Equilibration of quark-gluon plasma in heavy-ion collisions

Nuclear Physics Seminar

Brookhaven National Laboratory (Online)

April 26, 2022

Xiaojian Du | Bielefeld University

UNIVERSITÄT BIELEFELD

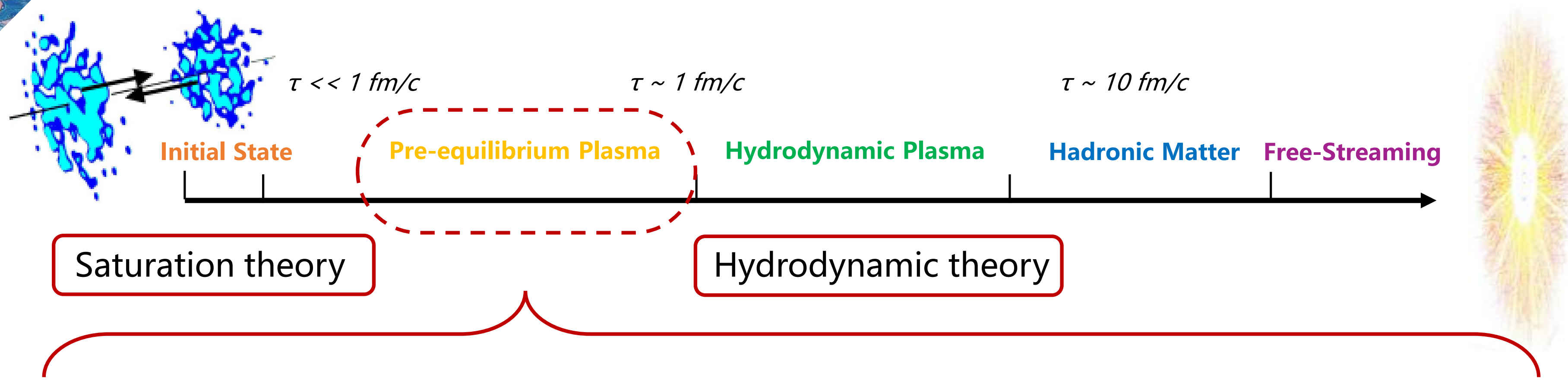
CRC–TR 211
Strong-interaction matter under extreme conditions

Outline

- **Introduction**
  - The motivation and the theoretical tool
- **Equilibration of QCD plasmas**
  - The turbulent nature of quark-gluon plasma (QGP)
- **Hydrodynamization of QCD plasmas**
  - Hydrodynamization and attractor of quark-gluon plasma
- **Early stage of heavy-ion collisions**
  - Application of attractor & phenomenology in heavy-ion collisions (HICs)
- **Space-time fluctuation**
  - Towards a complete picture of pre-equilibrium stage in HICs
- **Conclusions**

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011
[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, arXiv:2112.13876
[3] Others, in progress...

Pre-equilibrium stage in HICs



QGP equilibration in HICs

- Connects initial condition to equilibrium states
  - Off-thermal initial states into near-thermal hydrodynamic states (kinetics)
  - Saturated gluon fields into quark-gluon plasma (chemistry)

Kinetic Theory description of QGP equilibration

- Mechanism to thermalize states (kinetic equilibration)
- Include both gluon + quark degrees of freedom (chemical equilibration)

QCD Effective Kinetic Theory

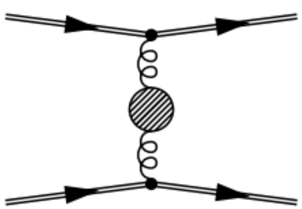
First principle QCD Effective Kinetic Theory (EKT)

$$\left(\frac{\partial}{\partial\tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

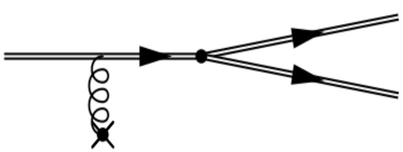
Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

Solving a set of coupled Boltzmann equations

- LO 2↔2 elastic scatterings & 1↔2 inelastic scatterings



2↔2: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation



1↔2: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation

- Gluon + all light quarks/antiquarks (finite net-baryon density) $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

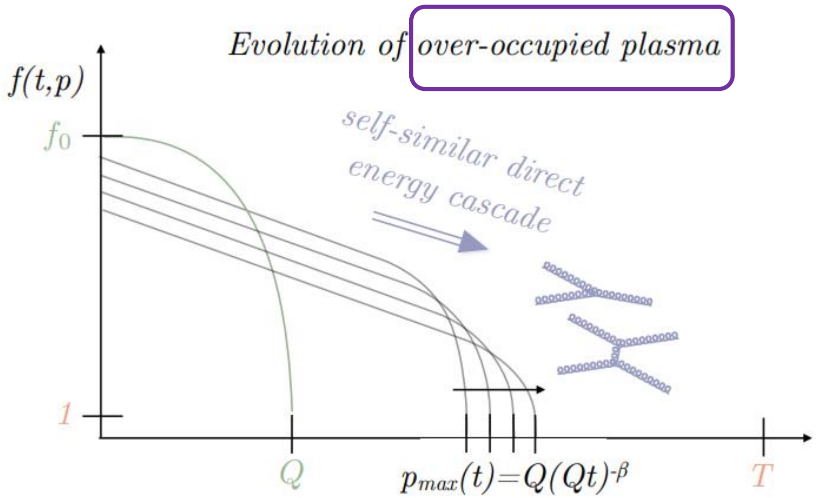
XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

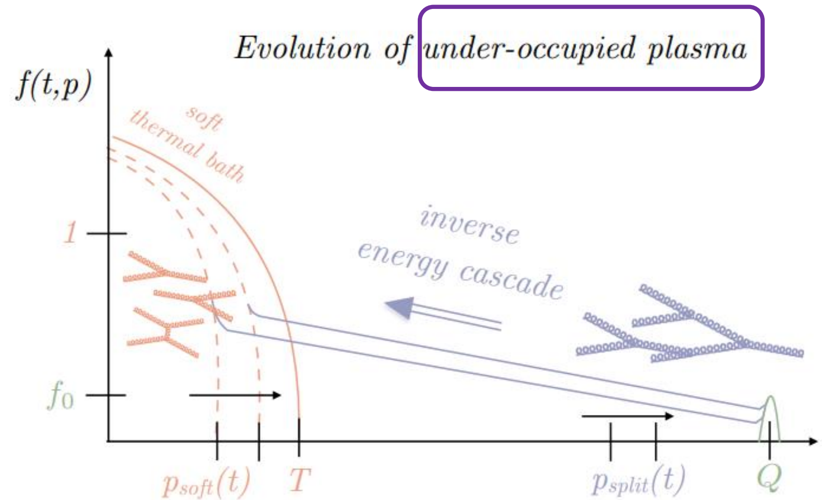
Equilibration of QCD plasmas

The turbulent nature of quark-gluon plasma

Equilibration of QCD plasma

Two typical far-from-equilibrium systems





Over-occupied plasma

- Separation of scale

  $\langle p \rangle_0 \ll T$

- Direct energy cascade

  low → high momentum

- **Initial state in HICs**

Under-occupied plasma

- Separation of scale

  $\langle p \rangle_0 \gg T$

- Inverse energy cascade

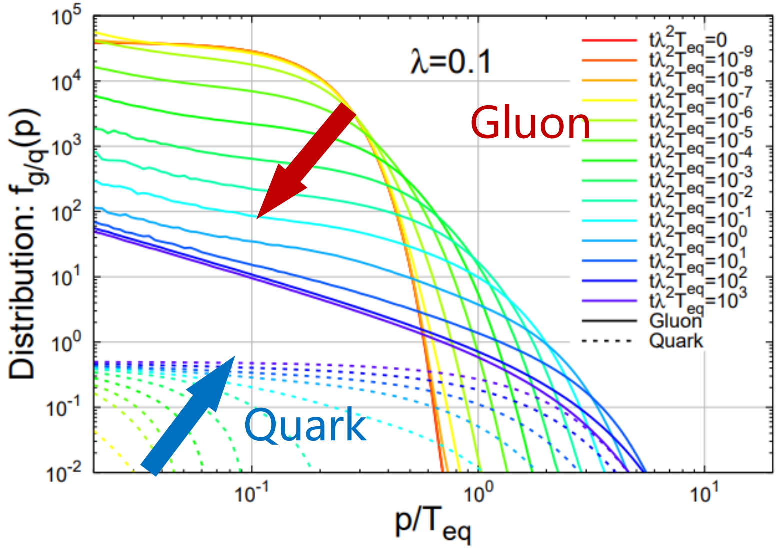
  high → low momentum

- **Jets in HICs**

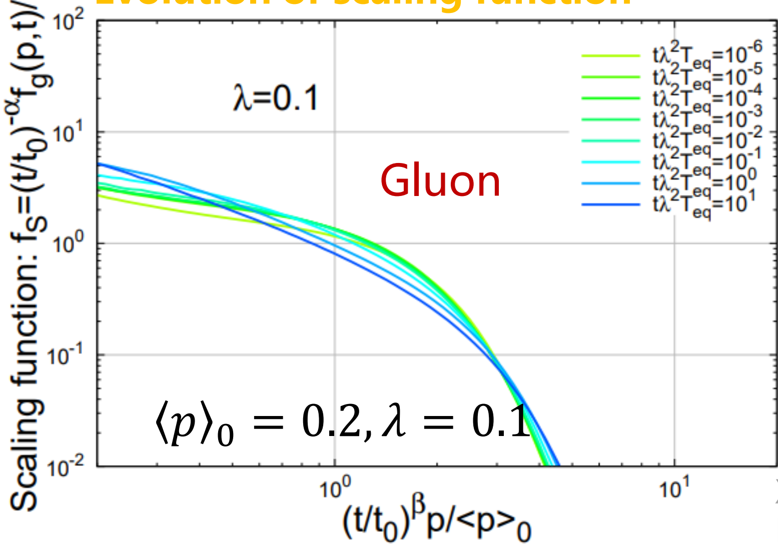
Schlichting, Teaney, ARNPS 69 (2019) 447

Over-occupied plasma

Evolution of distribution



Evolution of scaling function



Self-similar energy cascade

- Self-similar scaling spectra

$$f_g(p,t) = (t/t_0)^\alpha f_0 f_S\left((t/t_0)^\beta \frac{p}{\langle p\rangle_0}\right)$$

Universal Scaling Function

$$f_S\left((t/t_0)^\beta \frac{p}{\langle p\rangle_0}\right)$$

Scaling Exponents from Yang-Mills plasma

$$\alpha = -\frac{4}{7}, \beta = -\frac{1}{7}$$

- Also work for quark-gluon plasma

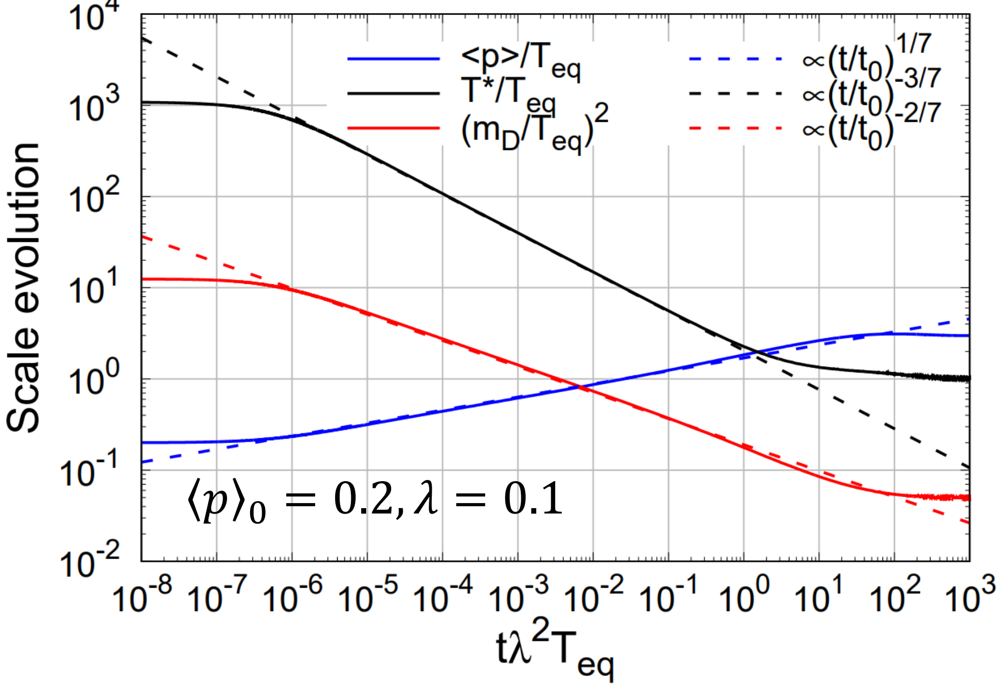
gluon dominated

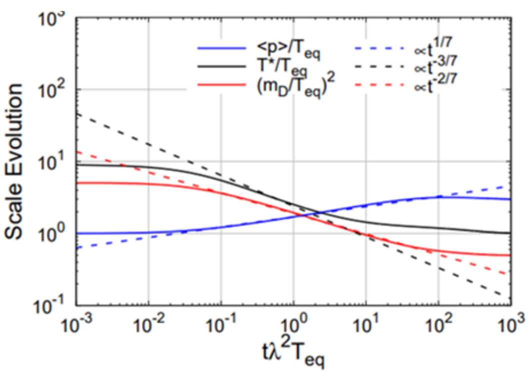
Quark spectra following gluon spectrum

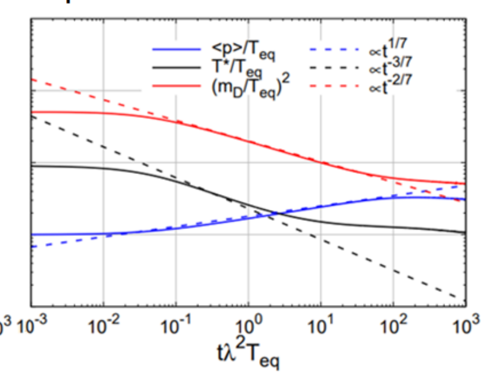
XD, Schlichting, PRD104(2021)054011
Berges, Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036

Over-occupied plasma

Evolution of scales







$\langle p\rangle_0 = 1, \lambda = 1$ $\quad$ $\langle p\rangle_0 = 1, \lambda = 10$

Self-similar scaling

$$f \sim f_0 \left(\frac{t}{t_0}\right)^{-\frac{4}{7}}$$

Power-law evolution

$$p \sim \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{\frac{1}{7}} \quad T \sim g^2 f_0 \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{-\frac{3}{7}}$$

$$m_D^2 \sim g^2 f_0 \langle p\rangle_0^2 \left(\frac{t}{t_0}\right)^{-\frac{2}{7}}$$

- Not limited to Yang-Mills plasma
But also for quark-gluon plasma

- Also work for stronger coupling

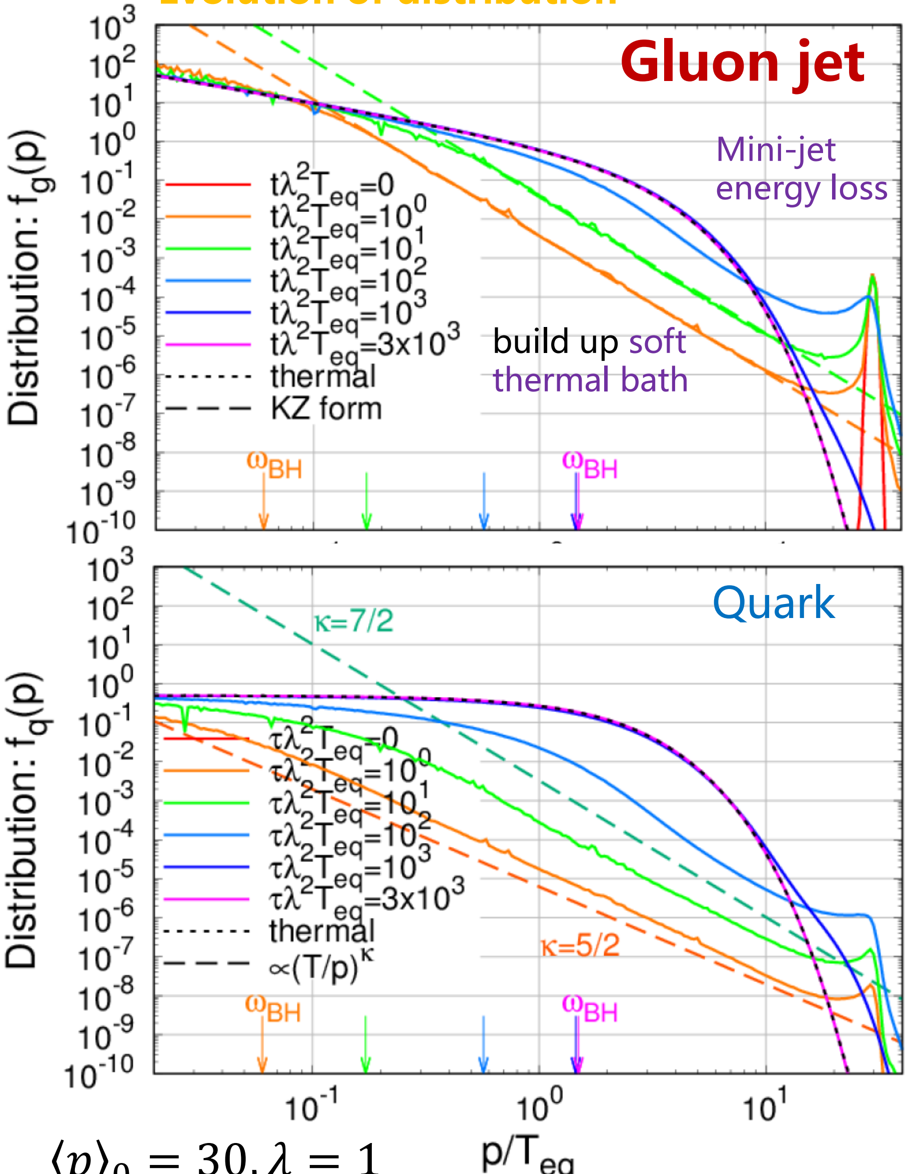
’t Hooft coupling

$$\lambda = 4\pi\alpha_s N_c$$

XD, Schlichting, PRD104(2021)054011

Under-occupied plasma

Evolution of distribution



$\langle p\rangle_0 = 30, \lambda = 1$

Wave turbulence

- Kolmogorov-Zakharov spectrum (exponent κ=7/2 for gluon)

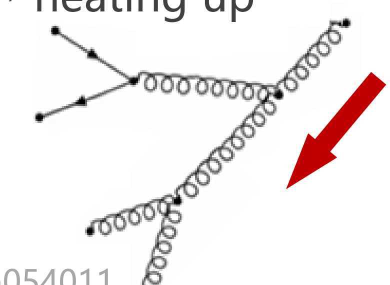
$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^\kappa$$

Blaizot, Iancu, Mehtar-Tani, PRL 111, 052001 (2013)
Mehtar-Tani, Schlichting, JHEP 09, 144 (2018)

Bottom-up thermalization

- Emission of (soft) quarks and gluon
- Radiative breakup by multiple branchings → build up soft thermal bath
- Mini-Jet energy loss → heating up thermal bath

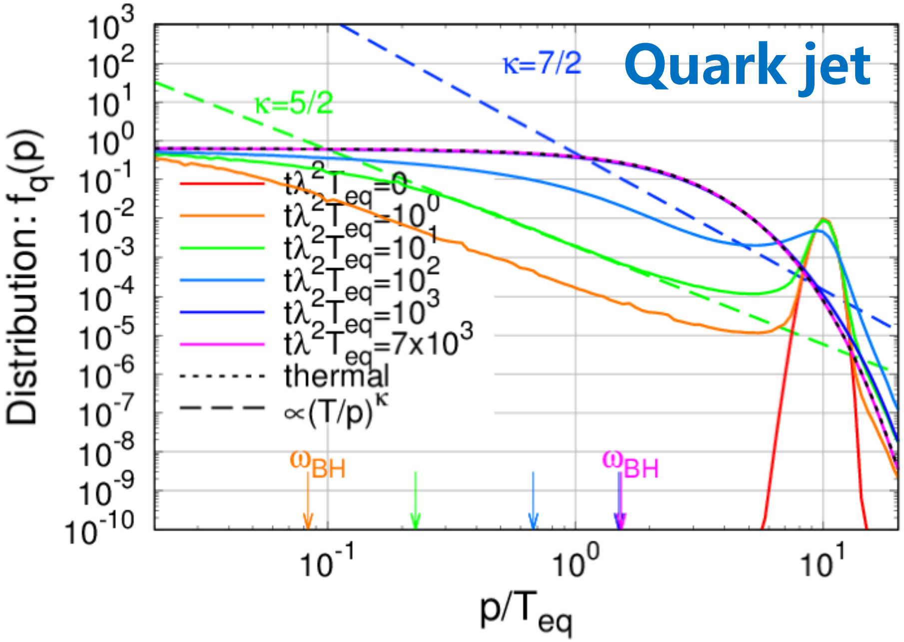
Baier, et al. PLB 502 (2001) 51

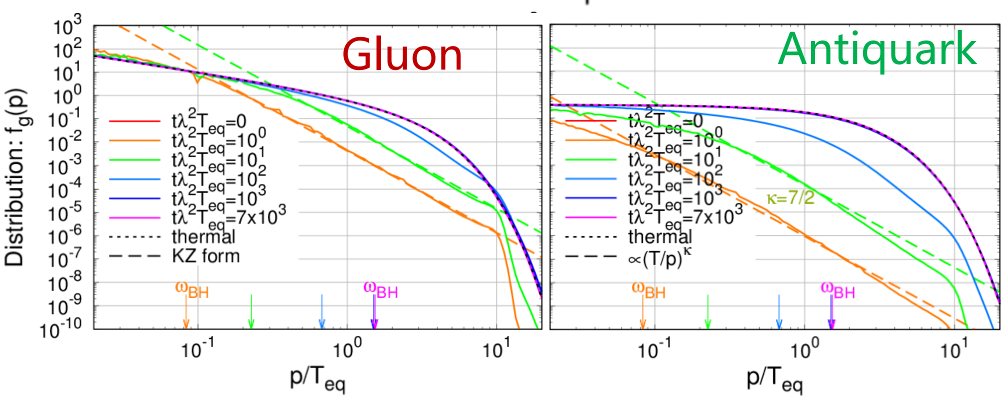


XD, Schlichting, PRD104(2021)054011

Under-occupied plasma

Evolution of distribution





$\langle p \rangle_0 = 10, \lambda = 1$

XD, Schlichting, PRD104(2021)054011

Wave turbulence

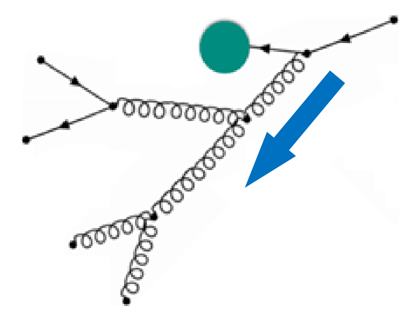
- Quark follows κ=5/2 to κ=7/2
- Gluon follows κ=7/2
- Antiquark follows gluon (secondary production)

$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p \rangle_0}{p}\right)^{\kappa}$$

Bottom-up thermalization

- Same pattern as for in-medium jet energy loss and equilibration

(with unified description of soft and hard sectors)



For turbulence from jet
Soudi, Schlichting, JHEP07(2021)077

Hydrodynamization of QCD plasmas

Hydrodynamization and attractor of quark-gluon plasma

Hydrodynamization in HICs

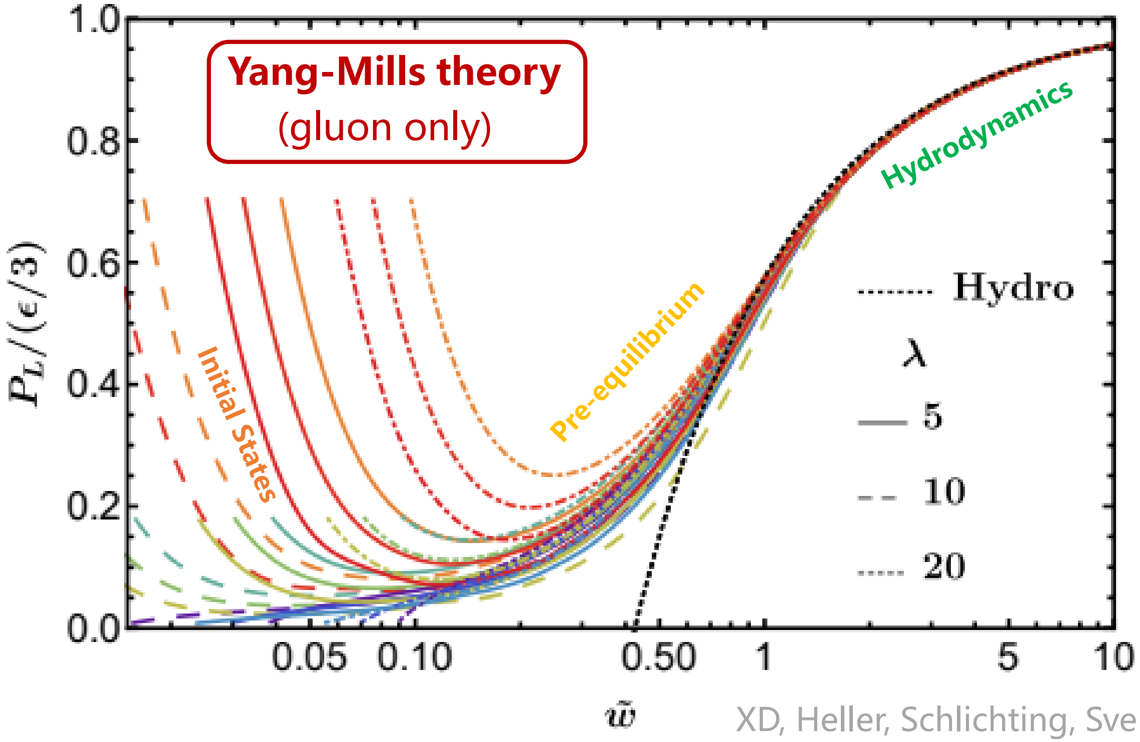
Kinetic equilibration

- First-order hydrodynamics

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}} \qquad \tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

Universal time scale

Evolution of pressure



XD, Heller, Schlichting, Svensson, arXiv:2203.16549

From EKT

- Effective constitutive relation

$$\frac{p_L}{e} = f(\tilde{\omega})$$

Pressure attractor

- Quick memory loss regardless of **initial states**
- Universal attractor towards **hydrodynamics**

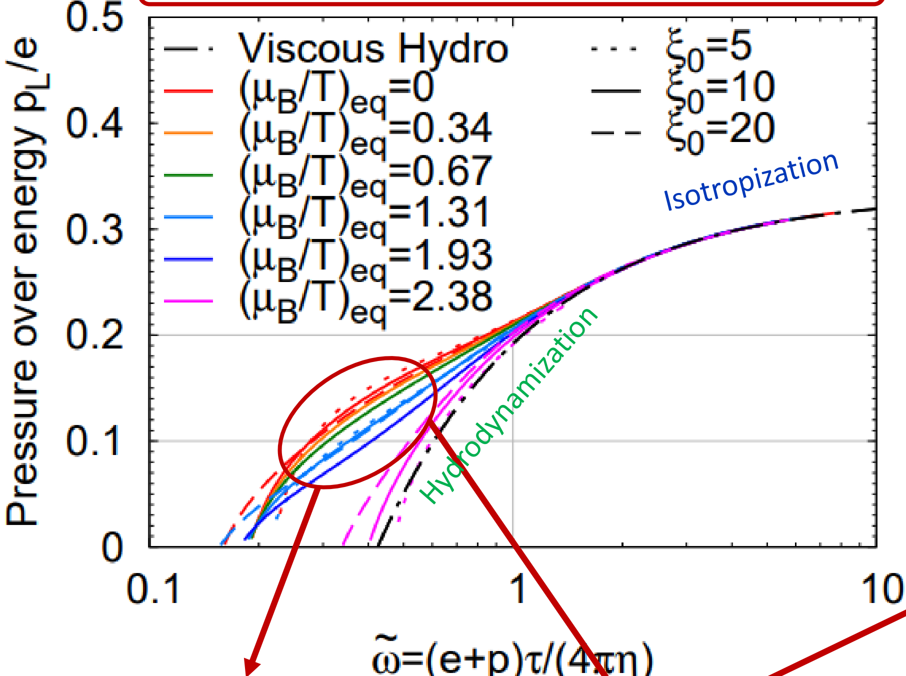
Hydrodynamization of QGP in HICs

Kinetic equilibration

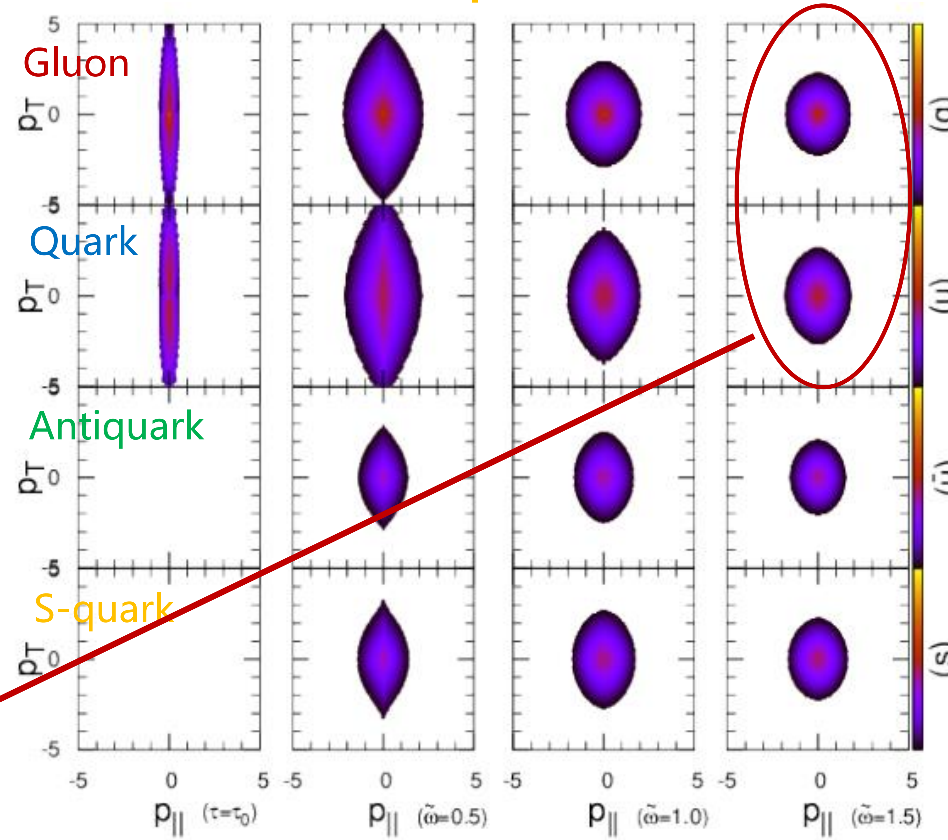
- Pressure anisotropy of QCD plasma still evolves to equilibrium
- Pressure anisotropy no longer guarantees an universal attractor

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Evolution of anisotropic distribution



Non-universal attractor

Quarks slow down equilibration
(gluons isotropy fast while quarks persist anisotropy)

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

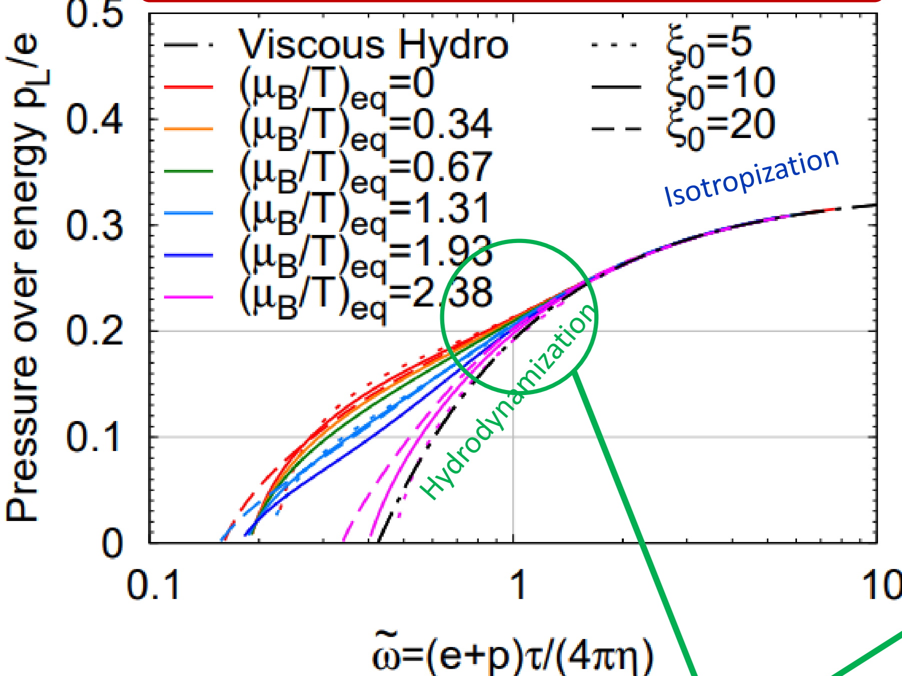
Hydrodynamization of QGP in HICs

Chemical equilibration

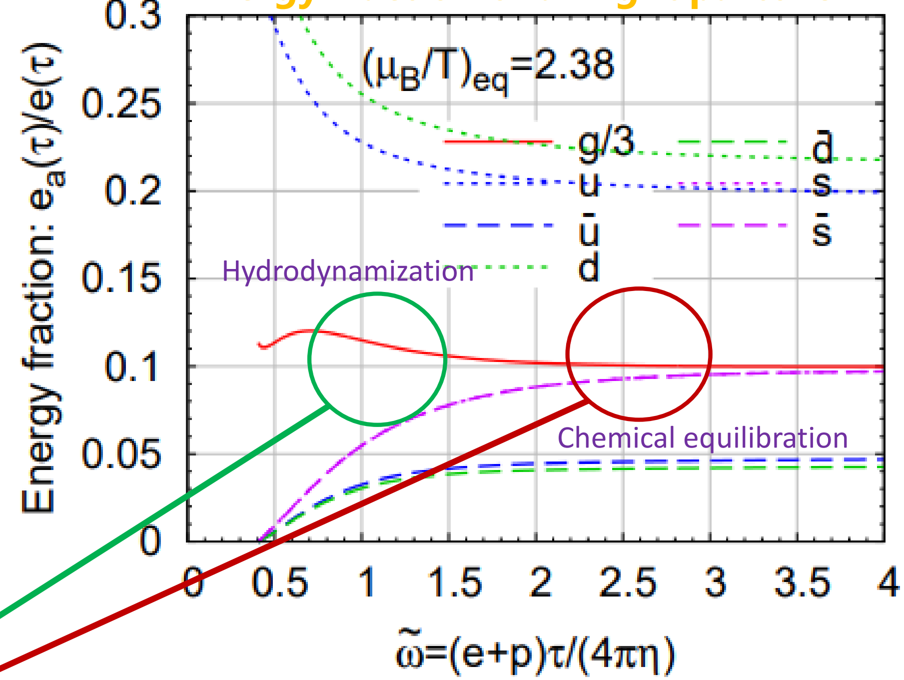
- Fractions of gluon/quark/antiquark change and evolve to equilibrium value
- Chemical equilibration persists after kinetic equilibration (hydrodynamization)

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Energy fraction of all light partons



Quarks slow down equilibration
(Chemical equilibration persists after hydrodynamization)

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

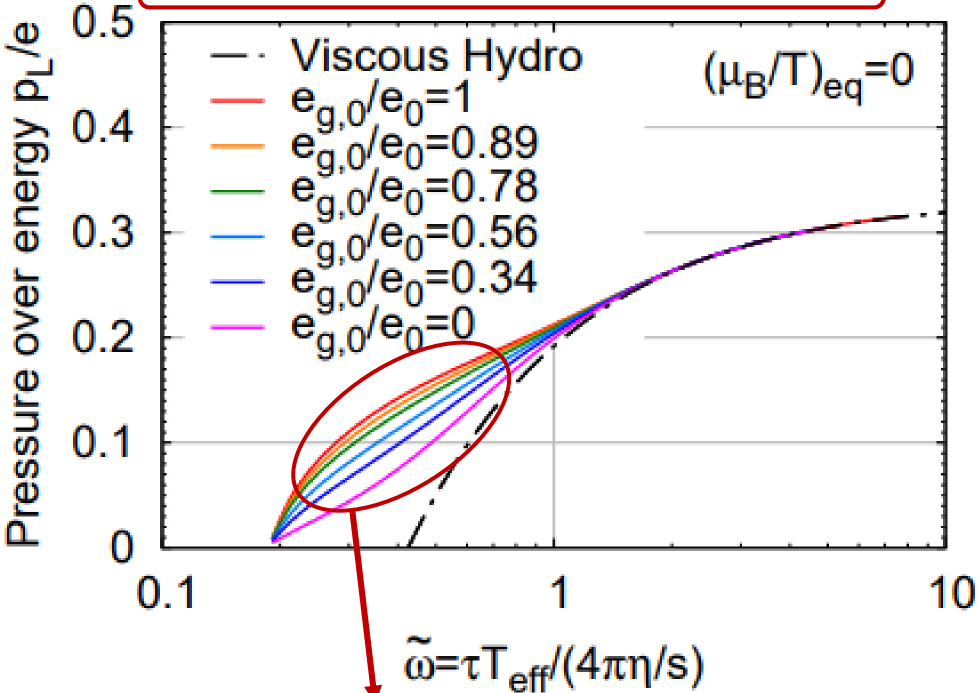
Hydrodynamization of QGP in HICs

Universal attractor with chemistry?

- Non-universal pressure (anisotropy) attractor
- Universal energy fraction (chemistry) attractor

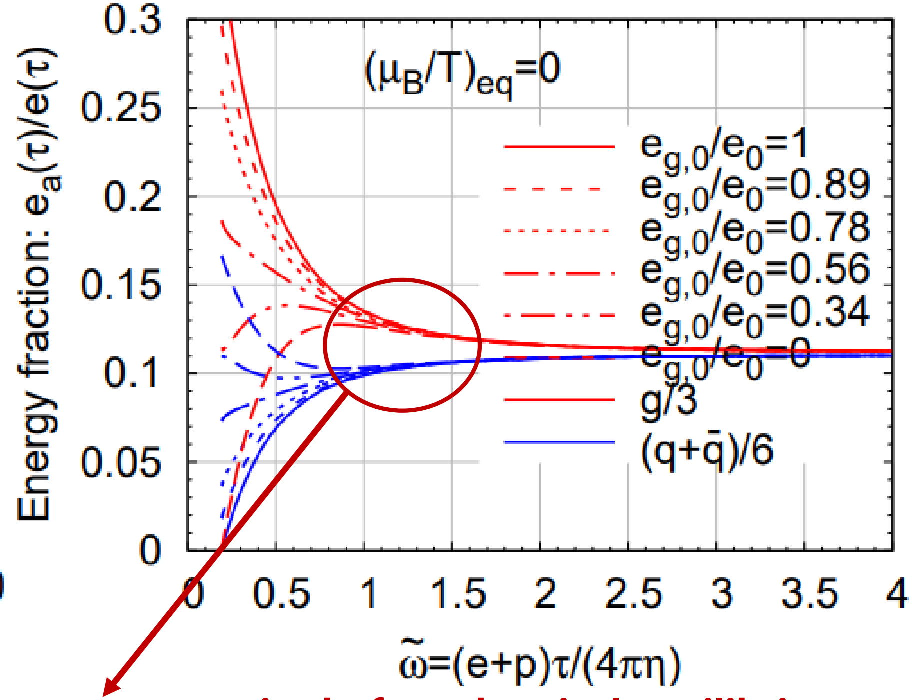
Evolution of pressure

QCD theory (gluon + quark/antiquark)



Spread before kinetic equilibrium

Evolution of energy fractions with different initial gluon fractions



Merge to one point before chemical equilibrium

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Hydrodynamization of QGP in HICs

Energy attractor and particle production

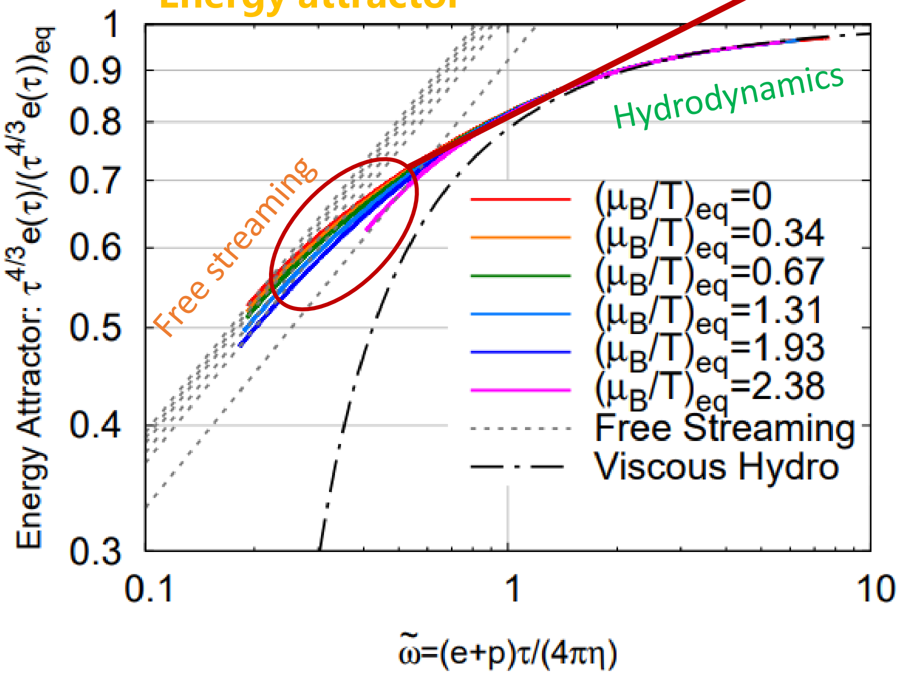
- **Pre-equilibrium** description connects initial states to hydrodynamics in HICs

$$\left(\tau^{4/3}e\right)_{\tilde{\omega}} = \left(4\pi\frac{\eta T_{\mathrm{eff}}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2}{30}\nu_{\mathrm{eff}}\right)^{\frac{1}{9}}(\tau e)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$\left(\tau\Delta n_f\right)_{\tilde{\omega}} = \left(\tau\Delta n_f\right)_0$$

Vary by different η/s
(Quarks slow down equilibration, increase η/s at larger density)

Energy attractor



Energy attractor

$$\mathcal{E}\left(\tilde{\omega}=\frac{(e+p)\tau}{4\pi\eta}\right)=\frac{\tau^{4/3}e}{(\tau^{4/3}e)_{eq}}$$

Two-way application

- Provide input for hydrodynamics
- Learn the past !(**pre-eq**, initial…)

Giacalone, Mazeliauskas, Schlichting PRL123(2019)262301
XD, Schlichting, PRL127(2021)122301

Early stage of heavy-ion collisions

Application of attractor & phenomenology in HICs

Pre-equilibrium QGP trajectory

Fix the final equilibrium quantities

- From EKT: entropy

$$(\tau s)_{eq} = \frac{\tau\left(e + p - \sum_f \mu_f \Delta n_f\right)}{T}$$

- From data: charged particle multiplicity

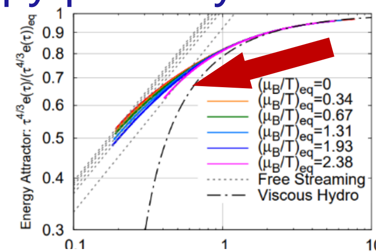
$$\frac{dN_{ch}}{d\eta} = \frac{N_{ch}}{JS} (\tau s)_{eq} S_T \approx 0.12 (\tau s)_{eq} S_T$$

- From EKT: net baryon number

$$\Delta n_B = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

- From data: entropy per baryon

$$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B}\right)_{eq}$$



Pre-equilibrium QGP

- Apply non-equilibrium attractor

$$\left(\tau^{4/3} e\right)_{\tilde{\omega}} = \mathcal{E}(\tilde{\omega})\left(\tau^{4/3} e\right)_{eq}$$

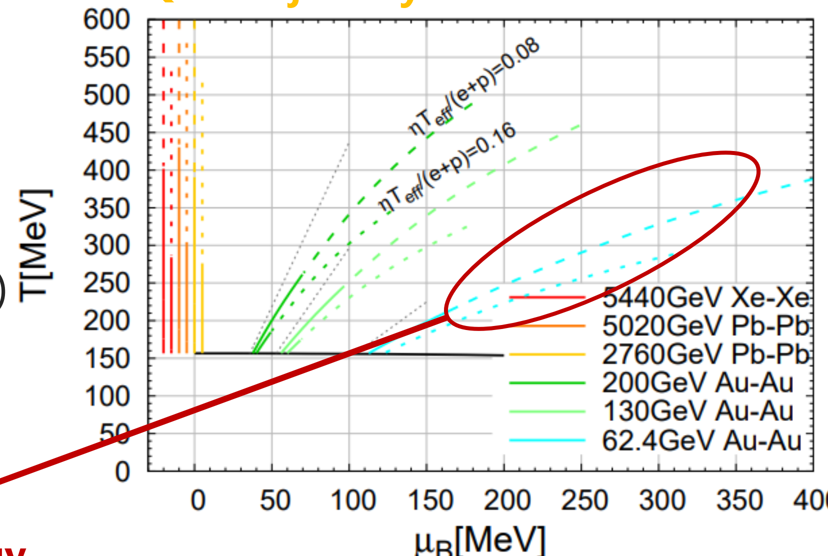
$$\left(\tau \Delta n_f\right)_{\tilde{\omega}} = \left(\tau \Delta n_f\right)_{eq}$$

- Define effective T and μ_B (Landau matching)

Non-equilibrium QGP trajectory

(at large baryon density)

QGP trajectory



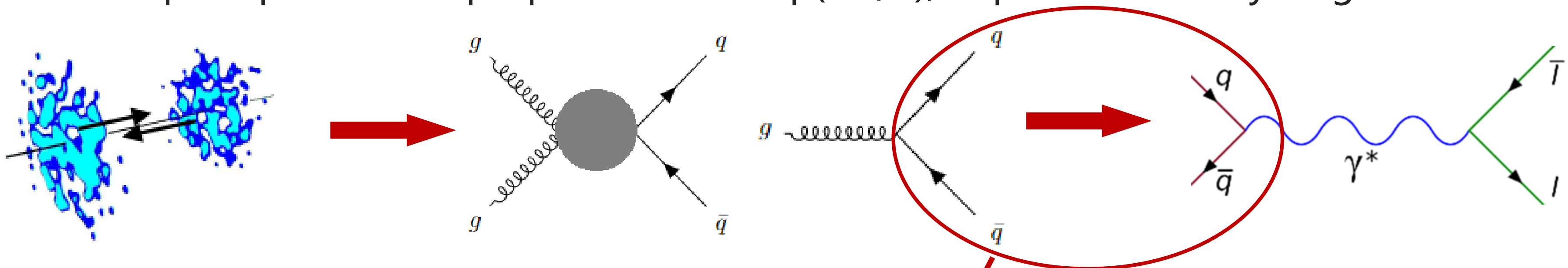
Pre-equilibrium stage important at RHIC energy

XD, Schlichting, PRL127(2021)122301

Pre-equilibrium di-lepton production

Electromagnetic probes

- Photon, di-lepton are produced through-out HICs, not interacting with QGP
- Di-lepton production proportional to exp(-M/T), important at early stage of HICs



Gluon saturated — **Gluon → quark/antiquark** — **Di-lepton production**

$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x,p_1) f_{\bar{q}}(x,p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-}\, \delta^{(4)}(K - P_1 - P_2)$$

Pre-equilibrium quark/antiquark distribution $f_{q/\bar{q}}(x,p)$

- Anisotropic distribution
- Quark/antiquark chemical production

Realistic space-time distribution complicated

(Parameterized distribution with fixed scale in equilibrium)

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Quark/antiquark distribution

- Kinetic equilibration

Gluon

$$f_g(\tau, p_t, p_z) = f_{BE}\left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)}\right)$$

Quark/antiquark

$$f_{q/\bar{q}}(\tau, p_t, p_z) = q(\tau) f_{FD}\left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)}\right)$$

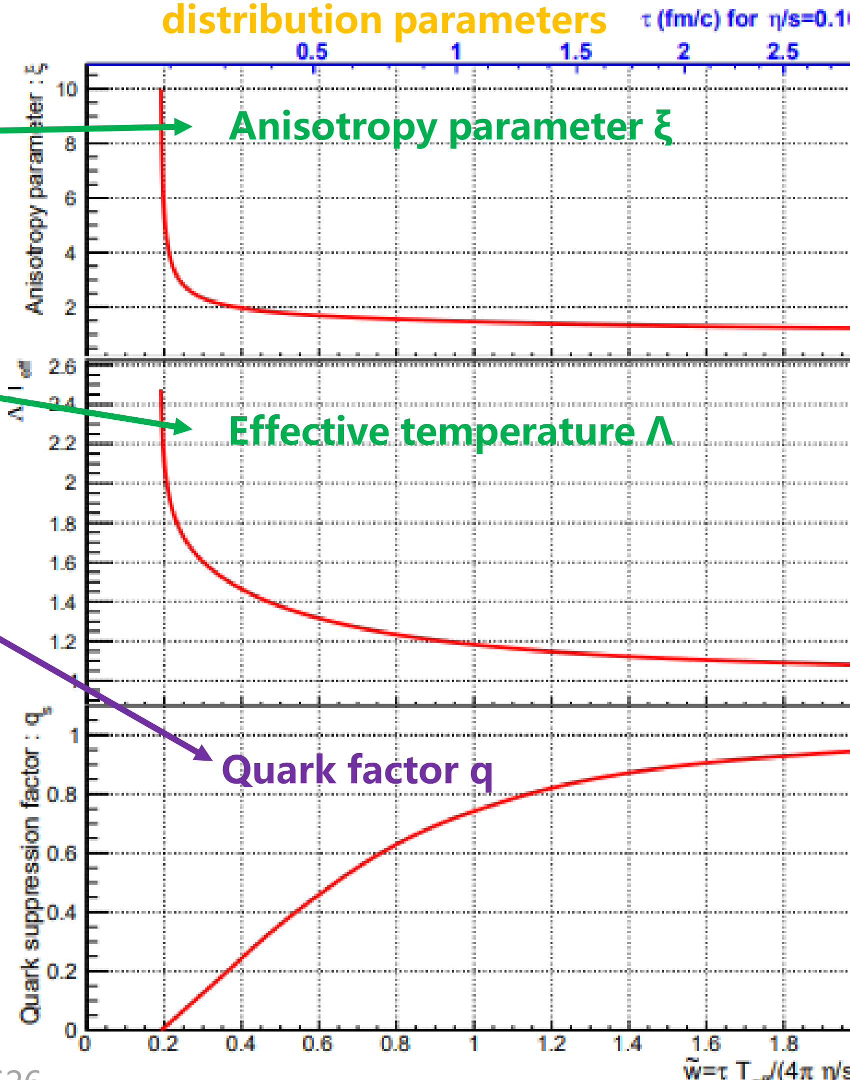
- Chemical equilibration

Quark factor

$$q(\tau) = \frac{e_g^{eq}}{e_q^{eq}} \frac{e_q(\widetilde{\omega})}{e_g(\widetilde{\omega})}$$

From QCD effective kinetic theory

Evolution of distribution parameters



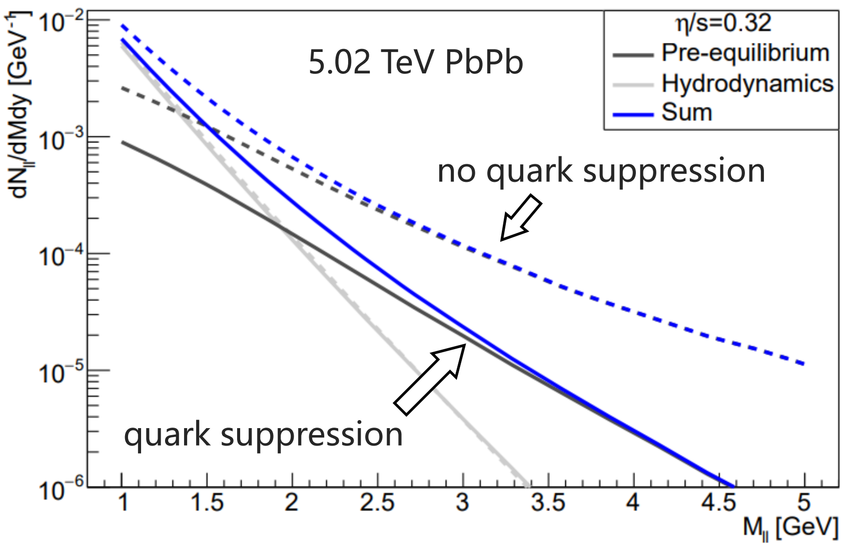
XD, Schlichting, PRL127(2021)122301
Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

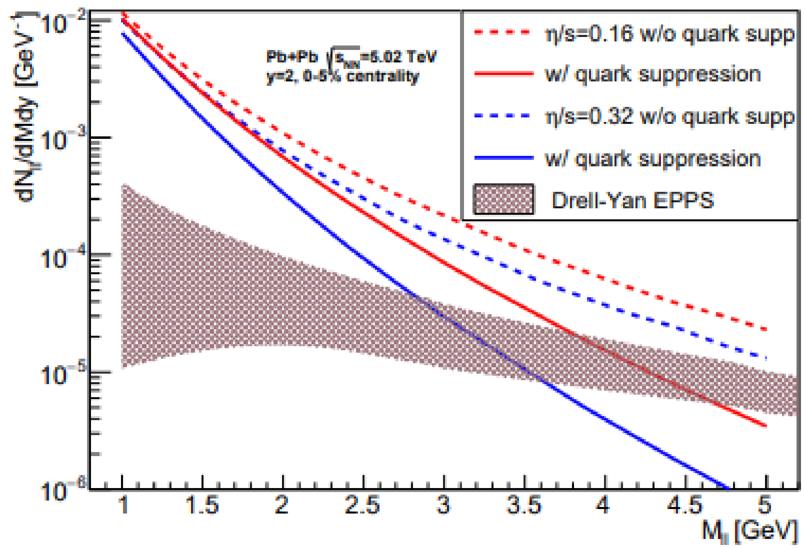
Pre-equilibrium di-lepton production

Di-lepton spectra in pre-equilibrium QGP

- Suppression compared to thermal quark (quark production)
- Suppression at weaker coupling medium (larger η/s, slower quark production)

Di-lepton spectra





- Pre-equilibrium production dominates at smaller invariant mass of di-lepton pair

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Di-lepton spectra and scale violation

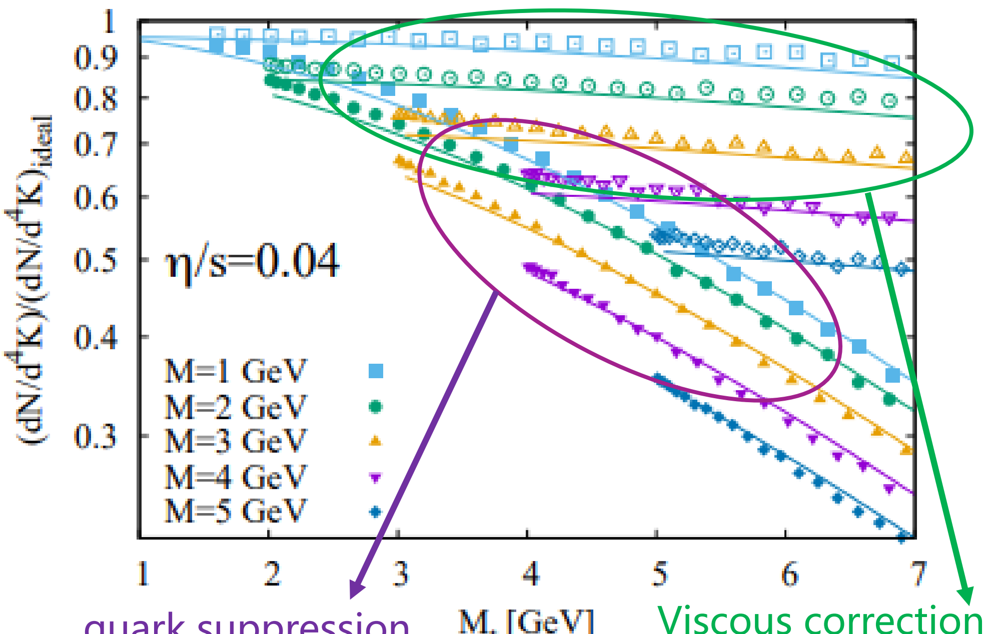
- Thermal and uniform QGP: McLerran-Toimela M_t^{-6} spectrum (ideal hydro)

$$\left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} = \frac{32N_c\alpha^2\sum_f q_f^2}{\pi^4}\frac{A_\perp(\tau T^3)^2}{M_t^6}$$

McLerran, Toimela, PRD31(1985)545

- Violate in pre-equilibrium QGP

Pre-equilibrium/thermal ratio



quark suppression (kinetic theory)

Viscous correction (N-S hydro)

Sensitivity of violation

- Depend on viscosity η/s, transverse mass M_t, invariant mass M

$$\frac{dN^{l^+l^-}}{d^4K} \simeq \left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} \frac{\left(1+a\frac{\eta}{s}M_t^2/n\right)^{-n}}{\sqrt{1+b\frac{\eta}{s}M^2}}$$

- Strong coupling limit η/s → 0, smoothly converges to McLerran-Toimela

Reynolds number

$$Re^{-1}(M_t) \equiv \frac{\eta}{s}\frac{M_t^2}{\tau T^3}$$

Coquet, XD, Ollitrault, Schlichting, Winn, arXiv:2112.13876

Sensitive to both kinetic (η/s) & chemical (quark factor) equilibration

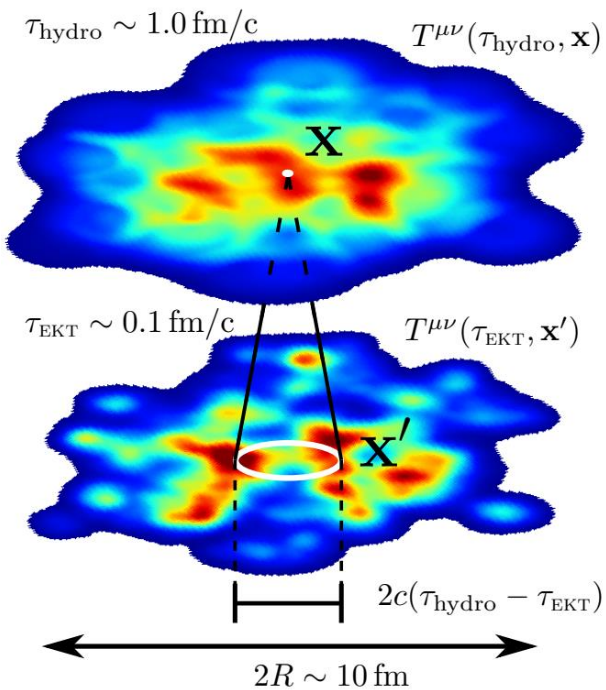
Space-time fluctuation

Towards a complete picture of pre-equilibrium stage in HICs

Propagation of space perturbation

Linearized QCD Effective Kinetic Theory

- Energy momentum tensor for inhomogeneous and anisotropic plasma



$$T^{\mu\nu}(\tau_{\mathrm{EKT}}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\mathrm{EKT}}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\mathrm{EKT}}, \mathbf{x}')$$

Background — Effective kinetic theory (EKT)

Perturbation — Linearized EKT

KøMPøST framework:
Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, PRL122 (2019) 12, 122302, PRC99 (2019) 3, 034910

Linear response/Green's function

- Propagate energy-momentum fluctuation in both position and momentum space (with Fourier transform)

$$\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\mathrm{hydro}}, \mathbf{x}) = \int d^2\mathbf{x}'\; G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\mathrm{hydro}}, \tau_{\mathrm{EKT}})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\mathrm{EKT}}, \mathbf{x}')\, \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\mathrm{hydro}})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\mathrm{EKT}})}$$

Extended from RTA: Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD102(2020)056003
Extended from Boltzmann: XD, Schlichting, in progress (both Yang-Mills and QCD)

Response functions

Response function in Fourier space

$$G^{\mu\nu}_{\alpha\beta}(\mathbf{x}-\mathbf{x}_0,\tau,\tau_0) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\tilde{G}^{\mu\nu}_{\alpha\beta}(\mathbf{k},\tau,\tau_0)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0)}$$

Energy perturbation

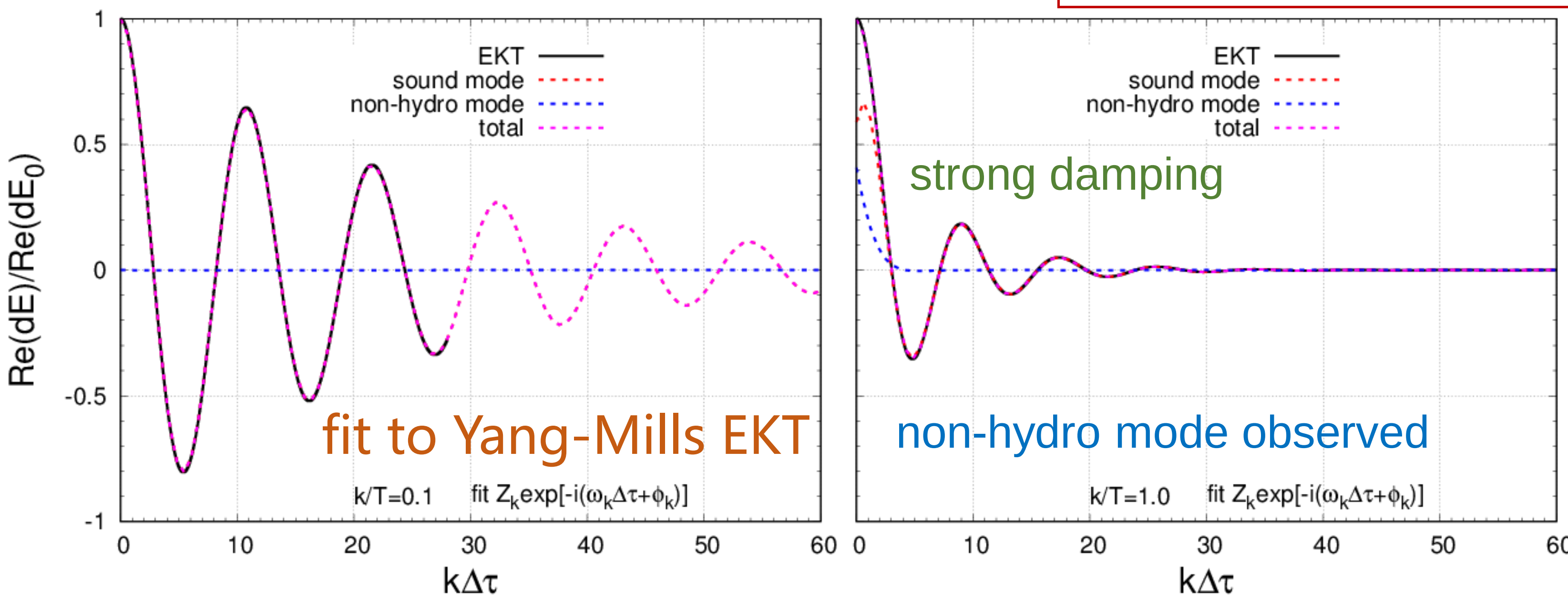
- Simple relation to calculate response function

$$\delta T^{\mu\nu}_{\mathbf{k},(s)}(\tau,\mathbf{k}) = \frac{T^{\tau\tau}(\tau)}{T^{\tau\tau}(\tau_0)}\tilde{G}^{\mu\nu}_{\alpha\beta}(\mathbf{k},\tau,\tau_0)\delta T^{\mu\nu}_{\mathbf{k},(s)}(\tau_0,\mathbf{k})$$

Initial conditions

$$f_a(\tau_0,\vec{p}) = \frac{1}{e^{p/T}-1}$$

$$\delta f_{\mathbf{k},a}(\tau_0,\vec{p}) = -\frac{\delta T}{T}p\partial_p f_a(\tau_0,\vec{p})$$

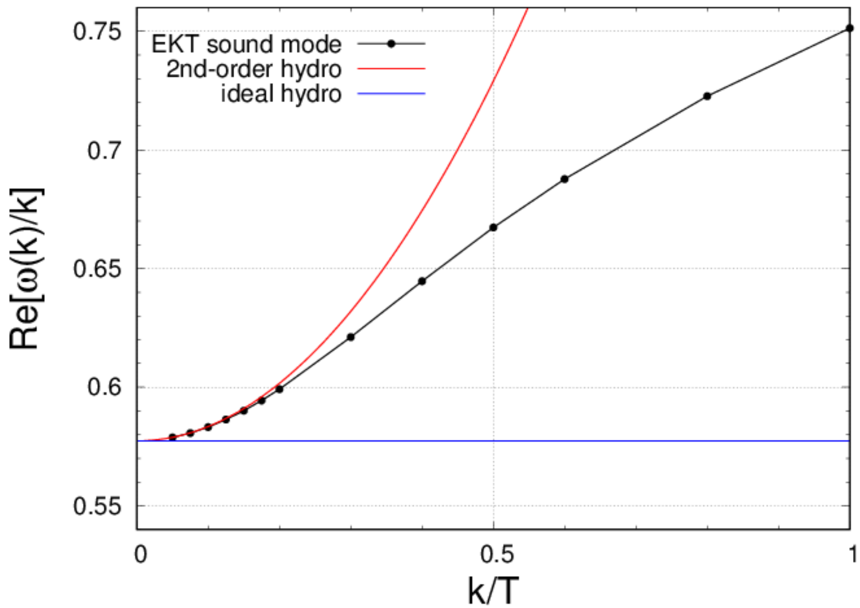


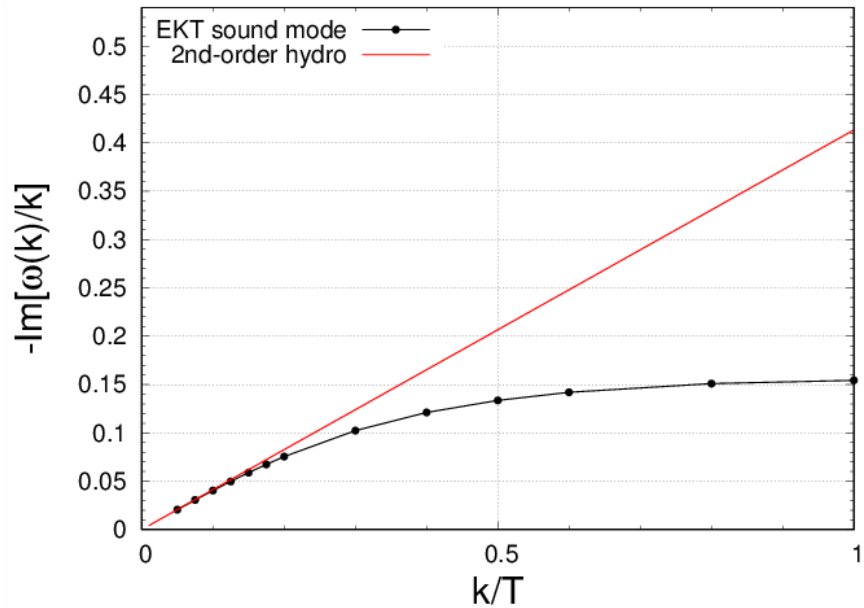
XD, Schlichting, in progress

Comparing to hydrodynamics

Dispersion relations in Yang-Mills EKT

- For different k-wave modes





Compare to 2nd-order hydrodynamic

- For different k-wave modes, 2nd-order hydro has

$$\omega_{1,2} = \pm c_s k - i\Gamma k^2 \pm \frac{\Gamma}{c_s}\left(c_s^2 \tau_\Pi - \frac{\Gamma}{2}\right) k^3 + \mathcal{O}(k^4)\,, \quad \Gamma = \frac{d-2}{d-1}\,\frac{\eta}{\varepsilon + P}$$

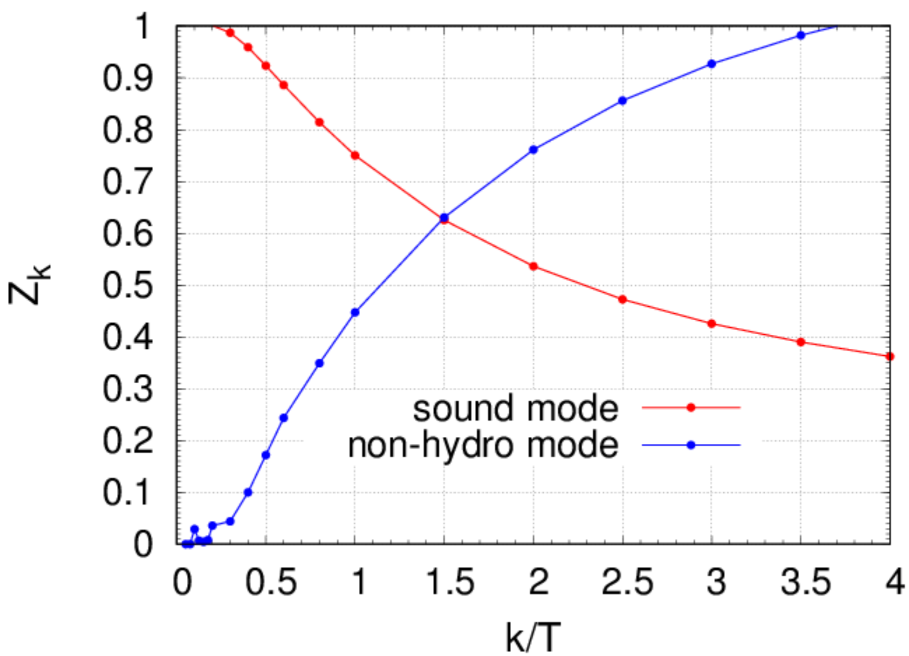
Baier, Romatschke, Son, Starinets, Stephanov, JHEP04(2008)100
Keegan, Kurkela, Mazeliauskas, Teaney, JHEP08(2016)171

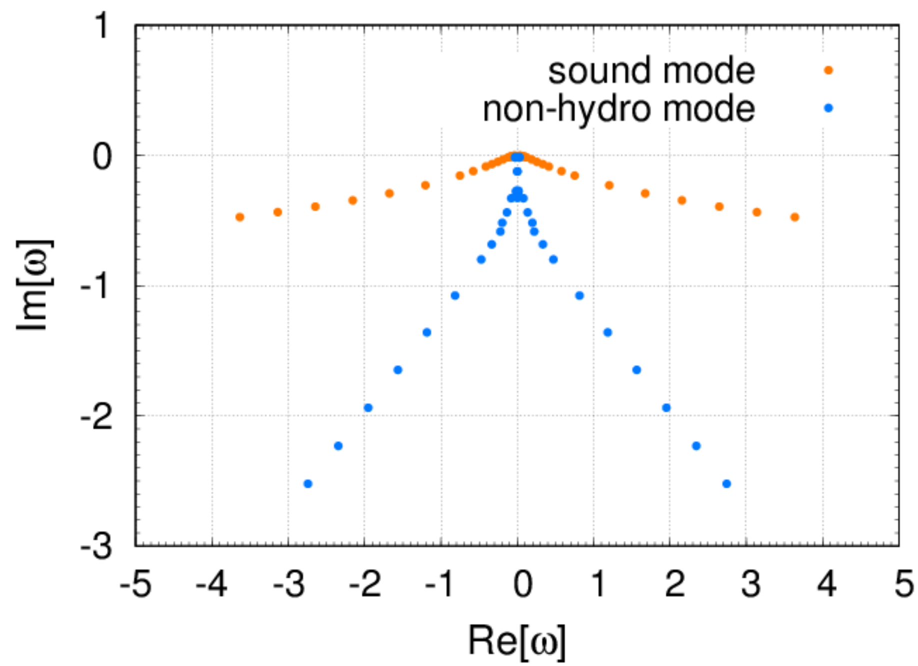
XD, Schlichting, in progress

Non-hydrodynamic modes

Residue and poles in the complex plane

- Pre-equilibrium Yang-Mills plasma described by a sound mode + a non-hydro mode





- More discussions:

RTA: Romatschke, EPJC76(2016)352, Kurkela, Wiedeman, EPJC79(2019)776

AdS/CFT: Buchel, Heller, Noronha, PRD94(2016)106011

XD, Schlichting, in progress

Conclusions

Summary and outlook

Conclusions

- **Non-equilibrium QCD**
  - QCD effective kinetic theory numerical solver at finite net-baryon density
- **Equilibration of QCD plasmas**
  - Self-similar scaling/Kolmogorov-Zakharov spectrum
  - Bottom-up thermalization
- **Hydrodynamization of QCD plasmas**
  - Hydrodynamization of QCD plasmas in Bjorken expansion
  - Non-equilibrium attractor and its universality in QCD plasmas
- **Early stage of heavy-ion collisions** **Importance of chemical equilibration**
  - Pre-equilibrium QGP Trajectory in HICs
  - Pre-equilibrium di-lepton production in HICs
- **Space-time fluctuation**
  - Including fluctuation with linearized EKT (now Yang-Mills plasma)
  - Yang-Mills plasma described by a sound mode + a non-hydro mode

❑ **Outlook: Towards a complete picture of pre-equilibrium stage in HIC:**

- Including charge fluctuation in QCD plasma

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011
[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, arXiv:2112.13876
[3] Others, in progress...