Advancing the Understanding of Non-Perturbative QCD Using Energy Flow

Stony Brook University, CFNS, 19-22 Sept 2022

Non-Perturbative Study of Isospin Symmetry Breaking

Abdel Nasser Tawfik

Future University in Egypt (FUE) & Egyptian Center for Theoretical Physics (ECTP)

a.tawfik@fue.edu.eg

Collaborator: Abdel Magied Diab

20th September 2022: 14:30 (20:30 GMT+2)

Agenda

Isobaric quantum spin

Non-perturbative QCD, at finite μ_b/μ_I

QCD-like effective model: Polyakov linear-sigma model

Some Thermodynamical Results

Final Remarks and Conclusions

Int.J.Mod.Phys.A 34 (2019) 31, 1950199, 1904.09890 [hep-ph]

Isobaric Quantum Spin Asymmetry

- To explain the neutron-proton symmetry, Heisenberg introduced isobaric spin (isospin) into nuclear physics.
- Wigner added more detail [2].
- The experimental confirmation of this quantum number was carried out in the thirties and fifties of the last century [3,4].

p & n are states of nucleon

light nuclei

proton : $q=+1$, $s=\frac{1}{2}$, $m=938.28\,\text{MeV}$ $\qquad p=\left|\frac{1}{2}+\frac{1}{2}\right\rangle$

neutron : $q=0$, $s=\frac{1}{2}$, $m=939.57\,\text{MeV}$ $\qquad n=\left|\frac{1}{2}-\frac{1}{2}\right\rangle$

[1] W. Heisenberg, Z. Physik, 77, 1-11 (1932)
[2] E.P. Wigner, Phys. Rev., 51, 106-19, 947-58 (1937)
[3] G. Breit, E. Condon, R.D. Present, Phys. Rev., 50, 825-45 (1936)
[4] D.H. Wilkinson, and G.A. Jones, Phil. Mag., 44, 542-47 (1953)

Isobaric Quantum Spin Asymmetry

Isobaric spin quantum number

Anderson et al. showed In 1961 that isobaric spin concepts were important in heavy nuclei as well [5]

The major difference between nucleons is that neutrons are uncharged. This "binary" nature of nucleon charge could be simply represented by charge operator Q and its associated eigenstates $\chi(N)$, wherein

$$Q\chi(N) = q\chi(N) \qquad 2.1.$$

with $q = 0$ and 1 being a measure of the charge on a neutron (n) or a proton (p) respectively.

[5] J.D. Anderson, and C. Wong, Phys. Rev. Letters, 7, 250-51 (1961),
[6] D. Robson, Isobaric Spin in Nuclear Physics, Annual Review Nucl. Sci. 16, 119-152 (1966)

Isobaric Quantum Spin Asymmetry

Isobaric spin quantum number

- Isospin *I* is a vector quantity.
- Both up and down quarks have *I*=1/2, I_3=+1/2 for up and I_3=−1/2 for down quarks.
- All other quarks have *I*=0.

$$I_3 = \frac{1}{2}(n_u - n_d)$$

valid for all hadrons

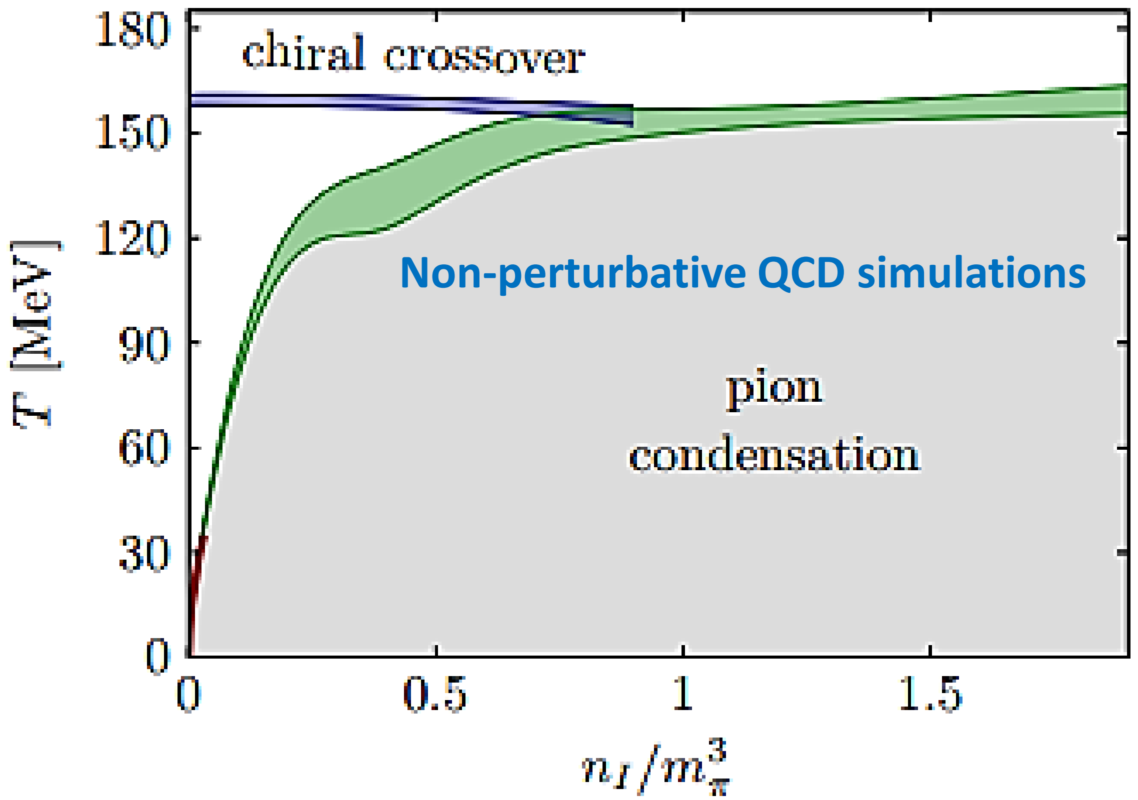
Finite *I* plays a major role in various physical systems, for instance, early Universe especially at large lepton asymmetry, compact stars with pion condensates, and spectroscopy of nuclei.

W. Greiner; B. Müller, Quantum Mechanics: Symmetries, (1994).

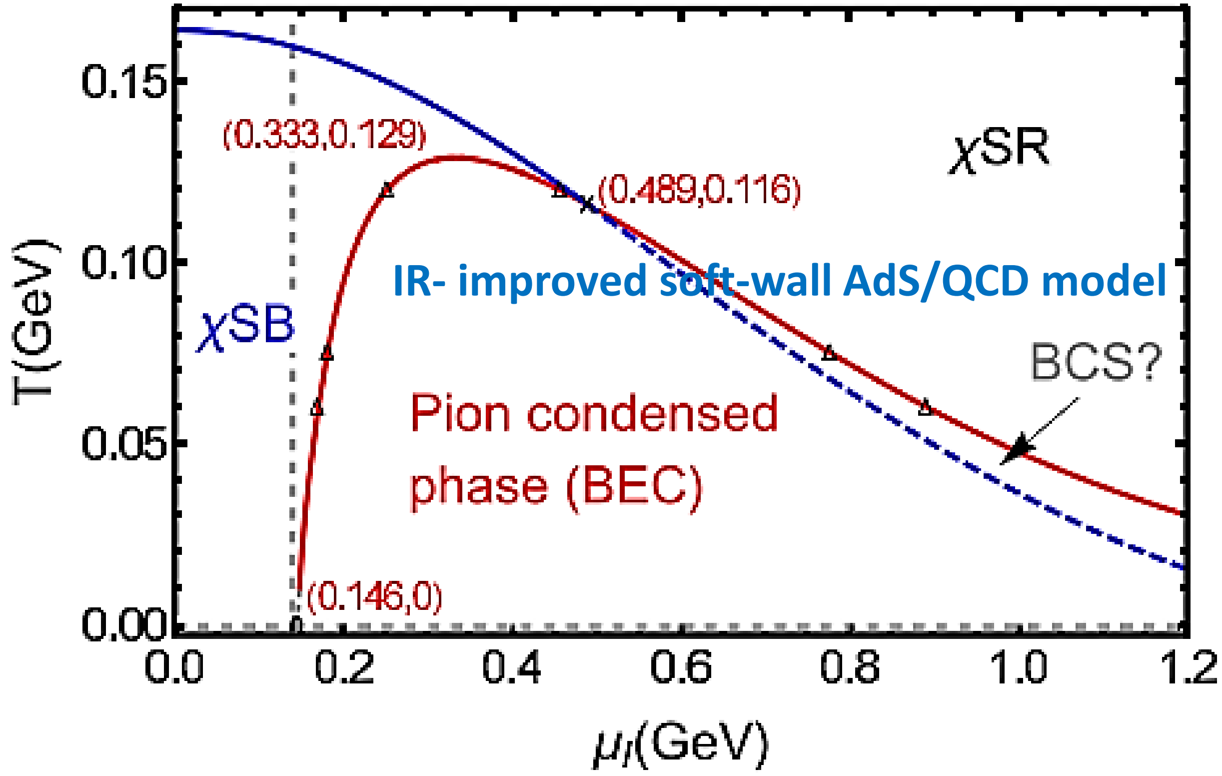
Isobaric Quantum Spin Asymmetry

Lattice QCD at finite Isospin asymmetry

Lattice QCD simulations at finite μ_I has a real and positive action and therefore can straightforwardly be implemented in the Monte Carlo techniques



LATTICE2021



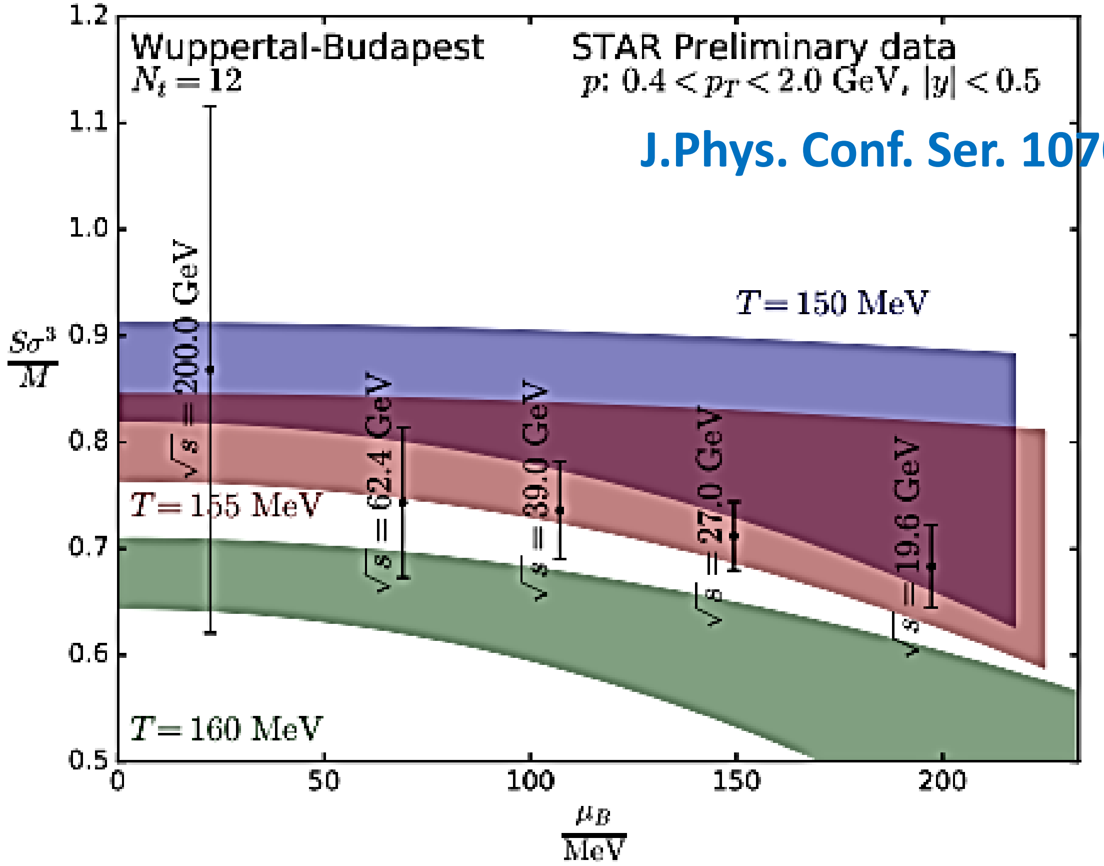
Chinese Physics C 44, (2020) 083106

Isobaric Quantum Spin Asymmetry

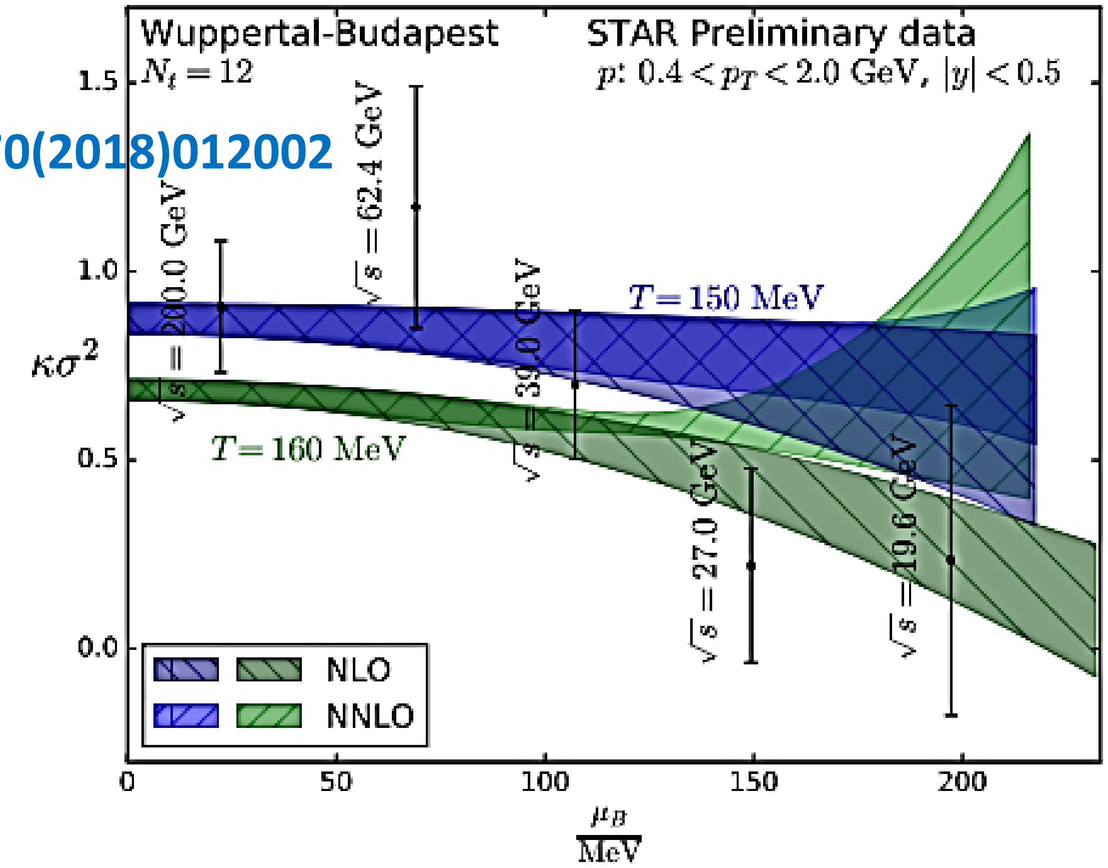
Lattice QCD at finite Isospin asymmetry

Extrapolated to finite baryon-chemical potential

Products of HM



J.Phys. Conf. Ser. 1070(2018)012002

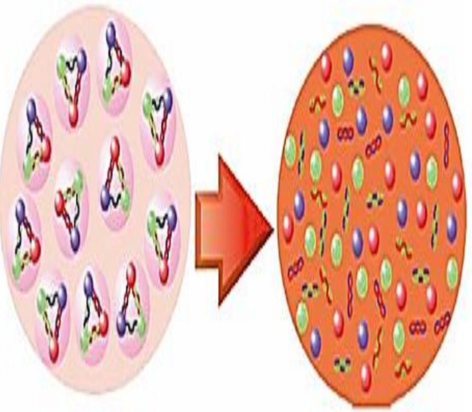


Finite μ_I-lattice QCD could be utilized to improve sign problem at finite μ_b

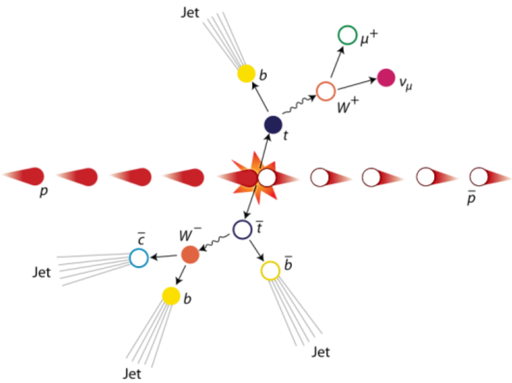
Isobaric Quantum Spin Asymmetry

Lattice QCD at finite Isospin asymmetry

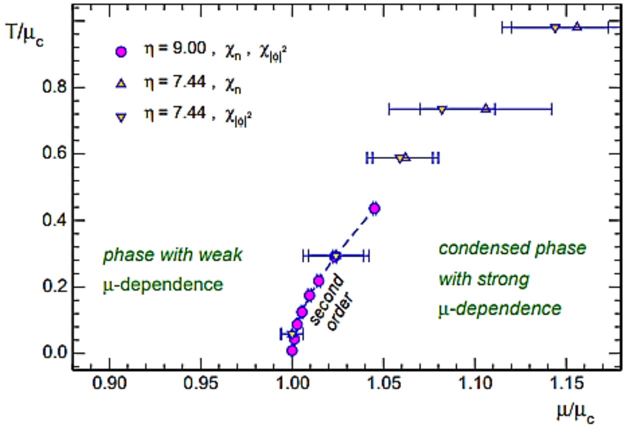
Lattice QCD at finite μ_I and finite μ_b obviously share some common features, such as, **deconfinement**, **particle creation**, **Silver-Blaze phenomenon**, and **Bose-Einstein condensation (BEC).**



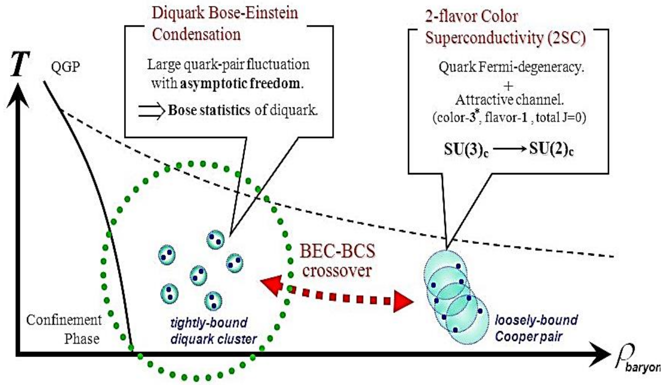
Deconfinement



Particle creation



Silver-Blade: At T=0, thermodin. are μ-independent up to μc



BEC

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

At finite isospin asymmetry in Minkowski space, the LSM Lagrangian density with Nf q-flavors can be incorporated with the Polyakov-loop potential

$$\mathcal{L}_{PLSM} = \mathcal{L}_{\text{chiral}} - \mathcal{U}(\phi, \bar{\phi}, T). \qquad (1)$$

The first term in (1) stands for LSM Lagrangian density in chiral limit

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\bar{\psi}\psi} + \mathcal{L}_m, \qquad (2)$$

where the first term counts for the contributions of the quarks (fermions) with Nc color dof, while the second term stands for the mesonic (bosonic) fields.

The second term in rhs of Eq. (1), $\mathcal{U}(\phi, \bar{\phi}, T)$ stands for the Polyakov-loop potential, which introduces the gluonic degrees-of-freedom and the dynamics of the quark-gluon interactions to the chiral LSM [37]. In the present calculations, we utilize a Polyakov-loop potential, which counts for strong coupling and includes higher orders of the Polyakov-loop variables.

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

$$\bar{\Phi} = \sum_{a=0}^{N_f^2-1} T_a(\bar{\sigma_a} + i\bar{\pi_a}) \tag{6}$$

where σ_a and π_a are the scalar and pseudoscalar fields, respectively. In vacuum state with $U(1)_A$ anomaly and as a result of the spontaneous symmetry breaking, the expectation values of mesonic fields, $\langle\Phi\rangle$, and of their conjugates, $\langle\Phi^\dagger\rangle$ are generated with the quantum numbers of the vacuum [48]. This leads to exact vanishing mean value of $\bar{\pi}_a$ but assures finite mean value of $\bar{\sigma_a}$ corresponding to the diagonal generators $U(3)$ as $\bar{\sigma_0} \neq \bar{\sigma_3} \neq \bar{\sigma_8} \neq 0$, where $\langle\Phi\rangle = T_0\bar{\sigma_0} + T_3\bar{\sigma_3} + T_8\bar{\sigma_8}$.

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

On the other hand, $\bar{\sigma}_3$ breaks the isospin asymmetry SU(2) , Furthermore, the potential of pure mesonic contributions in SU(N_f) can be written as ,

$$U(\bar{\sigma}) = \left(\frac{m^2}{2} - h_a\right)\bar{\sigma}_a - 3\mathcal{G}_{abc}\bar{\sigma}_b\,\bar{\sigma}_c - \frac{4}{3}\mathcal{F}_{abcd}\,\bar{\sigma}_b\,\bar{\sigma}_c\bar{\sigma}_d, \qquad (7)$$

where the coefficeints $\mathcal{G}_{abc}$ and $\mathcal{F}_{abcd}$ are given as

$$\mathcal{G}_{abc} = \frac{c}{6}\left[d_{abc} - \frac{3}{2}\left(d_{0bc}\delta_{a0} + d_{a0c}\delta_{b0} + d_{ab0}\delta_{c0}\right) + \frac{9}{2}d_{000}\delta_{a0}\delta_{b0}\delta_{c0}\right], \qquad (8)$$

$$\mathcal{F}_{abcd} = \frac{\lambda_1}{4}\left[\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{cd} + \delta_{ac}\delta_{bd}\right] + \frac{\lambda_2}{8}\left[d_{abn}d_{ncd} + d_{adn}d_{nbc} + d_{acn}d_{nbd}\right]. \qquad (9)$$

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

The explicitly symmetry breaking terms, h_0, h_3 and h_8, can be determined by minimizing the potential, Eq. (7), on tree level, $\partial U(\bar{\sigma})/\partial\bar{\sigma}_a = 0$. h_0 and h_8, can be determined from the partially conserved axial current (PCAC) relations

Hypothesis of *Partially Conserved Axial-Vector Current*

$$h_0 = \frac{1}{\sqrt{6}}\left(m_\pi^2 f_\pi + 2 m_K^2 f_K\right), \qquad (10)$$

$$h_8 = \frac{2}{\sqrt{3}}\left(m_\pi^2 f_\pi - m_K^2 f_K\right). \qquad (11)$$

The generator operator $\hat{T}_a = \hat{\lambda}_a/2$ in U(3) is a obtained from Gell-Mann matrices $\hat{\lambda}_a$] with the indices running as $a = 0, \cdots, 8$. From U(3) algebra, we have

$$\left[\hat{T}_a, \hat{T}_b\right] = i f_{abc} \hat{T}_c, \qquad (A1)$$

$$\left\{\hat{T}_a, \hat{T}_b\right\} = i d_{abc} \hat{T}_c, \qquad (A2)$$

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

where f_{abc} and d_{abc} are the standard antisymmetric and symmetric structure constants of SU(3), respectively. The symmetric structure constant d_{abc} can be defined as

$$d_{abc} = \frac{1}{4} Tr \left[\left\{ \hat{\lambda}_a, \, \hat{\lambda}_b \right\} \hat{\lambda}_c \right], \tag{A3}$$

$$d_{ab0} = \sqrt{\frac{2}{3}} \, \delta_{ab}. \tag{A4}$$

In PCAC relation, the decay constant f_a is related to the symmetric structure constant as

$$f_a = d_{aab} \bar{\sigma}_a. \tag{A5}$$

Accordingly, the decay constants of the charged and neutral pion mesons ($f_{\pi^\pm} = f_1, \; f_{\pi^0} = f_3$) and kaon meson ($f_{K^\pm} = f_4, \; f_{K^0} = f_6$) are given as

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

$$f_{\pi^0} = f_{\pi^\pm} = \sqrt{\frac{2}{3}}\bar{\sigma}_0 + \frac{1}{\sqrt{3}}\bar{\sigma}_8, \tag{A6}$$

$$f_{K^\pm} = \sqrt{\frac{2}{3}}\bar{\sigma}_0 + \frac{1}{2}\bar{\sigma}_3 - \frac{1}{2\sqrt{3}}\bar{\sigma}_8, \tag{A7}$$

$$f_{K^0} = \sqrt{\frac{2}{3}}\bar{\sigma}_0 - \frac{1}{2}\bar{\sigma}_3 - \frac{1}{2\sqrt{3}}\bar{\sigma}_8, \tag{A8}$$

where the isospin sigma field, $\bar{\sigma}_3$, is the difference between the decay constants of neutral and charged kaon mesons as,

$$\bar{\sigma}_3 = f_{K^\pm} - f_{K^0}. \tag{A9}$$

From the experimental and recent lattice review on physical constants , $f_{\pi^\pm} = f_{\pi^0} = 92.4$ MeV and $f_{K^\pm} = 113$ MeV, $f_{K^0} = 113.453$ MeV.

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

Thus, the explicit symmetry breaking term, h_3, can be deduced from $\partial U(\bar{\sigma})/\partial \bar{\sigma}_3 = 0$,

$$h_3 \;=\; \left[m^2 + \frac{c}{\sqrt{6}}\bar{\sigma}_0 - \frac{c}{\sqrt{3}}\bar{\sigma}_8 + \lambda_1\left(\bar{\sigma}_0{}^2 + \bar{\sigma}_3{}^2 + \bar{\sigma}_8{}^2\right) + \lambda_2\left(\bar{\sigma}_0{}^2 + \frac{\bar{\sigma}_3{}^2}{2} + \frac{\bar{\sigma}_8{}^2}{2} + \sqrt{2}\bar{\sigma}_0\bar{\sigma}_8\right)\right]\bar{\sigma}_3, \quad (12)$$

where the square brackets $[\cdots]$ is the squared mass of the a_0 meson and $\bar{\sigma}_3 = (f_{K^\pm} - f_{K^0})$

$$h_3 \;=\; m_{a_0}^2\left(f_{K^\pm} - f_{K^0}\right), \quad (13)$$

As a result of the finite isospin asymmetry, the masses of the quark flavors, as nature likely prefers, are not entirely degenerated, i.e. $m_u \neq m_d \neq m_s$. To assure this situation, we use the orthogonal basis transformation to convert the condensates from the original basis, σ_0, σ_3, and σ_8 to pure up

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

(σ_u), down (σ_d), and strange (σ_s) quark flavor basis, respectively,

$$\begin{bmatrix} \bar{\sigma_u} \\ \bar{\sigma_d} \\ \bar{\sigma_s} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} \sqrt{2} & 1 & 1 \\ \sqrt{2} & -1 & 1 \\ 1 & 0 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} \bar{\sigma_0} \\ \bar{\sigma_3} \\ \bar{\sigma_8} \end{bmatrix}. \tag{14}$$

Accordingly, the masses of u, d, and s quarks can be expressed as,

$$m_u = \frac{g}{2}\sigma_u, \qquad m_d = \frac{g}{2}\sigma_d, \qquad m_s = \frac{g}{\sqrt{2}}\sigma_s. \tag{15}$$

As mentioned above, the potential of the pure mesonic contributions can be obtained by substituting the mesonic field, Eq. (6), in the potential term of chiral LSM Lagrangian density, Eq. (7). The

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

potential of mesonic contributions can be given as,

$$U(\sigma_u, \sigma_d, \sigma_s) = \frac{m^2}{4}\left[\sigma_u^2 + \sigma_d^2 + 2\sigma_s^2\right] - \frac{c}{2\sqrt{2}}\sigma_u\sigma_d\sigma_s + \frac{\lambda_1}{16}\left(\sigma_u^2 + \sigma_d^2 + 2\sigma_s^2\right)^2 + \frac{\lambda_2}{16}\left(\sigma_u^4 + \sigma_d^4 + 4\sigma_s^4\right) - h_{ud}\frac{\sigma_u + \sigma_d}{2} - h_3\frac{\sigma_u - \sigma_d}{2} - h_s\sigma_s. \quad (16)$$

For symmetry breaking in vacuum, $H \neq 0$, $c \neq 0$ and $\lambda \neq 0$, the impacts of the isospin asymmetry violating SU(2), h_0, h_3 and h_8, have nonzero values, at the chiral masses $m_u \neq m_d \neq m_s \neq 0$.

m_σ [MeV]	c [MeV]	h_{ud} [MeV^3]	h_3 [MeV^3]	h_s [MeV^3]	m^2 [MeV^2]	λ_1	λ_2
800	4807.84	$(120.73)^3$	$-(78.31)^3$	$(336.41)^3$	$-(306.26)^2$	13.49	46.48

Values of the LSM parameters given in the mesonic Lagrangian, Eq. (4), as fixed at $m_\sigma = 800$ MeV

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

In the mean-field approximation (MFA), the PLSM thermodynamic potential can be related the to grand-canonical function $\mathcal{Z}$, which is given in dependence of the temperatures T and the chemical potentials of $f-$th quark flavor μ_f, see App. (B),

$$\Omega(T,\mu_f) = \frac{-T \cdot \ln[\mathcal{Z}]}{V} = U(\sigma_u, \sigma_d, \sigma_s) + \mathcal{U}_{\rm Fuku}(\phi, \bar{\phi}, T) + \Omega_{\bar{\psi}\psi}(T, \mu_f). \tag{17}$$

The chemical potentials μ_f are related to conserved quantum numbers of - for instance - baryon number (B), strangeness (S), electric charge (Q), and isospin (I) of each quark flavors,

$$\mu_u = \frac{\mu_B}{3} + \frac{2\mu_Q}{3} + \frac{\mu_I}{2}, \tag{18}$$

$$\mu_d = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \frac{\mu_I}{2}, \tag{19}$$

$$\mu_s = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \mu_S. \tag{20}$$

Isobaric Quantum Spin Asymmetry

Polyakov Linear-Sigma Model

In expression (17), the first term $U(\sigma_u, \sigma_d, \sigma_s)$; the potential of the pure mesonic contributions, was given Eq. (16), while the second term $\mathcal{U}_{\mathrm{Fuku}}(\phi, \bar{\phi}, T)$, the potential of Polyakov loop variables, was elaborated in Eq. (5). The last term refers to the quarks and antiquarks contributions to the PLSM potential [37, 51–53],

$$\Omega_{\bar{\psi}\psi}(T, \mu_f) \;=\; -2T \sum_{f=u,d,s} \int_0^{\infty} \frac{d^3\vec{P}}{(2\pi)^3} \ln\left[1 + n_{q,f}(T,\ \mu_f)\right] + \ln\left[1 + n_{\bar{q},f}(T,\ \mu_f)\right], \qquad (21)$$

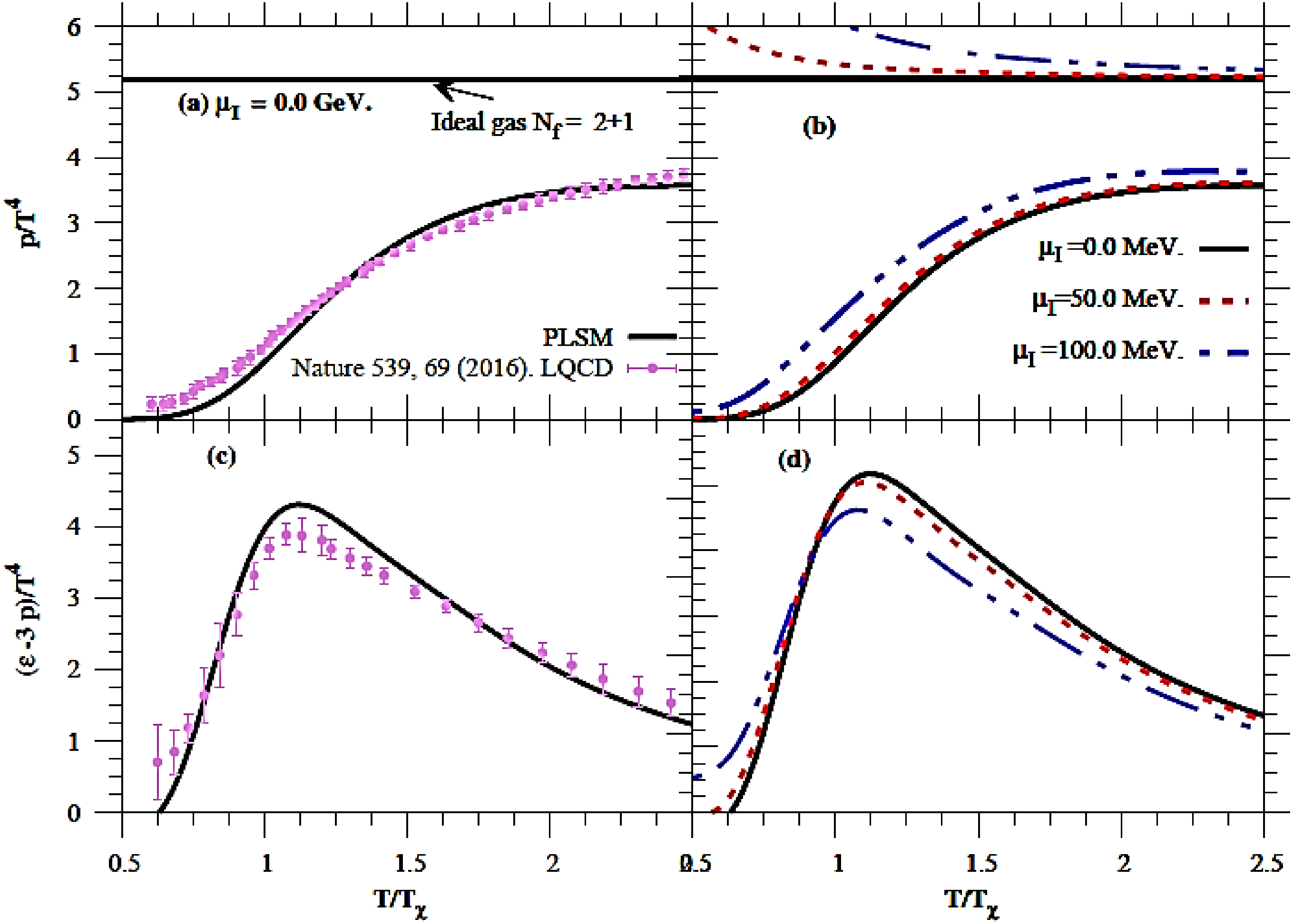
where the number density distribution for particle is given as

$$n_{q,f}(T,\ \mu_f) \;=\; 3\left(\phi + \bar{\phi} e^{-\frac{E_f-\mu_f}{T}}\right) \times e^{-\frac{E_f-\mu_f}{T}} + e^{-3\frac{E_f-\mu_f}{T}}, \qquad (22)$$

which is identical to that of anti-particle $n_{q,f}(T,\ \mu_f)$ with $-\mu_f$ replacing $+\mu_f$ and the order parameter ϕ by its conjugate $\bar{\phi}$ or vice versa. $E_f = (\vec{P}^2 + m_f^2)^{1/2}$ is the energy-momentum dispersion relation with m_f being the mass of f^{th} quark flavor.

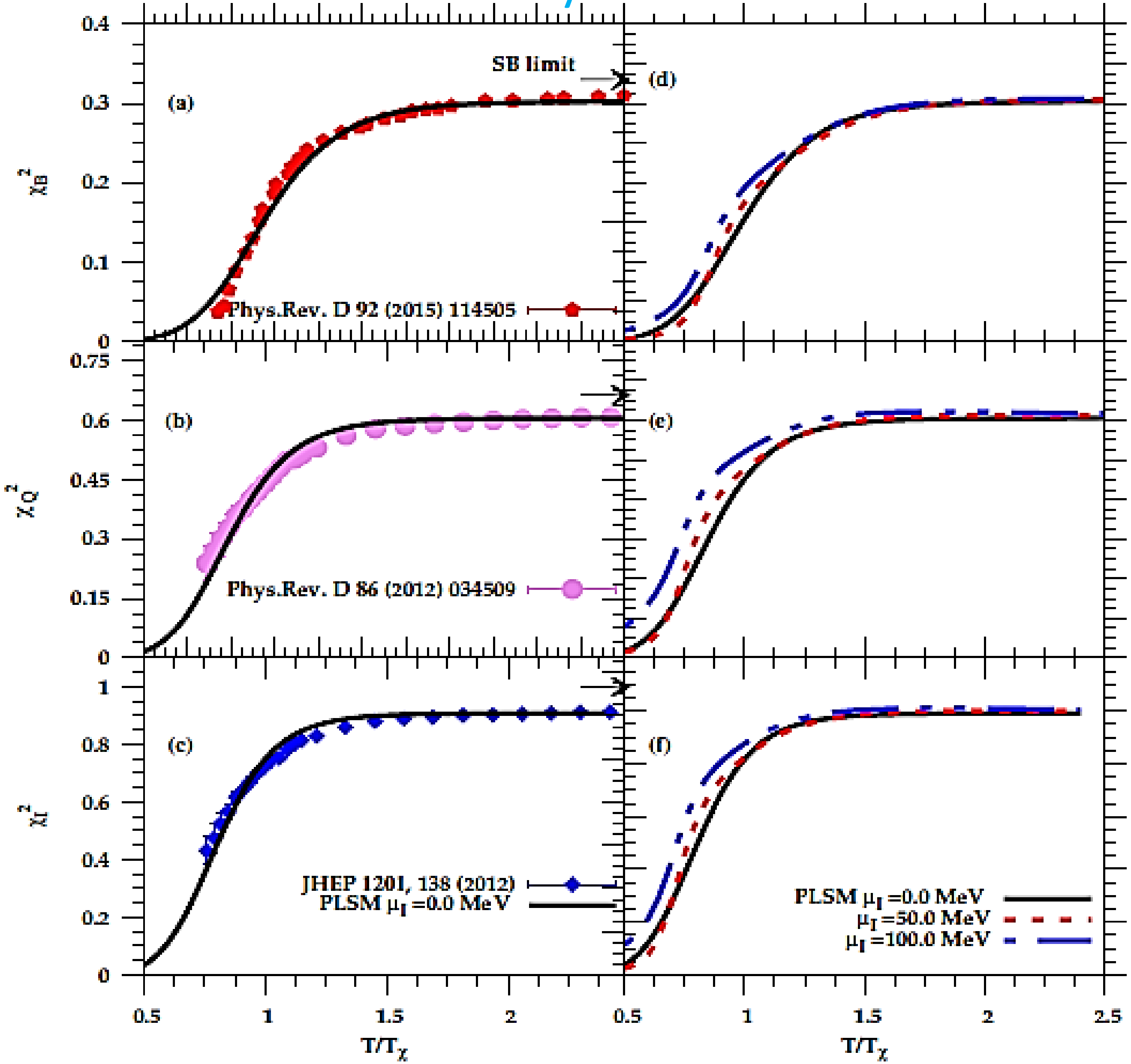
Isobaric Quantum Spin Asymmetry

Some Thermodynamic Results



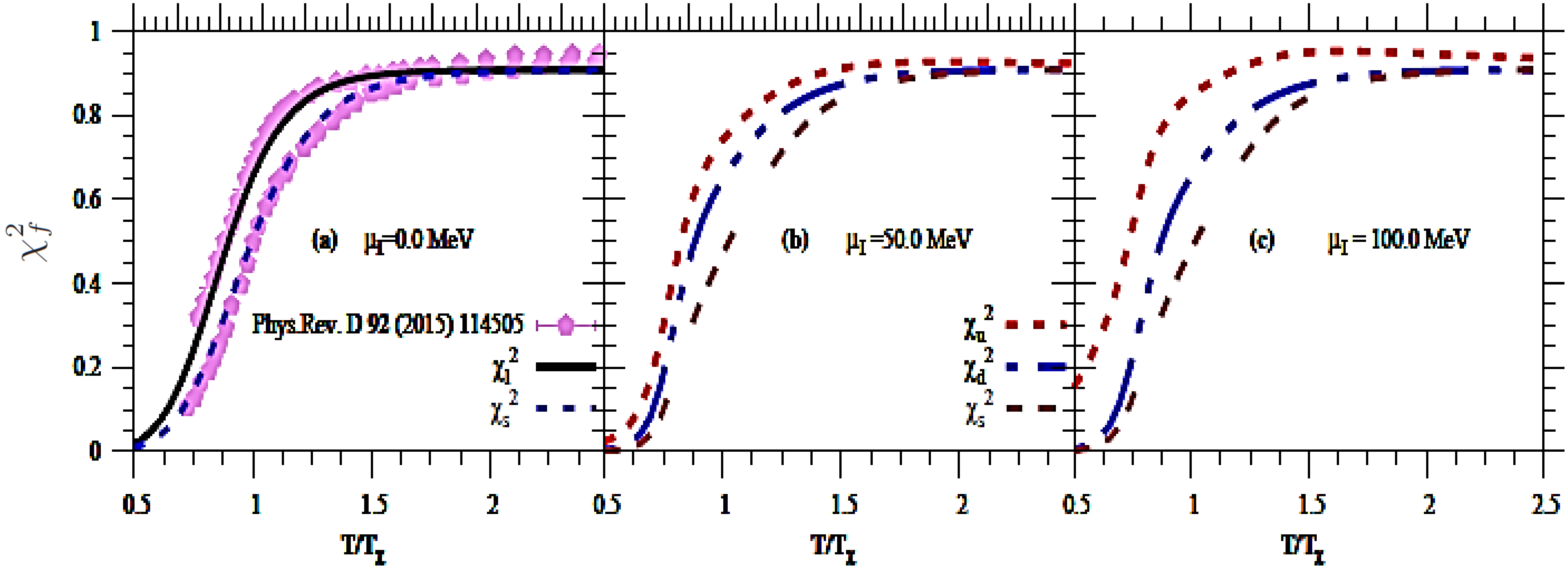
Isobaric Quantum Spin Asymmetry

Some Thermodynamic Results



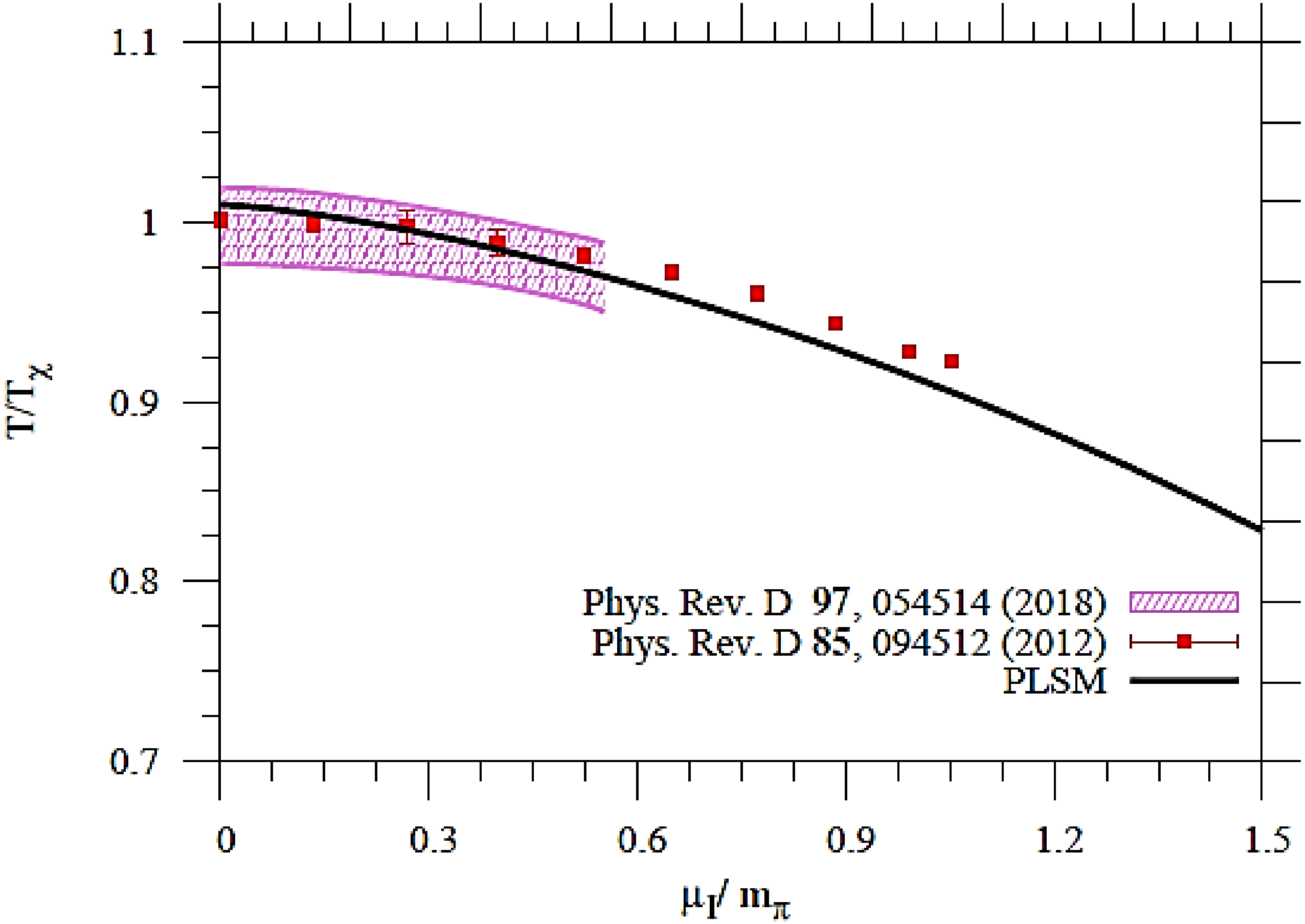
Isobaric Quantum Spin Asymmetry

Some Thermodynamic Results



Isobaric Quantum Spin Asymmetry

Some Thermodynamic Results



Conclusions

- In SU(3), finite isospin asymmetry makes the mean sigma fields $\bar{\sigma_a}$ having nonzero diagonal generators as $\bar{\sigma}_0 \neq \bar{\sigma}_3 \neq \bar{\sigma}_8 \neq 0$ and the parameters of explicitly symmetry breaking are non-vanishing $h_0 \neq h_3 \neq h_8 \neq 0.$
- This means that, the impacts of finite σ_3 and h_3 break SU(2) isospin asymmetry, where $\sigma_u = \sigma_l + \sigma_3$ and $\sigma_d = \sigma_l - \sigma_3$.
- To this end, we first driven the thermodynamic potential of pure mesonic contributions in SU(3) in basis of σ_u, σ_d and σ_s.
- Polyakov-loop potential in integrated in to assure integrating the gluonic dof in the chiral LSM and the gluon-quark interactions.