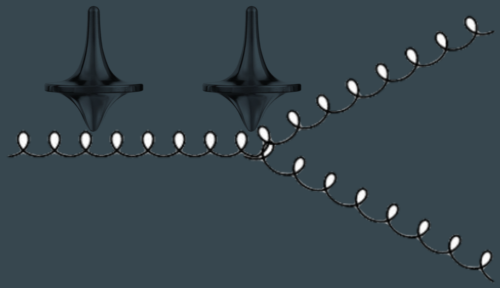
Quantum entanglement in hadronization

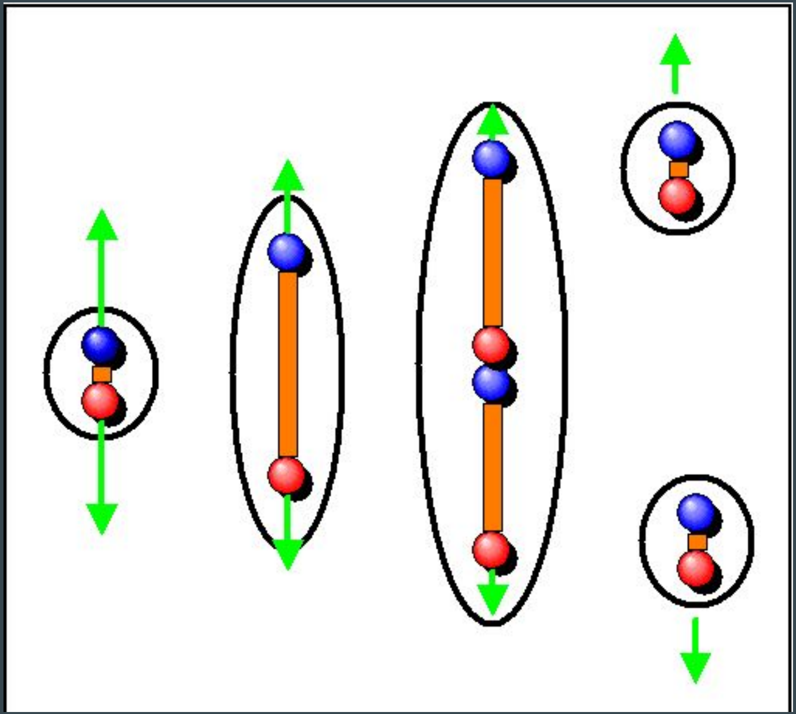
Kong Tu
BNL
09.21.2022



A big question in QCD

Confinement - we know it exists but we do not know why or details, e.g., how does hadron form (from partons)?

Theoretically (or mathematically), this is part of a millennium prize problem (CMI);



Confinement is elusive, entanglement is not (as much)

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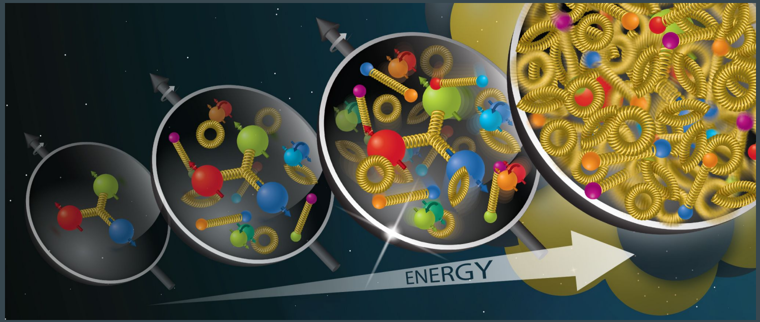
For example, a proton wave function CANNOT be written in terms of its subsystems in a *separable* way, (e.g., position, momentum, or spin, etc.)

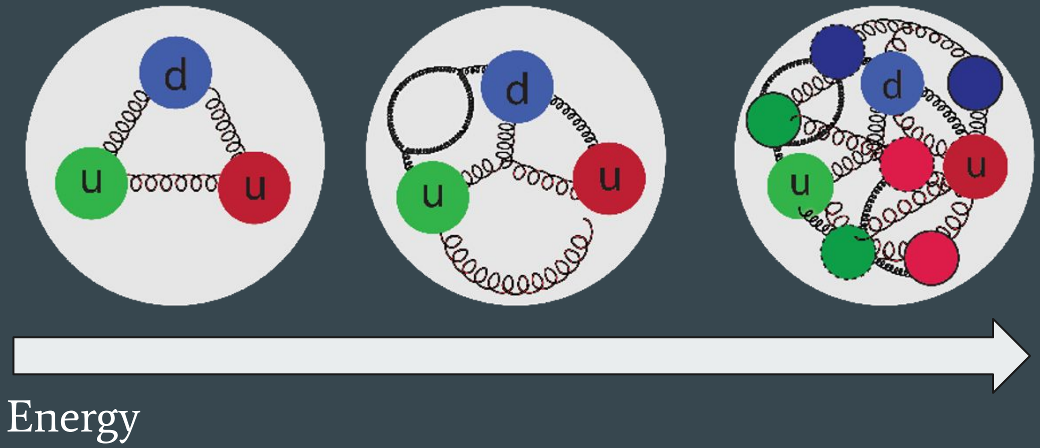
$$|\psi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle \neq$$

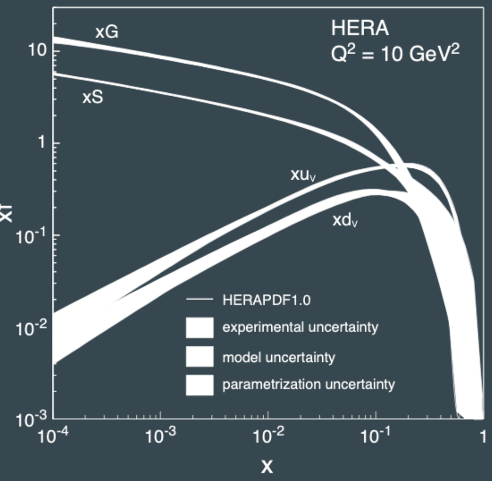


Therefore, it's quantum entangled.

Entanglement in hadron - is not an IF question but more *what it can tell us*!

One application of entanglement





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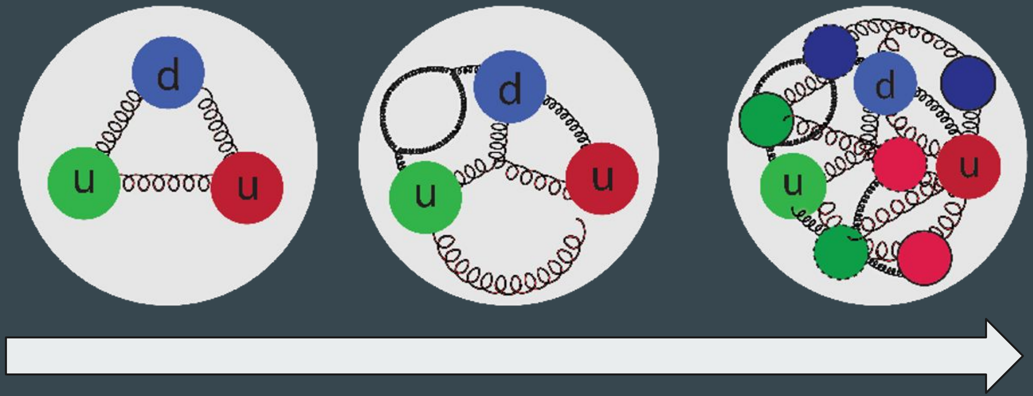
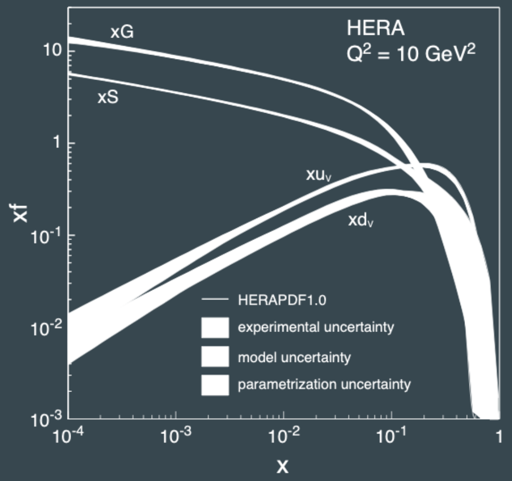


Energy

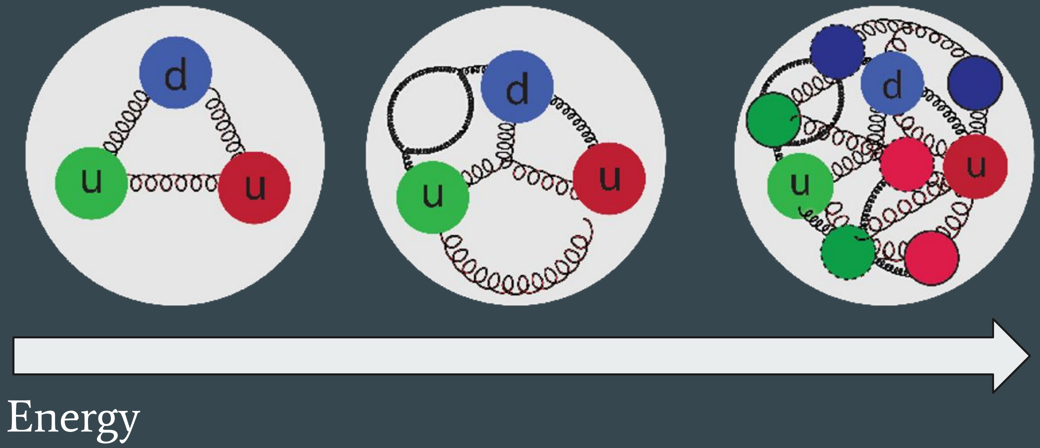


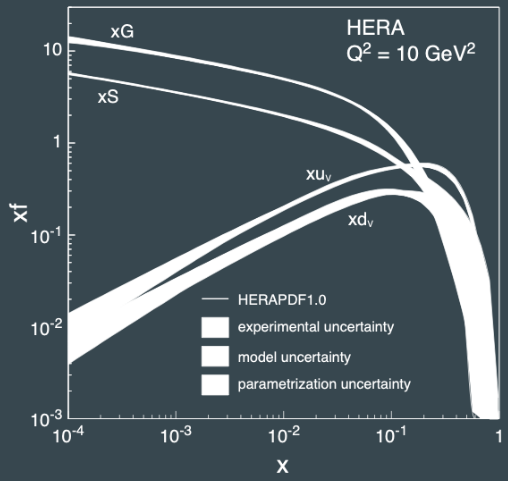
At low-x, it was found that the parton density (PDFs) and entanglement entropy are related, where entanglement entropy (EE) is a quantitative measure on how much a system is entangled.

$$S_{\text{partonic}}\left(x,Q^2\right)=\ln\left[xg\left(x,Q^2\right)+x\Sigma\left(x,Q^2\right)\right]$$

[Kharzeev, Levin (2017,2021), Hentschinski, Kutak (2021)]

One application of entanglement





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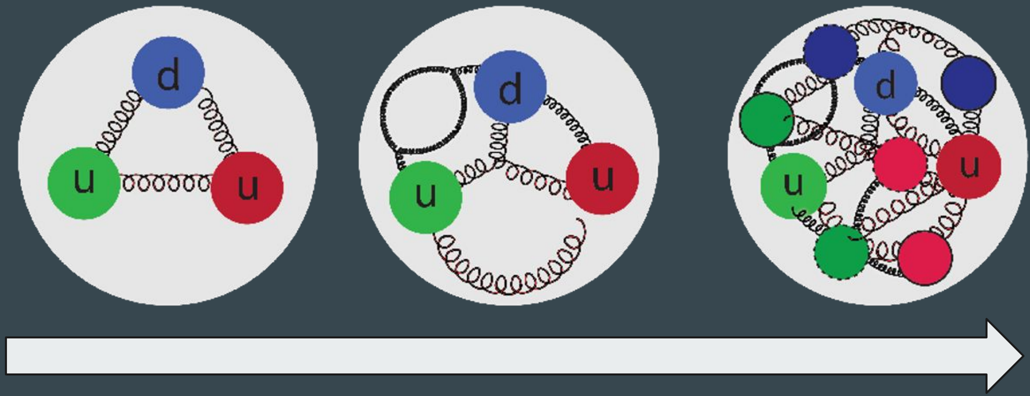
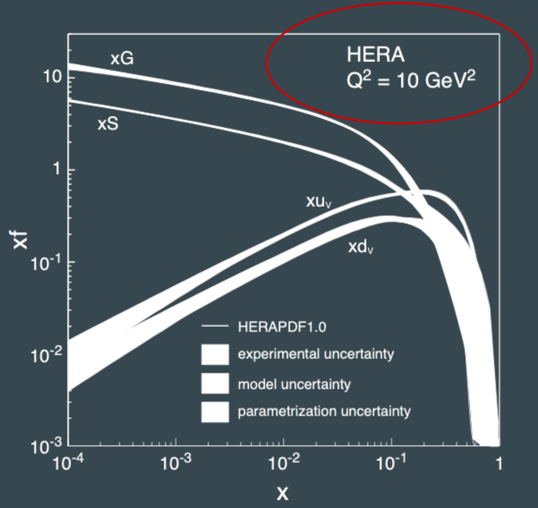
Lesson (1): at low-x, the proton is maximally entangled

One application of entanglement



Energy



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Lesson (2): at low-Q^2, EE applies to non-perturbative regime unlike PDFs

A quick example

Von Neumann entropy is defined by:

$$S = -tr[\rho ln \rho],$$

where ρ *is density matrix*

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(i) $|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right]$

$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \cdot \left[\langle\uparrow|_A + \langle\downarrow|_A\right].$

Not Entangled

$S_A = 0$

(ii) $|\Psi\rangle = \left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right] / \sqrt{2}$

$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A\right]$

Entangled

$S_A = \log 2$

(http://www2.yukawa.kyoto-u.ac.jp/~tadashi.takayanagi/CERNEE.pdf)

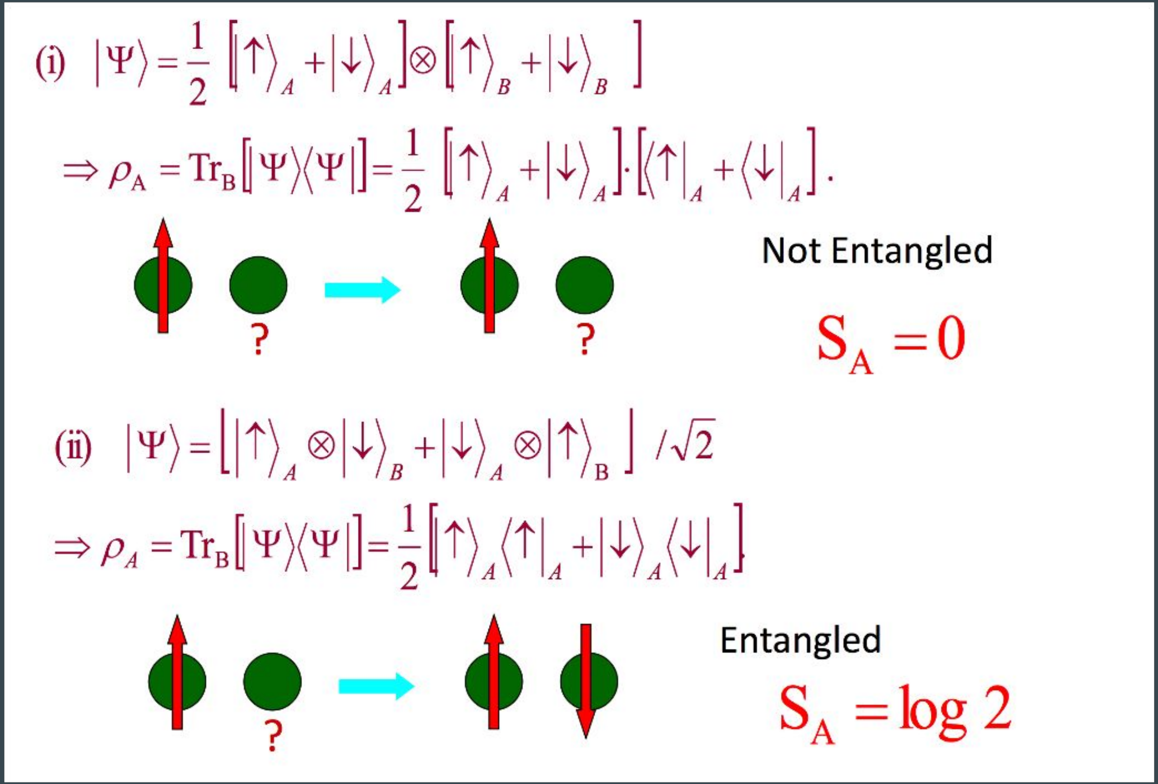
A quick example

Von Neumann entropy is defined by:

$$S = -tr[\rho ln\rho],$$

where ρ is density matrix

"Entanglement Entropy is a measure of how much a given state is quantum mechanically entangled…"



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How to measure EE?

EE has been proposed to be measured via hadron entropy [Kharzeev, Levin (2017)], [Tu, Kharzeev, Ullrich (2020)]

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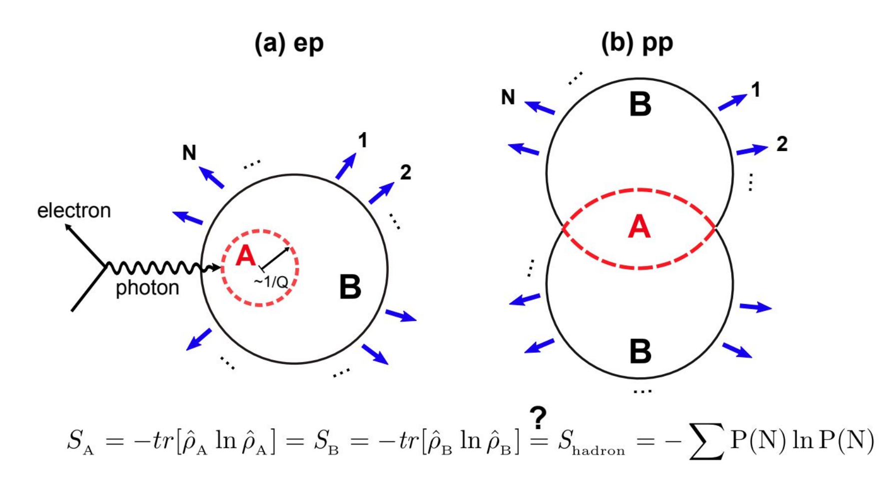

Phys. Rev. Lett. 124, 062001

Caveats:

- "Local parton-hadron duality"
- In DIS or minimum-biased pp, the system's entropy does not increase much.

EE should be less or equal to hadron entropy in the final state

$$S_B = -\sum P_N \log P_N$$

Observable of hadron entropy - Charge particle multiplicity distribution P(N)

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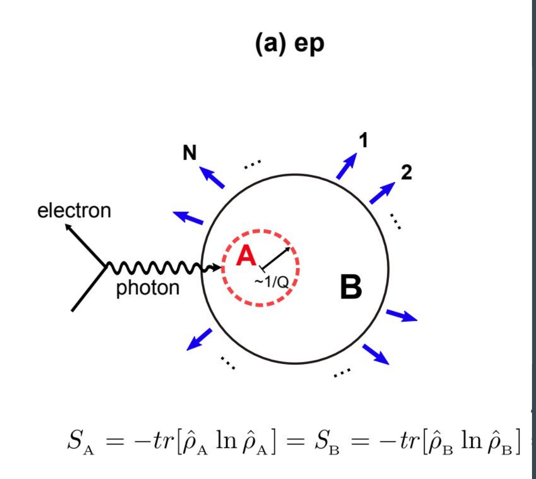
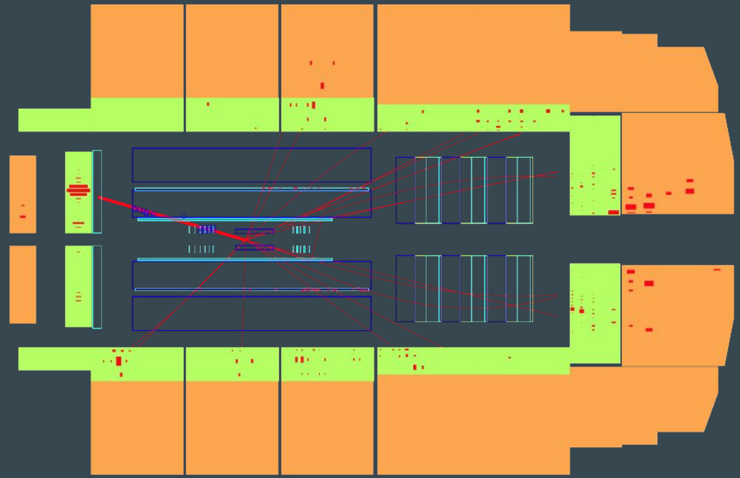


Phys. Rev. Lett. 124, 062001

H1 @ HERA event display ep 27x920 GeV DIS



Electron going

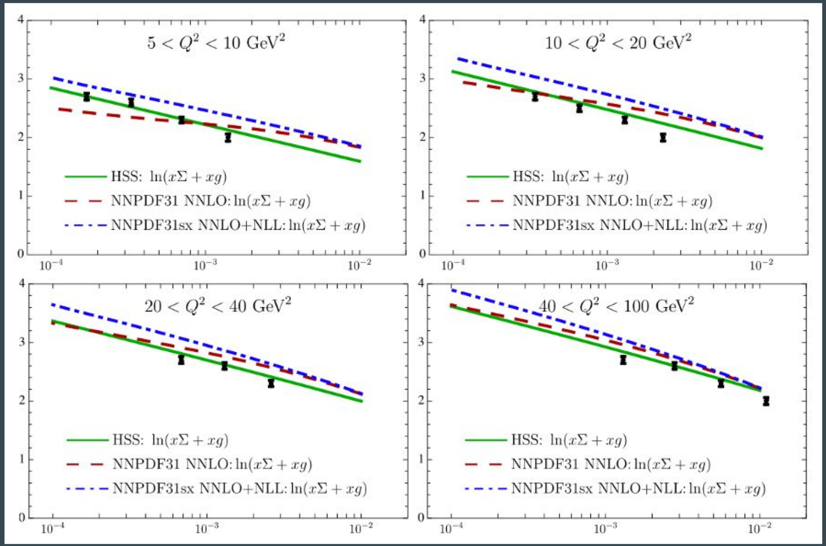
Proton going

Observable of hadron entropy - Charge particle multiplicity distribution P(N)

H1 data supports the entanglement picture

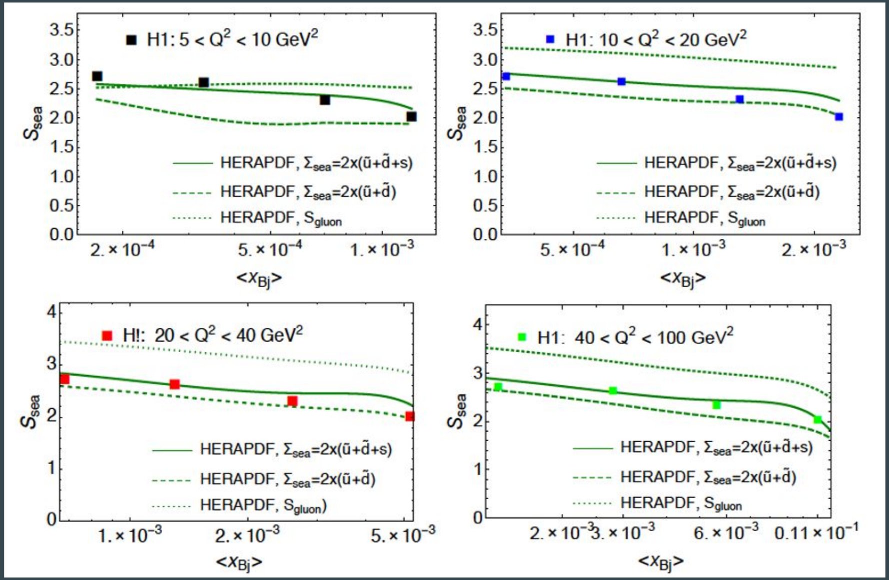
Different groups have made the predictions/calculations on the hadron entropy, which has been found to be consistent with the data. Data is from *Eur. Phys. J. C (2021) 81: 212*

Hentschinski, Kutak (2021)



Eur.Phys.J.C 82 (2022) 2, 111

Kharzeev, Levin (2021)



Phys.Rev.D 104 (2021) 3, L031503

So what?

We can use EE as a powerful tool to learn more

Data from ep DIS and pp hadronic collisions supported the entanglement description of the initial-state of the proton. There's more to it:

- EE has many known properties - more insights to nucleon and nuclear structure?
- How about going to non-perturbative regime, e.g., photoproduction? Nuclei and eA in EIC?
- *Maximally* entangled proton has a natural interpretation/connection to gluon saturation in the non-perturbative regime.

$$xG(x) \leq \text{const}\ \frac{1}{x^{1/3}}.$$

Kharzeev, Levin (2017)

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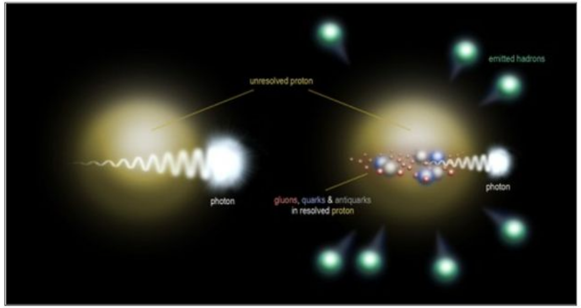
Interior of protons is maximally entangled

17 March 2022

EurekAlert!: [https://www.eurekalert.org/news-releases/946725]



If a photon carries too little energy, it does not fit inside a proton (left). A photon with sufficiently high energy is so small that it flies into the interior of a proton, where it 'sees' part of the proton (right). Maximum entanglement then becomes visible between the 'seen' and 'unseen' areas. (Source: IFJ PAN)

Fragments of the interior of a proton have been shown by scientists from Mexico and Poland to exhibit maximum quantum entanglement. The discovery, already confronted with experimental data, allows us to suppose that in some respects the physics of the inside of a proton may have much in common not only with wellknown thermodynamic phenomena, but even with the physics of... black holes.

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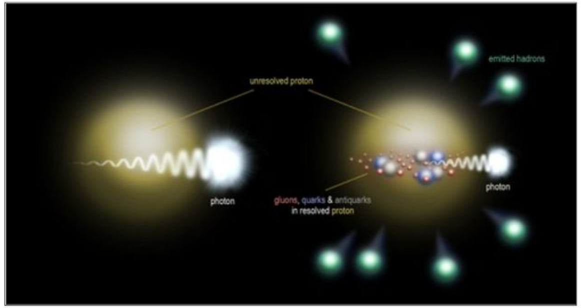

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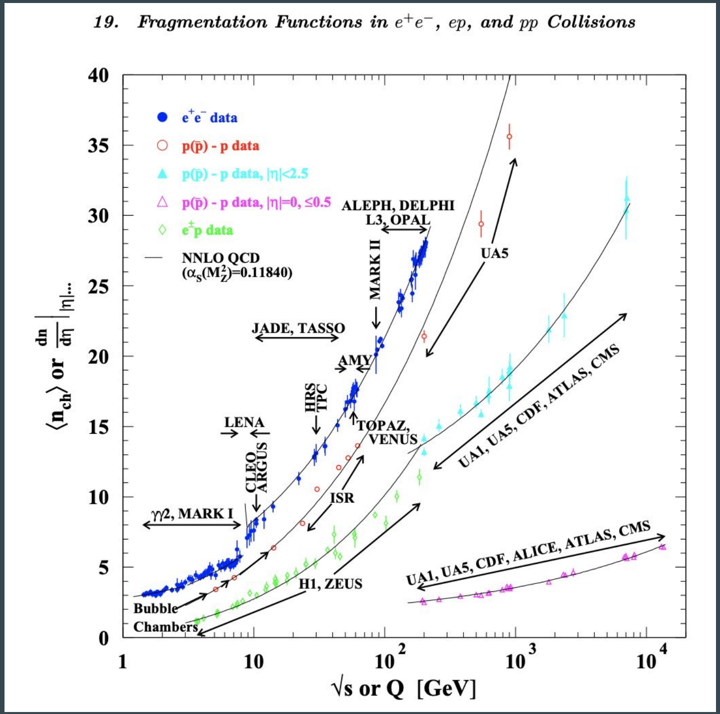
One major problem we have avoided - What role does the hadronization play?

PDF$\otimes$FF ≈ particle production

PDF⊗FF ≈ particle production

- EE seems to avoid the problem of hadronization/fragmentation by looking at the system as a whole, in terms of entropy.
  - *The system is highly/maximally entangled, which manifests itself in the total particle production or particle multiplicity distribution.*

PDG 2019 Fragmentation Function Review



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    - What can entanglement of two partons tell us about the hadron structure?
    - How does entanglement manifest in hadronization?

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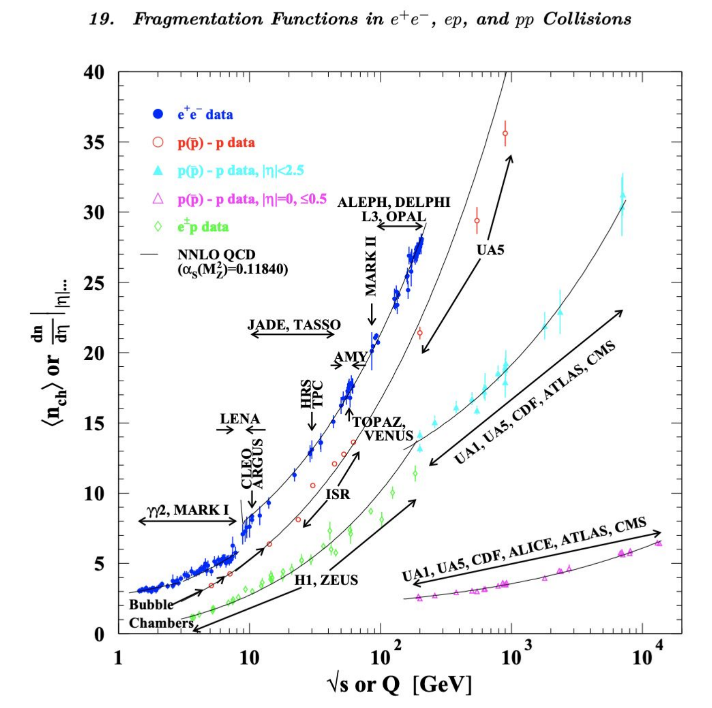

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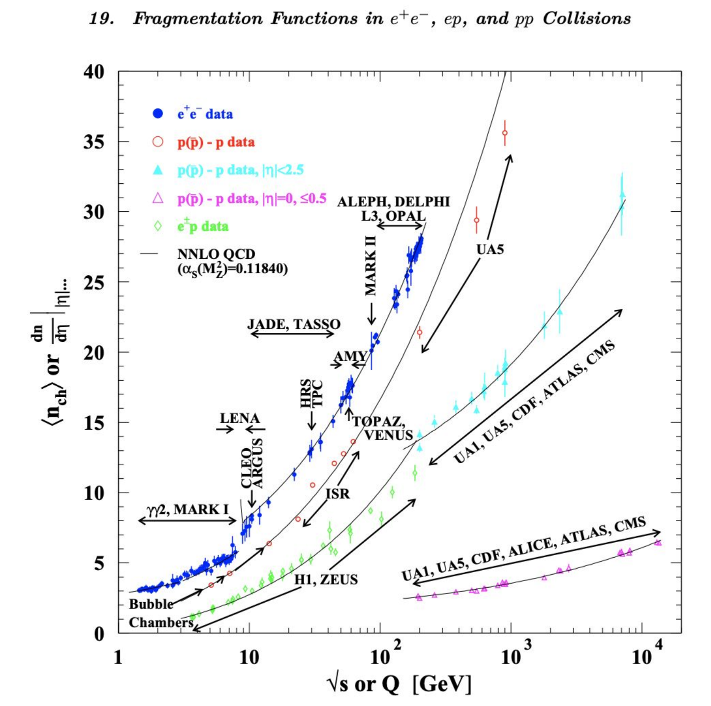
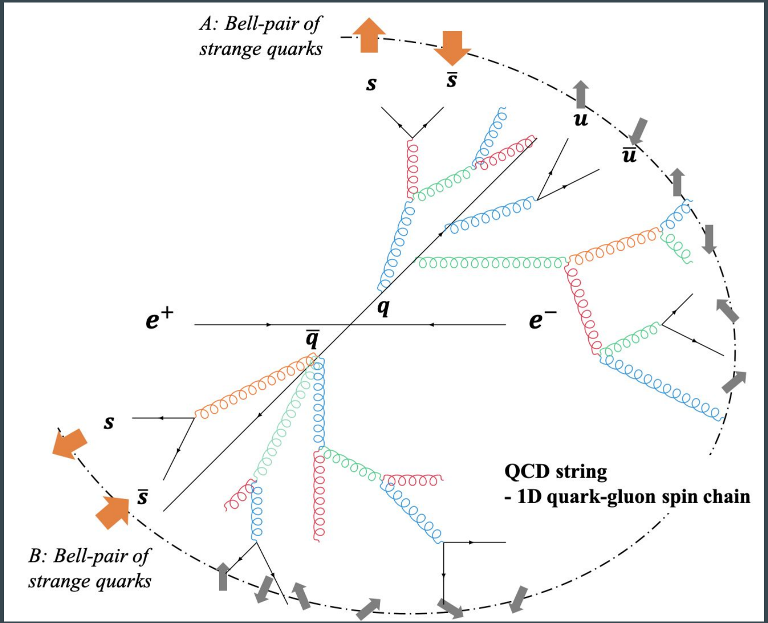


We have a new idea and proposal! 

parton spin entanglement in non-perturbative QCD strings

Assume N pairs of strange quark singlets, where their spin are quantum entangled - e,g., Bell pairs.



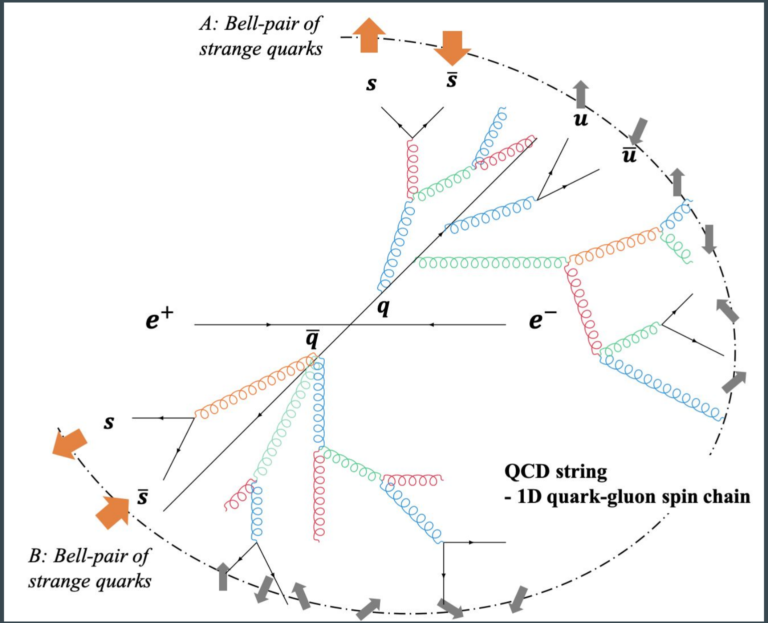
Same idea applies in DIS or pp collisions

parton spin entanglement in non-perturbative QCD strings

Assume N pairs of strange quark singlets, where their spin are quantum entangled - e,g., Bell pairs.

Questions to ask:

1. Is there a nonlocal effect between two strange quarks (s and/or sbar)?
2. How does the quark spin interact with the QCD string? How does it manifest in hadronization?
3. Where does the the quantum-to-classical transition take place and how?

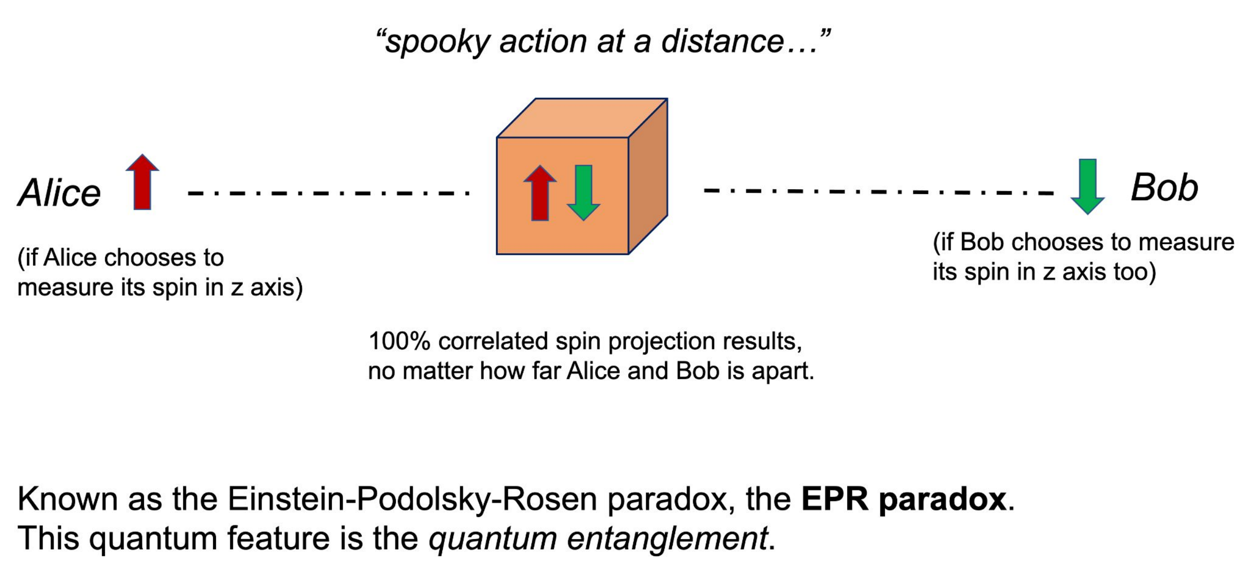


Same idea applies in DIS or pp collisions

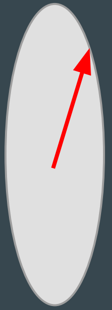
With a given entanglement, the goal is to see how it evolves/interacts with the QCD strings

CHSH inequality test

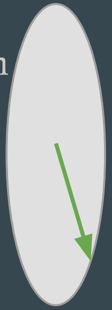
The EPR paradox:

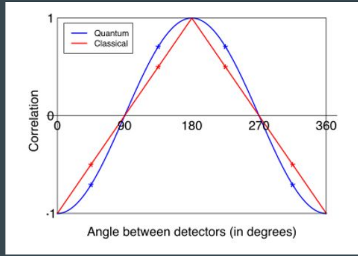


CHSH test:



Angle Θ_{ab} between the chosen axis of Alice and Bob

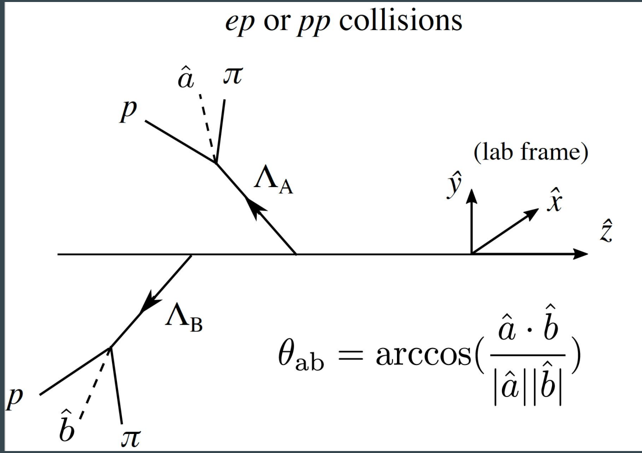




Similarly, we can try to use two entangled Λ particles in high energy collisions

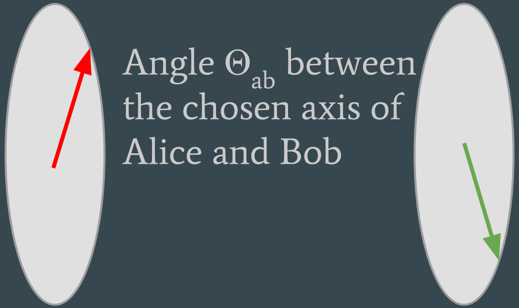
The beauty of Λ particle

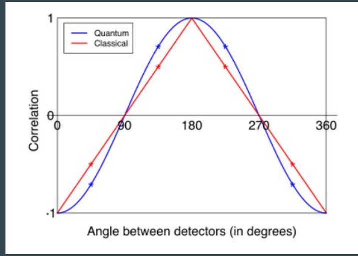
Λ polarization - the daughter particle's momentum in Λ's rest frame represents the spin of the Λ, and this "*chosen*" axis is not picked by experimentalists. Purely controlled by Λ's weak decay [Tornqvist 1981]



← Our proposal

CHSH test:





Of course, the measurability depends on the weak decay constant, etc.

A simple model on the quantum computer

Phys.Rev.D 106 (2022) 3, L031501

A modified CHSH inequality test with mixed state [Gong, Parida, Tu, Venugopalan 2022]:

$$\frac{P(|\hat{n}_1\rangle, |\hat{n}_2\rangle)}{P(|\hat{n}_1\rangle)P(|\hat{n}_2\rangle)} = 1 - \frac{a}{(a+b/2)^2}\cos(\theta_2 - \theta_1)$$

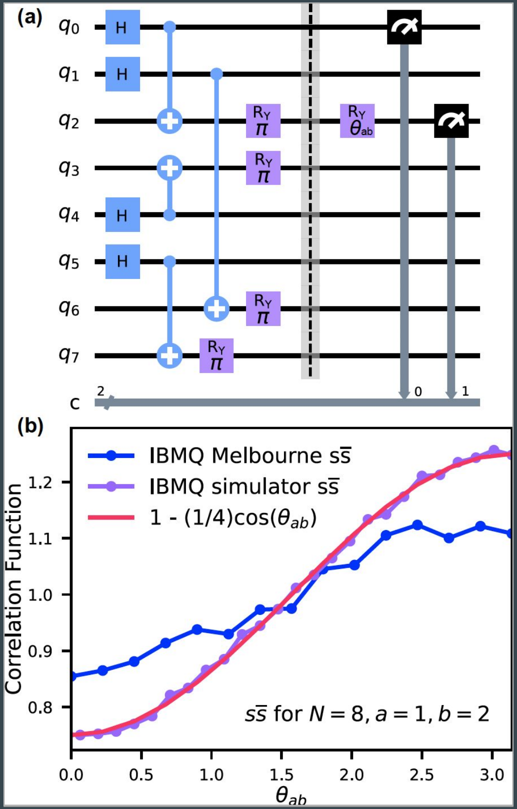
Correlation function   Angle* between the Λ daughters

a = #strange-antistrange pairs; b =#(light) quark pairs

Conclusion from this study:

A symmetric, rotationally invariant correlation function implies that the measured state violates its related CHSH inequality.

[Here we use strange quark to be equivalent to Λ particle]



Real-time evolution - the power of quantum computation

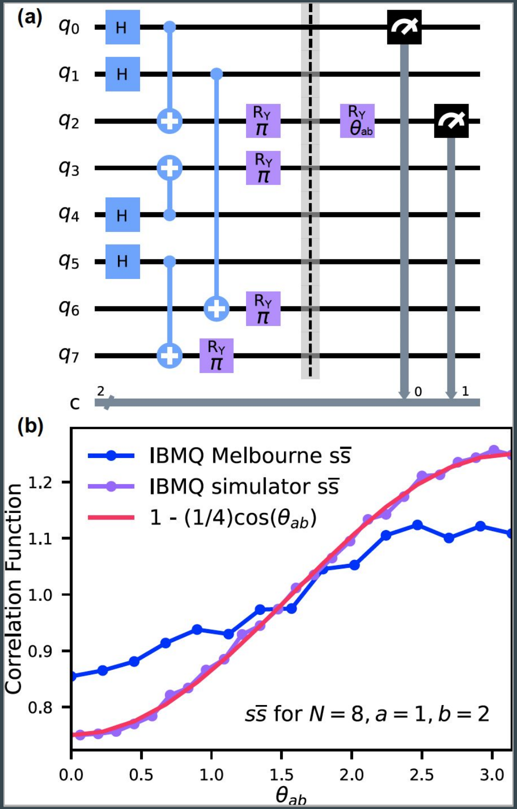
Phys.Rev.D 106 (2022) 3, L031501

Based on the simple setup, the power is to utilize quantum hardware to study a *real-time evolution* of the QCD string by mapping this problem to some known models[Barata, Gong, Tu, Venugopalan, work in progress].

An simple example:

$$H_{\text{Anderson}} = \sum_{k\sigma} \epsilon_k a^\dagger_{k\sigma} a_{k\sigma} + \epsilon_d d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow + \frac{\eta}{\sqrt{V}} \left(d^\dagger_\sigma a_{k\sigma} + a^\dagger_{k\sigma} d_\sigma \right) .$$

https://en.wikipedia.org/wiki/Anderson_impurity_model



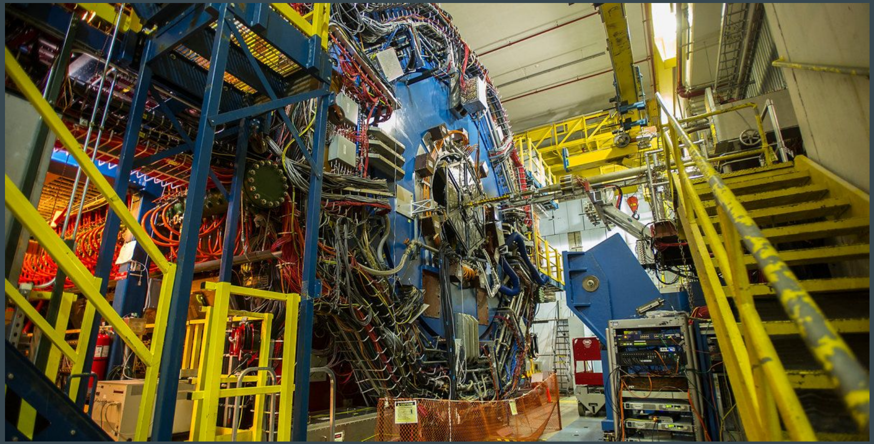
Other candidates are RKKY, Kondo model, etc.

Experiments & Measurements

- High energy experiments, e.g., Relativistic Heavy-Ion Collider (RHIC), can run different nuclear species, proton-proton collisions

- Future Electron-Ion Collider

- We can directly measure our proposed modified CHSH inequality:
    - Can Λ be polarized by their parton spin correlation?
    - Can we observe a clear experimental quantum-to-classical transition using Λ?



Ongoing search for double Λ polarization using STAR detector

(led by Jan Vanek, BNL)

This measurement will shed new light on non-perturbative QCD and hadronization

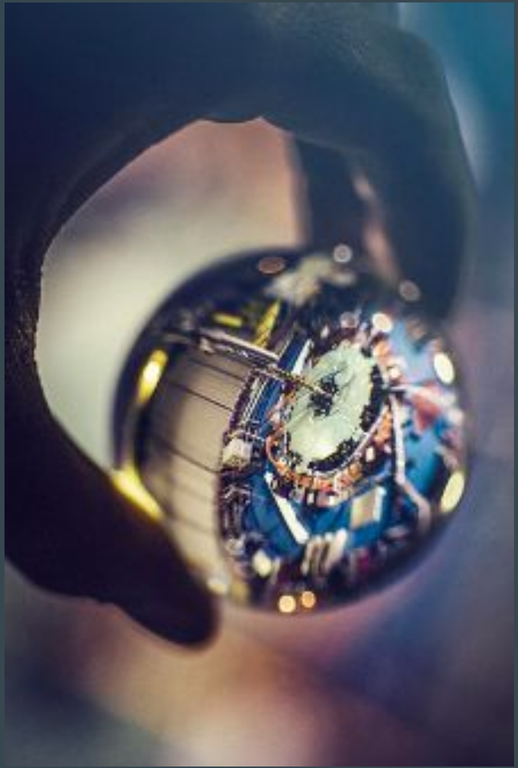
Summary and outlook

- Entanglement entropy has been found to be a great tool to study non-perturbative QCD in hadron structure
  - Established link between PDF and entanglement entropy
  - Data supported the picture of entanglement
  - Many applications can be followed.

- New direction: **Λ spin entanglement might be a new key to study the role of hadronization in non-perturbative regime, dynamics of QCD strings, and real-time evolution.**

  STAR experiment is actively searching for the spin entanglement. Stay tuned!

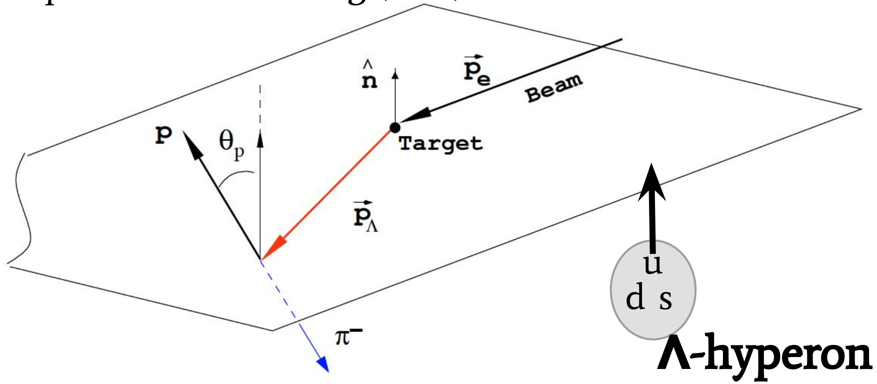
- Opportunities to utilize quantum hardware to study real-time evolution of QCD string, which provides more insights to the quantum many-body problem in high energy physics.



Backup

A related question, a longtime puzzle

Example: HERMES experiment of ep deep inelastic scattering (DIS).



Λ-polarization has been observed in p+p, NC/CC DIS, e+e, p+A, and AA* collisions.

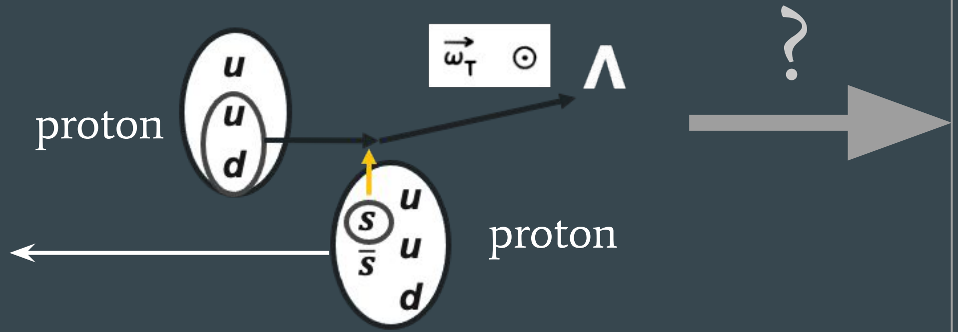
* only in heavy-ion AA collisions, people think it's of different origin.

Λ-hyperon polarization in high energy collisions was discovered ~ 50 years ago - still remains unknown to-date.

- when Λ is produced from *unpolarized target* (e.g., proton) or even e+e, Λ is polarized.
- however, the Λ is not polarized w.r.t the target when target is polarized.
- **Why?**

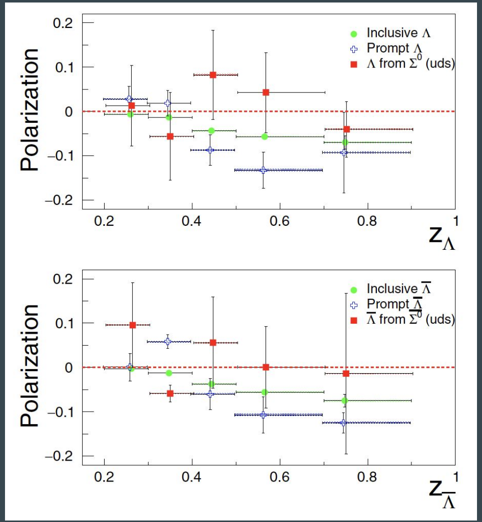
Polarization is a hadronization problem?

- Semi-classical model - Thomas spin precession effect in the quark-recombination process. (Degrand, Miettinen 1981)



- Although simple, it explains a lot of data!

Belle experiment e+e annihilation



PHYSICAL REVIEW LETTERS 122, 042001 (2019)

Popular trend: polarising Fragmentation function in the TMD frameworks

What role does the quark spin play?

What we learned so far:

- Λ polarization seems to have nothing to do with the target (species, spin, etc)!
- Λ polarization seems to relate to quark/hadron trans. momentum but not its spin!

Interesting questions to ask:

- Does the quark's spin have any effect in hadron polarization, or hadron formation?
- Is hadronization a process of quantum-to-classical transition?
- Can we use one observable to study both? Namely hadronization and polarization
  Two birds, one stone.

Spin entanglement measurement of Λ may help shine light on this ~50 years puzzle