[Rinaldi et al., arxiv:2108.02942]

Benchmarking quantum algorithms for quantum field theories

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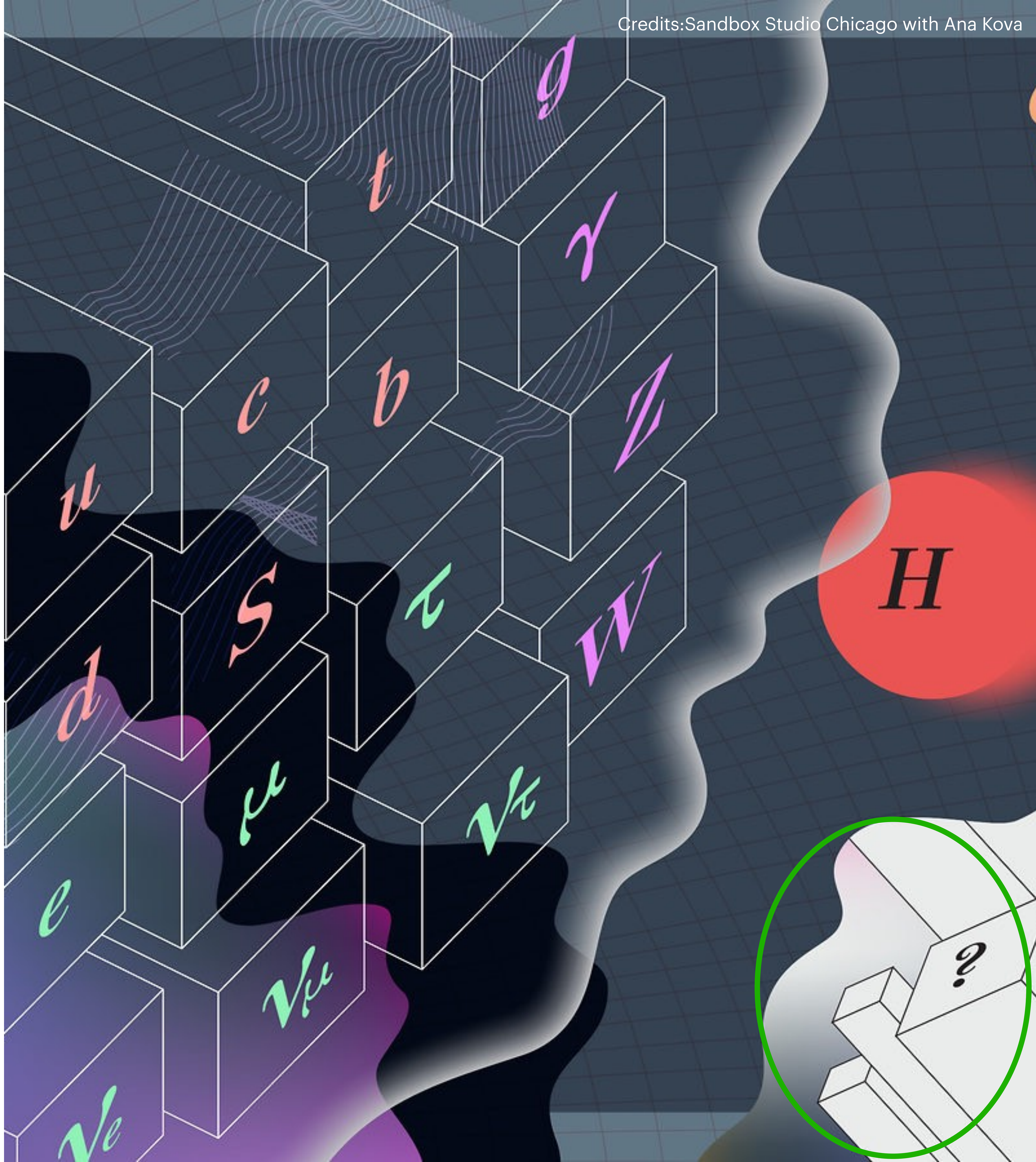
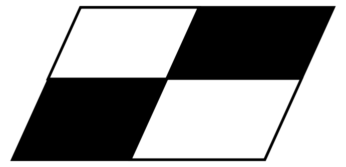
Quantum Field Theory

- The **Standard Model of particle physics** is our most precise description of the subatomic world
- It is a Quantum Field Theory, a very complicated many-body quantum system obeying the rules of quantum mechanics
- Some pieces of this description of the world are still missing:
  - What is the quantum theory for gravity?

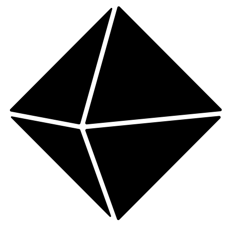


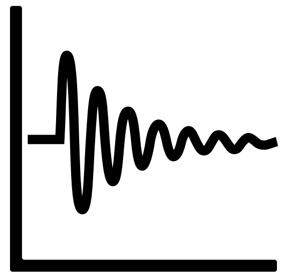
Outline

- Introduction to Matrix Models



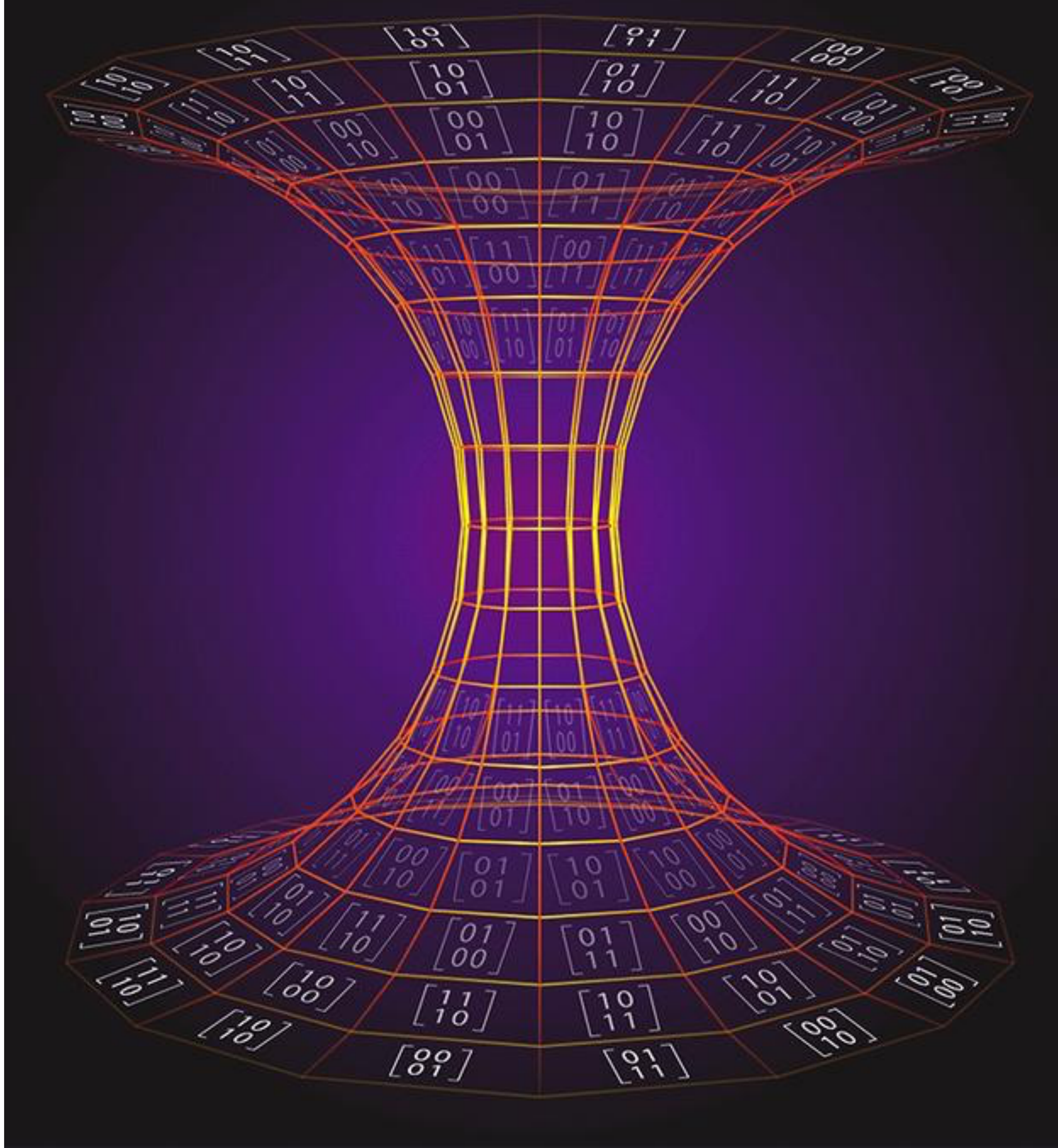
- **Numerical techniques for QC**:
  - Regularization: Truncated Hamiltonian
  - Quantum Computing Benchmarks: VQE





- Challenges for the future

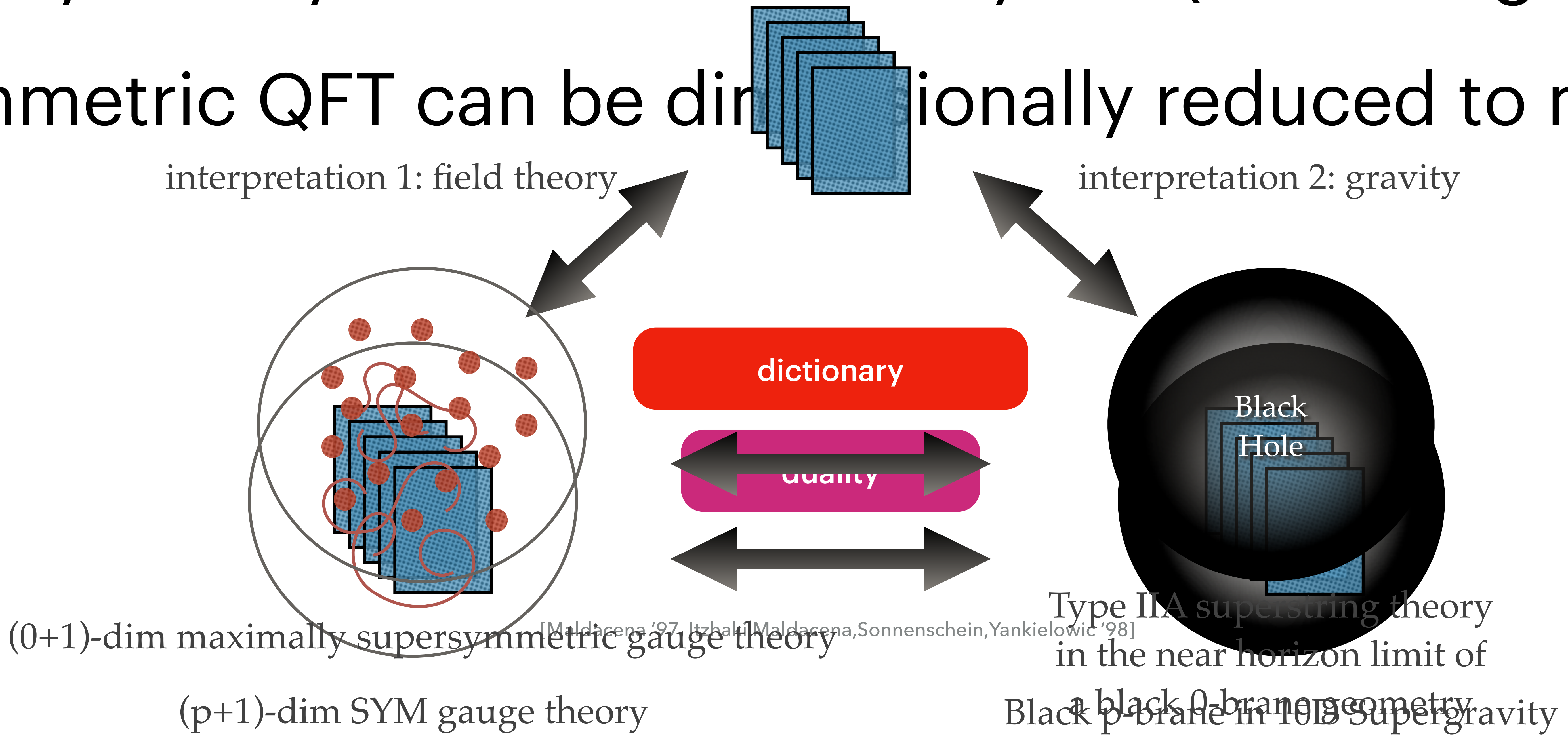
Credits:APS/Alan Stonebraker. Physics Magazine



Matrix Quantum Mechanics

Motivations

- Holographic duality → a quantum field theory **"is"** a gravitational theory
  - D0-branes and open strings ⇔ Black hole in Type IIA superstring
- Gauge/gravity duality → use QFT to study QG (*i.e.* emergent geometry)
  - Supersymmetric QFT can be dimensionally reduced to matrix QM



[Maldacena '97, Itzhaki,Maldacena,Sonnenschein,Yankielowicz '98]

Numerical Methods

- HPC simulations using Path Integral-based methods; Monte Carlo sampling of quantum mechanical paths.
- **Challenges**:
  - Sign problem → supersymmetric theories are hard
  - Wave function → physics applications require knowledge of entanglement (*i.e.* information paradox)

[Rinaldi et al., arxiv:2108.02942]

Numerical Methods for MQM

Prototypes

Bosonic Model

SU(N) Gauge Symmetry

Supersymmetric Model

FREE

BOS. INTERACTION

FERM.-BOS. INTERACTION

ZERO EN.

$$\hat{H}_B = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 + \frac{m^2}{2}\hat{X}_I^2 - \frac{g^2}{\ldots}\left[\ldots\right.\right.$$

$$\ldots \hat{X}_2, \hat{\xi}\right] + \frac{g}{2}\hat{\xi}^\dagger\left[-\hat{X}_1 + i\hat{X}_2, \hat{\xi}^\dagger\right] + \frac{3\mu}{2}\hat{\xi}^\dagger\hat{\xi}\right)$$

Challenge: numerical methods on quantum computers have a limited number of qubits!

$$\hat{X}_I = \sum_{\alpha=1}^{N^2-1} \hat{X}_I^\alpha \tau_\alpha \qquad I = 1,2$$

$$\hat{\xi} = \sum_{\alpha=1}^{N^2-1} \hat{\xi}^\alpha \tau_\alpha \qquad \hat{X}_I = \sum_{\alpha=1}^{N^2-1} \hat{X}_I^\alpha \tau_\alpha \qquad I = 1,2$$

$\hat{X}_I^\alpha$ → bosonic degrees of freedom

τ_α → generators of SU(N) group

$\hat{\xi}^\alpha$ → fermionic degrees of freedom

[Gharibyan et al., arxiv:2011.06573]

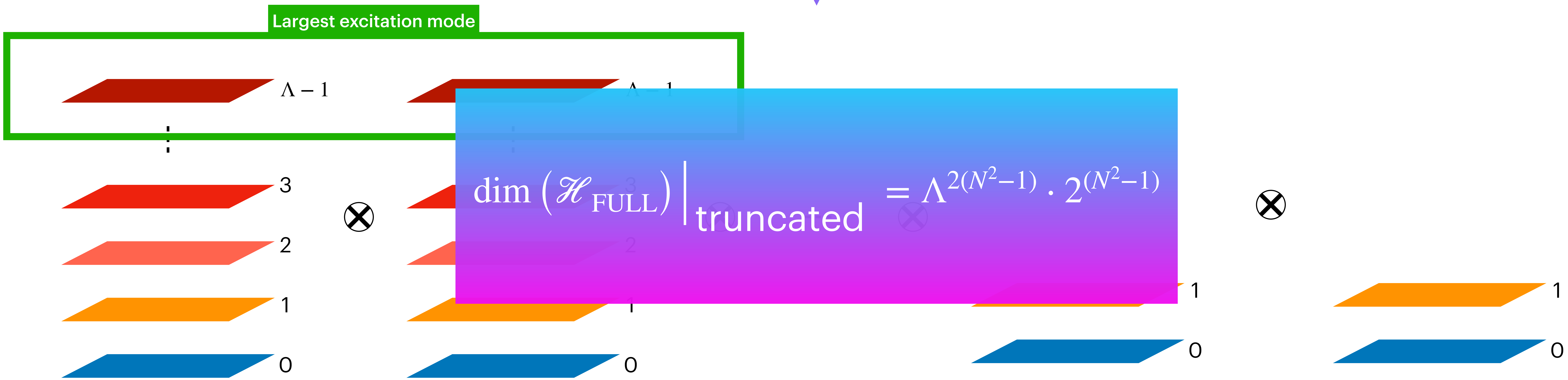
Hilbert space regularization

Truncation

$$\mathscr{H}_{\mathrm{FULL}} = \mathscr{H}_{\mathrm{BOS}} \otimes \mathscr{H}_{\mathrm{FER}}$$

Tensor product of each boson space

Tensor product of each fermion space



$$\dim\left(\mathscr{H}_{\mathrm{FULL}}\right)\Big|_{\text{truncated}} = \Lambda^{2(N^2-1)} \cdot 2^{(N^2-1)}$$

[Rinaldi et al., arxiv:2108.02942]

Truncated Hamiltonian

Challenge: physical results are at $\Lambda = \infty$!

- Write the **Hamiltonian** in the truncated Hilbert spac[e]
  - e.g. Bosonic 2MQM for SU(2) → Λ cutoff level → $\Lambda^6 \times \Lambda^6$ matrix (sparse)

$$\hat{H} = \sum_{\alpha,I}\left(\frac{1}{2}\hat{P}^2_{I\alpha} + \frac{m^2}{2}\hat{X}^2_{I\alpha}\right) + \frac{g^2}{4}\sum_{\gamma,I,J}\left(\sum_{\alpha,\beta} f_{\alpha\beta\gamma}\hat{X}^{\alpha}_{I}\hat{X}^{\beta}_{J}\right)^2 \qquad I = 1,2 \qquad \alpha = 1,2,3$$

SYMMETRIES

bosonic "sites"

1 2 3 $\quad I = 1$ $\quad g^2 = 0$

4 5 6 $\quad I = 2$ $\quad g^2 > 0$

$$|\mathrm{VAC}\rangle = \left(\otimes_{I,\alpha} |0\rangle_{I\alpha}\right)$$

[Rinaldi et al., arxiv:2108.02942]

Results

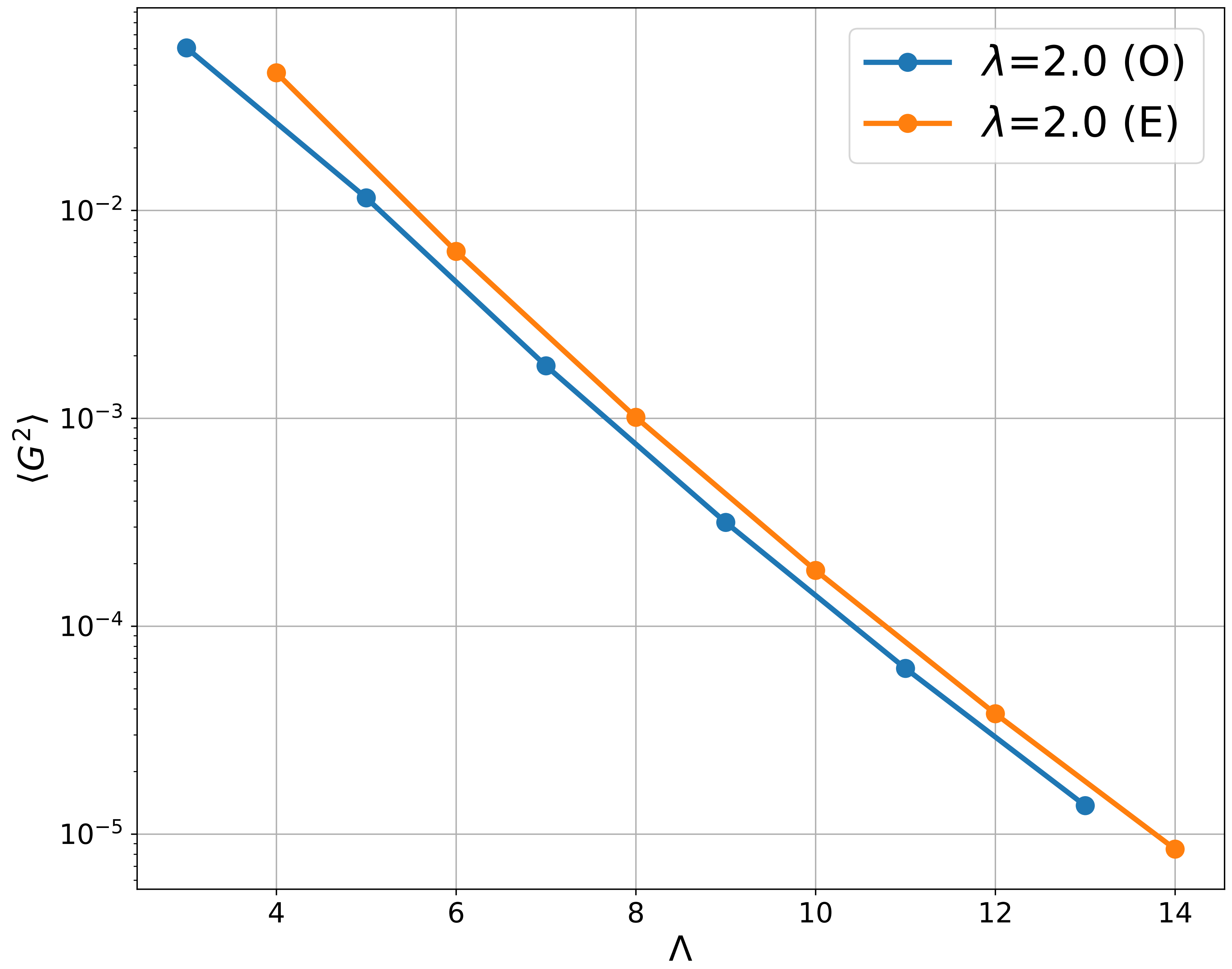
SU(2) Bosonic 2MQM

- **Benchmark**: compute the lowest states via exact diagonalization
- Study the convergence to $\Lambda \to \infty$
- Study the effects of different couplings
- Study the gauge-singlet constraint

$$\hat{G}_\alpha = i \sum_{\beta,\gamma,I} f_{\alpha\beta\gamma} \hat{a}^\dagger_{I\beta} \hat{a}_{I\gamma} \longrightarrow \hat{G}_\alpha \left(\otimes_{I,\beta} |0\rangle_{I\beta} \right) = 0$$

$$E_0 = \langle E_0 | \hat{H} | E_0 \rangle$$

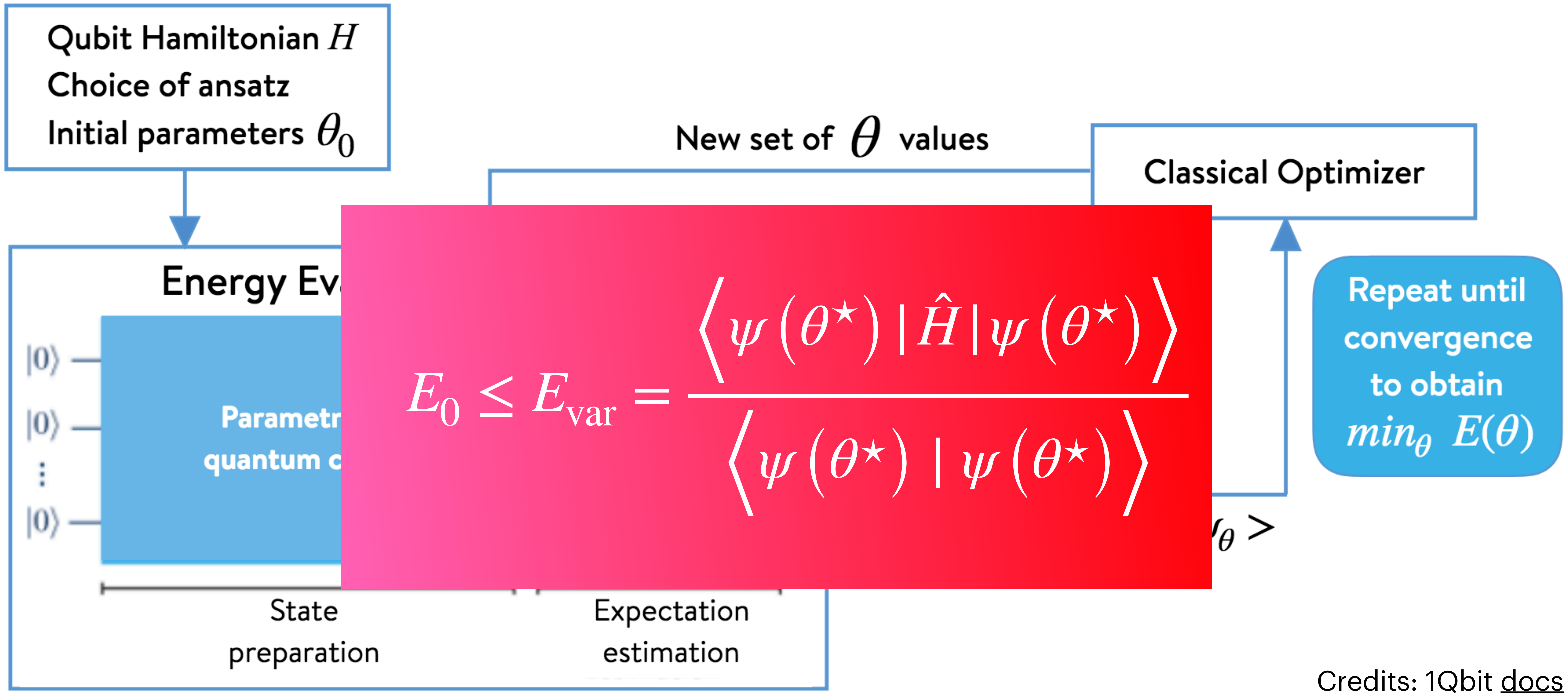
$$\sum_\alpha \langle E_0 | \hat{G}^2_\alpha | E_0 \rangle$$



[Peruzzo et al., arxiv:1304.3061]

Quantum Computing

Variational Quantum Eigensolver - VQE



$$E_0 \leq E_{\mathrm{var}} = \frac{\left\langle \psi(\theta^\star) \,|\, \hat{H} \,|\, \psi(\theta^\star) \right\rangle}{\left\langle \psi(\theta^\star) \,|\, \psi(\theta^\star) \right\rangle}$$

Credits: 1Qbit docs

PQC → Variational Ansatz for |Φ〉 → Evaluation of cost function → E(θ) → Optimize parameters → θ*

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

Qubitization of MQM

SU(2) Bosonic 2MQM

Truncation Level

Λ = 2	Each boson is 1 qubit	$\log_2 \Lambda^6$ = 6 qubits
Λ = 4	Each boson is 2 qubits	$\log_2 \Lambda^6$ = 12 qubits
Λ = 8	Each b…on … qubits	$\log_2 \Lambda^6$ = 18 qubits

limited resources

Annihilation operator for site "i"

$$\hat{a}_i = \hat{I}_1 \otimes \ldots \otimes \hat{I}_{i-1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \hat{I}_{i+1} \otimes \ldots \otimes \hat{I}_6$$

$$\hat{a}_i = \hat{I}_1 \otimes \ldots \otimes \hat{I}_{i-1} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes \hat{I}_{i+1} \otimes \ldots \otimes \hat{I}_6$$

$$\bullet \bullet \bullet$$

Build matrix Hamiltonian which gets mapped to qubits

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

SU(2) Bosonic 2MQM

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits → Choose Variational Ansatz → Choose Quantum Simulator → Choose Classical Optimizer

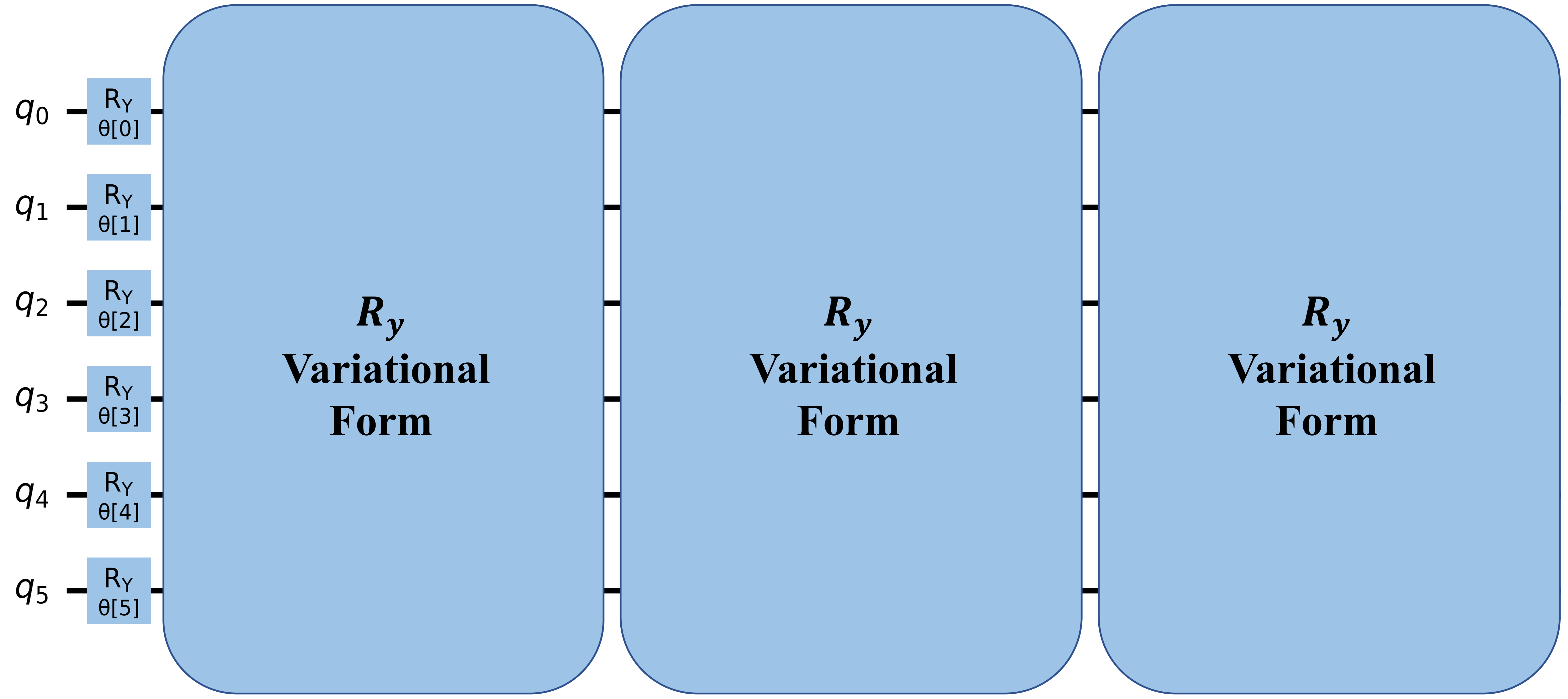
PQC → Variational Ansatz for $|\Phi\rangle$

depth = 3

parameters = 24

Evaluation of cost function → $E(\theta)$

Optimize parameters → θ^*



- Statevector simulator

- Least SQuares Programming optimizer (SLSQP)
- Constrained Optimization By Linear Approximation optimizer (COBYLA)
- Limited-memory BFGS Bound optimizer (L-BFGS-B)
- Nelder-Mead

Run each optimizer with a max. number of iterations

Run each multiple instances of PQC from different initial points

Qiskit

Results

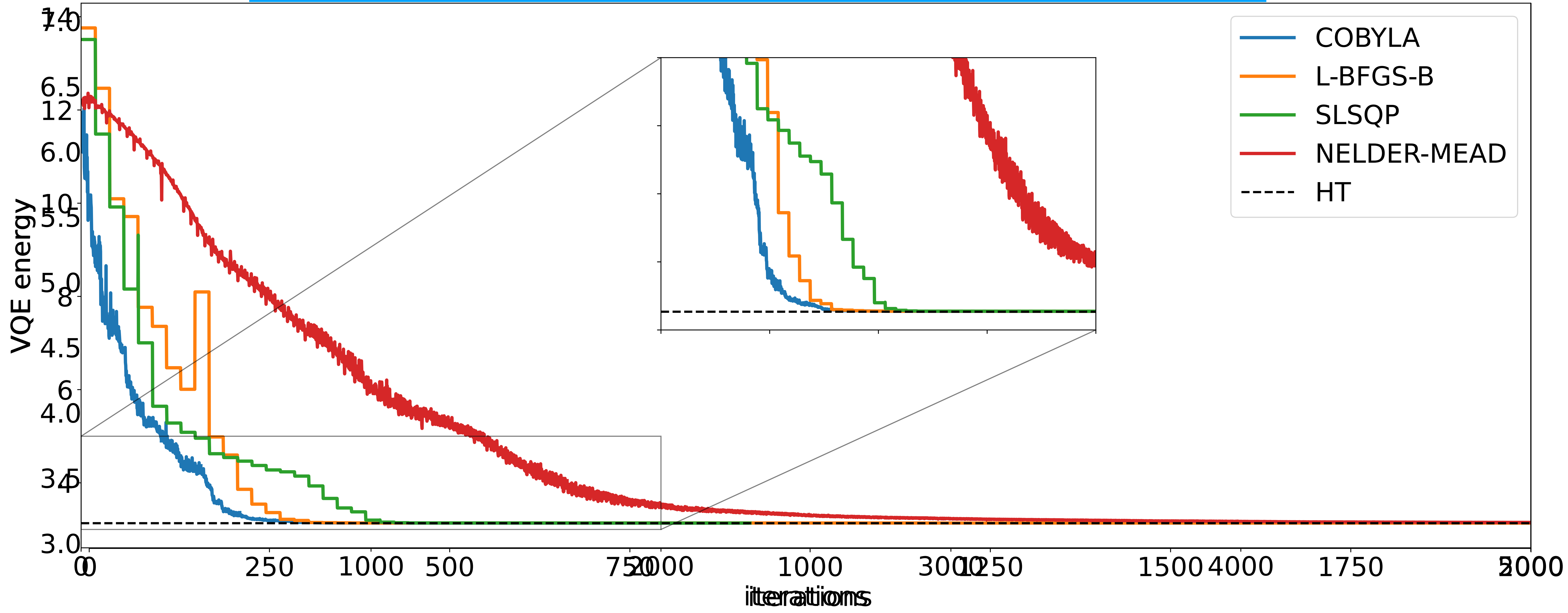
SU(2) Bosonic 2MQM

$\Lambda = 4$

$\log_2 \Lambda^6 = 12$ qubits

Optimizer	Var. form: R_y				Var. form: $R_y R_z$			
	Min.	Max.	Mean	Std.	Min.	Max.	Mean	Std.
COBYLA	**3.137059**	4.769101	3.251414	0.347646	3.137237	4.782013	3.378628	0.472015
L-BFGS-B	**3.137059**	5.769553	3.283462	0.434162	**3.137050**	4.286367	3.243110	0.307549
SLSQP	3.137060	5.769554	3.327706	0.471957	3.137059	4.232419	3.236925	0.290855
NELDER-MEAD	3.137471	5.713976	3.492673	0.478810	3.273614	6.443055	4.428032	0.758732

PQC with y rotation gates: depth = 3 → 24 parameters | Best out of 100 runs



Best VQE (100 runs): $E_0 = 3.137$

Exact Diagonalization: $E_0 = 3.13406$

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

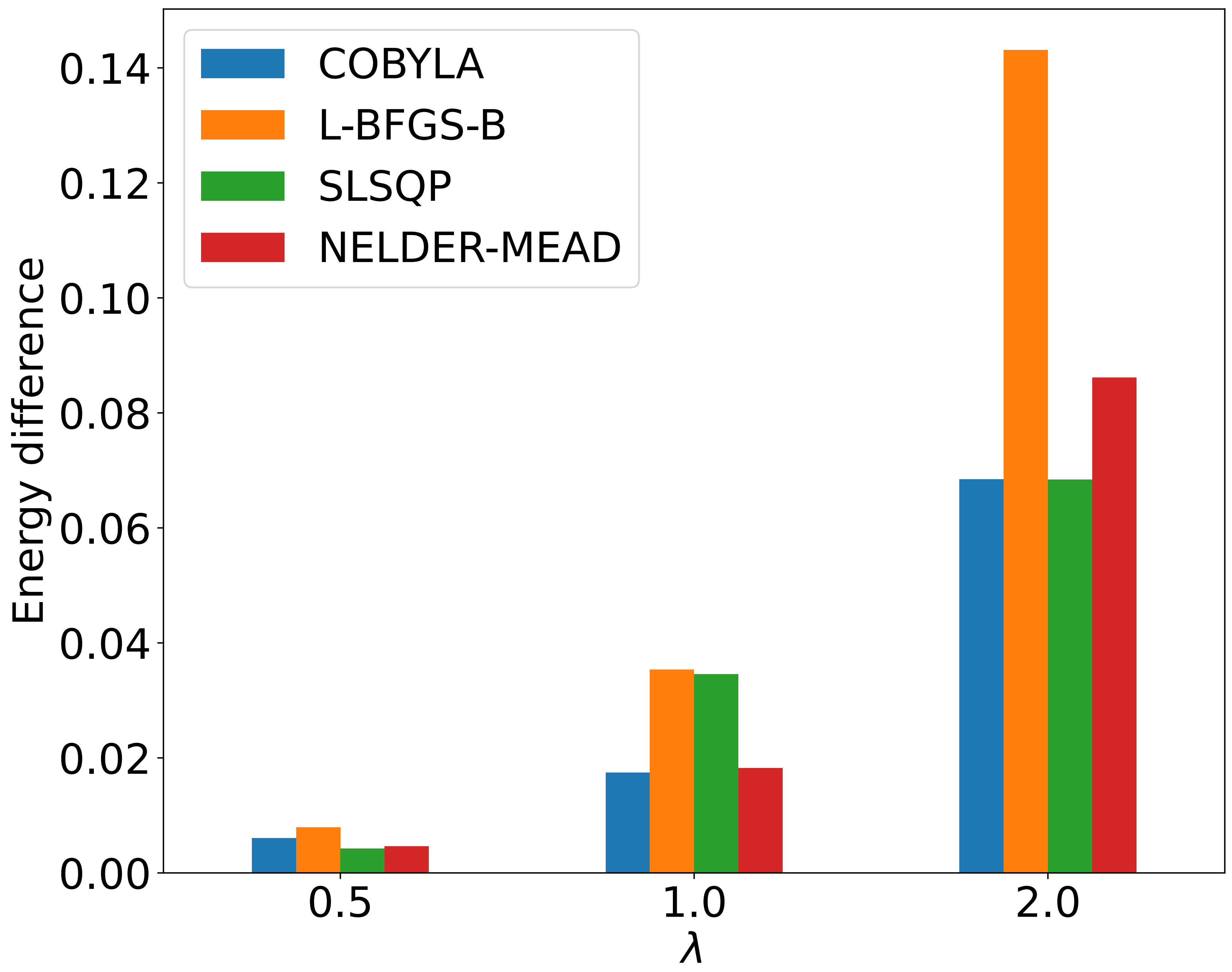
Qiskit

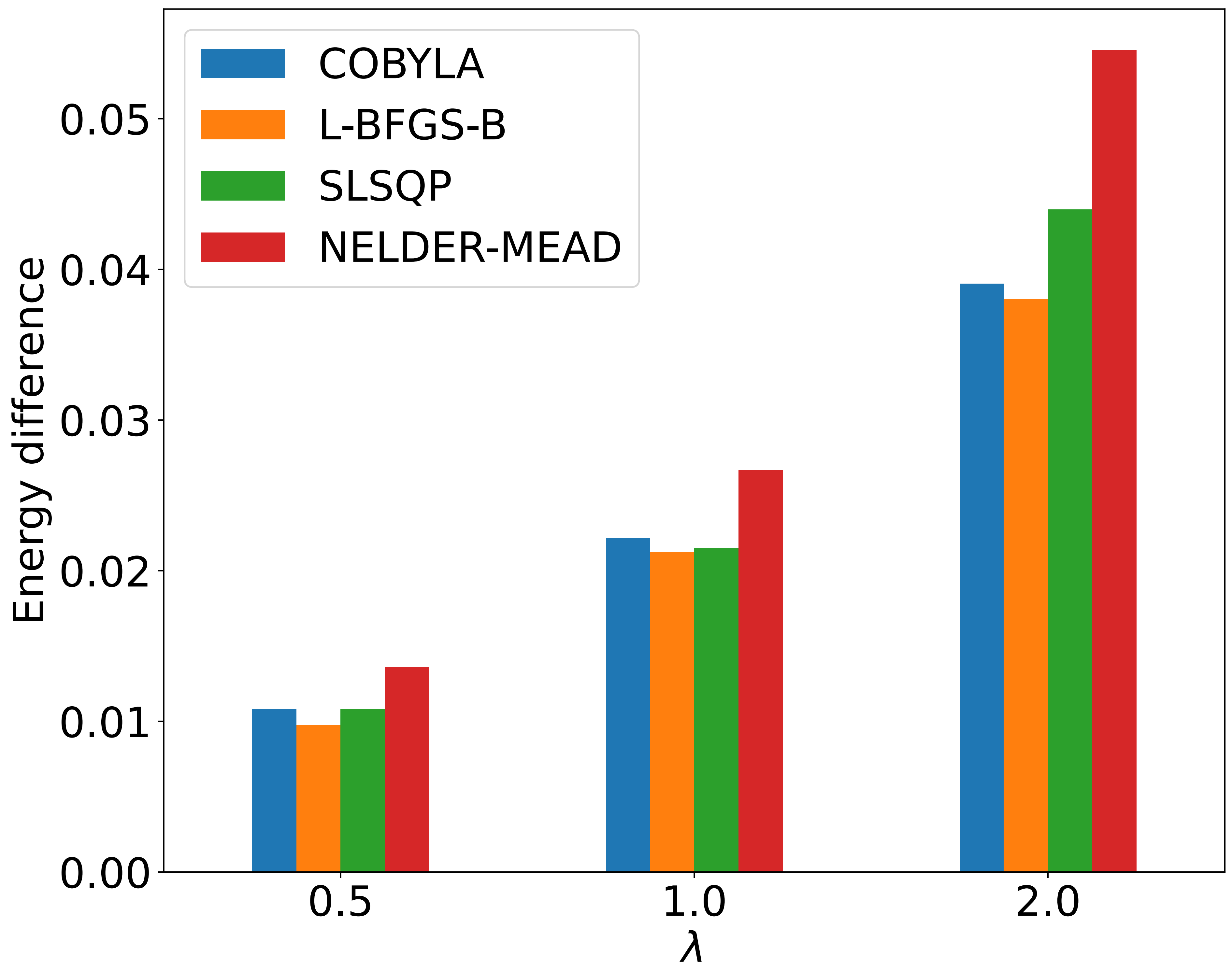
Results

SU(2) Bosonic 2MQM at large coupling

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits

$\Lambda = 4$ | $\log_2 \Lambda^6 = 12$ qubits





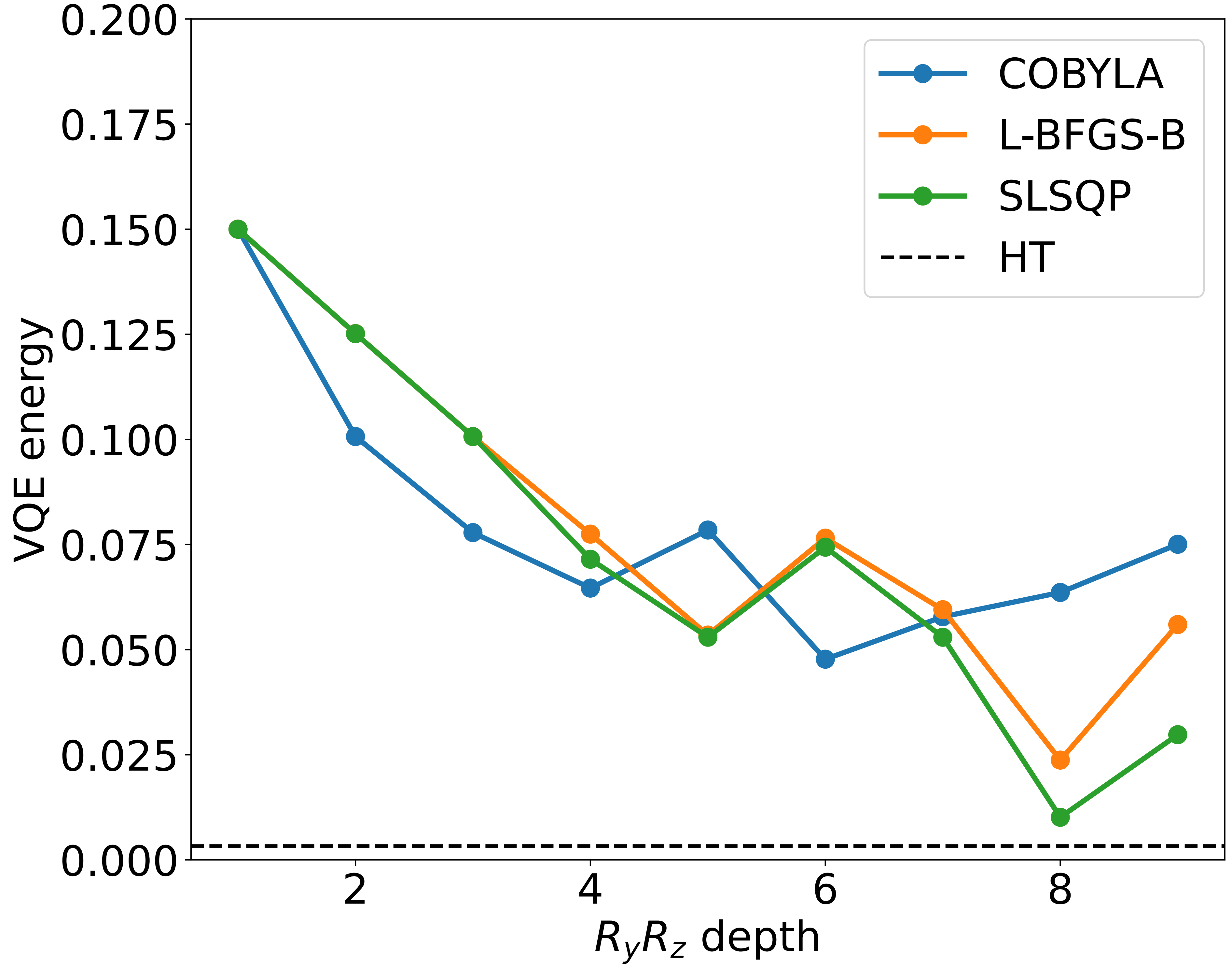
PQC with y rotation gates: depth = 3 → 24 parameters | Best out of 100 runs

Qiskit

Results

SU(2) Supersymmetric 2MQM at large coupling

$\Lambda = 2$ $\log_2 \Lambda^6 = 9$ qubits



	depth = 5				depth = 9	
λ	COBYLA	L-BFGS-B	SLSQP	NELDER-MEAD	Best	HT (exact)
0.5	0.088492	0.139702	0.134517	0.406003	0.02744	0.01690
1.0	0.135800	0.219268	0.308781	0.752459	0.07900	0.04829
2.0	0.387977	0.622704	0.522396	1.271939	0.17688	0.08385

Conclusions
and roadmap

- ✓ Quantum simulations can be used for addressing **QFT** questions
- ✓ Matrix models have favorable scaling of truncation errors with the number of qubits!
- ✦ Hybrid quantum-classical algorithms can be used on current quantum hardware
- ➡ Finding efficient parametrized quantum circuits for supersymmetric matrix models is very important
- ➡ Using machine learning or tensor network approximations to simplify quantum simulations could be crucial with current resources
- ➡ Error-mitigation will be important on real quantum hardware

Matrix-Model Simulations Using Quantum Computing, Deep Learning, and Lattice Monte Carlo

Enrico Rinaldi, Xizhi Han, Mohammad Hassan, Yuan Feng, Franco Nori, Michael McGuigan, and Masanori Hanada
PRX Quantum **3**, 010324