Freezing out fluctuations in Hydro+ near the QCD critical point

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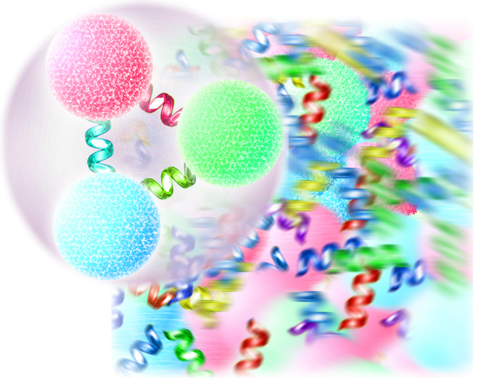
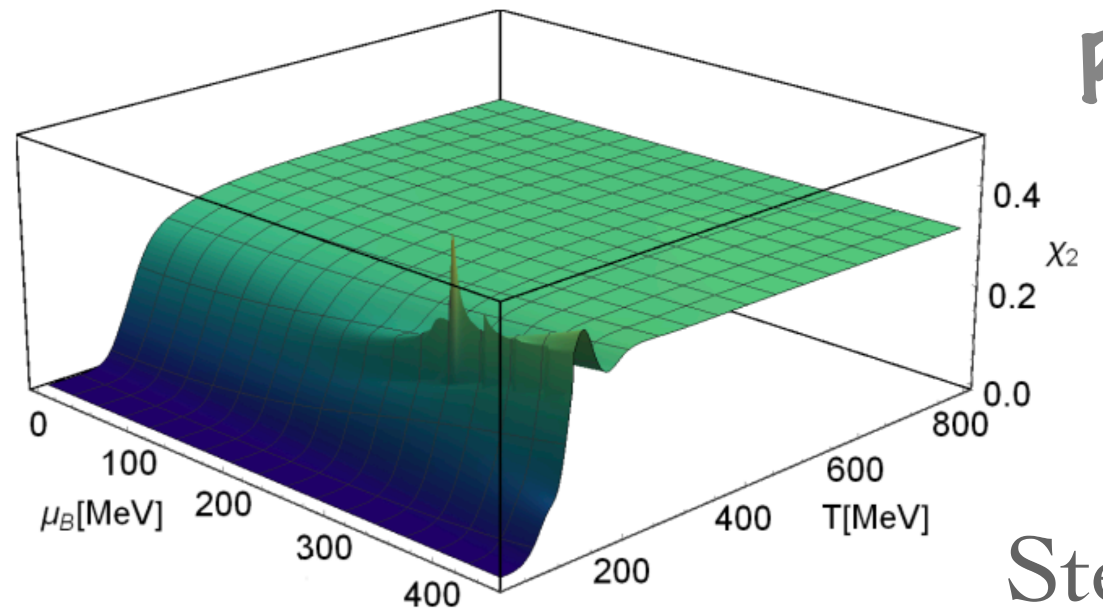
Overview



Hydrodynamic fluctuations of QGP



Parotto et al., 18

Stephanov.,08

$$\langle \delta N^2 \rangle \sim \xi^2 \,, \, \langle \delta N^3 \rangle \sim \xi^{4.5} \,, \quad \langle \delta N^4 \rangle_c \sim \xi^7$$

<u>Conservation laws</u>

<u>Finite time</u>

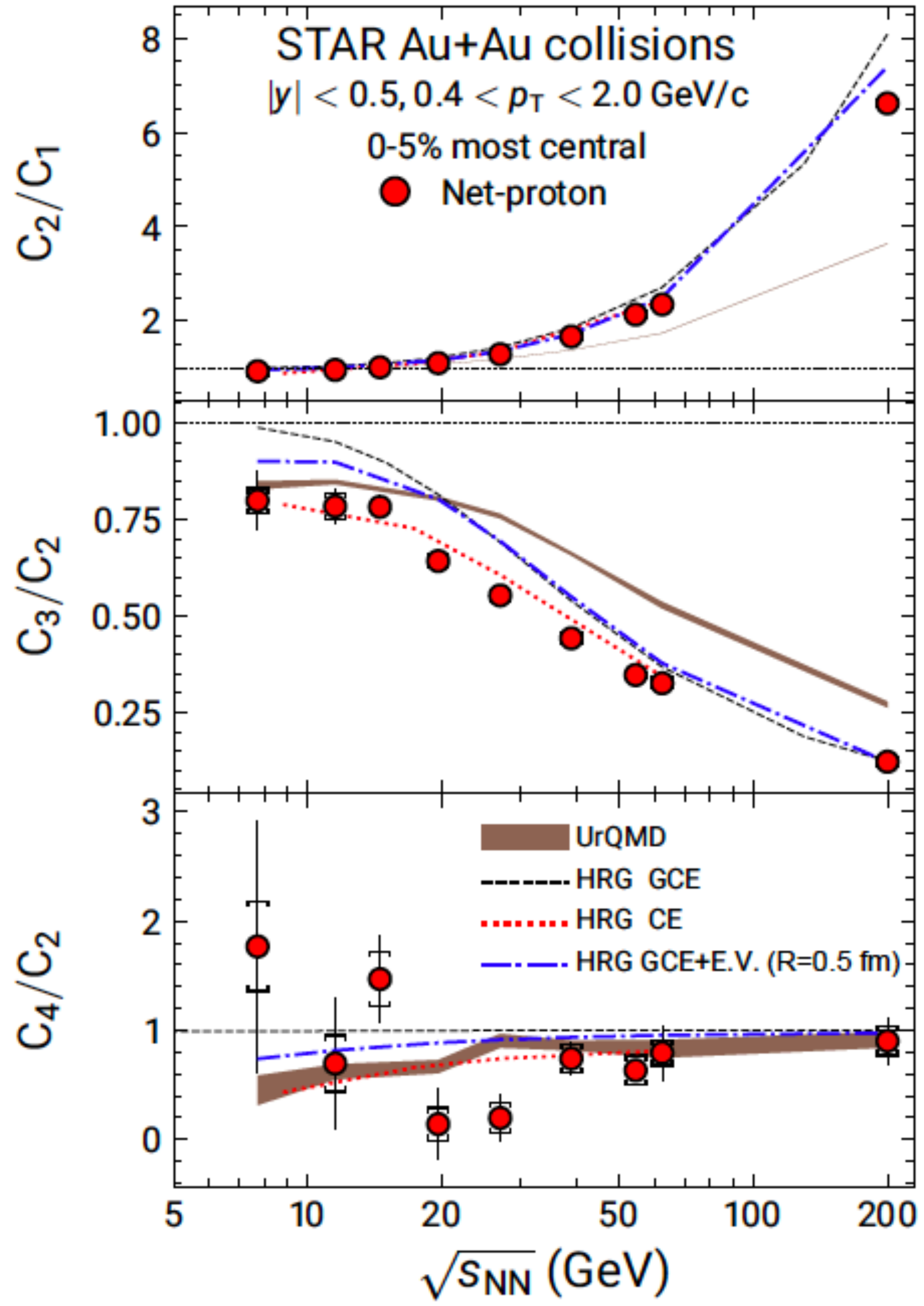
<u>Critical slowing down</u>

Fluctuations as observables for the QCD critical point

Non-monotonic deviation from baseline is suggestive of the presence of a critical point!

This talk : Freeze-out of Gaussian fluctuations

STAR Collaboration, 21



Cumulants of particle multiplicities

Dynamics of fluctuations near the critical point

In the previous talk, we heard about the stochastic description of fluctuations. Here, we'll use the complementary deterministic treatment.

Stochastic approach

$$\partial_t \breve{\psi} = -\nabla \cdot \left(\text{flux}\,[\breve{\psi}] + \text{noise} \right) \quad \text{(conservation)}$$

$$\langle \text{noise}(x)\text{noise}(y) \rangle \sim \delta^{(4)}(x-y) \quad \text{FDT}$$

Previous talk by M. Singh

Bluhm et al., 2020

Deterministic approach

$$\partial_t \psi = -\nabla \cdot \text{flux}\,[\psi, G],$$

$$\partial_t G = \text{relaxation}\left[G - G^{\text{eq}}; \psi\right]$$

This talk

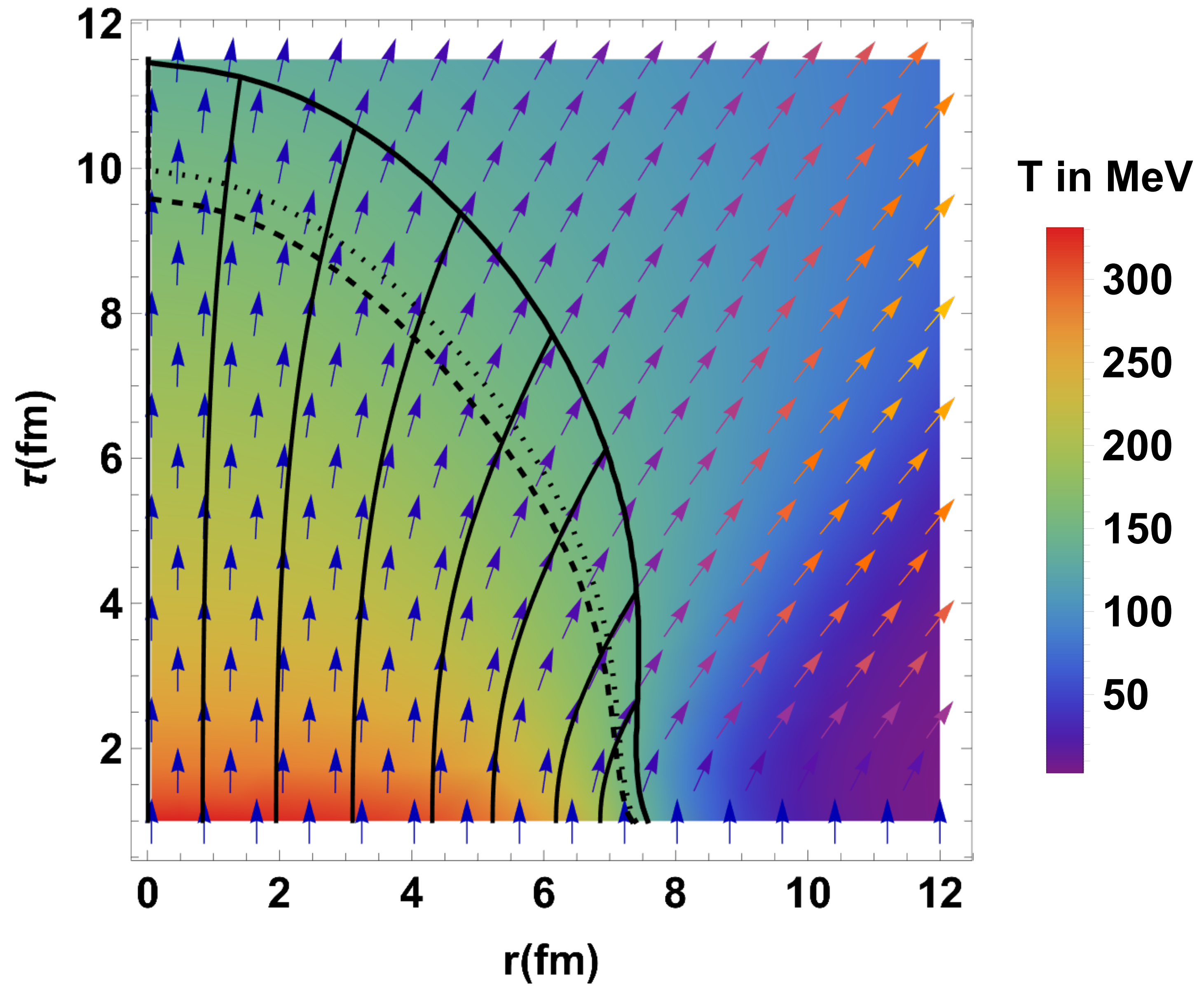
In particular, we use Hydro+ framework. We'll demonstrate the freeze-out in one of the available Hydro+ simulation.

Rajagopal et al, 19, Du et al. 20

Hydro+ simulation

* Hydrodynamics + relaxation equation for the slowest non-hydrodynamic mode

Stephanov & Yin, 2017



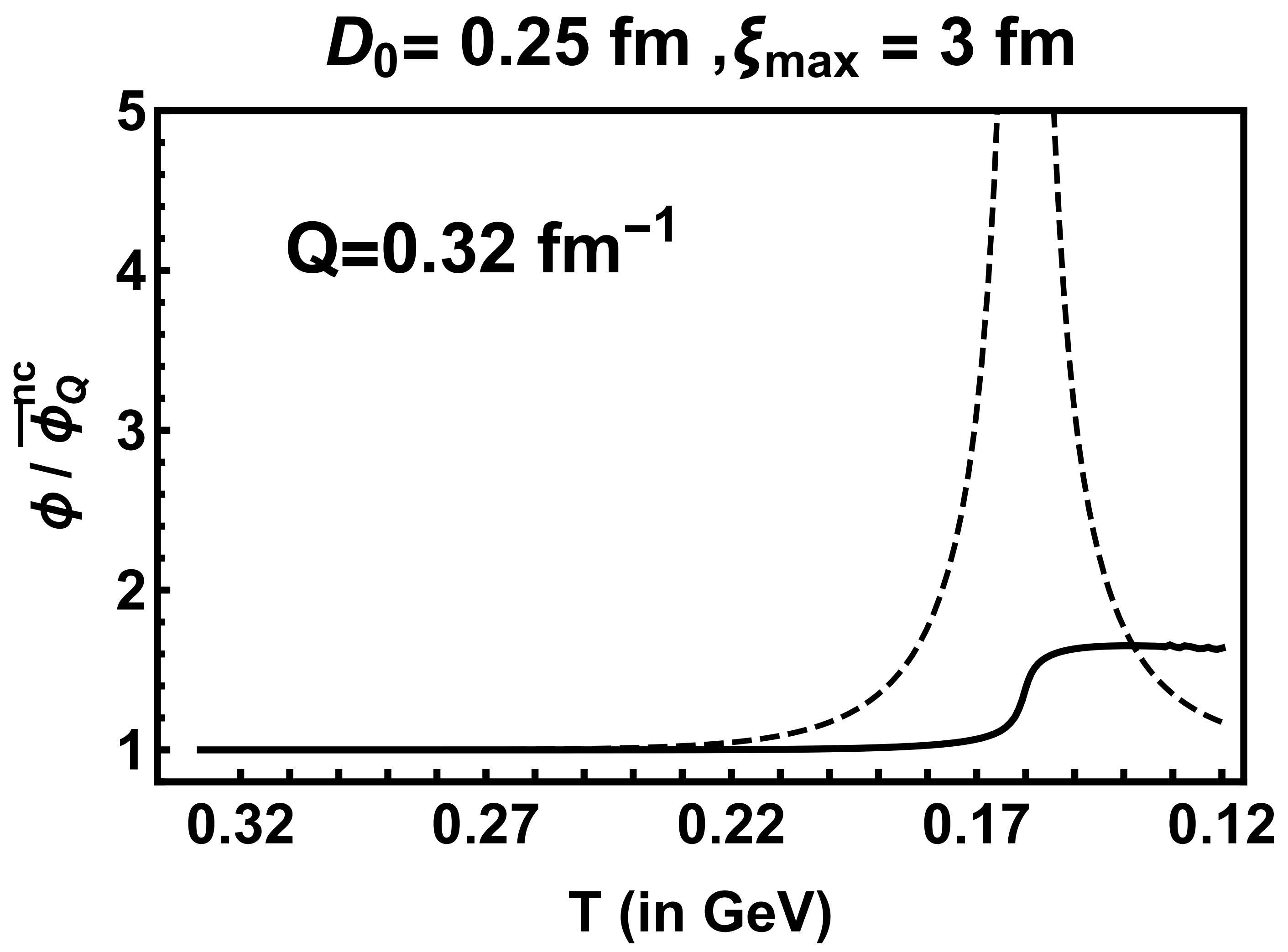
Baier and Romatschke, 2007

This talk :

Azimuthally symmetric, boost invariant hydrodynamic background with radial expansion with fluctuations discussed in Rajagopal, Ridgway, Weller, Yin, 19

Evolution of fluctuations

Stephanov & Yin, 2017



- The slowest and the most singular mode near the critical point corresponds to fluctuations of $\hat{s} \equiv \dfrac{s}{n}$
- The relaxation rate $\Gamma \sim \xi^{-3}$
- Equilibrium fluctuations $\propto C_p \sim \xi^2$

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$$\phi_{\mathbf{Q}} = \int_{\mathbf{\Delta x}} e^{-i\,\mathbf{Q}\cdot\mathbf{\Delta x}} \left\langle \delta\hat{s}(x_+)\, \delta\hat{s}(x_-) \right\rangle$$

$$u \cdot \partial \phi_{\mathbf{Q}} = -\Gamma\left(\mathbf{Q}\right) \left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}}\right)$$

$$\Gamma(\mathbf{Q}) = \frac{2D_0\xi_0}{\xi^3} K(|\mathbf{Q}\xi|), K(x) \sim x^2 \text{ for } x << 1$$

Zeroth mode doesn't evolve

Demonstrating critical slowing down

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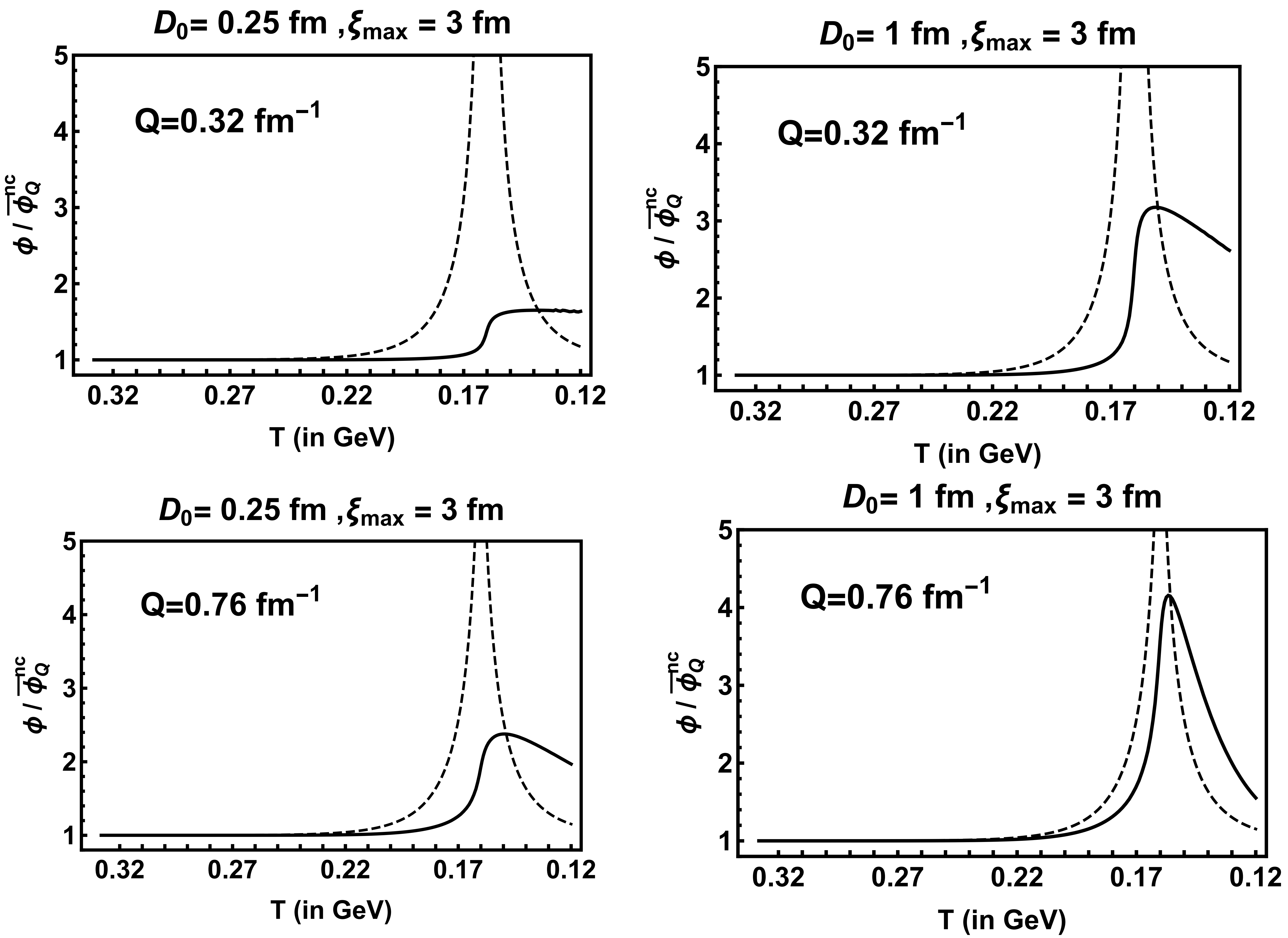
Lower Q modes are suppressed strongly due to conservation and relax more slowly

$$\bar{\phi}^{nc}_{\mathbf{Q}} \sim \frac{\xi_0^2}{1 + (\mathbf{Q}\xi_0)^2}$$

Normalized out-of-equilibrium fluctuations for two Q modes and two relaxation rates



Critical correlations in space

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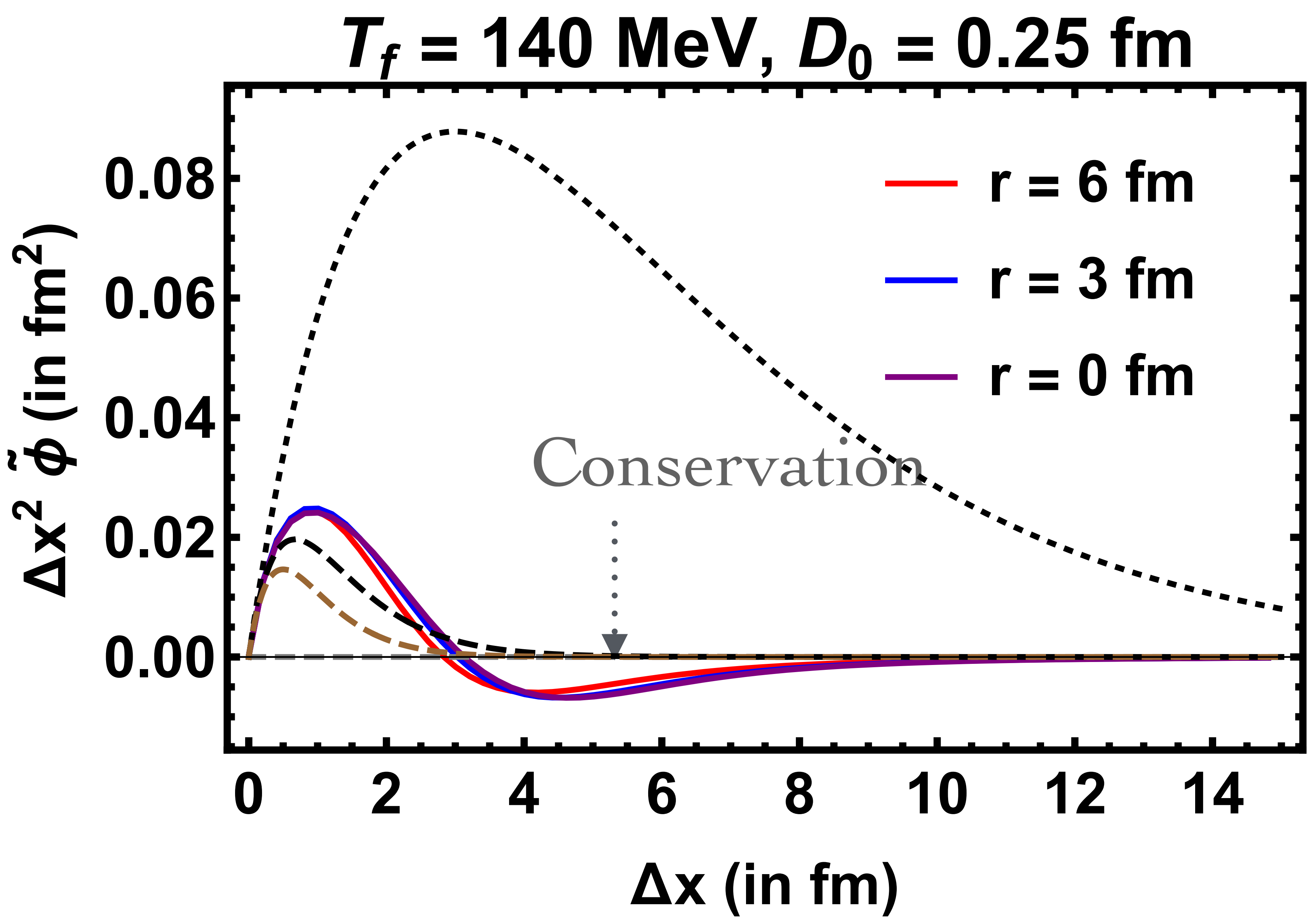
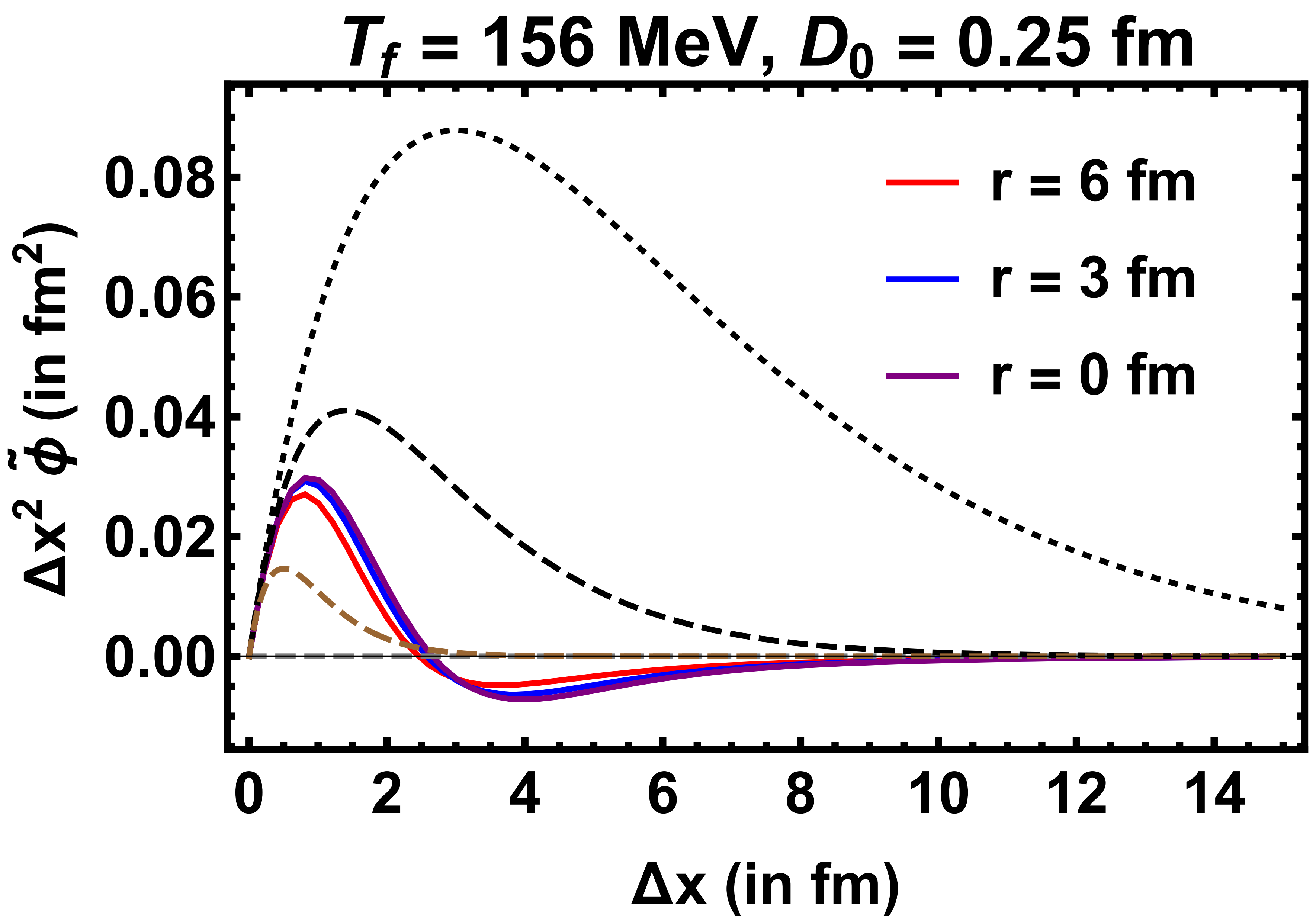
We consider two isothermal freeze-out scenarios: T=140 MeV and T=156 MeV





Memory

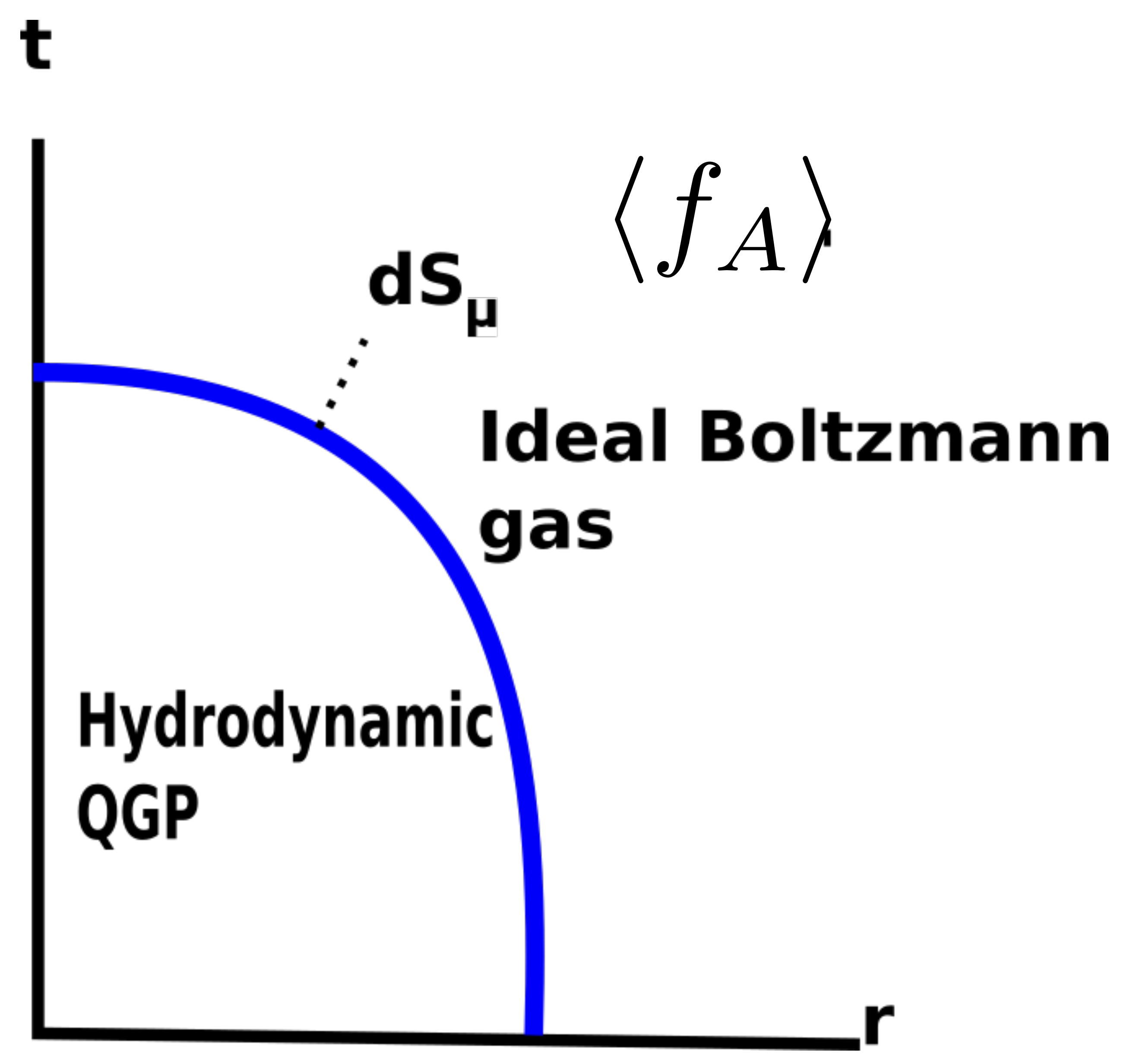
Out-of equilibrium fluctuations “remember” their past, so the difference between the two freeze-out scenarios is not too large

Conservation

$$\int d\Delta x\, \Delta x^2\, \tilde{\phi}(\Delta x) = \phi_0$$

Zeroth mode doesn’t evolve

Traditional Cooper-Frye freeze-out procedure



$$\langle N_A \rangle = \int dS_\mu \int Dp\, p^\mu \, \langle f_A(x,p) \rangle$$

Matches the averages of conserved densities before (hydrodynamic) and after (hadron resonance gas) freeze_out

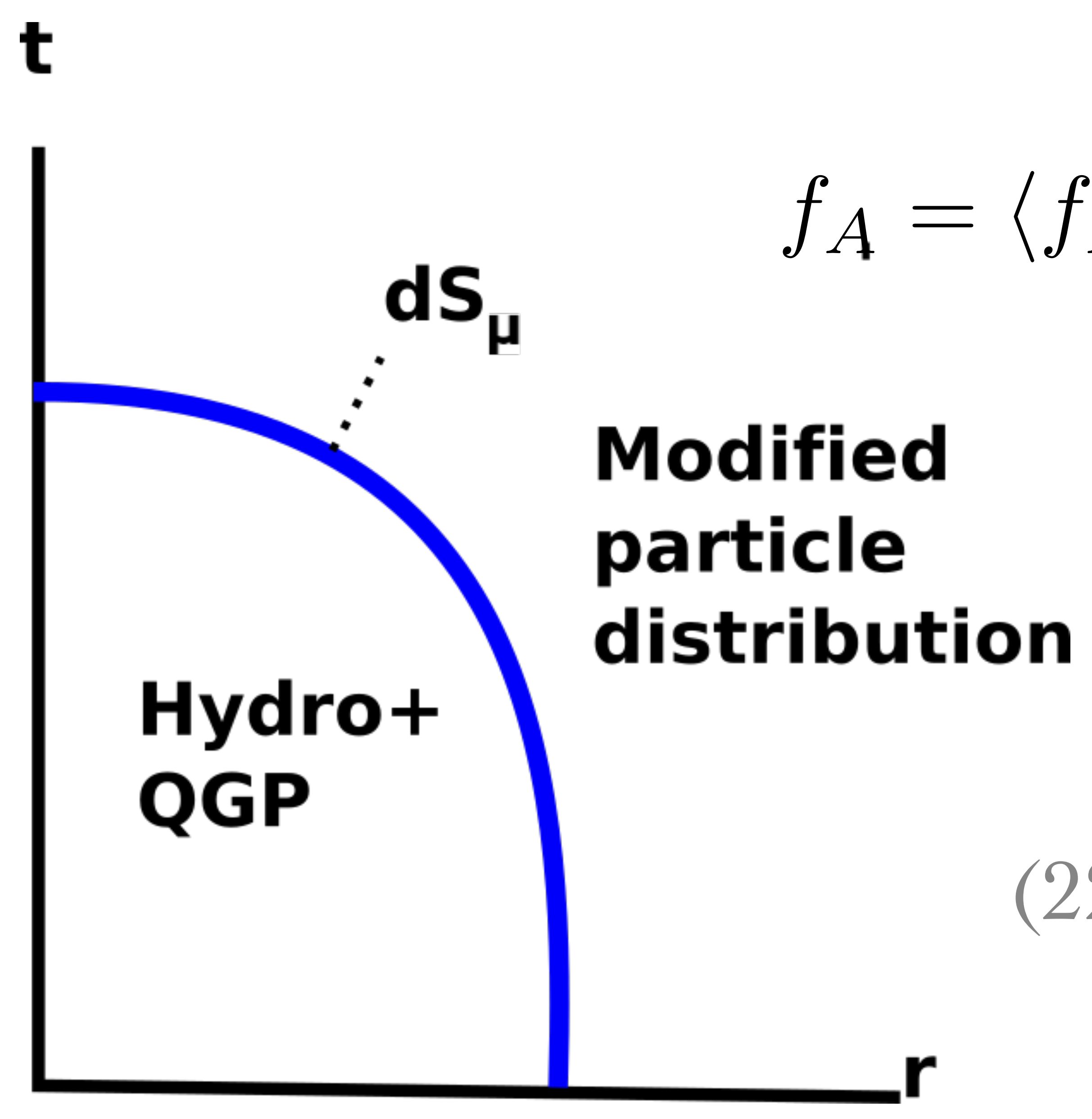
Does not describe fluctuations

Cooper and Frye, *74*

Critical fluctuations in hadron resonance gas

* We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with a critical sigma field

$$\delta m_A \approx g_A \sigma$$



$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

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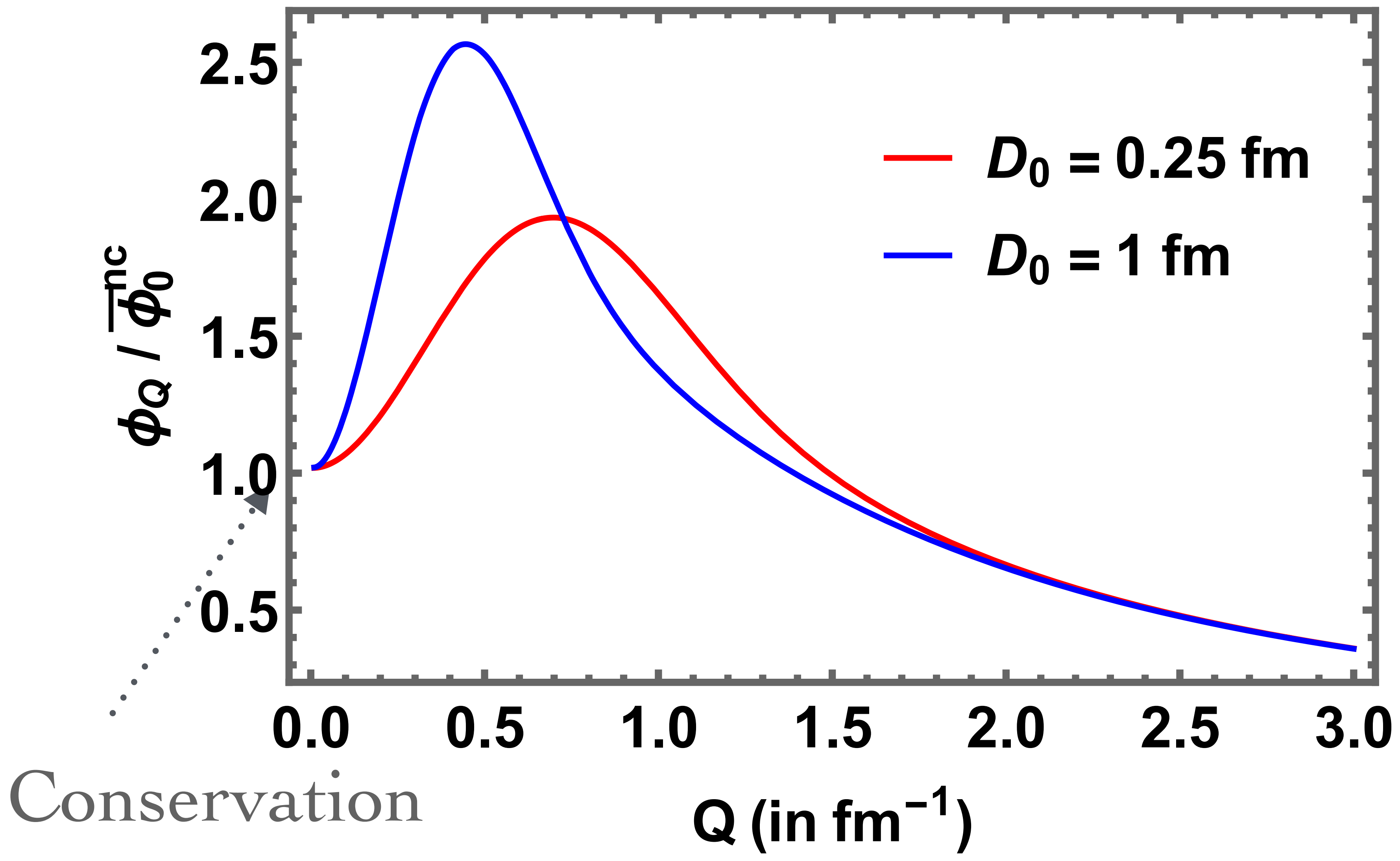
$$\langle \delta N_A^2 \rangle = \langle N_A \rangle + \langle \delta N_A^2 \rangle_\sigma$$

We match the two point function of σ to the two point function of the Hydro+ mode, $\hat{s} \equiv s/n$

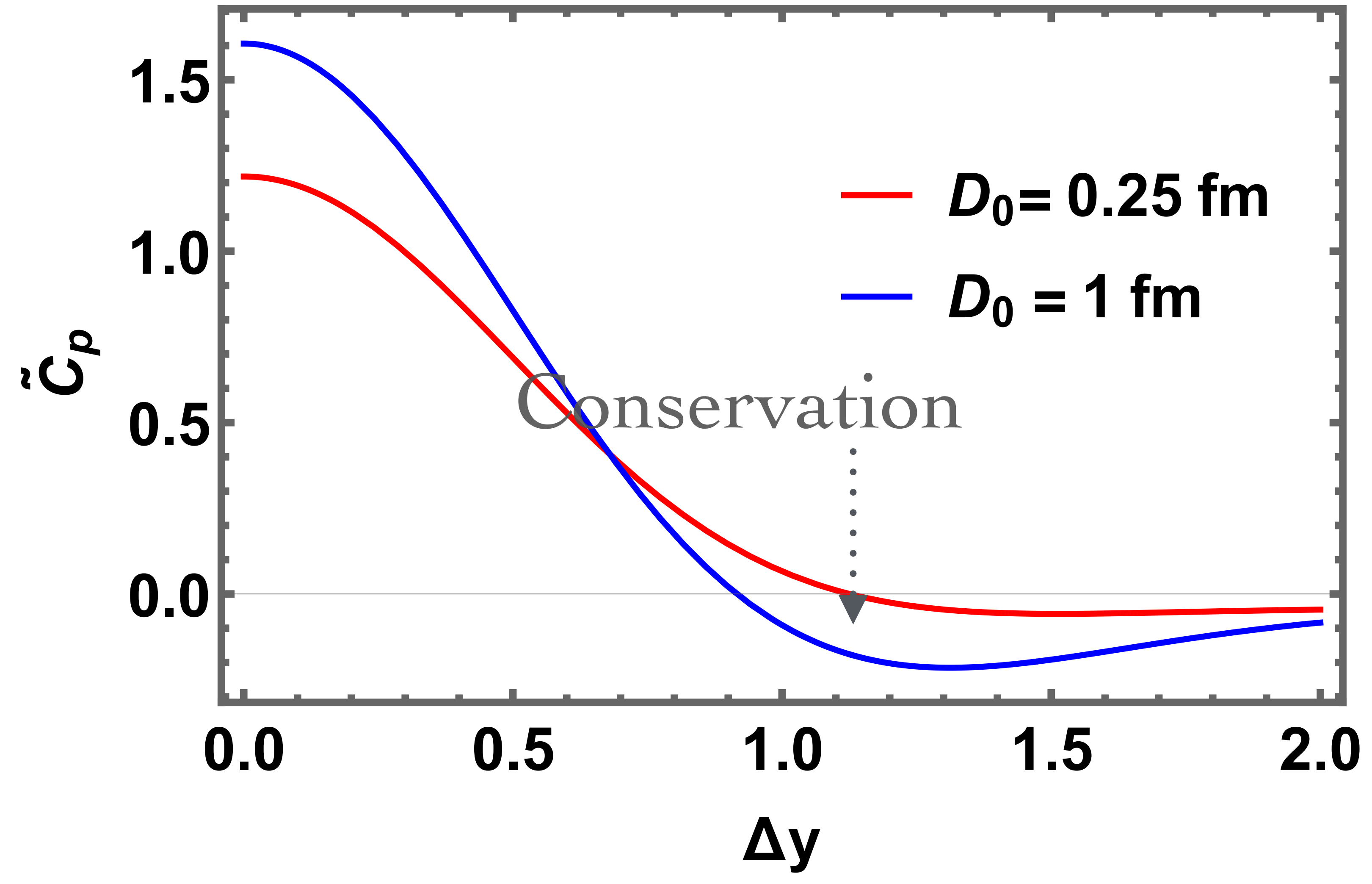
$$\left\langle \sigma(x_+)\sigma(x_-) \right\rangle \approx Z^{-1} \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

$$\left\langle \delta N_A^2 \right\rangle_\sigma = g_A^2 Z^{-1} \int dS_\mu J_A^\mu(x_+) \int dS_\nu J_A^\nu(x_-)\, \tilde{\phi}(x, \Delta\tilde{x})$$

Effect of conservation laws on particle (anti)correlations at freeze-out



$$C_A(y_+, y_-) = \left\langle \delta \frac{dN_A}{dy_+} \delta \frac{dN_A}{dy_-} \right\rangle$$



See next talk by J. Hammelmann for effects of conservation on cumulants during hadronic transport

Enhancement at low Δy, anti-correlations at large Δy

The low Q modes contribute the most to rapidity correlations

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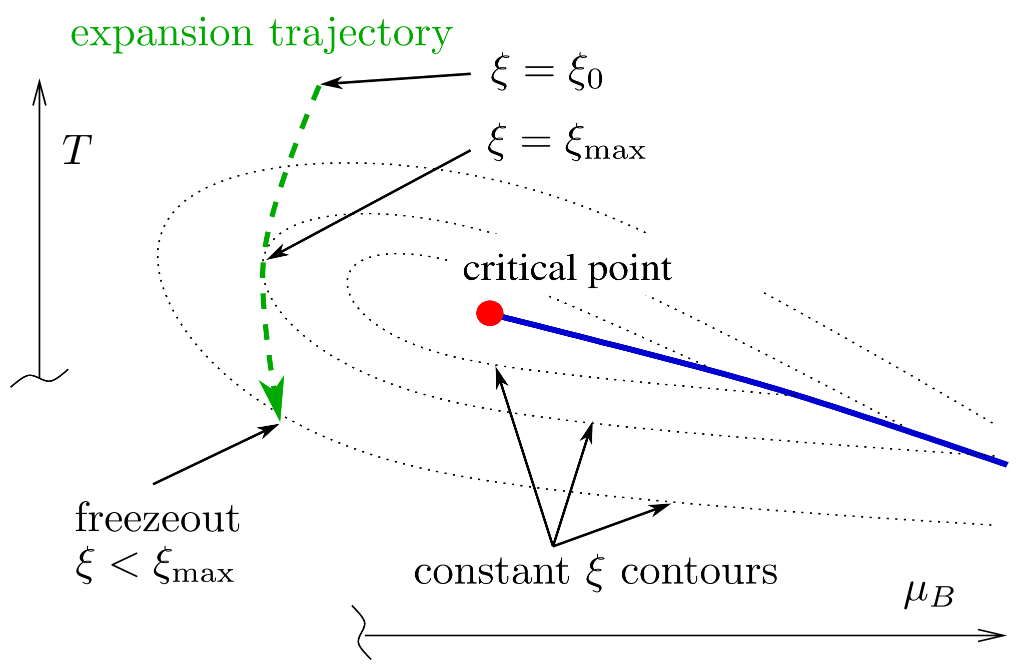
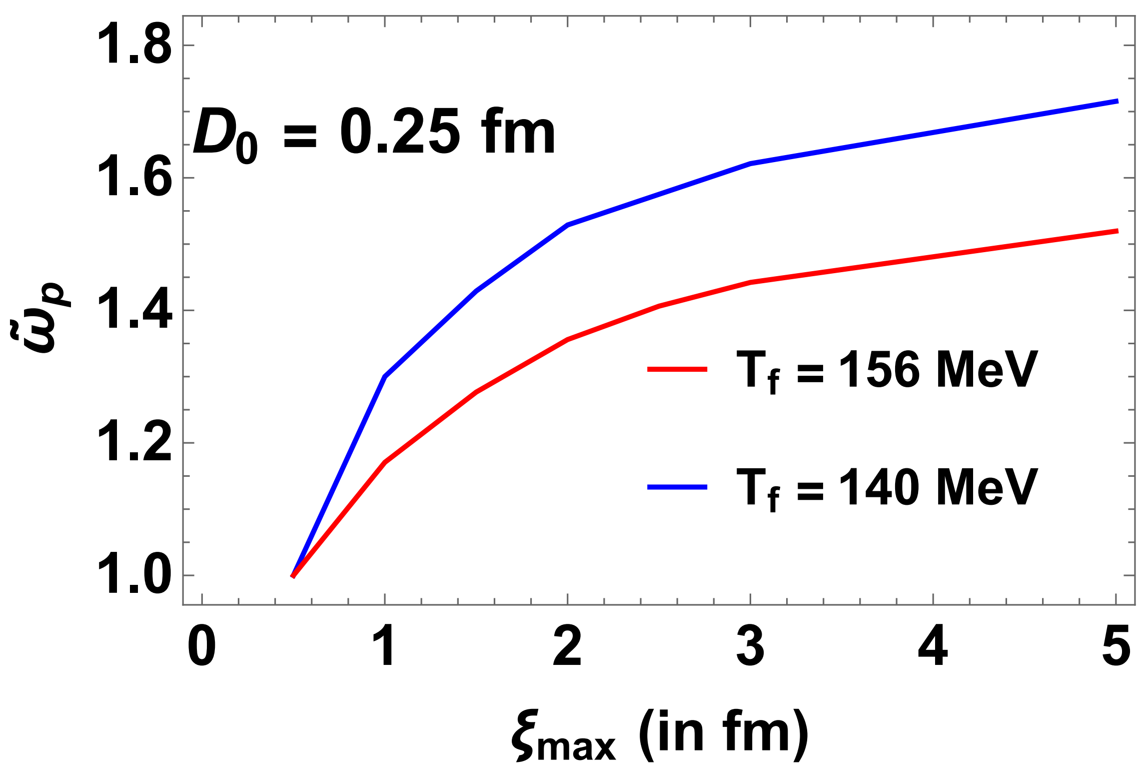
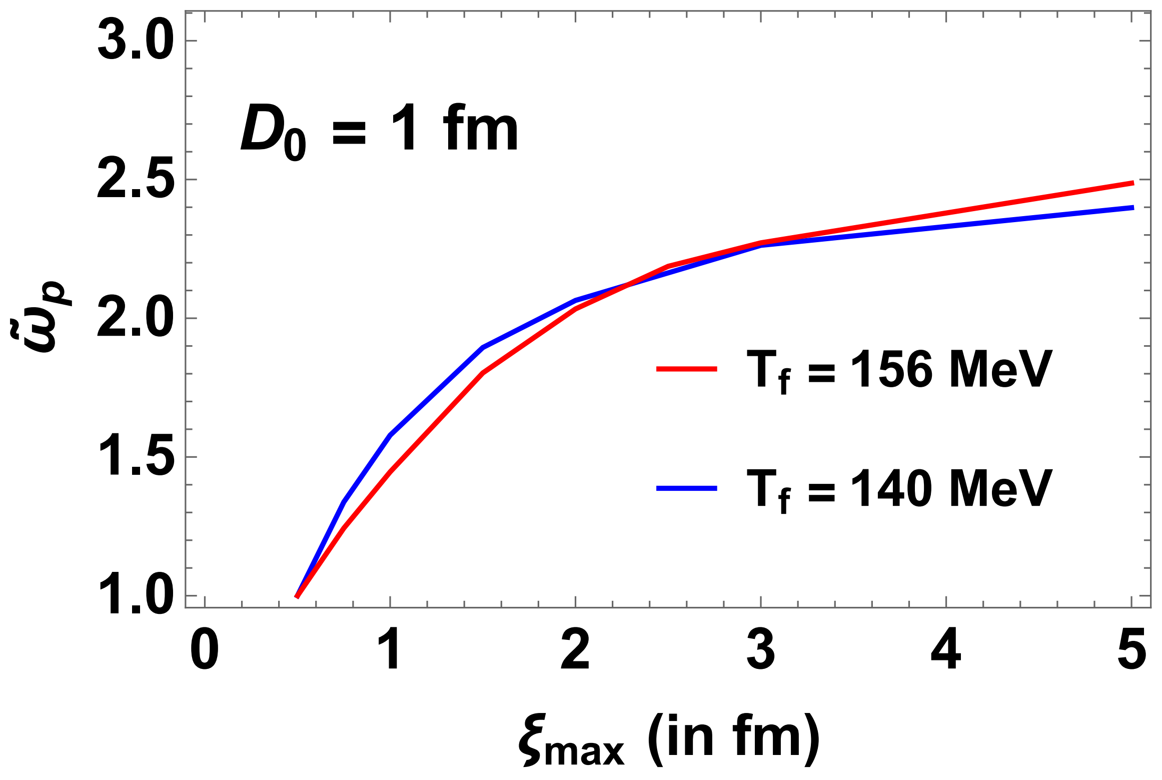
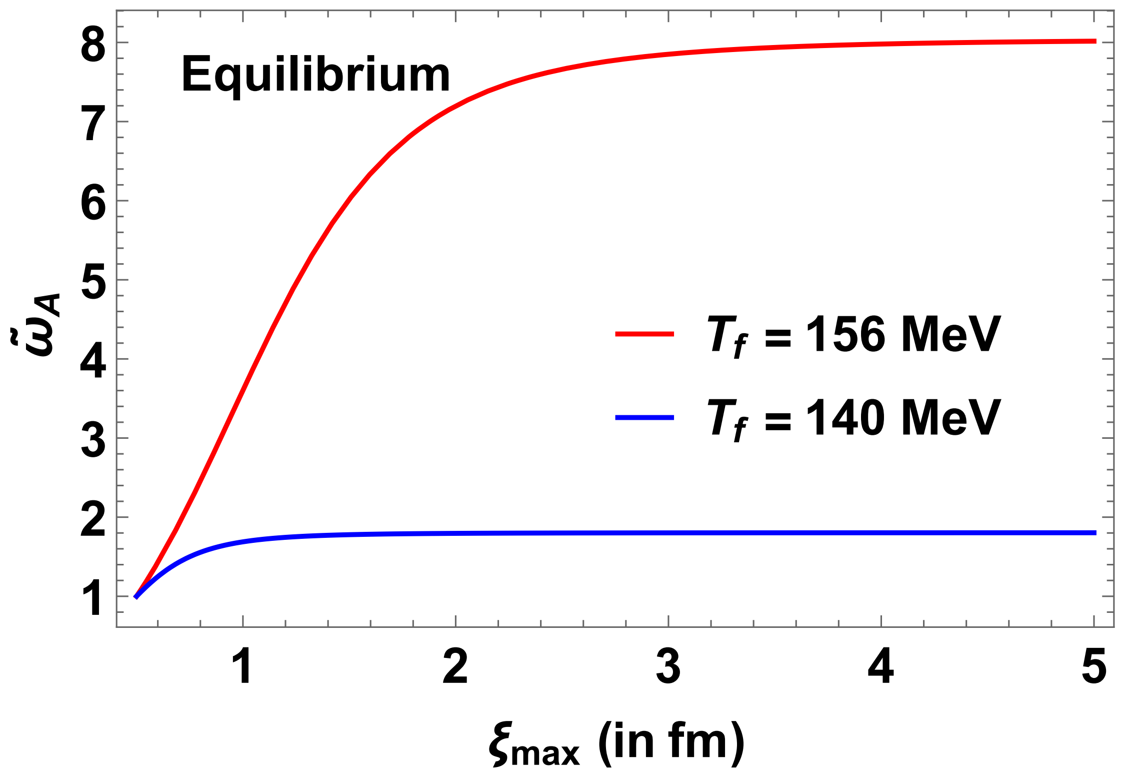
Critical contribution to variance of proton multiplicities

$$\omega_p \equiv \frac{\left\langle \delta N_p^2 \right\rangle_\sigma}{\left\langle N_p \right\rangle}$$









Max. ξ is achieved at T= 160 MeV

$$\tilde{\omega}_p \equiv \frac{\omega_p}{\omega_p^{\text{nc}}}$$

- The fluctuations are reduced relative to equilibrium value (due to conservation laws)
- The fluctuations are found to increase with D_0 (faster diffusion)
- Compared to the equilibrium scenario, the fluctuations are less sensitive to freeze-out temperature

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Outlook

* The freeze-out procedure developed here can already be integrated into the full numerical simulation of heavy ion collisions relevant for BES program

* Freeze-out of higher point fluctuations needs to be implemented and analyzed

* There procedure can be improved by adding less singular contributions and modes which are not critical

* Although there are estimates available, these estimates must be verified by determining the coupling constants gAs from the EoS

Thank you!