On sum rules for double and triple parton distribution functions and Pythia's model of multiple parton interactions

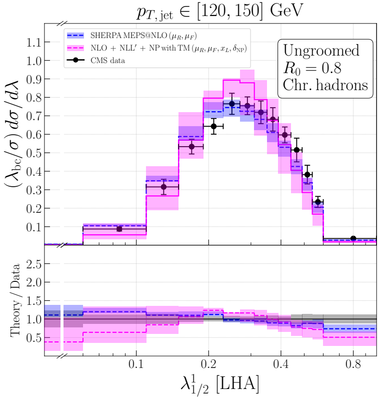
Oleh Fedkevych
in collaboration with Jonathan Gaunt

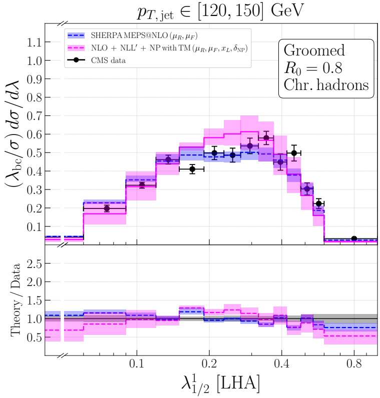
Università degli studi di Genova and INFN Sezione di Genova

21.09.22

Based upon 2208.08197

UNIVERSITÀ DEGLI STUDI DI GENOVA





Comparison of theoretic predictions (MC and resummation) against recent CMS data for the Jet Thrust angularity, $p_{T,\text{jet}} \in [120, 150]$ GeV

Data: 2109.03340; Theory: 2104.06920

Master formulas for DPS and TPS

A master formula for DPS can be schematically written as

$$\sigma_{hh'}^{\mathrm{DPS}} = \sum_{\mathrm{partons}} \int \prod_{i=1}^{2} dx_i \, dx'_i \, d^2 b \times$$
$$\times \quad \Gamma_h\left(\{x_i\}, \mathbf{b}, \{Q_i\}\right) \Gamma_{h'}\left(\{x'_i\}, \mathbf{b}, \{Q_i\}\right) [\cdots],$$

which for the TPS becomes

$$\sigma_{hh'}^{\mathrm{TPS}} = \sum_{\mathrm{partons}} \int \prod_{i=1}^{3} dx_i \, dx'_i \, d^2 b_i \, d^2 b \times$$
$$\times \quad \Gamma_h\left(\{x_i\}, \mathbf{b}_i, \{Q_i\}\right) \Gamma_{h'}\left(\{x'_i\}, \{\mathbf{b}_i - \mathbf{b}\}, \{Q_i\}\right) [\cdots].$$

A standard assumption on factorization into longitudinal and transverse parts reads
$\Gamma_h\left(\{x_i\}, \mathbf{b}_i, \{Q_i\}\right) \approx T_h\left(\{x_i\}, \{Q_i\}\right) \sum_i f(\mathbf{b}_i)$.

UNIVERSITÀ DEGLI STUDI DI GENOVA

The sum rules for dPDFs and tPDFs are

$$
\begin{aligned}
\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) &= (1 - x_1)\, f_{j_1}(x_1, Q), \\
\sum_{j_3} \int_0^{1-x_1-x_2} dx_3\;\, x_3\;\, T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) &= (1 - x_1 - x_2) D_{j_1 j_2}(x_1, x_2, Q), \\
\int_0^{1-x_1} dx_2\, D_{j_1 j_{2v}}(x_1, x_2, Q) &= \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q), \\
\int_0^{1-x_1-x_2} dx_3\;\, T_{j_1 j_2 j_{3v}}(x_1, x_2, x_3, Q) &= \left(N_{j_{3v}} - \delta_{j_3 j_1} - \delta_{j_3 j_2} + \delta_{\bar{j}_3 j_1} + \delta_{\bar{j}_3 j_2}\right) \times \\
&\times\; D_{j_1 j_2}(x_1, x_2, Q).
\end{aligned}
$$

UNIVERSITÀ DEGLI STUDI DI GENOVA

According to the PYTHIA8 model

$$
\begin{aligned}
T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) &= f^r_{j_1}(x_1, Q)\ f^{m\leftarrow j_1, x_1}_{j_2}(x_2, Q)\ f^{m\leftarrow j_1, x_1; j_2, x_2}_{j_3}(x_3, Q), \\
D_{j_1 j_2}(x_1, x_2, Q) &= f^r_{j_1}(x_1, Q)\ f^{m\leftarrow j_1, x_1}_{j_2}(x_2, Q).
\end{aligned}
$$

- To construct tPDFs one needs to access sPDFs used in PYTHIA8 at different generation stages.
- The tPDFs are constructed in a Monte Carlo way by taking an average over a large sample of "events".
- The approach can be applied to construct nPDFs as well!

As a baseline we use "naive" approach to tPDFs and dPDFs

$$
\begin{aligned}
T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) &= f^r_{j_1}(x_1, Q)\ f^r_{j_2}(x_2, Q)\ f^r_{j_3}(x_3, Q)\ \theta(1 - x_1 - x_2 - x_3). \\
D_{j_1 j_2}(x_1, x_2, Q) &= f^r_{j_1}(x_1, Q)\ f^r_{j_2}(x_2, Q)\ \theta(1 - x_1 - x_2).
\end{aligned}
$$

Let's check momentum rule first

x_1	x_2	j_1	j_2	PYTHIA tPDFs	"Naive" tPDFs
10^{-6}	10^{-4}	u	u	0.996	0.996
10^{-3}	10^{-4}	u	u	0.997	0.997
10^{-1}	10^{-4}	u	u	1.007	1.096
0.2	10^{-4}	u	u	1.008	1.195
0.4	10^{-4}	u	u	1.007	1.390
0.8	10^{-4}	u	u	1.002	1.626

Test of the momentum sum rule for the tPDFs.

Note that the factor $\theta(1 - x_1 - x_2 - x_3)$ in the definition of "naive" tPDFs does not imply that tPDFs obey the momentum sum rule!

UNIVERSITÀ DEGLI STUDI DI GENOVA

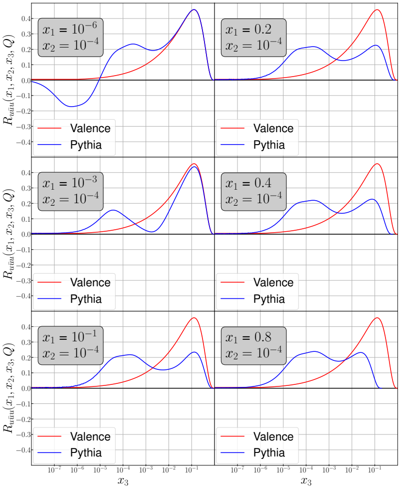
Now let's check number rule

We define

$$R_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) \equiv x_3 \, \frac{T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) - T_{j_1 j_2 \bar{j}_3}(x_1, x_2, x_3, Q)}{D_{j_1 j_2}(x_1, x_2, Q)},$$

which can be seen as a response of the valence sPDF $f_{j_{3v}}(x_3, Q)$ to the first two interactions.

Check of the number rule



The responses of the valence u-quark sPDF $R_{u\bar{u}u}(x_1, x_2, x_3, Q)$ as function of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The response functions are averaged over 10^7 function calls.

Check of the number rule

UNIVERSITÀ DEGLI STUDI DI GENOVA

The numerical integration over the response function yields

x_1	x_2	N_{u_v} PYTHIA	N_{u_v} "Naive"
10^{-6}	10^{-4}	2.019	2.006
10^{-3}	10^{-4}	2.005	2.006
10^{-1}	10^{-4}	2.001	2.005
0.2	10^{-4}	2.000	2.005
0.4	10^{-4}	1.999	1.997
0.8	10^{-4}	1.995	1.708

Integration over $R_{u\bar{u}u}$ response function with respect to x_3 at fixed x_1, x_2.

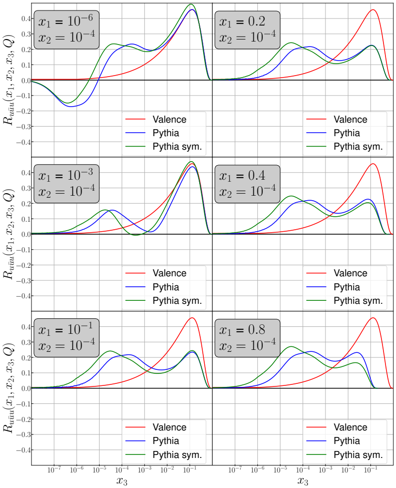
- Similar checks can be made for other flavour combinations.
- PYTHIA8 tPDFs preserve the sum rules at about 1% accuracy level.
- PYTHIA8 tPDFs do not obey DGLAP evolution equation and are asymmetric.

UNIVERSITÀ DEGLI STUDI DI GENOVA

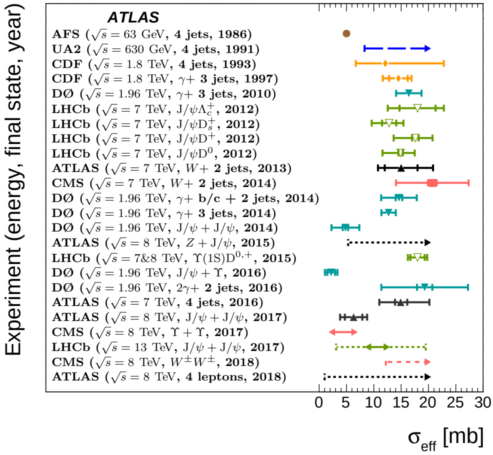
Summary and possible next steps

- We generalized the GS sum rules for the case of tPDFs.
- The sketch of proof of sum rules for "bare" and renormalized tPDFs is given.
- We demonstrated how one can construct asymmetric dPDFs, tPDFs (and nPDFs!) using PYTHIA8 code.
- Our attempt to construct symmetric tPDFs was not successful. However, the largest violations of the sum rules by symmetric tPDFs appear in the "deep valence region" ($x > 0.4$).
- Try to modify PYTHIA8 model to suppress large violations of the sum rules by symmetrized PYTHIA8 tPDFs.
- Make a phenomenological study with a tPDFs for different TPS production states. (e.g. to extend 4-jet DPS predictions of 2008.08347 to the TPS case using PYTHIA8 tPDFs).

Backup: Check of the number rule for symmetric tPDFs



The responses of the valence u-quark sPDF $R_{u\bar{u}u}(x_1, x_2, x_3, Q)$ as functions of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The green lines are symmetrized PYTHIA8 tPDFs.



From 1811.11094. See also recent *pp* 1909.06265 and *pA* 2007.06945 measurements. Also in CMS-PAS-BPH-21-004 the presence of TPS was observed!

Backup: GS sum rules

The sum rules for dPDFs were proposed by Gaunt & Stirling some time ago

$$\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q),$$

$$\int_0^{1-x_1} dx_2\, D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q).$$

- The GS sum rules state conservation of momentum and describe changes in the number of valence partons after a DPS processes.
- Originally were proved using the Light-cone representation of dPDFs
- More rigorous proof was given later in 1811.00289

Backup: GS sum rules for tPDFs

Let's start with momentum rule.

We get a chain of coupled equations:

$$\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q),$$

$$\sum_{j_3} \int_0^{1-x_1-x_2} dx_3\;\, x_3\;\, T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = (1 - x_1 - x_2) D_{j_1 j_2}(x_1, x_2, Q).$$

The Light-cone formalism for "bare" PDFs implies

$$
\begin{aligned}
D_{j_1 j_2}(x_1, x_2) &= \sum_{N,\{\beta_i\}} \int [dz]_N \, [d^2 \, \boldsymbol{k}]_N \, \left|\Phi_N(\{\beta_i, z_i, \, \boldsymbol{k}_i\})\right|^2 \times \\
&\times \sum_i^N \delta(x_1 - z_i) \, \delta_{j_1 p_i} \sum_{k \neq i}^N \delta(x_2 - z_k) \, \delta_{j_2 p_k}, \\
T_{j_1 j_2 j_3}(x_1, x_2, x_3) &= \sum_{N,\{\beta_i\}} \int [dz]_N \, [d^2 \, \boldsymbol{k}]_N \, \left|\Phi_N(\{\beta_i, z_i, \, \boldsymbol{k}_i\})\right|^2 \times \\
&\times \sum_i^N \delta(x_1 - z_i) \, \delta_{j_1 p_i} \sum_{k \neq i}^N \delta(x_2 - z_k) \, \delta_{j_2 p_k} \sum_{l \neq i,k}^N \delta(x_3 - z_l) \, \delta_{j_3 p_l},
\end{aligned}
$$

where

$$
\begin{aligned}
[dz]_N &\equiv \prod_{i=1}^N dz_i \, \delta\left(1 - \sum_i^N z_1\right), \\
\left[d^2 \, \boldsymbol{k}\right]_N &\equiv \prod_{i=1}^N d^2 \, \boldsymbol{k}_i \, \delta^2\left(\sum_i^N \, \boldsymbol{k}_i\right).
\end{aligned}
$$

Which can be written

$$\sum_{j_3} \int_0^{1-x_1-x_2} dx_3 \; x_3 \; T_{j_1 j_2 j_3}(x_1, x_2, x_3) = \sum_{N,\{\beta_i\}} \int [dz]_N \; [d^2 \; \boldsymbol{k}]_N \; |\Phi_N(\{\beta_i, z_i, \; \boldsymbol{k}_i\})|^2 \times$$

$$\times \quad \sum_{j_3} \int_0^{1-x_1-x_2} dx_3 \; \sum_i^N \delta(x_1 - z_i) \; \delta_{j_1 p_i} \; \sum_{k \neq i}^N \delta(x_2 - z_k) \; \delta_{j_2 p_k} \; \sum_{l \neq i,k}^N \delta(x_3 - z_l) \; \delta_{j_3 p_l} =$$

$$= \quad \sum_{N,\{\beta_i\}} \int [dz]_N \; [d^2 \; \boldsymbol{k}]_N \; |\Phi_N(\{\beta_i, z_i, \; \boldsymbol{k}_i\})|^2 \; \sum_i^N \delta(x_1 - z_i) \; \delta_{j_1 p_i} \times$$

$$\times \quad \sum_{k \neq i}^N \delta(x_2 - z_k) \; \delta_{j_2 p_k} \; \sum_{j_3} \sum_{l \neq i,k}^N z_l \; \delta_{j_3 p_l}.$$

The last sum in equation above can be written as

$$\sum_{j_3} \sum_{l \neq i,k}^N z_l \; \delta_{j_3 p_l} = \sum_l^N z_l - z_i - z_k = 1 - x_1 - x_2,$$

which allows to recover expression for the momentum sum rule!

UNIVERSITÀ DEGLI STUDI DI GENOVA

RGE can be used to proof the sum rules for renormalized tPDFs

$$
\begin{aligned}
T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = & \sum_{j_1' j_2' j_3'} Z_{j_1 j_1'}(Q) \otimes_1 Z_{j_2 j_2'}(Q) \otimes_2 Z_{j_3 j_3'}(Q) \otimes_3 T^B_{j_1' j_2' j_3'}(z_1, z_2, z_3) \\
& + \sum_{j_1' j_2'} Z_{j_1 j_1'}(Q) \otimes_1 Z_{j_2 j_3, j_2'}(Q) \otimes_{23} D^B_{j_1' j_2'}(z_1, z_2) + (\text{permutations}) \\
& + \sum_{j_1'} Z_{j_1 j_2 j_3, j_1'}(Q) \otimes_{123} f^B_{j_1'}(z_1)
\end{aligned}
$$

where

$$
\begin{aligned}
A \otimes_1 B &= \int \frac{dz}{z} A\left(\frac{x_1}{z}\right) B(z), \\
A \otimes_{12} B &= \int \frac{dz}{z^2} A\left(\frac{x_1}{z}, \frac{x_2}{z}\right) B(z), \\
A \otimes_{123} B &= \int \frac{dz}{z^3} A\left(\frac{x_1}{z}, \frac{x_2}{z}, \frac{x_3}{z}\right) B(z).
\end{aligned}
$$

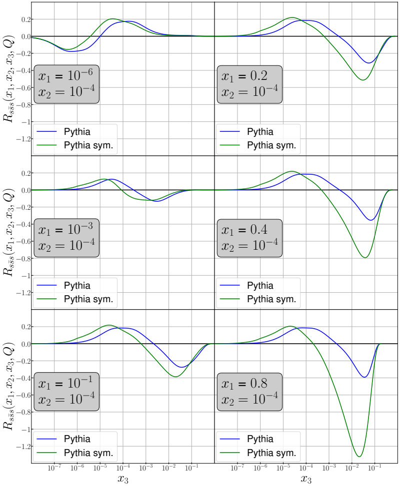
Backup: symmetric tPDFs

The numerical integration over the response function yields

x_1	x_2	N_{u_v} PYTHIA	N_{u_v} PYTHIA sym.	N_{u_v} "Naive"
10^{-6}	10^{-4}	2.019	2.542	2.006
10^{-3}	10^{-4}	2.005	2.154	2.006
10^{-1}	10^{-4}	2.001	2.188	2.005
0.2	10^{-4}	2.000	2.189	2.005
0.4	10^{-4}	1.999	2.161	1.997
0.8	10^{-4}	1.995	2.079	1.708

Integration over $R_{u\bar{u}u}$ response function with respect to x_3 at fixed x_1, x_2.

Backup: However, sometimes



The responses of the "valence" s-quark sPDF $R_{s\bar{s}s}(x_1, x_2, x_3, Q)$ as functions of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The green lines are symmetrized PYTHIA8 tPDFs.