



Image credit: Marguerite Tonjes

# Systematics

Nick Smith

Future Trends in Nuclear Physics Computing

29 September, 2022



# Outline

- What are systematics?
- Basic tasks in analysis
  - Modeling
  - Evaluation
  - Book-keeping
- Goals:
  - Review present thinking in
    - Technical design of systematics handling in analysis software and data flow
    - Approaches to systematics modeling and presentation
  - Encourage discussion
    - Good abstractions?
- My perspective:
  - CMS Higgs combination convener
  - Coffea analysis framework developer
  - Statistics hobbyist

# What are systematic effects?

- Ingredients in defining how  $p$  depends on  $\theta$ 
  - And the data  $y$
  - And the dimension of  $\theta$

( $\mu$  = parameters of interest)

Nuisance parameters are generally included in a model to take into account systematic uncertainties. Suppose that  $x$  are the *primary measurements* and have probability (density)  $p(x|\mu, \theta)$ . In order to constrain the nuisance parameters  $\theta$  we have a set of independent *auxiliary data*  $y$ , with probability  $p(y|\theta)$ . The joint probability of the data  $x$  and  $y$  is, therefore, given by

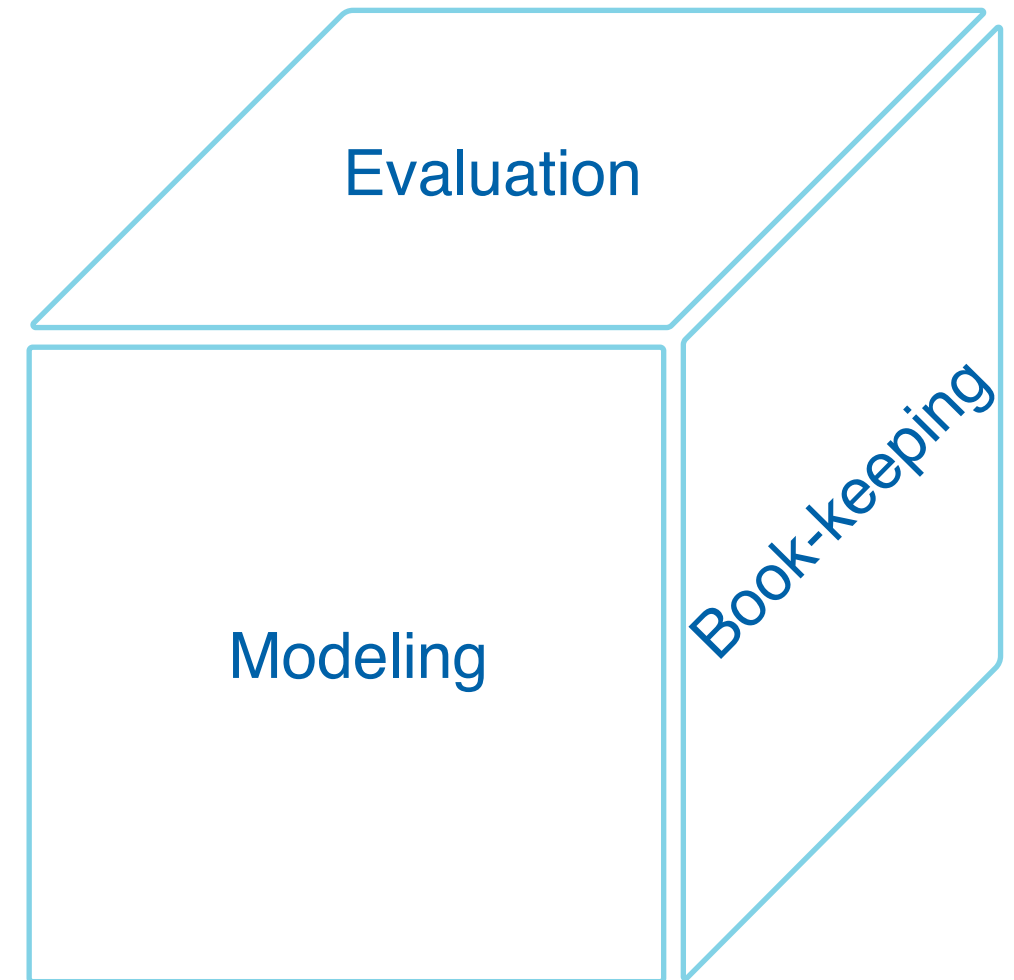
$$p(x, y|\mu, \theta) = p(x|\mu, \theta)p(y|\theta) . \quad (2)$$

[arxiv:2109.04981](https://arxiv.org/abs/2109.04981)

- Frequentist interpretation:  $p$  is likelihood
  - Profile or marginalize away  $\theta$
  - Formalism allows Bayesian approach as well
    - Know any PyMC3/BAT.jl/etc. analyses?

# Typical tasks

- Enumerate effects to get dimension of  $\theta$ 
  - Don't forget anything! Unknown unknowns?
- Choose a function class  $p_{\eta}(y|\theta)$ 
  - Also for  $p(x)$
- Evaluate  $\eta$ 
  - In practice: interpolate between several  $p(y|\theta_i)$
- Iterate
  - Compromise: fidelity/computability/practicality
    - Prune low-impact effects
  - Initial model might not fit observed  $y$  well
- We want to simplify these tasks





# Disclaimer

- My perspective: CMS Higgs combination
  - Mostly binned likelihood models
    - Cannot evaluate density, but can sample
    - Some unbinned  $p(x)$  estimated from samples
- Common ground: Poisson process?
  - One *can* model with binned approach
  - Unbinned = limit of small bins

$$\int_{\text{bin}} P(x) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x)}^N 1(x_i \in \text{bin})$$

~~$P(x) dx$~~

where else. Orear [1] makes plain what is happening by dividing the range of  $x$  into narrow bins of width  $\Delta x$ , so small that the probability of a bin containing more than one event is negligible. The probabilities for 0 and 1 events in a bin are given by Poisson statistics

$$P_0(x) = e^{-\Delta x P(x)},$$

$$P_1(x) = \Delta x P(x) e^{-\Delta x P(x)}.$$

The extended likelihood  $\mathcal{L}$  is thus the combined probability for a complete data sample

$$\mathcal{L} = \prod_i \Delta x P(x_i) \prod_j e^{-\Delta x P(x_j)},$$

where the first product is over all bins containing an event, and the second is over all bins.

In the limit  $\Delta x \rightarrow dx$ , the first term becomes  $\prod_i P(x_i) d^N x$ , analogously to eq. (1) – the  $d^N x$  merely expresses the fact that the functions are probability densities, not probabilities. The second is

$$e^{-\sum_j \Delta x P(x_j)} \rightarrow e^{-\int P(x) dx} = e^{-\mathcal{N}}.$$

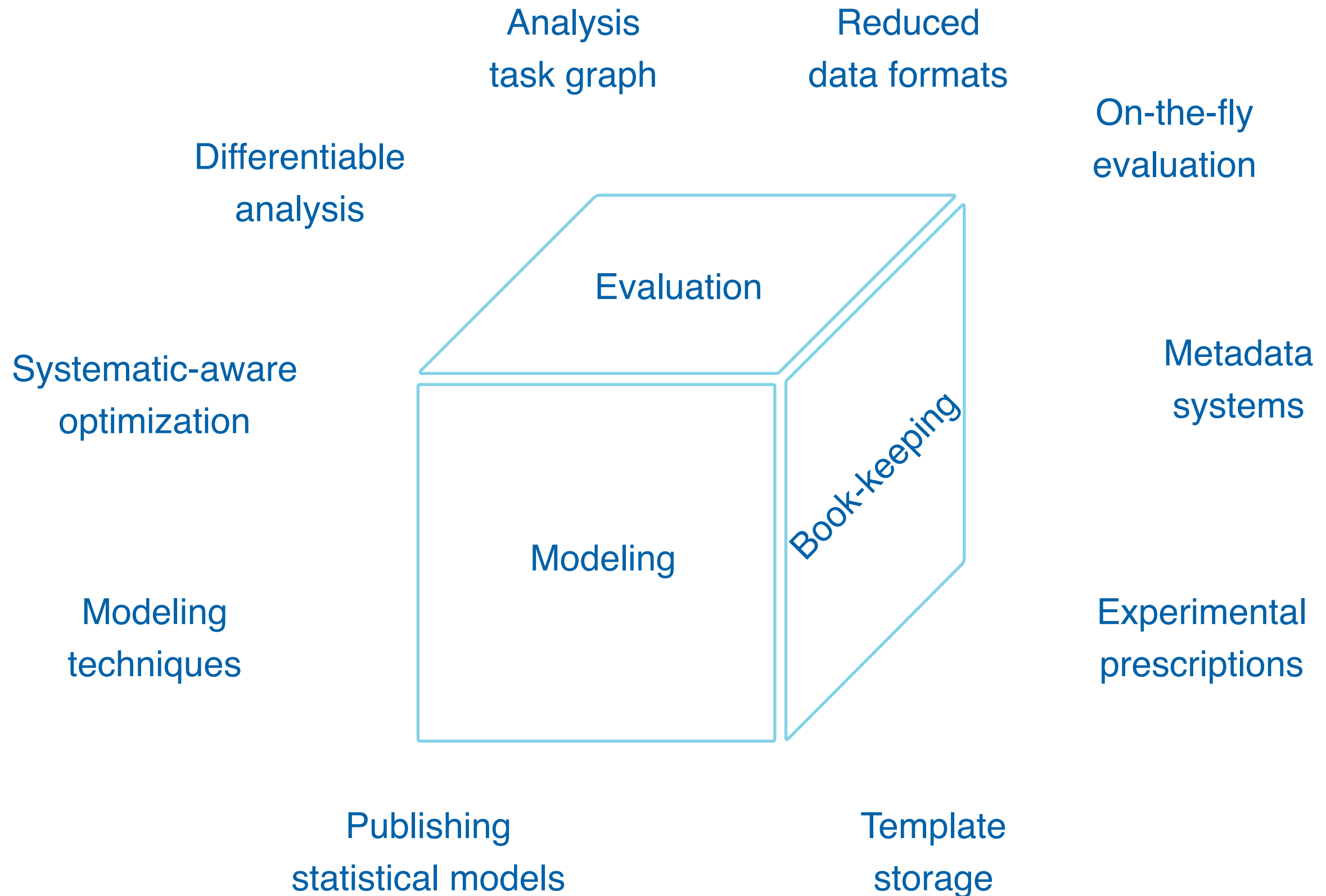
Thus the extended likelihood is given by

$$\mathcal{L} = \left[ \prod_i P(x_i) \right] e^{-\mathcal{N}}, \quad (5)$$

R. Barlow, Extended maximum likelihood

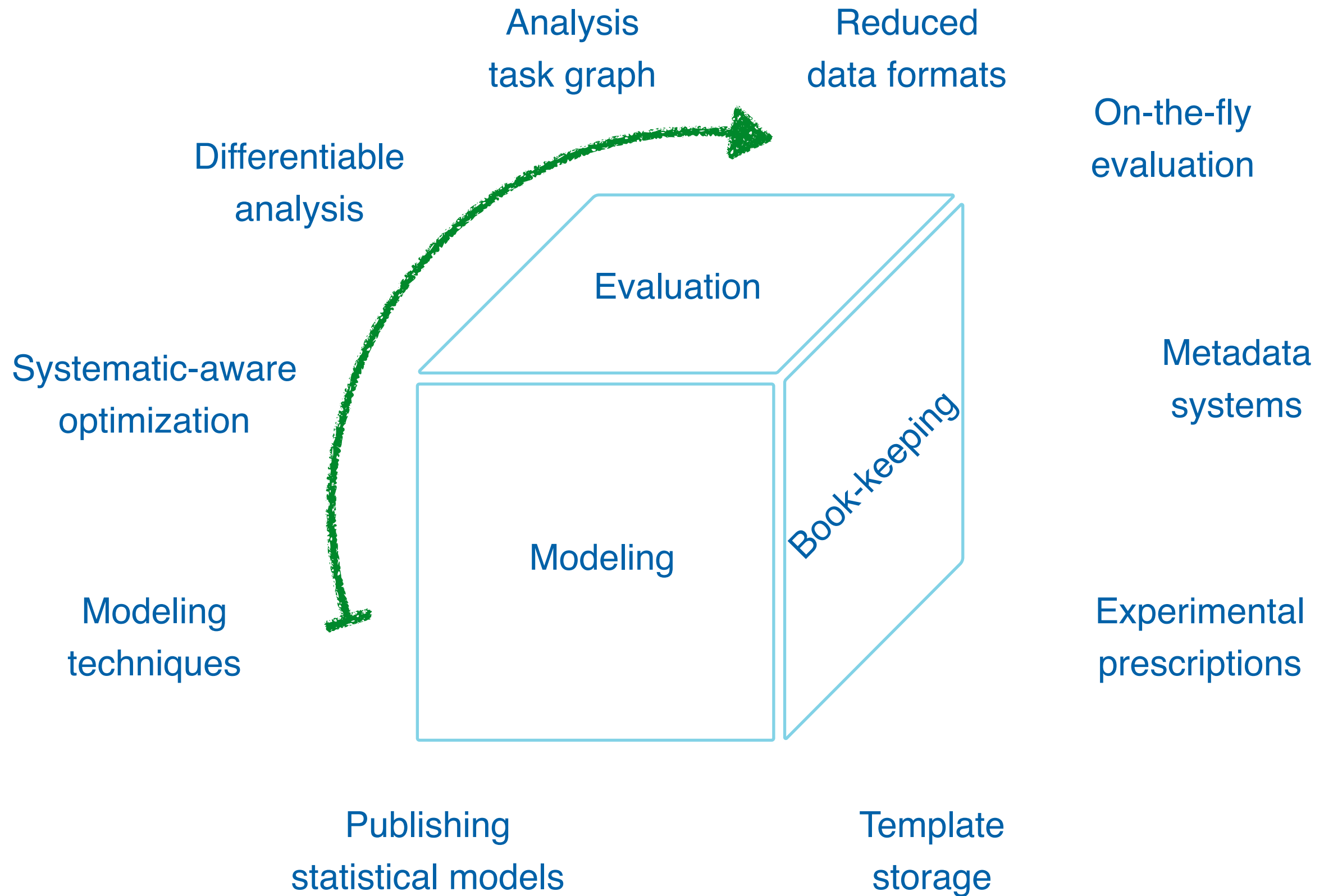
[doi:10.1016/0168-9002\(90\)91334-8](https://doi.org/10.1016/0168-9002(90)91334-8)

# Relevant topics





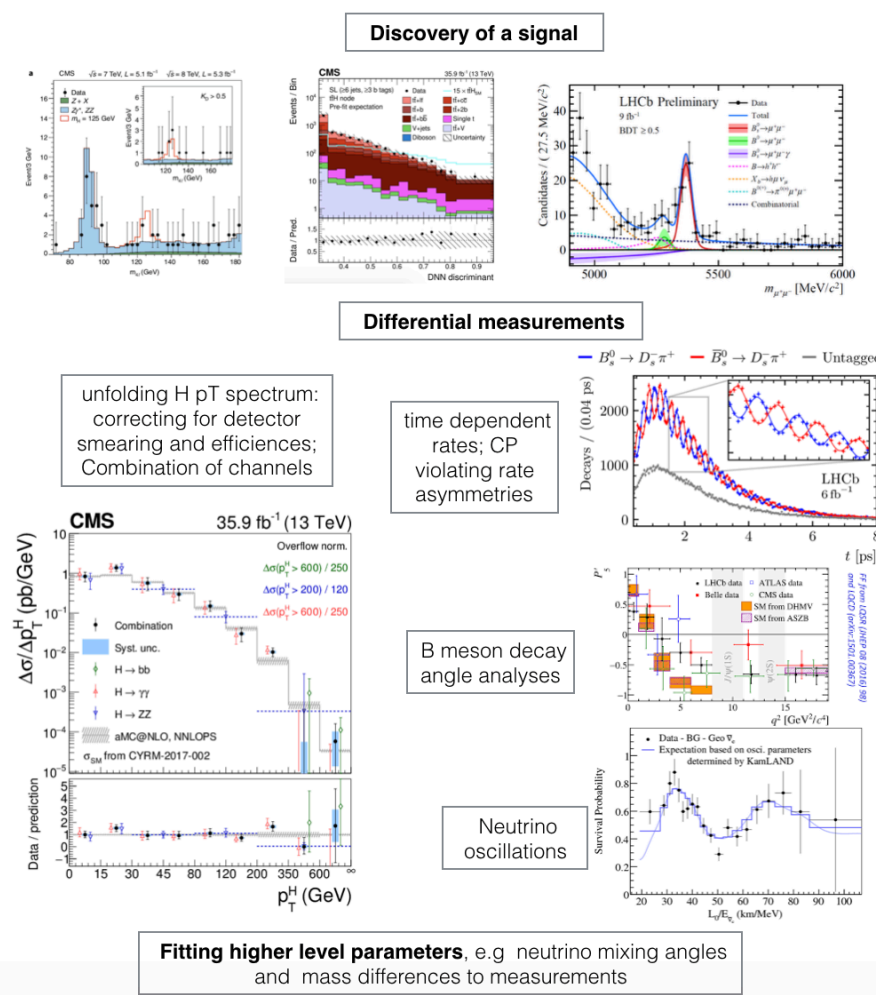
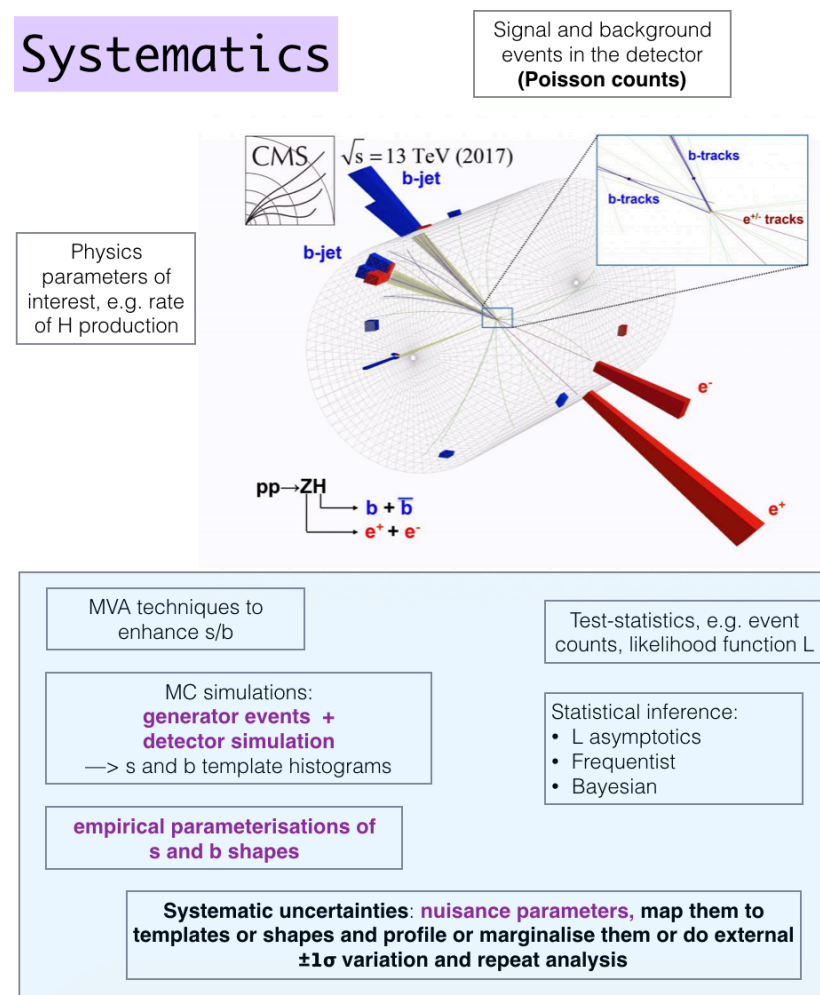
# Relevant topics



# Modeling techniques

- Can be a whole workshop
  - Was: [PHYSTAT-Systematics 2021](#)
  - Excellent presentations covering a wide range of techniques
  - A one-slide overview was [produced](#) (more on types than techniques)

## Systematics



## Nuisance pars:

### Luminosity

#### Detector:

- Acceptance
- Efficiency for specific particles
- Energy scales
- Resolutions

#### Signal process template:

- Theory modelling uncertainties
- Limited MC statistics

#### Background processes template:

- Theory total cross section uncertainty
- Theory modelling uncertainties
- Limited MC statistics

#### Empirical s and b shape modelling;

- Parameterisations
- Non-parametric
- smoothing and morphing of MC templates

#### Nuisance parameters can be constrained from:

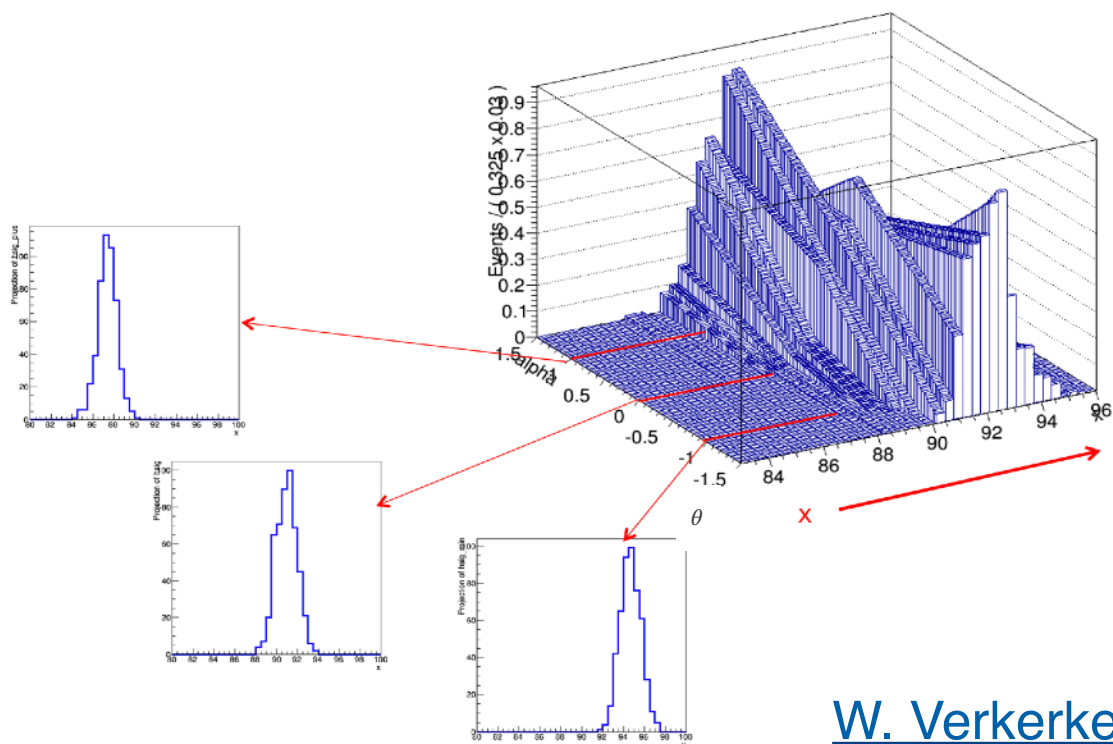
- Detector calibration data
- Control samples with different event selection
- from the data distributions
- measurements from other experiments
- theory calculations



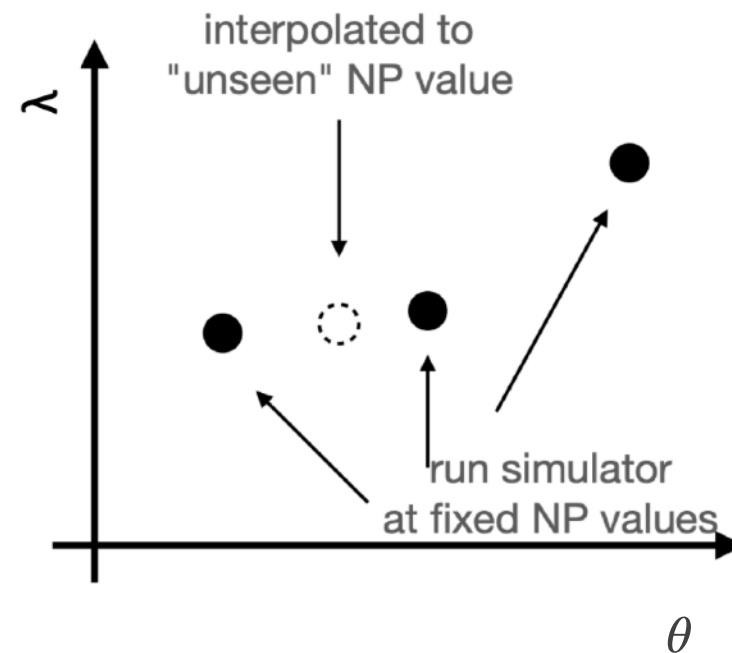
# Modeling techniques

- **Rich** set of interpolation/extrapolation techniques at end-stage
  - Morphing: vertical, horizontal, moment; splines; gaussian process; asymmetric shift interpolation; additive/multiplicative effects; MC stat uncertainty, [BB-lite](#); ...
  - i.e. what is done in [RooFit](#)/[pyhf](#)/[zfit](#)/[iMinuit](#)/[combine](#)/etc.
    - What features do each of these tools offer? Nobody has it all!

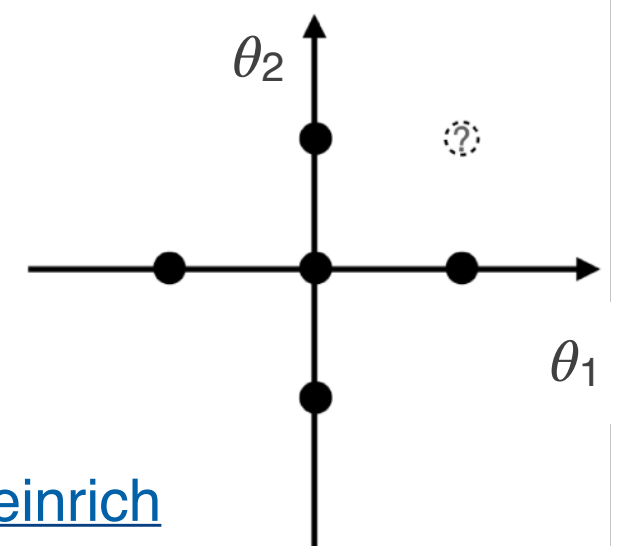
Visualization of bin-by-bin linear interpolation of distribution



[W. Verkerke](#)



Combining effects



[L. Heinrich](#)

# Modeling techniques

- Simpler taxonomy of techniques to get inputs to fitting tools?
  - This is the dominant analysis-stage computation expense (process billions of events)
- Posit three basic techniques
  - I think all of these can be done unbinned as well
  - Just need functions  $w(x, \theta)$  and  $\Delta(x, \theta)$

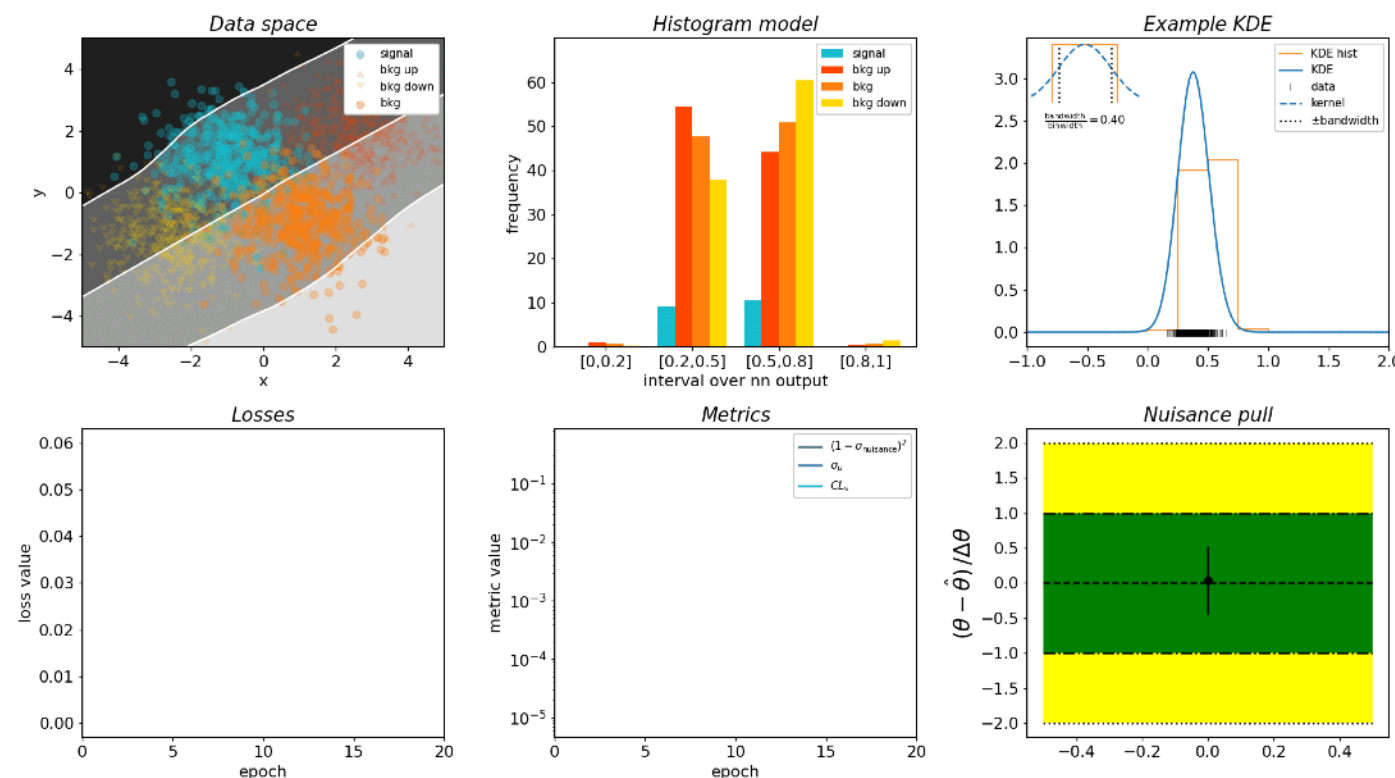
The diagram illustrates three modeling techniques branching from a nominal integral. On the left, the nominal integral is shown: 
$$\int_{\text{bin}} P(x) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x)}^N 1(x_i \in \text{bin})$$
 with the label "(nominal)" below it. Three arrows point from this integral to three alternative forms on the right. The top arrow points to: 
$$\int_{\text{bin}} P(x|\theta = \theta_1) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x|\theta=\theta_1)}^N 1(x_i \in \text{bin})$$
 with the label "(alternative sample, e.g. 2-point)" below it. The middle arrow points to: 
$$\int_{\text{bin}} P(x|\theta = \theta_1) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x)}^N w(x_i, \theta = \theta_1) 1(x_i \in \text{bin})$$
 with the label "(reweight, e.g. efficiency)" below it. The bottom arrow points to: 
$$\int_{\text{bin}} P(x|\theta = \theta_1) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x)}^N 1(x_i + \Delta(x_i, \theta = \theta_1) \in \text{bin})$$
 with the label "(shift, e.g. energy scale)" below it.



# Systematic-aware optimization

- Analysis design and optimization often involves ML these days
- Learn salient features, ignore features affected by nuisance params
- Dozens of proposals, see [HEPML LivingReview](#) sections:
  - Decorrelation methods allow for construction of control regions
  - Inference-aware: maximize sensitivity or exclusion power of POI in full likelihood model
    - Can be deployed in more “traditional” analyses for e.g. region/binning optimization
  - Domain adaptation: ensure marginalized observables are modeled well

[neos](#): N. Simpson, L. Heinrich



# Differentiable analysis

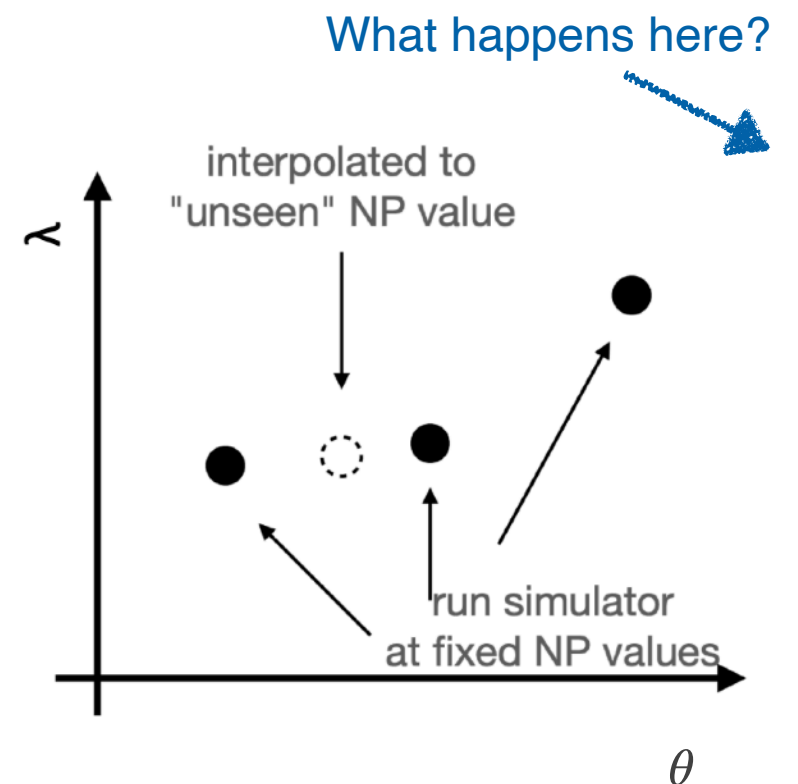
- Rather than  $\theta$  up/down variation, compute value and gradient
  - Auto-diff vs. finite-diff performance
- Higher order derivatives? How analytic are these things?
  - Need second order to get asymmetric (and it probably does not extrapolate well :)
- Lukas has said more

$$\lambda(\theta) = \frac{\sigma}{N} \sum_{x_i \sim P(x)} w(x_i, \theta) 1(x_i \in \text{bin})$$
$$\approx \lambda(\theta_0) + \left. \frac{d\lambda}{d\theta} \right|_{\theta=\theta_0} (\theta - \theta_0) + \dots$$

(reweight, e.g. efficiency) 👍

$$\lambda(\theta) = \frac{\sigma}{N} \sum_{x_i \sim P(x)} 1(x_i + \Delta(x_i, \theta) \in \text{bin})$$

(shift, e.g. energy scale) 🤔

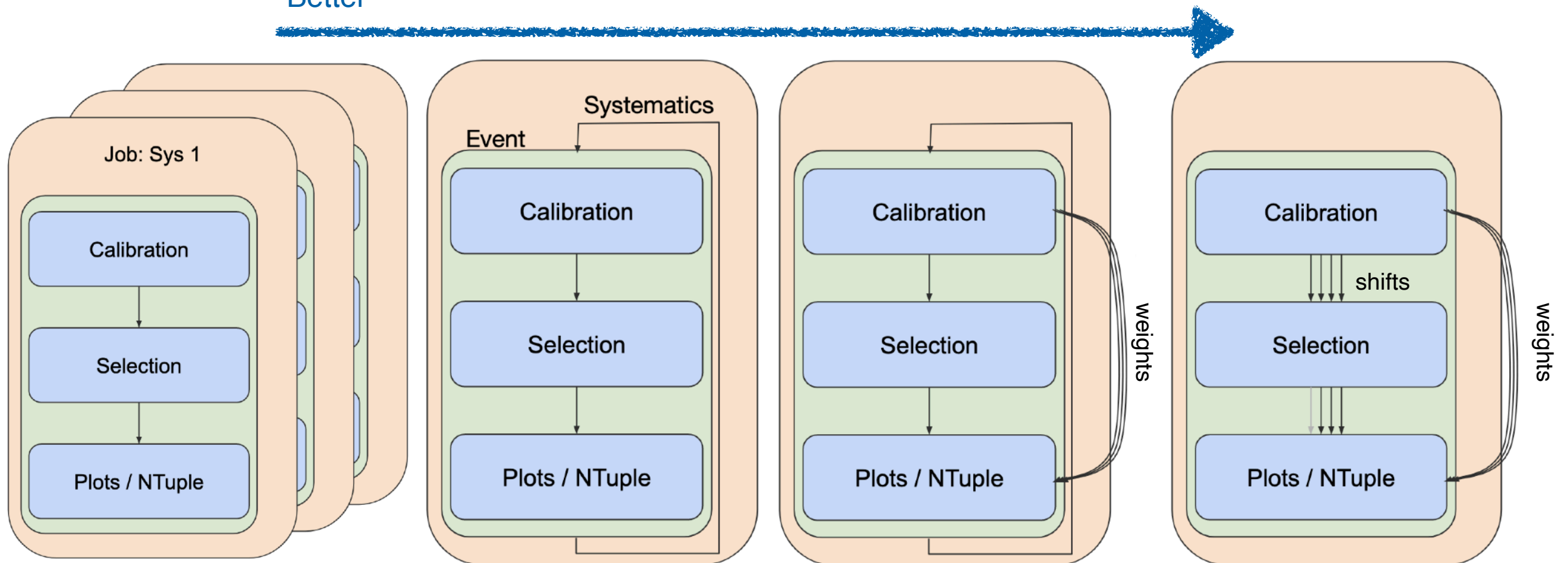




# Analysis task graph

- Simplest solution: re-run everything with alternate  $\theta$
- Better: loop over event while in-memory (likely CPU cache)
  - Why? Because IO is very expensive
- Best: compute all weights, compute shifts only as necessary

Better



S. Hageboeck

# Analysis task graph

- RDF Vary simplifies implementation!
  - Coffea as well once awkward v2 is out (to have full DAG)
    - Current: embed lazy-evaluated systematics in NanoEvents

Each `__getattr__` initiates evaluation

`events.jets.JES_jes.up.pt`

**In ROOT 6.26  
(experimental)**

proceed as usual,  
as if working with  
nominal values only

```
nominal_hx = attach an up/down variation to "pt"  
df.Vary("pt", "RVecD{pt*0.9, pt*1.1}", ["down", "up"])  
df.Filter("pt > k")  
df.Define("x", someFunc, ["pt"])  
df.Histo1D("x")
```

Python

```
hx = ROOT.RDF.VariationsFor(nominal_hx)  
hx["nominal"].Draw() obtain all variations  
hx["pt:down"].Draw("SAME")
```

[E. Guiraud](#)

# Reduced data formats

- Goal: maximize usability, minimize disk space
  - Keep minimal subset of observables  $x$
- Tradeoff with functions  $w(x, \theta)$  and  $\Delta(x, \theta)$ :
  - Large subset of  $x$  needed to evaluate: better to save output for  $\theta_0, \theta_1, \theta_2$
  - Small subset of  $x$  needed to evaluate: better to save those inputs, evaluate “on-the-fly”
  - Overlap with what is needed to identify the bin  $\rightarrow$  more likely on-the-fly
- CMS NanoAOD: calibrated objects, very few systematics
  - Keep only those too difficult to parameterize
    - Unclustered energy  $\Delta$  for MET: per-PF candidate species energy scale uncertainty
  - ATLAS DAOD\_PhysLite: similar goals
- Other considerations
  - CMS MiniAOD: lossy compression of track covariance matrices
  - Common weight trick: store  $1-w$  with reduced-precision mantissa

# On-the-fly evaluation

- Often calibrations and systematics go hand-in-hand
  - Can redefine  $p(y|\theta) = p(y+\Delta(y,\theta_0)|\theta-\theta_0)$  so auxiliary measurement is “spot-on”
- In CMS, corrections+uncertainty have long been parameterized
  - Lately, move towards standardizing to reduce proliferation of (often poorly-designed) serialization formats and (often slow) evaluation frameworks
- Correctionlib
  - A well-structured JSON data format for a wide variety of ad-hoc correction factors encountered in a typical HEP analysis and a companion evaluation tool suitable for use in C++ and python programs.
  - Development started Nov. 2020, all CMS analysis-stage corrections now compatible
  - Presented at PyHEP '22: [youtube](#)

## Python signature

```
def f(*args: str | int | float) -> float:  
    return ...
```

## C++ signature

```
double Correction::evaluate(const std::vector<std::variant<int, double, std::string>>& values) const;
```



# Metadata systems

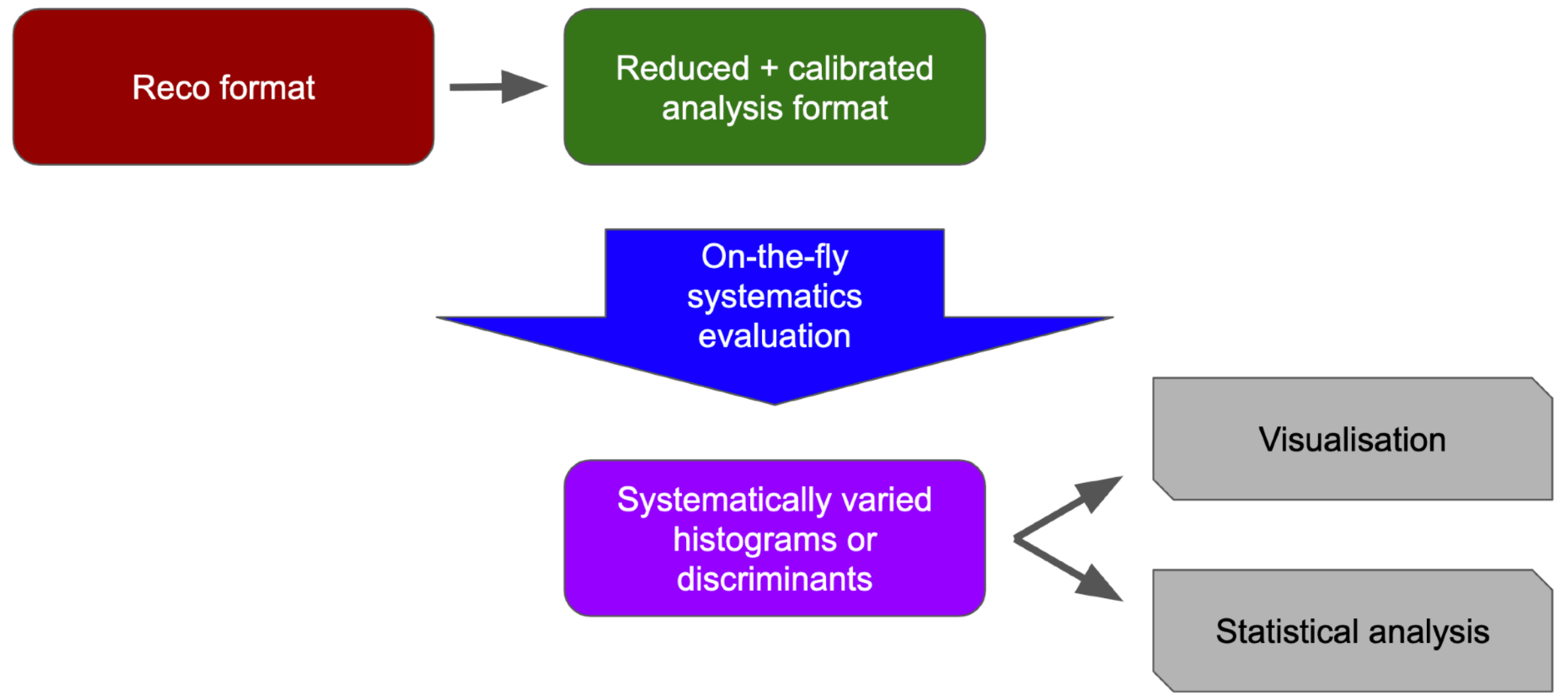
- Correctionlib json in database?
- Book-keeping alternative samples
  - At least in CMS, no automated access to generation config at analysis stage
  - Most book-keeping by hand: key on dataset name
- Paul has said more

$$\int_{\text{bin}} P(x|\theta = \theta_1) dx \approx \frac{\sigma}{N} \sum_{x_i \sim P(x|\theta=\theta_1)}^N 1(x_i \in \text{bin})$$

(alternative sample, e.g. 2-point)

# A flowchart

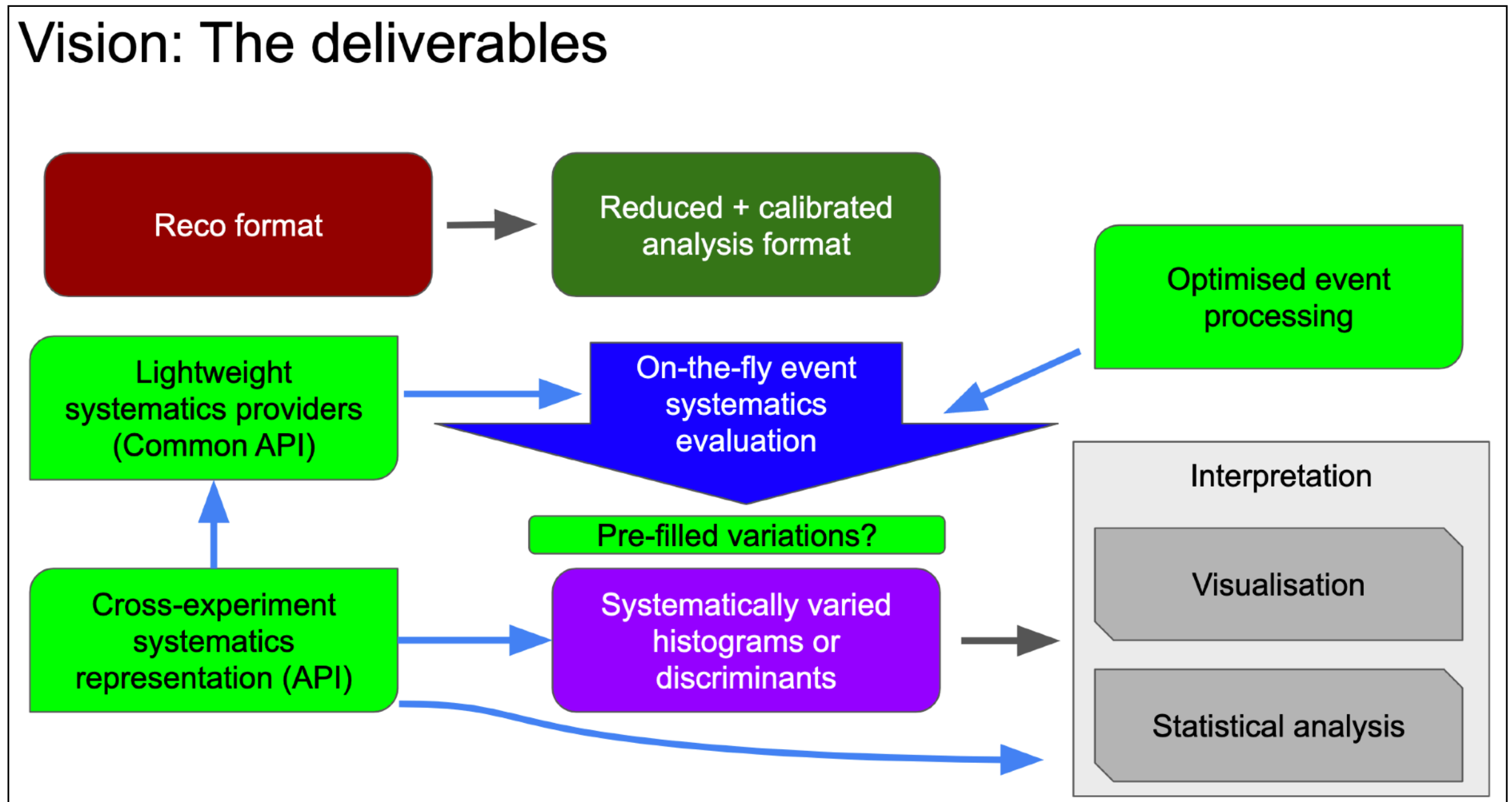
## Systematics: The Vision



P. Laycock, T.J. Khoo

# A flowchart

## Vision: The deliverables



P. Laycock, T.J. Khoo

# A challenge

## The “ttbar” systematics challenge

- Proposal for a DAWG follow-up
- Can ATLAS/CMS/LHCb do a joint (OpenMC) ttbar analysis?
  - Consistent MC events through collaboration reco
  - Prototype a common systematics representation
  - Propagate to (reasonable subset of) collaboration-specific systematics
  - Statistical combination of unfolded measurements
- Stretch goal: common cross-sections extracted from shared metadata API
- Show this off for AE3!?

[P. Laycock, T.J. Khoo](#)



# Experimental prescriptions

- Non-trivial to agree on parameterization, but crucial for combinations
- Correlate (i.e. use same subset of  $\theta$  for) common effects
  - Experimental effects (simplest: luminosity unc.)
  - Theory uncertainties for common processes
  - Etc.
  - Profit from increased sensitivity!
- CMS Higgs group: “datacards” (likelihood serialization format) are reviewed
  - Standard nuisances, naming conventions, sign, etc.
  - Simplifies combination later
- Is it worth establishing cross-experiment parameterization/nomenclature?
  - Test case:  $t\bar{t}$  challenge?

# Template storage

- Multi-dimensional histograms: axis for systematic variation
- Filling histograms with weights vector
  - Save repeated bin lookup for same observables
  - Planned feature for boost::histogram [boostorg/histogram#211](https://boostorg.org/docs/libs/histogram/)
- Better to have serializable object tailored to our use case
  - RDF has part of the answer:

```
hx = ROOT.RDF.VariationsFor(nominal_hx)
hx["nominal"].Draw()
hx["pt:down"].Draw("SAME")
```

obtain all variations

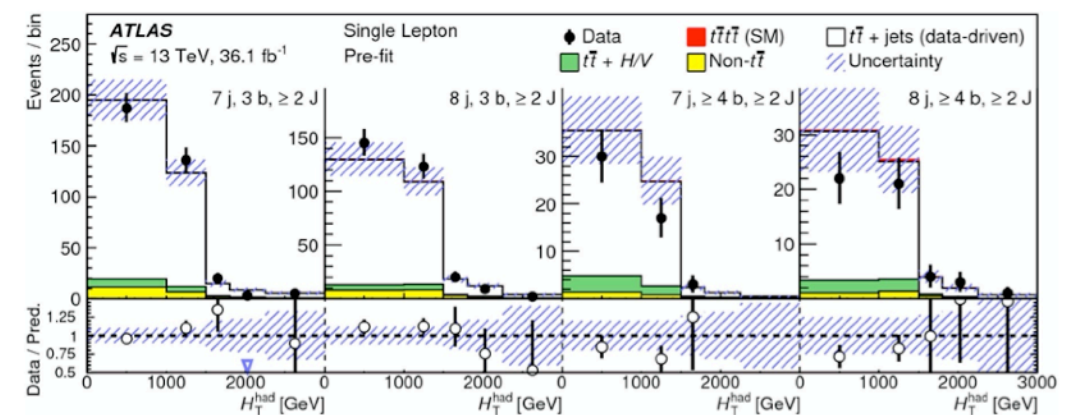
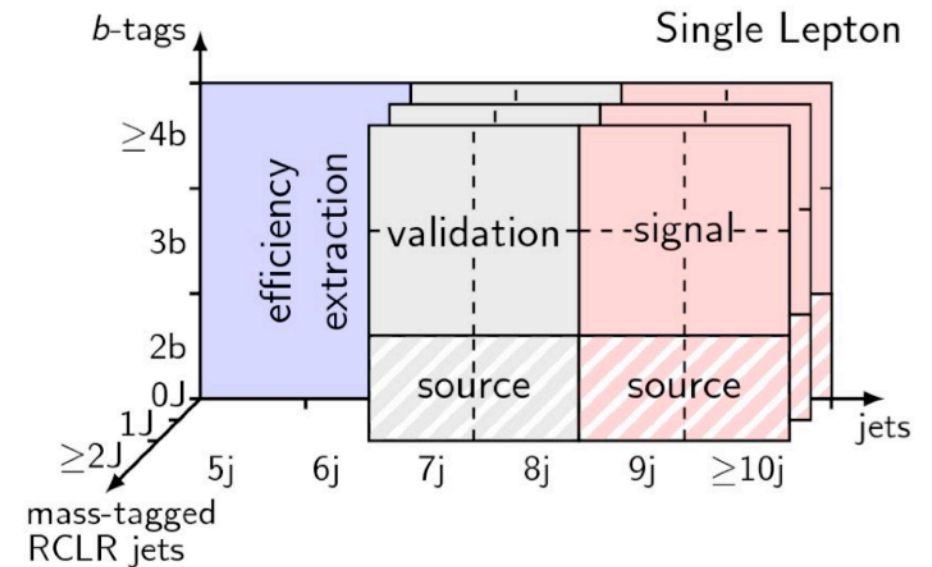
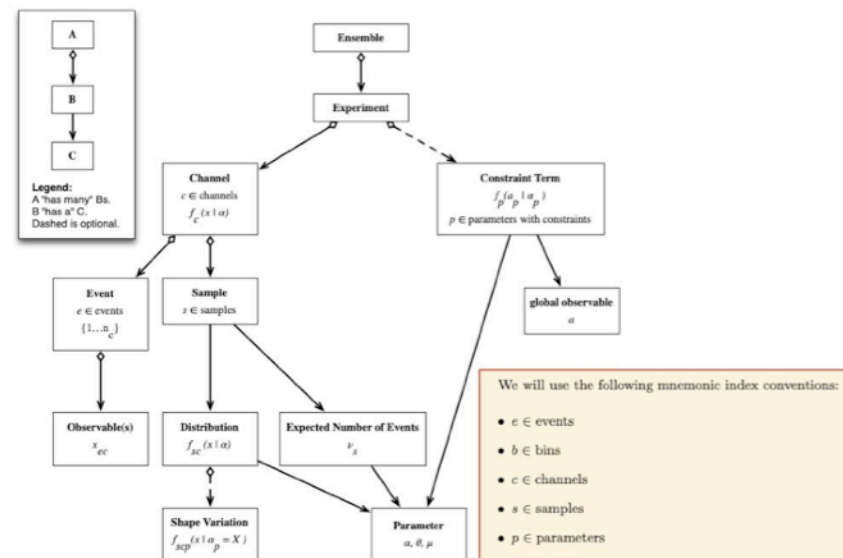
# Template storage

- [cabinetry](#) is a Python package to build and steer template fits

## A point of convergence

Several aspects of Analysis Systems converge in a typical physics plot:

- Specification of signal / validation / control regions
- Specification of variables to be used for stat analysis
- Reduction to that format running on data and MC
- Management of MC samples, data driven backgrounds, etc.
- Management of systematic variations
- Feed reduced data (eg. histograms) into specification for statistical model / likelihood function
- Fitting & statistical tools
- Publishing results & derived data products
- Analysis preservation & gateways targeting reinterpretation



A. Held

# Publishing statistical models

- More specifically the model  $p(x, y|\mu, \theta)$
- Why?
  - “The statistical models used to derive the results of experimental analyses are of incredible scientific value and are essential information for analysis preservation and reuse ... [and] can enhance the short- and long-term impact of experimental results.” ([arxiv:2109.04981](https://arxiv.org/abs/2109.04981))
- A goal now 22 years old ([K. Cranmer](#))
- Need a good data format, contenders:
  - pyhf JSON (HistFactory XML)
  - CMS combine datacard
  - [RooWorkspace json: HS3](#)
    - “A round-trip-capable, human-readable declarative format for statistical models was missing”



# HS3 specification

- Importers, exporters defined for many RooFit classes
  - User extendable
- Allows:
  - Easier conversion to other tools' formats
  - Faster/easier navigation in python code
  - Hand-editing
  - Simpler publication
- Many thanks to Carsten for starting this!

Transcript of a [RooArgusBG example](#), arbitrarily selected analytical example

```
"pdfs": {
  "background": {
    "mass": "mes",
    "power": "0.5",
    "resonance": "5.291",
    "slope": "argpar",
    "type": "ARGUS"
  },
  "model": {
    "coefficients": [
      "nsig",
      "nbkg"
    ],
    "dict": {
      "ModelConfig": "ModelConfig"
    },
    "summands": [
      "signal",
      "background"
    ],
    "tags": [
      "toplevel"
    ],
    "type": "pdfsum"
  },
  "signal": {
    "mean": "sigmean",
    "sigma": "sigwidth",
    "type": "Gaussian",
    "x": "mes"
  }
},
```

```
"variables": {
  "argpar": {
    "max": -1.0,
    "min": -100.0,
    "value": -20.0
  },
  "mes": {
    "max": 5.3,
    "min": 5.2,
    "value": 5.25
  },
  "nbkg": {
    "max": 10000.0,
    "min": 0.0,
    "value": 800.0
  },
  "nsig": {
    "max": 10000.0,
    "min": 0.0,
    "value": 200.0
  },
  "sigmean": {
    "max": 5.3,
    "min": 5.2,
    "value": 5.28
  },
  "sigwidth": {
    "max": 1.0,
    "min": 0.001,
    "value": 0.0027
  }
}
```

[C. Burgard](#)

# Combine $\leftrightarrow$ pyhf

- Meanwhile, active work to compare CMS combine and pyhf tools
  - In the end, differences in terminology and matters of taste

## Combine vs Pyhf: Validation

- ◆ Created a framework to perform the **combine $\rightarrow$ pyhf translation** with the **validation of results**
- ◆ Includes: maximum-likelihood fits, impacts of uncertainties, likelihood scans, etc.
- ◆ Using a generic toy example with **normalization, shape, and rate-parameter** nuisances to perform fast validation
- ◆ Achieved a **consistent implementation** using the two frameworks

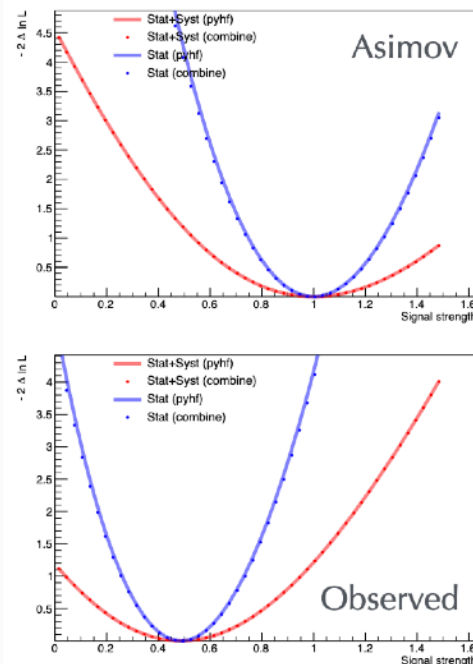
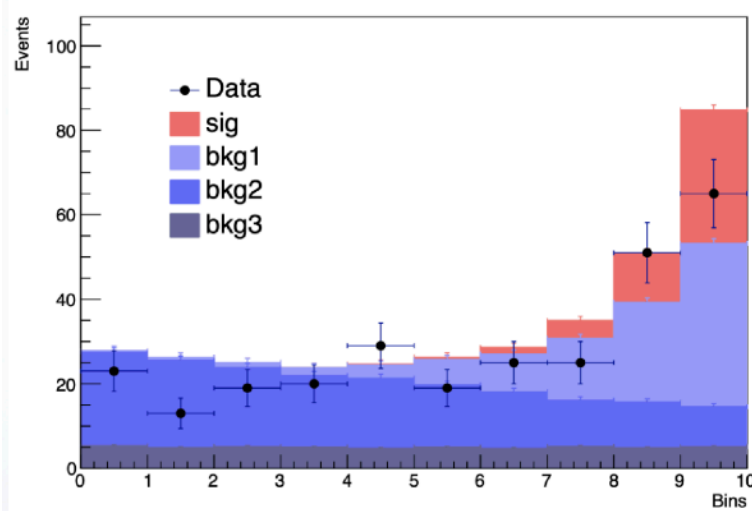
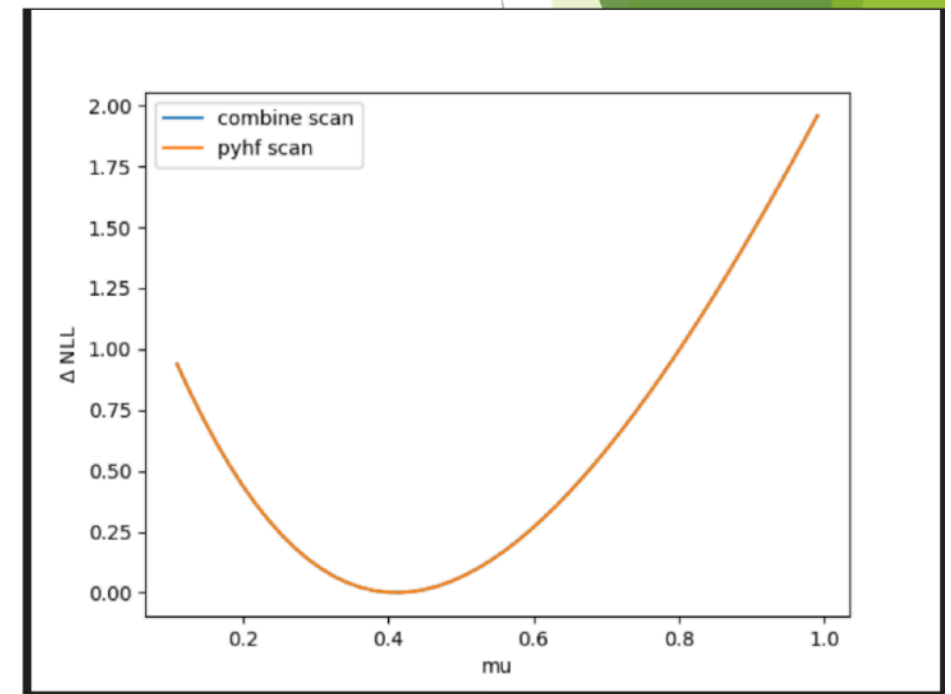


Figure 3: NLL scan for pyhf and Combine with “profiled” parameters (refits at each point)



[K. Skopven](#)

[P. Ridolfi](#)

# Summary

- Systematics influence **many** technical decision in analysis software
- Modern approaches to systematics handling are simplifying our life
  - More diversity in analyses will help us design better abstractions
- There is an active and enthusiastic community
- Further references:
  - [Systematics for data analysis](#)
  - [AE2 workshop: systematics, summary](#)
  - [PHYSTAT-Systematics 2021](#)
  - [Publication of statistical models: hands-on workshop](#)
  - [Coffea+RDF discussion on systematics](#)