Brookhaven National Laboratory

RBRC
RIKEN BNL Research Center

Azimuthal angular correlation of jets from soft gluon radiation

Yoshitaka Hatta

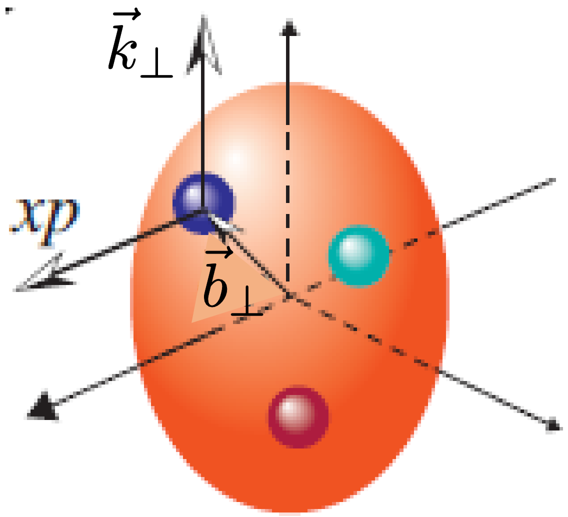
BNL & RIKEN BNL

in collaboration with Bowen Xiao, Feng Yuan, Jian Zhou

2010.10774 (PRL)
2106.05307 (PRD)

sPHENIX RBRC workshop July 20-22, 2022

Multi-dimensional tomography



$$u(x) = \int \frac{dz^-}{4\pi} \langle P|\bar{u}(0)\gamma^+ u(z^-)|P\rangle \, e^{ixP^+z^-}$$

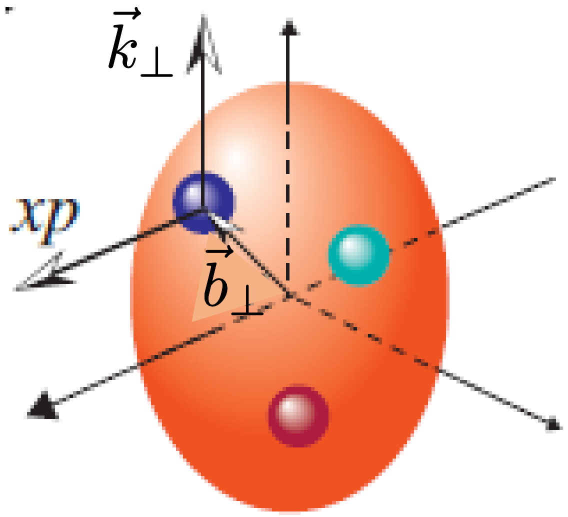
Ordinary parton distribution functions (PDF) can be viewed as the 1D tomographic image of the nucleon

The nucleon is much more complicated!
Partons also have transverse momentum $\vec{k}_\perp$
and are spread in impact parameter space $\vec{b}_\perp$

$u(x)$ →

- $u(x, \vec{k}_\perp)$ Transverse momentum dependent distribution (TMD) — 3D tomography
- $u(x, \vec{b}_\perp)$ Generalized parton distribution (GPD) — 3D tomography
- $u(x, \vec{k}_\perp, \vec{b}_\perp)$ Wigner distribution → (Fourier) GTMD $u(x, \vec{k}_\perp, \vec{\Delta}_\perp)$ — 5D tomography

Angular correlations



Rich angular correlations between impact parameter $\vec{b}_\perp$
and transverse momentum $\vec{k}_\perp$

$$W(x, \vec{b}_\perp, \vec{k}_\perp)$$
$$= W_0 + \cos 2(\phi_{b_\perp} - \phi_{k_\perp}) W_2 + \cos(\phi_{b_\perp} - \phi_{k_\perp}) W_O + \sin(\phi_{b_\perp} - \phi_{k_\perp}) W_{OAM} + \cdots$$

Elliptic Wigner — Odderon — Orbital angular momentum

Fourier transform → GTMD

$$W(x, \vec{\Delta}_\perp, \vec{k}_\perp) = W_0 + \cos 2(\phi_{\Delta_\perp} - \phi_{k_\perp}) W_2 + \cdots$$

Wigner distribution: Is it measurable?

In quantum optics, yes!

VOLUME 70, NUMBER 9 PHYSICAL REVIEW LETTERS 1 MARCH 1993

Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Rayme
Department of Physics and Chemical Physics Institute, University of Ore

A. Faridani
Department of Mathematics, Oregon State University, Corvalli
(Received 16 November 1992)

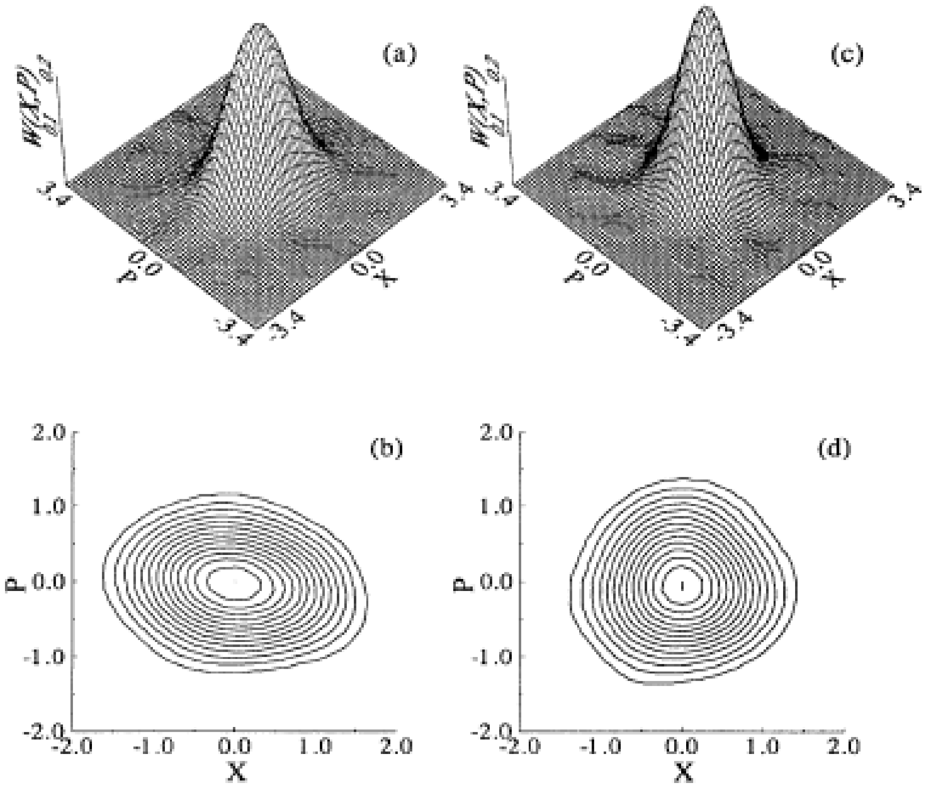
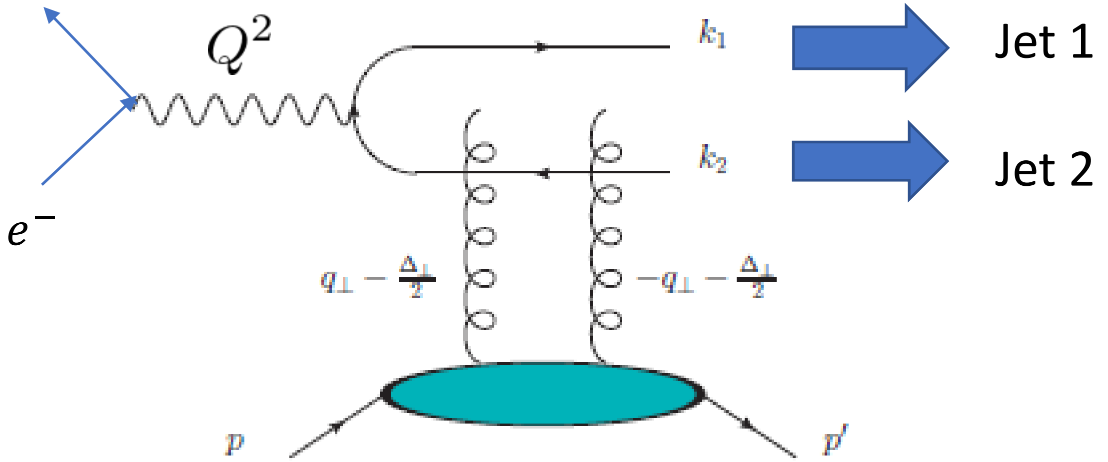


FIG. 1. Measured Wigner distributions for (a),(b) a squeezed state and (c),(d) a vacuum state, viewed in 3D and as contour plots, with equal numbers of constant-height contours. Squeezing of the noise distribution is clearly seen in (b).

Probing gluon Wigner in exclusive dijet production in DIS

YH, Xiao, Yuan (2016)



$\Delta^\mu = p'^\mu - p^\mu$ proton recoil

$\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ dijet total

$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$ dijet relative

GTMD

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \propto z(1-z)[z^2+(1-z)^2] \int d^2q_\perp d^2q'_\perp S(q_\perp, \Delta_\perp) S(q'_\perp, \Delta_\perp)$$

$$\times \left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2+\epsilon^2}\right] \cdot \left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2+\epsilon^2}\right] \qquad \epsilon^2 = z(1-z)Q^2$$

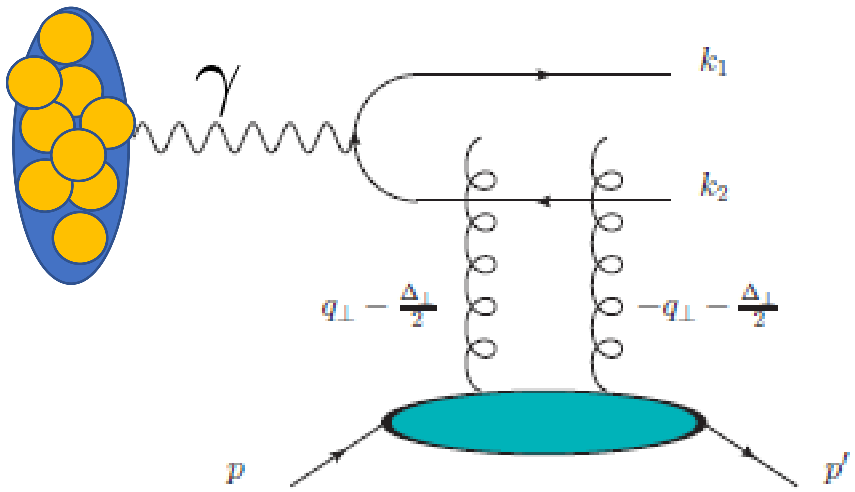
$$\sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta) d\tilde{\sigma}$$

`elliptic flow', expected to be a few percent effect

Exclusive dijet in UPC

Hagiwara, YH, Pasechnik, Tasevsky, Teryaev (2017)

UPC: where the heavy-ion and EIC communities can meet



$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega\frac{dN}{d\omega}\frac{N_c\alpha_{em}(2\pi)^4}{P_\perp^2}\sum_f e_f^2 2z(1-z)(z^2+(1-z)^2)\left(A^2 + 2\cos 2(\phi_P - \phi_\Delta)AB\right)$$

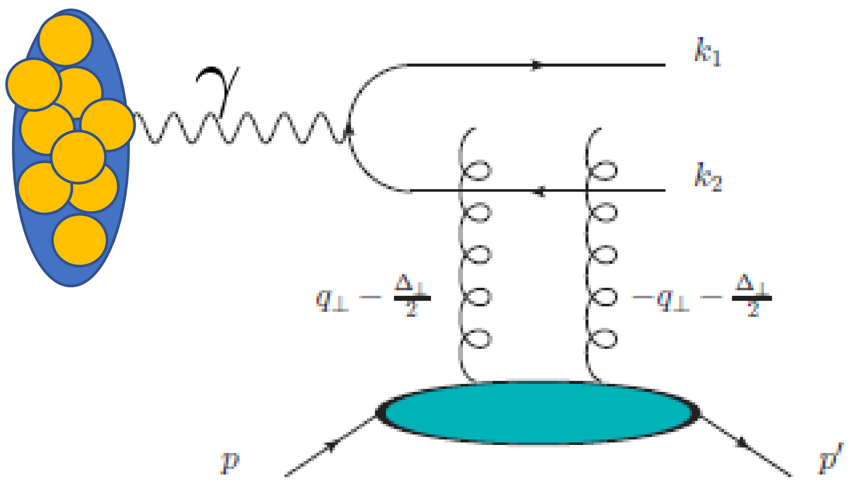
photon flux $\propto Z^2$

When $Q^2 \approx 0$ inversion can be done analytically. $S(\vec{q}_\perp, \vec{\Delta}_\perp) = S_0(q_\perp, \Delta_\perp) + 2\cos 2(\phi_q - \phi_\Delta)\tilde{S}(q_\perp, \Delta_\perp)$

$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp}\frac{\partial}{\partial P_\perp}A(P_\perp, \Delta_\perp). \qquad S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2}\int^{P_\perp^2}\frac{dP_\perp'^2}{P_\perp'^2}B(P_\perp', \Delta_\perp)$$

CMS dijet measurements

Partly motivated by our proposal, CMS measured dijet angular correlation in PbPb UPC

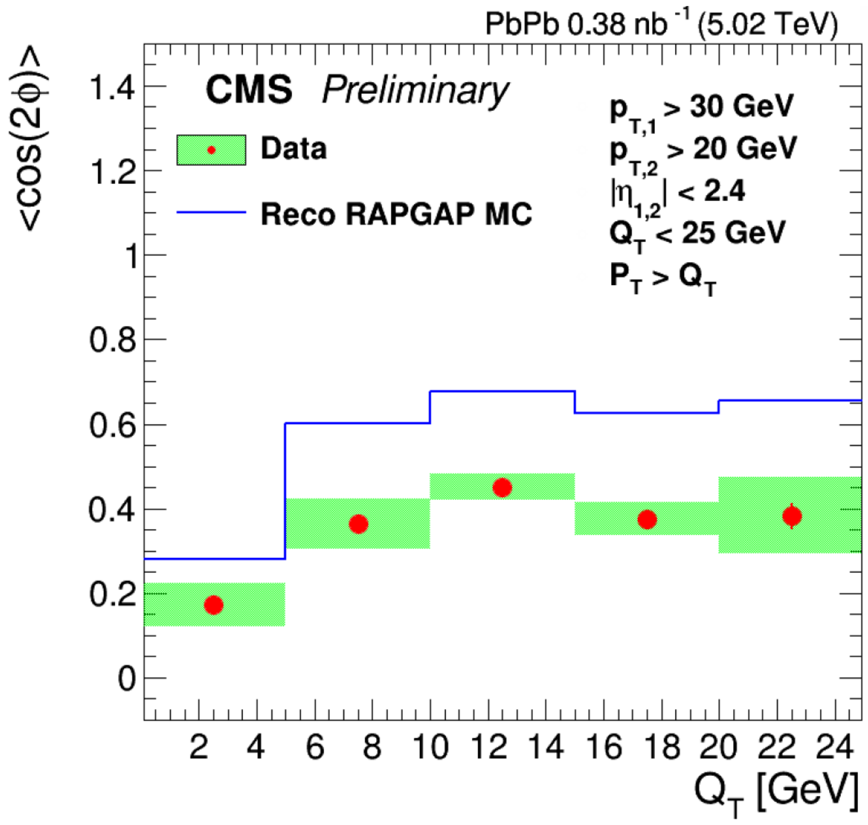


Use $\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ as a proxy for $\vec{\Delta}_\perp$

$\vec{\Delta}_\perp = -\vec{q}_\perp$ to leading order

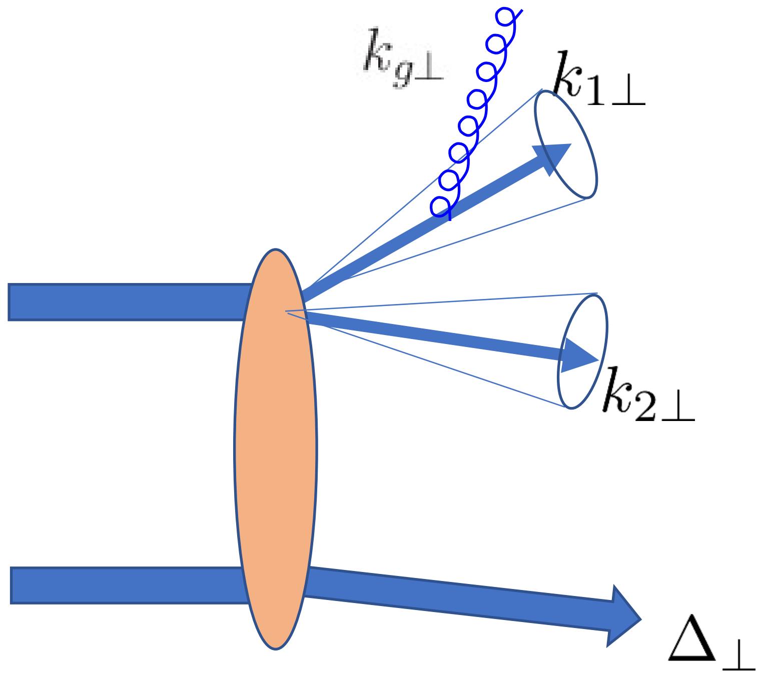
Measured the $\cos 2\phi$ correlation between $q_\perp$ and $P_\perp$, instead of that between $\vec{\Delta}_\perp$ and $P_\perp$

CMS-PAS-HIN-18-011



Very large asymmetry!
Anything to do with the elliptic Wigner?

Dijet with soft gluons: general consideration



To leading order, dijet total momentum is equal to the proton recoil momentum

$$q_\perp = k_{1\perp} + k_{2\perp} = -\Delta_\perp \qquad \frac{d\sigma}{d^2q_\perp} = \frac{d\sigma}{d^2\Delta_\perp}$$

With soft radiation, this becomes

$$\vec{q}_\perp = -\vec{\Delta}_\perp - \sum_i \vec{k}^i_{g\perp}$$

$k_{g\perp}$ tend to be along jet directions → $q_\perp$ tends to be along jet directions → $\cos 2\phi$

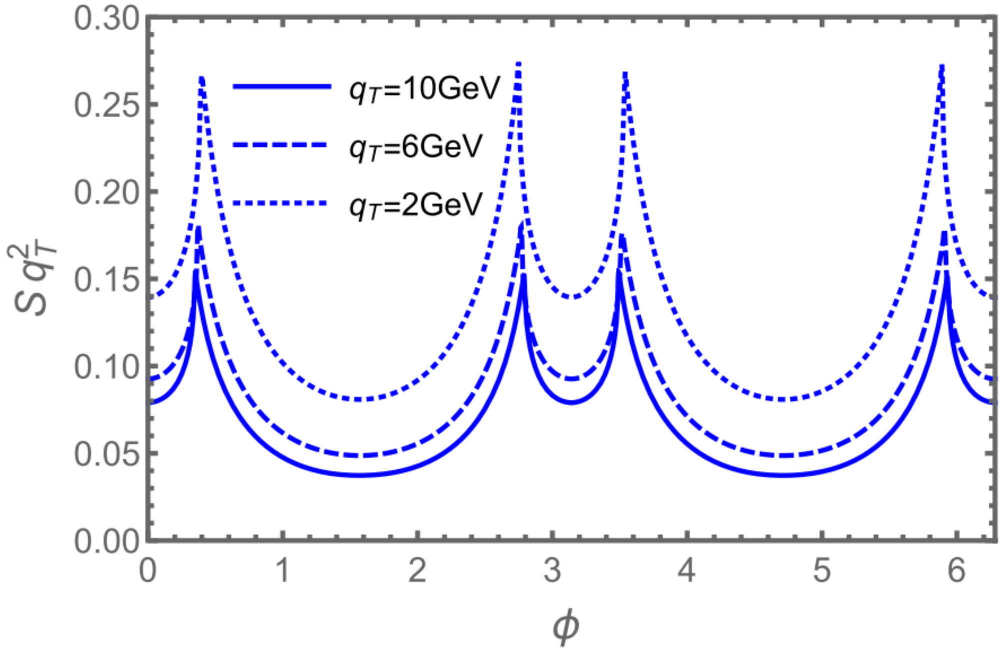
$$\frac{d\sigma}{d^2P_\perp d^2q_\perp} = \int d^2q'_\perp \frac{d\sigma_0}{d^2P_\perp d^2q'_\perp} S_J(q_\perp - q'_\perp),$$

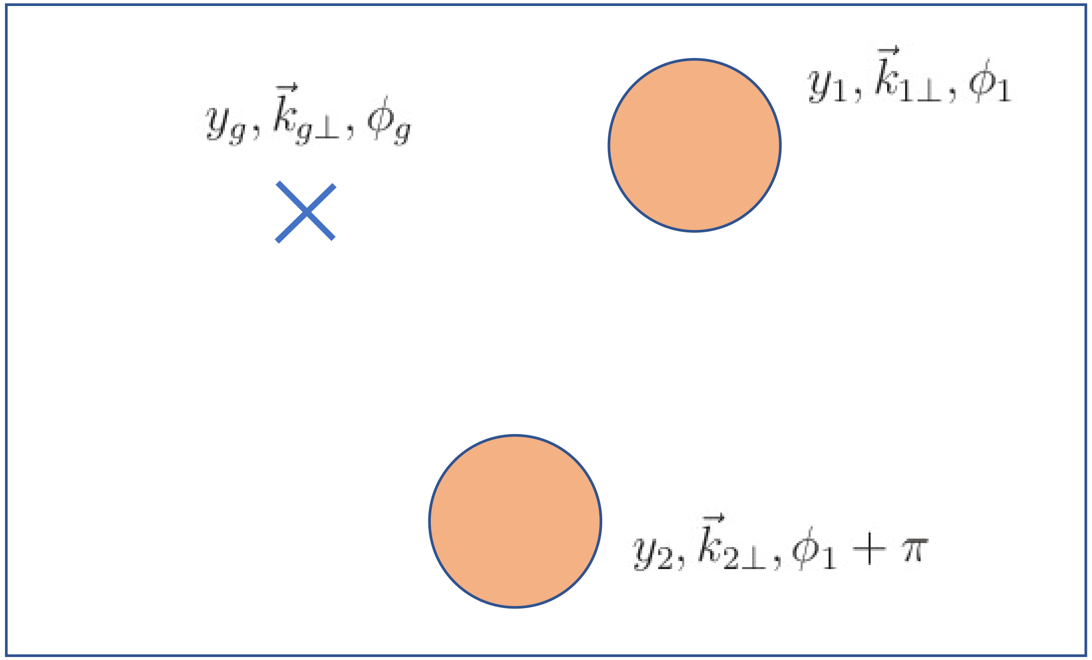
measured — primordial — Soft factor

Angular distribution of soft gluons around a dijet system

To one-loop,

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1\cdot k_2}{k_1\cdot k_g k_2\cdot k_g}\right)_{\vec{q}_\perp=-\vec{k}_{g\perp}}$$





Carefully integrate over the soft gluon phase space avoiding jet cones

→ all even harmonics $\cos 2n(\phi_{q_\perp} - \phi_{P_\perp})$

One-loop analytical result

$$g^2 \int \frac{d^3 k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F S_g(k_1, k_2)$$
$$= \frac{C_F \alpha_s}{\pi^2 q_\perp^2} [c_0^{\rm diff}(q_\perp^2) + 2\cos(2\phi) c_2^{\rm diff}(q_\perp^2) + \cdots].$$

$$c_n(q_\perp) = c_n(0) + \mathcal{O}(q_\perp^a / P_\perp^a)$$

power corrections

$$c_0^{\rm diff}(0) = \ln \frac{a_0}{R^2}, \qquad a_0 = 2 + 2\cosh(\Delta y_{12})$$
$$c_2^{\rm diff}(0) = \ln \frac{a_2}{R^2}. \qquad \ln a_2 = \Delta y_{12} \sinh \Delta y_{12} - \cosh \Delta y_{12} \ln [2(1 + \cosh \Delta y_{12})]$$

$$\frac{d\sigma}{d\phi} \sim \alpha_s \cos 2\phi \int \frac{dq_\perp^2}{q_\perp^2} = \infty$$

Angular dependent cross section divergent at fixed order.

Resummation

Catani, Grazzini, Torre, (2014), Catani, Grazzini, Sargsyan (2017)
YH, Yuan, Xiao, Zhou (2020~)

Fourier transform $q_\perp \to b_\perp$

Angular-independent part

$$\int \frac{d\vec{q}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} \left[\frac{1}{q_\perp^2}\right]_+ = -\frac{1}{4\pi}\ln\frac{b_\perp^2 P_\perp^2}{c_0^2}$$

IR divergent
→ regularized by the plus prescription

exponentiation →

$$\exp\left(-\frac{2C_F c_0}{\pi}\int_{\mu_b}^{P_\perp}\frac{d\mu}{\mu}\alpha_s(\mu)\right)$$

Angular-dependent part

$$\int \frac{d\vec{q}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} \frac{\cos 2\phi_{q_\perp}}{q_\perp^2} = -\frac{\cos 2\phi_{b_\perp}}{2\pi}\underbrace{\int_0^\infty \frac{dq_\perp}{q_\perp} J_2(b_\perp q_\perp)}_{=\frac{1}{2}}$$

No plus-prescription, but finite!

At higher orders, there are logarithms in the b-space even for the angular-dependent part.
→ Can be resummed by the same Sudakov factor

Fourier transform back to q-space

$$\frac{d\sigma}{dq_\perp^2 d\phi} = \cos n\phi \int_0^\infty db_\perp b_\perp J_n(b_\perp q_\perp) e^{-Sudakov} \ldots$$

$$\langle \cos n\phi \rangle \sim \frac{1}{q_\perp^2} \longleftrightarrow \langle \cos(n\phi) \rangle \propto q_\perp^n$$

before resummation — after resummation

Azimuthal asymmetries in QCD hard scattering: infrared safe but divergent

Stefano Catani,[a] **Massimiliano Grazzini**[b] **and Hayk Sargsyan**[b]

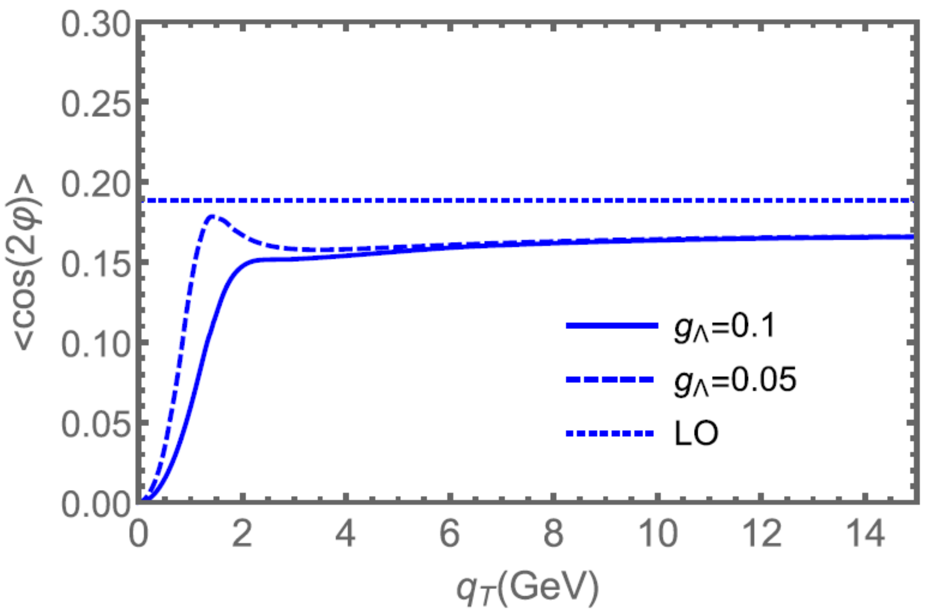
[a] *INFN — Sezione di Firenze and Dipartimento di Fisica e Astronomia, Università di Firenze, I-50019 Sesto Fiorentino, Florence, Italy*

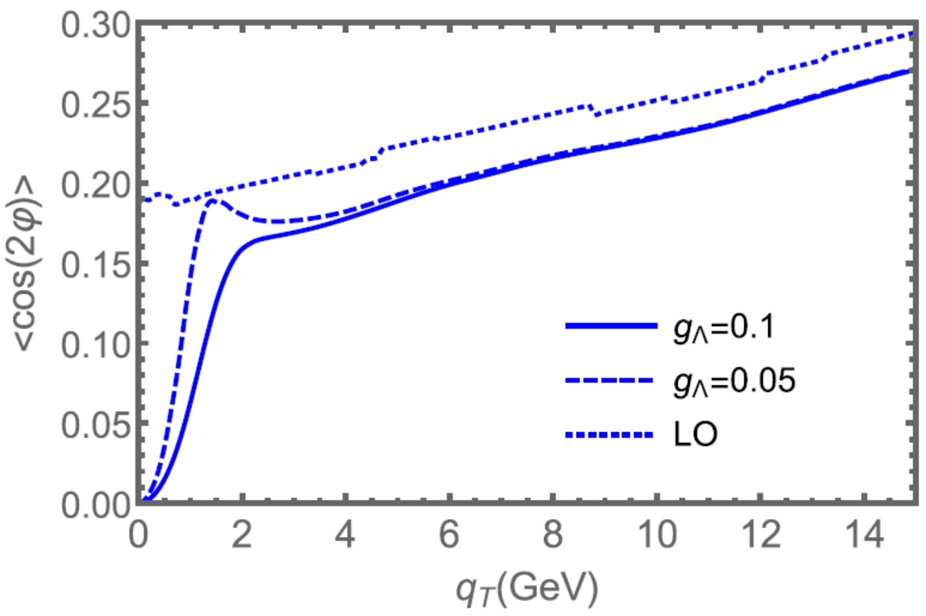
JHEP06(2017)017

Towards explaining the CMS data

2010.10774
2106.05307

UPC at the LHC, $P_\perp = 35\text{GeV}$ $R = 0.4$





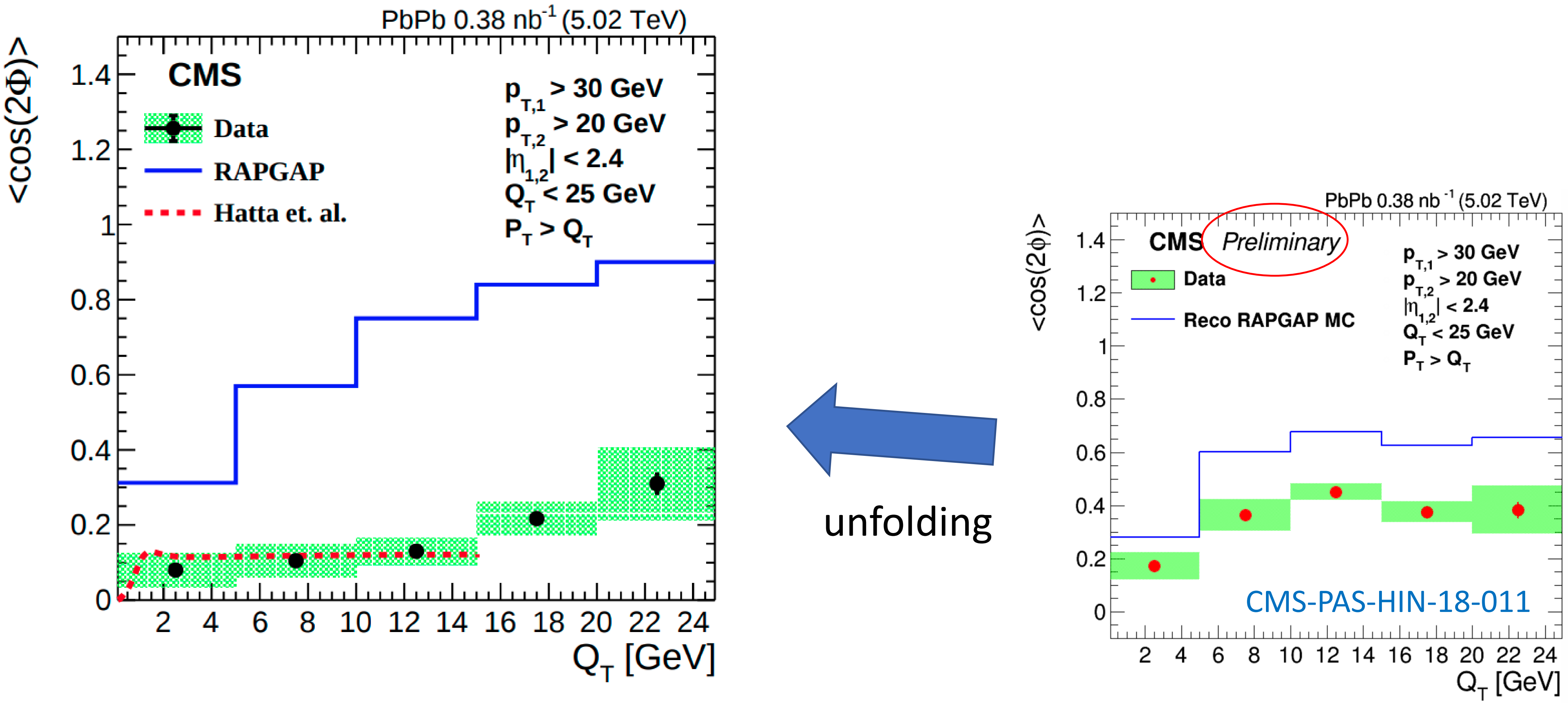
Part of power corrections included
→ monotonically rising behavior

Lesson learned:

$\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ cannot be a proxy for $\vec{\Delta}_\perp$.

CMS update

CMS-PAS-HIN-18-011
2205.00045 (updated plots)



Dijet in UPC pA/AA at sPHENIX

Take $\sqrt{s_{NN}} = 200\ \mathrm{GeV}$

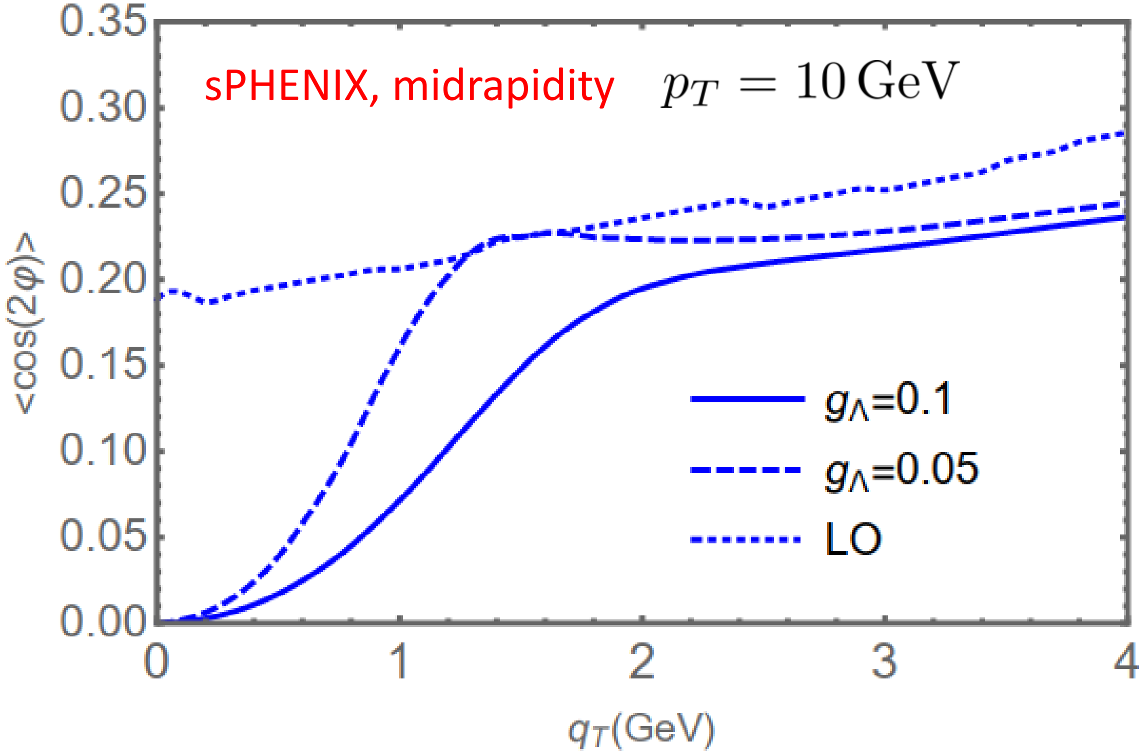
Typical WW photon energy in UPC $\omega = P_\perp e^y \sim 2\mathrm{GeV}$

Choose $y = -1$ and/or go to larger photon energy ω to make $P_\perp$ larger.

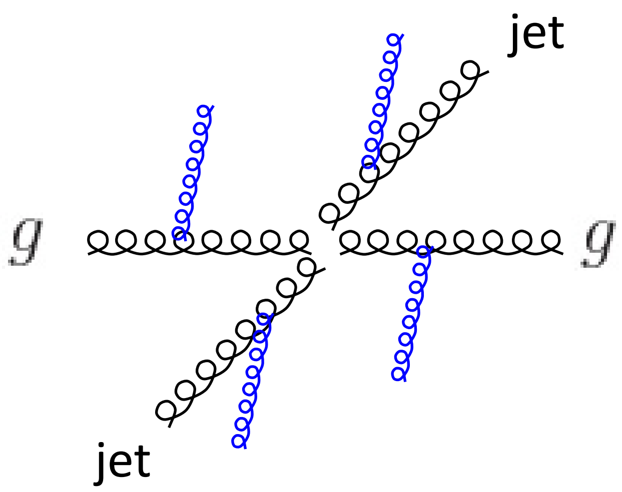
Stronger sensitivity to nonperturbative Sudakov factor

$$e^{-S_{sud}^{pert}(b_\perp)-2g_\Lambda b_\perp^2}$$

Can the recoiling proton momentum be measured in pA?



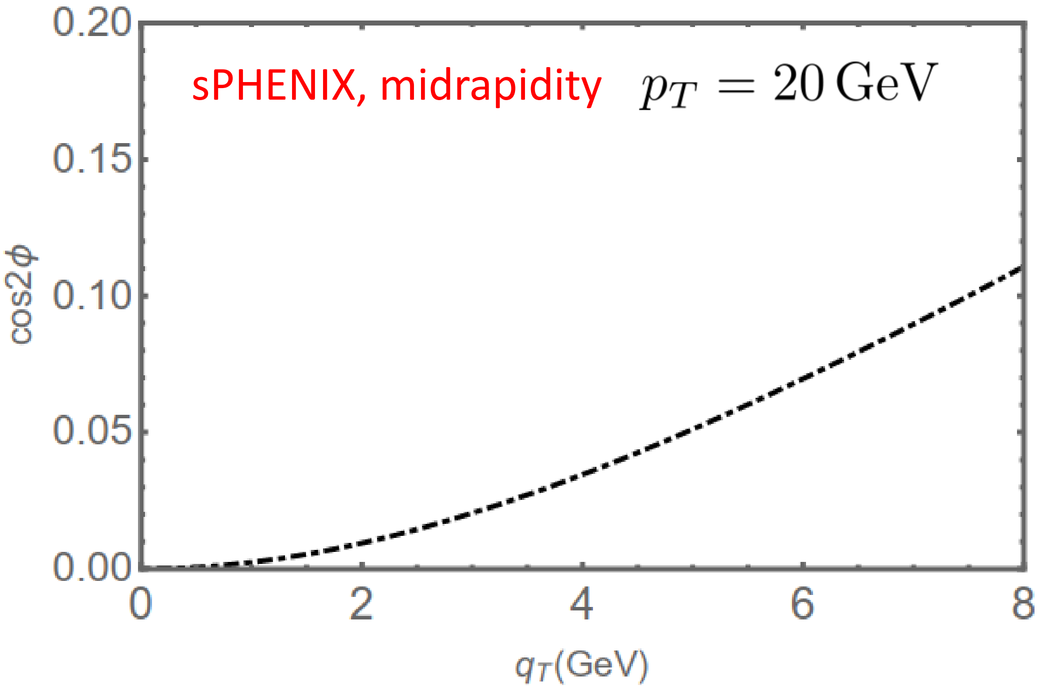
Inclusive dijet in pp at sPHENIX



$$\left.\frac{d^6\sigma}{d\Omega}\right|_{\rm soft} = \sigma_0^{gg} x_g f_g(x_g) \frac{\alpha_s}{2\pi^2} \frac{C_A}{q_\perp^2}\left[2\ln\frac{P_\perp^2}{q_\perp^2} + 2(c_0^{gg} + c_2^{gg} 2\cos(2\phi) + \cdots)\right]$$

$$c_2^{gg} = \ln\frac{a_1}{R^2} + \frac{\hat{t}^2+\hat{u}^2}{4(\hat{s}^2-\hat{t}\hat{u})}\ln\frac{a_2}{a_1}$$

Four colored partons

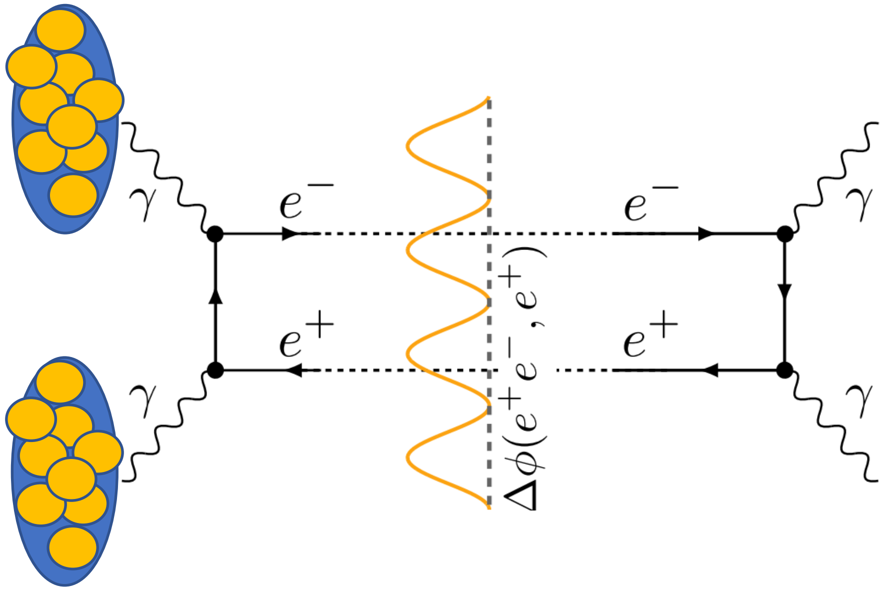


Double log resummation
→ universal, diagonal in color

Single log resummation
→ should be done in a matrix form in color space.

Ultraperipheral AA collisions (UPC) at RHIC and LHC

A large nucleus—copious source of linearly polarized photons



$$f(\Delta\phi) = C(1 + A_{2\Delta\phi}\cos 2\Delta\phi + A_{4\Delta\phi}\cos 4\Delta\phi)$$

Li, Zhou, Zhou (2019)

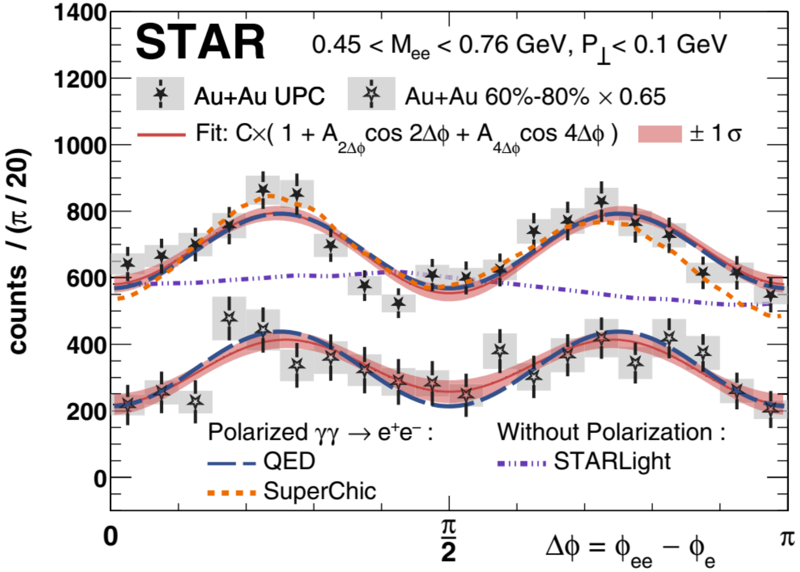
Dielectron production at midrapidity at low transverse momentum in peripheral and semi-peripheral Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

ALICE Collaboration

PHYSICAL REVIEW LETTERS **121,** 212301 (2018)

Observation of Centrality-Dependent Acoplanarity for Muon Pairs Produced via Two-Photon Scattering in Pb + Pb Collisions at $\sqrt{s_{NN}} = 5.02$ TeV with the ATLAS Detector

M. Aaboud *et al.**
(ATLAS Collaboration)

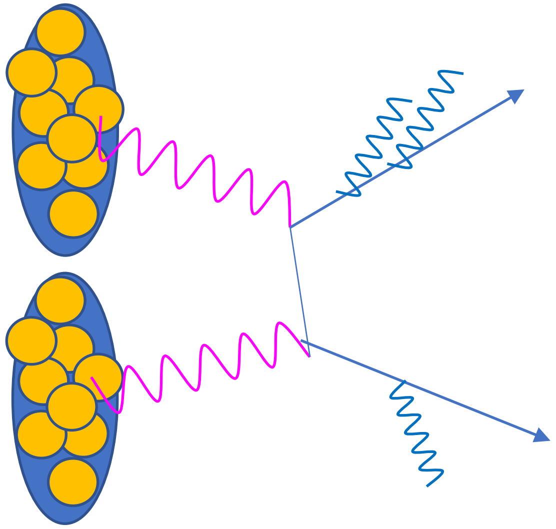


PHYSICAL REVIEW LETTERS **127,** 052302 (2021)

Measurement of e^+e^- Momentum and Angular Distributions from Linearly Polarized Photon Collisions

(STAR Collaboration)

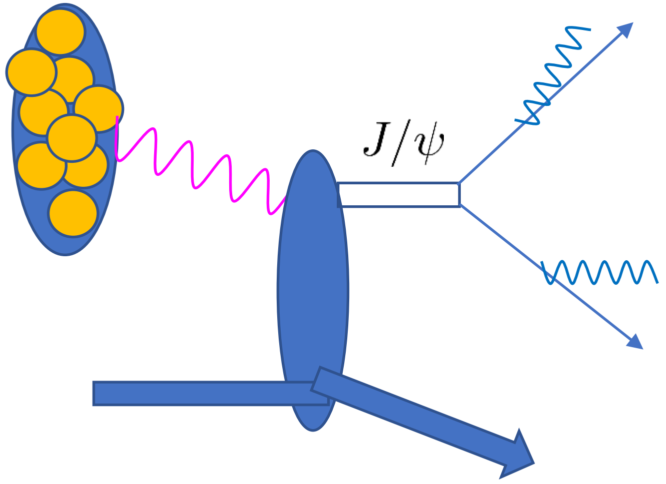
Soft photon resummation in UPC

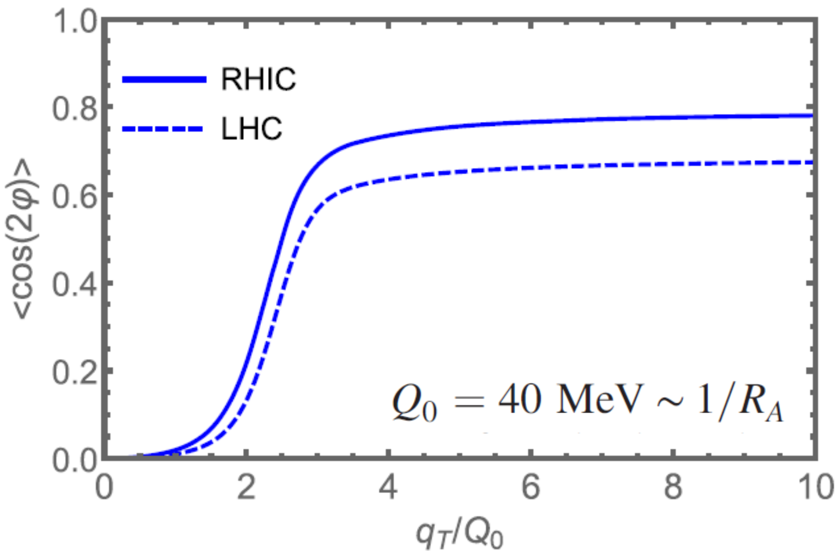


Linearly polarized photon→ $\cos 2\phi$ correlation in lepton pairs
Background from soft photon radiation

Large anisotropy $$\frac{c_2}{c_0} = \frac{\ln \frac{k_\perp}{2m}}{\ln \frac{2k_\perp}{m}} \sim \mathcal{O}(1) \qquad R \leftrightarrow m/k_\perp$$

Resummation Klein, Mueller, Xiao, Yuan (2020);
YH, Xiao, Yuan, Zhou (2021)





linearly polarized photon distribution dominates when

$$q_\perp < 100\,\mathrm{MeV}$$

At larger momentum, the final state radiation dominates.

Brandenburg et al, 2207.02478

Conclusions

- Multi-dimensional tomography (TMD, GPD, Wigner) important theme at the EIC, preview at sPHENIX
- New distributions often probed in jet angular correlations.
- Things took an interesting turn after the CMS measurement. Soft gluon radiations can diminish the signal. Interesting in its own right.
- Dijet total momentum $\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ very sensitive to higher order corrections, cannot be a proxy for $\vec{\Delta}_\perp$