Hadronization at Belle (II)

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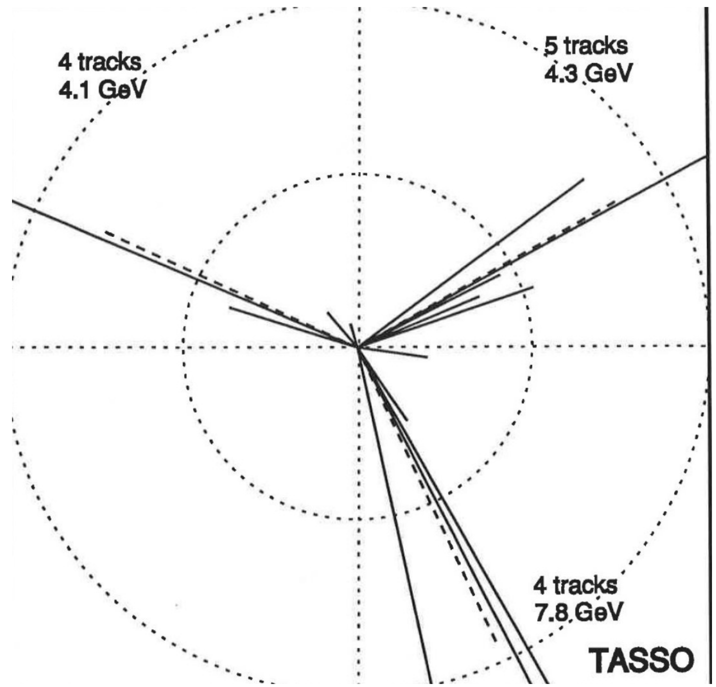
Duke UNIVERSITY
Jefferson Lab

Agenda

- Overview of Belle →Belle II
- Some recent highlights of the Belle program
  - Focus on overlap between perturbative and non-perturbative regimes
  - Non-perturbative parametrizations (FFs) ←→ models
- Outlook to Belle II program
- →"Opportunities for precision QCD physics in hadronization at Belle II -- a snowmass whitepaper", e-Print: 2204.02280 [hep-ex]

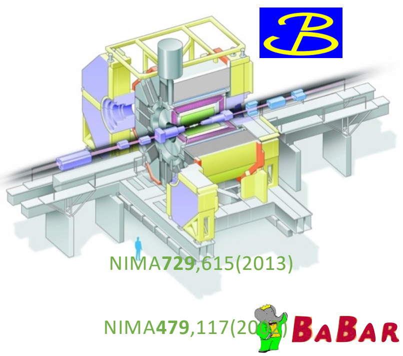
e^+e^- has a long history in studying QCD

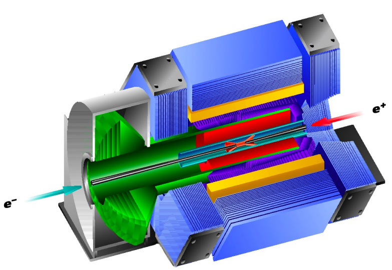
- PETRA at DESY →Discovery of the gluon (shown 1979)



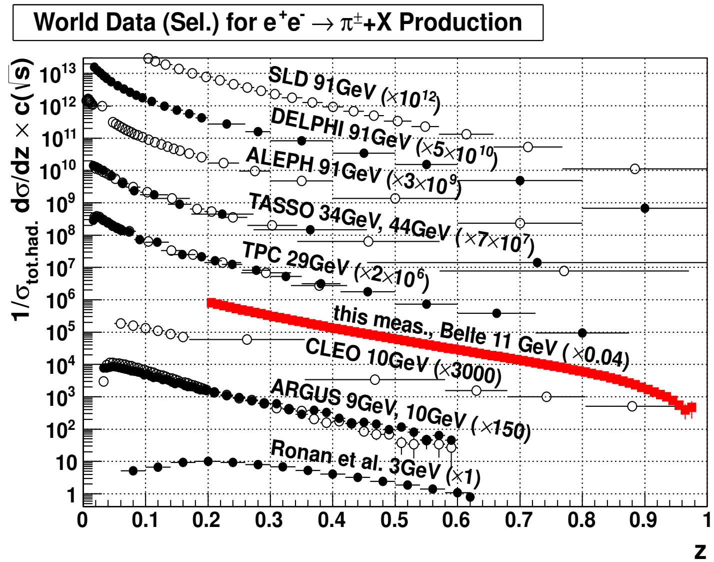
Role of *B*-factories

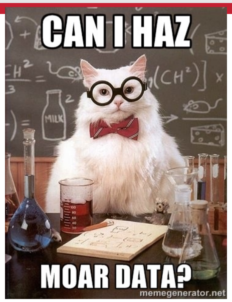
- Asymmetric-energy e^+e^- collider
- $\sqrt{s} \sim 10.6$ GeV ($\Upsilon(4S)$)
- $\beta\gamma = 0.425$
- $L \sim 1\ ab^{-1}$





- Asymmetric-energy e^+e^- collider
- $\sqrt{s} \sim 10.6$ GeV ($\Upsilon(4S)$)
- $\beta\gamma = 0.65$
- $L \sim 500\ fb^{-1}$

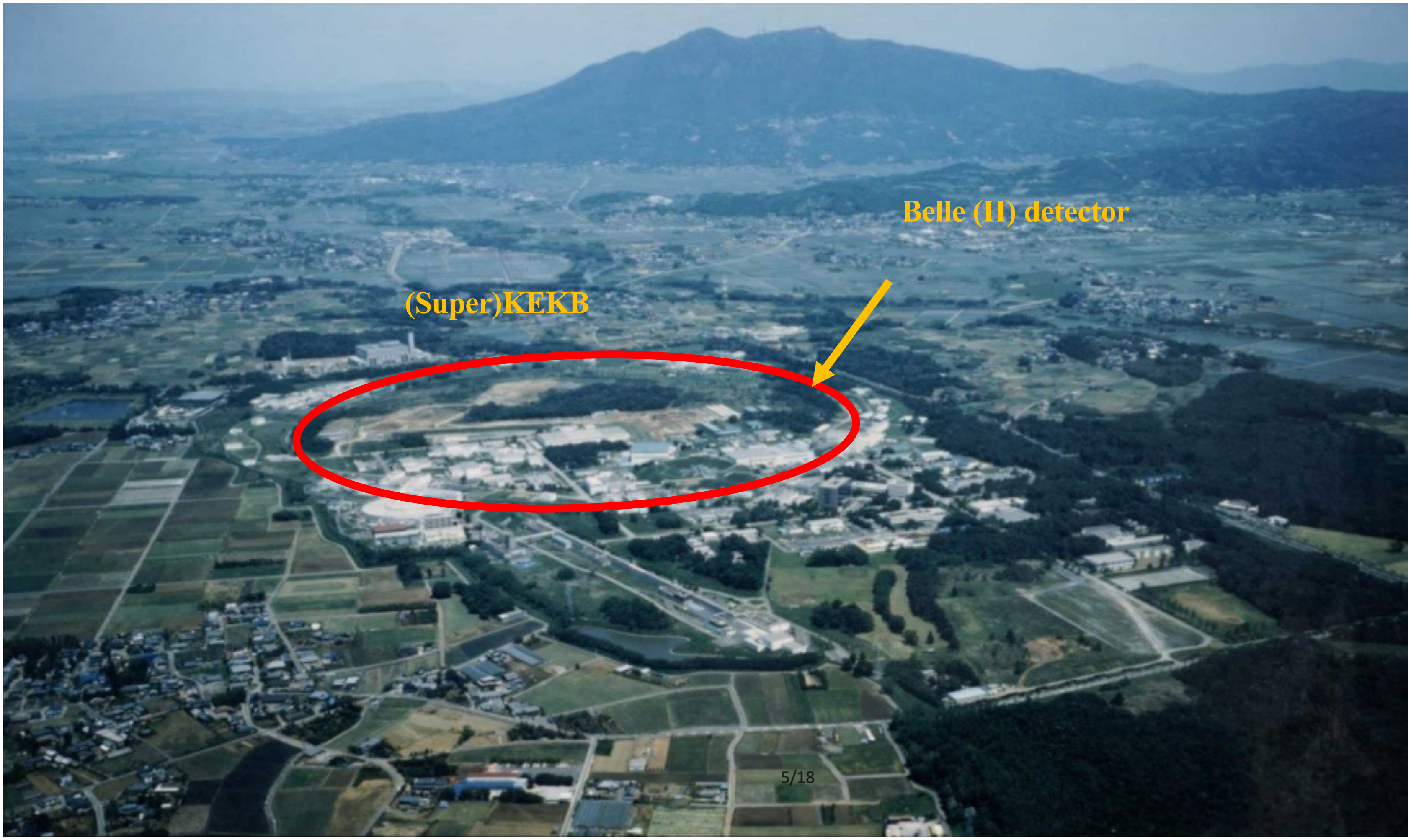




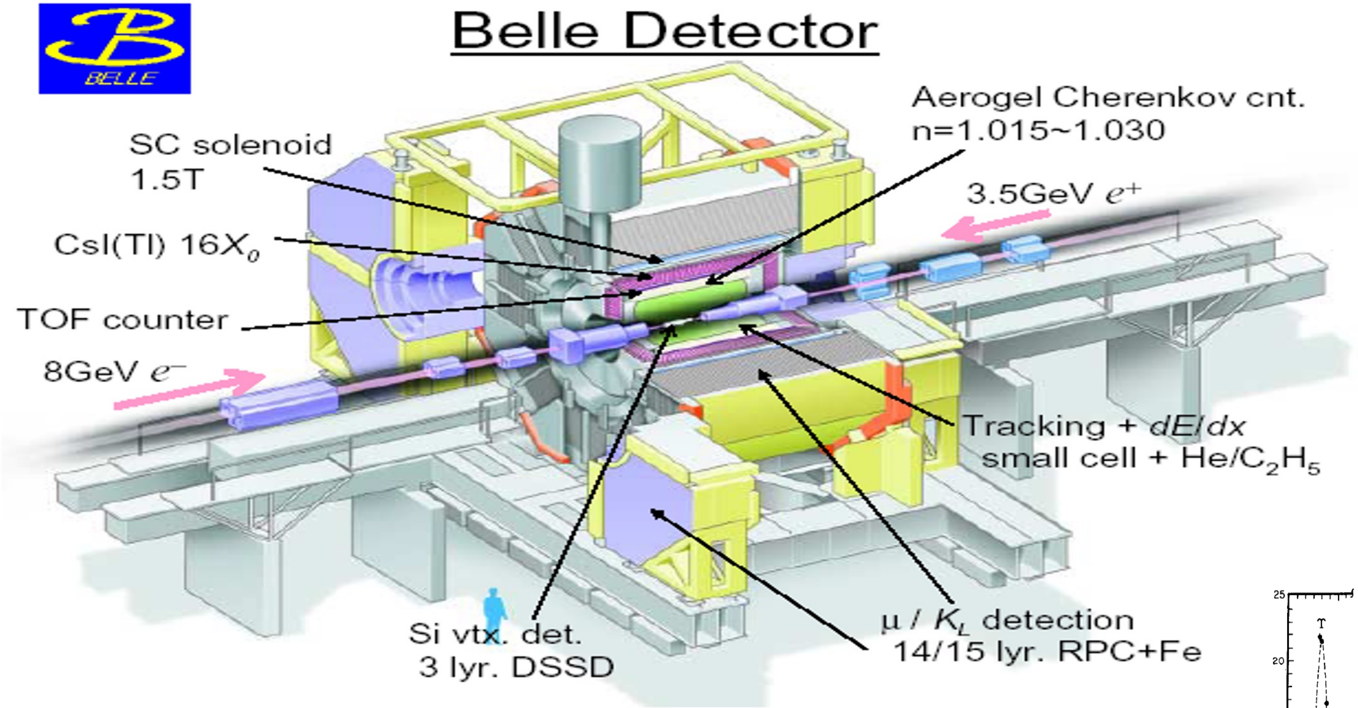
Phys.Rev.Lett. 111 (2013) 062002 (Belle)
Phys.Rev. D88 (2013) 032011 (BaBar)

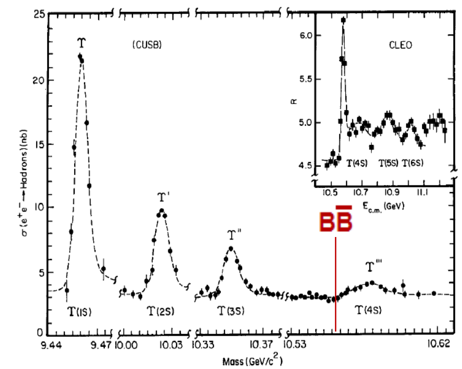
- Dominated by B factories
- Limited lever arm in $\sqrt{s}$ in particular at high z
- Precision data includes charged single hadrons π, K, p, D baryons…
- Well described at NNLO (e.g. DSS, NNFF)

KEK facility



Belle Experiment (1999 - 2010)





+About 4x10^6 events per fb in continuum

Experiment	Scans/ Off. Res. fb^{-1}	$\Upsilon(5S)$ 10876 MeV fb^{-1}	10^6	$\Upsilon(4S)$ 10580 MeV fb^{-1}	10^6	$\Upsilon(3S)$ 10355 MeV fb^{-1}	10^6	$\Upsilon(2S)$ 10023 MeV fb^{-1}	10^6	$\Upsilon(1S)$ 9460 MeV fb^{-1}	10^6
CLEO	17.1	0.4	0.1	16	17.1	1.2	5	1.2	10	1.2	21
BaBar	54	R_b scan		433	471	30	122	14	99	–	
Belle	100	121	36	711	772	3	12	25	158	6	102

The future is now: Next Generation B factory SuperKEKB

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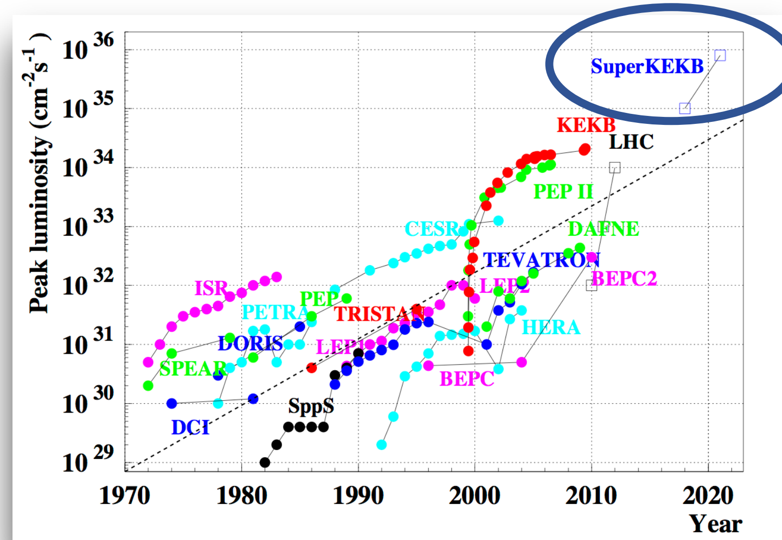
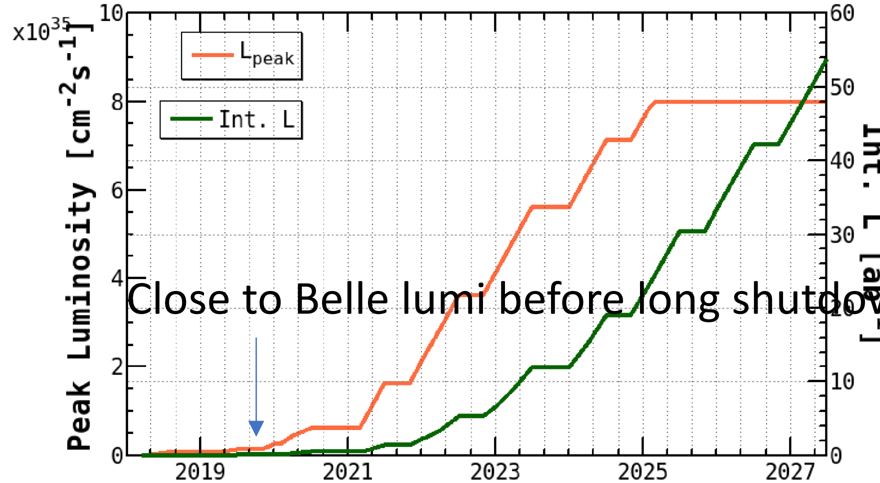


Belle/KEKB recorded ~1000 fb^{-1} . Now have to change units on the y-axis to ab^{-1}

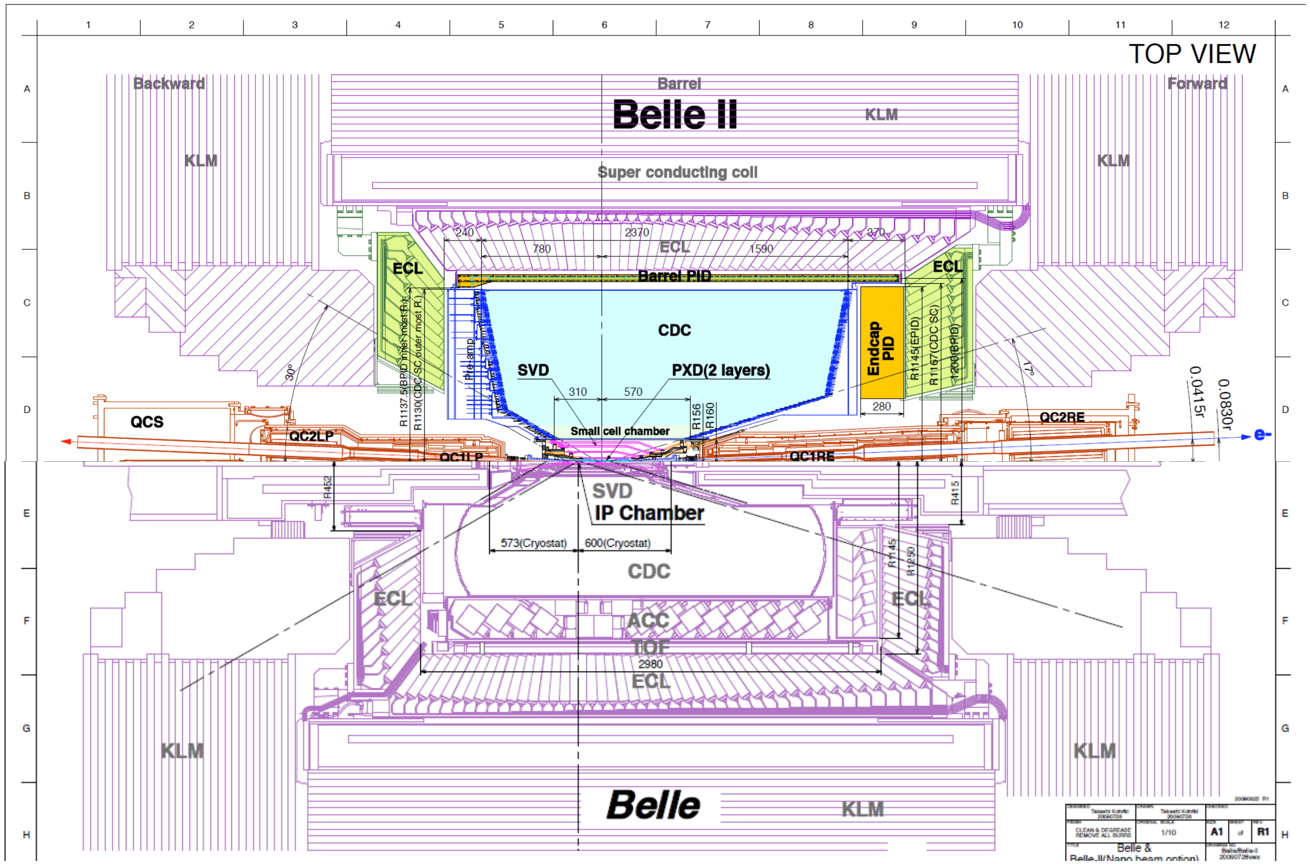


Close to Belle lumi before long shutdown

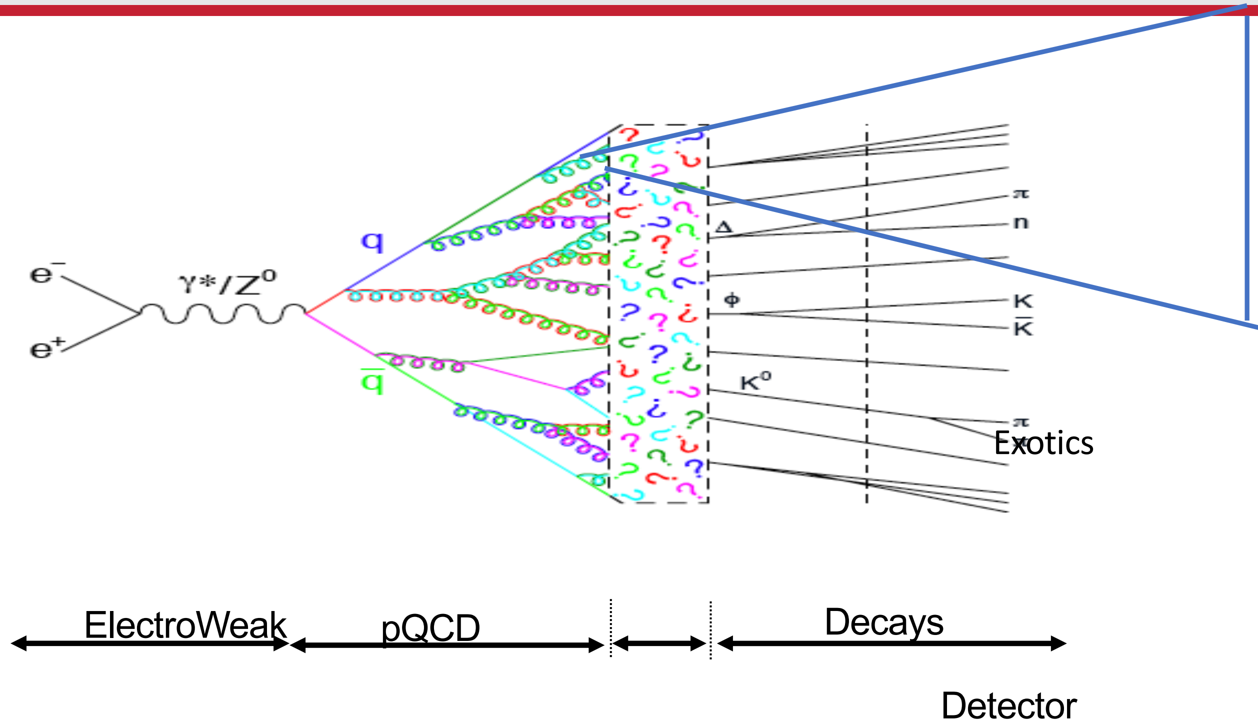
Beam currents *only* a factor of two higher than KEKB (~PEPII)

“nano-beams” are the key; vertical beam size is 50nm at the IP

Belle II Detector (comp. to Belle)



Hadronic Physics in e^+e^- B factories



- Study of hadronization factorized picture
  - Jets →integrate over many NP effects
  - hadronization dynamics→Fragmentation functions
  - Models
- Non-factorized
  - MCEGs
- Study how microscopic quark properties ←→ macroscopic hadron properties
- Study of the produced hadrons, spectroscopy

Non-perturbative effects in hadronization can be described in different frameworks

Factorized pQCD→Fragmentation Functions

- Essential for the extraction of PDFs
- Interpretable in QCD
- Focus on more '**inclusive**' measurements → factorization holds
- Recent activity in more exclusive measurements (in particular jets)
- Very precise extractions→Benchmark for MCEGs

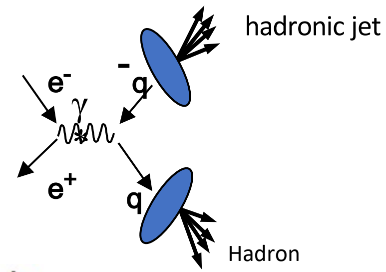
Hadronization Model in MCEG

- Exclusive final states
- "Hard" subprocesses well constrained by theory
- Measurements focusing on MCEG improvement different from measurements extracting hard physics (grooming) or FFs (more exclusive)
- Needed for FF extraction

- **Both needed**
- **Both need input from e^+e^-**

Access of FFs for light mesons in e^+e^- (spin averaged case)

- $$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{e^+e^-\to hX}}{dz} := \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2)+F_L^h(z,Q^2)\right),$$

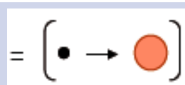
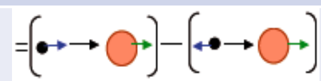
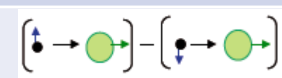
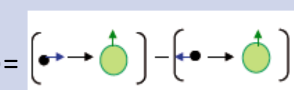
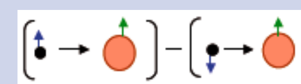


$$2F_1^h(z,Q^2) \;=\; \sum_q e_q^2\left(D_1^{h/q}(z,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\left(C_1^q\otimes D_1^{h/q}+C_1^g\otimes D_1^{h/g}\right)(z,Q^2)\right)$$

- Cleanest process→testbed for QCD calculations
- Limited access to flavor
  - (Use different couplings to γ^* and Z^0)
  - (Use polarization (SLD) and parity violating coupling)
  - **Use back-to-back correlations for different flavor combinations**
- Limited access to gluon FF
  - From evolution
  - From three jet events (but theory treatment not clear)

Single Hadron FFs

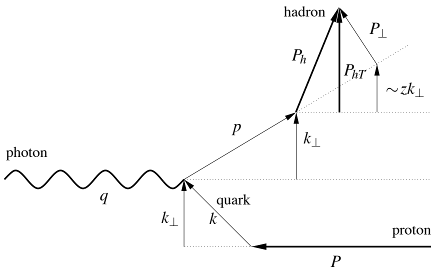
- Analogue →similar to PDFs encoding spin/orbit correlations
- Determining final state polarization needs self analyzing decay (Λ)
- Gluon FFs similar but with circular/linear polarization (not as relevant for e+e-)

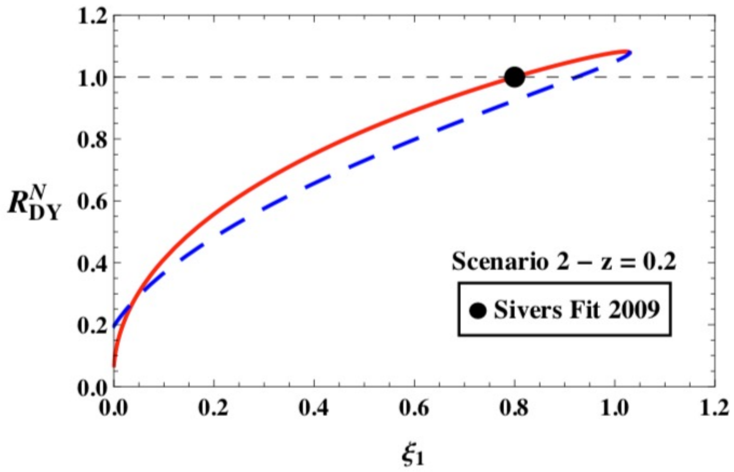
Parton polarization → Hadron Polarization ↓	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z, p_T)$ = 		$H_1^{\perp h/q}(z, p_T)$ = 
longitudinal		$G_1^{\Lambda/q}(z, p_T)$ = 	$H_{1L}^{h/q}(z, p_T)$ 
Transverse (here Λ)	$D_{1T}^{\perp\Lambda/q}(z, p_T)$ = 	$G_{1T}^{h/q}(z, p_T)$= 	$H_1^{\Lambda/q}(z, p_T)$ =  $H_{1T}^{\perp\Lambda/q}(z, p_T)$ = 

- Needed to access (spin dependent) parton structure of the nucleon
- Can probe fundamental QCD questions (e.g. $D_{1T}^{\perp} \leftrightarrow f_{1T}^{\perp}$)

Need for precise extraction of TMD FFs to extract TMD FFs

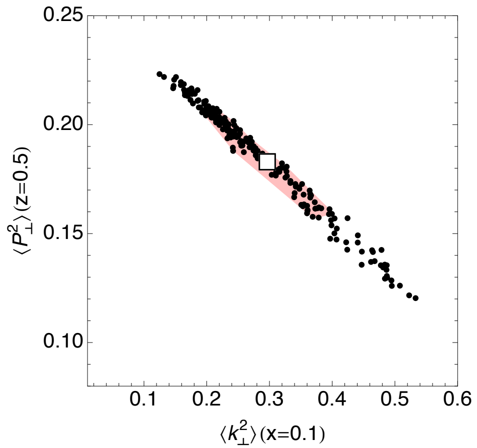
- Example: variation in Sivers effect varying $\xi = \frac{k_T}{p_T}$





Phys.Rev. D98 (2018) no.9, 094023

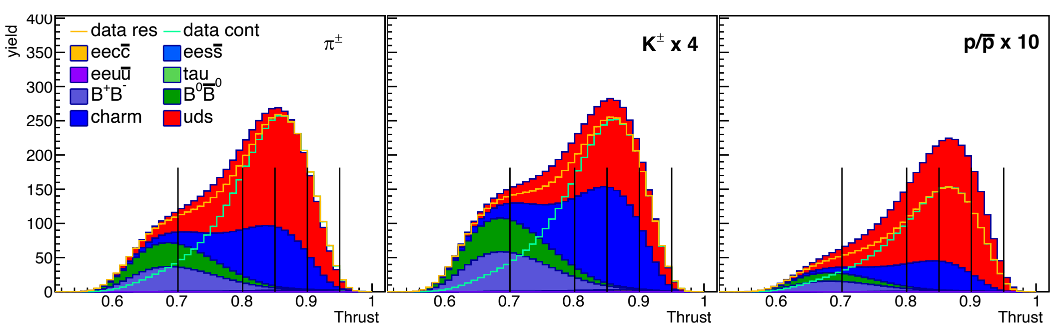
- SIDIS necessary to extract flavor dependence of FFs but cannot disentangle source of pT

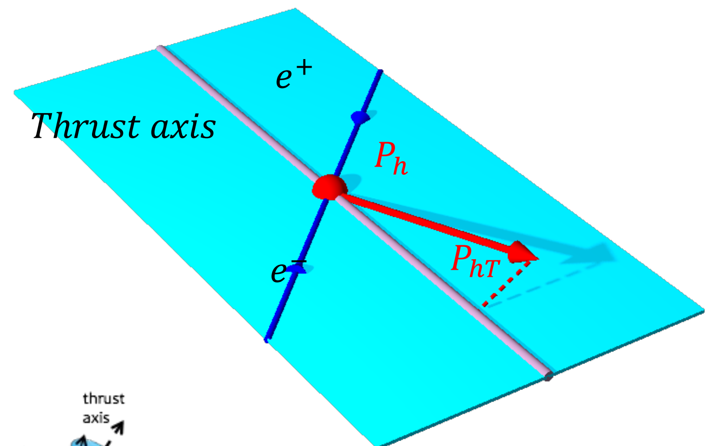


JHEP 1311 (2013) 194

P_T dependence of D_1

- Quasi inclusive hadron production gives access to transverse momentum in fragmentation
- **Transverse momentum measured with respect to thrust axis**
- Analysis performed differentially in bins of $z, P_{hT}, Thrust$ $(18\times20\times6)$

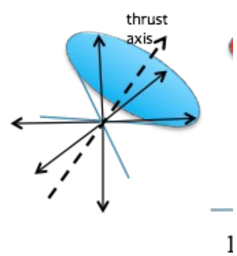




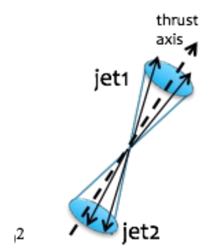
$$T = \sum_i \frac{|\mathbf{P}\cdot\hat{\mathbf{n}}|}{|P|}$$

thrust axis $\equiv \hat{\mathbf{n}}$

$0.5 \leq T \leq 1$



T~0.5

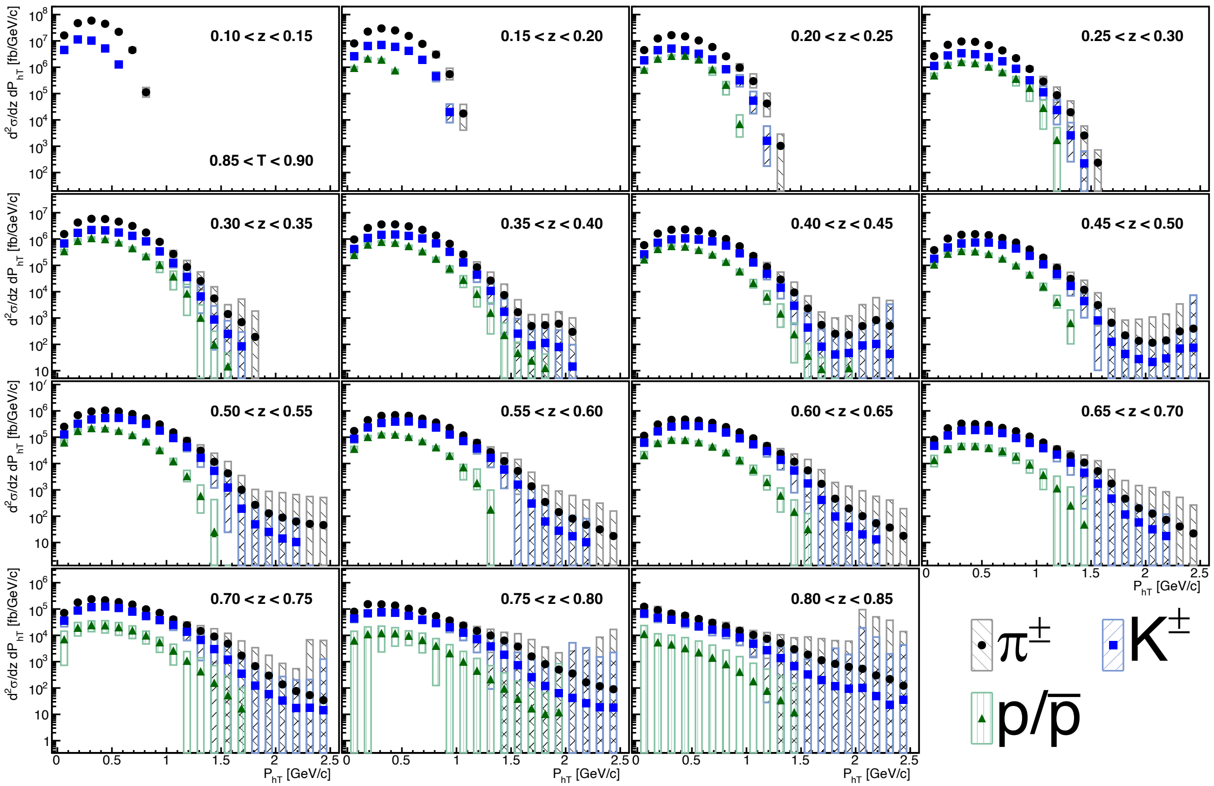


T~1

Phys.Rev. D99 (2019) no.11, 112006

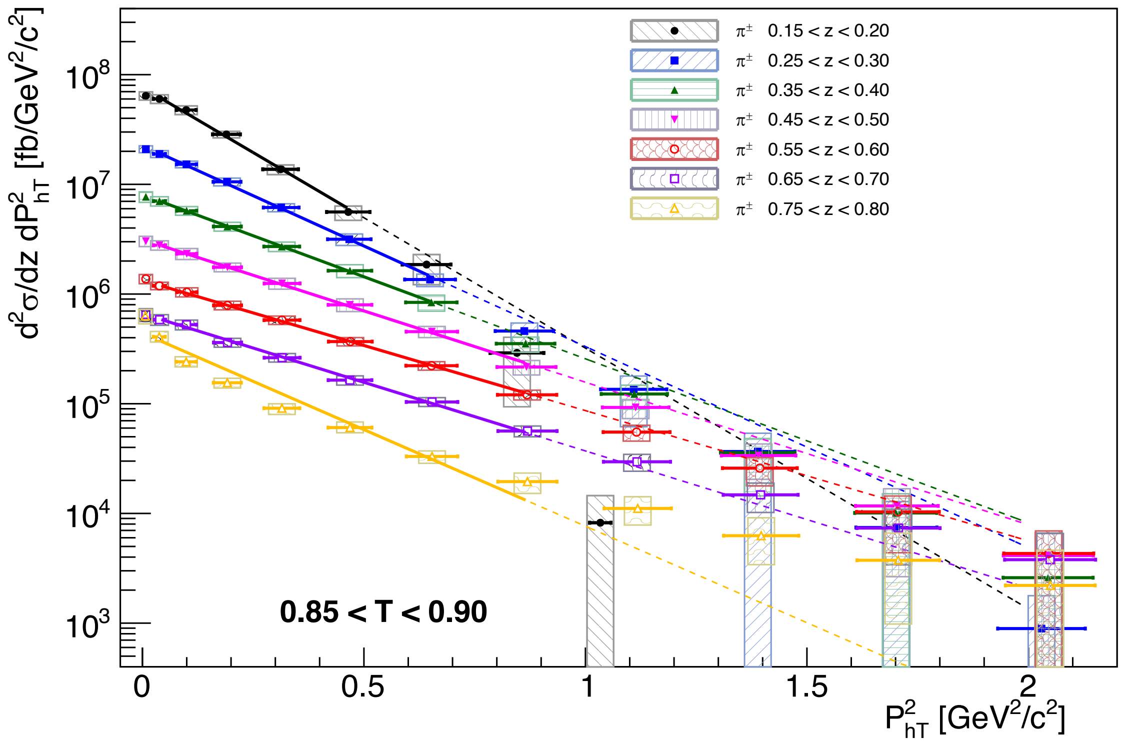
Transverse momentum distributions

- 0.85< Thrust T < 0.9
  - Transverse momenta mostly Gaussian
  - Possible deviations for large P_{hT} tails, but also large uncertainties



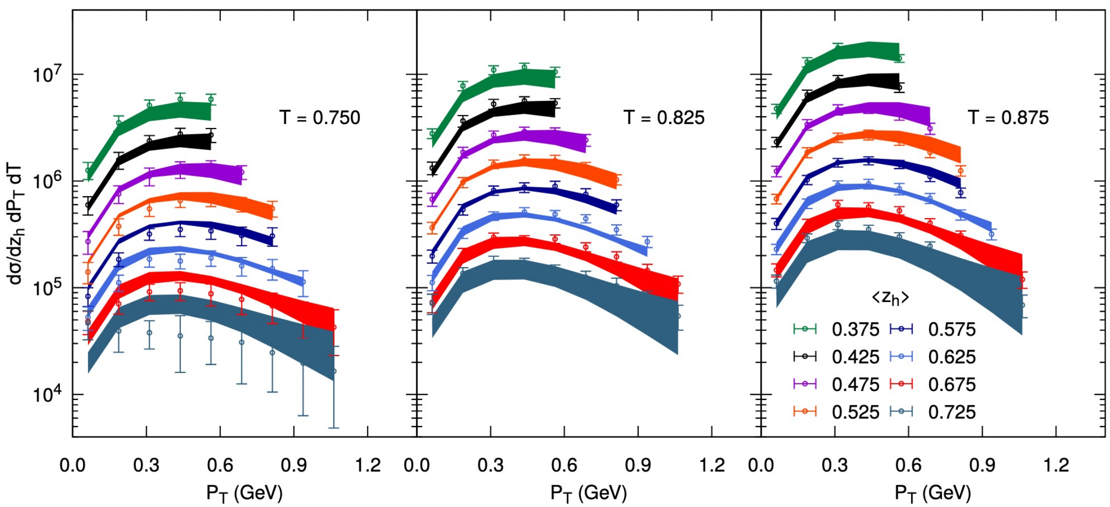
Transverse Momentum: Gaussian widths

- 0.85 < T < 0.9
  - Fit Gauss to low P_{hT} data
  - Mostly well described with possible exception at high z
  - Deviation from Gauss at large P_{hT}
  - Clear increase in width with z for low values of z

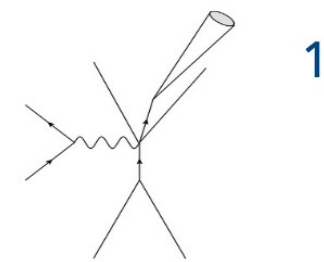


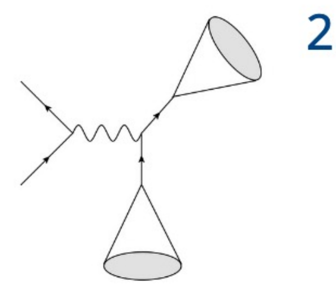
Data well described by Theory

- Subtle interplay between thrust, perturbative, nonperturbative part
- Extracted TMD has to be modified to fit than back-to-back or SIDIS definition
- Contrast with back-to-back hadrons (underway)



M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]





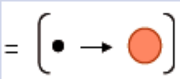
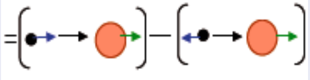
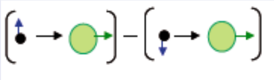
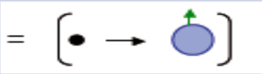
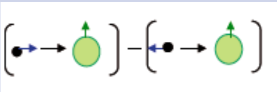
$$D = D^{\star}\sqrt{M_S}$$

See also

- Kang, Shao, Zhao and Makris, Ringer, Waalewijn

Single Hadron FFs

- Analogue →similar to PDFs encoding spin/orbit correlations
- Determining final state polarization needs self analyzing decay (Λ)
- Gluon FFs similar but with circular/linear polarization (not as relevant for e+e-)

Parton polarization → Hadron Polarization ⇣	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z,p_T)$ = 		$H_1^{\perp h/q}(z,p_T)$ = 
longitudinal		$G_1^{\Lambda/q}(z,p_T)$ = 	$H_{1L}^{h/q}(z,p_T)$ 
Transverse (here Λ)	$D_{1T}^{\perp\Lambda/q}(z,p_T)$ = 	$G_{1T}^{h/q}(z,p_T)$= 	$H_1^{\Lambda/q}(z,p_T)$ =  $H_{1T}^{\perp\Lambda/q}(z,p_T)$ = 

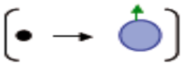
- Needed to access (spin dependent) parton structure of the nucleon
- Can probe fundamental QCD questions (e.g. $D_{1T}^{\perp} \leftrightarrow f_{1T}^{\perp}$)

Polarized Hyperon Production

- Large Λ transverse polarization in unpolarized pp collision

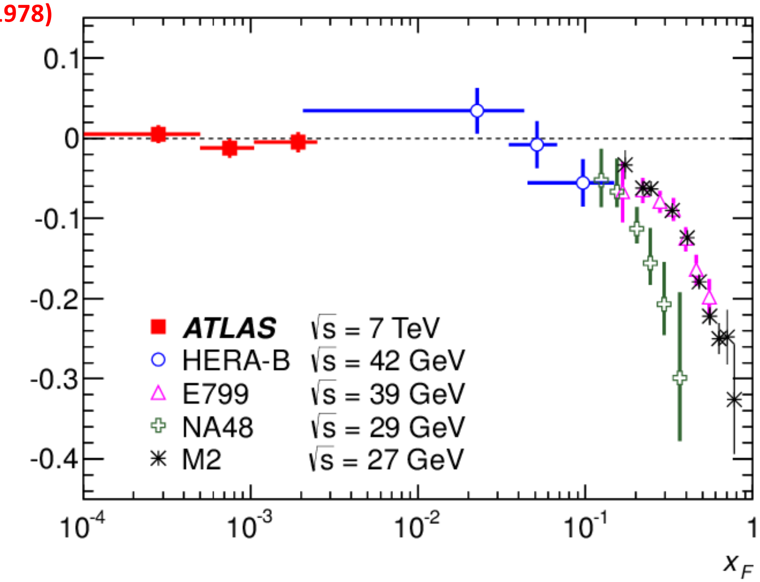
PRL36, 1113 (1976); PRL41, 607 (1978)

- Caused by polarizing FF

$D_{1T}^{\perp}(z, p_{\perp}^{2})$ =  ?

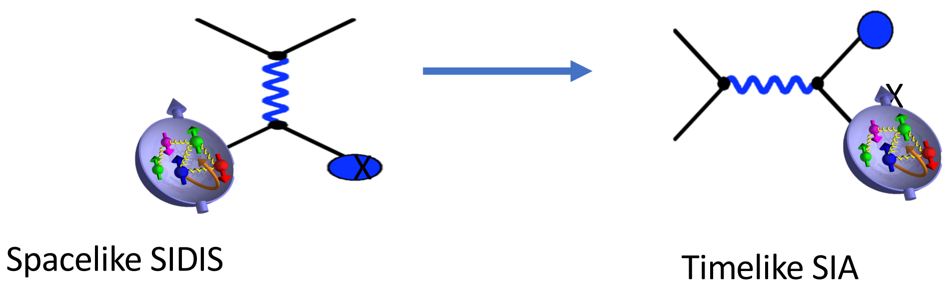
- OPAL experiment at LEP has studied transverse Λ polarization, no significant signal was observed.

Eur. Phys. J. C2, 49 (1998)



PRD 91, 032004 (2015)

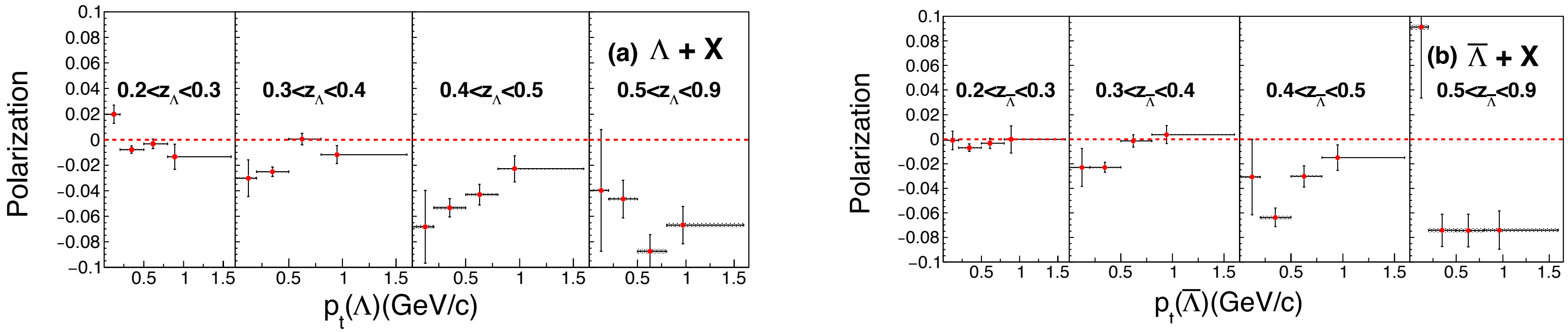
PDF in SIDIS $\Leftrightarrow$ FF in e^+e^-

- E.g. Sivers $\Leftrightarrow$ $\Lambda^{\uparrow}$ production
- 

- Sensitive to gauge structure of QCD→Test of universality
  (PRL105,202001（2010））

z_Λ, p_T Dependence of observed Λ polarization

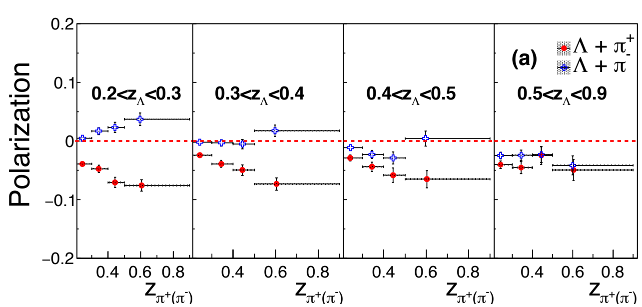
- Polarization rises with p_t in the lowest z_Λ and highest z_Λ bin. But the dependence reverses around 1 GeV in the intermediate z_Λ bins→**Unexpected!**
- Results are consistent between Λ and $(\bar{\Lambda})$
- Theory fits can describe data well
  - D'Alesio, Murgia, Zaccheddu, Phys. Rev.D 102, 054001 (2020)
  - Callos, Kang, Terry, Phys. Rev. D 102, 096007
  - Gamberg et al. P*hys.Lett.B* 818 (2021) 136371
  - Chen et al. arXiv:2102.00658 [hep-ph].

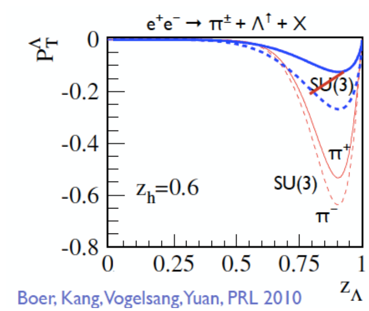


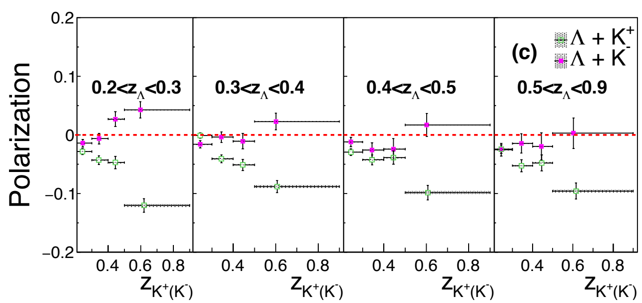
Phys. Rev. Lett. 122, 042001 (2019)

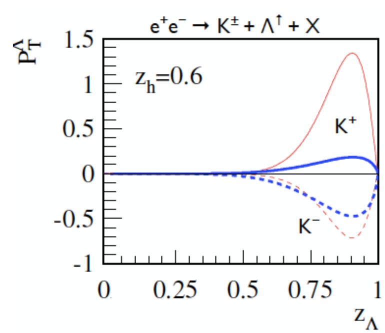
Tension with theory: ASSOCIATED PRODUCTION

- Correlation with opposite hemisphere light meson →quark flav/charge dependence
  - Sign of asymmetry dependent on quark charge cf Sivers
- Only experimental results on T-odd, chiral even FF →**Important to understand!**



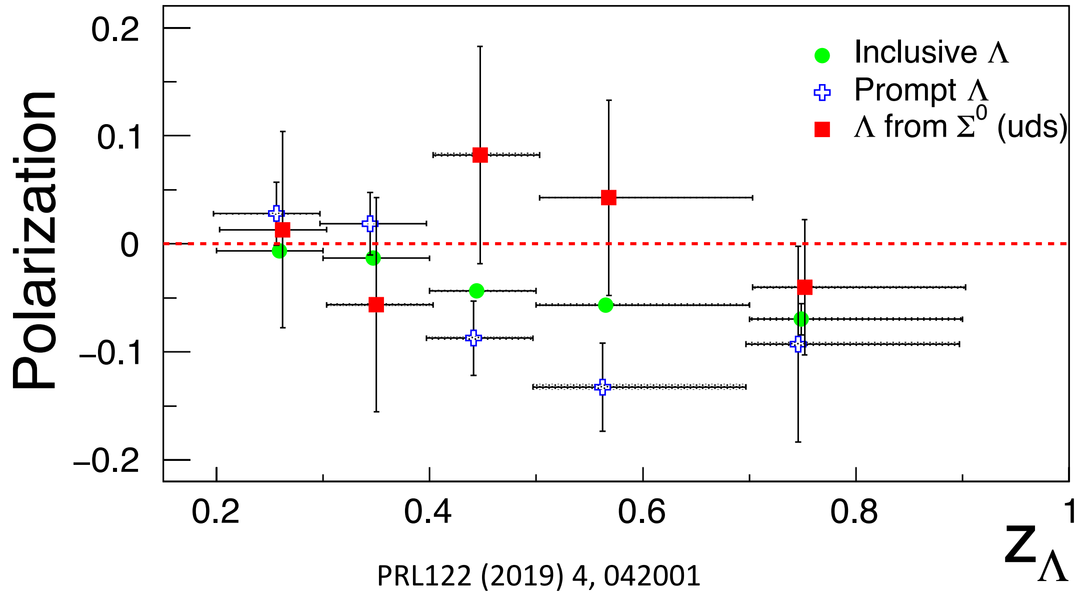






Belle II Makes Precision Λ program possible!

Feed-down corrected
$\Lambda^{\uparrow}$ at Belle
➔Large Uncertainties!

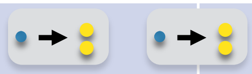
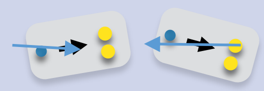
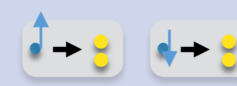


PRL122 (2019) 4, 042001

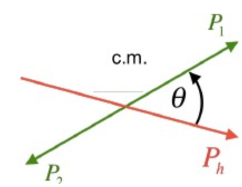
- Opportunities at Belle II:
  - Feed down correction for p_T dependence and associated production
    - (currently only for z dependence, introduces large uncertainties)
    - $\Lambda^{\uparrow} - \Lambda^{\uparrow}$ correlations
    - Extension to tensor polarized FFs: e-Print: 2206.11742 [hep-ph]
    - ….
  - Explore low p_T region (not shown here) with higher statistics and better tracking resolution

Di-hadron fragmentation Functions

Additional Observable:
$\vec{R} = \overrightarrow{P_1} - \overrightarrow{P_2}$:
The relative momentum of the hadron pair is an additional degree of freedom:
the orientation of the two hadrons w.r.t. each other and the jet direction can be an indicator of the quark transverse spin

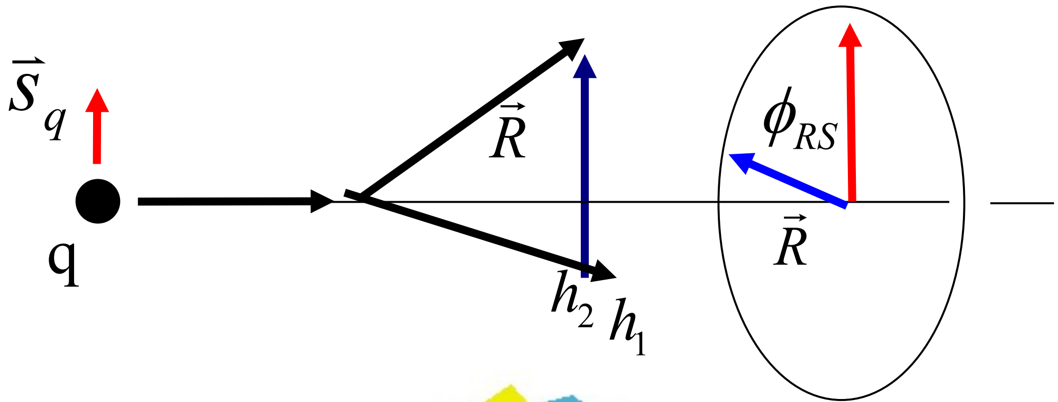
Parton polarization → **Hadron Polarization ↓**	**Spin averaged**	**longitudinal**	**transverse**
spin averaged	$D_1^{h/q}(z, M)$ 		$H_1^{\perp h/q}(z, p_T M, (P_h), \theta)$ 'Di-hadron Collins'
longitudinal			
Transverse		**$G_1^{\perp}(z, M, P_h, \theta)$=** T-odd, chiral-even →jet handedness QCD vaccum strucuture 	**$H_1^{∢}(z, M, (P_h), \theta)$=.** T-odd, chiral-odd Colinear 

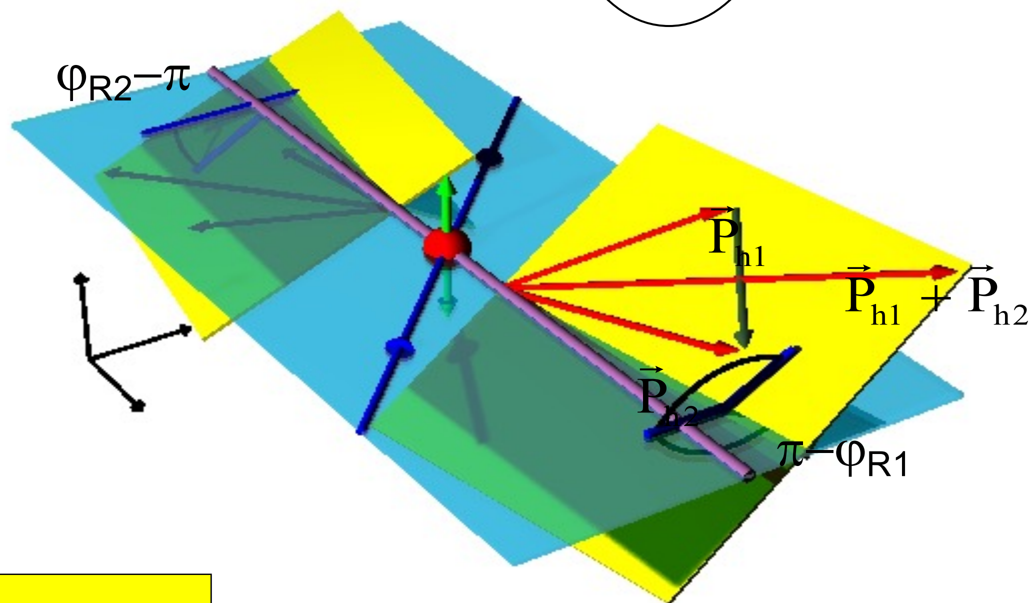
- Relative momentum of hadrons can carry away angular momentum
  - Relative and total angular momentum →In principle endless tower of FFs



More degrees of freedom→**More complete information about correlations in final state**

Di-Hadron Fragmentation

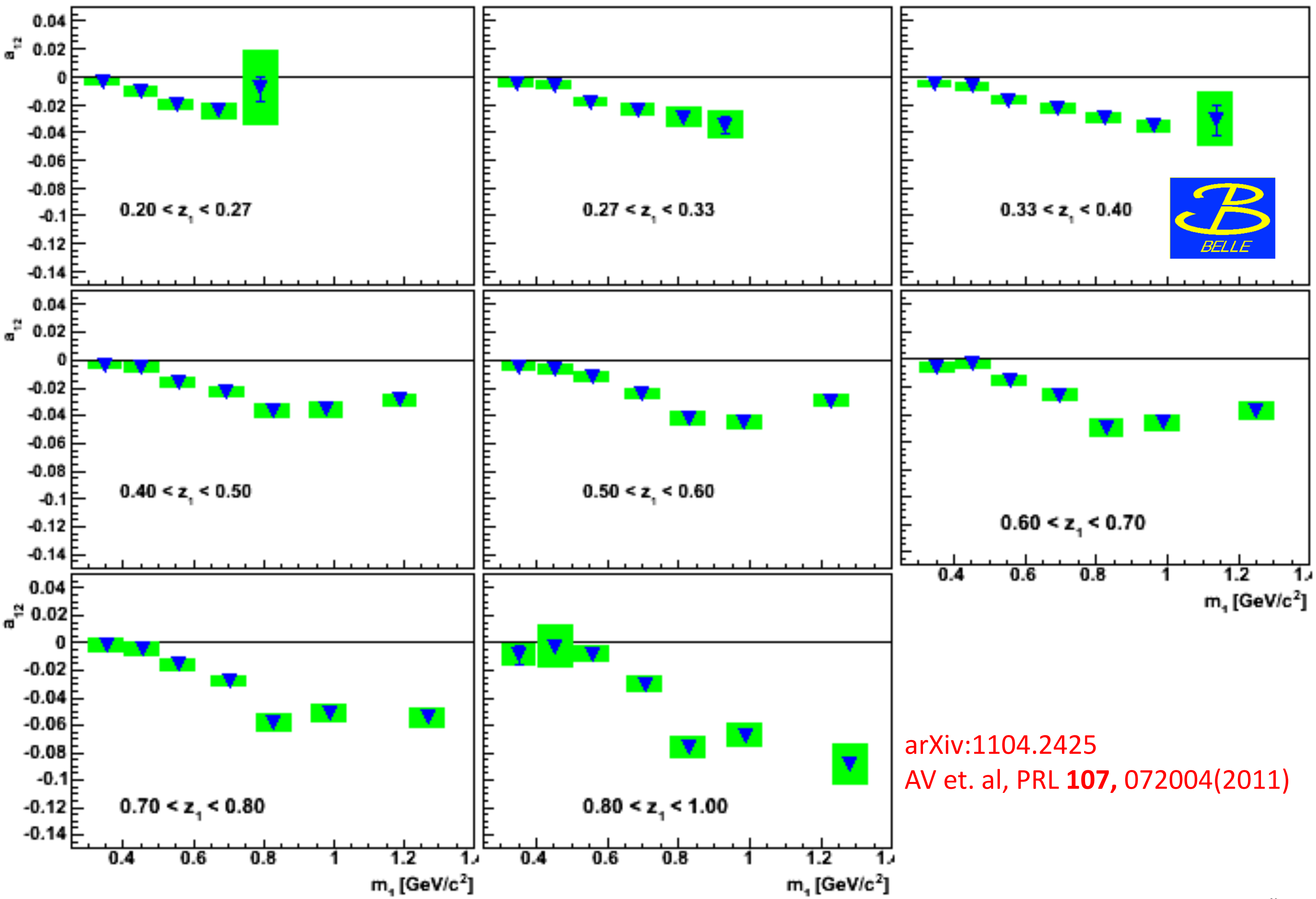




$$\sigma \propto H_1^{\angle} \cdot \bar{H}_1^{\angle}$$

First measurement of Interference Fragmentation Function

$a_{12} \propto H_1^{<} * H_1^{<}$

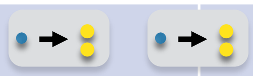
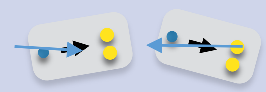
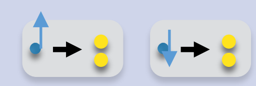


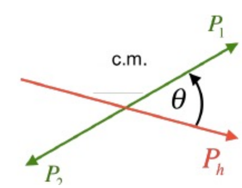
arXiv:1104.2425

AV et. al, PRL **107,** 072004(2011)

Di-hadron fragmentation Functions

Additional Observable:
$\vec{R} = \overrightarrow{P_1} - \overrightarrow{P_2}$:
The relative momentum of the hadron pair is an additional degree of freedom:
the orientation of the two hadrons w.r.t. each other and the jet direction can be an indicator of the quark transverse spin

Parton polarization → **Hadron Polarization ⇣**	**Spin averaged**	**longitudinal**	**transverse**
spin averaged	$D_1^{h/q}(z, M)$ 		$H_1^{\perp h/q}(z, p_T M, (Ph), \theta)$ 'Di-hadron Collins'
longitudinal			
Transverse		$G_1^{\perp}(z,M,P_h,\theta)$= T-odd, chiral-even →jet handedness QCD vaccum strucuture 	$H_1^{\sphericalangle}(z,M,(P_h),\theta)$=. T-odd, chiral-odd Colinear 

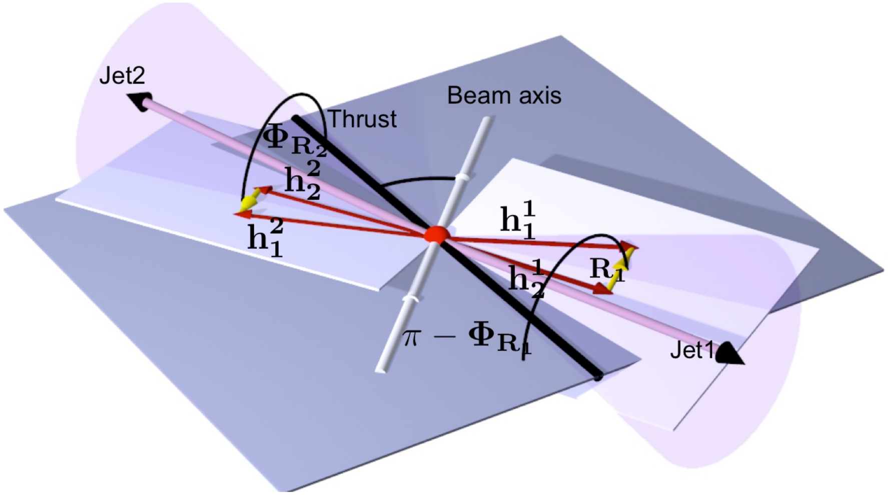


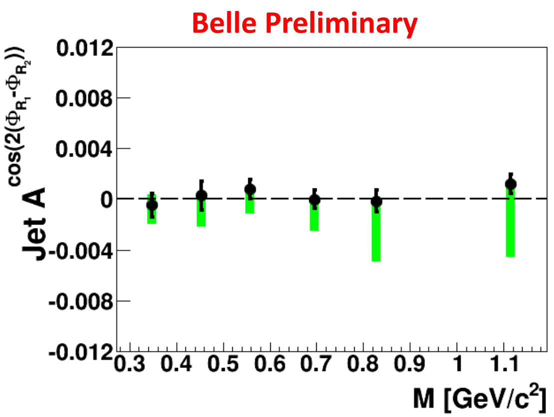
- Relative momentum of hadrons can carry away angular momentum
  - Relative and total angular momentum →In principle endless tower of FFs

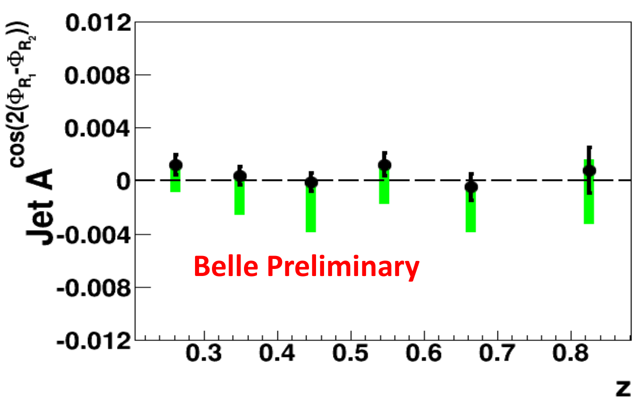
More degrees of freedom→**More complete information about correlations in final state**

Asymmetries for Cos(2(ϕ_{R1}-ϕ_{R2})) ($G_1^\perp$) small

- Turned out that the asymmetry is projected to be $\equiv 0$





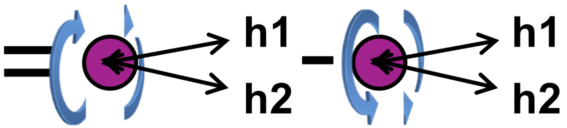


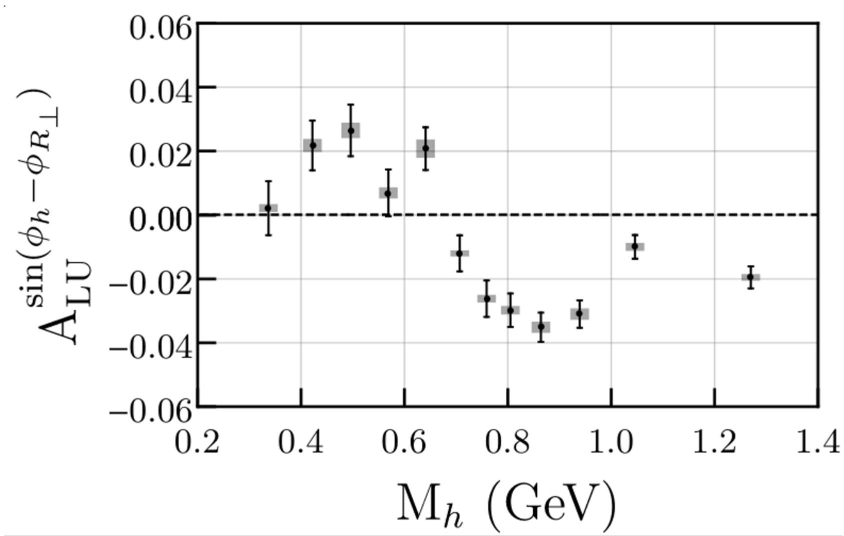
1505.08020 [hep-ex]

CLAS12 Beam Spin Asymmetry Measurements

Twist 2

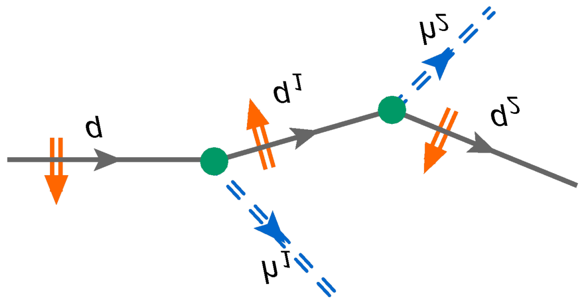
$A_{LU} \sim f_1 G_1^{\perp|\ell,m\rangle}$

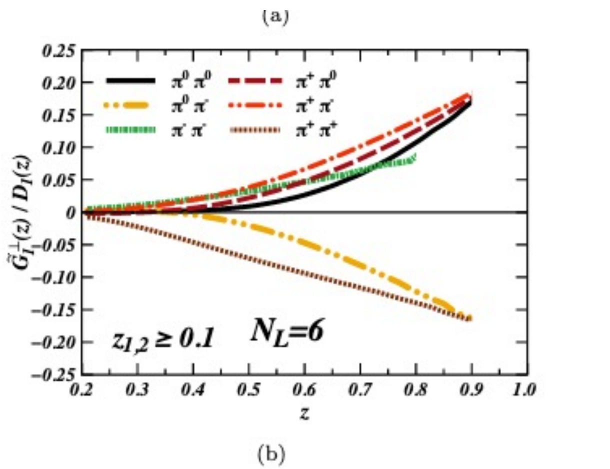
$G_1^{\perp|\ell,m\rangle}$ 



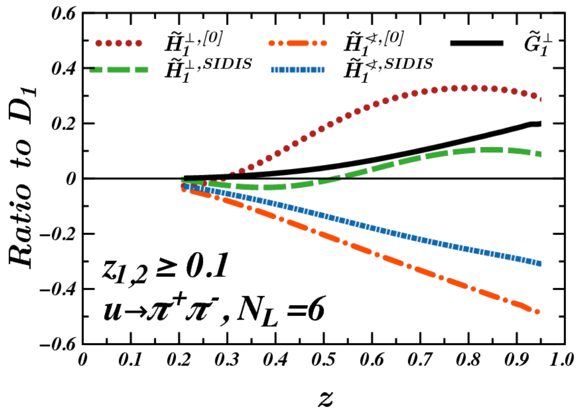
Di-hadron in MC models

- NJL Model implementation (Matevosjan et al. *Phys.Rev.D* 96 (2017) 7, 074010, *Phys.Rev.D* 97 (2018) 1, 014019





Phys.Rev.D 96 (2017) 7, 074010

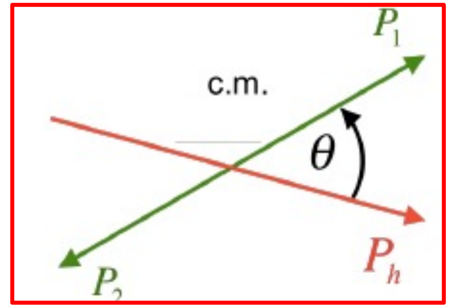


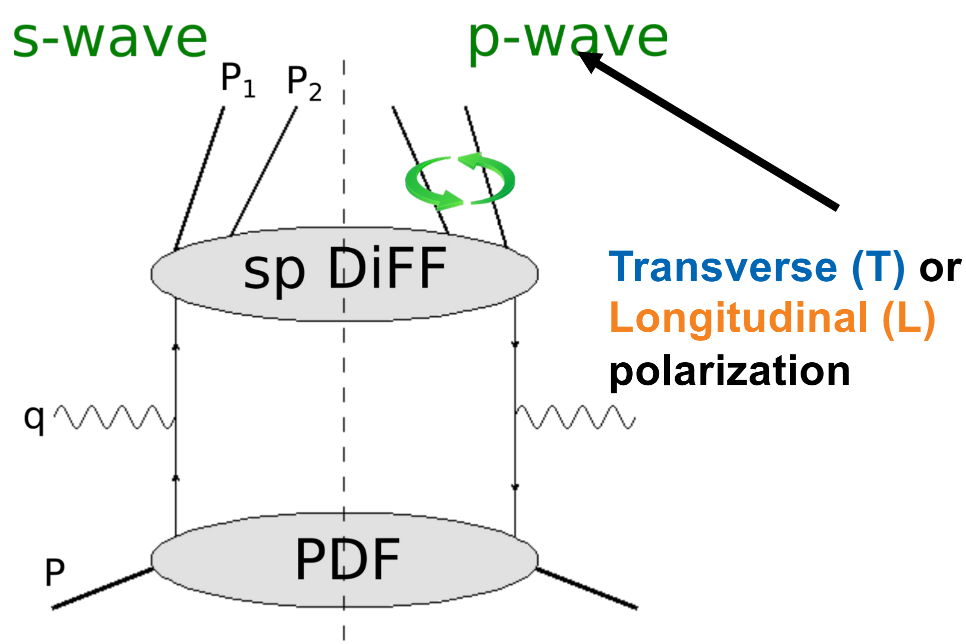
Phys.Rev.D 97 (2018) 1, 014019

Next Step: Partial Wave analysis

- Transverse polarization dependent effect comes from interference effect
  - Here interference between different partial waves
  - Dihadron FF expands on a basis of spherical harmonics
  - Angular momentum eigenvalues | ℓ, m >

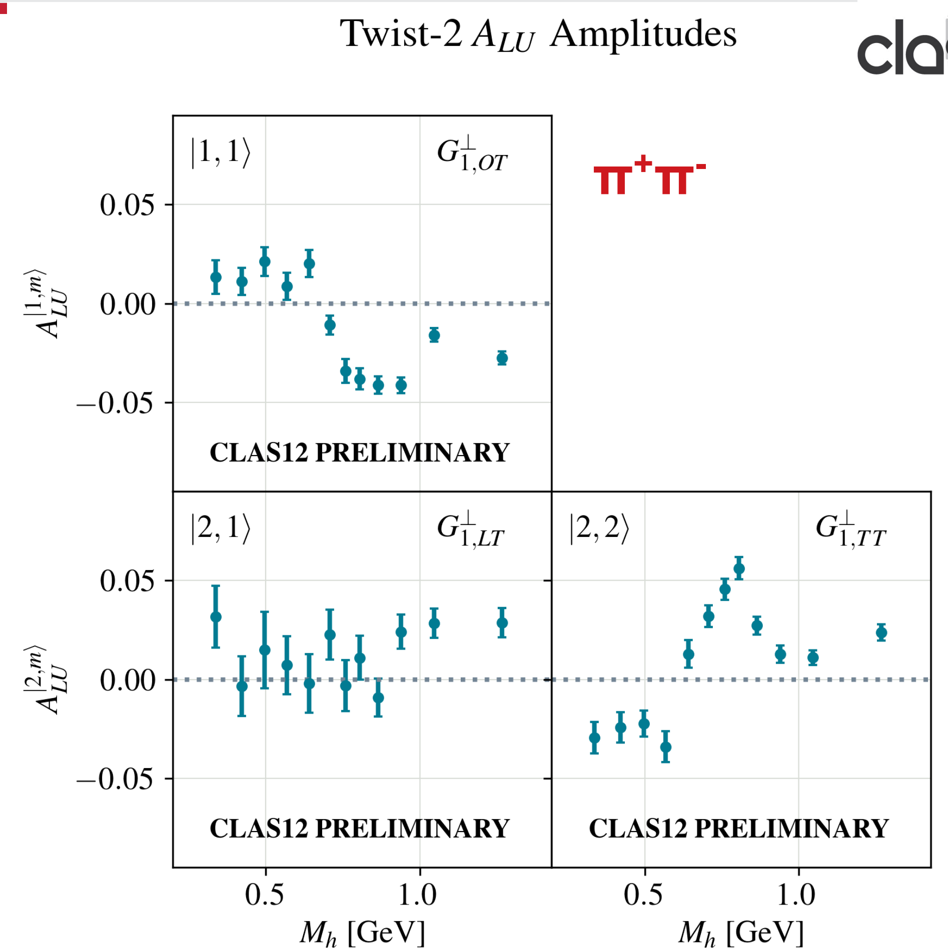
➔Explore dihadron fragmentation depending on relative angular momentum





Twist-2 A_{LU} at M_h Bins

Sensitive to $f_1 \cdot G_1^{\perp}$



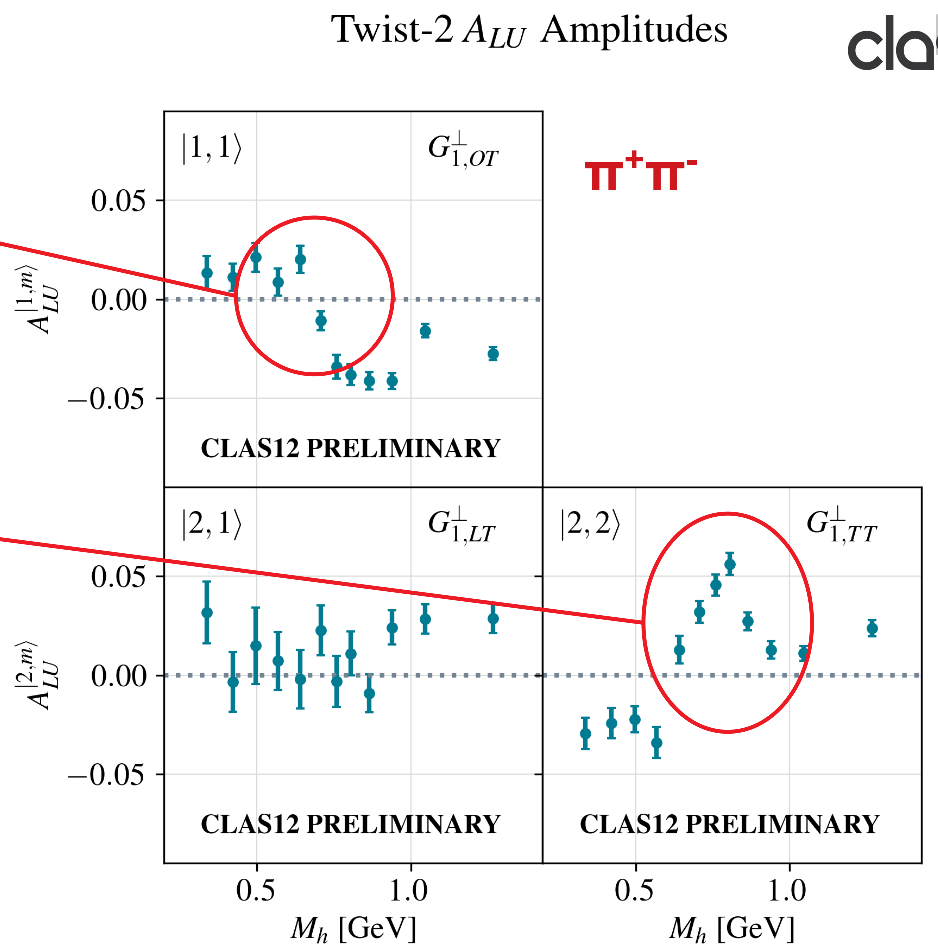
e-Print: 2107.12965 [hep-ex]

Twist-2 A_{LU} at M_h Bins

Sign change near ρ mass

Enhancement at ρ mass (and a sign change)

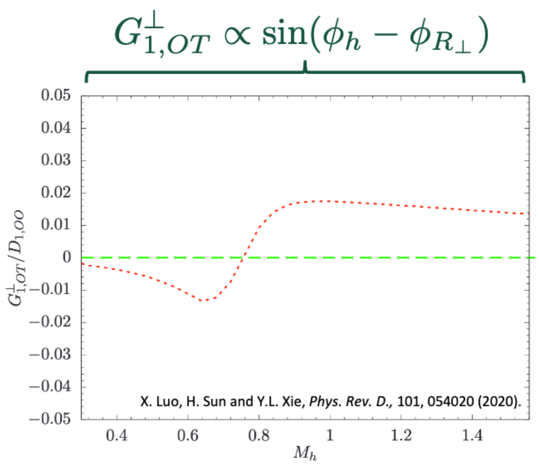
ρ meson → p-wave $\pi^+\pi^-$

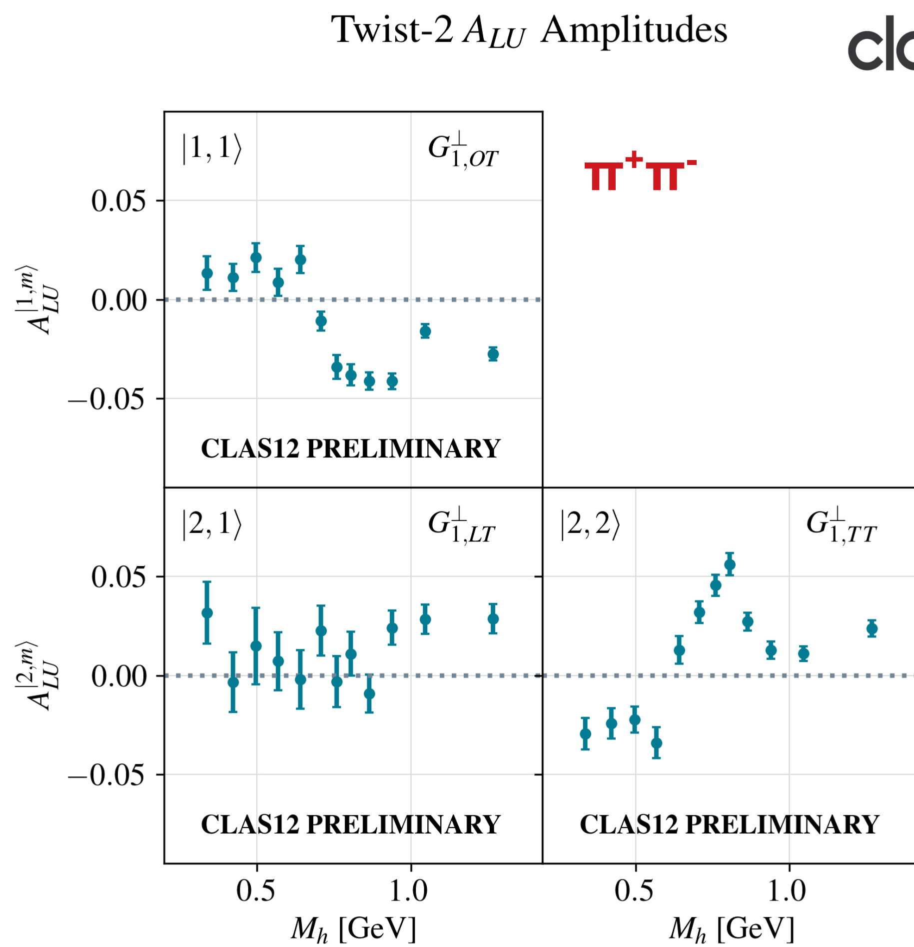


e-Print: 2107.12965 [hep-ex]

Twist-2 A_{LU} at M_h Bins

Sensitive to $f_1 \cdot G_1^{\perp}$



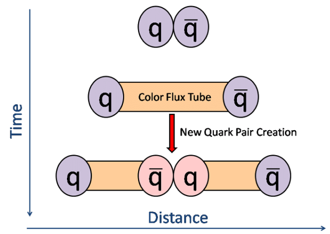


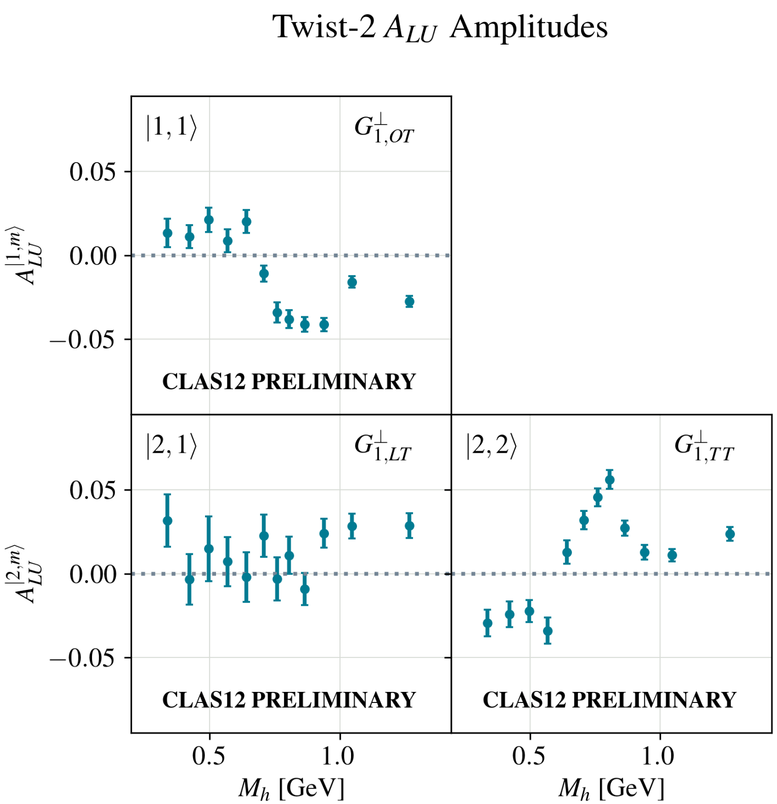
In good agreement with spectator
Model predictions

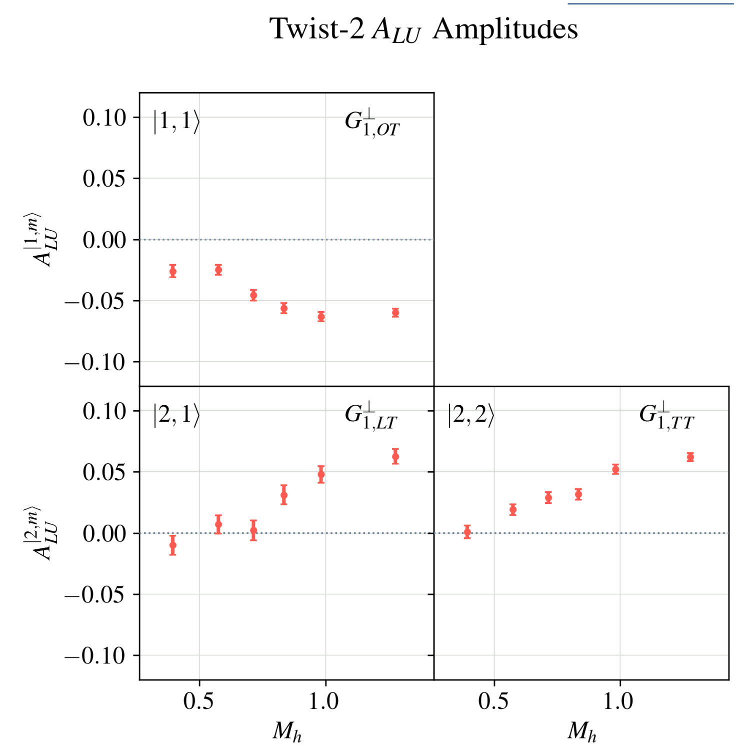
e-Print: 2107.12965 [hep-ex]

Compare Partial Wave Decomposition in MC and Data

- Comparing to Polarized Lund model here (StringSpinner, A. Kerbizi et al, *Comput.Phys.Commun.* 272 (2022))

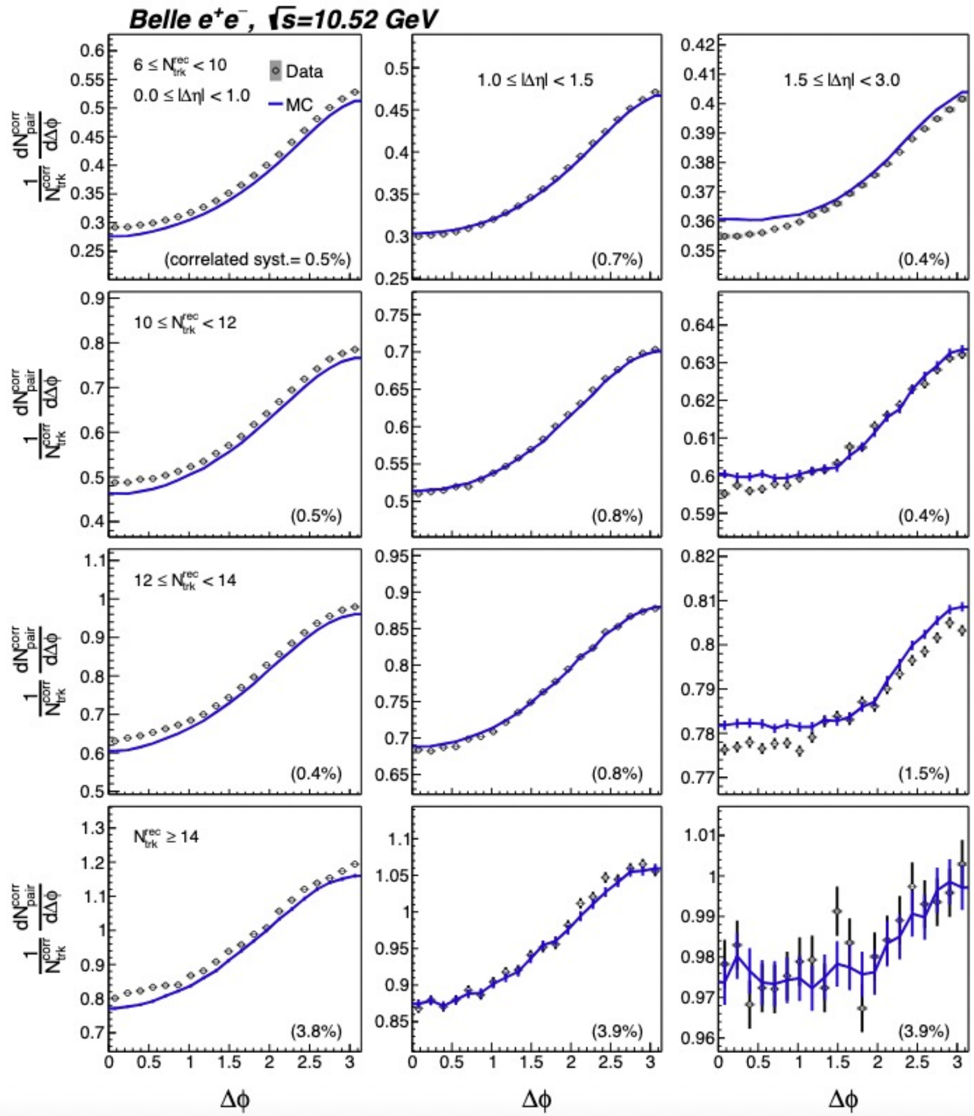






2-particle angular correlations at Belle

- Search for 'ridge-like' phenomenon
- Also interesting to compare correlations to model description

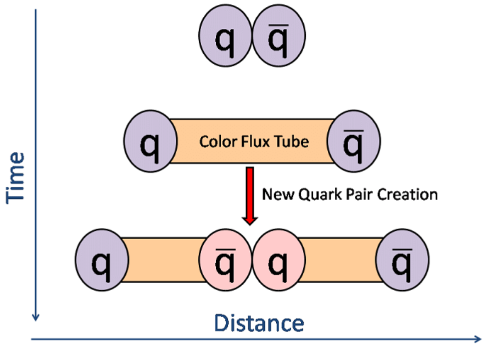


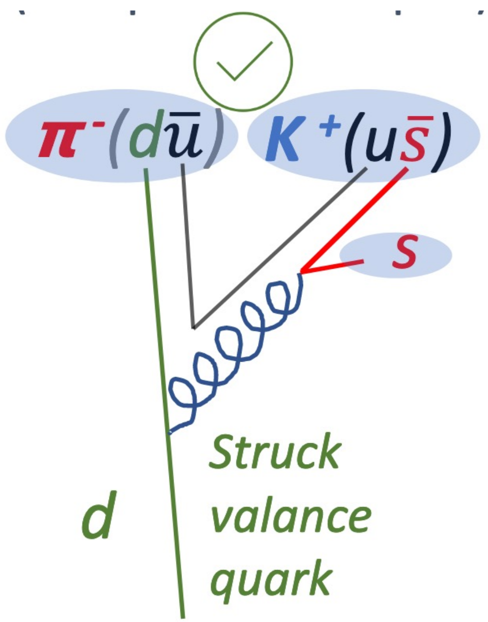
e-Print: 2206.09440 [hep-ex], Extention of Phys. Rev. Lett. 128 (2022) 142005, [2201.01694

Further tests of string fragmentation

- Correlations between leading and subleading particles

  → Mriganka's talk





MC tuning studies at Belle II

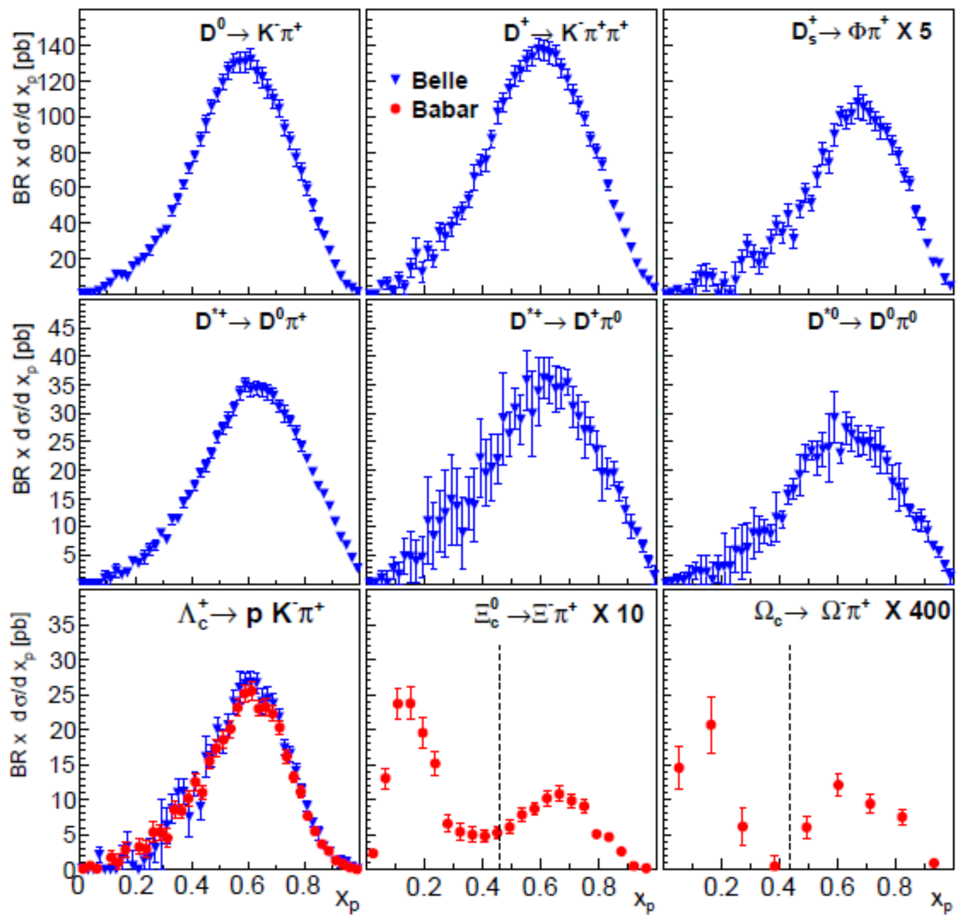
- Event Shapes
- Jet rates vs resolution, hemisphere,
- Event rates relative to event plane (and z, p_T), including baryons
- Multiplicities of resonance production ($\rho, \omega, K^*, \phi, \Lambda, \Sigma, \Xi, \Omega$)
  - Ratios between pseudo-scalar and vector mesons (also important for cosmic events)
- Charge/strangeness/baryon number compensation along event axis

➔ more details in the Snowmass Whitepaper

Current Status of HF FFs from Belle, BaBar

PRL.95, 142003 (2005)(Babar)
PRD73, 032002 (2006) (Belle)
PRD75, 012003 (2007)(Babar)
PRL 99, 062001 (2007)(Babar)

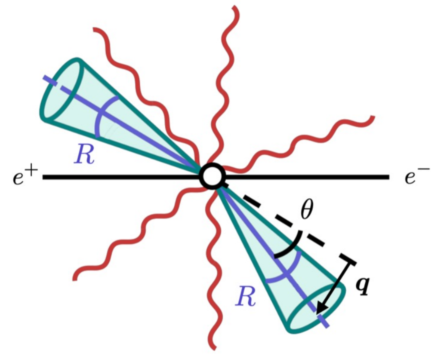
- Single hadron differential cross sections for Λ, Σ, Ξ, Ω, Λ_c, Σ_c, Ξ_c, Ω_c (etc) vs x_p available
- Heavier particles generally plotted vs normalized momentum
- Unlike light hadrons charmed hadrons contain large fraction of charm quark momentum
  ➔peaked at larger x_p
- Belle II prospects: Multidimensional extraction, p_T dependence

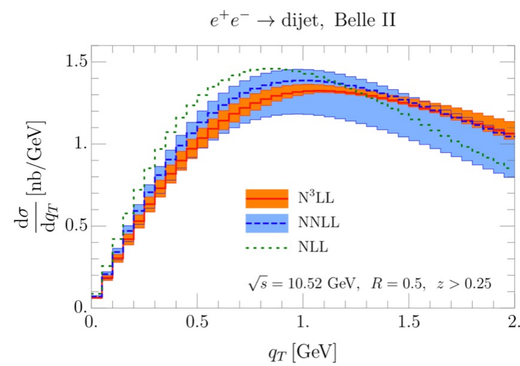


Jet Physics in e^+e^-

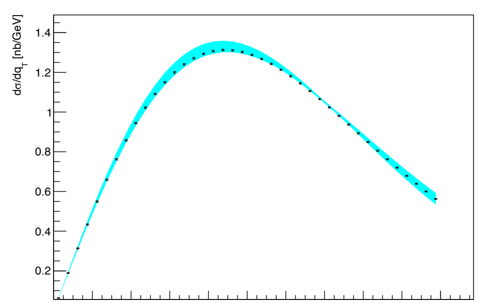
- Jet physics (will) play an important role at the EIC and LHC
- Precision measurements in e^+e^- annihilation will test current theoretical understanding
- Lower energies like Belle in particular sensitive to hadronization effects
- Example:
  NP hadronization
  ➔T-odd jets

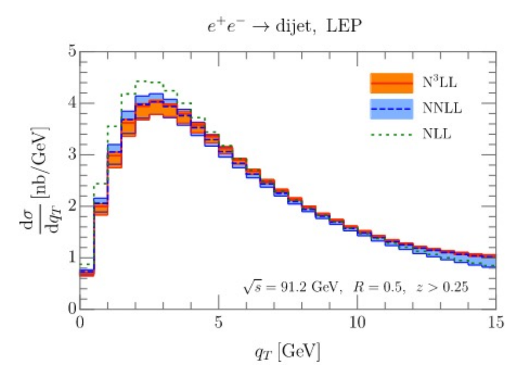
X. Liu and H. Xing, arXiv:2104.03328 [hep-ph]





Belle II projections (stat only) compared to theory

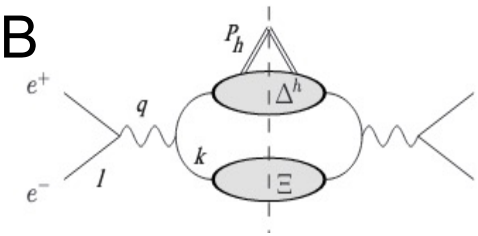




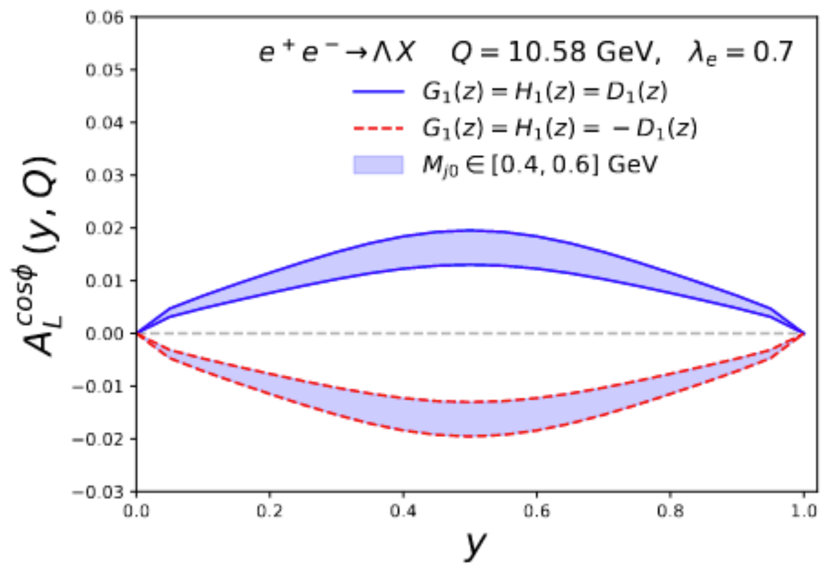
Jet mass

- Proposal to polarize electron beam at SuperKEKB

→ Whitepaper: e-Print: 2205.12847



- Can access jet mass
- ”QCD Higgs mechanism”



$$\frac{d\sigma^L}{d\Omega dz} \propto \lambda_e \sum_a \left\{ \frac{1}{2} C(y) G_1^{a\to\Lambda}(z,Q) + 2D(y)|S_T|\cos(\phi)\frac{M_\Lambda}{Q}\left(\frac{1}{z} G_T^{a\to\Lambda}(z,Q) + \frac{m_a^{dyn}}{M_\Lambda} H_1^{a\to\Lambda}(z,Q)\right)\right\}$$

See also A. Accardi, A. Signori, *Phys.Lett.B* 798 (2019) 134993

Summary and Outlook

- e^+e^- annihilation allow for precision studies of QCD
- Belle II sits at the intersection of non-perturbative and pQCD

- Belle II will provide world record statistics for
  - Precision measurements of fragmentation functions with complex final states
  - Tune MC generators➔Precise modeling of polarization dependent hadronization
  - Probe Jet calculations at low scales where hadronization effects play a significant role
  - Constrain HVP, Hlbl contributions to g-2
  - Constrain α_S
  - Test QCD calculations of event shapes
    - E.g. energy-energy correlators
  - …

Save the Date

- General FFs
- Di-hadrons : Pert →NP
- Jet →going lower →Pert →NP
- T-odd side
- Model comparisons

Non-perturbative effects in hadronization can be described in different frameworks

Factorized pQCD→Fragmentation Functions

- Essential for the extraction of PDFs
- Interpretable in QCD
- Focus on more '**inclusive**' measurements → factorization holds
- Recent activity in more exclusive measurements (in particular jets)
- Very precise extractions→Benchmark for MCEGs

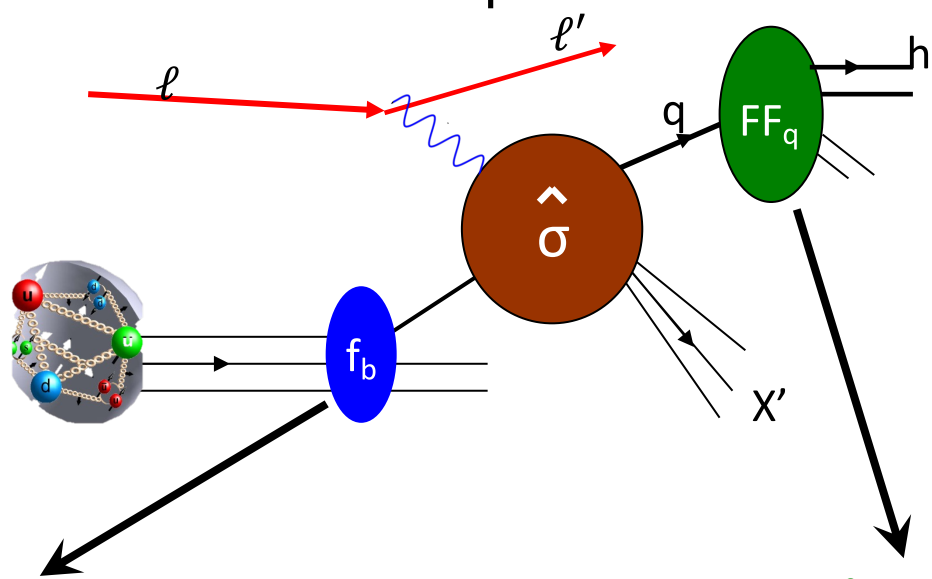
Hadronization Model in MCEG

- Exclusive final states
- "Hard" subprocesses well constrained by theory
- Measurements focusing on MCEG improvement different from measurements extracting hard physics (grooming) or FFs (more exclusive)
- Needed for FF extraction

- **Both needed**
- **Both need input from e^+e^-**

Fragmentation Functions appear almost always when accessing partonic structure of the nucleon

- Proton Structure extracted using QCD factorization theorem
- FFs contribute to virtually all processes
- Particular important for transverse spin structure

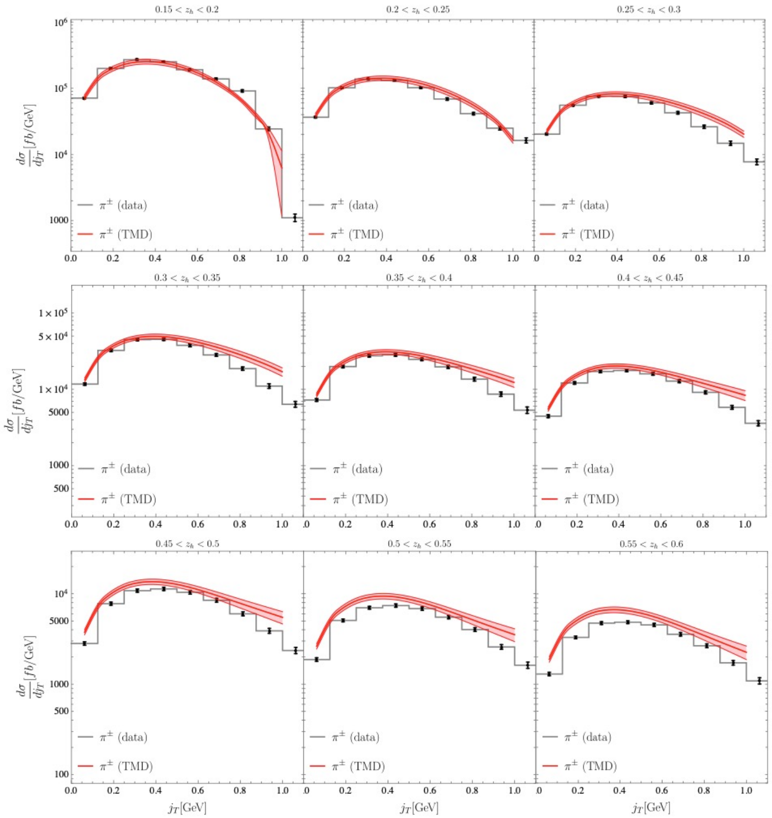


fragmentation function

Proton Structure

pQCD

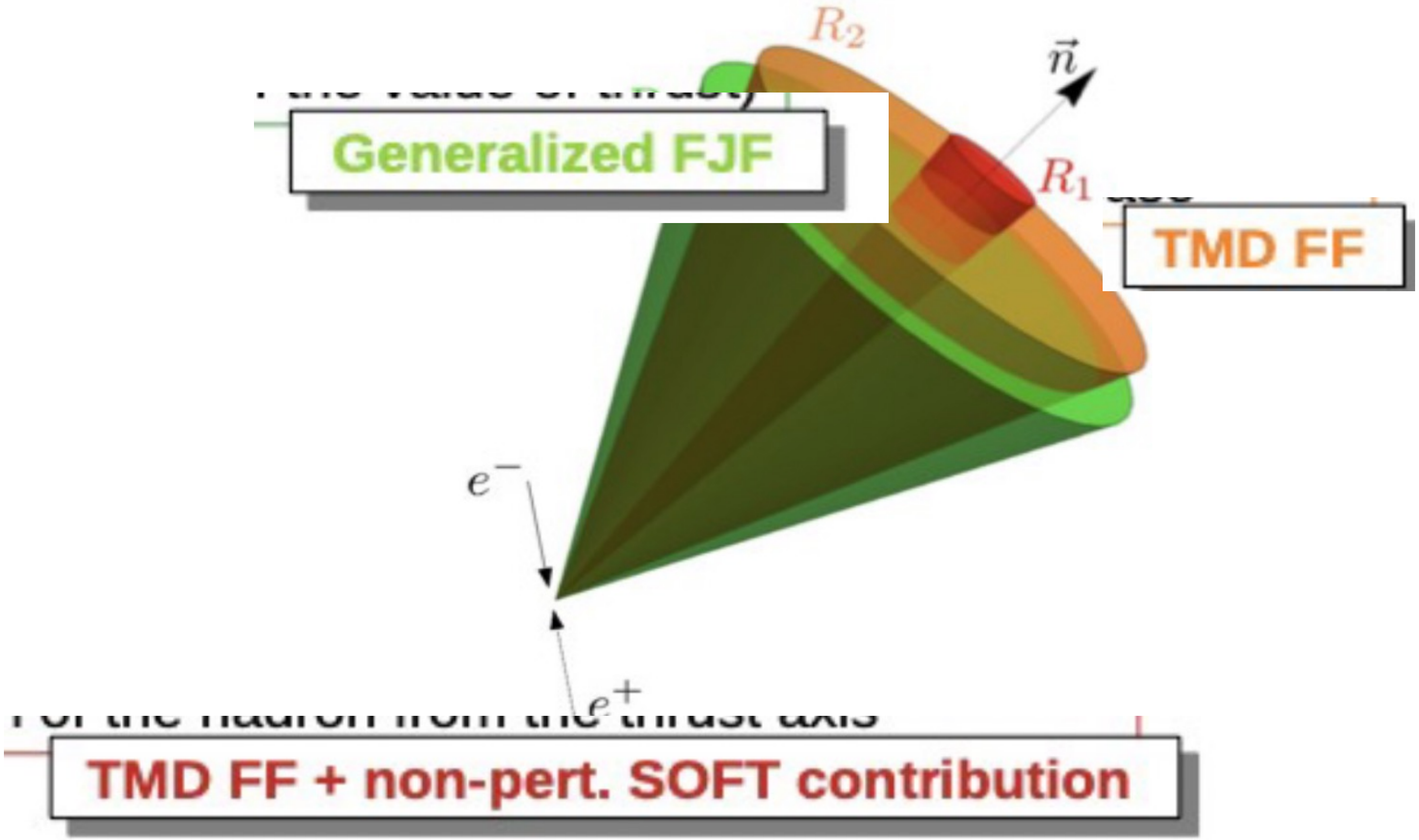
$$\frac{d^2\sigma(ep\to\pi X)}{dx\,dz}\propto q(x,k_T)\times\frac{d\sigma^2(e\,q\to e'q')}{dx}\times FF(z,p_T)$$



From Kang, Shao, Zhao
e-Print: [2007.14425](https://arxiv.org/abs/2007.14425) [hep-ph]

See also Boglione, Simonelli e-Print: [2007.13674](https://arxiv.org/abs/2007.13674) [hep-ph]
Makris, Ringer, Waalewijn [2009.11871](https://arxiv.org/abs/2009.11871) [hep-ph]

Three Regions:



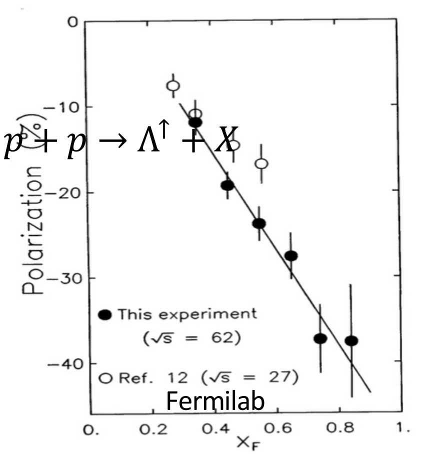
Polarized Hyperon Production

- Large Λ transverse polarization in unpolarized pp collision
- Caused by polarizing FF $D_{1T}^{\perp}(z, p_{\perp}^{2})$?
- Polarizing FF is chiral-even, has been proposed as a test of universa
- OPAL experiment at LEP has studied transverse Λ polarization, no s observed. PRL36, 1113 (1976); PRL41, 607 (1978)

- FF counterpart of the Sivers function. PRL105,202001 (2010)

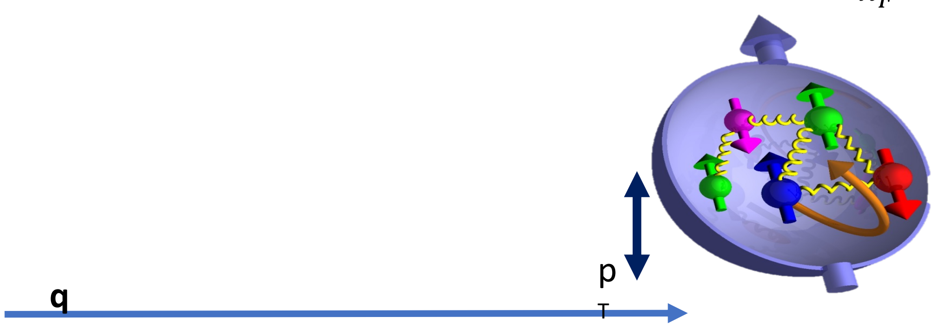
Eur. Phys. J. C2, 49 (1998)

$p + p \to \Lambda^{\uparrow} + X$



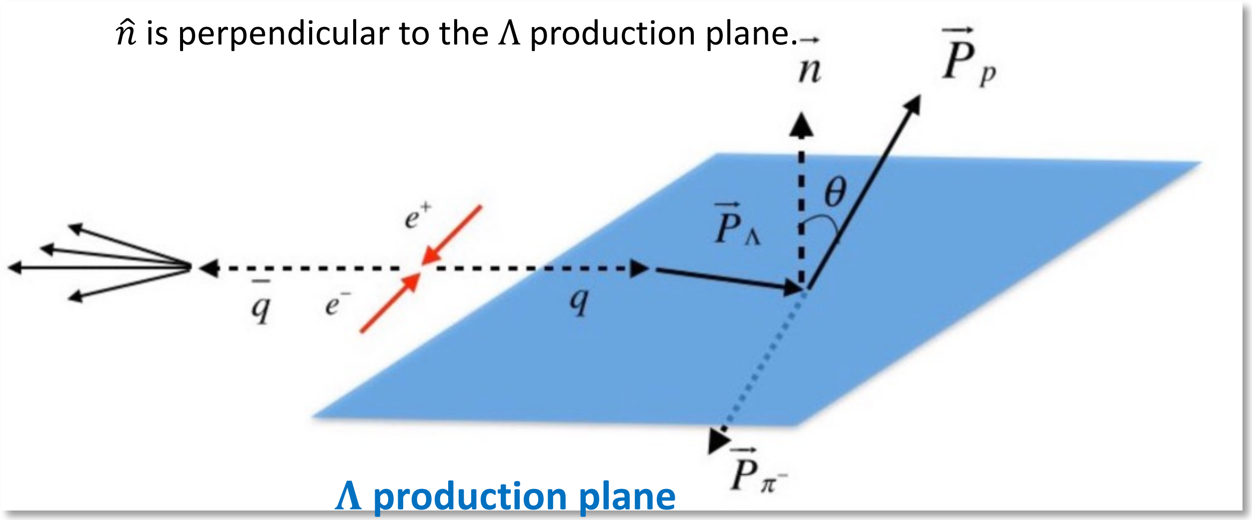
ISR data
(Phys.Lett. B185 (1987) 209)

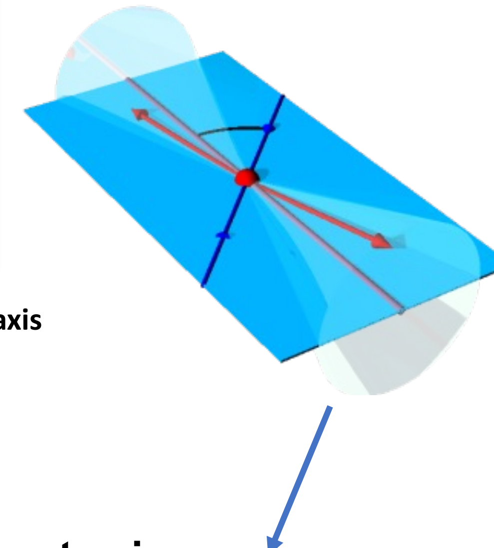
$x_F = p_L / \max p_L \sim_{LO} x_1 - x_2 \sim_{forward} x_1$



Observables in Λ Restframe

- Self-analyzing decay leads to polarization dependent distribution



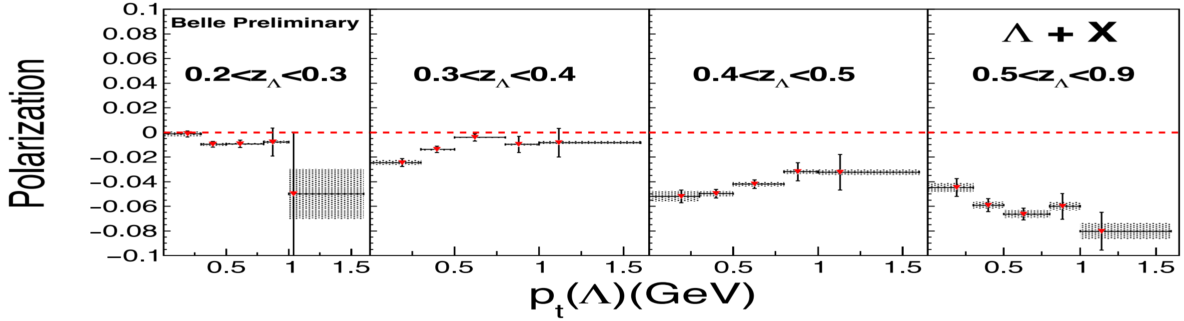


- where α is the decay parameter: α_+=0.642 $\pm$ 0.013 for Λ and α_-=-0.71 $\pm$ 0.08 for $\bar{\Lambda}$ (PDG).
- The p_t is measured as the transverse momentum of Λ relative to the **thrust axis**

$$\frac{1}{N}\frac{dN}{d\cos\theta} = 1 + \alpha P \cos\theta$$

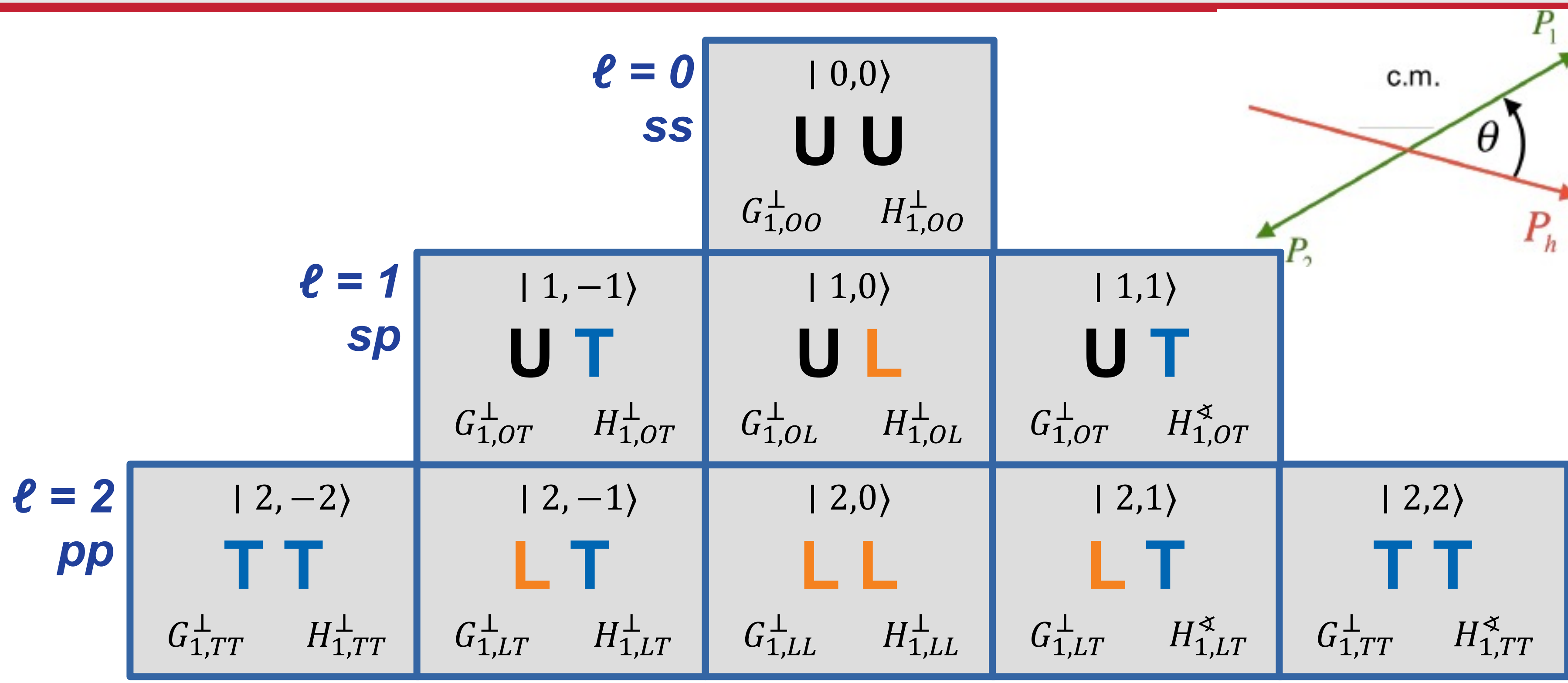
Belle II Prospects

- Explore low p_T region with higher statistics and better tracking resolution
- Feed down correction for p_T dependence and associated production
  - (currently only for z dependence, introduces large uncertainties)
  - $\Lambda^{\uparrow} - \Lambda^{\uparrow}$ correlations
  - ….



Y. Guan, arXiv:1611.0668

Partial Wave Decomposition



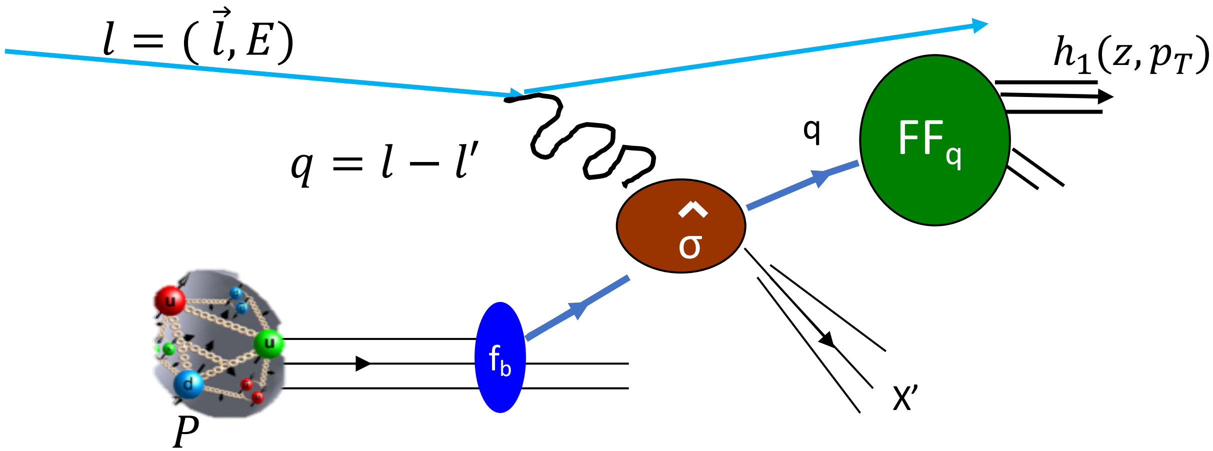
- Full partial wave decomposition ➔full description of two-particle correlations in hadronization

➔Describe hadronization dynamics

➔Bridge between FFs and MCEGs

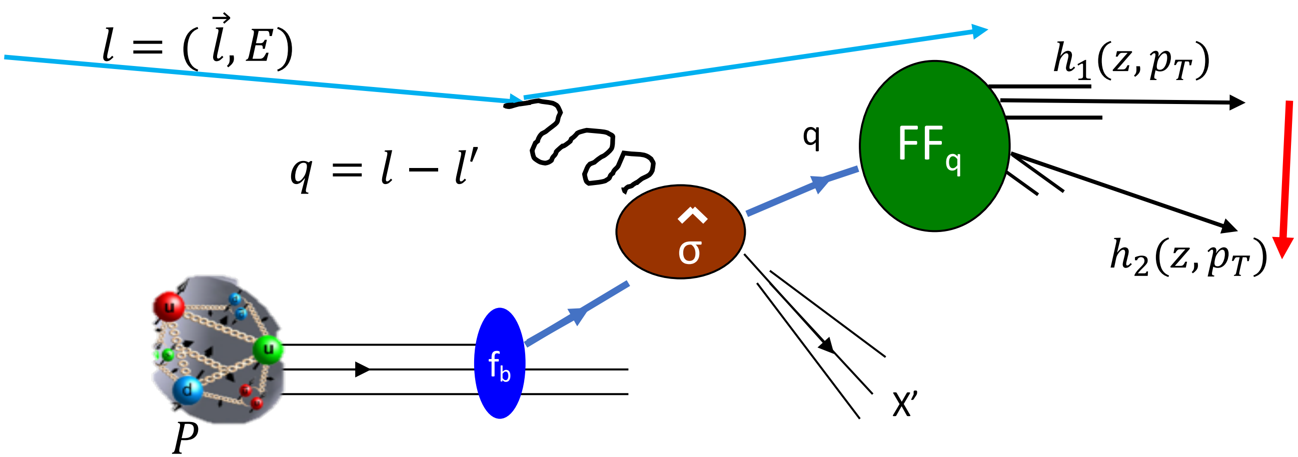
Graphic by C. Dilks

In a factorized picture, hadronization can be described by Fragmentation functions



- LO/LT: probability of finding hadron with momentum fraction z
- <u>Observables:</u>
  - z: fractional energy of the quark carried by the hadron
  - $p_{h,T}$: transverse momentum of the hadron wrt the quark direction: **TMD FFs**

In a factorized picture, hadronization can be described by Fragmentation functions

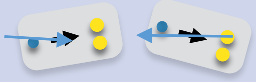
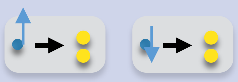


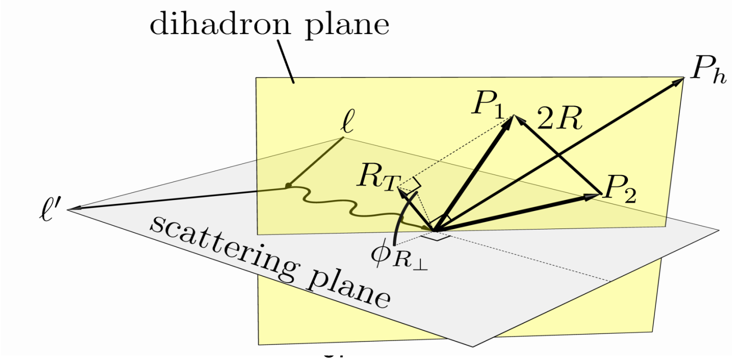
$\overrightarrow{\boldsymbol{P_h}} = \overrightarrow{\boldsymbol{P_1}} - \overrightarrow{\boldsymbol{P_2}}$

$\overrightarrow{\boldsymbol{R}} = \overrightarrow{\boldsymbol{P_1}} - \overrightarrow{\boldsymbol{P_2}}$

Di-hadron fragmentation Functions

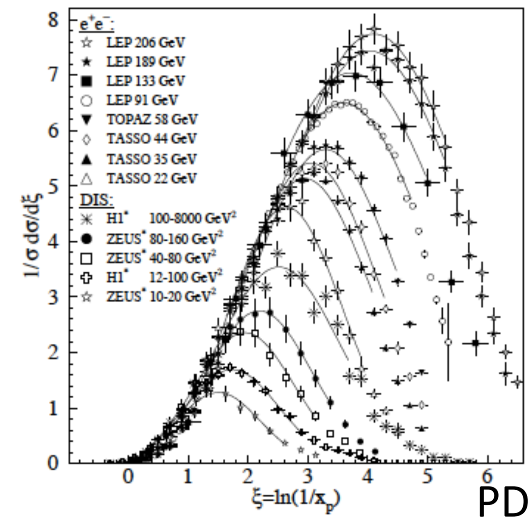
More degrees of freedom→More information about correlations in final state

Parton polarization →	Spin averaged	longitudinal	transverse
		$G_1^{\perp}(z,M,P_h,\theta)$= T-odd, chiral-even →jet handedness QCD vaccum strucuture 	**$H_1^{\sphericalangle}(z,M,(P_h),\theta)$=.** T-odd, chiral-odd Colinear 

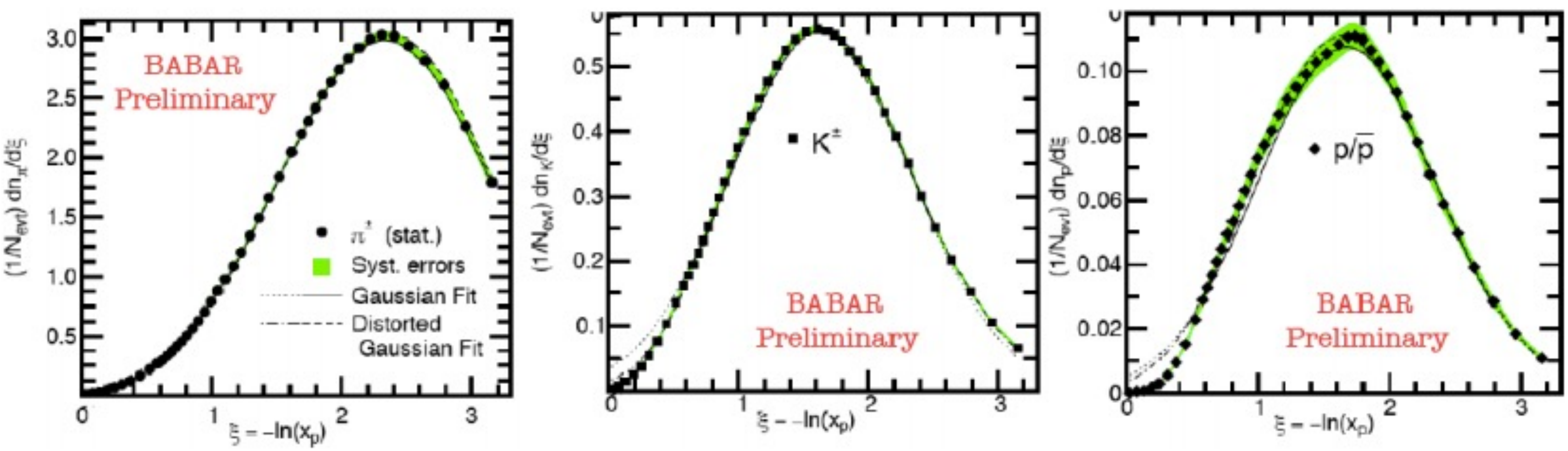


Perturbative QCD tests

- Time like splitting functions have singulari
  like important for DIS)
- MLLA → test for resummation
- Observed shape consistent with QCD calc
- FCC-ee might go to lower z. Impact?



PDG



Belle result on di-hadron cross-sections

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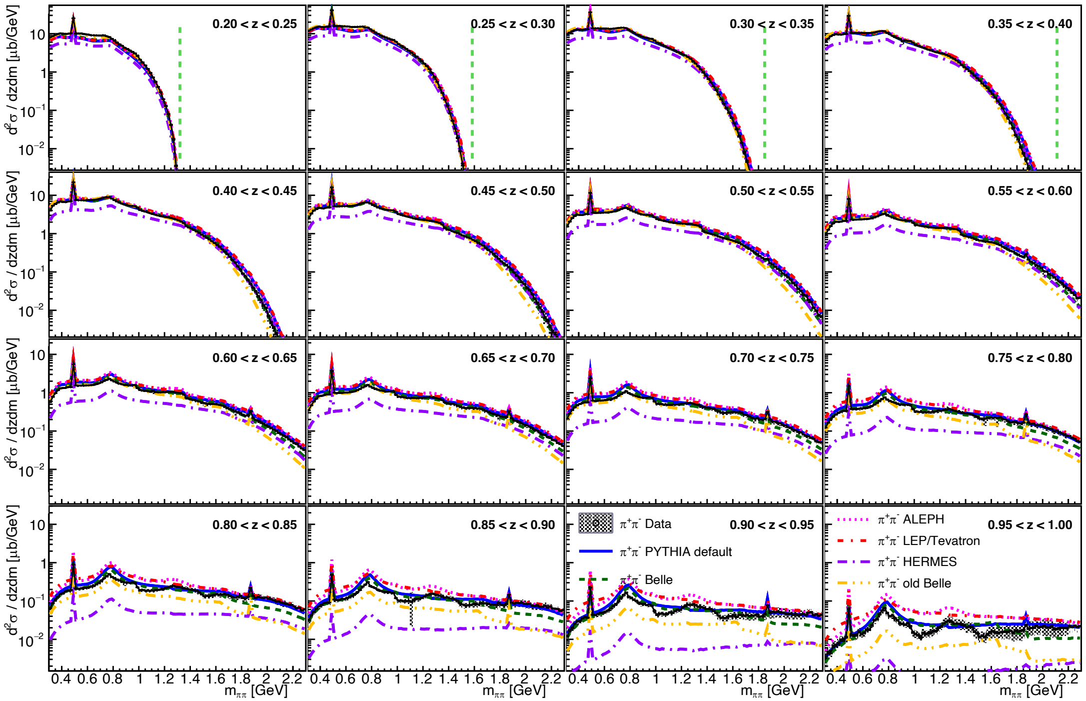
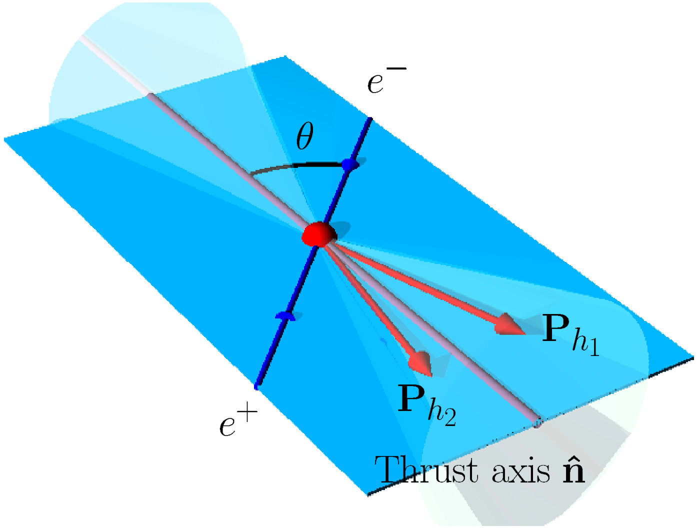
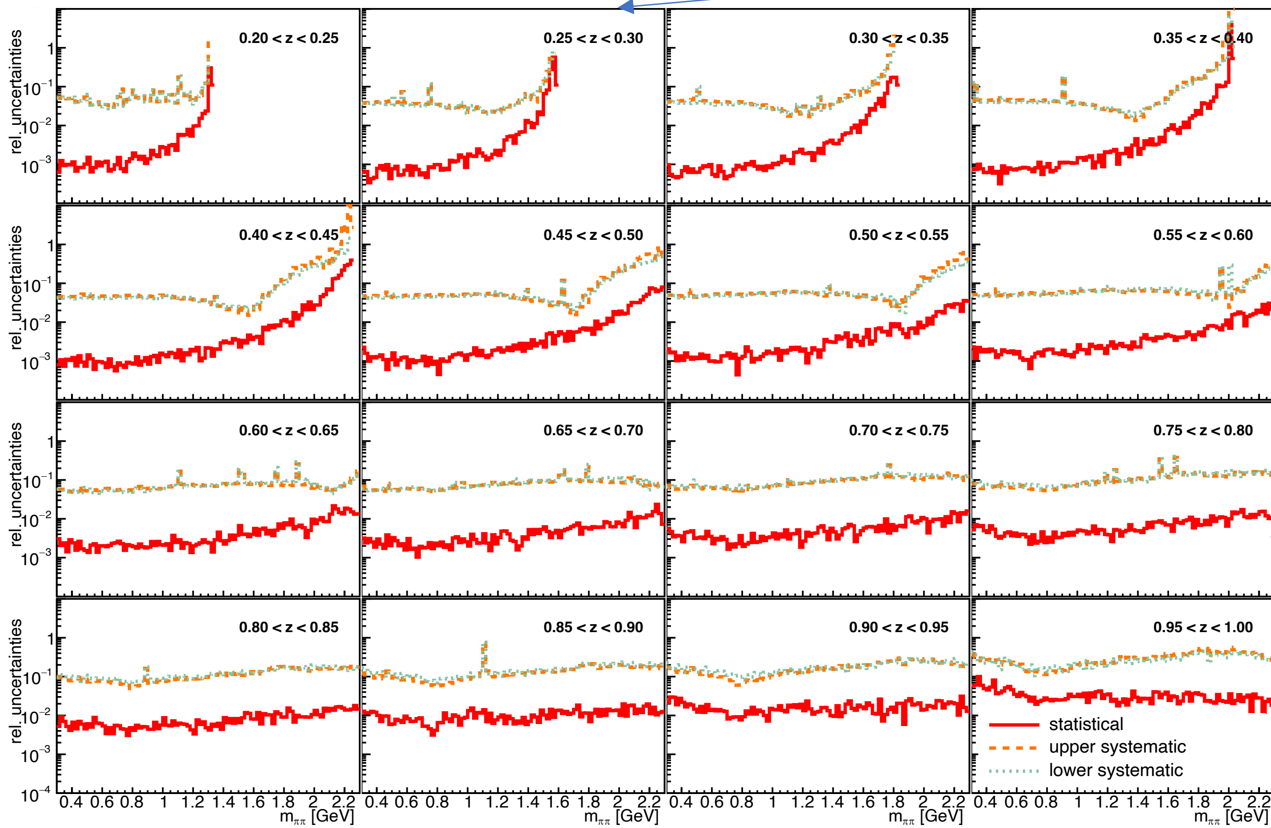




Results Systematics Dominated



- Low z:
  Dominated by PID uncertainties
  Belle II prospects: Improved PID, higher statistics to improve uncertainties on PID
- High z:
  Dominated by ISR uncertainties
  Belle II prospects: better understanding of ISR radiation with better statistics
-

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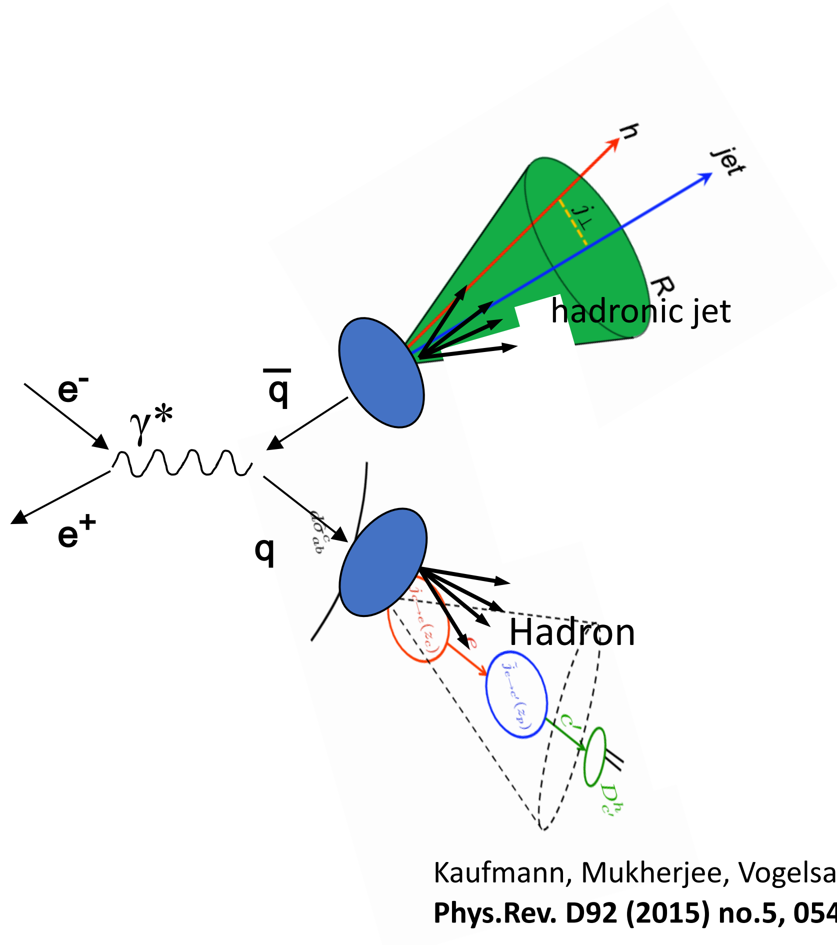
More recently: Interest in determining TMD FFs from Hadrons in Jets



Kaufmann, Mukherjee, Vogelsang,
Phys.Rev. D92 (2015) no.5, 054015

CLAS12 Beam Spin Asymmetry Measurements

Twist 2

$A_{LU} \sim f_1 G_1^{\perp|\ell,m\rangle}$

$G_1^{\perp|\ell,m\rangle}$ =