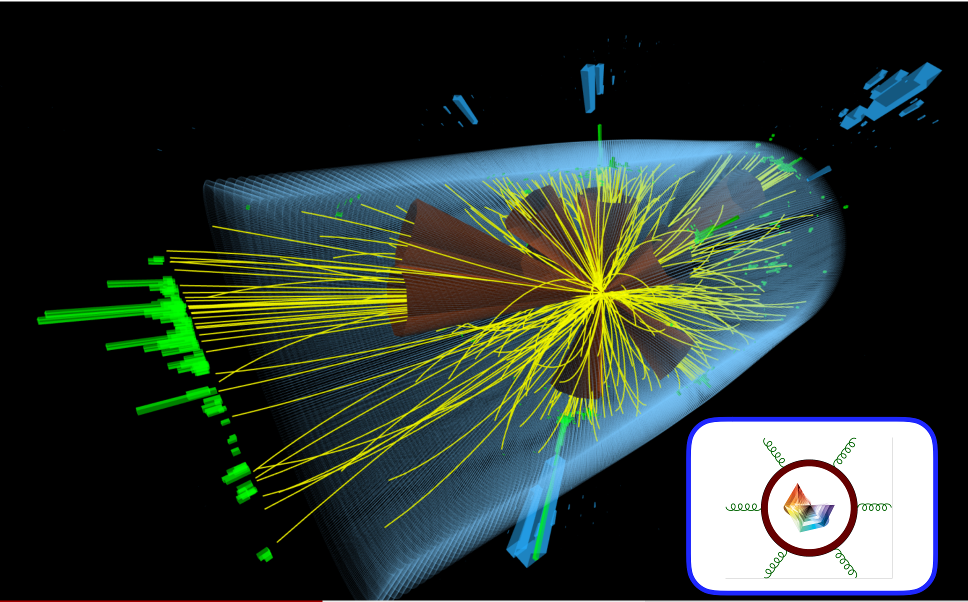
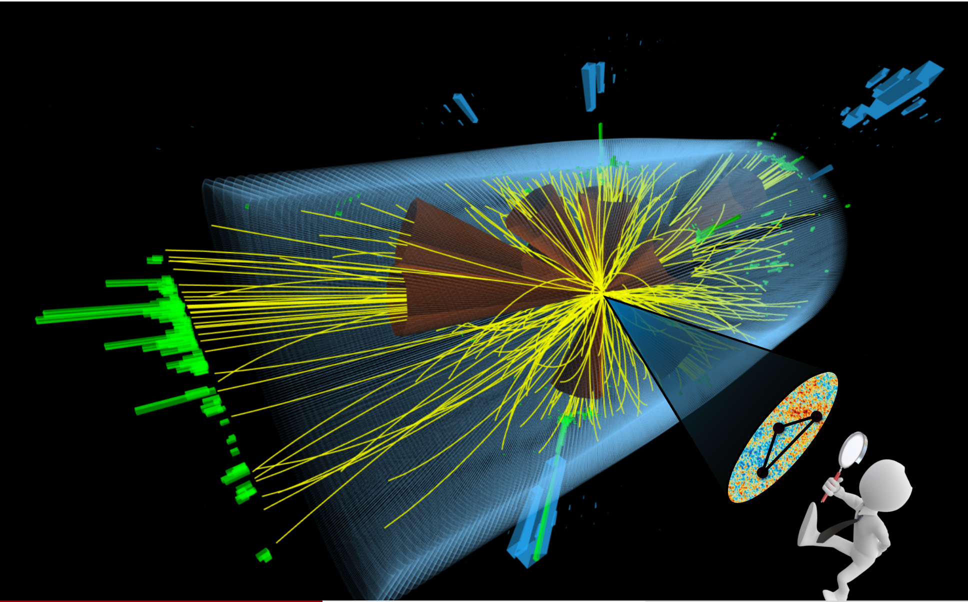
Conformal Colliders Meet the LHC

Ian Moult
Yale

Jets!

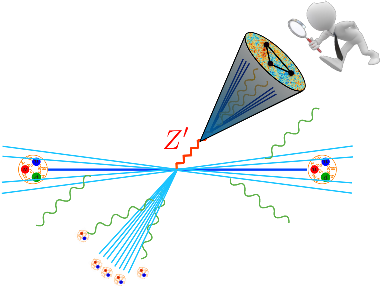


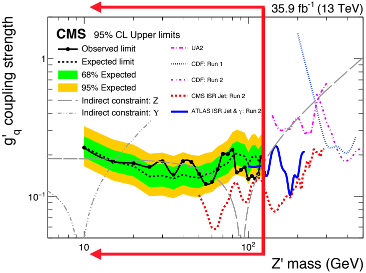
Jet Substructure!



Jet Substructure: Searches

- Jet Substructure uses the internal structure of jets to provide qualitatively new ways to study physics at the LHC.

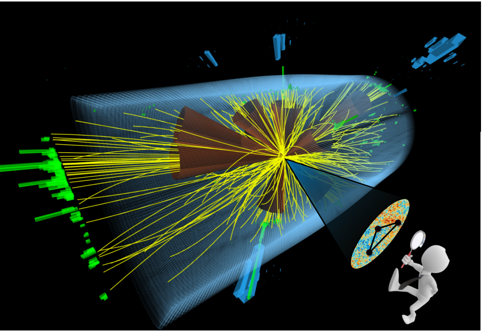




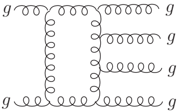
- Its introduction in 2008 by Butterworth, Davison, Rubin and Salam, along with anti-k_T by Cacciari, Soyez, Salam reinvigorated the study of jets in QCD.

Changing the Perspective

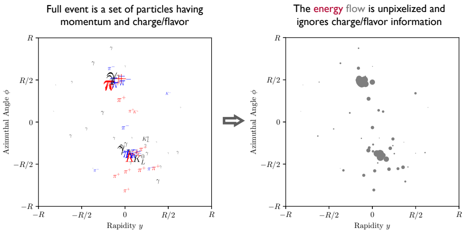
- This changes the problem from studying the production of jets (S-matrix elements) to studying the statistical properties of energy flux within jets.



S-Matrix



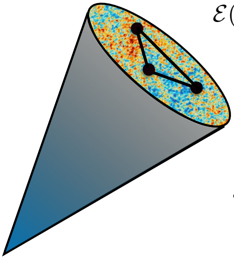
Energy Flux



- Requires the development of a new set of theoretical tools and new ways of thinking about jets.

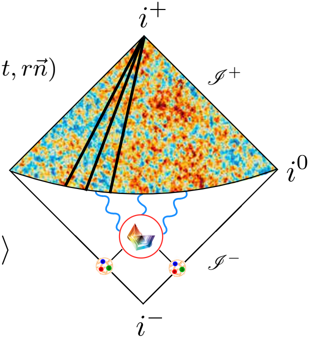
Insights from Conformal Field Theory

- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators. [Hofman, Maldacena] [Korchemsky, Sterman] [Ore, Sterman]



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\vec{n})$$

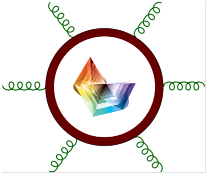
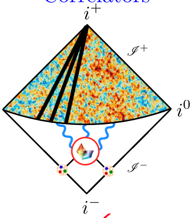
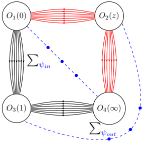
$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



- From the perspective of QFT, jet substructure is the study of correlation functions of energy flow operators.

Energy Correlators

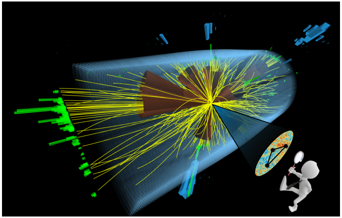
- Correlation functions of energy flow operators take an interesting intermediate position between amplitudes and correlation functions.

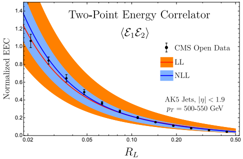
	Amplitudes	Energy Correlators	Correlation Functions
			
Asymptotic States	✓	✓	
IR Finite		✓	✓

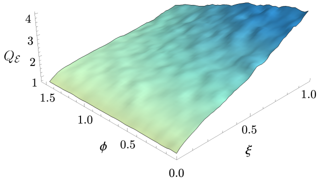
- Despite their physical importance, much less explored.

Conformal Colliders Meet the LHC

- Progress in the understanding of lightray operators allows the calculation and measurement of the shapes and scalings of multipoint correlators, inside high energy jets at the LHC.

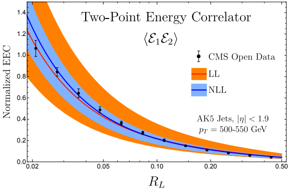




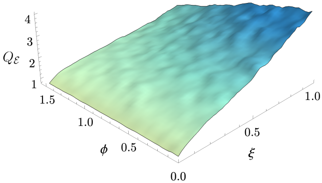


Outline

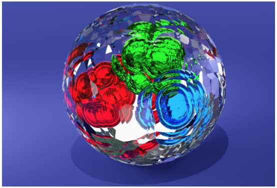
- Scaling Behavior in Jet Substructure



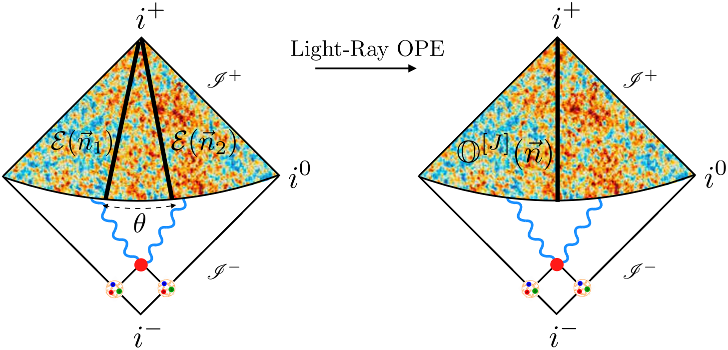
- Non-Gaussianities in Energy Flux



- Resolving the Scales of the QGP

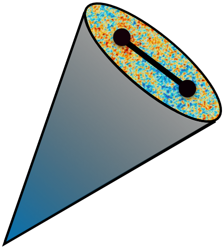


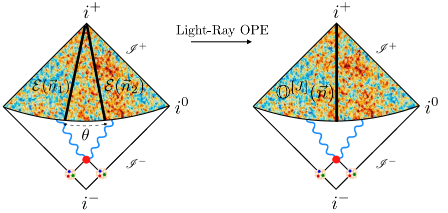
Scaling Behavior in Jet Substructure



The OPE Limit of Lightray Operators

- Energy flow operators admit an OPE!
- Jet Substructure is the study of the OPE limit of lightray operators.





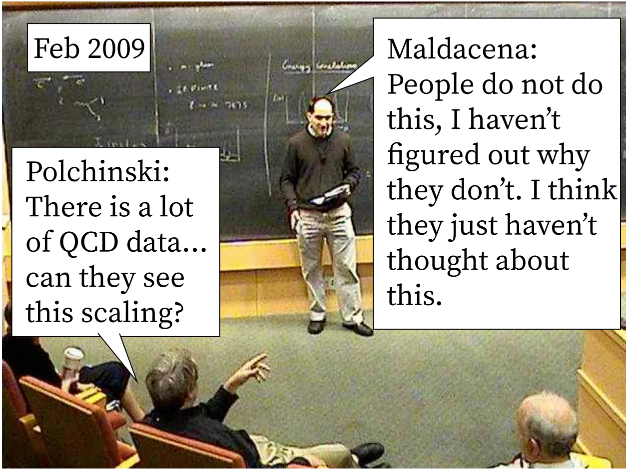
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

- Allows a completely new approach to jet substructure as the study of the symmetry and OPE structure of these operators.

Theory-Experiment Gap

- OPE scaling is the most basic prediction of QFT for jet substructure.

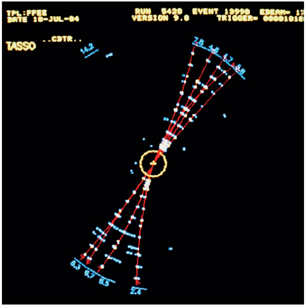


- Shockingly, still true as of 2022...

Reason I: Sociology

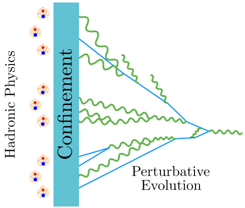
- The classic approach to studying jets is to use "jet shapes", which are resolution variables about an underlying S-matrix element. e.g. "thrust" by Farhi in 1977.

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

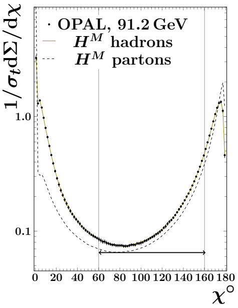


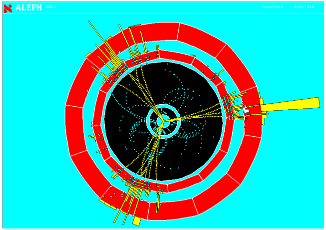
- Goal of jet substructure is to study structure of energy flow, NOT underlying S-matrix element, but historical prejudice very strong...
- Jet shapes are not correlation functions.

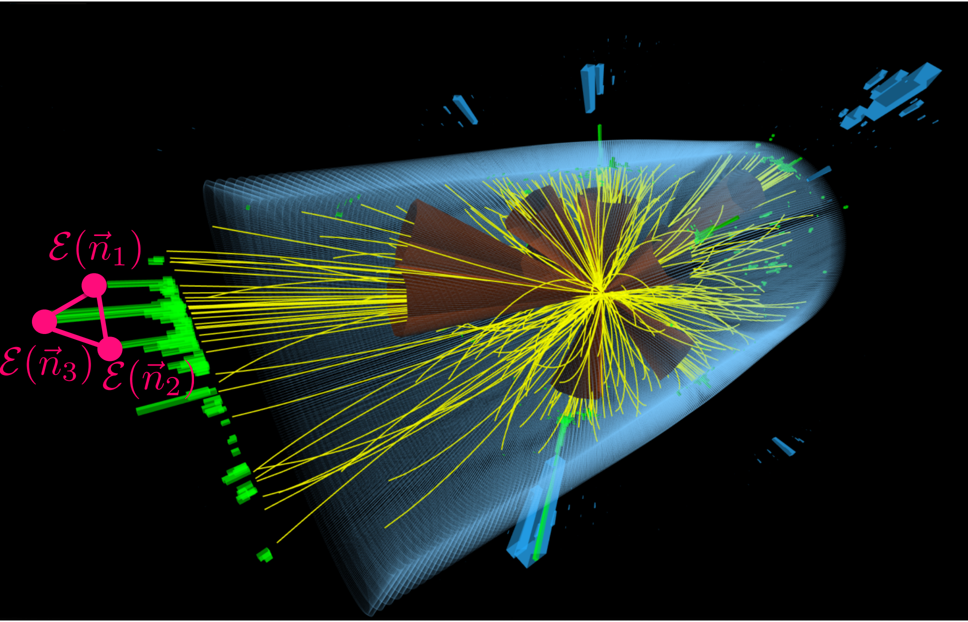
Reason II: LEP Energies



- (Un)Fortunately, QCD is not a conformal theory.
- At energy scales of LEP, hadronization has a large impact:
  - Large corrections to structure of energy flow.
  - Jets consist of a small ~ 5 number of particles.

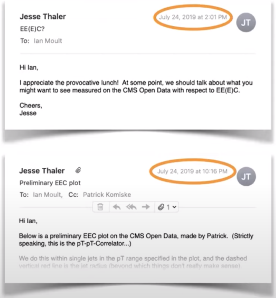





$\mathcal{E}(\vec{n}_1)$
$\mathcal{E}(\vec{n}_3)$
$\mathcal{E}(\vec{n}_2)$

CMS Open Data

- CMS has released a sample of high quality data for public use.
  $\implies$ Perfect for jet studies.
- Packaged in "MIT Open Data":



Jesse Thaler — July 24, 2019 at 2:01 PM — JT
EE(E)C?
To: Ian Moult

Hi Ian,

I appreciate the provocative lunch! At some point, we should talk about what you might want to see measured on the CMS Open Data with respect to EE(E)C.

Cheers,
Jesse

Jesse Thaler — July 24, 2019 at 10:16 PM — JT
Preliminary EEC plot
To: Ian Moult, Cc: Patrick Komiske

Hi Ian,

Below is a preliminary EEC plot on the CMS Open Data, made by Patrick. (Strictly speaking, this is the pT-pT-Correlator...)

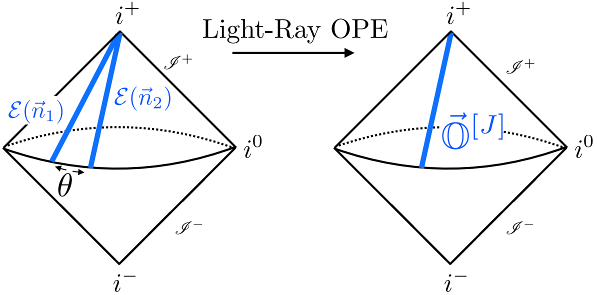
We do this within single jets in the pT range specified in the plot, and the dashed vertical red line is the jet radius (beyond which things don't really make sense).

- Provided by Jesse Thaler and Patrick Komiske.
- Ideal for rapid testing of new theory ideas!
  $\implies$ Can then be done more carefully by professionals.

The Lightray OPE

- In CFTs, the lightray OPE is a convergent, and rigorous expansion. [Hofman, Maldacena; Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$



- To describe the leading scaling at the LHC, we can restrict to the leading term in the OPE $\implies$ twist-2 light ray operators.
- To understand what we can hope to see at the LHC, must look at the structure of these operators.

The Leading Twist Lightray OPE

[Hofman, Maldacena]
[Chen, IM, Zhu]

- The twist-2 operators in QCD are characterized by a spin-J and a transverse spin $j = 0, 2$.
- These can be light-transformed to obtain a vector of twist-2 lightray operators parametrized by spin-J:

Local Operators

[Kravchuk, Simmons Duffin]

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$ (for λ)

$$\xrightarrow{\lim_{r\to\infty} r^2 \int_0^\infty dt} \quad \vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

unpolarized: $\mathbb{O}_q^{[J]}(\vec{n})$, $\mathbb{O}_g^{[J]}(\vec{n})$

polarized: $\mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n})$, $\mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n})$

- The anomalous dimensions of these operators,

$$\frac{d}{d\ln\mu^2}\vec{\mathbb{O}}^{[J]}(\hat{n}_1) = \hat{\gamma}(J)\vec{\mathbb{O}}^{[J]}(\hat{n}_1)$$

determines the leading behavior of jet substructure.

Explicit Structure

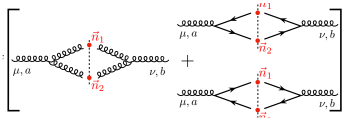
[Chen, IM, Zhu]

- The OPE coefficients can be expressed in terms of a matrix, $\hat{C}_\phi(J)$, whose entries are analytic functions of J:

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\vec{\mathcal{J}}\left[\widehat{C}_\phi(2) - \widehat{C}_\phi(3)\right]\vec{\mathbb{O}}^{[3]}(\hat{n}_1) + \cdots ,$$

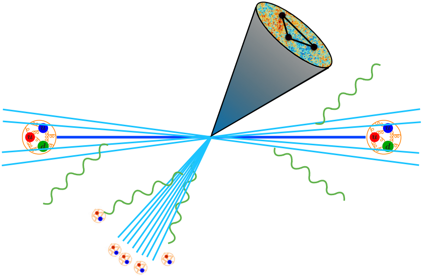
$$\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \cdots$$

- Perhaps surprisingly, these play a crucial role: OPE coefficients onto polarized operators are suppressed, and proportional to $(C_A - n_f)$.



Factorization Theorem at the LHC

- Can derive a factorization theorem in the LHC environment extending the proofs of Collins-Soper-Sterman for inclusive fragmentation:



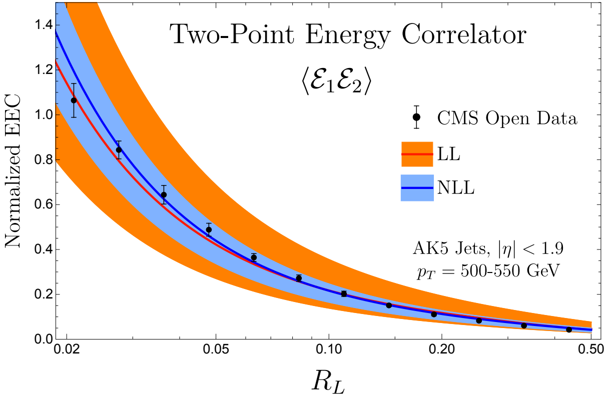
$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T\,\mathrm{d}\eta\,\mathrm{d}\{\zeta\}} = \sum_i \mathcal{H}_i\left(p_T/z, \eta, \mu\right) \otimes \int_0^1 dx\, x^N\, \mathcal{J}_{ij}(z, x, p_T R, \mu)\, J_j^{[N]}(\{\zeta\}, x, \mu)\,.$$

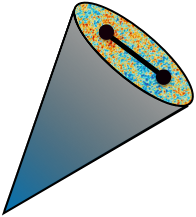
[Lee, Mecaj, Moult]

The OPE Limit in Data

[Komiske, Moult, Thaler, Zhu]
[Dixon, Moult, Zhu]
[Lee, Mecaj, Moult]

- The $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$ OPE inside high-energy jets!

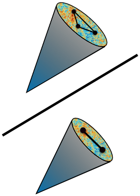




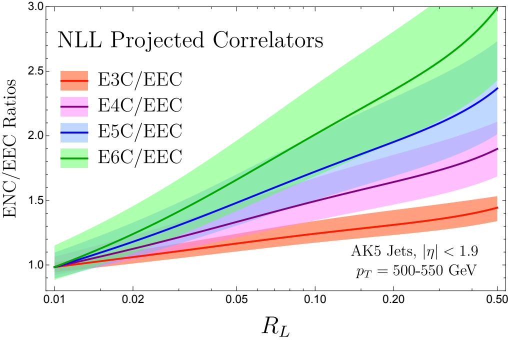
- Beautiful illustration of the universality of the OPE limit in QFT!
- Universality allows calculations in the complicated LHC environment.

Higher Point Scaling

- The light-ray OPE predicts that the N-point correlators develop an anomalous scaling that depends on N.



$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

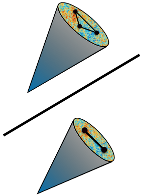


- Directly probes the spectrum of (twist-2) lightray operators in QCD.

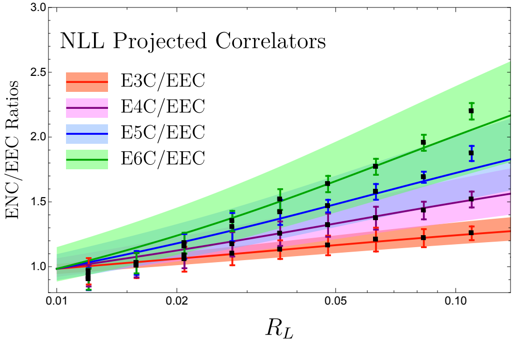
The Spectrum of a Jet

[Chen, Moult, Zhang, Zhu]
[Lee, Mecaj, Moult]

- Measurements of asymptotic energy flux directly extract the spectrum of (twist-2) lightray operators in QCD at the quantum level!



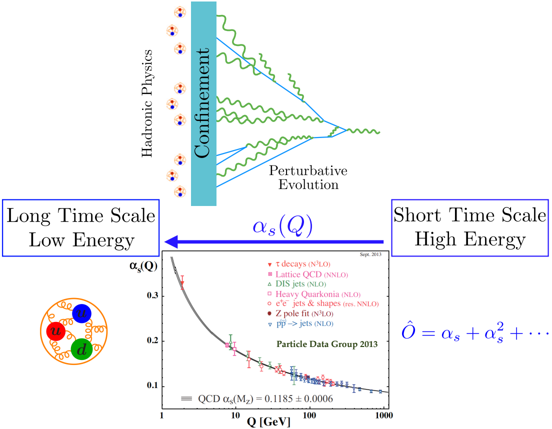
$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$



- A never before observed feature of QFT, accessible due to the high energies and remarkable detectors of the LHC.

The Confinement Transition

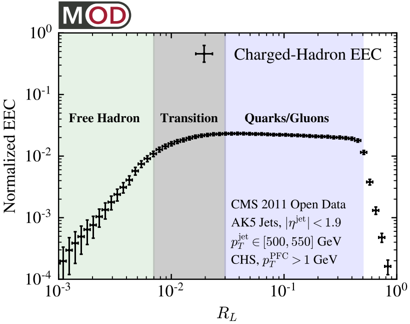
- Jets exhibit a transition from weakly coupled quarks and gluons to freely propagating hadrons.

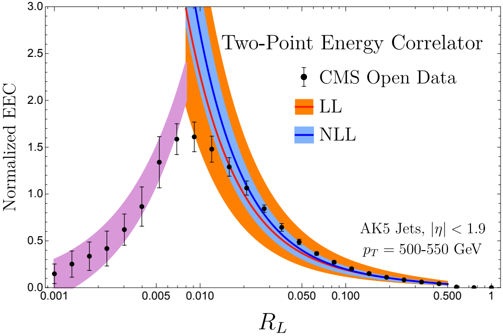


- Can it be directly imaged?

The Confinement Transition

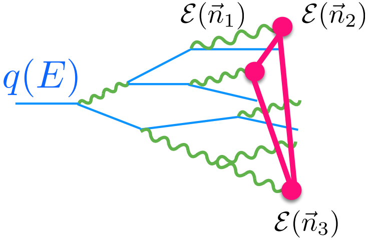
- Distinct scalings associated with interacting quarks and gluons and free hadrons clearly visible!

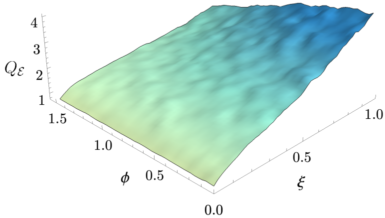




- Precision measurements of the confinement transition possible.
- https://www.youtube.com/watch?v=ORwDv1KTB5U

Non-Gaussianities in Energy Flux

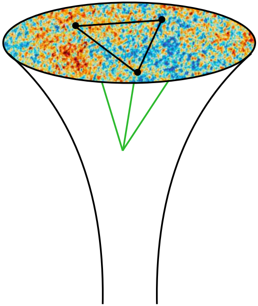


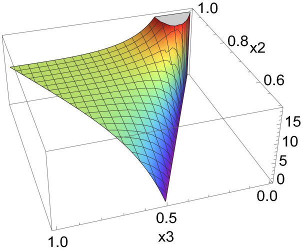


[Chen, Moult, Thaler, Zhu]

Non-Gaussianities

- Higher-point correlators probe more detailed aspects of interactions.
- e.g. Non-Gaussianities allow one to distinguish models of inflation.
- Three-point function, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$, first computed by Maldacena.





[Cabass, Pajer, Stefanysyzn, Supel]

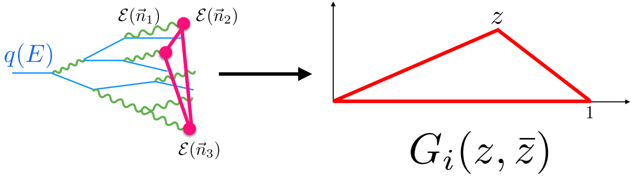
- Can we compute higher-point functions of energy flux?

Multipoint Correlators

- The only explicit results for correlators with $N > 2$ are the remarkable strong coupling results of Hofman and Maldacena:

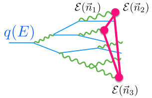
$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_n)\rangle = \left(\frac{q}{4\pi}\right)^n \Bigg[1+\sum_{i<j}\frac{6\pi^2}{\lambda}[(\vec{n}_i.\vec{n}_j)^2-\frac{1}{3}]+ \\ +\frac{\beta}{\lambda^{3/2}}[\sum_{i<j<k}(\vec{n}_i.\vec{n}_j)(\vec{n}_j.\vec{n}_k)(\vec{n}_i.\vec{n}_k)+\cdots]+o(\lambda^{-2})\Bigg]$$

- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



Perturbative Calculation

- To compute in perturbation theory, one integrates over the energy fraction of particles, with the angles of observed particles fixed.
- At lowest order in perturbation theory, one has:



$$\int d\omega_1 d\omega_2 d\omega_3 \; \delta(1-\omega_1-\omega_2-\omega_3) \; (\omega_1\omega_2\omega_3) \; P_{1\to 3}$$

- Consider for illustration a simple Mandelstam invariant in the splitting function $P_{1\to 3} \supset \frac{1}{s_{123}}$.
- One obtains integrals of the form:

$$\int d\omega_1 d\omega_2 d\omega_3 \delta(1-\omega_1-\omega_2-\omega_3) \frac{\omega_1\omega_2\omega_3}{\omega_1\omega_2 z_{12}^2 + \omega_1\omega_3 z_{13}^2 + \omega_2\omega_3 z_{23}^2}$$

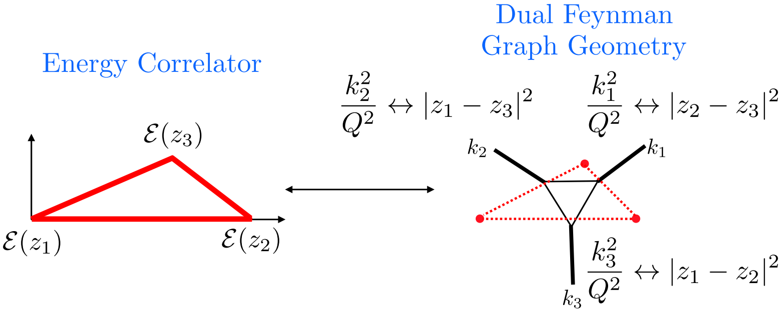
- This is immediately recognized as a Feynman parameter integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \,, x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \,,$$
$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$

Perturbative Calculation

- This is recognized as a dual Feynman loop integral, where the $|z_{ij}|^2$ are the dual coordinates:

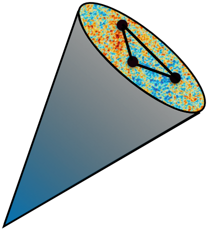
$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \,, x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \,,$$
$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$



- Structure of such integrals well understood.

Multi-point Correlators at Weak Coupling

- Turn out to have an elegant perturbative structure. e.g. in $\mathcal{N}=4$



[Chen, Luo, Moult, Yang, Zhang, Zhu]

$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v)$$
$$-(1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z)$$
$$+\frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

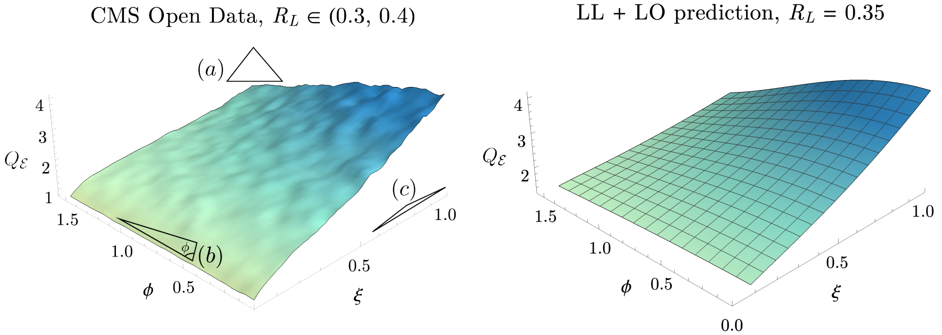
- Here Φ and D_2^+ are polylogarithmic functions

$$\Phi(z) = \frac{2}{z-\bar{z}}\left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\left(\log(1-z) - \log(1-\bar{z})\right)\log(z\bar{z})\right)$$
$$D_2^+(z) = \mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\log(|1-z|^2)\log(|z|^2)$$

- Real world QCD involves more complicated polynomials, but is otherwise similar.

Shape Dependence of Non-Gaussianity in Data

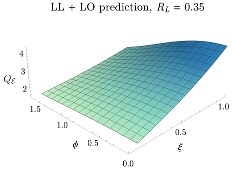
- Can directly study non-gaussianities inside high energy jets.

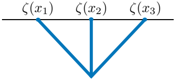


- Illustrates theoretical control over multi-point correlations!

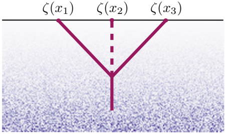
Shape Dependence of Non-Gaussianity in Data

- Enhancement in squeezed limit/ general behavior analogous to that of cosmological correlators.

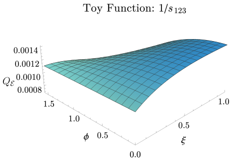


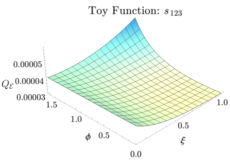


Quantum Vacuum



Classical





[Green, Porto]

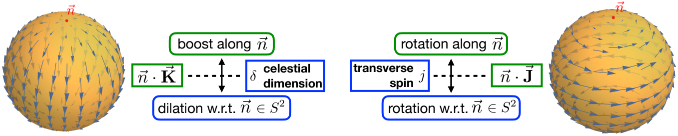
Multi-Point Correlators in Data

- Why is this so neat (at least to me)? This is an honest to goodness correlation function living on the celestial sphere!
- It has all the nice theoretical properties one could want, but in the real physical observable!
- e.g. amplitudes are beautiful, jet cross section at LHC ugly.
- One simple example to highlight this structure: Celestial Block decomposition

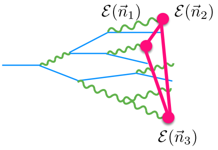
$$\boxed{g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})}$$

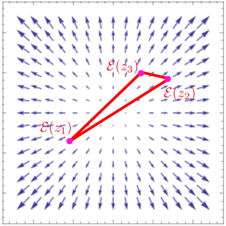
- Exactly analogous to decomposition of $2 \to 2$ scattering into partial waves with SO(3) quantum numbers.

- Are derived by studying the action of the Lorentz group on the celestial sphere:



- In the "jet substructure" limit, reduces to living in the plane transverse to the jet:





Casimir Differential Equations

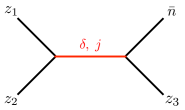
- Celestial blocks derived by solving Casimir differential equations

$$G(z_1, z_2, z_3, \bar{n}) = \frac{1}{(z_1 \cdot z_2)^3} \frac{1}{(z_3 \cdot \bar{n})^4} \left(\frac{z_1 \cdot z_3}{z_1 \cdot \bar{n}} \right) g(z, \bar{z})$$

Symmetry: Lorentz Group

Representation labels: δ **celestial dimension** ------ $\vec{n} \cdot \vec{\mathbf{K}}$; j **transverse spin** ------ $\vec{n} \cdot \vec{\mathbf{J}}$

Quadratic Casimir: $\frac{1}{2} M_{\mu\nu} M^{\mu\nu} \xrightarrow{\text{eigenvalue}} -\left(\delta(\delta-2) + j^2\right)$



$$g(z, \bar{z}) = \sum_{\delta, j} c_{\delta, j} g_{\delta, j}(z, \bar{z})$$

Casimir Equation: acting Casimir operator on $z_1,\ z_2$

$$\mathcal{L}^{\mu\nu}(z_1, z_2) \mathcal{L}_{\mu\nu}(z_1, z_2) G_{\delta, j} = -(\delta(\delta-2) + j^2) G_{\delta, j}$$

[Dolan, Osborn, 2003]

$$\mathcal{L}^{\mu\nu}(z_1, z_2) \equiv \sum_{i=1,2} \left(z_i^\mu \frac{\partial}{\partial z_{i\nu}} - z_i^\nu \frac{\partial}{\partial z_{i\mu}} \right)$$

Solutions (2D Conformal Blocks):

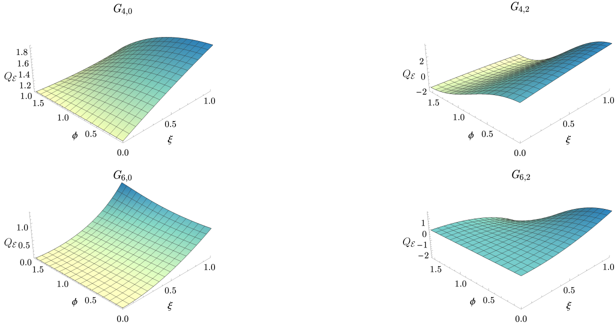
$$g_{\delta, j}(z, \bar{z}) = \frac{1}{1 + \delta_{j,0}} \left(k_{\delta - j}(z) k_{\delta + j}(\bar{z}) + k_{\delta + j}(z) k_{\delta - j}(\bar{z}) \right)$$

[Notations] In our case, $a = 0, b = -1$

$$k_\beta(x) \equiv x^{\beta/2}\ {}_2F_1\left(\frac{\beta}{2} + a, \frac{\beta}{2} + b, \beta, x \right)$$

Celestial Partial Waves

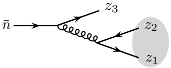
- These are partial waves living on the detector:



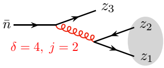
- Celestial block expansion converges rapidly.

Perturbative Data from Lightray OPE

- Interesting interplay with structure of Feynman diagrams.
- e.g. In QCD can choose color structures to isolate internal parton states.



- Only $j = 0, 2$ contribute. Leading twist $j = 2$ block uncontaminated by higher twist contributions.



Contributing Operator

$F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$

twist-2, transverse spin-2 gluonic operator

highest transverse spin series

$$-z^3\bar{z}\ {}_2F_1\left(3,2,6,z\right)$$

$$-z^3\bar{z}\ (\cos 2\phi) \quad +\frac{39}{10}z^2\bar{z}^2 \quad -z\bar{z}^3$$

$$-z^4\bar{z}\ (\cos 3\phi) \quad +\frac{39}{20}z^3\bar{z}^2 \quad +\frac{39}{20}z^2\bar{z}^3 \quad -z\bar{z}^4$$

$$-\frac{6}{7}z^5\bar{z}\ (\cos 4\phi) \quad +\frac{229}{140}z^4\bar{z}^2 \quad -\frac{211}{140}z^3\bar{z}^3 \quad +\frac{229}{140}z^2\bar{z}^4 \quad -\frac{6}{7}z\bar{z}^5$$

$$-\frac{5}{7}z^6\bar{z}\ (\cos 5\phi) \quad +\frac{207}{140}z^5\bar{z}^2 \quad -\frac{233}{140}z^4\bar{z}^3 \quad -\frac{233}{140}z^3\bar{z}^4 \quad +\frac{207}{140}z^2\bar{z}^5 \quad -\frac{5}{7}z\bar{z}^6$$

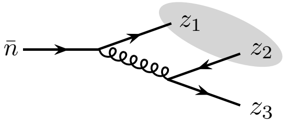
$$\ldots \qquad \ldots \qquad \ldots$$

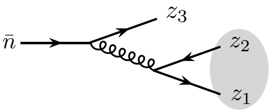
- Non-trivial interplay with structure of transcendental function space.

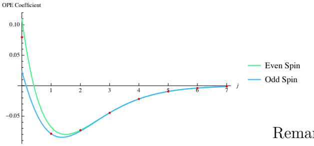
Towards a Bootstrap

[Chang, Simmons-Duffin]
[Chen, Moult, Sandor, Zhu]

- Crossing equations recently studied in the large (transverse) spin limit.
- OPE data analytic in light-ray transverse spin.







Snowmass White Paper:
The Analytic Conformal Bootstrap

Thomas Hartman,[1] Dalimil Mazáč,[2] David Simmons-Duffin,[3] Alexander Zhiboedov[4]

Remarkably, the machinery of the OPE [49, 65, 71, 72] and crossing equations [24, 25] can be generalized to light-ray operators in a nontrivial way. The light-ray OPE has interesting applications in the study of jet substructure in QCD [73, 74]. Developing a better understanding of the space of light-ray operators and associativity of the light-ray OPE is an important open problem in our quest for understanding nonperturbative Lorentzian dynamics of CFTs.

- Recent interest in Lorentzian dynamics of CFT has strong overlap with real world collider physics!

OPE Data

- Exact equivalence with a CFT four point function allows us to directly read off the OPE data using Lorentzian Inversion $\implies$ it is an analytic function of the transverse spin on the celestial sphere.

Gribov-Froissart Formula

Partial wave expansion

$$A(s,t) = \sum_{\ell=0}^{\infty}(2\ell+1)\, a_\ell(s)\, P_\ell(z), \quad z = \cos\theta$$

Gribov-Froissart formula

$$a_\ell(s) = \frac{1}{2\pi}\int_1^\infty dz\, Q_\ell(z)[\mathrm{Disc}_t A(s,z) + (-1)^\ell \mathrm{Disc}_u A(s,-z)]$$

partial wave ← **Discontinuity of amplitude**

Conformal block expansion $\quad g(z,\bar z) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar z)$

Lorentzian inversion $\quad c(\delta,j) = c^t(\delta,j) + (-1)^j c^u(\delta,j)$

$$c^t(\delta,j) = \frac{\kappa_{\delta+j}}{4}\int_0^1 dz d\bar z \mu(z,\bar z) g_{j+d-1,\delta+1-d}(z,\bar z)\mathrm{dDisc}\,[g(z,\bar z)]$$

$A(s,t)$	$g(z,\bar z)$
$a_\ell(s)$	$c_{\delta,j}$
$\mathrm{Disc}A$	$\mathrm{dDisc}\ g$

$P_\ell(z)$ $\longrightarrow$	$Q_\ell(z)$
$g_{\delta,j}$ $\longrightarrow$	$g_{j+d-1,\delta+1-d}$

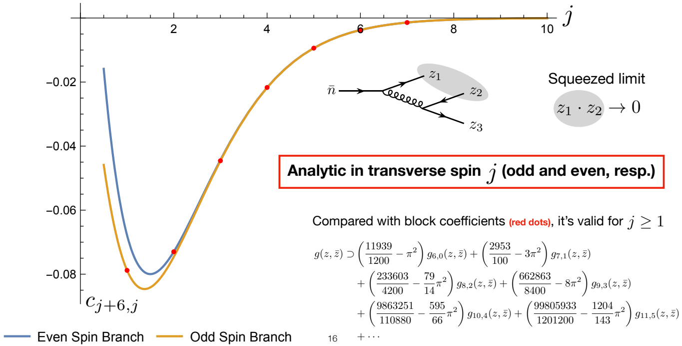
Double Discontinuity $\quad \mathrm{dDisc}\, g(z,\bar z) = \cos(\pi(a+b))g(z,\bar z) - \frac{1}{2}e^{i\pi(a+b)}g^{\circlearrowleft}(z,\bar z) - \frac{1}{2}e^{-i\pi(a+b)}g^{\circlearrowright}(z,\bar z)$

15

- Allows us to analyze and understand jet substructure observables at the LHC using same language as local correlation functions.

Perturbative Data for Lightray OPE

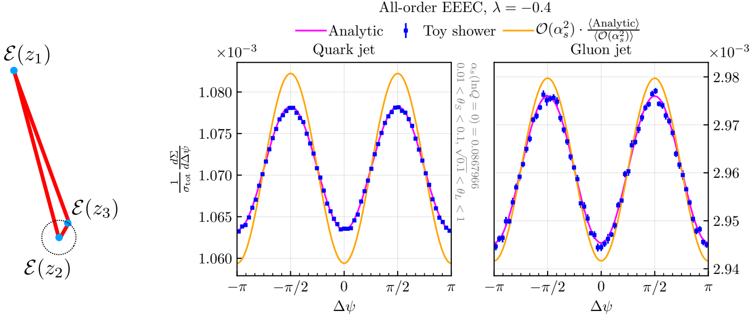
- e.g. spectrum of operators for an OPE channel of a quark jet:



- Provides new handles for studying higher twist structure of collinear limits in QCD.

Parton Shower Development

- Illustrates complete control of three-point correlations in jets.
- Crucial for validating implementations of higher order effects in parton showers. e.g. Spin Correlations (transverse spin operators)



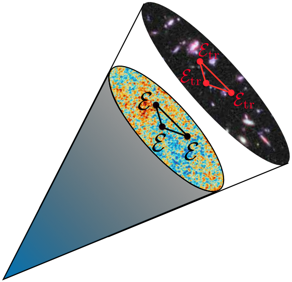
[Karlberg, Salam, Scyboz, Verheyen]

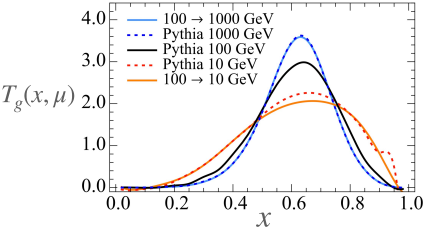
- Full incorporation of higher-point correlations in parton showers will play an important role in enhancing the LHC search program.

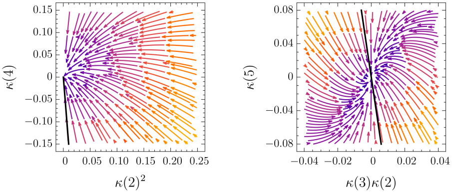
Track Functions

[Chang, Procura, Thaler, Waalewijn]
[Li, Moult, Van Velzen, Waalewijn, Zhu]
[Jaarsma, Li, Moult, Waalewijn, Zhu]

- A key in the ability to study higher point correlations has been the development of QFT formalisms for performing calculations on charged particles (tracks).

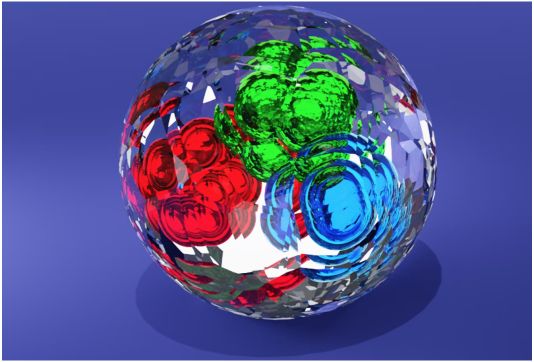






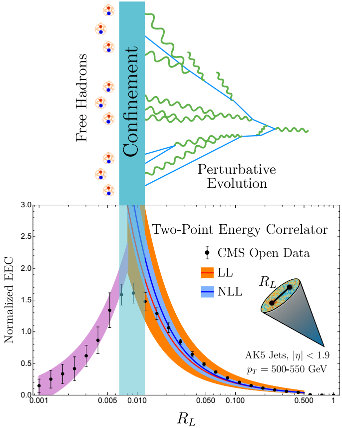
- Described by non-perturbative track functions satisfying non-linear RG evolution, encoding correlations in the hadronization process.

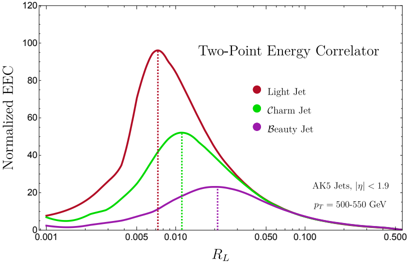
Resolving the Scales of the QGP



The Confinement Transition

- Energy correlators allow the hadronization process to be directly imaged inside high energy jets.



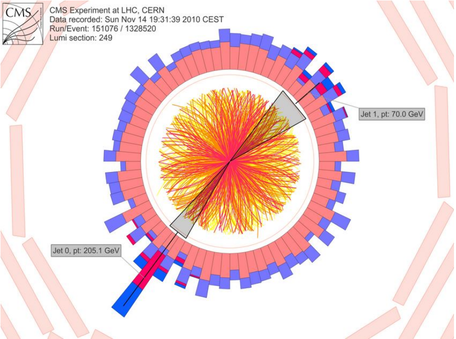


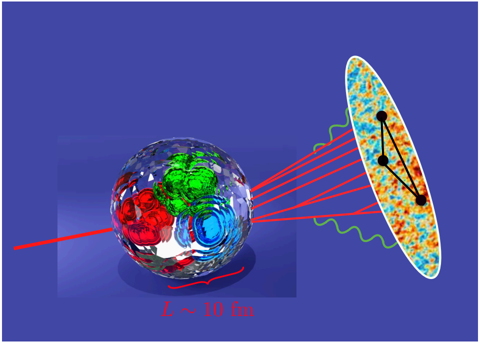
[Lee, Mecaj, Moult]

- Studying the transition for heavy mesons/baryons/onia will provide new insights into confinement.

From Jets to Jet Substructure in Heavy Ion

- Resolving the mystery of how asymptotically free quarks and gluons conspire to form a strongly coupled fluid is a primary goal of the heavy ion program.

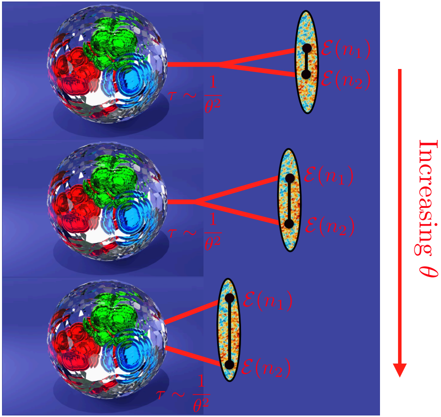


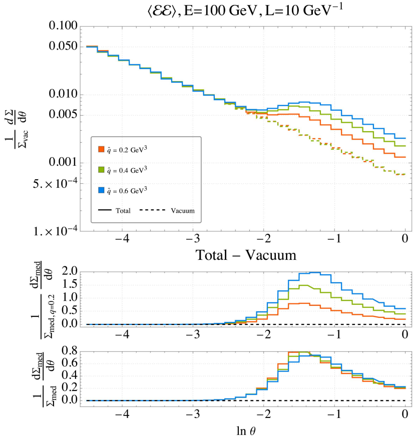


- Jets are multi-scale probes $\Longrightarrow$ scales of the QGP are imprinted at characteristic scales in the substructure of jets.

Resolving the Scales of the QGP

- QGP scales cleanly imprinted in two-point correlation.



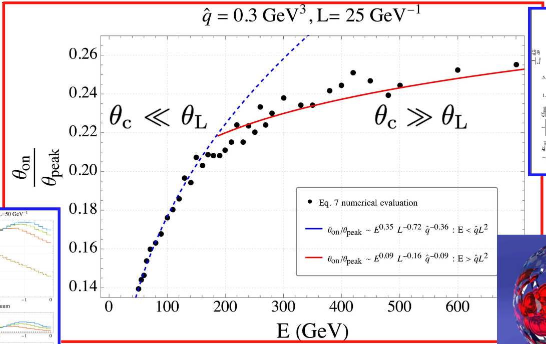


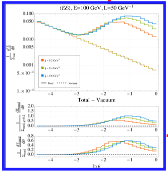
Increasing θ

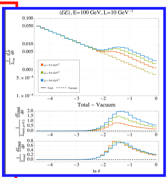
[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]

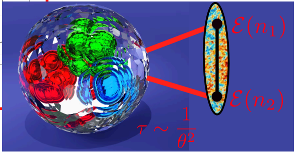
Resolving the Scales of the QGP

- Detailed shape of the transition can extract whether the medium interacts with the partons in the jet coherently.





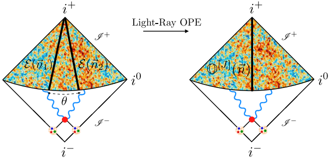




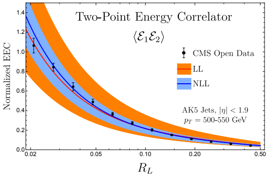
- Jet Substructure provides a new lens through which to view the QGP.

Summary

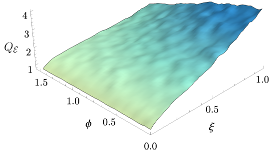
- Insights from formal theory are transforming the way we think about jet substructure.

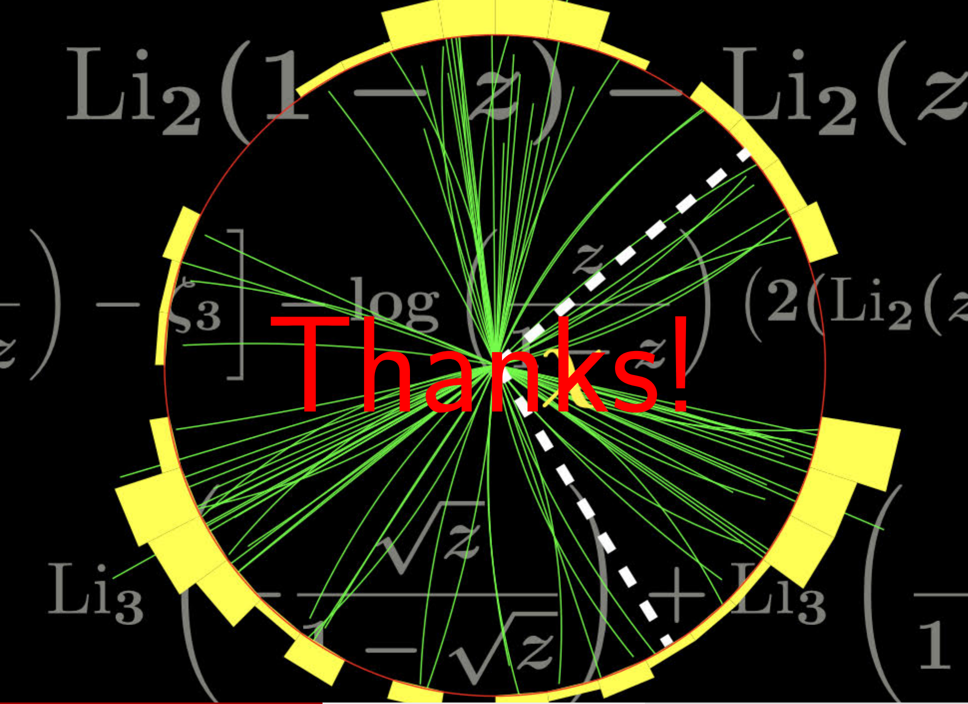


- Jet Substructure provides a physical realization of the OPE limit of lightray operators $\Longrightarrow$ direct bridge between recent field theory developments and QCD phenomenology.



- Opens the door to a precision physics program using jet substructure, and many new opportunities to search for new physics!





Thanks!