Light and heavy flavor probes of dense QCD matter at sPHENIX

Ivan Vitev
In place of introduction

One crisp New York morning in 1999 I walked to the Apple Tree Supermarket ...

The world did not end in 2000 when RHIC started (around 1995 strangelets were all the rage)

However,
- The heavy ion program (at high energies) in the US is coming to an close
- It is important to maximize the scientific output of RHIC before the EIC phase transition
- sPHENIX and STAR will not exist in isolation, connection to LHC, transition to EIC

- QGP formation in small and large systems / hadron production
- Light flavor jets and jet correlations - photon tagged jets
- Heavy flavor jets, di jets, and jet substructure
- Conclusions
I. Hadron production and Large/Small systems

"I think you should be more explicit here in step two."
Production of light and heavy hadrons

QCD factorization approach is well established. Still large uncertainties remain related to non-perturbative physics / hadronization (fragmentation functions). This is especially true for heavy flavor.

\[
\frac{d\sigma^{H_1 H_2 \to h X}}{dp_T d\eta} = \frac{2p_T}{S} \sum_{a b c} f_a^{H_1} \otimes f_b^{H_2} \otimes d\hat{\sigma}_{ab} \otimes D_c^h
\]

Specific applications include LO, NLO, + resummation and parton showers. Also PYTHIA baseline (LO+PS).

In the presence of nuclear matter – initial-state (CNM) and final-state (QGP effects)

Calculate those effects dynamically (vs parameterize them from data)
Cold nuclear matter effects

Process dependent corrections to QCD factorization

\[
\frac{d\sigma}{d\Omega d\mu} = \frac{4}{s} \sum_{ij} \int d\eta_{c.m.} \int d^2k_i f_i/A(x_i + \Delta x_i, k_i; \mu_F) \int d^2k_j f_j/B(x_j + \Delta x_j, k_j; \mu_F)
\times \frac{d\sigma_{ij \rightarrow k}}{d\cos \theta} (x_ix_js, \cos \theta_{c.m.}; \mu_R).
\]

- Cronin effect (and of course isospin)

\[
g(k) = \exp(-k^2_T/(\langle k^2_T \rangle_{pp})/\pi \langle k^2_T \rangle_{pp},
\langle k^2_T \rangle_{pp} \approx \langle k^2_T \rangle_{pp} + L_A \frac{\mu^2}{\lambda} \ln(1 + c p_T^2/\mu^2)
\]

- CNM energy loss

\[
\Delta x/x = \epsilon_\text{fl} \int_{m_N/p^+}^{1} dx \int_{x m_N \leq |k| \leq x p^+} d^2k x \frac{dN_{IS}}{dxd^2k}.
\]

- Coherent power corrections

\[
\Delta x_i/x_i \sim \mu^2 A^{1/3}/(-u) \quad \Delta x_j/x_j \sim \mu^2 B^{1/3}/(-t)
\]

Calculated corrections appear as kinematic modifications

J. Qiu et al. (2005)

M. Gyulassy et al. (2002)

W. Kei et al. (2022)

Parton level results at RHIC compared to nPDF parameterization

I.V. (2007)
**QGP effects**

**Final-state collisional and radiative processes**

- **Collisional energy loss**
  \[
  \frac{dE_{el}}{d\Delta z} = \frac{C_F}{4} \left(1 + \frac{N_f}{6}\right) \alpha_s(ET) g_s^2 T^2 \ln \left( \frac{ET}{m_D^2} \right) \left( \frac{1}{v} \frac{1 - v^2}{2v^2} \ln \frac{1 + v}{1 - v} \right)
  \]

- **In-medium splitting functions / radiative energy loss**
  \[
  \frac{dN_{qq}^{med}}{dxdk^2} \equiv P_{qq}(x,k^2) \int_0^\infty d\Delta z \int d^2q \frac{dR_g(\Delta z)}{d^2q} \\
  \left\{ \left[ \frac{B}{B^2} \cdot \frac{1}{N_c^2} \frac{B}{C^2} \cdot \frac{A}{A^2} - \frac{B}{B^2} \right] [1 - \cos(\omega_1 \Delta z)] \\
  + \frac{C}{C^2} \cdot \left[ \frac{2C}{C} - \frac{A}{A^2} - \frac{B}{B^2} \right] [1 - \cos(\omega_2 \Delta z)] + \frac{B}{B^2} \cdot \frac{C}{C^2} [1 - \cos(\omega_3 \Delta z)] \\
  - \frac{A}{A^2} \cdot \left( \frac{A}{A^2} - \frac{D}{D^2} \right) [1 - \cos(\omega_4 \Delta z)] - \frac{A}{A^2} \cdot \frac{D}{D^2} [1 - \cos(\omega_5 \Delta z)] \right\}
  \]

  Also evaluated branching for heavy flavor and the energy loss limit

**System size dependence (expanding QGP)**

\[
\frac{\Delta E_{el}}{E} \propto \int_{\tau_0}^{\tau_0 + L} \mu^2 d\Delta z \propto L^{1/3} \quad \frac{\Delta E_{rad}}{E} \propto \int_{\tau_0}^{\tau_0 + L} \frac{\mu^2}{\lambda_g} \Delta z d\Delta z \propto L
\]

Much weaker path length dependence of collisional vs radiative E-loss. Implies increased importance in small systems.

**Hydro medium and TRENTO initial conditions**

J. Bernhard (2018)

M. Sievert et al. (2019)
Light and heavy flavor fragmentation and evolution

Light – DSS, heavy - Lund-Bowers

\[ D(z) = z^{-1 - b M_{11}^2} (1 - z)^a e^{-b M_{11}^2 / z} \]

M. Bowers (1981)

In-medium evolution

Heavy flavor specific

\[ c_{gh}(r) = F \left( \frac{1 + \sqrt{1 - 4r^2}}{2} \right) - F \left( \frac{1 - \sqrt{1 - 4r^2}}{2} \right) - 2r \sqrt{1 - 4r^2}, \]

\[ F(x) = -x^4 + \frac{4}{3} x^3 - x^2, \]

\[ c_{HH}(r) = \frac{1}{1 + r^2} + \frac{2r^2 + 1}{2(1 + r^2)^2} + \frac{2r^2}{1 + r^2} - 2 \ln \frac{1}{1 + r^2}. \]

Additional medium-induced scaling violations

\[ P'_{ii} \rightarrow P''_{ii} + k^2 \frac{dN^{medl}_{ji}}{d\Omega}, \text{ with } x \rightarrow 1 - x. \]

\[ d_{ji}(Q^2) \rightarrow d_{ji}(Q^2) + d_{ji}^{med}(Q^2). \]
Phenomenological results

Large systems

Radiative processes dominate

Theoretical results agree with existing light hadron and D meson measurements at RHIC and LHC. True for both central and peripheral collisions.

There is tension with the B meson production (or non-prompt J/ψ). May be dissociation?

Small systems

Centrality determination in p/d+A challenging.
No room for quenching effects in p+Pb

W. Ke et al. (2022)
QGP in small systems?

Correlation between multiplicity and number of collisions can be vastly improved in collisions of small nuclei (such as O+O). If there is even a small chance, it should be considered at RHIC.

From jet quenching perspective whether QGP is produced or not can be easily distinguished in small systems (assuming good determination of centrality).
II. Inclusive jet production, correlations and substructure
Jets – the next step in understanding the QCD with nuclei

- One can leverage the differences between the vacuum parton showers and the medium-induced showers to predict jets to experimental signatures of parton interaction in matter.

\[ |J_i(\epsilon_i)| = \frac{1}{\left(1 - [1 - f(R_i, p_{T_i}^{\text{min}})_{q,g}]\epsilon_i\right)} \]

\[ \frac{\sigma^{AA}(R, \omega^{\text{min}})}{d^2 E_T dy} = \int_{\epsilon=0}^{1} d\epsilon \sum_{q,g} P_{q,g}(\epsilon) \frac{1}{(1 - (1 - f_{q,g}) \cdot \epsilon)^2} \frac{\sigma^{NN}(R, \omega^{\text{min}})}{d^2 E_T dy} \]

Y. Chien et al. (2015)

I. V. et al. (2009)

Rather flat inclusive jet suppression and distinct radius dependence
Predicted in 2009, there are still no conclusive measurements for inclusive jets at RHIC. This brings us to photon-tagged jets

- Gamma-jets give cleaner constraints on the E-loss of jets
- Transition from enhancement to suppression or different $p_T$ dependence

\[
\frac{1}{\langle N_{bin} \rangle} \frac{d\sigma^{AA}}{dp_T^\gamma dp_T^{jet}} = \sum_{q,g} \int_0^1 d\epsilon \frac{P_{q,g}(\epsilon)}{1 - [1 - f(R)]\epsilon} \\
\times R_{q,g} \frac{d\sigma^{CNM}}{dp_T^\gamma dp_T^{jet}} \left(\frac{p_T^\gamma}{1 - [1 - f(R)]\epsilon}\right).
\]

Similar physics, different flavor composition and underlying cross section

Indication of different shape and of R dependence

STAR / N. Shahoo, et al. (2022)
While we now have photon/pion tagged jet $I_{AA}$'s, the momentum imbalance at RHIC has not been measured even in preliminary form – an area where sPHENIX can give an impact.

- Define average momentum imbalance and direct constraints in energy loss
  $$\frac{d\sigma}{dz_{J\gamma}} = \int^{p_{T\text{jet}}\text{max}}_{p_{T\text{jet}}\text{min}} dp_{T\gamma} \frac{p_{T\text{jet}}}{z_{J\gamma}^2} \frac{d\sigma[z_{J\gamma},p_{T\gamma}(z_{J\gamma},p_{T\text{jet}})]}{dp_{T\gamma}dp_{T\text{jet}}}$$

- Define average momentum imbalance and direct constraints in energy loss

<table>
<thead>
<tr>
<th>System</th>
<th>$\langle z_{J\gamma}\rangle_{\text{LHC}}$</th>
<th>$\langle z_{J\gamma}\rangle_{\text{RHIC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p+p</td>
<td>0.94</td>
<td>0.90</td>
</tr>
<tr>
<td>A+A, CNM</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>A+A, $g_{\text{med}}=1.8$ ,Rad.+Col</td>
<td>0.84</td>
<td>0.78</td>
</tr>
<tr>
<td>A+A, $g_{\text{med}}=2.0$,Rad.+Col</td>
<td>0.80</td>
<td>0.74</td>
</tr>
<tr>
<td>A+A, $g_{\text{med}}=2.2$,Rad.+Col</td>
<td>0.71</td>
<td>0.70</td>
</tr>
</tbody>
</table>

W. Dai et al. (2012)
Jet shape – most closely related to the jet cross sections

\[ \psi_{\text{tot.}}(r/R) = \frac{1}{\text{Norm}} \int_{\epsilon=0}^{1} d\epsilon \sum_{q,g} P_{q,g}(\epsilon) \left( \frac{1}{(1 - (1 - f_{q,g}) \cdot \epsilon)^3} \right) \]
\[ \times \frac{\sigma_{qg}^{NN}(R, \omega_{\text{min}})}{d^2E_T'dy} \left[ (1 - \epsilon) \psi_{\text{vac.}}(r/R) + f_{q,g} \cdot \epsilon \psi_{\text{med.}}(r/R) \right] \]

We can define an observable that characterizes the mean width of the energy flow

\[ \langle r/R \rangle = \int_0^1 d(r/R)(r/R)\psi(r/R) \]
\[ \Delta \langle r/R \rangle = (\langle r/R \rangle_{\text{tot.}} - \langle r/R \rangle_{\text{vac.}}) / \langle r/R \rangle_{\text{v}} \]


<table>
<thead>
<tr>
<th></th>
<th>Vacuum</th>
<th>Medium</th>
<th>Total</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au+Au</td>
<td>0.271</td>
<td>0.601</td>
<td>0.283</td>
<td>4%</td>
</tr>
<tr>
<td>Cu+Cu</td>
<td>0.271</td>
<td>0.640</td>
<td>0.272</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

Observables that characterize mean intra-jet properties are modified very little. Larger modification can be seen in the periphery of energy and particle flow distributions.
Light jet substructure – jet splitting functions

A. Larkoski et al. (2015)

\[ r_g = \Delta R_{12} \]

\[ p_{T2} \]

\[ p_{T1} \]

\[ z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R_0} \right)^\beta \]

\[ \frac{dN_j^{\text{vac},\text{MLL}}}{dz_g d\theta_g} = \sum_i \left( \frac{dN_{\text{vac}}}{dz_g d\theta_g} \right)_{j \rightarrow i} \exp \left[ - \int_{\theta_g}^{1} d\theta \int_{z_{\text{cut}}}^{1/2} dz \sum_i \left( \frac{dN_{\text{vac}}}{dz d\theta} \right)_{j \rightarrow i} \right] \]

Sudakov Factor

\[ p(\theta_g, z_g)_{j} = \frac{dN_j^{\text{vac},\text{MLL}}}{dz_g d\theta_g} \int_0^1 d\theta \int_{z_{\text{cut}}}^{1/2} dz \frac{dN_j^{\text{vac},\text{MLL}}}{dz d\theta} \]

Groomed, soft dropped momentum sharing distributions - directly proportional to the splitting functions, + resummation for small angles

H. Li et al. (2018)

We find only small modification
III. Heavy flavor jets, dijet mass, and substructure
There are significant differences due to the **heavy quark mass** between massless and massive splitting functions.

Higher orders in opacity have minimal effect on heavy flavor splitting.

Different dead cone effect for different splittings.

\[
\left( \frac{dN_{\text{med}}}{dx \, d^2k} \right)_{Q \to \bar{Q}} = \frac{\alpha_s}{2\pi^2} C_F \int_0^1 \frac{dz}{\lambda_{x_1}(z)} \int d^2q_\perp \frac{1}{x} \frac{d\sigma_\text{med}^2}{d^2q_\perp^2} \left\{ \frac{1 + (1 - x)^2}{x} \right\} \left[ \frac{B_1}{B_1 + v^2} \right. \\
\left. \times \left( \frac{B_1}{B_1 + v^2} - \frac{C_1}{C_1 + v^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{C_1}{C_1 + v^2} \right. \\
\left. \times \frac{B_1}{B_1 + v^2} \right] (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) \right. \\
\left. + \frac{A_1}{A_1 + v^2} \left( \frac{D_1}{D_1 + v^2} - \frac{A_1}{A_1 + v^2} \right) (1 - \cos[\Omega_4\Delta z]) - \frac{A_1}{A_1 + v^2} \cdot \frac{D_1}{D_1 + v^2} (1 - \cos[\Omega_5\Delta z]) \right. \\
\left. + \frac{1}{N_c^2} \left( \frac{B_1}{B_1 + v^2} - \frac{C_1}{C_1 + v^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \ldots \right\} 
\]
**Enhancing the heavy quark mass effect**

- We can look at the di b-jets and their momentum imbalance but it is relatively small.

<table>
<thead>
<tr>
<th>Kinematics</th>
<th>dijet flavor</th>
<th>$\langle z_J \rangle_{PP}$</th>
<th>$\langle z_J \rangle_{AA}$</th>
<th>$\Delta \langle z_J \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMS [25]</td>
<td>b-tagged</td>
<td>$0.661 \pm 0.003$</td>
<td>$0.601 \pm 0.023$</td>
<td>$0.060 \pm 0.025$</td>
</tr>
<tr>
<td>Experiment</td>
<td>inclusive</td>
<td>$0.669 \pm 0.002$</td>
<td>$0.617 \pm 0.027$</td>
<td>$0.052 \pm 0.024$</td>
</tr>
<tr>
<td>LHC theory</td>
<td>b-tagged</td>
<td>$0.685$</td>
<td>$0.626 \pm 0.013$</td>
<td>$0.059 \pm 0.013$</td>
</tr>
<tr>
<td></td>
<td>inclusive</td>
<td>$0.701$</td>
<td>$0.605 \pm 0.022$</td>
<td>$0.096 \pm 0.022$</td>
</tr>
<tr>
<td>sPHENIX</td>
<td>b-tagged</td>
<td>$0.730$</td>
<td>$0.665 \pm 0.012$</td>
<td>$0.065 \pm 0.012$</td>
</tr>
<tr>
<td></td>
<td>inclusive</td>
<td>$0.743$</td>
<td>$0.643 \pm 0.005$</td>
<td>$0.100 \pm 0.005$</td>
</tr>
</tbody>
</table>

Quantitative differences quite small (we react to differences in shape)

- Examine instead the dijet mass, where effects are additive.

$$\frac{d\sigma}{dm_{12}} = \int dp_1T dp_2T \frac{d\sigma}{dp_1T dp_2T} \delta \left( m_{12} - \sqrt{\langle m_1^2 \rangle + \langle m_2^2 \rangle + 2p_1T p_2T \langle \cosh(\Delta \eta) - \cos(\Delta \phi) \rangle} \right)$$

Individual mass modification is negligible, no change.

Z. Kang et al. (2018)
When it comes to dijet mass modification the results are very encouraging – RHIC example. Best seen at masses under 100 GeV.
Also works well at LHC in this mass range and even to a few hundred GeV.
Will be an extremely valuable measurement to make (try it)

Ideal measurement to make at RHIC. Suppression of the inclusive di jet mass distribution by an order of magnitude
Heavy flavor di jet measurements at RHIC

Great way to study the effect of mass on parton energy loss

- Light and heavy flavor di jet mass show different $m_{12}$ dependence. At the LHC for $m_{12} > 100$ GeV we find the same dependence
- Differences between light and heavy di jet mass modification can reach nearly a factor of 10. At the LHC they are very small, if any.

sPHENIX will have excellent reach to measure heavy flavor jets. It will be very important to complement such measurements with heavy flavor jet substructure that will be described next

Z. Kang et al. (2018)
Inverting the mass hierarchy of jet quenching effects

A regime where splitting function-dependent dead cone effect alters the longitudinal structure of the shower

\[
\left( \frac{dN^{\text{vac}}}{dz d^2k_{\perp}} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{2\pi^2} \frac{C_F}{k_{\perp}^2 + z^2 m^2} \times \left( \frac{1 + (1 - z)^2}{z} - \frac{2z(1 - z)m^2}{k_{\perp}^2 + z^2 m^2} \right)
\]

\[
\left( \frac{dN^{\text{vac}}}{dz d^2k_{\perp}} \right)_{g \rightarrow Q\bar{Q}} = \frac{\alpha_s}{2\pi^2} \frac{T_R}{k_{\perp}^2 + m^2} \times \left( z^2 + (1 - z)^2 + \frac{2z(1 - z)m^2}{k_{\perp}^2 + m^2} \right)
\]

- At RHIC jet energies, and at lower jet energies at the LHC there is a unique reversal of the mass hierarchy effects on \( b > c >= u,d \). (Single B,D meson tag)
- Modification of the double B,D meson tag is small. Allows us to get a new handle on mass correction

One example, but expect that jet substructure will be more significantly modified for heavy flavor jets (especially b-jets)

\[\sqrt{s_{NN}}=200 \text{ GeV} \]
\[\text{ant-}k_T \ R=0.4\]
\[10<P_{T,j}<30 \text{ GeV} \]
\[\text{SD: } \beta=0 \quad z_{\text{cut}}=0.1\]

H. Li et al. (2018)
Important progress has been made in the theory of hard probes (QCD, SCET, NRQCD) – precise high order and resumed calculations standard. A+B collisions provide new opportunities to study many-body QCD, and have led to emergence of EFTs in matter. Progress toward medium motion effects, gradient corrections – leading subeikonal effects can be studied at RHIC. Hadron production has been instrumental in the discovery of jet quenching and jet tomography at RHIC. First to benefit from modern QCD/SCET techniques in matter (evolution, NLO). An important question is whether QGP can be produced in small (p-sized) systems, jet quenching does not support that hypothesis at present. This can be tested with small symmetric vs asymmetric systems at LHC and should be explored if at all possible at RHIC. Jet production and substructure are a step forward in jet quenching studies. Require precise theoretical control on parton showers. Predictions are for flat suppression of inclusive jets (significant) and distinct radius dependence, but very small modification of light jet substructure. Photon-tagged jets show different $p_T$-dependent suppression driven by the trigger and exhibit significant momentum imbalance similar to the one seen at LHC. Heavy flavor provides a new mass scale (“dead cone effect”) RHIC led to many of the important developments in HF physics and is ideally suited to study the mass effect. Predictions for heavy flavor jets quenching and di jet imbalance, but more importantly heavy jet substructure modification (momentum sharing distributions) can show different mass hierarchy of nuclear effects at moderate $p_T$. Di-jet mass calculations have shown that this observable can enhance otherwise more subtle jet quenching effects. The exploration of the extreme phases of matter has also fascinated the general public. RHIC should make the most out of the remaining years of running.
The jet charge

\[ Q_{\kappa, \text{jet}} = \frac{1}{(p_T^\text{jet})^\kappa} \sum_{h \text{ in jet}} Q_h (p_T^h)^\kappa \]

\[ \langle Q_{\kappa,q} \rangle = \frac{\tilde{J}_{qq}(E, R, \kappa, \mu)}{J_q(E, R, \mu)} \tilde{D}_q^\kappa(\kappa, \mu) \]

\[ \tilde{J}_{qq}(E, R, \kappa, \mu) = \int_0^1 dz \, z^\kappa J_{qq}(E, R, z, \mu) \]

\[ \tilde{D}_q^\kappa(\kappa, \mu) = \int_0^1 dz \, z^\kappa \sum_{h} Q_h D_h^\kappa(z, \mu) \]

The components of the factorization formula receive in-medium corrections

\[ \langle Q_{qq}^{pp} \rangle (1 + \tilde{J}_{qq}^\text{med} - J_q^\text{med}) \exp \left[ \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{\pi} \tilde{D}_q^\text{med} \right] + O(\alpha_s^2, \chi^2) \]

\[ \tilde{J}_{qq}^\text{med} - J_q^\text{med} = \frac{\alpha_s(\mu)}{2\pi^2} \int_0^1 dx \, (x^\kappa - 1) \int_0^{2E_x(1-x)\tan R/2} \frac{d^2k_{\perp}}{k_{\perp}^2} P_{\text{med, real}}(x, k_{\perp}) \]

- Medium-induced scaling violation of the individual flavor and average jet charge
- The CMS collaboration has inverted the problem to determine quark/gluon jet fraction and found no significant difference between pp and AA

A. Sirunyan et al. (2020)

R. Field et al. (1978)

SCET approach

D. Krohn et al. (2012)

H. Li et al. (2019)
There have been evolving measurements at RHIC – from ones suggesting equal D and B meson suppression to ones inferring measurable differences.

Radiative energy loss is not dominant below 10 GeV for heavy quarks/heavy mesons. Especially when bottom quarks are included.

- S. Wicks et al. (2007)
- V. Greco et al. (2008)
- A. Adil et al. (2006)
Taking a closer look at the dijet mass

- Approximating the dijet cross section with individual jet \( p_T \), rapidity, mass and angular distributions (which we simulate from PYHIA )
- We have checked that any difference are \(< 10\%\), also cancel in \( R_{AA} \) ratios

\[
\frac{d\sigma}{dm_{12}} = \int dp_{1T} dp_{2T} \frac{d\sigma}{dp_{1T} dp_{2T}} \delta \left( m_{12} - \sqrt{\langle m_1^2 \rangle + \langle m_2^2 \rangle + 2p_{1T} p_{2T} (\cosh(\Delta \eta) - \cos(\Delta \phi))} \right)
\]

2-D nuclear modification factor needed

2-D distributions for inclusive dijets and Di-bjets

Differences very small
Production of hadrons and jets can be understood from the broader and softer splitting functions.

Holds to higher orders in opacity.

Most importantly – additional medium-induced contribution to factorization formulas (final-state) – Additional scaling violation due to the medium-induced shower. Additional component to jet functions.
At the level of the Lagrangian

\[ \mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A_{G/C}^{\mu, a}) + \mathcal{L}_{g-G/C}(A_{G/C}^{\mu, b}, A_{G/C}^{\mu, a}) + \psi \leftrightarrow \chi \]

Possible scaling for the virtual gluons interacting with the heavy quarks

\[ q_G \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2) \sim (\lambda^2, \lambda^2, \lambda^1) \]

1. \[ q_G \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2) \sim (\lambda^2, \lambda^2, \lambda^1) \]

- Energy component must always be suppressed
- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

Depends on the type of the source of scattering in the medium

Leading medium corrections

\[ \mathcal{L}_{Q-G/C}^{(0)}(\psi, A_{G/C}^{\mu, a}) = \sum_{p,q_T} \hat{\psi}_p^{\dagger}(-gA_{G/C}^{0})\psi_p \text{ (collinear/static/soft).} \]

Sub-leading medium corrections

\[ \mathcal{L}_{Q-G/C}^{(1)}(\psi, A_{G/C}^{\mu, a}) = g \sum_{p,q_T} \hat{\psi}_p^{\dagger}\left(\frac{2A_{G}^{n}(\mathbf{n} \cdot \mathbf{P}) - i(\mathbf{P} \times \mathbf{n})A_{G}^{n}}{2m}\right)\psi_p \text{ (collinear)} \]

\[ \mathcal{L}_{Q-G/C}^{(1)}(\psi, A_{G/C}^{\mu, a}) = 0 \text{ (static)} \]

\[ \mathcal{L}_{Q-G/C}^{(1)}(\psi, A_{G/C}^{\mu, a}) = g \sum_{p,q_T} \hat{\psi}_p^{\dagger}\left(\frac{2A_{G} \cdot \mathbf{P} + [\mathbf{P} \cdot \mathbf{A}_G] - i[\mathbf{P} \times \mathbf{A}_G] \cdot \sigma}{2m}\right)\psi_p \text{ (soft)} \]

Y. Makris et al. (2019)
QCD evolution in the soft gluon energy loss limit and beyond

Advances in understanding in-medium parton showers. Beyond energy loss

If a connection is to be found between the energy loss and the evolution approach, it is in the soft gluon limit

Analytic solution to DGLAP evolution

\[
D_{h/c}^{\text{med.}}(z, Q) = D_{h/c}(z, Q)e^{-[n(z)-1] \langle \frac{\Delta E}{E} \rangle_z - \langle N g \rangle_z}.
\]
Effect of medium motion and inhomogeneities

- In the QGP - transverse and longitudinal expansion, rotation at non-zero impact parameter, fluctuations
- Cold nuclear matter – orbital motion of nucleons, breakup of the nucleus, color charge fluctuations

Several selected results

Scattering

\[ \frac{E}{d^2k_{\perp} \, dx \, d^2p_{\perp}} \frac{dN^{(1)}}{dE} = \frac{\alpha_s \, N_c}{\pi^2 \, x} \left( \frac{E}{d^2p_{\perp}} \frac{dN^{(0)}}{dE} \right) \int_0^L dz \, \rho \int d^2q_{\perp} \bar{\sigma}(q_{\perp}^2) \times \left\{ \frac{2k_{\perp} \cdot q_{\perp}}{k_{\perp}^2(q_{\perp}^2)^2} \left( 1 - \cos \left( \frac{(k - q_{\perp})^2}{2x \, E \, z} \right) \right) + \frac{q_{\perp}^2}{k_{\perp}^2(q_{\perp}^2 + \mu^2)} \frac{u_{\perp} \cdot k_{\perp}}{2(1 - u_z) x E} \right\} \]

Radiation

\[ \left\langle \frac{k_{\perp}}{k_{\perp}^2} \right\rangle = \frac{N_c \, L}{C_F \, \lambda \, 8(1 - u_z) x E} \]

Should appear on the ArXiv tonight, I think

A. Sadofyev et al. (2021)
A useful modern way (though not unique) to calculate jet cross sections

Factorization formula

\[
\frac{d\sigma_{pp \to \text{jetX}}}{dpTd\eta} = \frac{2p_T}{s} \sum_{a,b,c} \frac{1}{x_a} f_a(x_a, \mu) \int_{x_b}^{1} \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c}^{1} \frac{dz_c}{z_c^2} \frac{d\sigma_{ab}^c(s, p_T, \hat{\eta}, \mu)}{dvdz} J_c(z_c, \omega_j, \mu)
\]

\[
\mu_j = \omega_j \tan \frac{R}{2} = (2p_T \cosh \eta) \tan \left( \frac{R}{2 \cosh \eta} \right) \approx p_T R
\]

In-medium jet functions

\[
J_{q,med,1}(z, \omega R, \mu) = \int_{z(1-z)\mu \tan(R/2)}^{\mu} dq_\perp P_{qq}(z, q_\perp)
\]

\[
+ \int_{z(1-z)\mu \tan(R/2)}^{\mu} dq_\perp P_{gg}(z, q_\perp).
\]

- Stable in numerical implementation
- Implemented at fixed order - NLO

Cross section contribution

\[
d\sigma_{\text{PbPb}}^{\text{jet,med}} = \sum_{i=q,\bar{q},g} \sigma_i^{(0)} \otimes J_i^{\text{med}}
\]

Z. Kang et al. (2016)