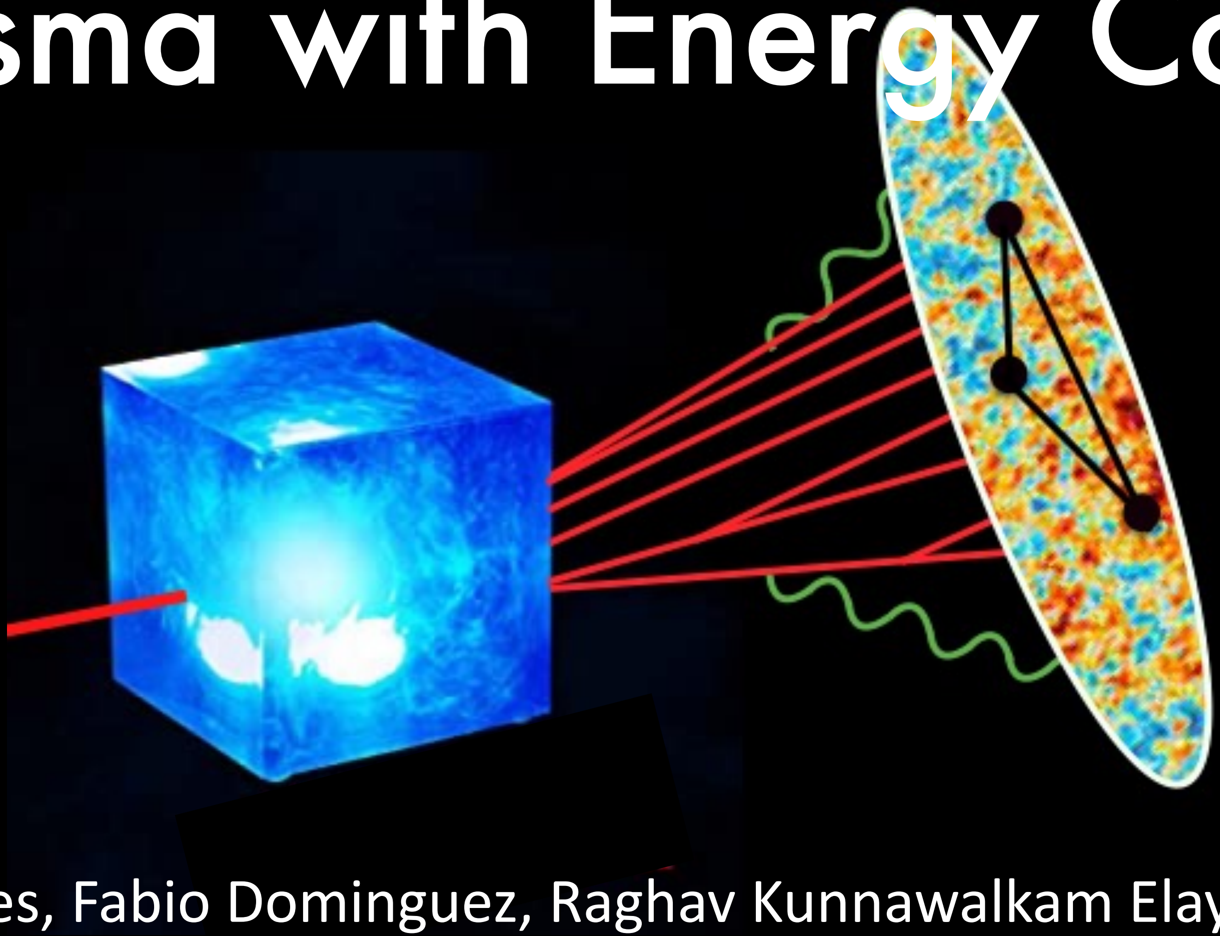
Resolving the Scales of the Quark-Gluon Plasma with Energy Correlators



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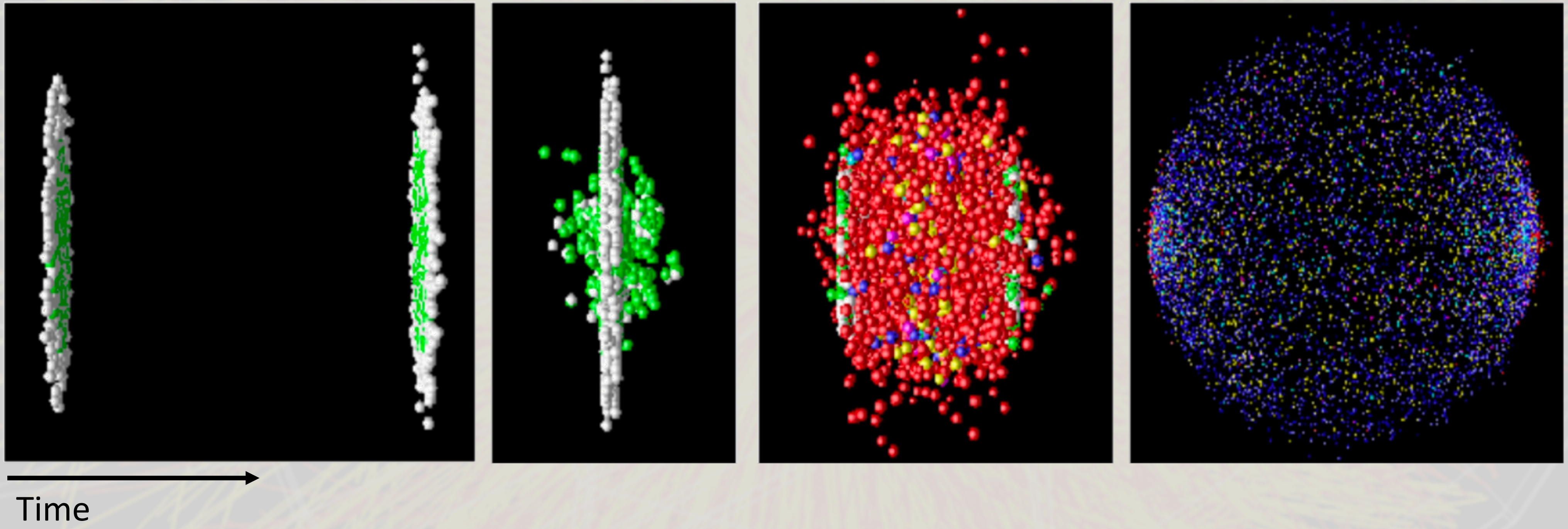
21/07/2022

Outline

1. Introduction
2. Correlation functions of $\mathcal{E}(\vec{n})$
3. The observable analytically
4. The observable numerically

Part 1: Introduction



We want to study the QGP in HIC.
– 20 years of HIC at RHIC, 10 years of HIC at the LHC, sPHENIX coming soon.

We need to study the QGP with sensitive QCD probes with good theoretical control…

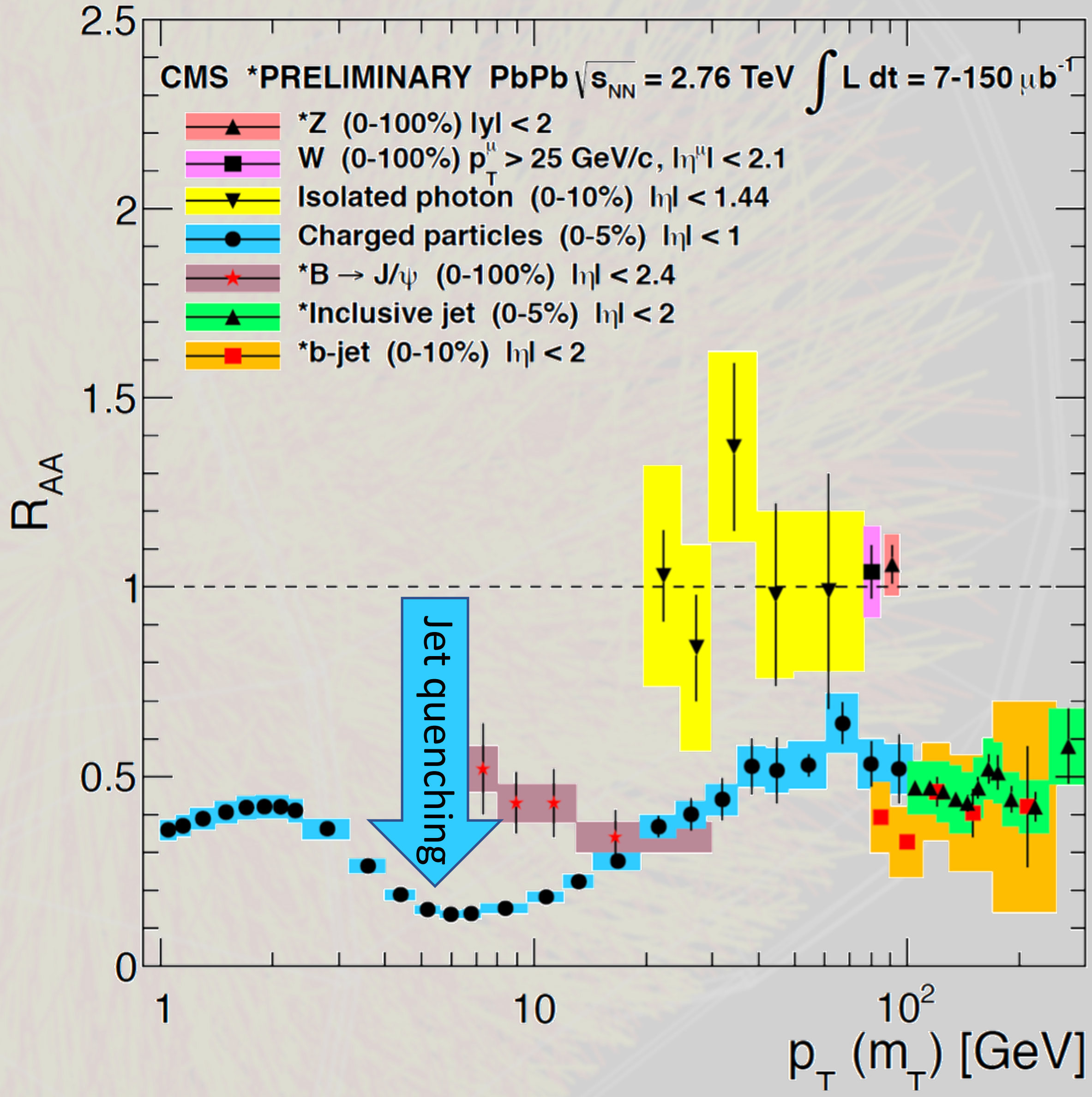
Introduction

Prototypical observable:

$$R_{AA} = \frac{dN_{AA}/d^2p_T\,dy}{\langle N_{coll}\rangle\, dN_{pp}/d^2p_T\,dy}$$

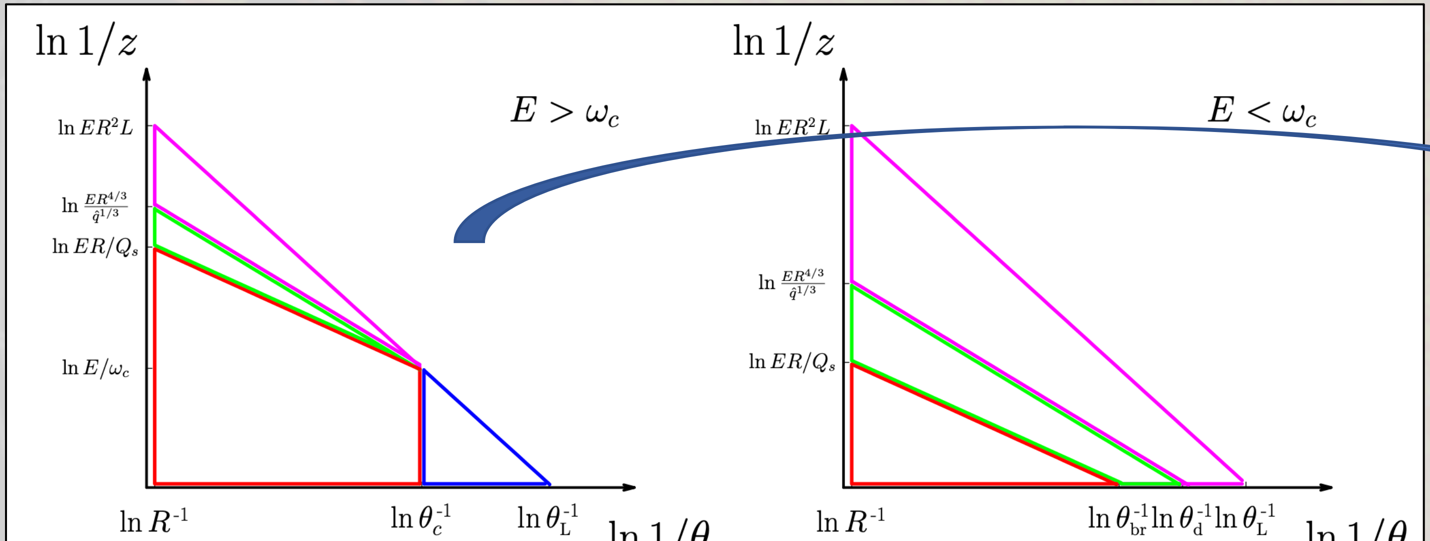
$R_{AA} \neq 1$ for coloured probes.

Principle mechanism is energy loss due to medium induced radiation.



[A. Florent - Hard Probes 2013]

Introduction



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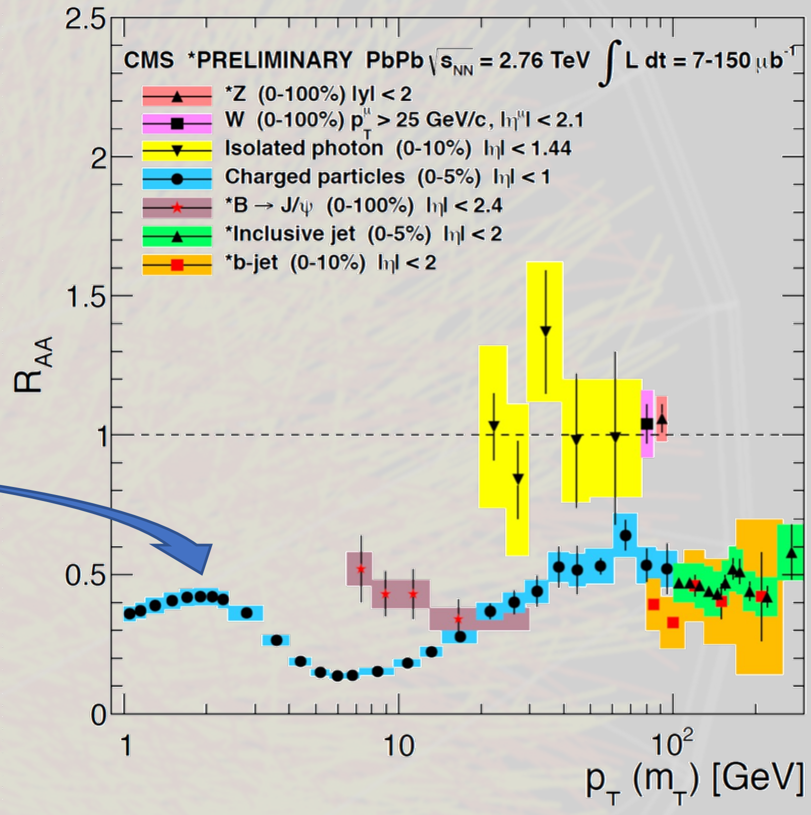
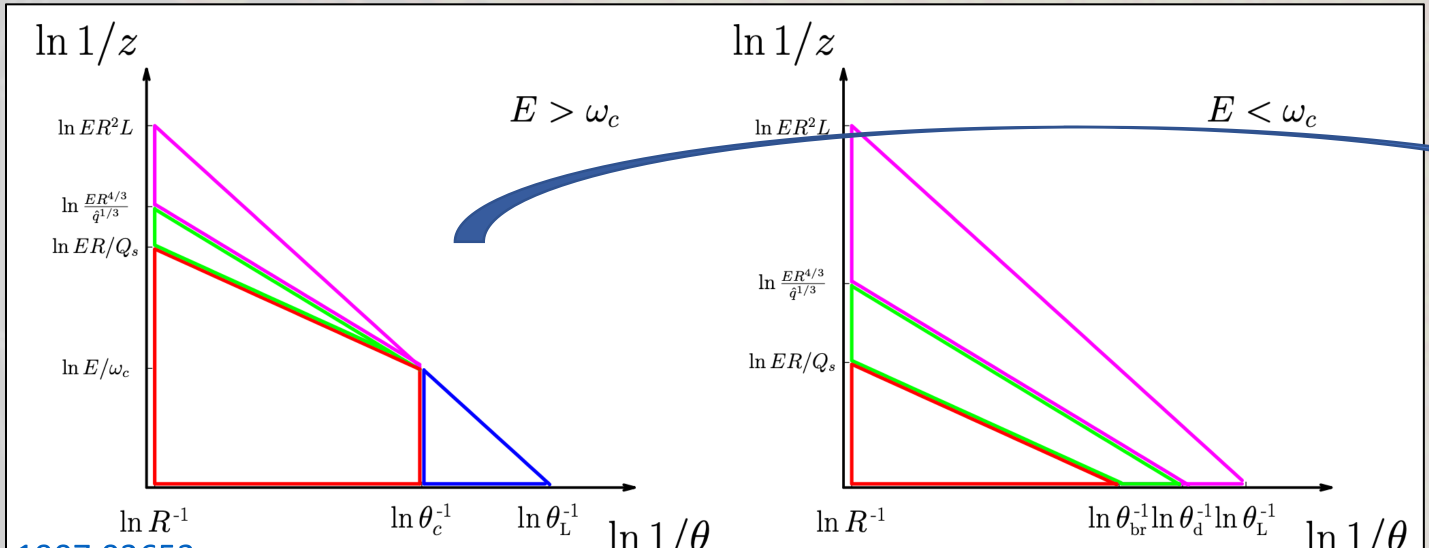


[A. Florent - Hard Probes 2013]

Problem:

Jet quenching is a multi-scale process. It is difficult to unambiguously resolve the scales/properties of the QGP involved within current approaches.

Introduction



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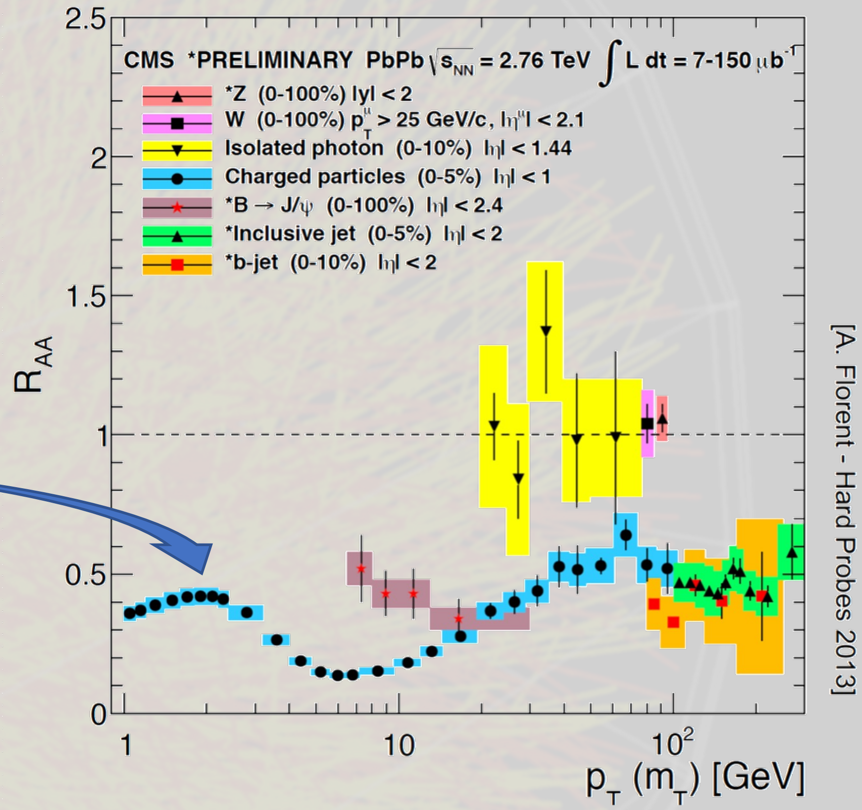
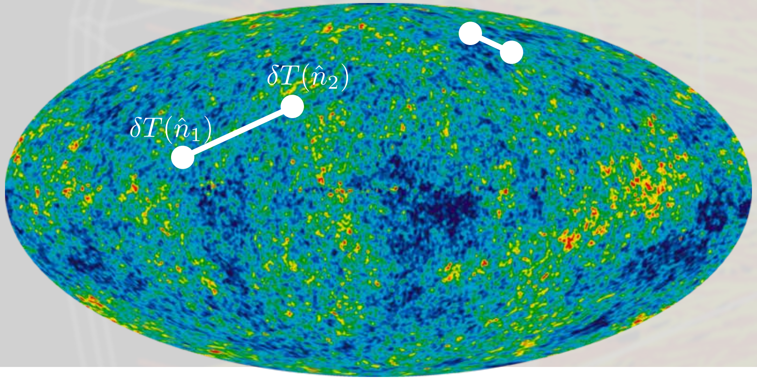


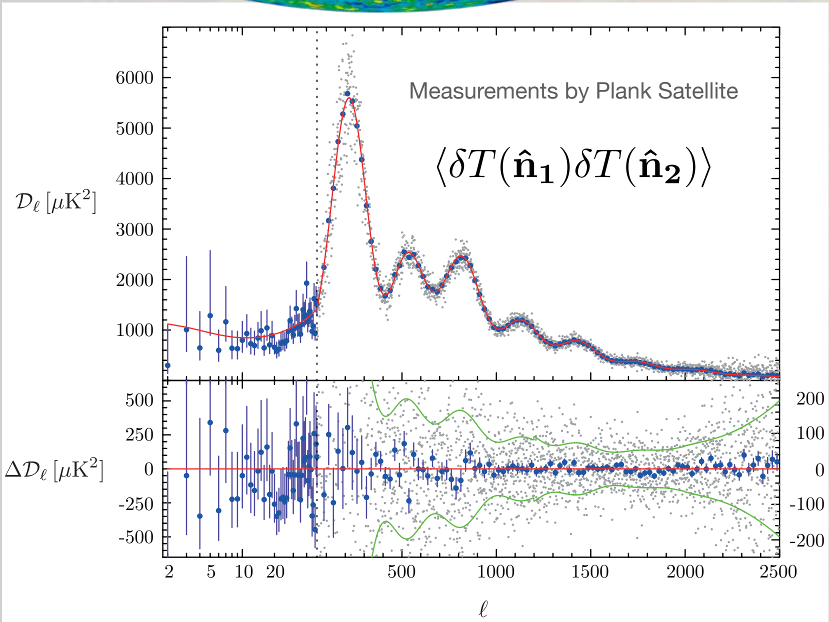
There has been a lot of work introducing observables.

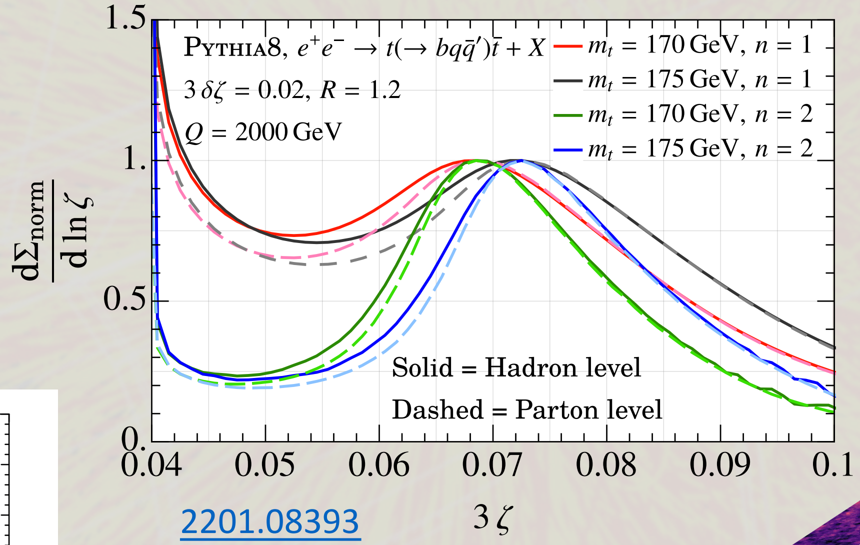
1512.08107, 1710.03237, 1812.05111, 2010.00028, and more

We would like to present a new approach to add to this body of work.

Part 2: Correlation functions of $\mathcal{E}(\vec{n})$







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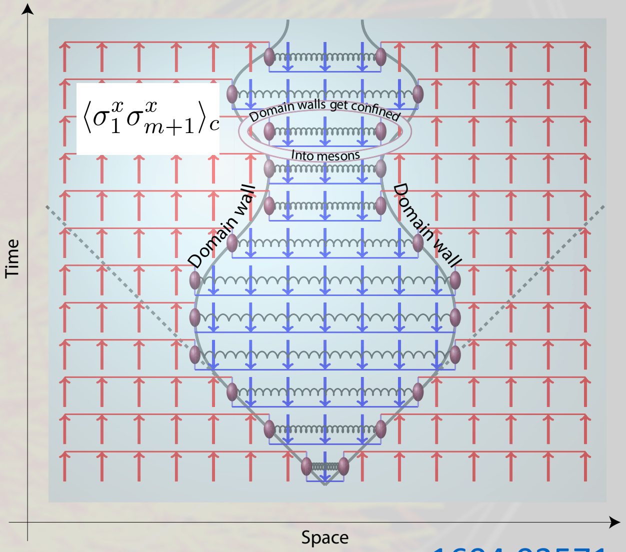

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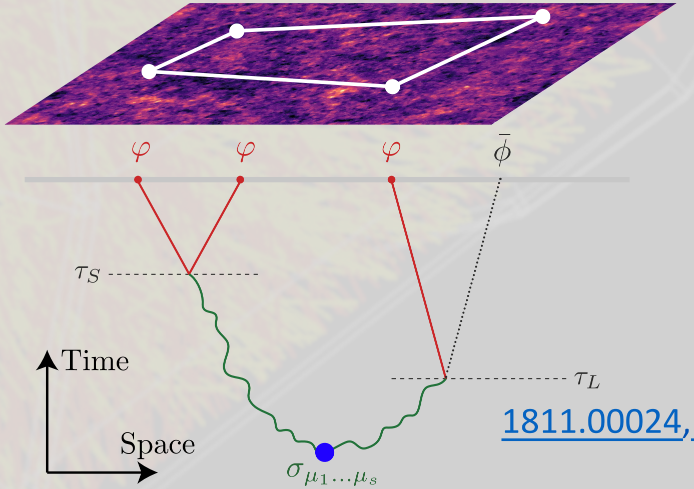

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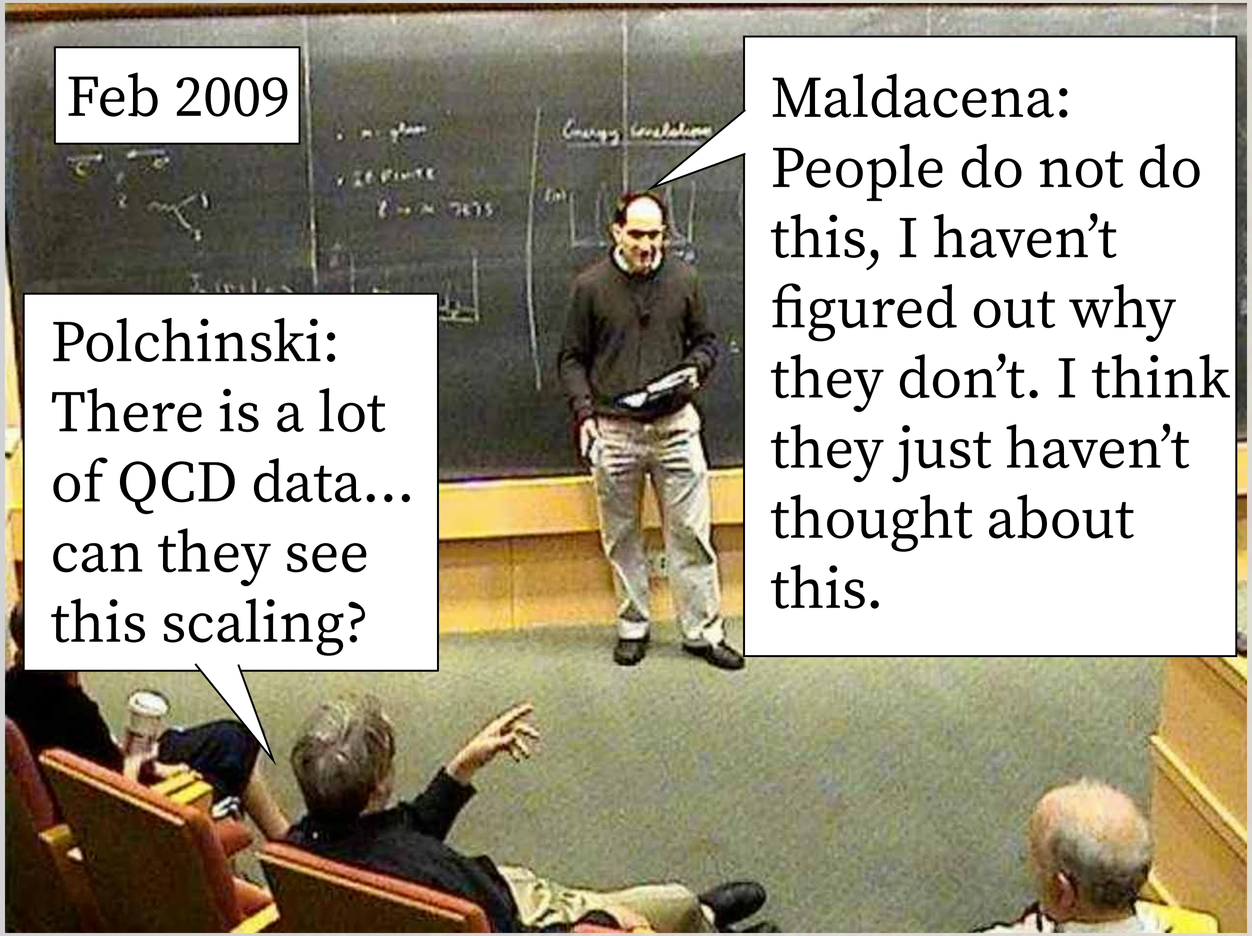
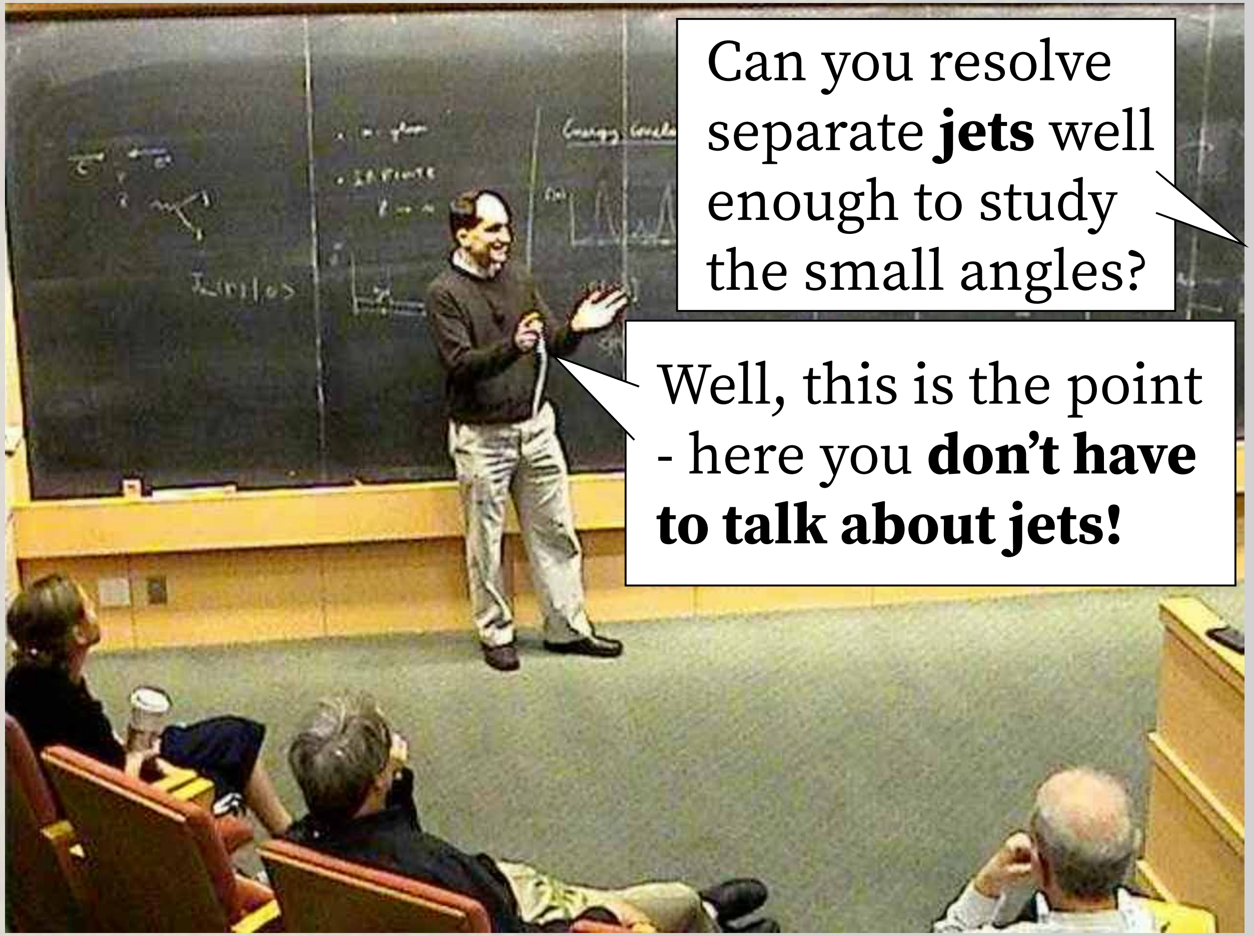
Correlation functions of $\mathcal{E}(\vec{n})$





Correlation functions of $\mathcal{E}(\vec{n})$

Recap

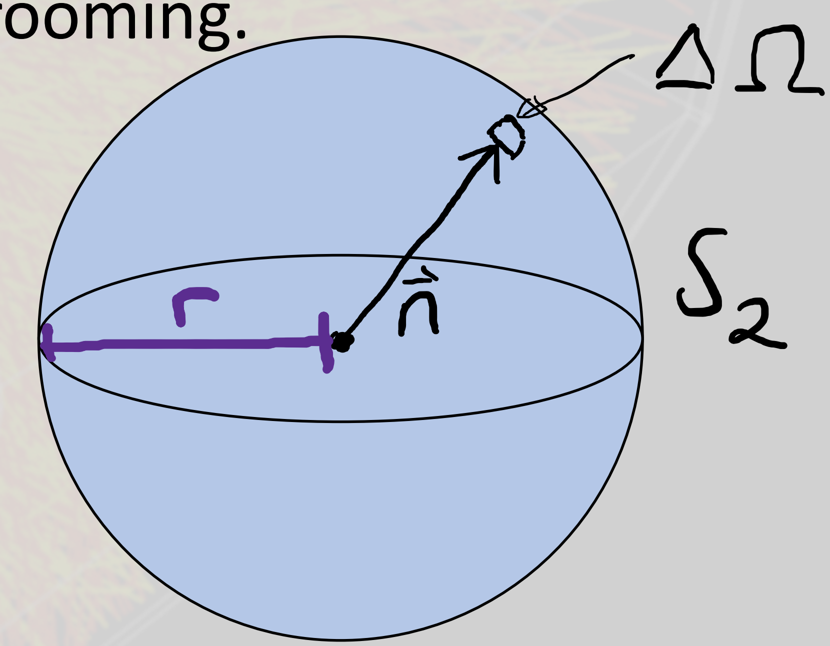
- Correlation functions in statistics:
  - $\mathrm{Corr}_2(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$ (also just the covariance)
  - $\mathrm{Corr}_3(X, Y, Z) = \langle XYZ \rangle - \langle X \rangle \big(\langle Y \rangle \langle Z \rangle - \mathrm{Corr}_2(Y, Z) \big)$
  - …
- In physics we usually refer to $\langle X_1 \ldots X_n \rangle$ as an n point correlator. This is just conventional and has origins in that often $\langle X_i \rangle = 0$.
- QFT correlators (propogators) relate back to these statistical correlators through the path integral and statistical mechanics…

Correlation functions of $\mathcal{E}(\vec{n})$

- Generally one can define correlators of any quantum charge or conserved quantity.
- For QCD, correlators of energy flux are usually of most interest – these naturally remove soft physics without grooming.

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) \simeq \int_0^\infty dt\ E_{\text{flux through } \Delta\Omega}(t)$$



0803.1467

Correlation functions of $\mathcal{E}(\vec{n})$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

So what does $\hat{T}_{0i}$ look like?

Most simply with confined particle states:

$n^i \langle \hat{T}_{0i} \rangle(t) \propto \sum_i E_i$ where E_i is the energy of a particle i which passes through the $\Delta\Omega$ normal to n^i at a time t.

In short, $\langle \hat{\mathcal{E}}(\vec{n}) \rangle$ is just a QFT definition of a calorimeter.

Correlation functions of $\mathcal{E}(\vec{n})$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\, r^2 n^i T_{0i}(t, r\vec{n})$$

In fact, one can show that all collider observables can be expressed as a weighted sum over energy correlators:

$$\langle O \rangle = \sum_i C_i(O) \int \mathrm{d}\, \vec{n}_{1,\ldots,i} \langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_i) \rangle.$$

2004.11381, 2205.06818

Perhaps not surprising when one thinks of a $\mathcal{E}(\vec{n})$ as providing the idealised output of a calorimeter.

Also intuitively, higher point correlators are more differential and so provide more information on the process at hand.

Correlation functions of $\mathcal{E}(\vec{n})$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

$$\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\vec{n}_i \mathrm{d}\vec{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\vec{n}_i - \vec{n}_1)\delta^{(2)}(\vec{n}_j - \vec{n}_2)$$

inclusive cross section to produce two particles, i, j, and anything else!

Correlation functions of $\mathcal{E}(\vec{n})$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

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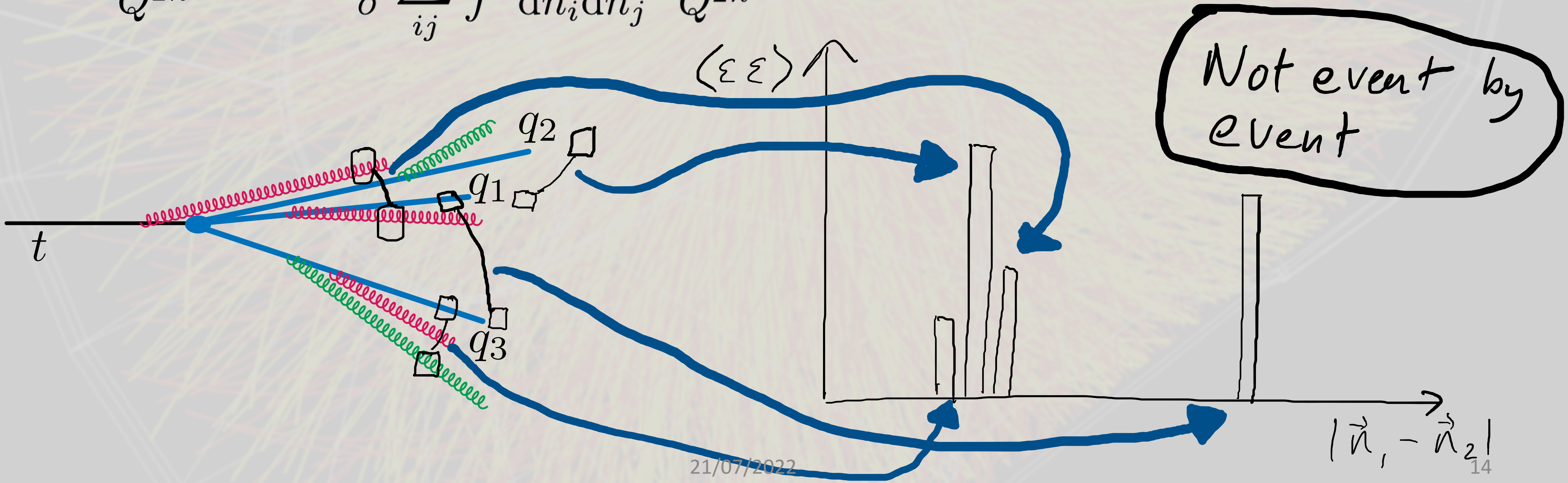


Correlation functions of $\mathcal{E}(\vec{n})$

Pros:

- Defined on inclusive cross-sections and can be made insentive to soft radiation. Textbook example of where pp CSS factorisation can be used without any violation.

$$\frac{d\sigma}{d\zeta} = \int dE_J\, E_J^2 H(E_J) J_{\mathrm{EEC}}(\zeta, E_J) + \text{power corrections}\,,$$ 2109.03665

- Well studied by CFT community. Powerfull techniques exist for calculations: light-ray OPE, celestial Blocks, lorentzian inversion. 2202.04085
- Including the only calculation of jet substructure at strong coupling. 0803.1467

Cons:

- Tend to be reliant on high stats.
- Not event-by-event so cannot be directly used to tag.

Case Study: vacuum jets

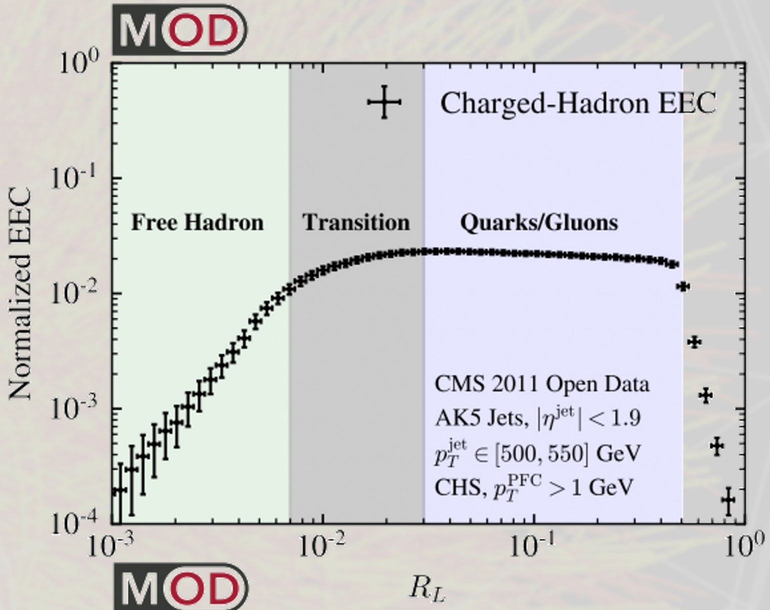
In summary: *The small angle behavior of the energy correlation functions is determined by the spin $j = 3$ non-local operators that appear in the OPE*

0803.1467

$$\langle \mathcal{E}(\theta_1)\mathcal{E}(\theta_2)\cdots\rangle \sim \sum_n |\theta_{12}|^{\tau_n - 4}\langle \mathcal{U}_{3-1,n}(\theta_2)\cdots\rangle \quad (2.19)$$

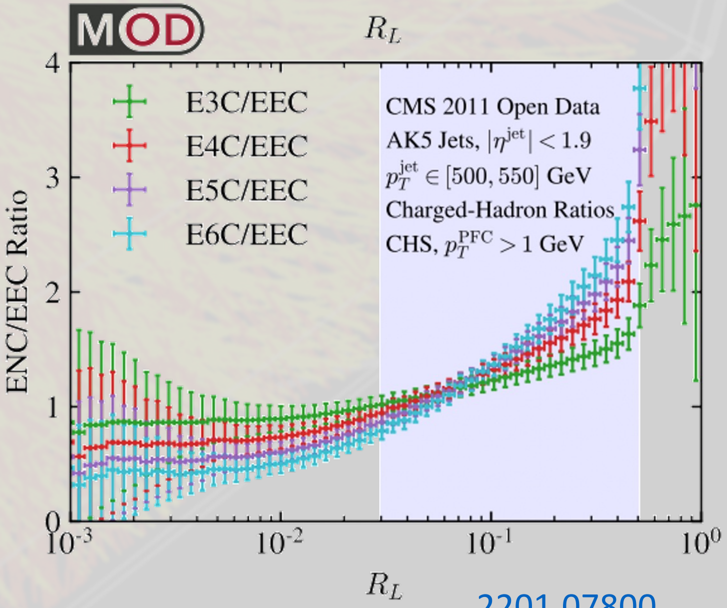
$\tau_n(j) = 2 - \gamma(j)$, in the formula above $j = 3$

$R_L \sim \theta$



In a CFT $\gamma(j)$ is a constant whilst in QCD the running coupling causes $\gamma(j)$ to have logarithmic scale dependence but at LL the structure is otherwise unchanged.

$\gamma(j) > \gamma(j-1)$



An n-point correlator has a $\tau(j = n+1) = 2 - \gamma(j = n+1)$ scaling

$R_L \sim \frac{\Lambda_{\text{QCD}}}{p_{T\,\text{jet}}}$ breaks the OPE scaling of approx assympotically free dynamics.

2201.07800

Correlation functions of $\mathcal{E}(\vec{n})$

Which correlation function is the one for us?

- In the previous slide the 2-point correlator gives a sentive pobe of hadronisation.
- In 2201.08393 the 3-point provided a sentive probe to the top mass.

Look to what is currently done and sucessful.

- R_{AA} can be expressed as a function of one-point correlators + corrections:
  - $R_{AA} = \langle N_{AA}\rangle / (\langle N_{\mathrm{Coll}}\rangle \langle N_{pp}\rangle)$. $\langle N\rangle$ is the one point correlator of the number opertator and due to momentum conservation $\langle N\rangle \approx \langle\mathcal{E}\rangle/\langle Q\rangle$.
- In effect, R_{AA} gives access to the simplest but also least sensitive correlator. Let us increase the sentivity (at the expense of a little more complexity) by looking directly at the 2-point correlator.

Part 3: The observable analytically

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\vec{n}_{1,2} \frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} \delta(\vec{n}_2 \cdot \vec{n}_1 - \cos\theta).$$

The observable analytically

Equiv. to $t_f < L$

Vacuum $\theta \ll 1$ resummation

$$\frac{\mathrm{d}\Sigma^{(1)}}{\mathrm{d}\theta} \sim \frac{1}{\theta^{1-\gamma(3)}} + \mathcal{O}(\theta^0)$$

$\theta > (EL)^{-1/2}$ Medium induced quenching

$$\left.\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta}\right|_{\theta \gtrsim \theta_L} = \frac{1}{\sigma_{qg}} \int \mathrm{d}z \, \frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta \mathrm{d}z} z^n (1-z)^n \times \left(1 + \mathcal{O}(\alpha_\mathrm{s} \ln \theta_L^{-1}) + \mathcal{O}\left(\alpha_\mathrm{s} \frac{\mu_\mathrm{s}^n}{E^n}\right)\right)$$

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta \mathrm{d}z} = \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z}(1 + F_{\mathrm{med}}(z, \theta, \hat{q}, L))$$ 1907.03653, 2107.02542

$$\frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z} \approx \frac{\alpha_\mathrm{s} C_\mathrm{F} \sigma}{\pi} \frac{1 + (1-z)^2}{z \; \theta} + \mathcal{O}(\alpha_\mathrm{s}^2, \theta^0)$$

The observable analytically

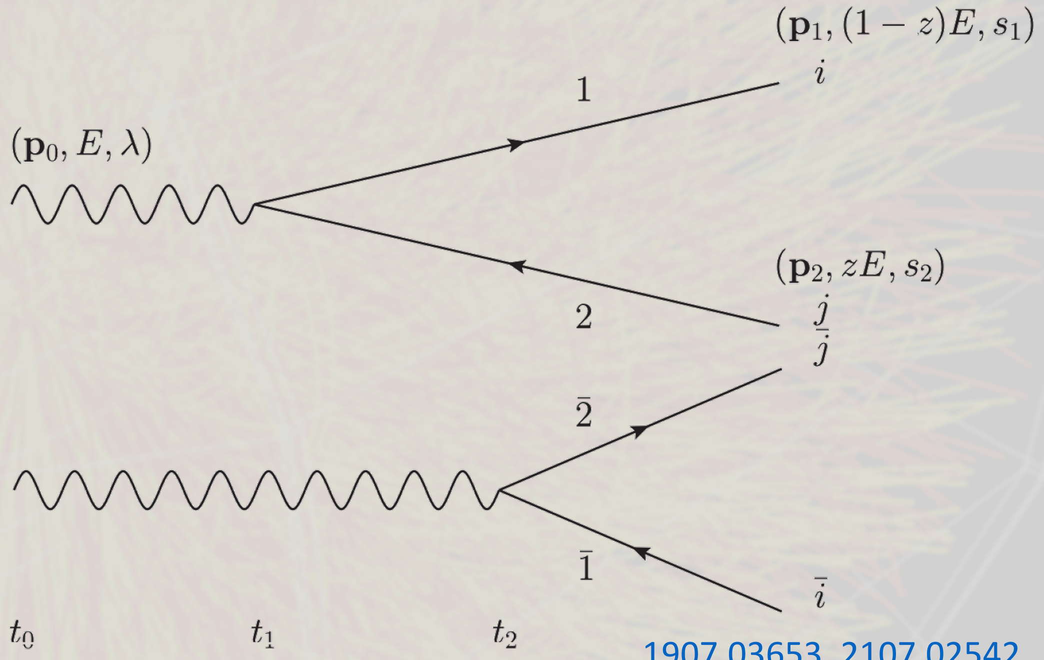
$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta \mathrm{d}z} = \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z}\left(1 + F_{\mathrm{med}}(z, \theta, \hat{q}, L)\right)$$

Static brick medium: length L, transport coefficient $\hat{q}$.

We assume up to one hard splitting occurs in the medium: q->qg.

The initial quark is considered to have large light-cone energy, as do both its decay products.
Formally, $E \to \infty$ and $0 \ll z \ll 1$.

All three particles undergo broadening and energy loss by interacting with the medium. Broadening and energy loss are resumed in the BDMPS-Z formalism with a harmonic oscillator potential.



1907.03653, 2107.02542

We study quark jets with substructure formed from the q->qg process. The diagrams are drawn for **γ**->qq processes without meaningful loss of information: 2107.02542 considered **γ**->qq, q->qg, and g->gg processes.

The observable analytically

Vacuum $\theta \ll 1$ resummation  $\theta > (EL)^{-1/2}$ Medium induced quenching

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \frac{\mathrm{d}z}{\sigma_{qg}} \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z} \left(g^{(n)}(\theta, \alpha_{\mathrm{s}}) + F_{\mathrm{med}}(z, \theta, \hat{q}, L)\right) \quad (7)$$

$$\times \, z^n (1-z)^n \left(1 + \mathcal{O}(\alpha_{\mathrm{s}} \ln \theta_L^{-1}) + \mathcal{O}\left(\alpha_{\mathrm{s}} \frac{\mu_{\mathrm{s}}^n}{E^n}\right)\right),$$

where $g^{(1)} = \theta^{\gamma(3)}$ at fixed coupling The expression for $g^{(n)}$ with $n > 1$ is more complicated. However, crucially one still has that $g^{(n)} \to 1$ as $\alpha_{\mathrm{s}} \ln \theta^{-1} \to 0$.

Part 4: The observable numerically

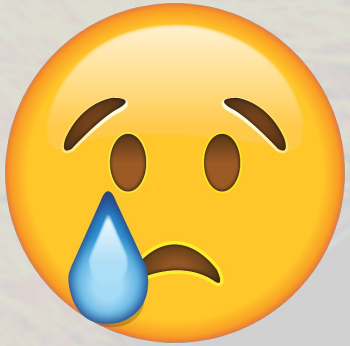
$$F_{\text{med}} = 2\int_0^L \frac{\mathrm{d}t_1}{t_\mathrm{f}} \left[\int_{t_1}^L \frac{\mathrm{d}t_2}{t_\mathrm{f}} \cos\left(\frac{t_2 - t_1}{t_\mathrm{f}}\right) \mathcal{C}^{(4)}(L, t_2)\mathcal{C}^{(3)}(t_2, t_1) - \sin\left(\frac{L - t_1}{t_\mathrm{f}}\right) \mathcal{C}^{(3)}(L, t_1) \right]$$

$$\mathcal{C}^{(3)}_{gq}(t_2, t_1) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{tr}[V_2^\dagger V_1]\, \mathrm{tr}[V_0^\dagger V_2] - \frac{1}{N_c} \mathrm{tr}[V_0^\dagger V_1] \right\rangle .$$

$$\mathcal{C}^{(3)}_{gq}(t_2, t_1) = \mathrm{e}^{-\frac{1}{2}\int_{t_1}^{t_2} \mathrm{d}s\, n(s)[N_c(\sigma_{02}+\sigma_{12}) - \frac{1}{N_c}\sigma_{01}]}$$
$$= \mathrm{e}^{-\frac{1}{12}\hat{q}(t_2 - t_1)^3\theta^2\left(1 + z^2 + \frac{2z}{N_c^2 - 1}\right)} .$$

$$\mathcal{C}^{(4)}_{gq}(L, t_2) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{tr}[V_{\bar 1}^\dagger V_1 V_2^\dagger V_{\bar 2}]\, \mathrm{tr}[V_{\bar 2}^\dagger V_2] - \frac{1}{N_c} \mathrm{tr}[V_{\bar 1}^\dagger V_1] \right\rangle ,$$

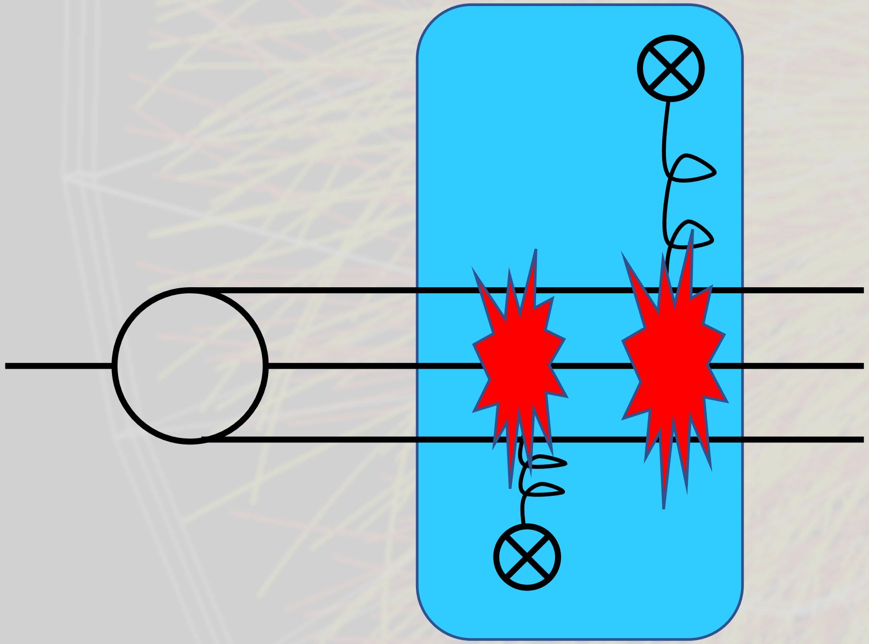
$$\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar 2} V_{\bar 1}^\dagger]\, \mathrm{tr}[V_2 V_{\bar 2}^\dagger]\rangle \simeq \mathrm{e}^{-\frac{1}{4}\hat{q}\theta^2(t - t_2)(t_2 - t_1)^2(1 - 2z + 3z^2)}$$
$$\times \left(1 - \frac{1}{2}\hat{q}\theta^2 z(1 - z)(t_2 - t_1)^2 \int_{t_2}^t \mathrm{d}s\, \mathrm{e}^{-\frac{1}{12}\hat{q}\theta^2\left[(s - t_2)^2(2s - 3t_1 + t_2) + 6z(1 - z)(s - t_2)(t_2 - t_1)^2\right]}\right)$$



The observable numerically

$\theta_{\mathrm{c}} \gg \theta_{\mathrm{L}}$

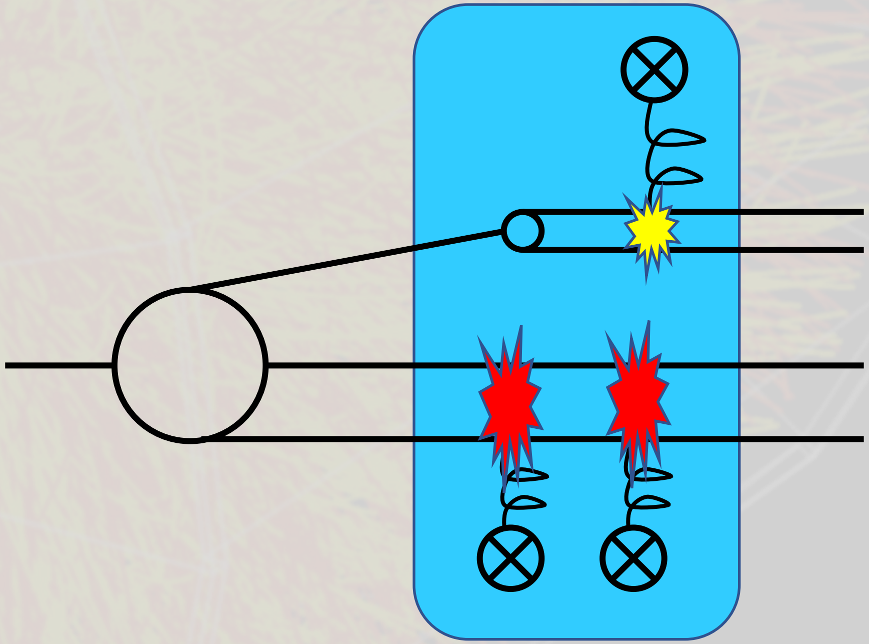
$E \gg \hat{q}L^2$



For angles $\theta_{\mathrm{c}} \gg \theta \gg \theta_{\mathrm{L}}$, the quark jet undergoes some energy loss but the substructure is not resolved.

$\theta_{\mathrm{c}} \ll \theta_{\mathrm{L}}$

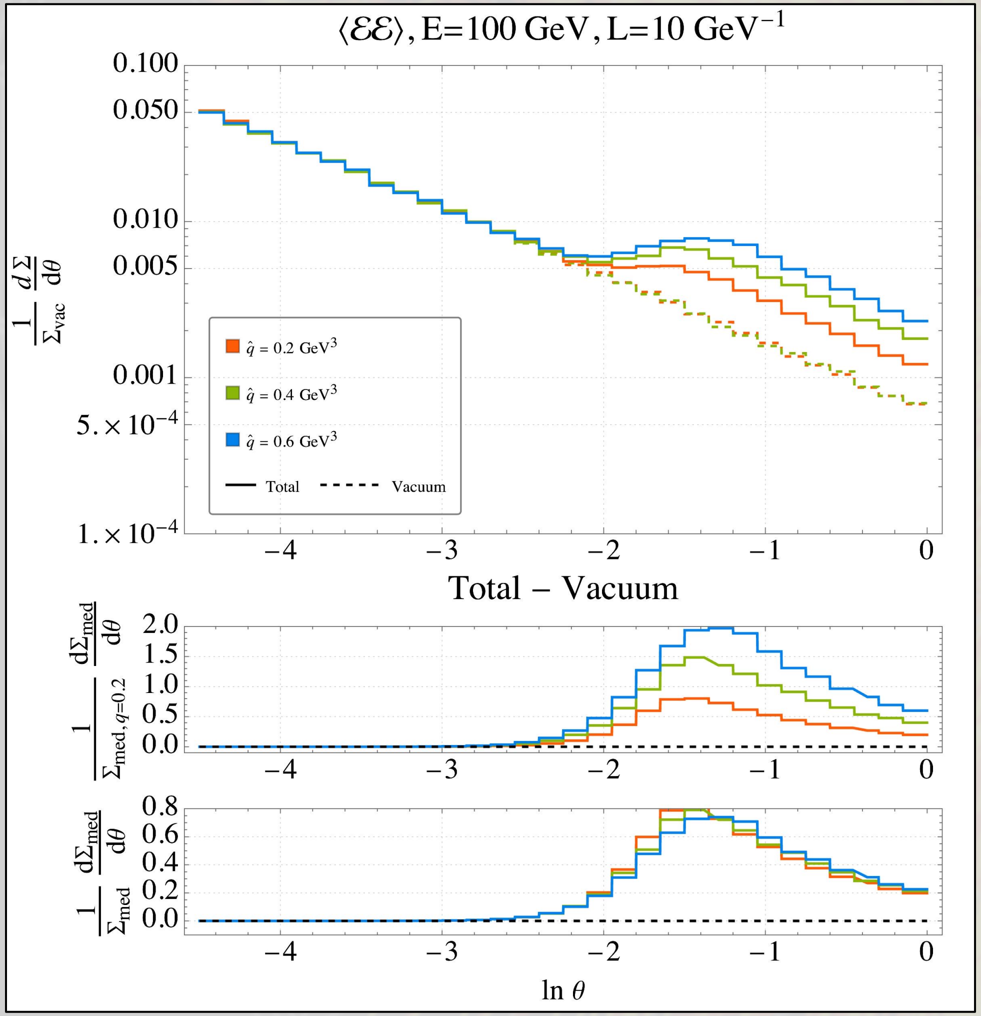
$E \ll \hat{q}L^2$

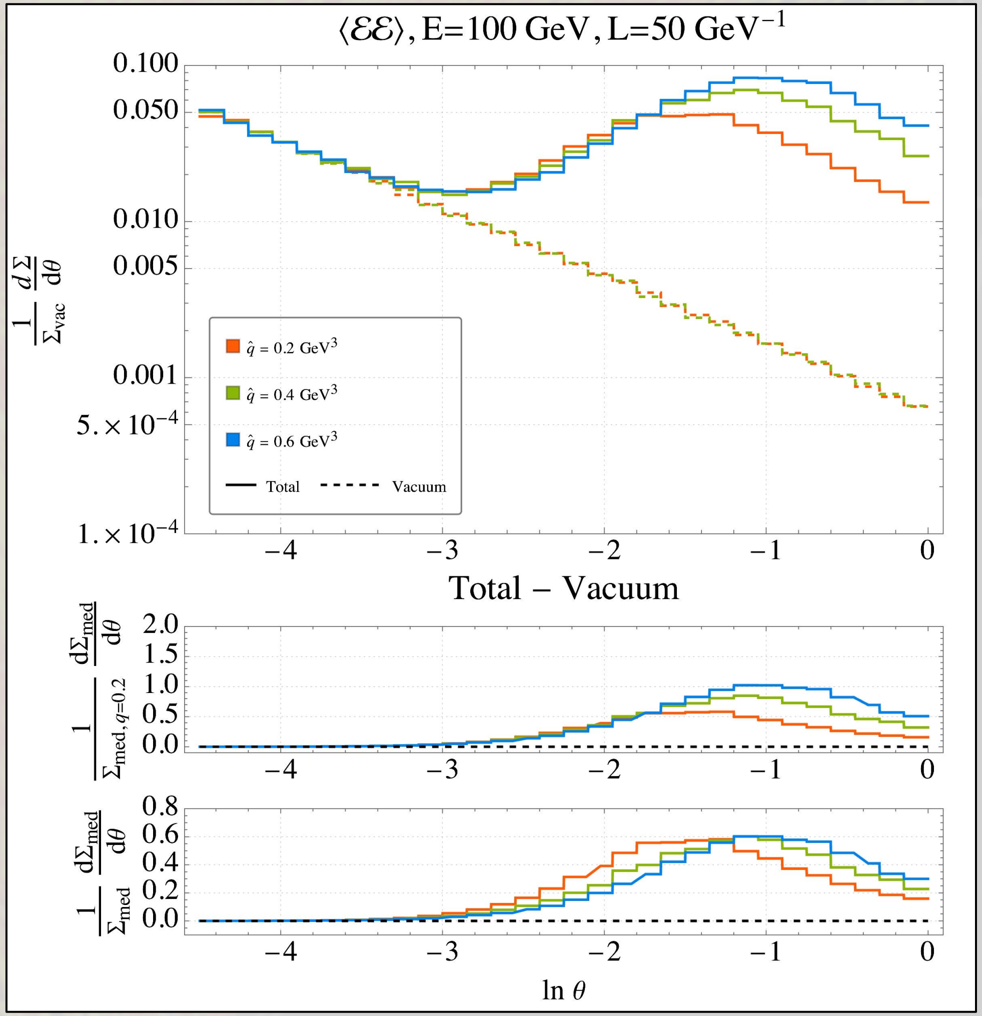


Initial splitting can be resolved by the medium when $\theta \gg \theta_{\mathrm{L}}$. Broadening and energy loss occur.

The observable numerically

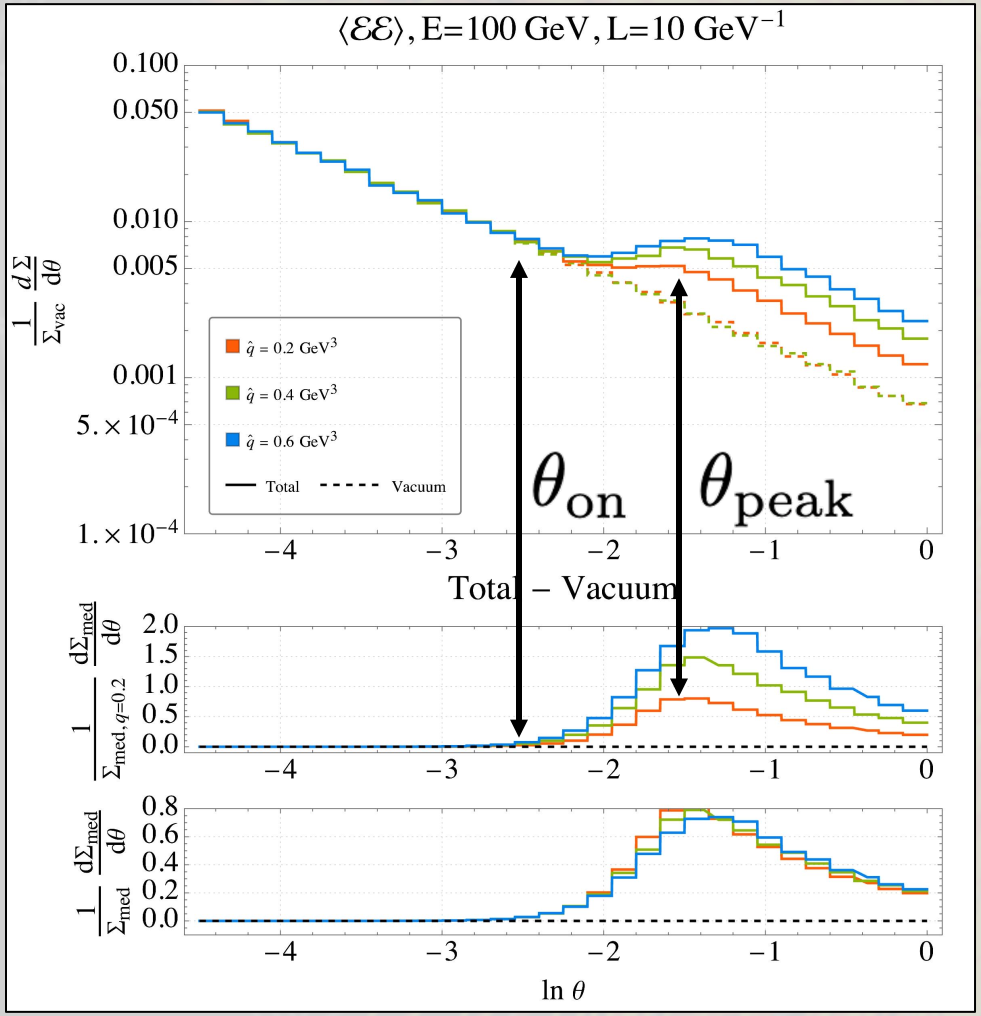
$\theta_c \gg \theta_L$ $\quad L = 10\ \text{GeV}^{-1} \equiv 2\ \text{fm}$ $\quad \theta_c \ll \theta_L$

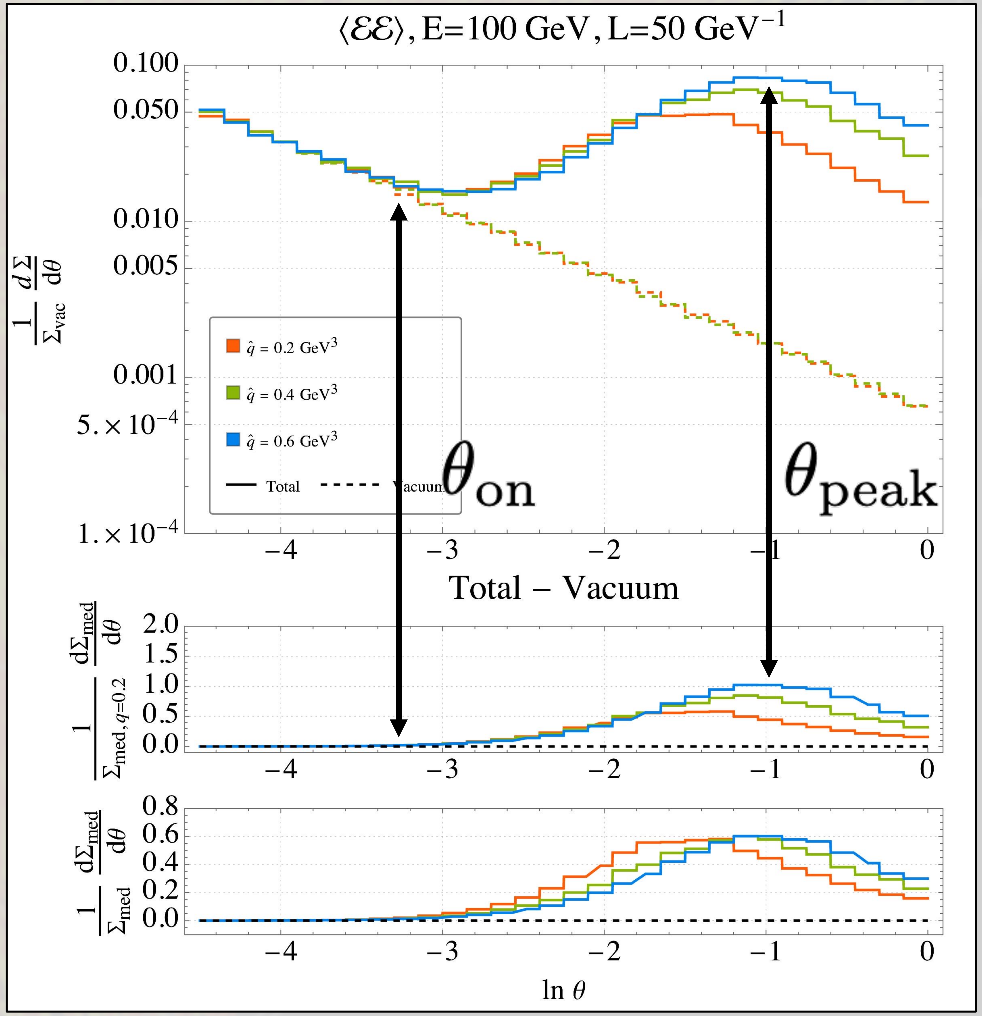




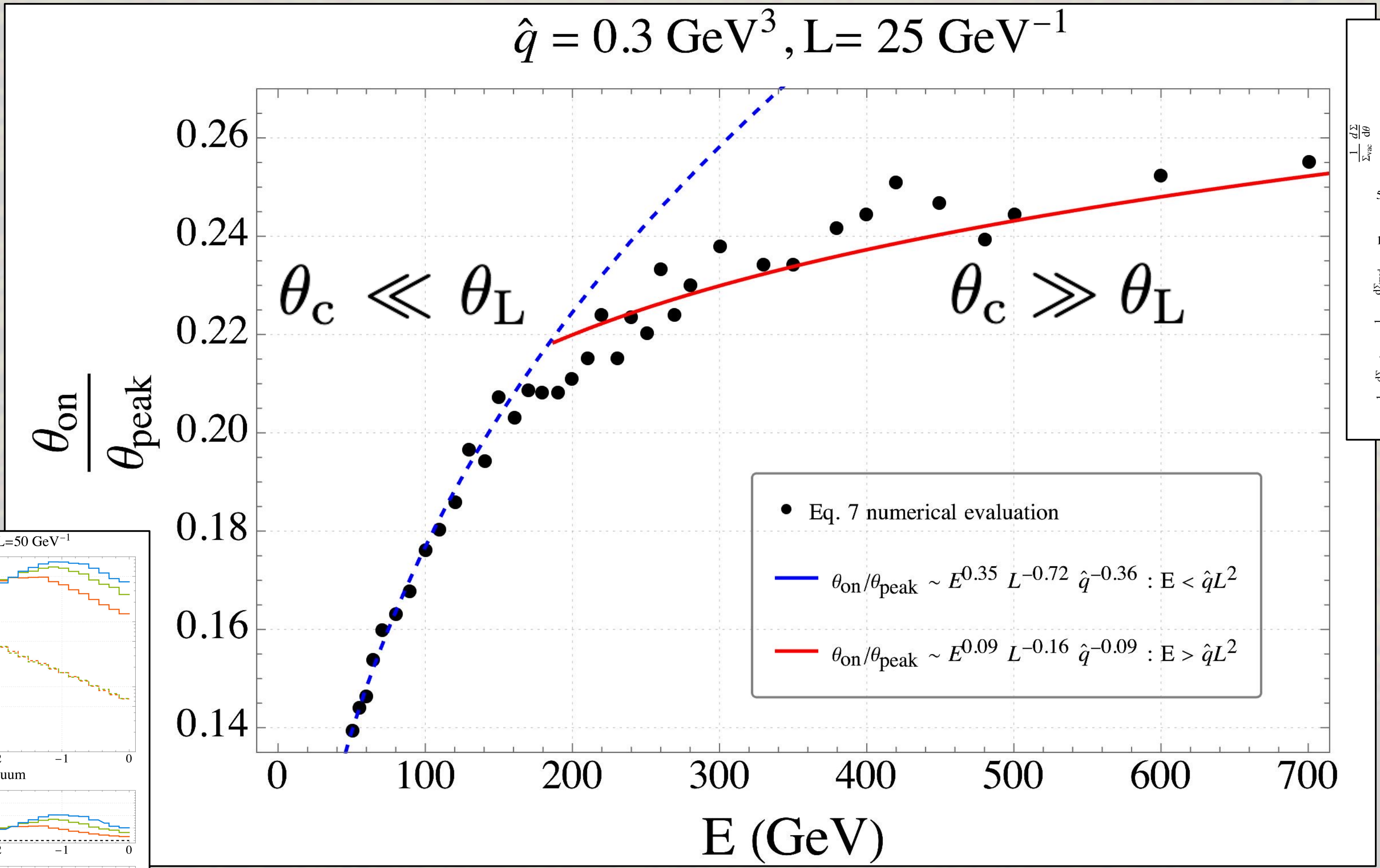
The observable numerically

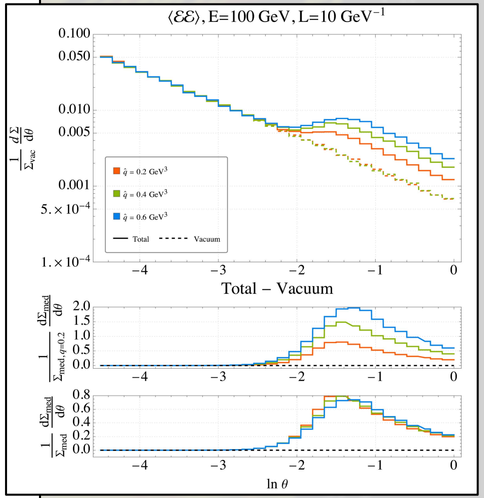
$\theta_c \gg \theta_L$ $\quad L = 10\ \text{GeV}^{-1} \equiv 2\ \text{fm}$ $\quad \theta_c \ll \theta_L$

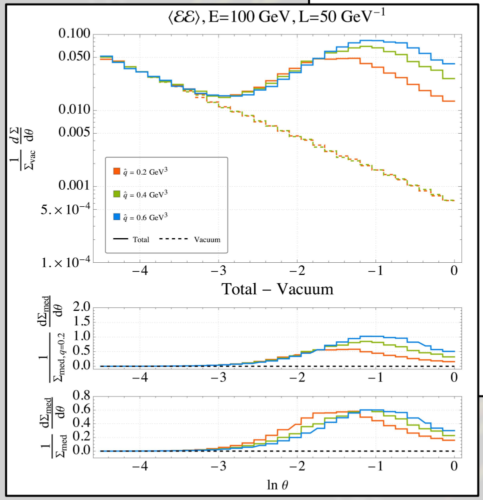




The observable numerically

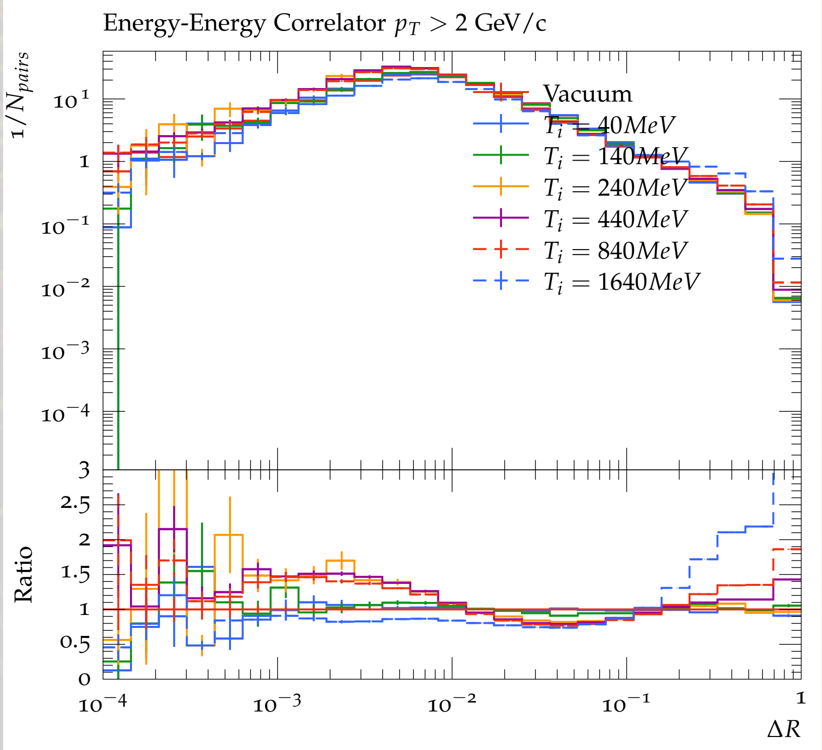


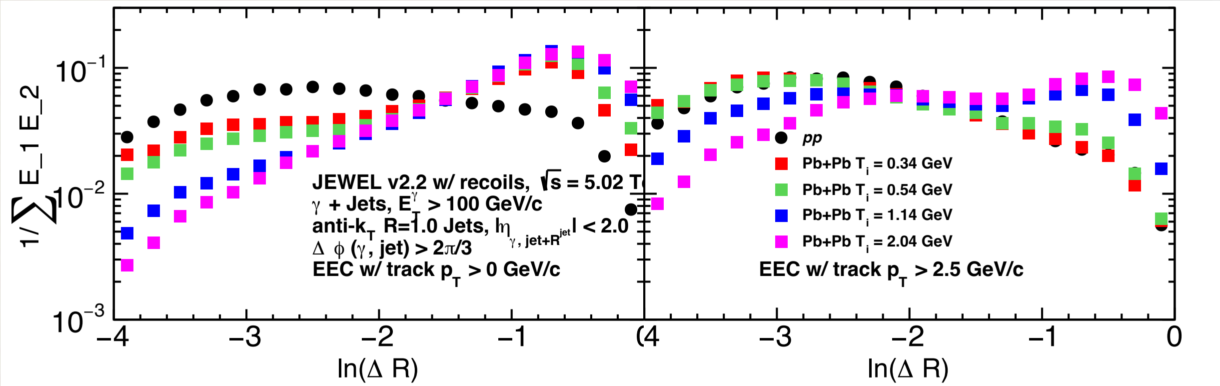




The observable numerically

An analysis in JEWEL is now also under way.





Early results indicate the main features of the curves are resilient against a hadron pt cut $p_t \gtrsim 2$ GeV.

Complimentarity between be measurement at sPHENIX and LHC.

Conclusions

Energy Correlators are cool and fun! 😁

- Our early results suggest properties of the QGP can be resolved by using energy correlators for jet substructure.
- Our initial analysis uses the BDMPS-Z model for the numerics. However, the basic features should be model independent, they are set by formation times and uncertainty relations. Could not be explained by changing q/g fraction.
- Correlators are naturally insentive to low scale physics – hadronisation, background amd soft corrections typically are sub-leading.
- We are optimistic for a future measurement at sPHENIX and are studying feasibility in JEWEL.

Part N/A: Supplemental Material

$$\mathcal{M}_{\gamma\to q\bar{q}} = \frac{e}{E}\mathrm{e}^{i\frac{\boldsymbol{p}_1^2}{2zE}L+i\frac{\boldsymbol{p}_2^2}{2(1-z)E}L}\int_0^\infty \mathrm{d}t \int_{\boldsymbol{k}_1,\boldsymbol{k}_2}\left[\mathcal{G}(\boldsymbol{p}_1,L;\boldsymbol{k}_1,t|zE)\,\bar{\mathcal{G}}(\boldsymbol{p}_2,L;\boldsymbol{k}_2,t|(1-z)E)\right]_{ij}$$

$$\times\,\gamma_{\lambda,s,s'}(z)\boldsymbol{k}\cdot\boldsymbol{\epsilon}_\lambda^*\,\mathcal{G}_0(\boldsymbol{k}_1+\boldsymbol{k}_2,t|E)$$

$$\mathcal{G}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0) = \int_{\boldsymbol{x}_1,\boldsymbol{x}_2}\mathrm{e}^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}_1+i\boldsymbol{p}_0\cdot\boldsymbol{x}_0}\mathcal{G}(\vec{x}_1,\vec{x}_0)$$

$$\mathcal{G}(\vec{x}_1,\vec{x}_0) = \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t_1)=\boldsymbol{x}_1}\mathcal{D}\boldsymbol{r}\,\exp\left[i\frac{E}{2}\int_{t_0}^{t_1}\mathrm{d}s\,\dot{\boldsymbol{r}}^2\right]V(t_1,t_0;[\boldsymbol{r}])$$

$$V(t_1,t_0;[\boldsymbol{r}]) = \mathcal{P}\exp\left[ig\int_{t_0}^{t_1}\mathrm{d}t\,\mathbf{t}^a A^{-,a}(t,\boldsymbol{r}(t))\right]$$

$$\frac{\mathrm{d}N^{\mathrm{med}}}{\mathrm{d}z\mathrm{d}\boldsymbol{p}^2} = \frac{1}{4(2\pi)^2\,z(1-z)}\langle|\mathcal{M}_{\gamma\to q\bar{q}}|^2\rangle = \frac{1}{4(2\pi)^2\,z(1-z)}\langle\left|\mathcal{M}^{\mathrm{in}}_{\gamma\to q\bar{q}}+\mathcal{M}^{\mathrm{out}}_{\gamma\to q\bar{q}}\right|^2\rangle$$

Part N/A: Supplemental Material

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta \mathrm{d}z} = \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z}(1 + F_{\mathrm{med}}(z, \theta, \hat{q}, L))$$

$$F_{\mathrm{med}} = 2\int_0^L \frac{\mathrm{d}t_1}{t_\mathrm{f}}\left[\int_{t_1}^L \frac{\mathrm{d}t_2}{t_\mathrm{f}} \cos\left(\frac{t_2 - t_1}{t_\mathrm{f}}\right)\mathcal{C}^{(4)}(L, t_2)\mathcal{C}^{(3)}(t_2, t_1) - \sin\left(\frac{L - t_1}{t_\mathrm{f}}\right)\mathcal{C}^{(3)}(L, t_1)\right]$$

$$\mathcal{C}_{gq}^{(3)}(t_2, t_1) = \frac{1}{N_c^2 - 1}\left\langle \mathrm{tr}[V_2^\dagger V_1]\,\mathrm{tr}[V_0^\dagger V_2] - \frac{1}{N_c}\,\mathrm{tr}[V_0^\dagger V_1]\right\rangle .$$

$$\mathcal{C}_{gq}^{(3)}(t_2, t_1) = \mathrm{e}^{-\frac{1}{2}\int_{t_1}^{t_2}\mathrm{d}s\, n(s)[N_c(\sigma_{02}+\sigma_{12}) - \frac{1}{N_c}\sigma_{01}]}$$
$$= \mathrm{e}^{-\frac{1}{12}\hat{q}(t_2-t_1)^3\theta^2\left(1+z^2+\frac{2z}{N_c^2-1}\right)} .$$

$$\mathcal{C}_{gq}^{(4)}(L, t_2) = \frac{1}{N_c^2 - 1}\left\langle \mathrm{tr}[V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}]\,\mathrm{tr}[V_{\bar{2}}^\dagger V_2] - \frac{1}{N_c}\,\mathrm{tr}[V_{\bar{1}}^\dagger V_1]\right\rangle ,$$

$$\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger]\rangle \simeq \mathrm{e}^{-\frac{1}{4}\hat{q}\theta^2(t-t_2)(t_2-t_1)^2(1-2z+3z^2)}$$
$$\times\left(1 - \frac{1}{2}\hat{q}\theta^2 z(1-z)(t_2-t_1)^2\int_{t_2}^{t}\mathrm{d}s\,\mathrm{e}^{-\frac{1}{12}\hat{q}\theta^2\left[(s-t_2)^2(2s-3t_1+t_2)+6z(1-z)(s-t_2)(t_2-t_1)^2\right]}\right)$$