\boldsymbol{x})$

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The cross section

$$\frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2} = \frac{1}{(2\pi)^4} \int d^2u_1 d^2u_2 d^2\bar{u}_1 d^2\bar{u}_2 e^{-i\boldsymbol{q}_1(\boldsymbol{u}_1-\bar{\boldsymbol{u}}_1)} e^{-i\boldsymbol{q}_2(\boldsymbol{u}_2-\bar{\boldsymbol{u}}_2)}$$
$$\times \langle\gamma^*|C^\dagger \hat{S}^\dagger C a^\dagger_{i,a}(\eta,\boldsymbol{u}_1) a^\dagger_{j,b}(\xi,\boldsymbol{u}_2) a_{i,a}(\eta,\bar{\boldsymbol{u}}_1) a_{j,b}(\xi,\bar{\boldsymbol{u}}_2) C^\dagger \hat{S} C|\gamma^*\rangle$$

where $C = C_\xi C_\eta$, and $\eta \gg \xi$,

$$C_\eta \simeq 1 + i\sqrt{2}\int d^2v_1 \hat{b}^i_{Da}(\boldsymbol{v}_1)\left[a^{i\dagger}_a(\eta,\boldsymbol{v}_1) + a^i_a(\eta,\boldsymbol{v}_1)\right]$$
$$C_\xi \simeq 1 + i\sqrt{2}\int d^2v_2 \left(\hat{b}^j_{Db}(\boldsymbol{v}_2) + \delta\hat{b}^j_b(\eta,\boldsymbol{v}_2)\right)\left[a^{j\dagger}_b(\xi,\boldsymbol{v}_2) + a^j_b(\xi,\boldsymbol{v}_2)\right]$$

- $C|\gamma^*\rangle$ Initial state
- $\hat{S}$ S-matrix
- $Ca_{j,b}(\xi,\bar{\boldsymbol{u}}_2)C^\dagger$ dressed gluons in the final state

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Continue the calculation of Σ

Use Σ_2 as example, in coordinate space,

$$\Sigma_2 = 4\int d^2\boldsymbol{x}\int d^2\bar{\boldsymbol{x}} f^i(\bar{u}_1-\boldsymbol{x})f^i(u_1-\bar{\boldsymbol{x}})f^j(\bar{u}_2-\bar{u}_1)f^j(u_2-u_1)\langle\rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})\rangle_P$$
$$\left\langle\Big[[U^\dagger(u_1)T^aU(u_1)][U^\dagger(u_2)-U^\dagger(u_1)][U(\bar{u}_2)-U(\bar{u}_1)][U^\dagger(\bar{u}_1)T^aU(\bar{u}_1)]\Big]_{d'd}\right\rangle_T$$

where $f^i(\boldsymbol{x}) = \frac{g}{(2\pi)^2}\frac{x_i}{x^2}$.

- Kinematic factors (Eikonal emission vertices)
- Projectile (photon)
- Target (nucleus)

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Expectation values for projectile and target

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Dipole expectation values

- Expectation values for $q\bar{q}$

$$\langle q\bar{q}|\hat{\rho}_{d'}(\bar{\boldsymbol{x}})\hat{\rho}_d(\boldsymbol{x})|q\bar{q}\rangle = \frac{\delta^{dd'}}{2}\left(\delta^2(\bar{\boldsymbol{x}}-z_1)-\delta^2(\bar{\boldsymbol{x}}-z_2)\right)\left(\delta^2(\boldsymbol{x}-z_1)-\delta^2(\boldsymbol{x}-z_2)\right)$$

$$\begin{aligned}&\langle q\bar{q}|\hat{\rho}^a(\boldsymbol{x_1})\hat{\rho}^b(\boldsymbol{x_2})\hat{\rho}^c(\boldsymbol{x_3})|q\bar{q}\rangle\\ =&\frac{if_{abc}}{4}\left(\delta^{(2)}(\boldsymbol{x_2}-\boldsymbol{z_1})+\delta^{(2)}(\boldsymbol{x_2}-\boldsymbol{z_2})\right)\prod_{i=1,3}\left(\delta^{(2)}(\boldsymbol{x_i}-\boldsymbol{z_1})-\delta^{(2)}(\boldsymbol{x_i}-\boldsymbol{z_2})\right)\end{aligned}$$

$\boldsymbol{z}_1, \boldsymbol{z}_2$ are the transverse coordinates of quark and anti-quark.

- Average over different dipole size $\boldsymbol{r} = \boldsymbol{z}_1 - \boldsymbol{z}_2$

$$\langle\rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})\rangle_P \approx \sum_{s_1}\int_z\int d^2\boldsymbol{r}\Psi_\lambda^{T*}(z,r,s_1)\Psi_\lambda^T(z,r,s_1)\langle q\bar{q}|\rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})|q\bar{q}\rangle$$

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MV model projectile average

- MV model describes the distribution of classical color source not quantum operators.

$$W(\rho_a) = \exp\left\{ -\int_{\boldsymbol{x}} \frac{\rho_a(\boldsymbol{x})\rho_a(\boldsymbol{x})}{2\mu^2} \right\}$$

- Two and three point correlators

$$\langle \hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y})\rangle_{\text{MV}} = \langle \rho_a(\boldsymbol{x})\rho_b(\boldsymbol{y})\rangle_{\text{MV}} = \mu^2\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta_{ab}$$

$$\langle \hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y})\hat{\rho}_c(\boldsymbol{z})\rangle_{\text{MV}} = -\frac{1}{2}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta^{(2)}(\boldsymbol{y}-\boldsymbol{z})T^a_{bc}\,\mu^2$$

- Symmetrization of $\hat{\rho}s$

$$\begin{aligned}\hat{\rho}_a(x)\hat{\rho}_b(y) &= \{\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})\} + [\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})] \\ &= \rho_a(\boldsymbol{x})\rho_b(\boldsymbol{y}) - \frac{1}{2}\delta^{(2)}(x-y)T^c_{ab}\rho_c(\boldsymbol{x})\end{aligned}$$

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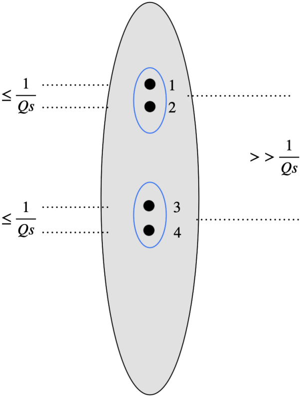
Target average(I)



- "Dipole" approximation

  Phys. Rev. D 96, 074018, Kovner, Rezaeian

- Dense target → Saturated

- $\frac{1}{Q_s}$ serves the role of correlation length in transverse plane

- For the example configuration

  $\mathrm{Tr}\left[U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)\right]$

  $\approx$

  $\frac{1}{N_c^2-1}\,\mathrm{Tr}\left[U(x_1)U^\dagger(x_2)\right]\,\mathrm{Tr}\left[U(x_3)U^\dagger(x_4)\right]+$

  ...

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Target average (II)

We only have one type of Wilson line correlator in momentum space

$$
\begin{aligned}
&\left\langle \mathrm{Tr}\left[U(k_1)T^aU^\dagger(k_2)U(k_3)T^aU^\dagger(k_4)\right]\right\rangle_T \\
=&T^a_{bc}T^a_{de}\left\langle\left[U^{fb}(k_1)U^{\dagger cg}(k_2)U^{gd}(k_3)U^{\dagger ef}(k_4)\right]\right\rangle_T \\
\approx&T^a_{bc}T^a_{de}(\frac{(2\pi)^2}{N_c^2-1})^2\Big\{(N_c^2-1)\delta^{bc}\delta^{de}\delta^{(2)}(k_1-k_2)D(k_1)\delta^{(2)}(k_3-k_4)D(k_3) \\
&+(N_c^2-1)\delta^{bd}\delta^{ce}\delta^{(2)}(k_1+k_3)D(k_1)\delta^{(2)}(k_2+k_4)D(-k_2) \\
&+(N_c^2-1)^2\delta^{be}\delta^{cd}\delta^{(2)}(k_1-k_4)D(k_1)\delta^{(2)}(k_2-k_3)D(-k_2)\Big\}
\end{aligned}
$$

here the dipole $D(p)$ is defined as

$$
D(p)=\frac{1}{N_c^2-1}\int dx^2 e^{ipx}\langle\mathrm{Tr}\Big(U^\dagger(x)U(0)\Big)\rangle_T
$$

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The structure of analytic results(Σ_2)

- Dipole model

$$\frac{dN^{(2)}}{d\eta dq_1^2 d\xi dq_2^2} \propto \int_{k,\bar{k}} S_\perp \mathcal{I}(\varepsilon_f, |q_1+q_2+k+\bar{k}|) D(-\bar{k}) D(-k) \frac{(N_c^2-1)\Gamma(q_2, q_2+k, q_2+k) - \Gamma(q_2, k+q_2, \bar{k}+q_2)}{(q_1+q_2+k+\bar{k})^2}$$

  - $S_\perp$-Transverse area
  - $\mathcal{I}(\varepsilon_f, |\boldsymbol{P}|) = \int dr r \varepsilon_f^2 K_1^2(\varepsilon_f r)(1 - J_0(|r||\boldsymbol{P}|))$

- MV model

$$\frac{dN_2}{dq_1^2 dq_2^2 d\eta d\xi} \propto \int d^2x \mu^2(x) \int_{k_1,k_2} D(k_1+q_1) D(-k_2+q_2) \Big\{ (N_c^2-1)\Big(\frac{(k_2-k_1)\cdot L(q_2,k_2)}{(k_2-k_1)^2}\Big)^2 - \frac{(k_2-k_1)\cdot L(q_2,k_2)}{(k_2-k_1)^2} \frac{(k_2-k_1)\cdot L(q_2, q_2-k_2-k_1)}{(k_2-k_1)^2} \Big\}$$

  - $L^i(\boldsymbol{p},\boldsymbol{q}) = \frac{\boldsymbol{p}^i}{\boldsymbol{p}^2} - \frac{\boldsymbol{q}^i}{\boldsymbol{q}^2}$ (Lipatov vertex)
  - $\Gamma(\boldsymbol{p},\boldsymbol{k},\boldsymbol{q}) = L(\boldsymbol{p},\boldsymbol{k}) \cdot L(\boldsymbol{p},\boldsymbol{q})$

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