First principles' calculations of the gluon structure of hadrons

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Gluon structure

Gluons are key to understanding the visible universe

- Dominant contribution to mass of the visible universe
- Significant contribution to the spin of hadrons
- Fundamental to understanding a new form of matter: color glass condensate

Complete tomography of hadrons needs detailed understanding of gluon structure!

EIC provides great opportunity for collaboration and interplay between theory and experiment

Background image: CERN Courier

Outline

1. Gluon structure of hadrons - what do we know?
2. Gluon structure from first principles
3. Unpolarised gluon PDF in the pseudo distribution framework
4. Polarised gluon PDF in the pseudo distribution framework

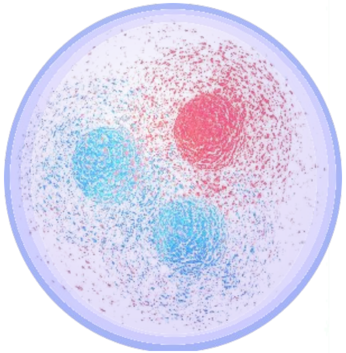
Gluon structure of hadrons - what do we know?

Hadron structure at a glance

Spin → Form factors → Parton distribution functions (PDFs) → Generalised parton distribution functions (GPDs) → Wigner functions

What are the internal correlations of the proton?

How is the spin of the hadron split amongst its constituents?



How is the momentum of the constituents correlated with their transverse structure?

How does the proton interact with the other fundamental forces?

How is the momentum of a fast moving hadron spread amongst its constituent quarks and gluons?

Hadron structure at a glance

Spin

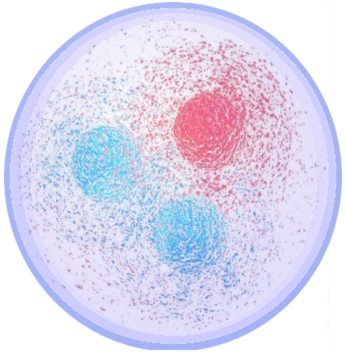
Form factors

Parton distribution functions (PDFs)

Generalised parton distribution functions (GPDs)

Wigner functions

How is the spin of the hadron split amongst its constituents?



How is the momentum of a fast moving hadron spread amongst its constituent quarks and gluons?

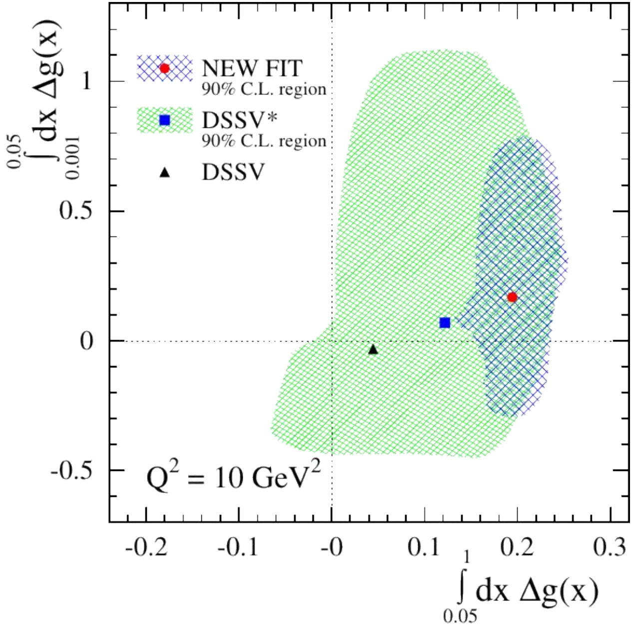
Aidala et al., RMP 85 (2013) 655
Deur et al., Rep. Prog. Phys. 82 (2019)
Ji et al., Nat. Rev. Phys. 3 (2021) 27

The origin of hadron spin

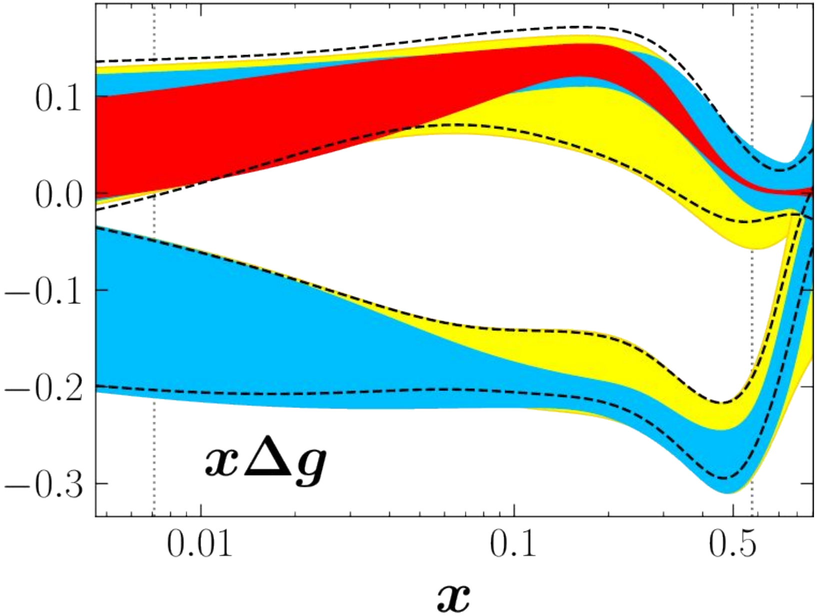
30 years after the EMC experiment precipitated the "proton spin crisis", experimental picture still unclear

We do know quarks carry approximately 30% of the proton's spin, gluon picture is much less clear

de Florian et al., PRL 113 (2014) 012001



Zhou et al., PRD 105 (2022) 074022

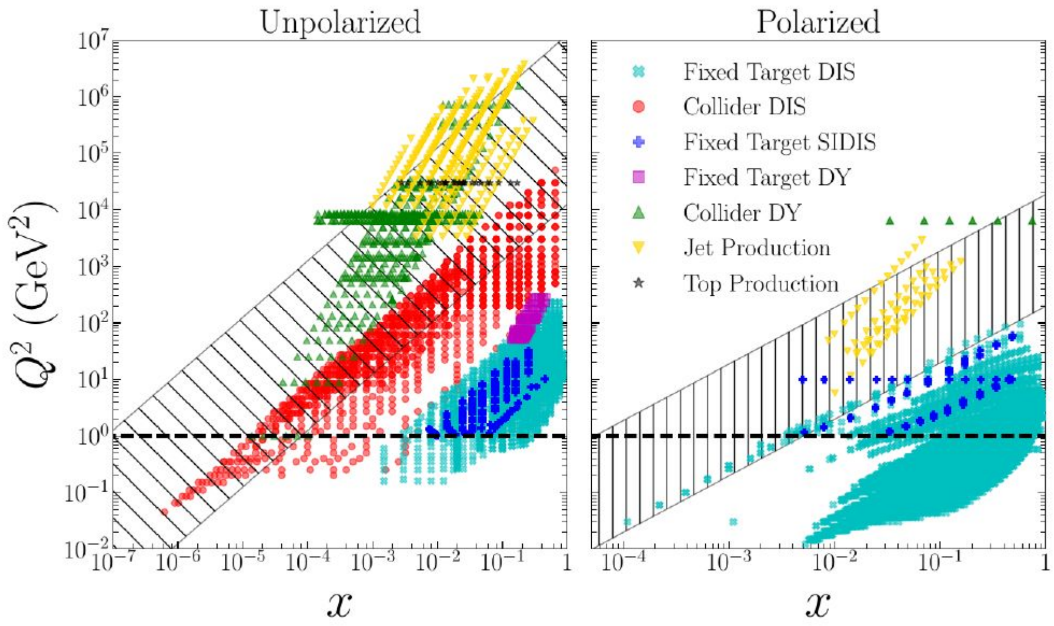


Gluon PDFs

How is the momentum of a fast-moving hadron spread amongst its constituents?

LHC has considerably improved our knowledge of gluon PDFs

EIC and LHeC will expand this considerably!

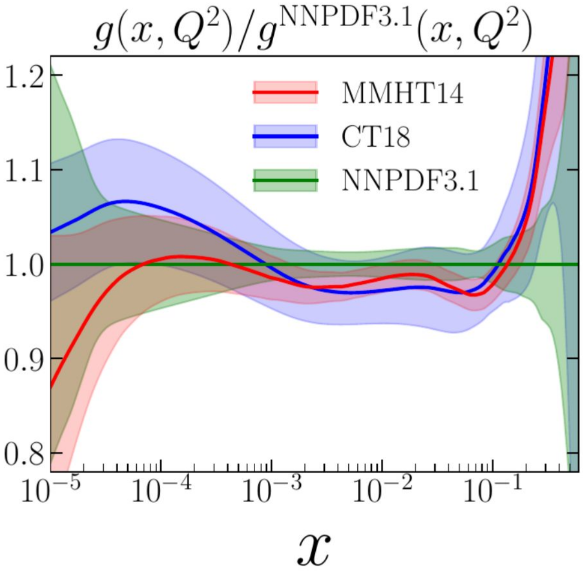


Ethier and Nocera, Ann.Rev.Nucl.Part.Sci. 70 (2020) 43

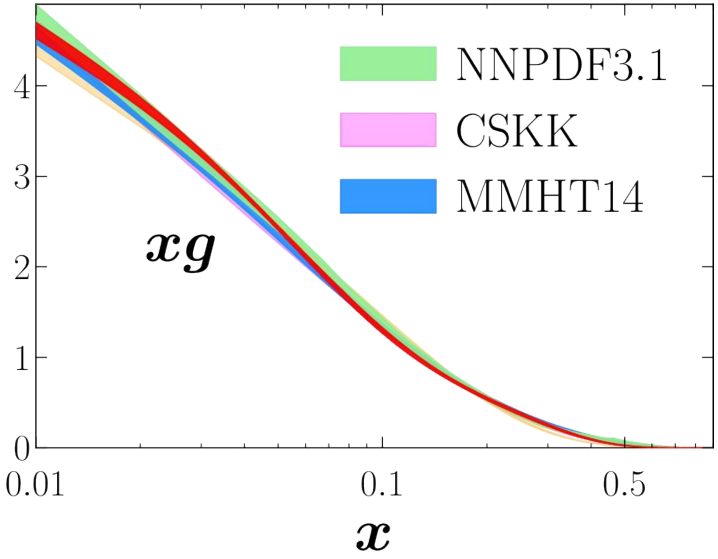
Gluon PDFs: experimental status

LHC has considerably improved our knowledge of unpolarised gluon PDFs

Large uncertainties remain at large and small Bjorken-x



Ethier and Nocera, Ann.Rev.Nucl.Part.Sci. 70 (2020) 43

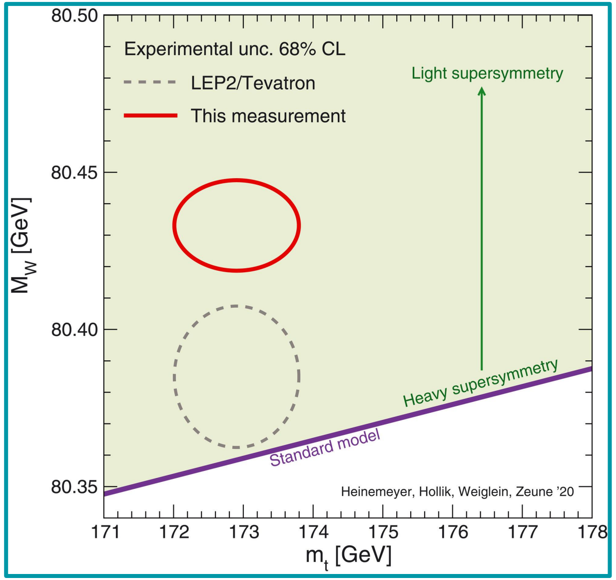


Zhou et al., PRD 105 (2022) 074022

Unpolarised gluon PDFs

Unpolarised gluon PDFs an important source of theoretical uncertainty at LHC

- Higgs couplings
- Certain search channels for BSM particles
- Mass of the W boson



Recent measurement from Tevatron

$$m_W^{(\text{Tevatron 2022})} = 80.4335(94)\ \text{GeV}$$

Significant (7 sigma) tension with standard model expectation

$$m_W^{(\text{SM})} = 80.357(6)\ \text{GeV}$$

And previous experimental results

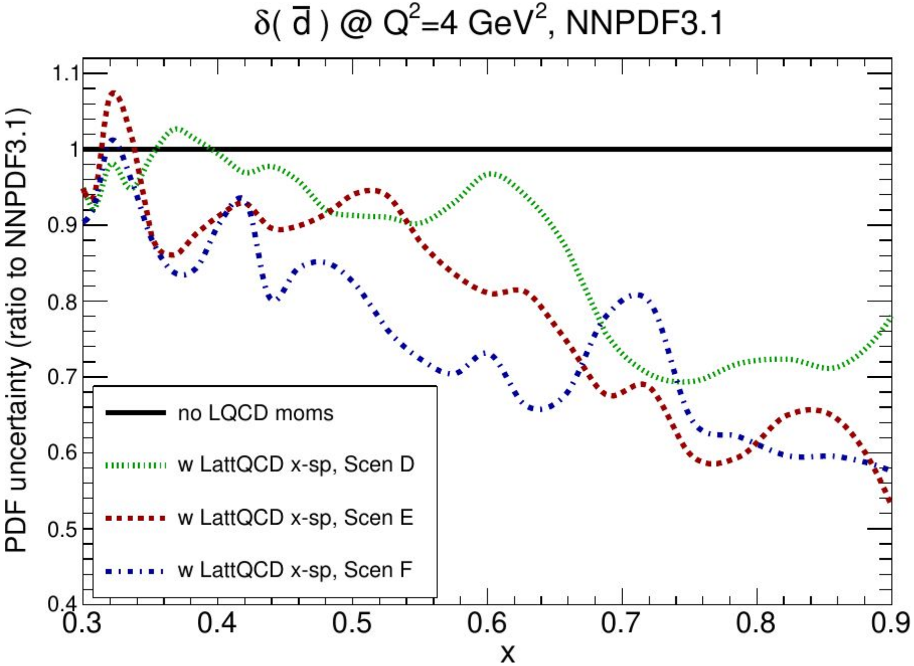
$$m_W^{(\text{LEP+Tevatron})} = 80.385(15)\ \text{GeV}$$

$$m_W^{(\text{ATLAS})} = 80.370(19)\ \text{GeV}$$

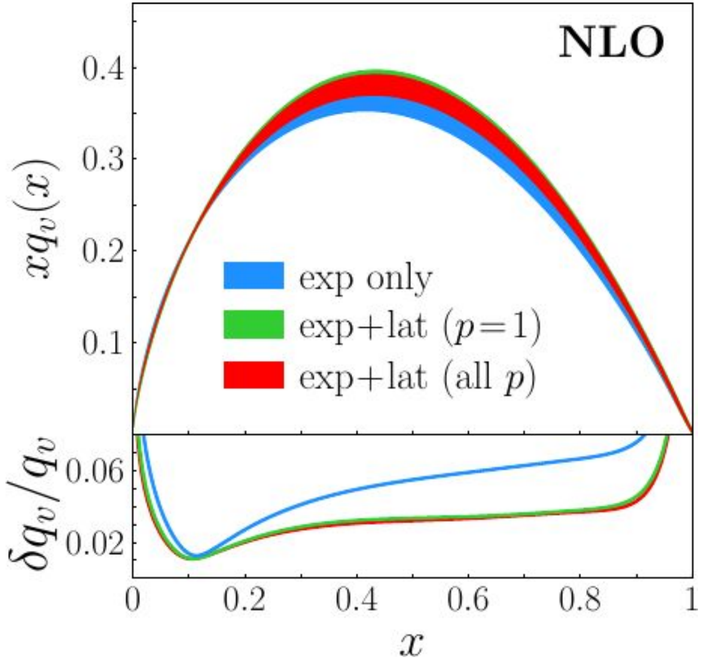
CDF, Science 376 (2022) 170

PDFs from QCD

First principles calculations complement, and inform, JLab 12 GeV, the LHC and the EIC



Lin et al., Prog. Nucl. Part. Phys. 100 (2018) 107



Barry et al., PRD 105 (2022) 114051

Gluon structure from first principles - spin

Liu, AAPPS Bull. (2022) 32
Ji et al., Nat. Rev. Phys. 3 (2021) 27

The origin of hadron spin

Two decompositions of the spin of the proton

$$\frac{\Delta\Sigma}{2} + L_q + J_g = \frac{1}{2}$$

Frame-independent decomposition

“Ji sum rule”

Expressed in terms of the quark helicity and orbital angular momentum contributions, and gluon contribution

$$\frac{\Delta\Sigma}{2} + \Delta G + \ell_q + \ell_g = \frac{1}{2}$$

Infinite-momentum frame decomposition

“Jaffe-Manohar sum rule”

Expressed in terms of the quark and gluon helicities, and the quark and gluon twist-3 orbital-angular-momentum

Partonic interpretation

The origin of hadron spin

Broadly speaking - two approaches to first principles calculations of these contributions

$$\frac{\Delta\Sigma}{2} + L_q + J_g = \frac{1}{2}$$

Obtained from the axial current ($\frac{\Delta\Sigma}{2}$)

Defined through TMDs or through form factors of the energy-momentum tensor ($L_q + J_g$)

$$\frac{\Delta\Sigma}{2} + \Delta G + \ell_q + \ell_g = \frac{1}{2}$$

Obtained from the axial current ($\frac{\Delta\Sigma}{2}$)

Defined through local operators in IMF or through TMDs ($\ell_q + \ell_g$)

The origin of hadron spin

Broadly speaking - two approaches to first principles calculations of these contributions

$$\frac{\Delta\Sigma}{2} + L_q + J_g = \frac{1}{2}$$

$$\frac{\Delta\Sigma}{2} + \Delta G + \ell_q + \ell_g = \frac{1}{2}$$

Defined through the local operator in the infinite-momentum frame

$$S_G = \int \mathrm{d}^3x \,\mathrm{Tr}\,[\mathbf{E} \times \mathbf{A}_{\mathrm{phys}}]$$

Both challenging for lattice QCD!

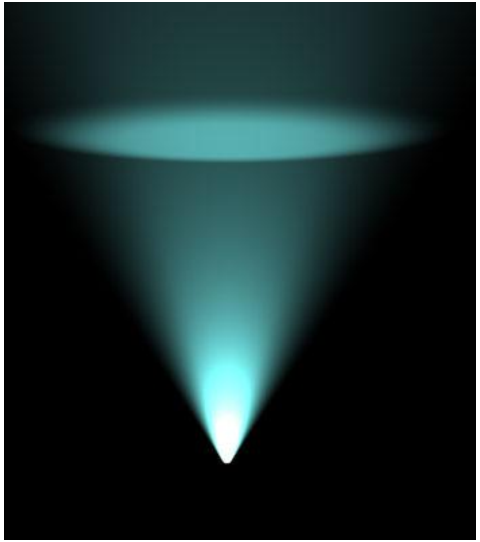
Defined through the integral of the helicity PDF

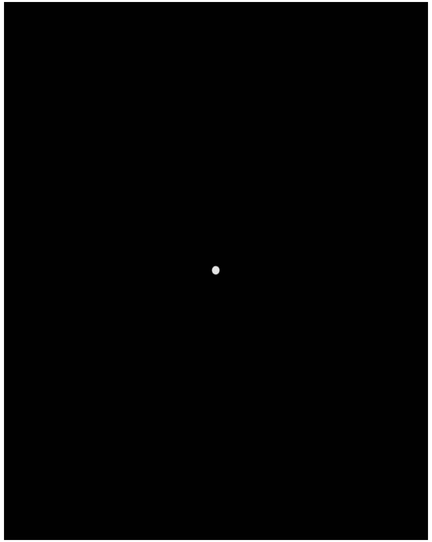
$$\Delta G(Q^2) = \int_0^1 \mathrm{d}x \,\Delta g(x, Q^2)$$

The challenge

Lattice QCD is formulated in a finite volume on a discretised Euclidean spacetime lattice

- Light cone not accessible





The challenge

Lattice QCD is formulated in a finite volume on a discretised Euclidean spacetime lattice

- Light cone not accessible

Path integral is sampled stochastically via Markov chain Monte Carlo

- Infinite momentum not accessible numerically!

Static quantities extracted in the long Euclidean-time limit

- Noise-to-signal ratio increases exponentially with Euclidean time
- Particularly challenging for gluons

Large-momentum effective theory (LaMET)

Framework to relate lattice-calculable to infinite-momentum quantities

Ji et al., PRL 111 (2013) 112002

- originally introduced to enable the calculation of the gluon spin contribution
- now a general framework for first principles' calculations

Effective theory: infinite momentum limit does not commute with removing the regulator

Relies on perturbative matching

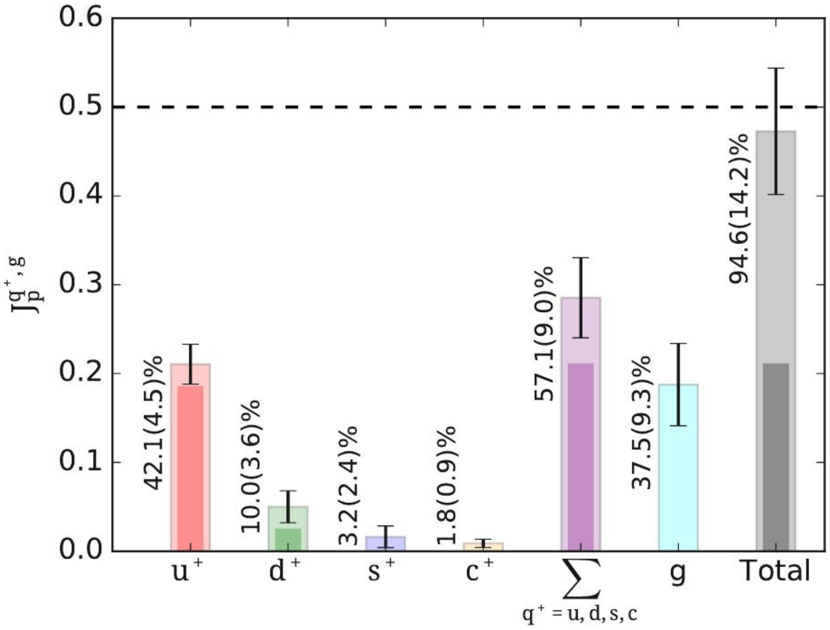
Ji et al., RMP 93 (2021) 35005

- Quantities required to have the same infrared behaviour
- For example, at one loop in the MS-bar scheme

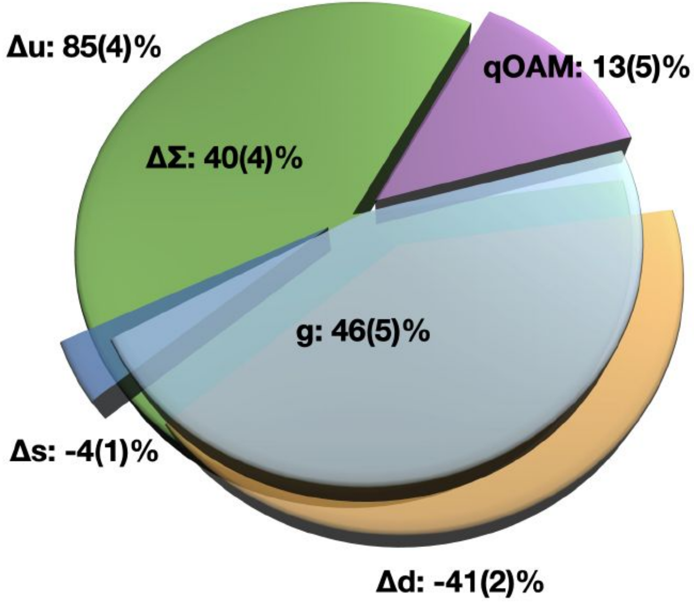
$$\Delta G^{\overline{\mathrm{MS}}}(\mu) = \frac{\alpha_s}{3\pi}\left[3\ln\frac{\mu^2}{m^2} + 7\right] \qquad \Delta G^{\overline{\mathrm{MS}}}(\mu, P_z) = \frac{\alpha_s}{3\pi}\left[\frac{5}{3}\ln\frac{\mu^2}{m^2} - \frac{1}{9} + \frac{4}{3}\ln\frac{P_z^2}{m^2}\right]$$

First calculations of the spin of the proton

Two state-of-the-art decompositions from lattice QCD



Alexandrou et al., PRD 101 (2020) 094513



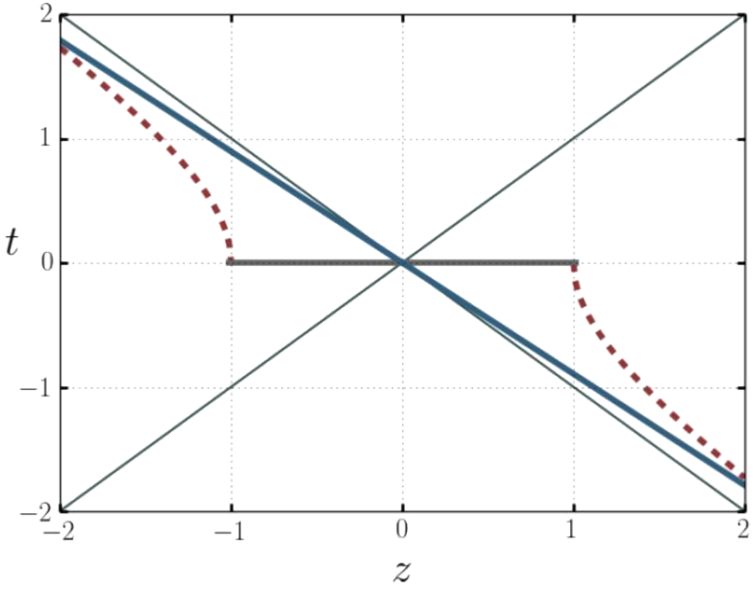
Wang et al., PRD 106 (2022) 014512

Gluon structure from first principles - PDFs

Ji et al., PRL 110 (2013) 262002
Ji, Sci. Ch. Phys. Mech. Ast. 57 (2014) 1407
Ji et al., RMP 93 (2021) 35005

Large-momentum effective theory (LaMET)

Intuitive picture for PDFs - defined through operators of light-like separated fields



Ji et al., PRL 110 (2013) 262002
Ji, Sci. Ch. Phys. Mech. Ast. 57 (2014) 1407
Ji et al., RMP 93 (2021) 35005

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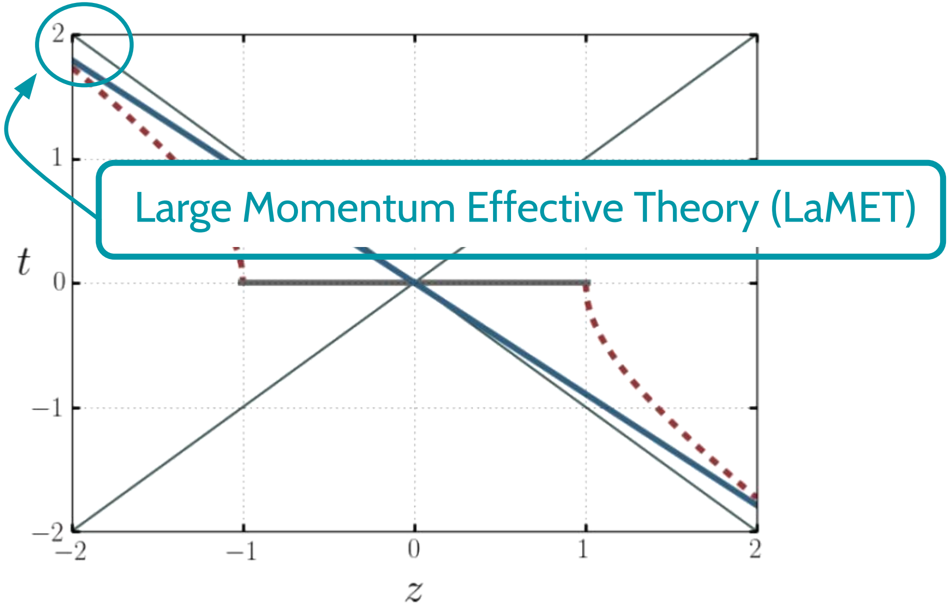

Distributions galore

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu} \mathcal{M}^{(0)}_{g/H}(\nu, 0)$$

PDFs

$$n^2 = 0 \qquad \xi^-$$

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P, n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha, 0)G_{\rho\sigma}|H(P)\rangle$$

Distributions galore

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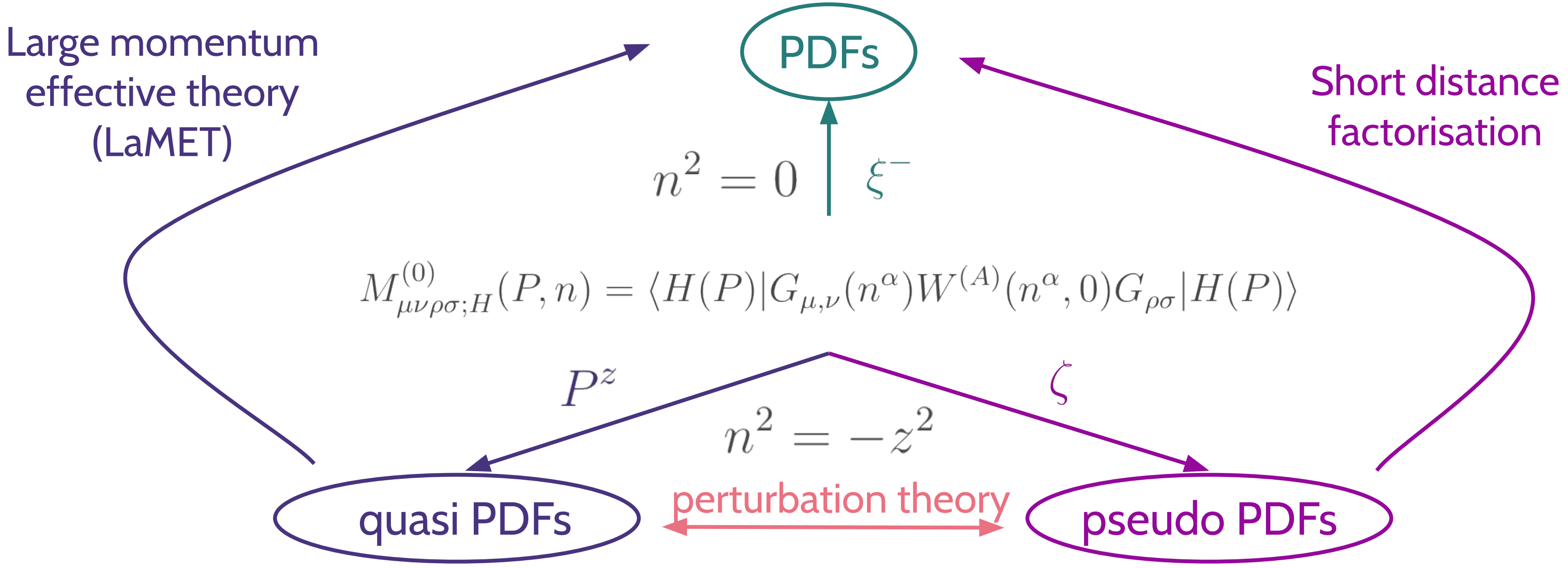


$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)| G_{\mu,\nu}(n^\alpha) W^{(A)}(n^\alpha, 0) G_{\rho\sigma} |H(P)\rangle$$

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \langle H(P) | \overline{\psi}(0, z, \mathbf{0}_\mathrm{T}) W(z,0) \Gamma_j \psi(0) | H(P) \rangle$$

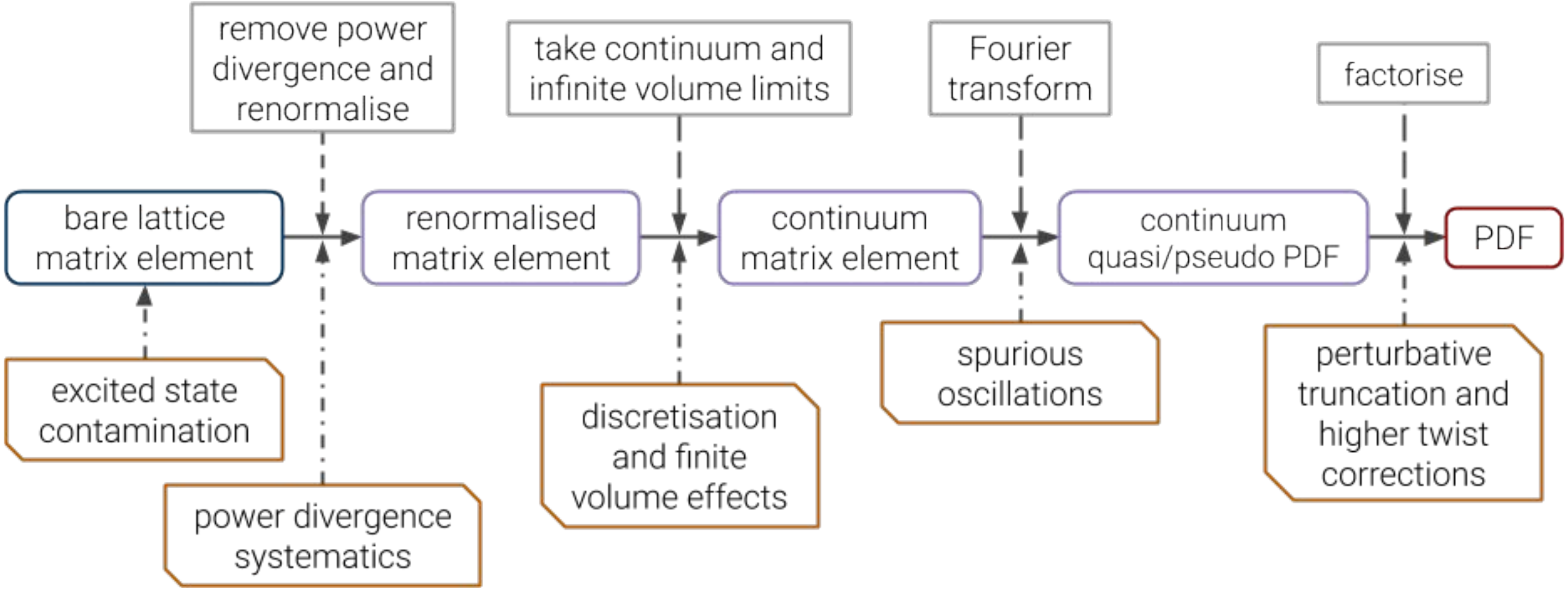
$$\widetilde{p}^{(0)}_{j/H}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \langle H(P) | \overline{\psi}(0, z, \mathbf{0}_\mathrm{T}) W(z,0) \Gamma_j \psi(0) | H(P) \rangle$$

Ji, PRL 110 (2013) 262002

Radyushkin, PRD 96 (2017) 034025

Distributions in practice

CJM, POS(LATTICE2018) 018



Methods galore

Quasi and pseudo PDFs

Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025

Factorisable matrix elements

Braun & Müller, EPJC 55 (2008) 349
Ma & Qiu, 1404.6860

Fictitious heavy quarks

Detmold & Lin, PRD 73 (2006) 014501

Compton amplitude

Chambers et al., PRL 118 (2017) 242001

Euclidean hadronic tensor

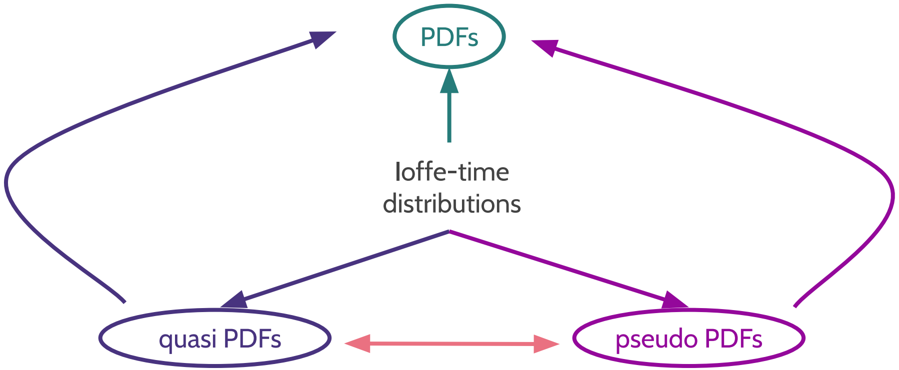
Liu & Dong, PRL 72 (1994) 1790

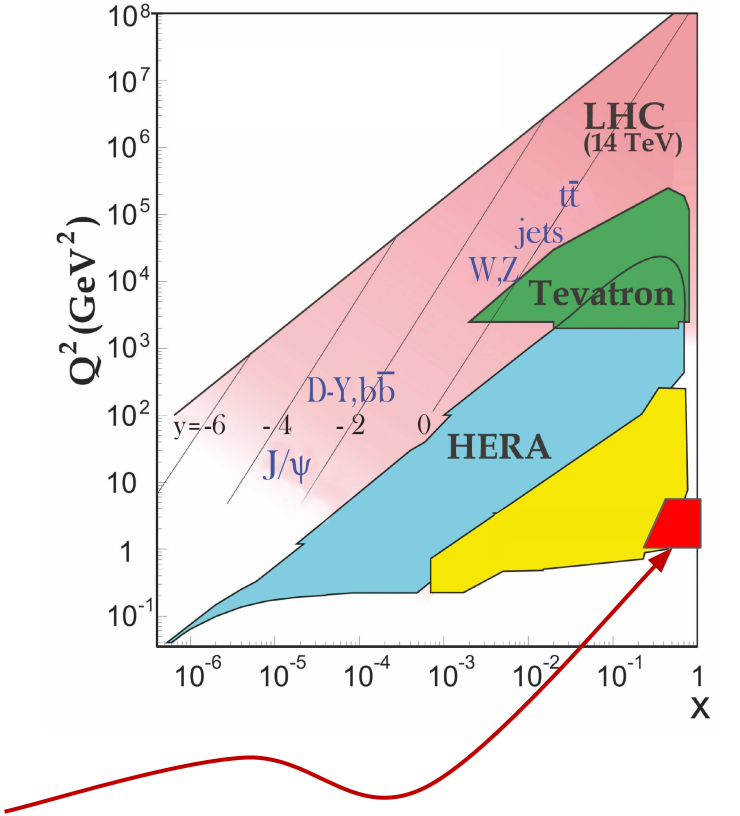
Many review articles!

Lin et al., PPNP 100 (2018) 107
CJM, POS(LATTICE2018) 018
Zhao, IMJPA 33 (2019) 1830033

Cichy & Constantinou, AHEP (2019) 3036904
Constantinou et al., PPNP 121 (2021) 130908
Ji et al., RMP 93 (2021) 035005
Constantinou et al., 2202.07193

Lattice “cross-sections” and global fits





“Lattice ~~cross-sections~~” factorisable matrix elements

$$\sigma_{j/H}(\xi, \cdots) \in \left\{ \widetilde{f}^{(0)}_{j/H}(\xi, P^z), \widetilde{p}^{(0)}_{j/H}(\xi, z^2), \cdots \right\}$$

Braun & Müller, EPJC 55 (2008) 349
Ma & Qiu, PRD 98 (2018) 074021

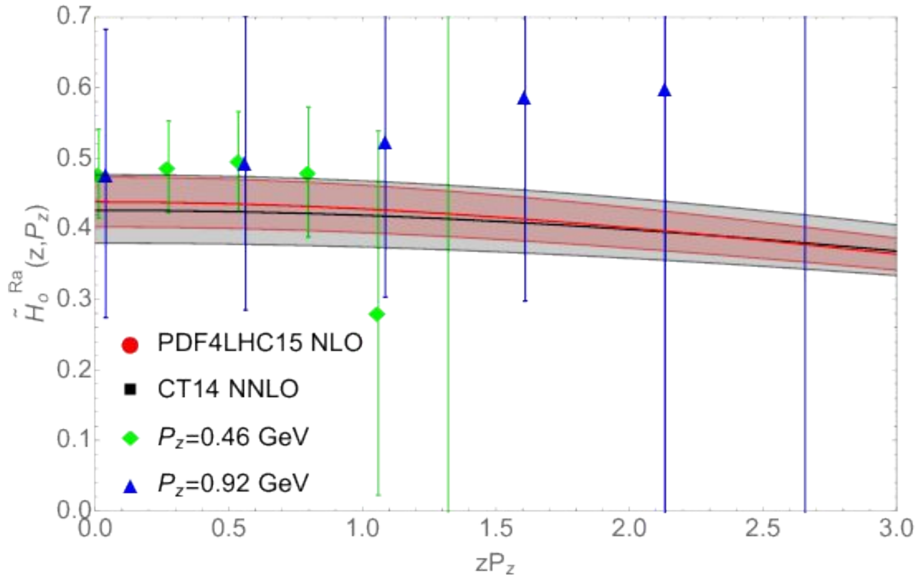
Gluon PDFs: lattice calculations

Gluon observables provide significant challenges for lattice calculations

- significant signal-to-noise issues
- nonperturbative renormalisation challenging

First proof-of-principle calculation using LaMET illustrates the challenges

Fan *et al.*, PRL 121 (2018) 242001



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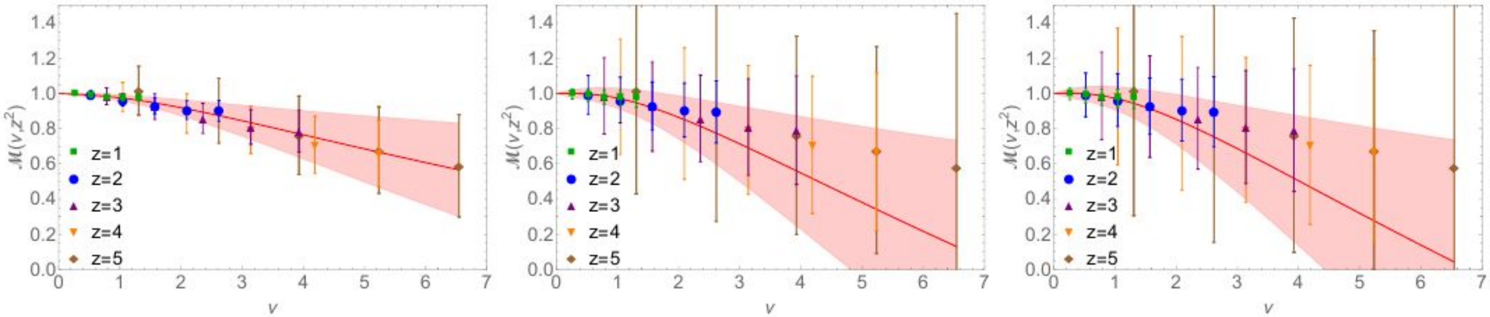
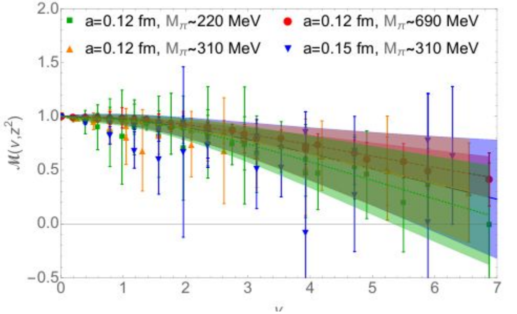
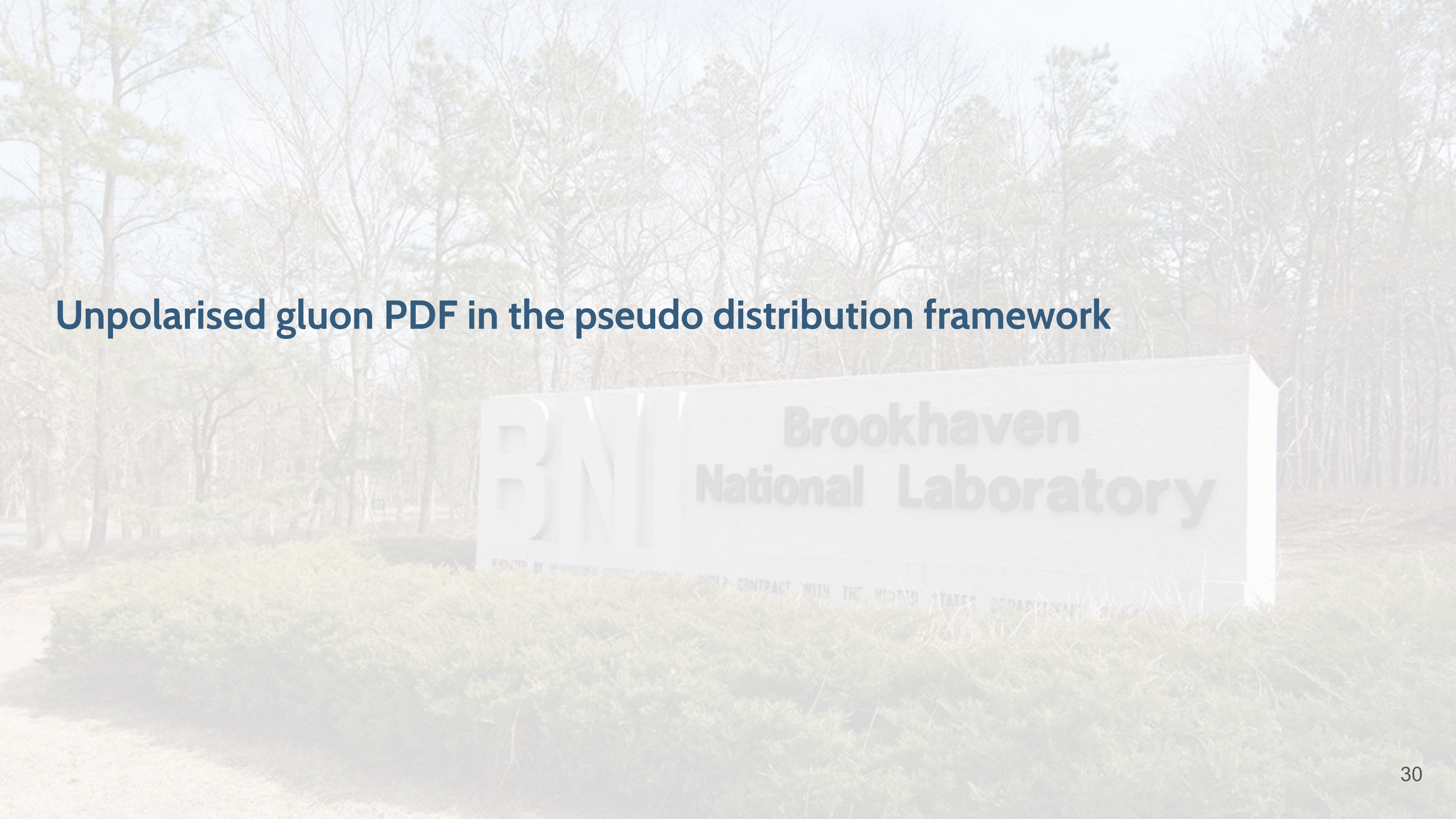


Fan et al., Int.J.Mod.Phys.A 36 (2021) 13



Fan and Lin, PLB 823 (2021) 136778

Essentially the challenge is the noise-to-signal ratio of gluon observables



Unpolarised gluon PDF in the pseudo distribution framework

Khan et al., PRD 104 (2021) 034507

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$$W^{(A)}(n^\alpha,0) = \mathcal{P}\exp\left\{ig\int_0^n \mathrm{d}y^\mu A^{(A)}_\mu(y)\right\}$$

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$n^2 = 0$

$$\mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu}$$

PDFs $\quad \xi^-$

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)$$

Gluon PDFs: pseudo-distribution formalism

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$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$n^2 = -z^2$

$$\mathcal{M}^{(0)}_{g/H}(\nu,z^2) = \frac{1}{2E_P^2}\left[M^{(0)}_{0ii0;H}(P,z) - M^{(0)}_{jiij;H}(P,z)\right]$$

$$\mathcal{M}^{(\text{red.})}_{g/H}(\nu,z^2) = \left(\frac{\mathcal{M}^{(0)}_{g/H}(\nu,z^2)}{\mathcal{M}^{(0)}_{g/H}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}^{(0)}_{g/H}(0,z^2)|_{p=0}}{\mathcal{M}^{(0)}_{g/H}(0,0)|_{p=0,z=0}}\right)$$

ζ pseudo PDFs

$$\mathcal{M}^{(\text{red.})}_{g/H}(\nu,z^2) = \int_0^1 \frac{\mathrm{d}\xi\,\xi}{\langle\xi\rangle^2(\mu)}\left[c_{gg}(\xi\nu,\mu^2 z^2)f_{g/H}(\xi,\mu^2) + \frac{P^z}{E_P}c_{gq}(\xi\nu,\mu^2 z^2)f_{S/H}(\xi,\mu^2)\right]$$

$n^2 = 0$

$$\mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu}$$

PDFs ξ^-

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)$$

HadStruc lattice implementation

Gluons provide significant signal-to-noise challenges for lattice calculations

Mitigated through three strategies

1. Gradient flow smearing reduces ultraviolet fluctuations
2. Distillation and summed GEVP method improves operator overlap and reduces excited state contamination
3. Reduced Ioffe-time distribution reduces correlated uncertainties through ratio

In the following, we neglect mixing with scalar quark distribution

- note this appears in factorisation to PDF

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q(x,\mu^2) \\ x f_g(x,\mu^2) \end{pmatrix} = \begin{pmatrix} P_{qq}(x,\mu^2) & P_{qg}(x,\mu^2) \\ P_{gq}(x,\mu^2) & P_{gg}(x,\mu^2) \end{pmatrix} \otimes \begin{pmatrix} f_q(\mu^2) \\ x f_g(\mu^2) \end{pmatrix}$$

Smearing

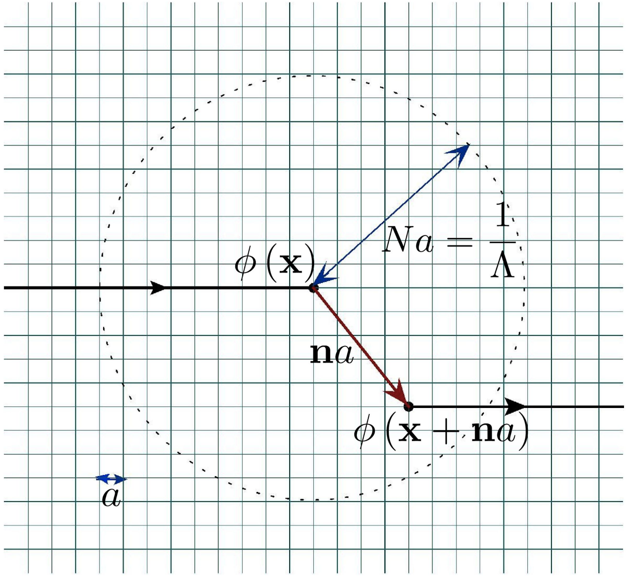
“Smearing” partially restores rotational symmetry: widely-used lattice technique

- construct operators with improved continuum limits, *i.e.* reduced systematic uncertainties

- suppresses operator mixing

- precisely identify hadronic excited states

- reduce statistical noise



Davoudi & Savage, Phys. Rev. D 86 (2012) 054505

Gradient flow smearing

Gradient flow: deterministic evolution of fields in "flow time" τ toward classical

$$\frac{\partial}{\partial\tau}B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\nu = \partial_\nu + [B_\nu, \cdot]$$

$$\frac{\partial}{\partial\tau}\chi(\tau,x) = D_\mu D^\mu \chi(\tau,x) \qquad D_\mu = \partial_\mu + B_\mu$$

Dirichlet boundary conditions

$$B_\mu(\tau=0,x) = A_\mu(x) \qquad \chi(\tau=0,x) = \psi(x)$$

Can be implemented on the lattice and solved nonperturbatively

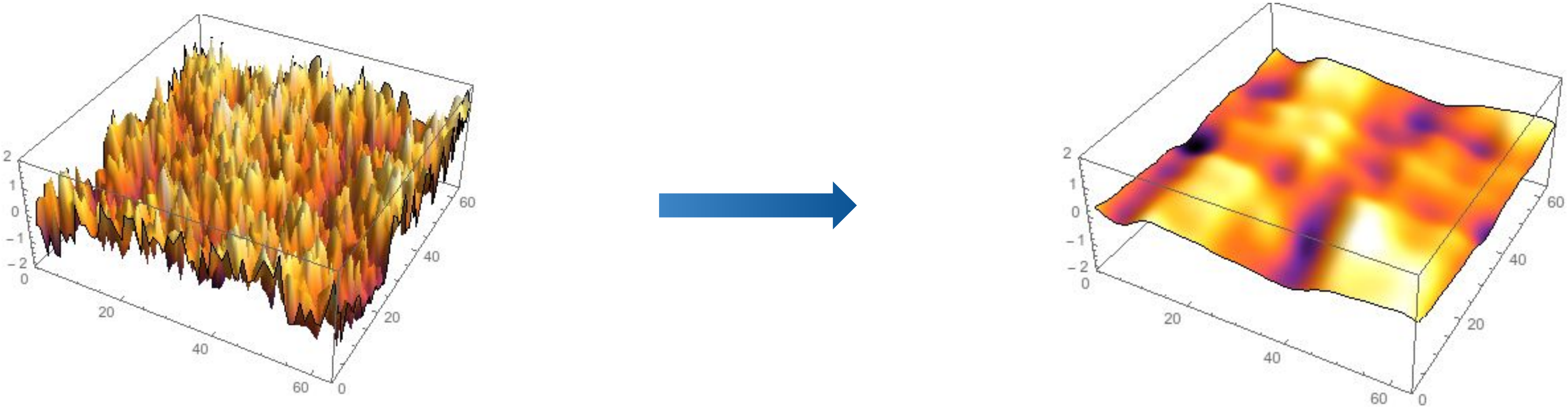
$$\frac{\partial}{\partial\tau}V_\mu(\tau,x) = -g_0^2\left\{\partial_{x,\mu}S\big[V_\mu(\tau,x)\big]\right\}V_\mu(\tau,x)$$

Narayanan & Neuberger, JHEP 0603 064
Lüscher, JHEP 1008 071
Lüscher, JHEP 04 (2013) 123

Gradient flow smearing

Gradient flow: deterministic evolution of fields in “flow time” τ toward classical minimum

Evolution in flow time corresponds to exponential damping of UV modes



Key result: correlation functions remain finite at finite flow time

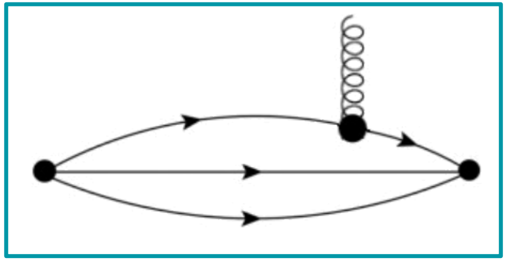
Lüscher & Weisz, JHEP 02 (2011) 051
Lüscher, JHEP 04 (2013) 123

Correlators

Signal-to-noise ratio improved and excited state effects reduced through sGEVP

Typical lattice calculation based on 3-point function (and ratio with 2-point function)

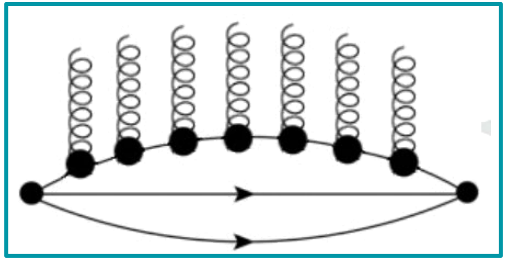
$$\langle C_{3pt}(t, t_g)\rangle = \langle 0|T\{O_N(t)\, O_g(t_g)\, \bar{O}_N(0)\}|0\rangle$$



Summation method

$$C^{i,s}_{3pt}(t) = \sum_{t_g=1}^{t-1} C^{i}_{3\mathrm{pt}}(t, t_g)$$

$$C^{i}_{3pt}(t, t_g) = \Big(C^{i}_{2pt}(t) - \langle C_{2pt}(t)\rangle\Big)\Big(O^{i}_{g}(t_g) - \langle O_g(t_g)\rangle\Big)$$



Leads to

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\, t \exp(-\Delta E\, t)$$

Correlators

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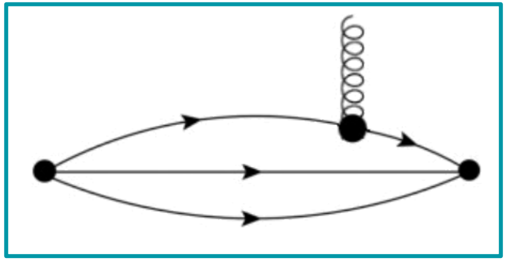


Summation method

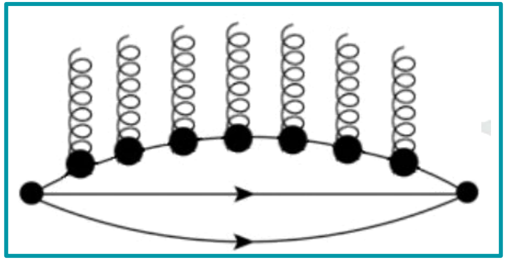
$$C^{i,s}_{3pt}(t) = \sum_{t_g=1}^{t-1} C^{i}_{3\mathrm{pt}}(t,t_g)$$

$$C^{i}_{3pt}(t,t_g) = \left(C^{i}_{2pt}(t) - \langle C_{2pt}(t)\rangle\right)\left(O^{i}_{g}(t_g) - \langle O_g(t_g)\rangle\right)$$

Leads to

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\,t\exp(-\Delta E\,t)$$

what we want

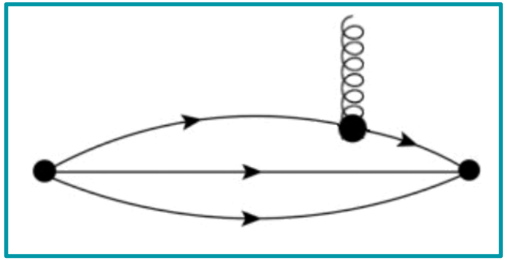


Correlators

Signal-to-noise ratio improved and excited state effects reduced through sGEVP

Typical lattice calculation based on 3-point function (and ratio with 2-point function)

$$\langle C_{3pt}(t, t_g)\rangle = \langle 0|T\{O_N(t)\, O_g(t_g)\, \bar{O}_N(0)\}|0\rangle$$



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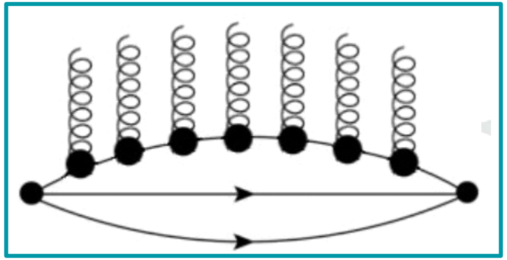


Leads to

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\, t\, \exp(-\Delta E\, t)$$

Combined with distillation and GEVP method for operator construction

Bouchard *et al.*, PRD 96 (2017) 014504

Reduced Ioffe-time distribution

Double ratio removes correlated uncertainties and need for renormalization

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) \Big/ \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0,z=0}} \right)$$

Connect this ratio to the Ioffe-time distribution through factorisation

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, \frac{\mathcal{I}_g(u\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \left\{ \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) B_{gg}(u) + 4 \left[\frac{u + \ln(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3} \left[1 - u^3 \right]_+ \right\} - \frac{\alpha_s C_F}{2\pi} \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) \int_0^1 dw\, \frac{\mathcal{I}_S(w\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)}\, \mathfrak{B}_{gq}(w)\,.$$

with

$$\mathcal{I}_g(\nu, \mu^2) = \frac{1}{2} \int_{-1}^1 dx\, e^{ix\nu}\, x\, g(x, \mu^2) \qquad B_{gg}(u) = 2 \left[\frac{(1 - u\bar{u})^2}{1 - u} \right]_+ \qquad \mathfrak{B}_{gq}(w) = \left[1 + (1 - w)^2 \right]_+$$

Extracting the PDF

Determining the PDF from limited, discrete lattice data is an ill-posed inverse problem

Treat this inverse problem by parameterising the PDF

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)}$$

Test systematic effects by modifying parameterisation

transformed Jacobi polynomial

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, x^\alpha (1-x)^\beta \left(\frac{1}{B(\alpha+1, \beta+1)} + d_1^{(\alpha,\beta)}\, J_1^{(\alpha,\beta)}(x) \right)$$

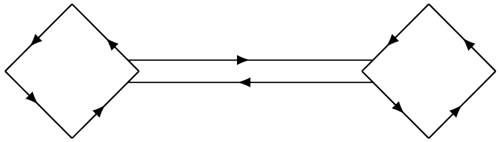
$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)} + \left(\frac{a}{|z|} \right) P_1(\nu)$$

Results

Results calculated on a single lattice ensemble of 2+1 stout-smeared Wilson-improved clover fermions and tree-level tadpole-improved Symanzik gauge action

ID	a (fm)	M_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	64

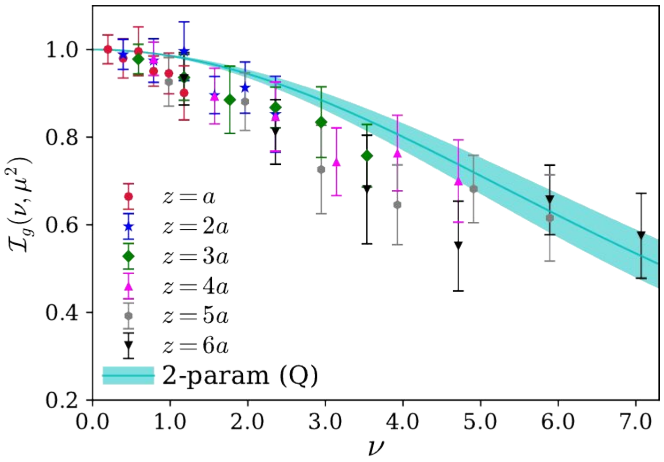
- Implement gradient flow via the Wilson flow
- Momentum smearing for interpolating operators at nonzero momentum
- Unimproved field strength operator

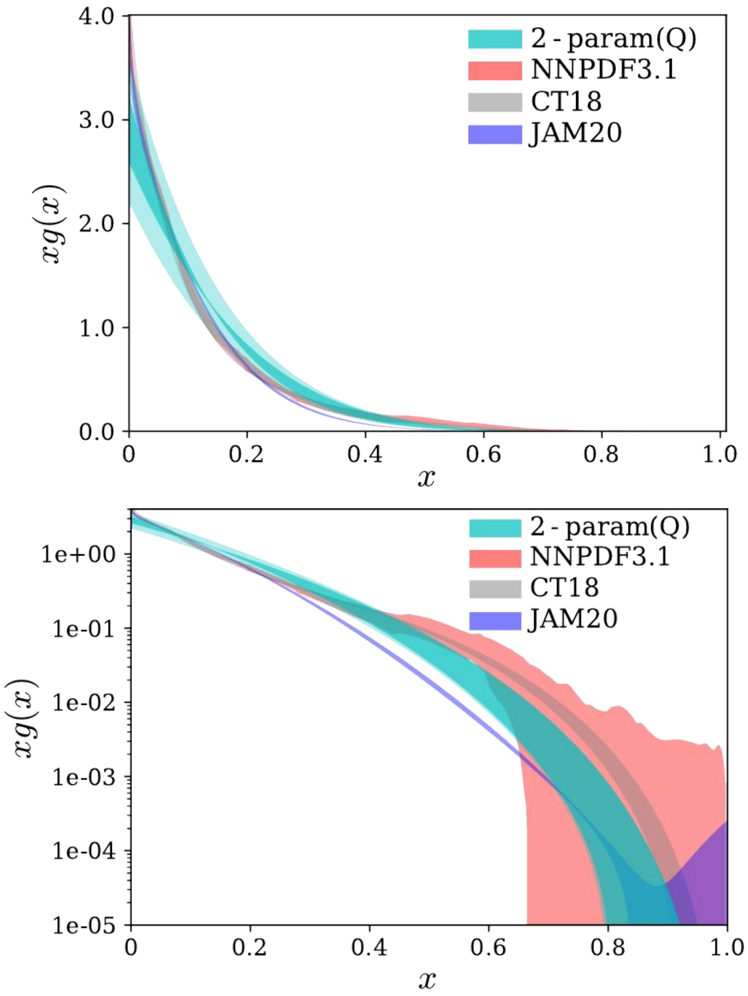


$$-\frac{i}{2}\left[P_{\mu\nu}^{(1+1)} - \left[P_{\mu\nu}^{(1+1)}\right]^\dagger - \frac{1}{3}\operatorname{Tr}\left\{P_{\mu\nu}^{(1+1)} - \left[P_{\mu\nu}^{(1+1)}\right]^\dagger\right\}\right] = ag^2\left[G_{\mu\nu} + \mathcal{O}(a^2, a^2g^2)\right]$$

$$P_{\mu\nu}^{(1+1)} = U_\mu(x)U_\nu(x+a\widehat{\mu})U_\mu^\dagger(x+a\widehat{\nu})U_\nu^\dagger(x)$$

Results: unpolarised gluons





Unpolarised gluon PDFs: the take-home message

Challenging calculations, requiring a suite of sophisticated approaches, but controlled extractions with moderate precision at moderate Bjorken-x feasible in the near future

Polarised gluon PDF in the pseudo distribution framework

Egerer et al., 2207.08733

Gluon PDFs: pseudo-distribution formalism for polarised gluons

Unpolarised case

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,z) = \langle H(P)|G_{\mu\nu}(0,z,\mathbf{0}_\mathrm{T})W^{(A)}(z,0)G_{\rho\sigma}(0)|H(P)\rangle$$

becomes

Balitsky, Morris and Radyushkin, JHEP 02 (2022) 193

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,z) = \langle H(P)|G_{\mu\nu}(0,z,\mathbf{0}_\mathrm{T})W^{(A)}(z,0)\widetilde{G}_{\rho\sigma}(0)|H(P)\rangle$$

Decompose into invariant amplitudes, but a higher twist term cannot be removed

$$M^{(0)}_{0i;0i}(P,z) + M^{(0)}_{ij;ij}(P,z) = -2P^z E_H \mathcal{M}_{\Delta g/H}(\nu,z^2) + 2E_H^3 z \mathcal{M}_{pp}(\nu,z^2)$$

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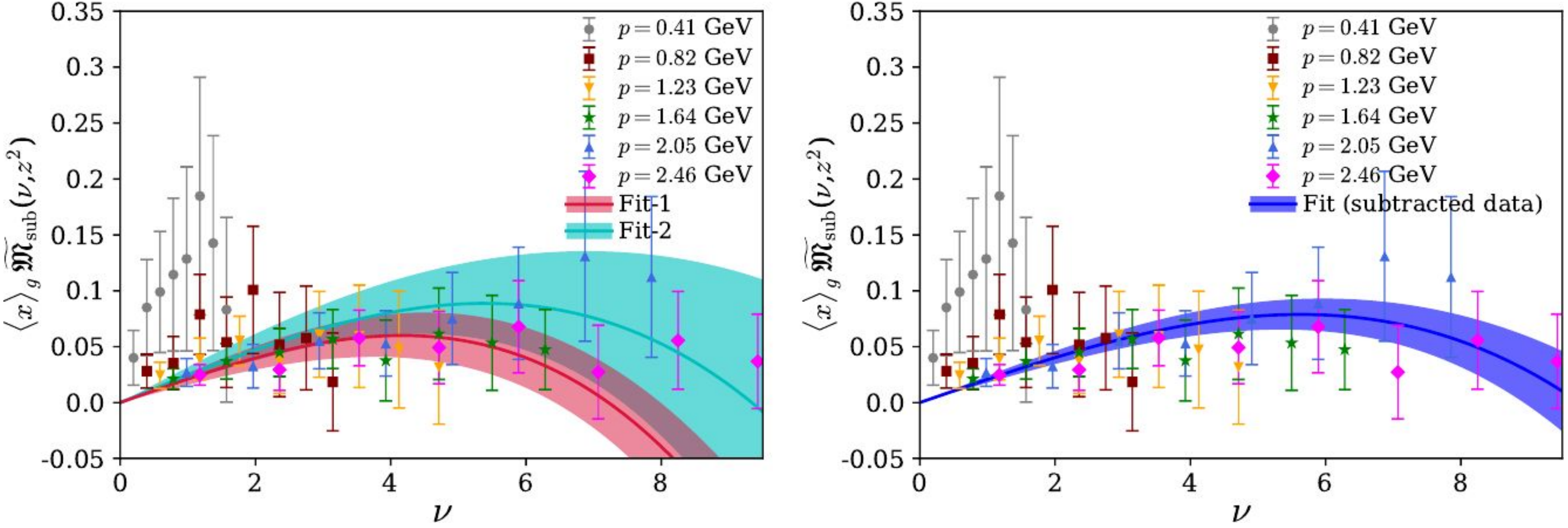
what we want

Complicates the analysis!

Results: polarised gluons



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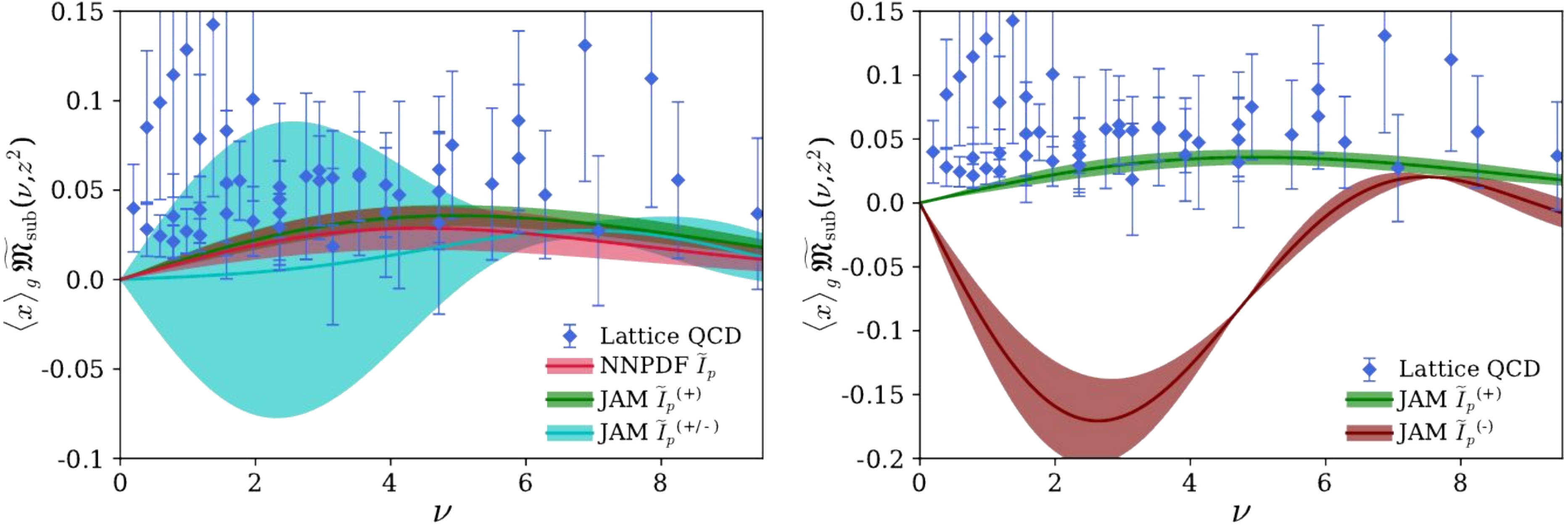


Polarised gluon PDFs: the take-home message

Very challenging calculations, requiring high statistics, but clear opportunity for meaningful contributions from controlled calculations, even with large uncertainties

Summary

Precise extraction of the unpolarised gluon PDF from pseudo-distribution framework

First extraction of the polarised gluon PDF from lattice QCD

evidence for meaningful contributions to our picture, even with large uncertainties

Significant improvement in precision using:

gradient flow smearing; distillation and summed GEVP method; ratio method

Future improvements needed:

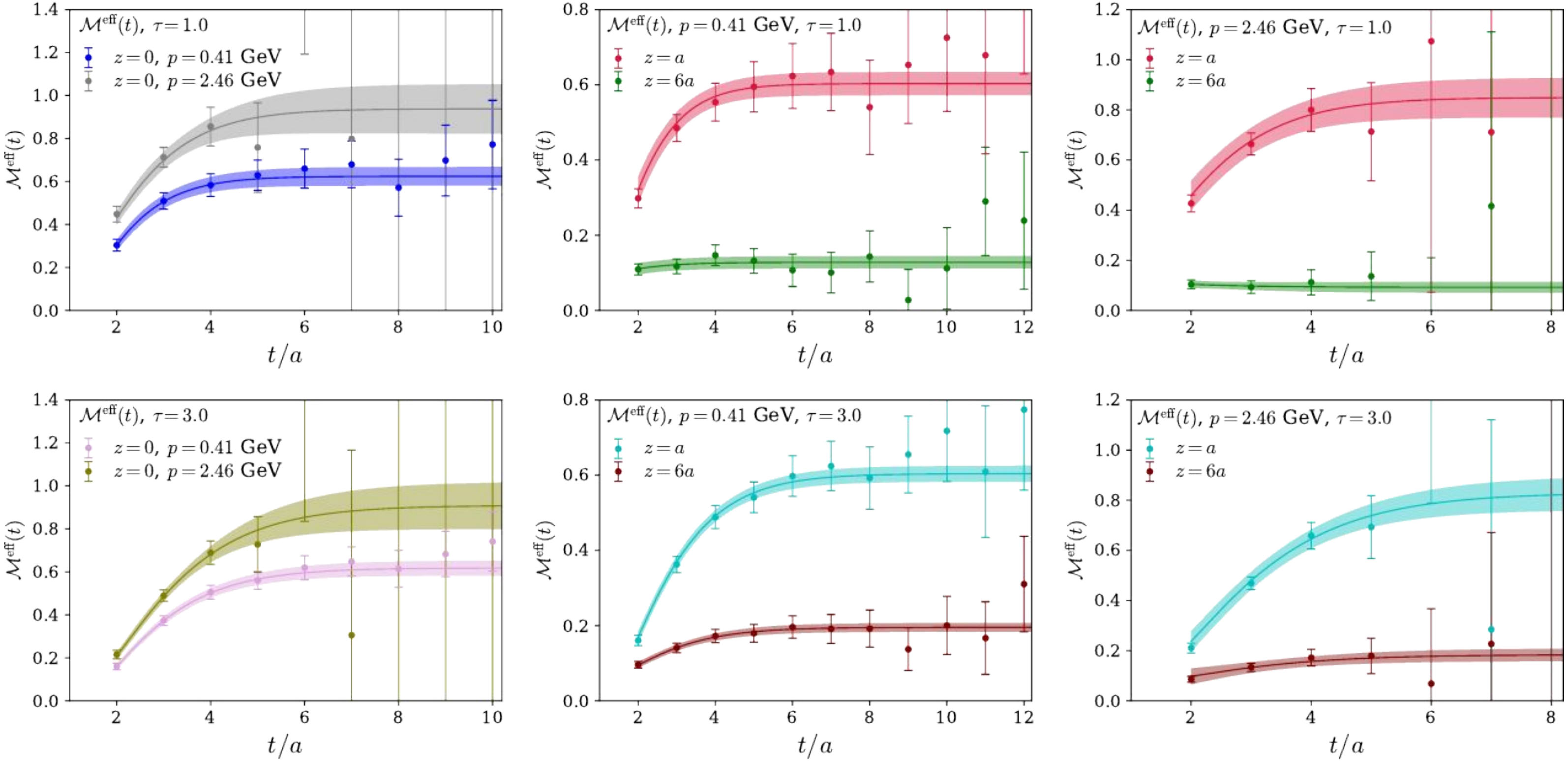
1. Increased statistics
2. Calculation of gluon momentum fraction
3. Long-term goal - combine with isoscalar quark PDF

Thank you!

Chris Monahan

cjmonahan@wm.edu

Results: unpolarised gluons



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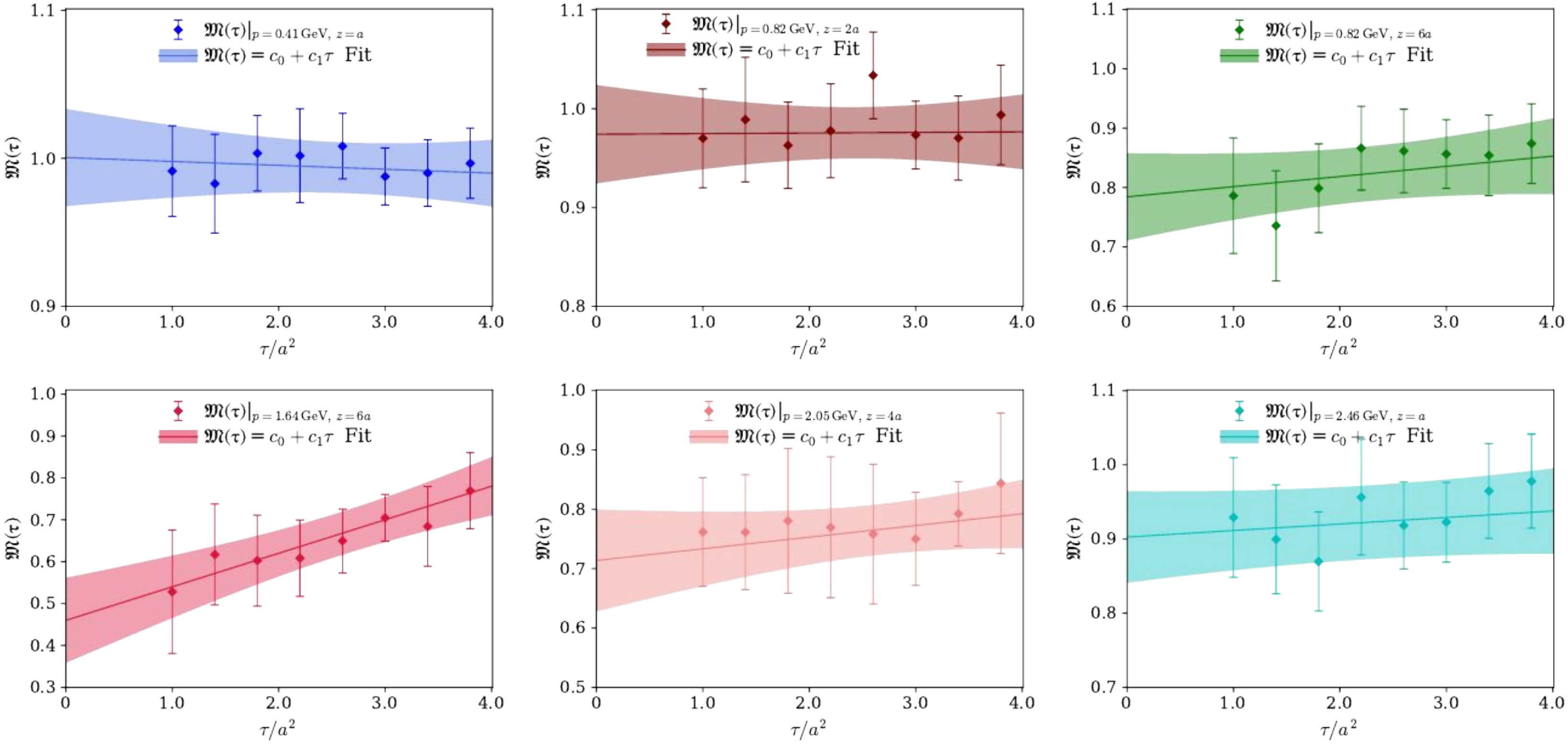

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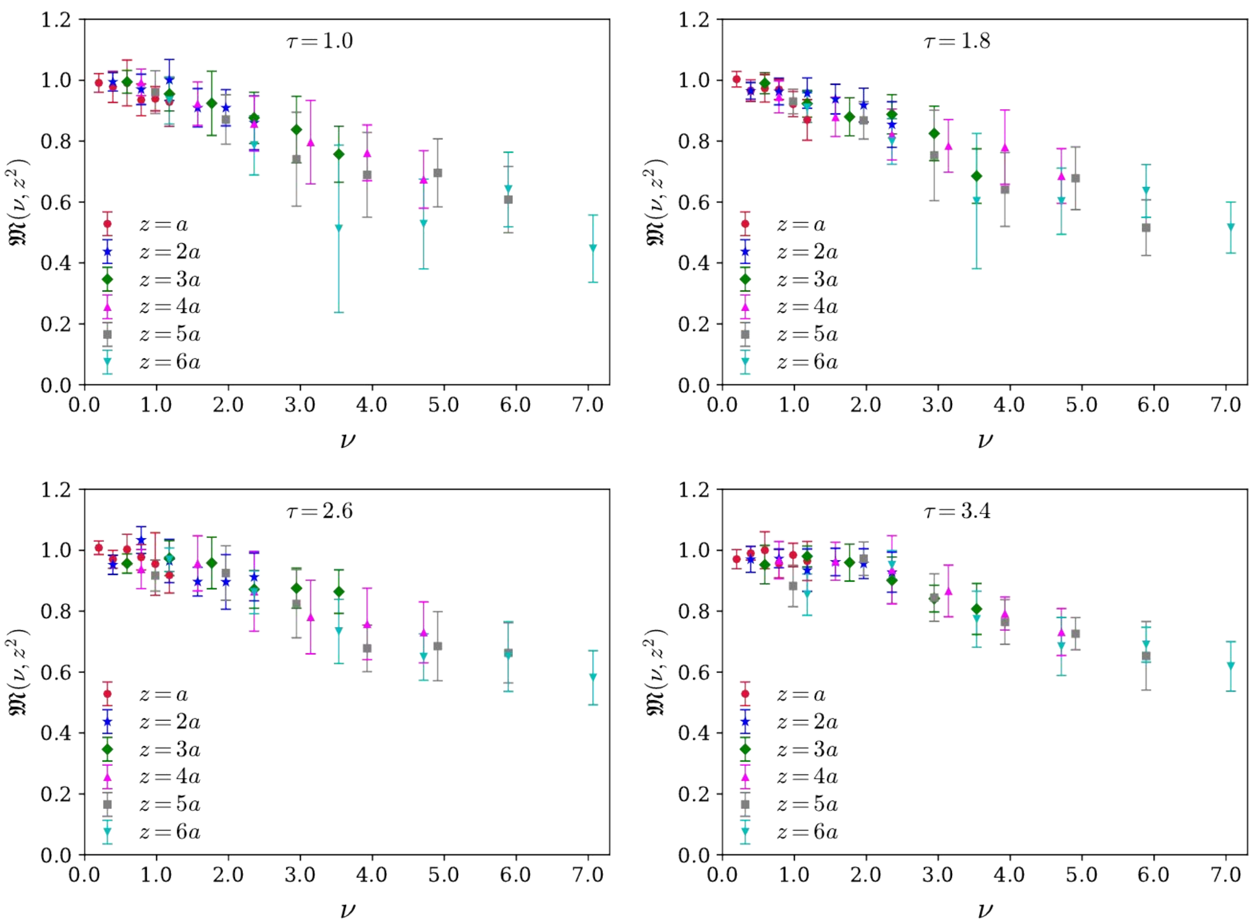
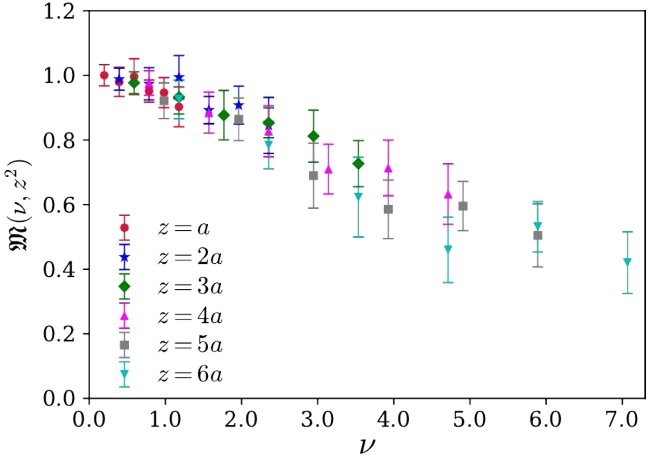
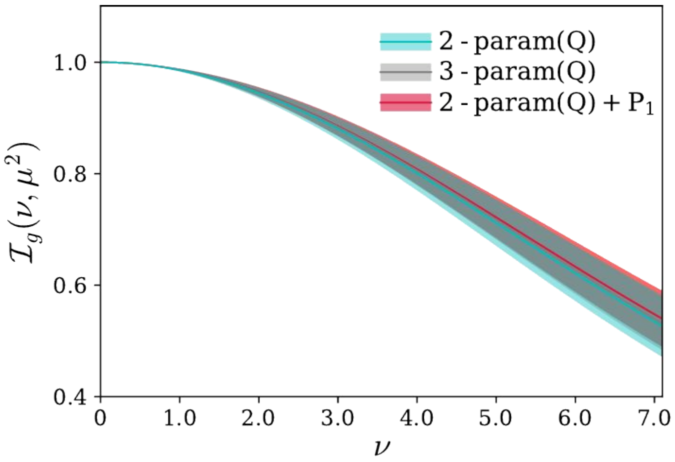
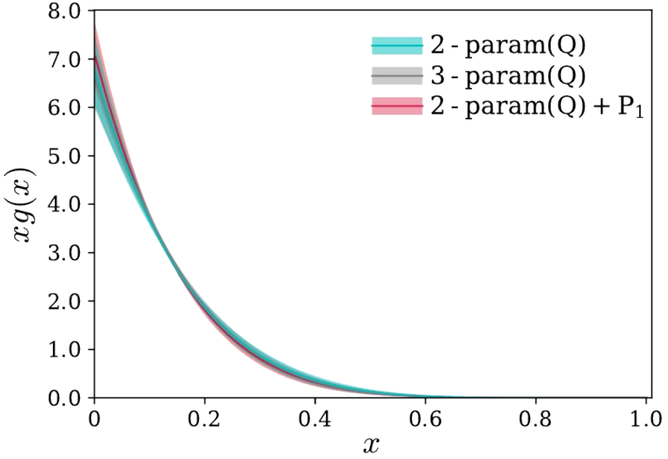


$\tau = 1.0$
$\tau = 1.8$
$\tau = 2.6$
$\tau = 3.4$
$\mathfrak{M}(\nu, z^2)$
ν
$z = a$
$z = 2a$
$z = 3a$
$z = 4a$
$z = 5a$
$z = 6a$

Results: unpolarised gluons







Gradient flow theory in D+1 dimensions

Gradient flows are, broadly speaking, any solution of steepest descent:

- energy landscape
- field configuration space
- probability measure space

Typically expressed through solutions to PDEs.

E.g. in Euclidean space, gradient flows along f:R →R are solutions to

$$\frac{\mathrm{d}x_t}{\mathrm{d}t} = -\nabla f(t)$$

Leads to the gradient descent algorithm for finding local minima

Lüscher & Weisz, JHEP 1102 (2011) 51
Lüscher, JHEP 1304 (2013) 123

Gradient flow theory in D+1 dimensions

Gradient flow for boundary theory can be implemented in a D+1 dimensional Lagrangian

$$S_{GF} = -2 \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^D x \, \mathrm{Tr}\left\{ L_\mu(\tau,x) \left[\partial_\tau B_\mu(\tau,x) - D_\nu G_{\nu\mu}(\tau,x) \right] \right\}$$

Lagrange multiplier field couples to the boundary gauge field only through the bulk field

Variation of the action with respect to Lagrange multiplier field imposes gradient flow equation in the bulk

Formalism suited to studying formal properties of the flow; not typically practical

- Generalised BRST invariance constrains counterterms in the bulk
- Guarantees renormalised correlation functions remain finite, up to a fermionic wavefunction renormalisation

Lüscher & Weisz, JHEP 1102 (2011) 51
Lüscher, JHEP 1304 (2013) 123

Gradient flow: perturbation theory

Gradient flow in QCD

$$\frac{\partial}{\partial\tau}B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\nu = \partial_\nu + [B_\nu, \cdot]$$

Dirichlet boundary conditions

$$B_\mu(\tau=0,x) = A_\mu(x) \qquad \chi(\tau=0,x) = \psi(x)$$

Tree-level expansion and "flow propagator"

$$B_\mu(\tau,x) = \int \mathrm{d}^4y \Big\{ K_\tau(x-y)_{\mu\nu}A_\nu(y) + \int_0^\tau \mathrm{d}\sigma K_{\tau-\sigma}(x-y)_{\mu\nu}R_\nu(\sigma,y)\Big\}$$

where

$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{e^{ipx}}{p^2}\Big\{(\delta_{\mu\nu}p^2 - p_\mu p_\nu)e^{-\tau p^2} + p_\mu p_\nu\Big\}$$

$$R_\mu(\tau,x) = 2[B_\nu,\partial_\nu B_\mu] - [B_\nu,\partial_\mu B_\nu] - [B_\mu,\partial_\nu B_\nu] + [B_\nu,[B_\nu,B_\mu]]$$

Renormalised boundary theory requires no further renormalisation in the bulk

Lüscher & Weisz, JHEP 1102 (2011) 51
Makino & Suzuki, arXiv:1410.7538