Hydrodynamic fluctuations in heavy-ion collisions

RHIC-AGS Annual Users' Meeting 2022

Workshop on the Beam Energy Scan

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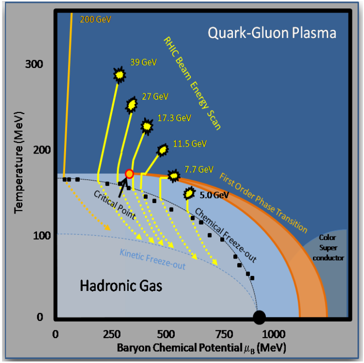
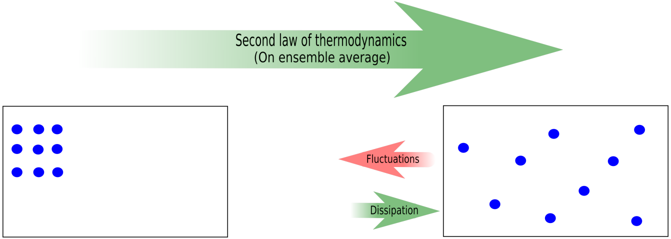
Driven to Discover®

In collaboration with Chun Shen, Sangyong Jeon and Charles Gale

QCD phase diagram and the hydrodynamic fluctuations





- Dynamical fluctuations near critical point provide signatures for identifying the critical point
- Fluctuations are intricately related to dissipation by the fluctuation-dissipation theorem

Approaches to study thermal fluctuations

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   - Hydro-kinetics (Y. Akamatsu A. Mazeliauskas and D. Teaney PRC 2017, PRC 2018; M. Martinez and T. Shaefer PRC 2019; X. An, G. Basar, M. Stephanov and H.-U. Yee, PRC 2019, PRC 2020 ...)
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Deterministic Approach

- A dissipative scale k_* is obtained

$$k_* = \left(\frac{\omega}{\gamma}\right)^{1/2}.$$

$\gamma = \gamma_\eta = \frac{\eta}{\varepsilon+P}$. Waves with wavenumbers $k >> k_*$ are damped too fast compared to the timescale $2\pi/\omega$

- Two point correlators of fluctuating hydrodynamic fields are defined

$$N_{ab}(t, \mathbf{x}, \mathbf{y}) = \langle \phi_a(t, \mathbf{x}) \phi_b(t, \mathbf{y}) \rangle.$$

- Hydrodynamic equations are expanded up to linear order in thermal fluctuations with an added noise term governed by the FDT

Deterministic approach

- Evolution equations for N_{ab} are written in momentum space. These evolution equations, coupled with the hydrodynamic equations can now be evolved in a $3+3+1$ D space
- Divergences at $k >> k_*$ are absorbed in transport coefficients and in hydrodynamic fields
- Advantage: Saves computational resources, elegant renormalization procedure
- Disadvantage: Complicated system of equations in $3+3+1$ D space, difficult to extend beyond two-point correlators
- Challenge: How to freeze-out correlators (See next talk by M. Pradeep)

Stochastic hydrodynamics

$$\partial_\mu (T^{\mu\nu}_{\text{ideal}} + \pi^{\mu\nu} + \Pi \Delta^{\mu\nu} + S^{\mu\nu}_{\pi} + + S_\Pi \Delta^{\mu\nu}) = 0,$$

$$(u.\partial)\pi^{\mu\nu} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - \eta \nabla^{\langle\mu} u^{\nu\rangle} + \ldots),$$

$$(u.\partial)\Pi = -\frac{1}{\tau_\Pi}(\Pi - \zeta(\partial.u) + \ldots),$$

$$(u.\partial)S^{\mu\nu} = -\frac{1}{\tau_\pi}(S^{\mu\nu} - \xi^{\mu\nu} + \ldots),$$

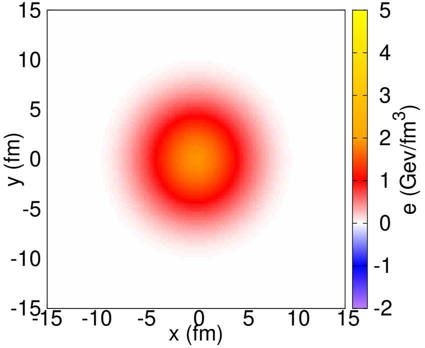
$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = 2\eta T \left[\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha} - \frac{2}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right] \delta^4(x - x'),$$

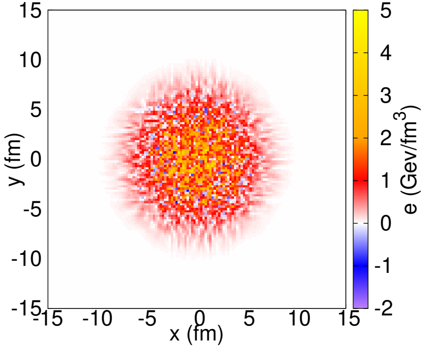
$$(u.\partial)S_\Pi = -\frac{1}{\tau_\Pi}(S_\Pi - \Xi + \ldots),$$

$$\langle \Xi(x_1)\Xi(x_2)\rangle = 2T\zeta\delta^4(x_1 - x_2).$$

Hydrodynamic Noise
(we need to be careful)

- Arbitrarily large gradients
- Potentially negative energy densities

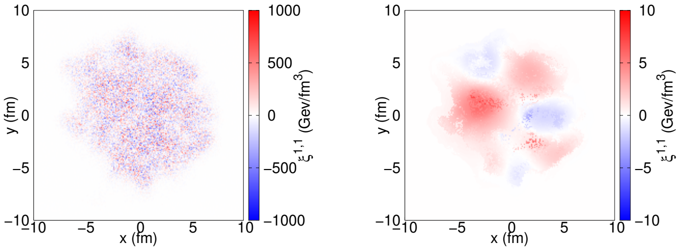




Removing high **k** modes

- We do not need all modes
- Modes above p_{cut} decay on small time scales

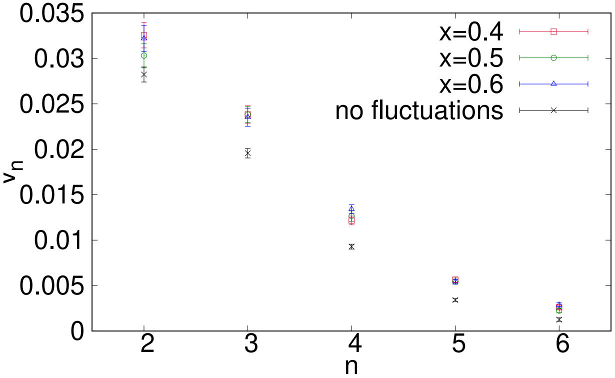
$$\sqrt{\frac{\omega}{\tau_\pi}} \ll p_{cut} \qquad p_{cut} = \frac{x}{\tau_\pi}$$



Noise Sampling before and after removal of high k modes

Choosing a p_{cut}

- A natural scale is $p_{cut} \sim 1/\tau_\pi$
- Charged hadron v_n are independent for three choices of p_{cut}
- Energy conservation is verified
- We use $p_{cut} = 0.5/\tau_\pi$ in calculations shown here



0-5% centrality. IP-Glasma + MUSIC + UrQMD

Our Model

IP-Glasma initial conditions include sub-nucleonic color charge fluctuations and classical Yang-Mills evolution

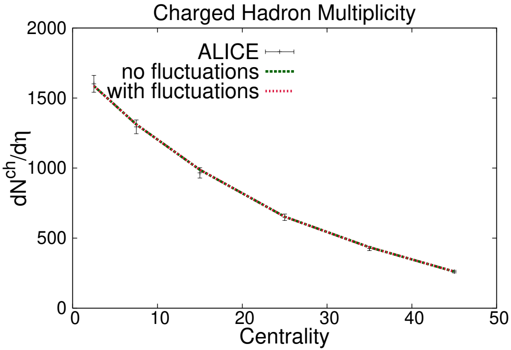
MUSIC is a second order 3+1D relativistic hydrodynamics package where stochastic terms were added

- τ_{sw} =0.4 fm
- EOS: hotQCD
- Constant η/s
- Temperature dependent bulk viscosity
- Shear Fluctuations
- T_{sw} = 145 MeV

UrQMD does hadronic re-scatterings and resonance decays after freeze-out

Effects of fluctuations on observables

- Multiplicity and lower v_n are unaffected



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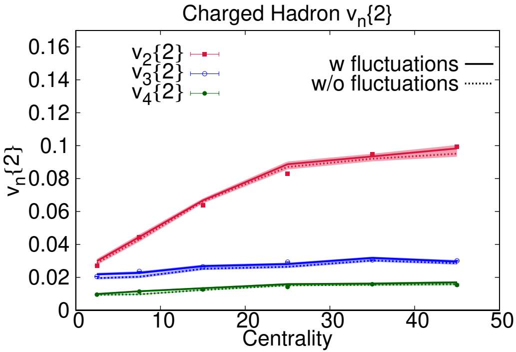

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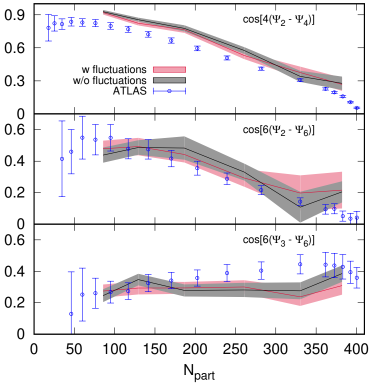
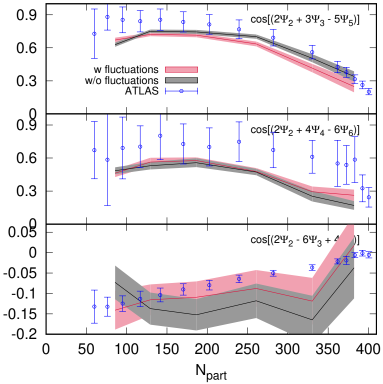
Effects of fluctuations on observables

2 plane event-plane correlators



Data from: ATLAS, PRC, 90, 024905, 2014

3 plane event-plane correlators



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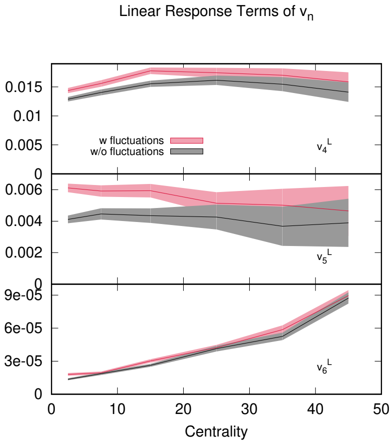
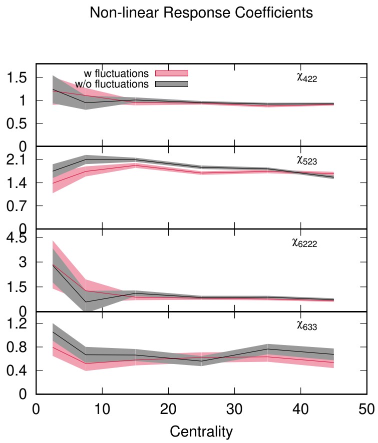
Effects of fluctuations on observables

$$V_n = V_n^L + \sum_{n=p+q} \chi_{npq} V_p V_q$$

$$V_6 = V_6^L + \chi_{6222}(V_2)^3 + \chi_{633}(V_3)^2$$





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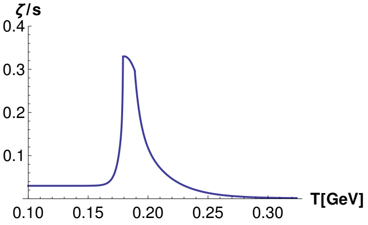
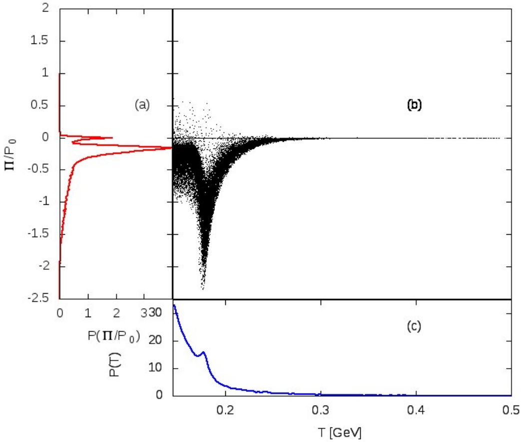
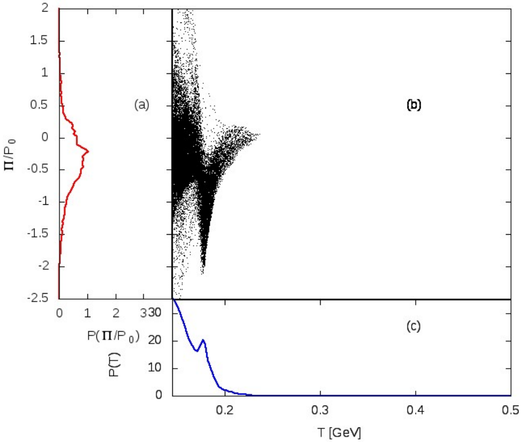


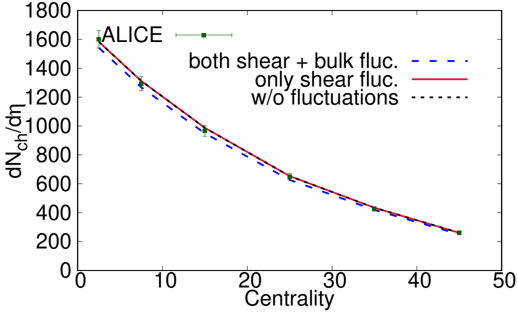
UrQMD does hadronic re-scatterings and resonance decays after freeze-out

Evolution history with bulk fluctuations





Observables with shear + bulk fluctuations



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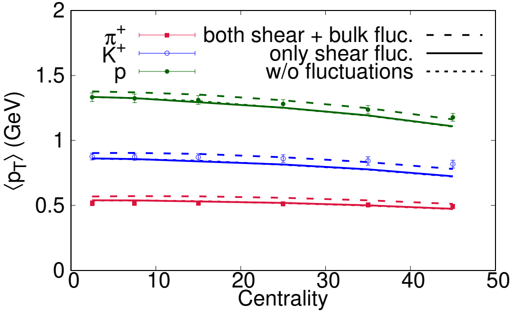

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Summary

- Hydrodynamic fluctuations in HIC simulations could be included by removing high wavenumber modes
- Effects of shear fluctuations are visible in the linear and non-linear response coefficients of v_n
- Bulk fluctuations affect the total lifetime of the fireball. Consequently, they affect total entropy production and mean transverse momentum of particles