Effective Field Theory and Leuscher Finite Volume Methods

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LQCD Group

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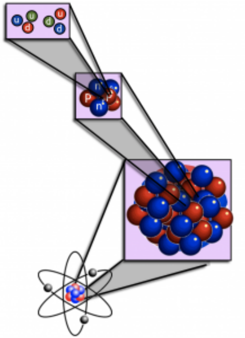
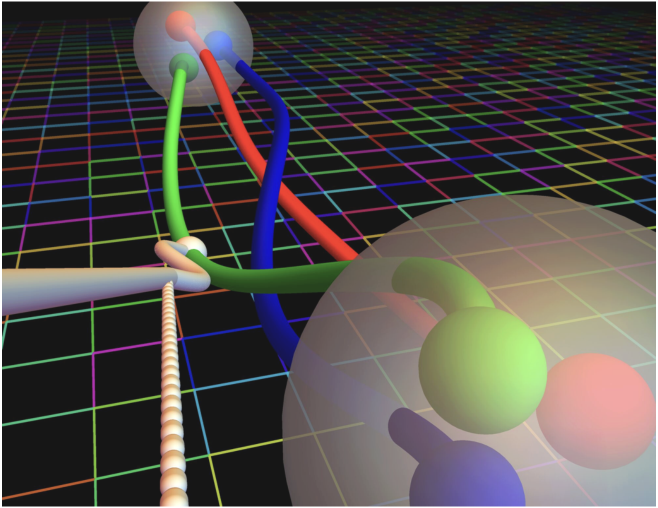


$$\mathcal{L}_{QCD} = \overline{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m\delta_{ij})\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- hadrons
- nuclei
- nuclear matter

Compute observables from lattice simulations - a difficult task!

Symmetries and Interactions from Lattice Gauge Theory [Mil+22]



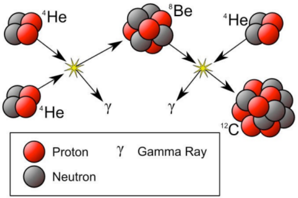
[Nic+18]

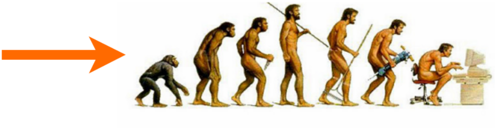
NN interactions at low energies

THE DEUTERON: LONG STANDING PUZZLE!

1. understanding nuclear physics from first principles
2. benchmark for multi-nucleon systems
3. extraction of two-nucleon scattering phase shifts

Why Deuterium





Binding Energy: 2.2 MeV, resulting in weakly bound deuteron. Requires fine tuning of pion-exchange and short-range interactions!
AGAINST expectation of binding energy from pion decay constant, f_π

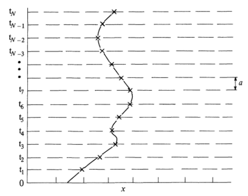
1. BBN
2. nucleosynthesis
3. many-body theory

Our Methods

Tools that we use!

Lattice:

Regularization by discretization!



$$N = \frac{T}{a}$$

$$Z = \int (\Pi_i dx_i) e^{-S}$$

Effective Field Theory:

1. Spontaneous symmetry breaking

$$m_u - m_d \approx 0$$

approximate symmetry under SU(2) isospin.

2. Global Chiral Symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

3. Non-relativistic QM

NN interactions

$\not\pi$ EFT

Effective Field Theory

1. Establish a power counting scheme
2. Focus on relevant symmetries
3. Write a Langrangian that encapsulates the important aspects

Definition 1 (Goldstone Theorem)

Goldstone's Theorem
When a continuous symmetry is spontaneously broken, meaning its currents are conserved but the ground state is not invariant under an action of the charges, then new **massless** (or light, if not exact symmetry) scalar particles appear in the spectrum.

EFT for Meson Analysis

Global Flavor Symmetry **broken** by non-zero quark condensate $< q\,\bar{q} >$

Chiral Perturbation Theory

$$N_f = 3: \quad \Theta(x) = \sum_{a=1}^{8} \lambda_a \Theta_a(x) \equiv \begin{bmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{bmatrix}$$

$$N_f = 2: \quad \Theta(x) = \sum_{i=1}^{3} \sigma_i \Theta_i(x) \equiv \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{bmatrix}$$

Include $(m_u - m_d)$ for explicit symmetry breaking in expansion parameter ϵ

baryonic EFT

1. expansion parameter $\epsilon \sim \frac{m_\pi}{\Lambda}, \frac{p}{\Lambda}$

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_\tau + \frac{\nabla^2}{2M} \right) \psi + g_0 \left(\psi^\dagger \psi\right)^2 + \frac{g_2}{8} \left[(\psi\psi)^\dagger \left(\psi \overleftrightarrow{\nabla}^2 \psi \right) + \text{h.c.} \right] + \cdots \ ,$$

2. LECS scale $\sim \Lambda^{-dim(\mathcal{O})}$
3. relevant parameters: scattering length, a

compare g0 to s-wave scattering

scattering theory

S-matrix for nonrelativistic scattering

$$S = 1 + \frac{iMp}{2\pi} A$$

← Scattering Amplitude

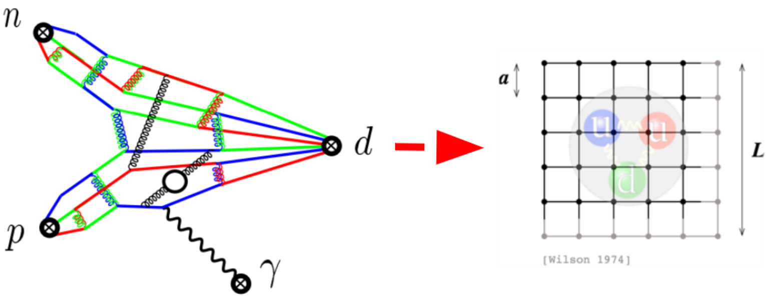
s-wave $$A = \frac{4\pi}{M} \frac{1}{p \cot \delta - ip}$$

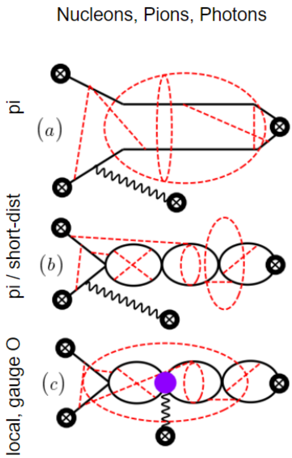
δ is the s-wave scattering phase shift

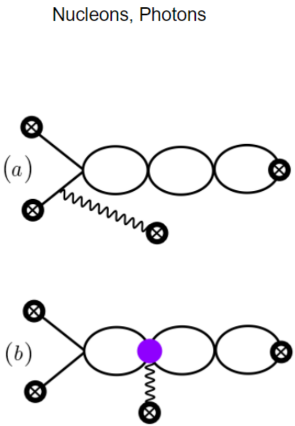
short-range two-body potential

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + r_1 p^4 + \cdots$$

Deuteron




Nucleons, Pions, Photons
pi
(a)
pi / short-dist
(b)
local, gauge O
(c)


Nucleons, Photons
(a)
(b)

$\not\pi$-EFT / EFT$_{\not\pi}$

1. path integral formulation of quantum field theory in Euclidean spacetime
2. discretized on a periodic lattice size $L^3 \times N_\tau$. Zero temp $\rightarrow$ large N_τ.
3. Lattice spacings defined by dimensional analysis b_s and $b_\tau = \frac{b_s^2}{M}$

Free Theory $\mathcal{L}(\psi^\dagger, \psi) = \psi^\dagger (\partial_\tau - \mu) \psi + \mathcal{H}[\psi^\dagger, \psi]$

$$Z = \int \mathcal{D}\psi^\dagger \mathcal{D}\psi e^{-\int d\tau d^3x [\mathcal{L}(\psi^\dagger, \psi)]}$$

$$S_{\text{free}} = \sum_{\tau,\tau'} \frac{1}{b_\tau} \psi^\dagger_{\tau'} [K_0]_{\tau,\tau'} \psi_\tau \qquad K_0 \equiv \begin{pmatrix} D & -1 & 0 & 0 & \cdots \\ 0 & D & -1 & 0 & \cdots \\ 0 & 0 & D & -1 & \cdots \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & . & . & . & . \end{pmatrix}$$

$$\mathcal{L}_{\text{int}} = \sum_n g_0 \psi_{n,\uparrow} \psi_{n,\uparrow} \psi_{n,\downarrow} \psi_{n,\downarrow}$$

$D \equiv 1 - \frac{b_s^2 \nabla_L^2}{2}$ contains the spatial Laplacian

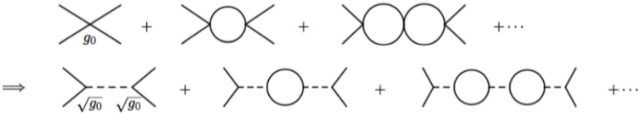
H-S transformation $$e^{b_\tau g_0 \psi^\dagger_\uparrow \psi_\uparrow \psi^\dagger_\downarrow \psi_\downarrow} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\phi^{-\phi^2/2 - \phi\sqrt{b_\tau g_0}(\psi^\dagger_\uparrow \psi_\uparrow + \psi^\dagger_\downarrow \psi_\downarrow)}$$

Computing two-particle spectrum

We utilize the transfer matrix method! (Analytic Computation)

Transfer Operator $\hat{T} = e^{-ia\hat{H}}$

$$\langle pq|\mathcal{T}|p'q'\rangle = \frac{\delta_{pp'}\delta_{qq'} + \frac{g_0}{V}\delta_{p+q,p'+q'}}{\sqrt{\xi(p)\xi(q)\xi(q')\xi(p')}}, \qquad \xi(p) \equiv \frac{\Delta(q)}{M}$$



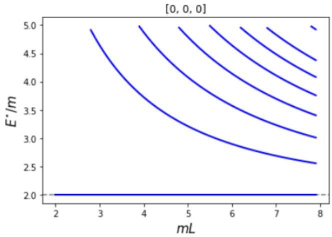
Extracting the spectrum

Leuscher Method for Scattering in Periodic volumes

1. Asymptotic scattering states
2. Finite volume effects on energies
3. spectrum in box is quantized

$$\langle \mathbf{2}|S-1|\mathbf{2}\rangle = (2\pi)^4\delta(P'-P)\, i\mathcal{M}(E^\star,\theta^\star)$$

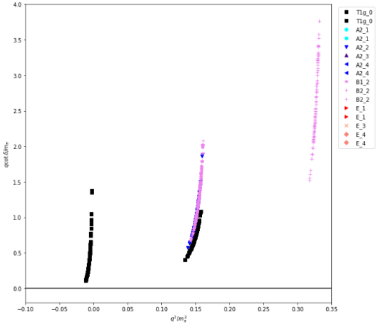
$$\mathcal{M}(s) = \frac{8\pi\sqrt{s}}{\xi}\frac{1}{q^\star \cot\delta(s) - iq^\star}$$

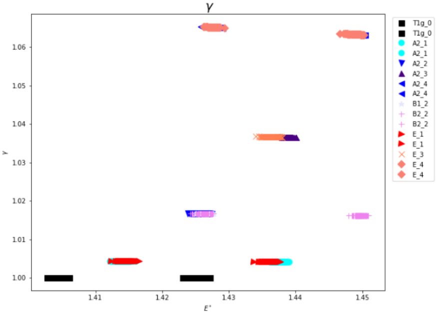


1. Leuscher's method extracts scattering phase shift
2. spectrum has cubic symmetry group, NOT spherical symmetry

$$q^* \cot \delta(q^*) = \frac{2}{\gamma L \sqrt{\pi}} Z_{00}^{\mathbf{d}} \left(1, \frac{q^{*2} L^2}{4\pi^2}\right) \qquad \mathbf{d} \equiv \frac{L}{2\pi}\mathbf{P}$$

$$Z_{00}^{\mathbf{d}}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{r} \in P_{\mathbf{d}}} (\mathbf{r}^2 - q^2)^{-s}$$





Calculating Deuteron Form Factor

1. Calculate form factor using LEFC Transfer Matrix
2. Analyze the spectrum using Lellouch-Luscher
3. Use formalism for 2 + J -> 2 scattering

define matrix elements of a vector current, $\mathcal{J}^\mu(x)$, in terms of two Lorentz scalars

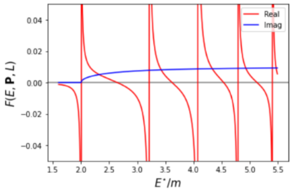
$$_\infty\langle P_f|\,\mathcal{J}^\mu(x=0)\,|P_i\rangle_\infty = (P_f+P_i)^\mu F_1(Q^2) + (P_f-P_i)^\mu F_2(Q^2)$$

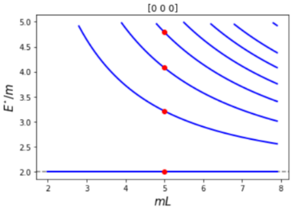
$$\boxed{_\infty\langle P_f|\,\mathcal{J}^\mu(0)\,|P_i\rangle_\infty = (P_f+P_i)^\mu F(Q^2)}$$

1. Use Finite Volume Function that relates to spectrum
2. Luescher's quantization condition
3. Calculate spectrum with FVF

$$\mathcal{M}^{-1}(P_n^2) + F(P_n, L) = 0$$

$$F(P, L) = \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{D((P-k)^2)}{2\omega_{\mathbf{k}}}$$





To be done

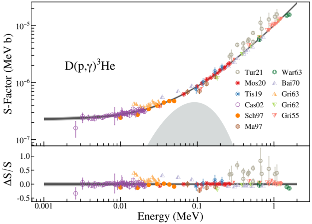
1. Map the calculated spectrum to irreps of cubic group
2. Confirm spectrum of deuteron form factor

Spin j	Decomposition into Irreps of O
0	A_1
1	T_1
2	$E \oplus T_2$
3	$A_2 \oplus T_1 \oplus T_2$
4	$A_1 \oplus E \oplus T_1 \oplus T_2$
5	$E \oplus 2T_1 \oplus T_2$
6	$A_1 \oplus A_2 \oplus E \oplus T_1 \oplus 2T_2$
7	$A_2 \oplus E \oplus 2T_1 \oplus 2T_2$
8	$A_1 \oplus 2E \oplus 2T_1 \oplus 2T_2$
9	$A_1 \oplus A_2 \oplus E \oplus 3T_1 \oplus 2T_2$
10	$A_1 \oplus A_2 \oplus 2E \oplus 2T_1 \oplus 3T_2$
11	$A_2 \oplus 2E \oplus 3T_1 \oplus 3T_2$
12	$2A_1 \oplus A_2 \oplus 2E \oplus 3T_1 \oplus 3T_2$

Conclusion

1. Use of LQCD allows investigation of low-energy properties of nucleons
2. $\not\pi$ EFT used to investigate phenomena at energies below mass of pion
3. calculation of deuteron form-factor from first principles will be benchmark for further computations
4. Work done with HadSpec group https://jeffersonlab.github.io/hadspec/

EXTRA: Work on $d\ p \to \gamma\ He$ thermonuclear rate for BBN [Mos+21]



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